Liquidity and the Market for Ideas*

Rafael Silveira Randall Wright

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Abstract

We study markets where innovators sell ideas to entrepreneurs who may be better at implementing them. These markets are decentralized, with random matching and bilateral bargaining. Entrepreneurs hold liquid assets (e.g. cash) lest potentially profitable opportunities may be lost. We extend search-based models of the demand for liquidity along several dimensions, including allowing agents with insufficient money to put deals on hold while they try to raise the funds. Given liquidity costs (e.g. interest rates) we determine which ideas get traded in equilibrium, compare this to the efficient outcome, and discuss the optimal response of monetary policy.

*Both authors are affiliated with the Department of Economics, University of Pennsylvania. Email: rafaels@econ.upenn.edu; rwright@econ.upenn.edu. We thank Ken Burdett, Boyan Jovanovic, Tom Holmes, Richard Rogerson, Rob Shimer, and participants in seminars at the NBER, the Money and Payments Workshop at the Cleveland Fed, the Bank of Canada, the Philadelphia S&M (Search and Matching) Club, and the Research on Money and Markets Conference in Toronto for their input. Wright thanks the NSF for financial support.

1 Introduction

We take it for granted that people understand that the development and implementation of new ideas is one of the major factors underlying economic performance.[1] In this vein, the concept of *technology transfer* is important, both to innovators and entrepreneurs looking to come up with and commercialize new technologies, and to governments seeking to spur economic development. The issue is this: When innovators come up with new inventions (or ideas or projects), should they try to implement them themselves, say through start-up firms? Or should they try to sell them, perhaps to established firms, or more generally to entrepreneurs who are better at implementing these ideas?

If agents are heterogeneous in their abilities to come up with ideas and to extract their returns, one can imagine that some will specialize in innovation while others will specialize in implementation or commercialization. A superior allocation of resources will generally emerge when those who have the ideas are not necessarily those who implement them. People in the "knowledge transactions field" share the view that the transfer of ideas from innovators to entrepreneurs leads to a more efficient use of resources, making all parties better off and increasing the incentives for investments

[1]Both the inputs to and outputs of this process are important. On the input side, research and development expenditures account for 3% of US GDP, and according to a survey by the Association of University Technology Managers, the licensing of innovations just by universities, hospitals, research institutions, and patent management firms added more than $40 billion to the economy in 1999 and supported 270,000 jobs. On the output side, it is obvious that new ideas and technologies are essential to production and growth, and going back to Schumpeter (1934) it is often said that the creation of new firms is a significant mechanism through which new technologies are implemented.

in research. Obviously, however, this requires some mechanism – say, some market – for the exchange of ideas, and the details of how this mechanism works could in principle have a big impact on outcomes. This is the subject of the current study.[2]

Our analysis is related to the well-known model of Holmes and Schmitz (1990, 1995), although we also deviate considerably from their approach. What we share with them is, in their words, the following: "The model has two key features. The first crucial assumption is that opportunities for developing new products repeatedly arise through time... The second key feature is that we assume that individuals differ in their abilities to develop emerging opportunities." Hence, "There are two tasks in the economy, developing products and producing products previously developed" (Holmes and Schmitz 1990, p. 266-7). Where we differ is the way we envision the market where ideas get traded. While they model it as a competitive equilibrium, we take seriously the notion that there are considerable frictions in this market.

We think it is clear that there is really no centralized market for ideas – innovators cannot simply choose a quantity of new ideas to supply to maximize profit taking as given the competitive price, and entrepreneurs do not simply choose how many new ideas to buy at a given price. The

[2] A common idea in this literature is that inventor-founded startups are often second-best solutions, since innovators do not have the entrepreneurial skills to commercialize new products. Of course, one could imagine innovators trying to buy implementation expertise from entrepreneurs, but the usual view is that such expertise is largely tacit and difficult to measure, so it seems more natural for ideas to be sold to entrepreneurs. See Teece et al. (1997), Pisano and Mang (1993), and Shane (2002).

idea market is in our view much more *decentralized*. Hence, we model it using search theory, with random matching and bilateral bargaining between innovators and entrepreneurs. Also, we take the position that *liquidity* may be critical in this market, and to capture this, we make use of some recent results in search-based monetary economics.

When there are imperfect markets for the exchange of ideas, it is not only important who you meet and what they know, there is also the issue of how to pay for it. The fact that you may be better than me at implementing my project means very little if you have nothing to offer in exchange. This is especially important in highly decentralized markets, where it is easy to imagine reasons why I would be reluctant to give up an idea for a promise of future payment (e.g., once I give it up it is hard to get it back). Hence, it is easy to imagine reasons why quid pro quo is the order of the day: "You want my idea? Show me the money."

Given this, entrepreneurs may choose to keep liquid assets, cash on hand being the purest example, in case they come across a potentially profitable opportunity that may be lost if there cannot be a quick agreement. Naturally, how much liquidity they choose to keep on hand depends on its cost, e.g. the nominal interest rate, which is at least in part determined by monetary policy, as well as other factors including anything that affects the willingness of innovators to sell their ideas and the willingness of entrepreneurs to invest in these opportunities. Our goal is to sort out the role of some of these factors, and hence sort out what determines how many and which opportunities get traded.

The view that financial constraints matter in this context is by no means new. For example, Evans and Jovanovic (1989), among many others, argue for the importance of liquidity or borrowing constraints by showing that the decision to become an entrepreneur depends positively on one's wealth, and interpreting this as evidence of financial constraints.[3] We believe that our approach is broadly consistent with this literature. However, those papers do not incorporate any notion of liquidity as it is modeled in monetary economics, and focus more on various credit market imperfections that are imposed in sometimes rather ad hoc ways.[4]

We want to explore an alternative approach where agents are constrained by their liquidity, which is *an endogenous choice*, depending on variables like interest rates. This allows us to introduce monetary policy considerations into the discussion of innovation and technology transfer, and we think that this may be more important than is commonly understood. Moreover, our approach generates some different implications, that may be worth exploring, compared to the models mentioned above. For instance, if the problem in the market is borrowing constraints, high interest rates could help by increasing savings, but according to our approach high interest rates make things worse by raising the cost of maintaining liquid assets.

[3] See also Evans and Leighton (1989), Holtz-Eakin et al. (1994), Fairlie (1999), Quadrini (1999), Gentry and Hubbard (2000), Lel and Udell (2002), Paulson and Townsend (2000), and Guiso, Sapienza and Zingales (2001).

[4] Some people simply assume there is no credit (Lloyd-Ellis and Bernhardt 2000 and Buera 2005), some assume credit is exogenously limited to a fixed multiple of wealth (Evans and Jovanovic 1989), some model it as the solution to a moral hazard problem (Aghion and Bolton 1996), and some use asymmetric information (Fazzari et al. 1988, 2000).

As we said, our framework is similar to some work in the search and matching literature. We think that using this approach to study technology transfer is a neat application of the theory, and a natural way to look at the substantive issues. Since we allow liquidity to potentially play a prominent part, our framework is especially close to recent monetary search theories, which are all about the role of liquidity in decentralized trade. In particular, our environment shares features with the model of monetary exchange in Lagos and Wright (2005), where sometimes agents trade in centralized markets and sometimes in decentralized markets. But while we borrow from that model, we also extend it in a number of ways.

First, in our market, ideas are indivisible and have random valuations. Together with the liquidity problem, this means the bargaining problem may be nonconvex, and hence we need to consider the possibility of randomized trade using lotteries. Second, we extend the framework, in what we think is a very realistic way, to allow agents with insufficient liquidity to try to put deals on hold until they can raise funds in the centralized market, which may or may not work. In this way we capture both theories where liquidity is crucial, and those where it plays no role at all, as special cases. Third, we consider the case where there is a public good aspect to ideas – i.e. the fact that I give you my idea does not necessarily mean that I cannot also use it – as well as the case where they are purely private goods.

Fourth, when ideas are intermediate inputs into some production process, we stress that whether or not the idea market functions well, which determines the extent to which the most efficient agents are implementing projects, feeds back to aggregate variables like wages and employment. Finally, in part as a result of the points made above, we argue that monetary policy may be more potent than is commonly understood or than is predicted by models where agents are simply trading consumption goods. So while we have things in common with modern monetary theory, we go well beyond what has been done in terms of introducing some technical extensions, discussing new economic interpretations, and deriving policy implications that arise when we model the idea market in this way.

The rest of the paper is organized as follows. Section 2 lays out our basic assumptions. Section 3 discusses the centralized market, and Section 4 discusses the decentralized market where ideas are traded. Section 5 puts things together to characterize equilibrium. Section 6 discusses efficiency and policy considerations. Section 7 takes up various extensions. Section 8 concludes. Many technical results are relegated to the Appendix.[5]

2 Basic Assumptions

Time is discrete and continues forever. As in Lagos and Wright (2005), we assume that alternating over time there are two types of markets: a

[5] We mention some other related work. Several studies consider the transfer of ideas as a strategic action among firms, including Katz and Shapiro (1986), Gallini and Winter (1985), and Shepard (1994). Baccara and Razin (2004) consider strategic behavior among agents forming a team to implement an idea. Anton and Yao (1994, 2002) study markets where buyers do not know the value of an idea, and sellers are reluctant to reveal it because buyers may not pay afterwards. Others focus on licensing contracts in terms of incentives, including Aghion and Tirole (1994) and Arora (1995). There is a literature that focuses on university inventions, including Lowe (2003), Shane (2002), and Jensen and Thursby (2001). den Haan, Ramey and Watson develop a matching model of entrpreneurs and lenders, and cite related work. Serrano (2005) studies empirically the market for patent transfers, and cites previous papers along the same line.

centralized market, denoted CM, where agents perform the usual activities of producing, consuming and adjusting their assets; and a decentralized market, denoted DM, where agents meet bilaterally and may trade ideas. Ideas traded in one DM yield returns realized in the next CM. Agents have discount factor β between one DM and the next CM, and discount factor δ between the CM and the next DM, $\delta\beta < 1$. There are two types of agents: innovators, denoted i, who are relatively good at coming up with ideas, and entrepreneurs, denoted e, who may be better at implementing them. For now the numbers of each type, N_i and N_e, are exogenous.

Every time the DM opens, an innovator i gets some idea (for free) that has value $R_i \geq 0$ if he implements it himself, where R_i is drawn from CDF $F_i(\cdot)$. To keep things simple, if not implemented in one period, an idea's value next period is an i.i.d. draw from F_i; hence if innovator i finds himself in the CM with an idea, he will implement it, since he gets a new draw in any event. If i with an idea worth R_i to him in the DM meets entrepreneur e, it has value $R_e \geq 0$ to e, where R_e is drawn from $F_e(\cdot|R_i)$. When convenient, for a few results below, we assume F_j' exists, is continuous, and has support with finite upper bound $\hat{R}$.[6] Whenever i and e meet and realize $R_e > R_i$, e has a better capacity or ability to implement the project.

One may ask, *what exactly is an idea*? One view is that an idea I is an intermediate input in some production process that can be implemented by

[6]As a special case, we can have R_i and R_e independent, which can be interpreted as saying there is nothing special about the idea but only the match between the idea and the agent. We can also have $R_i = \bar{R}_i$ with probability 1, including the case $\bar{R}_i = 0$, where i is are purely an "idea man" who cannot implement anything.

agent j with technology $f_j(h, I)$, where h is labor input. Given I, j solves

$$R_j(I) = \max_h \{f_j(h, I) - wh\},$$

where w is the wage (more generally, h and w can be vectors of inputs and prices). This is important because it shows that the allocation of ideas affects wages, hours and other variables in general equilibrium – having the wrong agents implementing I can have a big impact on economic aggregates. However, to ease the presentation, we begin with the case where $R_j = f_j(I)$ does not require outside labor, and return to the general specification later.

In any case, when i and e meet and $R_e > R_i$, there are gains from trade.[7] We assume ideas are indivisible – either I tell you or I don't. Also, there is no private information: both agents *know* (R_i, R_e), even though e cannot implement the idea without i giving him the details. For example, if my idea is for a restaurant with some new cuisine, I can let you taste it without showing you the recipe. We abstract from informational frictions not because they are uninteresting, but because we want to focus on new issues (several of the papers mentioned in the Introduction consider private information). Also, we assume for now that if i gives e the idea then i does not also implement it – say, because there is only room for one new restaurant – but we also explore below the alternative case where both can implement it, which is relevant to the extent that ideas have a public good component.

[7] Note that i will never prefer to not trade an idea for speculative reasons – i.e. in hope of meeting another e with a bigger R_e – since he will get a new idea anyway, and the value of any idea is i.i.d. across periods. This is the same as the reason agents never choose to not implement an idea in the CM.

The price at which an idea is traded is determined by bargaining. This price is in terms of money, by which we do not necessarily mean cash *per se*, but relatively liquid assets generally, including e.g. checking deposits.[8] If the price at which they would otherwise trade is greater than the amount of liquidity m that e has on hand, several things could happen: i could walk away and keep the idea for himself; they could settle for a lower price; or they could agree to try to meet again in the next CM, where e can raise funds. But with probability γ the meeting in the next CM fails to happen. Rather than go into details, we prefer to remain agnostic and simply label the event an 'exogenous breakdown.' The fact that it is not certain they can put a deal together next period provides incentive for entrepreneurs to keep liquidity on hand, lest potentially profitable opportunities fall through.

While there may well be other ways to model the idea market, we think our setup is reasonable. Some of the assumptions are similar to those in standard monetary theory, and are made to generate a role for liquidity. Thus, i will not give up his idea before he is paid, hoping to get paid in the future, say, because after e has the idea he will renege.[9] We understand that there are many ways in the real world for innovators to try to get people

[8] He, Huang and Wright (2005) introduce banks and checking explicitly into an otherwise standard search model of monetary exchange. Presumably something similar can be done here, but for the sake of focus we prefer not to go into all the requisite details. Hence, we frame the discussion in terms of money, but it is understood that in principle the point should apply to liquid assets broadly.

[9] Obviously we assume there is no problem with a simultaneous (quid pro quo) exchange. Of course, it is important that we cannot use reputation to enforce payment, since otherwise credit could work and eliminate the role for liquidity. A standard way to rule out reputation in these kinds of models is to assume some form of anonymity in the DM; see Kocherlakota (1998), Wallace (2001) or Corbae et al. (2003) for details.

who are good at implementation involved in their projects: hiring managers; forming partnerships; licensing; and so on. We focus on the case where they *sell* the idea, which is not the only possibility but surely an interesting case. For one thing, it is consistent with the extensive evidence of financial constraints on entrepreneurial activity discussed in the introduction.

3 The CM

Let $W_j(m, R)$ be the the value functions for type $j = i, e$ agents entering the CM, with m dollars and a project in hand with value R (for i this would be his own idea if he did not sell it in the previous DM, and for e this would be an idea that he purchased). We use $R = 0$ to indicate either a project with 0 return or no project (for i this would be because he sold his idea, and for e this would be because he failed to buy one). Let $V_j(m)$ be the value function for agents entering the DM with m dollars before the random values of the ideas are drawn.

Then for $j = i, e$, the CM problem is

$$\begin{aligned} W_j\left(m,R\right) &= \max_{X,H,\hat{m}} \left\{U(X) - h + \delta V_j(\hat{m})\right\} \\ \text{s.t. } X &= \varepsilon + wh + \phi(m - \hat{m} + \pi M) + R, \end{aligned} \tag{1}$$

where X is consumption, h labor supply, $\hat{m}$ money taken out of the CM, ε an endowment, w the real wage, and ϕ the value of money (i.e. $1/\phi$ is the nominal price level). The term πM is a lump sum cash transfer, with M the aggregate money stock when the CM opens, which therefore evolves over time according to $M' = (1+\pi)M$.[10] We impose the usual conditions on utility U. Also, for now we assume a representative firm with a linear technology, so the real wage is pinned down and can be normalized to $w = 1$.

Eliminating h using the budget equation, we rewrite (1) as

$$\begin{aligned} W_j(m, R) &= \varepsilon + \phi m + \phi \pi M + R + \max_X \{U(X) - X\} \\ &\quad + \max_{m'}\{-\phi \hat{m} + \delta V_j(\hat{m})\}. \end{aligned} \tag{2}$$

Now several results follow immediately. First, W_j is linear in (m, R), with $\partial W_j / \partial m = \phi$ and $\partial W_j / \partial R = 1$. Second, X is given by the solution to $\partial U(X)/\partial X = 1$, independent of (m, R) or any other variable. Third, $\hat{m}$ is given by the solution to

$$-\phi + \delta \frac{\partial V_j(\hat{m})}{\partial \hat{m}} \leq 0, = 0 \text{ if } \hat{m} > 0, \tag{3}$$

independent of (m, R). This implies all agents of a given type j take the same amount of money $\hat{m}_j$ out of the CM, independent of the (m, R) with which they enter.[11]

4 The DM

Let α_j be the DM arrival rate (probability of a meeting) for $j = i, e$. Normalizing $N_e = 1$, the only restriction on arrival rates is $\alpha_e = \alpha_i N_i$, so we

[10] As is standard, equilibrium requires $\pi \geq \beta - 1$ as otherwise there will be arbitrage opportunities. By the Fisher equation, this is equivalent to $i \geq 0$ where i_n is the nominal interest rate. As is also standard, we assume the inequality is strict, but we do consider the limiting case where $\pi \to \beta - 1$, or $i_n \to 0$, which is called the Friedman rule.

[11] The results in this paragraph assume an interior solution for h and the strict concavity of V_j. One can generalize the assumptions and arguments in Lagos and Wright (2005) to guarantee that this is valid.

can take them to be exogenous, for now. If an e does not meet an i, he enters the next CM with his money but no project, $(\hat{m}_e, 0)$. Similarly, if i does not meet an e, he enters the next CM with $(\hat{m}_i, R_i)$. If an e and i do happen to meet, several things can happen. If $R_e \leq R_i$ there are no gains from trade; and if $R_e > R_i$ there are, and two cases need to be considered. On the one hand, suppose $\hat{m}_e \geq p$ where p is the price they would agree to if there were no issues of liquidity (e.g. if e had access to unlimited funds). Then they can trade immediately at p.

On the other hand, suppose $\hat{m}_e < p$. In this case the bargaining problem is nonconvex, and in principle they may want to trade using lotteries; for now we assume that this not an option, but we revisit lotteries later and show that the basic qualitative results are the same. Hence, they can either settle for $\hat{m}_e$ now, or they could try to meet again in the next CM, where e can always raise the funds. If they do meet again, they can renegotiate the price to p', but we will see $p' = p$. In any case, meeting in the next CM is not a sure thing: with probability γ, there is an exogenous breakdown. Hence, i may or may not prefer the chance at p' to the sure thing of $\hat{m}_e$.[12]

We now analyze the bargaining problems in more detail. As is common in the related literature, we make use of the generalized Nash solution, where threat points are given by continuation values and θ denotes the bargaining power of e. To begin, consider what happens if they put the deal on hold and meet again in the next CM. Given the value function next period W_j',

[12] Also, i could walk away and keep the idea, but if $R_e > R_i$ this is dominated trying to meet in the CM. Note i cannot trade the idea for $\hat{m}_e$ plus a promise of additional payment in the next CM, since e will not honor the promise.

the bargaining solution is:

$$\max_{p'}[W'_e(\hat{m}_e - p', R_e) - W'_e(\hat{m}_e, 0)]^\theta [W'_i(\hat{m}_i + p', 0) - W'_i(\hat{m}_i, R_i)]^{1-\theta} \quad (4)$$

Since W'_j is linear, $W'_e(\hat{m}_e - p', R_e) - W'_e(\hat{m}_e, 0) = R_e - \phi' p'$ and $W'_i(\hat{m}_i + p', 0) - W'_i(\hat{m}_i, R_i) = \phi' p' - R_i$, so (4) reduces to:

$$\max_{\hat{p}} \left(R_e - \phi' p'\right)^\theta \left(\phi' p' - R_i\right)^{1-\theta} \quad (5)$$

This immediately yields $p' = [\theta R_i + (1-\theta) R_e] / \phi'$.

Now consider what happens in the DM this period. A difference from the situation analyzed above is that the threat points are no longer given by the continuation values of not trading, but by the expected values of putting the deal on hold,

$$\overline{W}'_e = \gamma W'_e(\hat{m}_e - p', R_e) + (1-\gamma) W'_e(\hat{m}_e, 0) \quad (6)$$

$$\overline{W}'_i = \gamma W'_i(\hat{m}_i + p', 0) + (1-\gamma) W'_i(\hat{m}_i, R_i). \quad (7)$$

Again using the linearity of W'_j, the bargaining problem becomes:

$$\max_{p}[-\phi' p + \gamma \phi' p' + (1-\gamma) R_e]^\theta [\phi' p - \gamma \phi' p' - (1-\gamma) R_i]^{1-\theta} \quad (8)$$

A second difference from the previous situation is that now we have the constraint $p \leq \hat{m}_e$, since e cannot pay more than he has in the DM.

Suppose first that the constraint $p \leq \hat{m}_e$ does not bind. Then it is simple to show $p = p'$, the same as the solution in the CM next period. In this case the agents settle immediately. Suppose now that $\hat{m}_e < p'$, which is equivalent to $R_e \geq B(R_i) \equiv \frac{\phi' \hat{m}_e - \theta R_i}{1-\theta}$ (the label B stands for the fact that the constraint just *binds*). In this case e wants to pay $\hat{m}_e$ and close the deal, but i may prefer to put it on hold and try to meet in the next CM. In fact, he prefers to trade now rather than put the deal on iff $W_i'(\hat{m}_i+\hat{m}_e,0) \geq \overline{W}_i'$, which simplifies easily to $R_e \leq H(R_i) \equiv \frac{\phi'\hat{m}_e - R_i(1-\gamma+\theta\gamma)}{\gamma(1-\theta)}$ (the label H stands for the fact that the innovator is just willing to putting the deal on *hold*).

Taking $z = \phi'\hat{m}_e$ as given for now, Figure 1 shows the possible outcomes in (R_i, R_e) space, partitioned into the following regions. Below the 45^o line, in region A_1, there are no gains from trade and hence no trade. Above the 45^o line there are gains from trade, and several outcomes are possible. Below $R_e = B(R_i)$, in A_2, the constraint $p \leq \hat{m}_e$ does not bind and there is immediate trade. Above B and below $R_e = H(R_i)$, in A_3, there is immediate trade and e gets the idea for $\hat{m}_e$ since i does not want to risk putting the deal on hold. Above H, the deal is put on hold. This occurs when R_i and R_e are both high because: (i) then p' is high; and (ii) when R_i is high there is less downside risk for i in case they do not meet again.

Figure 1 about here.

We now describe the DM value function for e. We break the presentation into parts by writing

$$V_e(\hat{m}) = (1-\alpha_e)\beta W_e'(\hat{m},0) + \alpha_e\beta\sum_{j=1}^{5} V_e^j(\hat{m}), \tag{9}$$

where the first term is the expected value of no meeting, and for $j = 1, ...5$, $V_e^j(\hat{m})$ is the expected value of a meeting with (R_i, R_e) in region A_j of Figure

1. To begin, integrate over region A_1 to get

$$V_e^1(\hat{m}) = \int_0^\infty \int_0^{R_i} W_e'(\hat{m}, 0) dF_e(R_e|R_i) dF_i(R_i),$$

which is the outcome when $R_e < R_i$, which means no trade.

Now consider regions where $R_e > R_i$. In A_2,

$$V_e^2(\hat{m}) = \int_0^{\phi'\hat{m}} \int_{R_i}^{B(R_i)} W_e'(\hat{m} - p, R_e) dF_e(R_e|R_i) dF_i(R_i)$$

is the expected outcome when they trade at $p \leq \hat{m}$. In A_3,

$$V_e^3(\hat{m}) = \int_0^{\phi'\hat{m}} \int_{B(R_i)}^{H(R_i)} W_e'(0, R_e) dF_e(R_e|R_i) dF_i(R_i)$$

is the expected outcome when they trade at $\hat{m}$. And in A_4 and A_5,

$$\begin{aligned} V_e^4(\hat{m}) &= \int_0^{\phi'\hat{m}} \int_{H(R_i)}^{\infty} \overline{W}_e' dF_e(R_e|R_i) dF_i(R_i) \\ V_e^5(\hat{m}) &= \int_{\phi'\hat{m}}^{\infty} \int_{R_i}^{\infty} \overline{W}_e' dF_e(R_e|R_i) dF_i(R_i) \end{aligned}$$

with $\overline{W}_e'$ given in (6). We show in Appendix A how to reduce all of this to

$$\begin{aligned} V_e(\hat{m}) &= \beta W_e'(\hat{m}, 0) + \alpha_e \beta \theta \int_0^{\phi'\hat{m}} \int_{R_i}^{B(R_i)} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i) \\ &+ \alpha_e \beta \int_0^{\phi'\hat{m}} \int_{B(R_i)}^{H(R_i)} (R_e - \phi'\hat{m}) dF_e(R_e|R_i) dF_i(R_i) \qquad (10) \\ &+ \alpha_e \beta \gamma \theta \int_0^{\phi'\hat{m}} \int_{H(R_i)}^{\infty} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i) \\ &+ \alpha_e \beta \gamma \theta \int_{\phi'\hat{m}}^{\infty} \int_{R_i}^{\infty} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i). \end{aligned}$$

A similar exercise can be performed for i. We do not provide the details, however, because it turns out that

$$V_i(\hat{m}) = \beta\phi'\hat{m} + v, \tag{11}$$

where v does not depend on $\hat{m}$. Intuitively, for i, neither the probability of trade nor the terms of trade depend on his own money holdings (they depend on the money on the other side of the market). So whatever money i brings to the DM, he simply takes back to the next CM.

5 Equilibrium

We now combine the DM and CM to get equilibrium. The key condition from the CM is the FOC for $\hat{m}$, given by (3). All we need to do is insert the derivative of the DM value function V_j to determine the choice of $\hat{m}_j$. For $j = i$ this is easy: by (11), $\partial V_i/\partial \hat{m} = \beta\phi'$, so (3) becomes

$$-\phi + \delta\beta\phi' \leq 0, = 0 \text{ if } \hat{m} > 0.$$

As in standard in these models (see Lagos and Wright 2005), we only consider equilibria where $\delta\beta\phi' < \phi$, and hence we conclude that $\hat{m}_i = 0$.

In case it is not clear, the reason we only consider equilibria satisfying this condition is that when $\delta\beta\phi' > \phi$ no equilibrium exists, and when $\delta\beta\phi' = \phi$ equilibrium is indeterminate. One way to understand this is to use the Fisher equation: $1 + i_n = (1 + i_r)\phi/\phi'$, where i_n is the nominal interest rate, i_r the real rate, and ϕ/ϕ' the inflation rate between two meetings of the CM. In this model, $1 + i_r = 1/\beta\delta$, since preferences are quasi-linear. Hence the condition $\delta\beta\phi' < \phi$ simply says that the nominal interest rate is positive. To restate what we said above, $i_n < 0$ is inconsistent with equilibrium and $i_n = 0$ implies equilibrium is indeterminate. Although we do not allow $i_n = 0$, we do consider the limit as $i_n \to 0$, which is called the Friedman rule. In any case we have $\hat{m}_i = 0$.

A similar exercise for $j = e$ is less simple. First, in Appendix A, we derive from (10)

$$\partial V_e/\partial \hat{m} = \beta\phi' \left[1 + \ell(\phi'\hat{m})\right], \tag{12}$$

where for any $z = \phi'\hat{m}$ we define $l(z)$ as follows:

(i) if $\gamma > 0$ and $\theta < 1$ then

$$\begin{aligned} \ell(z) \equiv\; & (1-\gamma)\alpha_e \int_0^z \tfrac{z-R_i}{\gamma^2(1-\theta)^2} F_e' \left[\tfrac{z-R_i(1-\gamma+\theta\gamma)}{\gamma(1-\theta)} | R_i\right] dF_i(R_i) \\ & -\alpha_e \int_0^z \left\{ F_e\left[\tfrac{z-R_i(1-\gamma+\theta\gamma)}{\gamma(1-\theta)} | R_i\right] - F_e\left[\tfrac{z-\theta R_i}{1-\theta} | R_i\right]\right\} dF_i(R_i); \end{aligned} \tag{13}$$

(ii) if $\gamma = 0$ then

$$\begin{aligned} \ell(z) \equiv\; & \alpha_e F_i'(z) \int_z^\infty (R_e - z) dF_e(R_e|z) \\ & -\alpha_e \int_0^z \left\{1 - F_e\left[\tfrac{z-\theta R_i}{1-\theta} | R_i\right]\right\} dF_i(R_i); \end{aligned} \tag{14}$$

(iii) $\theta = 1$ then

$$\ell(z) \equiv (1-\gamma)\alpha_e F_i'(z) \int_z^\infty (R_e - z) dF_e(R_e|z). \tag{15}$$

Obviously the reason for the different cases is that we have to be careful about dividing by 0 in (13). In any case, $\ell(z)$ is simply the expected marginal benefit of having more cash in a meeting.[13]

Inserting (12) into (3), we get

$$-\phi+\delta\beta\phi'\left[1+\ell(\phi'\hat{m})\right]\leq 0, =0 \text{ if } \hat{m}>0. \tag{16}$$

Given any path for $\{M\}$, equilibria can be defined in terms of a path for $\{\phi\}$, satisfying (16) plus some side conditions, but to simplify the discussion we focus on steady state equilibria where the growth rate of the money supply π and real balances $z=\phi M$ are constant. Since $\hat{m}_e=M'$ (entrepreneurs hold all the money), again using the Fisher equation, (16) can be simplified in steady state to

$$\ell(z)\leq i_n,\ =i_n \text{ if } z>0. \tag{17}$$

Note that in this condition i_n is exogenous – it is a policy variable.[14]

From (16), a nonmonetary steady state with $\phi=z=0$ always exists. A monetary steady state exists at any $z>0$ such that $\ell(z)=i_n$ and, in addition, $\ell'(z)=\partial^2 V_e/\partial\hat{m}^2\leq 0$, which is obviously necessary because otherwise $\hat{m}_e$ yields a minimum rather than a maximum in the CM problem. For the record we formalize this as:

Definition: A monetary steady state equilibrium is a $z>0$ such that $\ell(z)=i_n$ and $\ell'(z)\leq 0$.

[13] Consider $\gamma=0$. The first term in (14) is the probability of meeting i with idea $R_i=z$, which is $F_i'(z)$, times the net gain for e from buying the idea, R_e-z, integrated over R_e. And the second term is the probability of $(R_i,R_e)\in A_3$ times -1, since in in A_3 the constraint binds and a marginal dollar is simply taken by i. A similar intuition applies to (13), except things are complicated by the fact that sometimes deals are put on hold. Notice also that $\partial V_e/\partial\hat{m}=[1+\ell(z)]\,\partial V_i/\partial\hat{m}$, since for e the return on $\hat{m}$ includes a liquidity component that is not there for i. See Lagos (2005) for an extensive analysis of similar equations in a model of liquidity.

[14] The central bank can either set i_n directly and let the money growth rate π adjust, or they can fix π and $\phi/\phi'=1+\pi$ will pin down i_n through the Fisher equation.

Once we know z, the other endogenous variables can be easily recovered. Hence, we concentrate on finding solutions to $\ell(z) = i_n$ with $\ell'(z) \leq 0$.

Consider the generic case, $\gamma \in (0,1)$ and $\theta \in (0,1)$. In the Appendix we verify that $\ell(0) = 0$ and $\lim_{z \to \infty} \ell(z) = 0$. Under standard conditions, ℓ is continuous and $\ell(z) > 0$ for some $z > 0$.[15] Hence there exist a solution to $\ell(z) = i_n$ iff i_n is not too big, and these solutions generically come in pairs. For each pair of solutions, the higher z constitutes a monetary equilibrium, while the lower one does not, because it violates the second order condition $\ell'(z) \leq 0$. The main point is that a steady state monetary equilibrium exists iff i_n is not too big. Figure 2 shows two examples. The first panel is drawn assuming F_i and F_e are independent lognormal distributions; the second assuming they are independent uniform distributions.

Figure 2 about here.

For completeness, consider the case where $\gamma \notin (0,1)$ or $\theta \notin (0,1)$. The results are essentially the same when $\theta = 1$ or $\gamma = 0$, except that we may have $\ell(0) > 0$, but this is irrelevant for the economics.[16] When $\gamma = 1$, however, things are quite different, because then $\ell(z) = 0$ for all z and the only equilibrium is the nonmonetary equilibrium, $z = 0$. Given $i_n > 0$, if $\gamma = 1$ then e has no demand for liquidity, since agents can always raise funds in the next CM without fear that a deal will fall through. The friction $\gamma < 1$ is

[15] Continuity is not not necessarily needed (for existence e.g.) but it makes the presentation easier.

[16] If $\ell(0) > 0$, as opposed to $\ell(0) = 0$, we may lose the first solution to $\ell(z) = i_n$, but this is irrelevant because this solution is not an equilibrium, due to the fact that it violates the second order condition $\ell'(z) \leq 0$.

crucial. Also, perhaps surprisingly, when $\theta = 0$ a monetary equilibrium can still exist, contrary to the typical search and bargaining model. Usually when $\theta = 0$ money cannot be valued because the buyer gets 0 surplus from trade. However, here e still gets positive surplus in region $A3$, where the constraint $p \leq \hat{m}$ is binding. This is because in $A3$ there is really no bargaining: i simply swaps his idea for $\hat{m}$ iff $\phi' \hat{m} > R_i$.[17]

In any case, when $\gamma < 1$, given $R_e > R_i$, i and e want to trade, but not every deal can get done in the next CM. Without liquidity, with probability $1-\gamma$ the less efficient agent i will implement the project. It is clear what the role for money is here: if e had sufficient cash on hand in the DM, they could close the deal then and there and not risk it falling through. With money it is less likely that the less efficient agent will have to implement the project. Yet money does not overcome the frictions in this market completely: even in the monetary equilibrium, however, not all potentially profitable deals get done.

The optimal monetary policy minimizes the probability that deals fall through. This requires e to carry sufficient liquidity to close deals with $R_e > R_i$ with probability 1. By running the Friedman rule, $i_n = 0$, monetary policy can make liquidity essentially free. This minimizes the probability that deals fall through, but notice that it does not necessarily achieve the first best outcome where agents trade whenever $R_e > R_i$, unless $\theta = 1$. It is obvious from Figure 2 that when $\theta < 1$ the equilibrium has z too low to

[17] The indivisibility of the idea is what drives this result; it is not true when we introduce lotteries below.

yield the first best even when $i_n = 0$. It is also obvious from (15) that when $\theta = 1$ we do get the first best at $i_n = 1$.

Summarizing the results of this section:

Proposition: A nonmonetary equilibrium always exists. A monetary equilibrium exists iff i_n is not too big. In any monetary equilibrium, $\partial z / \partial i_n \leq 0$. Monetary equilibrium yields the first best outcome iff $\theta = 1$ and $i_n = 0$.

6 Extensions

6.1 An Explicit Example

Consider the case where $R_i = R$ with probability 1, while for now $F_e(\cdot)$ is general (but of course we do not need to condition on R_i explicitly since it is degenerate). There are two relevant cases to consider: $z < R$ and $z > R$. In the former case $z < R$, there are essentially only two outcomes, (R_i, R_e) is either in A_1 or A_5, and[18]

$$V_e(\hat{m}) = \beta W_e'(\hat{m}, 0) + \alpha_e \beta \gamma \theta \int_R^\infty (R_e - R) dF_e(R_e).$$

In the other case $z < R$, (R_i, R_e) may be in A_1, A_2, A_3 or A_4, and hence

$$\begin{aligned} V_e(\hat{m}) &= \beta W_e'(\hat{m}, 0) + \alpha_e \beta \theta \int_R^{B(R)} (R_e - R) dF_e(R_e) \\ &+ \alpha_e \beta \int_{B(R)}^{H(R)} (R_e - \phi' \hat{m}) dF_e(R_e) + \alpha_e \beta \gamma \theta \int_{H(R)}^{\infty} (R_e - R) dF_e(R_e). \end{aligned}$$

[18] Notice that having a little more money does not improve one's trading prospects in this case, because any profitable deal will be put on hold anyway.

A little calculus yields $V_e' = \beta\phi' [1 + \ell(z)]$ where $\ell(z) = 0$ if $z < R$, while if $z > R$

$$\ell(z) = \frac{\alpha_e(1-\gamma)(z-R)}{\gamma^2(1-\theta)^2} F_e' [H(R)] - \alpha_e \{F_e [H(R)] - F_e [B(R)]\}.$$

The first term is gain from relaxing the probability of just hitting the H condition, times the gain to closing the deal now rather than taking a chance on meeting later; the second term is the loss to ending up in region A_3, where any additional cash is simply taken by i. It is clear that $\ell(R) = 0$, and that $\ell(z) > 0$ for some $z > R$ if F_e' is continuous and $F_e(R) < 1$. Hence there is a monetary equilibrium as long as i_n is not too high.

Suppose R_e is uniform on $[0, 1]$. Straightforward algebra yields

$$\ell(z) = \begin{cases} 0 & z < R \\ \frac{\alpha_e(1-\gamma)(1-\gamma+\gamma\chi)(z-R)}{\gamma^2(1-\theta)^2} & R < z < z_H \\ \frac{\alpha_e(z-\theta R-1+\theta)}{1-\theta} & z_H < R < z_B \\ 0 & R > z_B \end{cases}$$

where

$$z_H = \gamma(1-\theta) + (1-\gamma+\gamma\theta)R \text{ and } z_B = 1-\theta+\theta R$$

solve $H(z_H) = 1$ and $H(z_B) = 1$ (this is a relevant value because 1 is the upper bound of R_e). As seen in the first panel of Figure 3, $\ell(z)$ is piece-wise linear with a discontinuity at z_H.

Figure 3 about here.

Hence, for any $i_n < \hat{\imath}_n$, where

$$\hat{\imath}_n = \frac{\alpha_e(1-\gamma)(1-\theta+\theta\gamma)(1-R)}{\gamma(1-\theta)},$$

there is a unique monetary equilibrium at $z = z_H$. At $z = z_H - \varepsilon$ the marginal value of additional liquidity exceeds i_n, at $z = z_H + \varepsilon$ marginal value is actually negative, and so e chooses $z = z_H$. This example is nice because it is easy to solve for everything explicitly, but it has the property that the equilibrium z is insensitive to i_n (up to $\hat{\imath}_n$). The second panel in Figure 3 shows the case where everything is the same except R_e is log-normal; here $\ell(z)$ is continuous and it is clear that the equilibrium z is smoothly decreasing in i_n.

6.2 Lotteries

When ideas are indivisible, based on previous work in monetary theory and elsewhere, one might think that agents should be allowed to trade using lotteries.[19] Appendix C shows that lotteries are never be used in the CM, because even if ideas are indivisible, when there is no liquidity constraint the bargaining problem is still convex; but they are used in the DM when the constraint $p \leq \hat{m}_e$ binds. Appendix C also shows that deals are not put on hold if we have lotteries: when the constraint $p \leq \hat{m}_e$ binds, e gives i all his money in exchange for a probability $\mu \in (0, 1)$ of transferring the idea. If e does not win this lottery he does not get the idea, but if they meet again in the next CM he gets it $p' = [\theta R_i + (1-\theta) R_e] / \phi'$. Hence e potentially pays twice: once for the lottery in the DM, and again if he loses the lottery but meets i in the next CM.

[19] The analysis here follows Berentsen, Molico and Wright (2002), although that paper only considers the simple case where agents are restricted to $\hat{m} \in \{0, 1\}$.

The payoff for e from this lottery is

$$\mu W_e'(\hat{m}_e - p, R_e) + (1-\mu)[\gamma W_e'(\hat{m}_e - p - p', R_e) + (1-\gamma) W_e'(\hat{m}_e - p, 0)],$$

and his threat point is $\gamma W_e'(\hat{m}_e - p', R_e) + (1-\gamma) W_e'(\hat{m}_e, 0)$. By linearity of W_e', his surplus is $-\phi' p + \mu(1-\gamma) R_e + \mu\gamma\phi' p'$. Similarly, the surplus for i is $\phi' p - \mu(1-\gamma) R_i - \mu\gamma\phi' p'$. Hence the bargaining problem reduces to

$$\max_{p,\mu} \left[-\phi' p + \mu(1-\gamma) R_e + \mu\gamma\phi' p'\right]^{\theta} \left[\phi' p - \mu(1-\gamma) R_i - \mu\gamma\phi' p'\right]^{1-\theta}$$

subject to the constraints $p \le \hat{m}_e$ and $\mu \le 1$ (as well as nonnegativity, but this will not be binding as long as $R_e > R_i$).

Ignoring the constraints, the FOC wrt p and μ are

$$\begin{aligned} 0 &= \theta\left[\phi' p - \mu(1-\gamma) R_i - \mu\gamma\phi' p'\right] \\ &\quad -(1-\theta)\left[-\phi' p + \mu(1-\gamma) R_e + \mu\gamma\phi' p'\right] \qquad (18) \\ 0 &= \theta\left[\phi' p - \mu(1-\gamma) R_i - \mu\gamma\phi' p'\right]\left[(1-\gamma) R_e + \gamma\phi' p'\right] \\ &\quad -(1-\theta)\left[-\phi' p + \mu(1-\gamma) R_e + \mu\gamma\phi' p'\right]\left[(1-\gamma) R_e + \gamma\phi' p'\right] \qquad (19) \end{aligned}$$

These cannot both hold when $R_e > R_i$; hence we cannot have $p < \hat{m}_e$ and $\mu < 1$. If $\mu = 1$ and $p < \hat{m}_e$ then (18) implies $p = p'$. If $p = \hat{m}_e$ and $\mu < 1$ then (19) implies $\mu = \Omega\phi'\hat{m}_e$, where

$$\Omega = \frac{(\theta + \gamma - 2\gamma\theta) R_e + (1 - \theta - \gamma + 2\gamma\theta) R_i}{[(1-\theta\gamma) R_e + \gamma\theta R_i][\gamma(1-\theta) R_e + (1-\gamma+\theta\gamma) R_i]}.$$

Appendix C verifies that $\partial\Omega/\partial R_i < 0$ and $\partial\Omega/\partial R_i < 0$, and that $\mu = \Omega\phi'\hat{m}_e < 1$ iff $R_e > B(R_i)$, where B is the same as in the model without lotteries. Appendix C also shows that $\partial\mu/\partial\theta > 0$ and $\partial\mu/\partial\gamma < 0$.

FIGURE 4 ABOUT HERE.

All of this implies that the outcome is as depicted in Figure 4, which shows the bargaining solution (p, μ) as a function of R_e for a given R_i, assuming $R_i < \phi' \hat{m}_e$, which guarantees $\mu = 1$ and $p < \hat{m}_e$ when $R_e = R_i$. As R_e increases, p increases while μ stays at 1, until p hits $\hat{m}_e$, after which μ decreases while p stays at $\hat{m}_e$. The main impact of introducing lotteries is to allow trade to potentially occur in what was region $A_4 \cup A_5$ in Figure 1, where previously the deal was necessarily put on hold. However, the lottery only allows the idea to be transferred with probability μ; with probability $1-\mu$ the agents can only hope to reconvene next period to exhaust the gains from trade. And note that it is still the best deals that have the greatest risk of falling through; indeed, $\mu \to 0$ as $R_e \to \infty$.

One can write the DM value function with lotteries in a way similar to (10), and differentiate to get $V_e'(\hat{m})$ as before. The same method leads to an equilibrium condition with the same form, $i_n = \ell(z)$, except $\ell(z)$ is slightly different. For the record, with lotteries we have

$$\begin{aligned}
\ell(z) \quad \equiv \quad & \alpha_e \int_0^z \left\{ \tfrac{\theta(1-\gamma)(z-R_i)}{(1-\theta)^2} - \tfrac{\theta(1-\gamma)(z-R_i)}{(1-\theta)^2[z\gamma+R_i(1-\gamma)]} \right\} F_e' \left[B(R_i)|R_i\right] dF_i(R_i) \\
& + \alpha_e \int_0^z \int_{B(R_i)}^\infty \frac{\theta(1-\gamma)(R_e - R_i)}{\gamma(1-\theta)R_e + (1-\gamma+\theta\gamma)R_i} dF_e(R_e|R_i)\; dF_i(R_i) \\
& + \alpha_e \int_z^\infty \int_{R_i}^\infty \frac{\theta(1-\gamma)(R_e - R_i)}{\gamma(1-\theta)R_e + (1-\gamma+\theta\gamma)R_i} dF_e(R_e|R_i)\; dF_i(R_i).
\end{aligned}$$

6.3 Non-rival Ideas

One thing potentially true about (some) good ideas is that I can let you use it without reducing its value to me. Thus, assume now that an idea can be implemented by the innovator and the entrepreneur simultaneously. For example, if I give you the idea for a very good restaurant, you can open up for business in a different part of town, and we do not compete, while in out baseline model there is only room for one. Of course intermediate cases are also interesting, but for the sake of example here we take up the purely non-rival case.

Assume that there is a first mover advantage, in which if only one agent decides to implement the idea, the other agent will get zero return from entering the market late. As before, if the idea is not implemented, the innovator receives an i.i.d update of the return every time he enters the decentralized market. Thus, innovators will implement their idea in every period. By consequence of the assumptions above, every time an entrepreneur gets a project he will also choose to implement it right away.

Here we will highlight the main differences from the original model. Because of the non-rivalry assumption, even when $R_e < R_i$ there may be gains from trade. The bargaining problems are modified as follows. Indeed, as long as $R_e > 0$, e would be willing to pay something for the idea. The bargaining problem in the next CM becomes:

$$\max_{p'} \left(R_e - \phi p'\right)^{\theta} \left(\phi p'\right)^{1-\theta}$$

Solving this, we get $\phi p' = (1-\theta)R_e$. The DM bargaining problem becomes:

$$\max_p [-\phi p + \gamma\phi p' + (1-\gamma)R_e]^\theta [\phi p - \gamma\phi p']^{1-\theta}$$

subject to $p \le m$. The solution is

$$\phi p = \min\left\{\phi m, (1-\theta)R_e\right\}.$$

The constraint $m \le p$ binds iff $R_e \ge B = \frac{\phi m}{1-\theta}$, and i prefers to put the deals on hold rather than trade for m now rather iff $R_e \le H = \frac{\phi m}{\gamma(1-\theta)}$. Also, we have

$$V_e(m) = (1-\alpha_e)\beta W_e(m,0) + \alpha_e\beta\sum_{j=1}^{3} V_e^j(m),$$

where the first term is the expected value of not meeting someone and going to the next CM with $(m, 0)$, and $V_e^j(m)$ is the expected value of a meeting when (R_i, R_e) falls in region $\bar{A}_j$ of Figure 5, $j = 1, 2, 3$. Notice there are only three relevant regions in this version of the model.

FIGURE 5 ABOUT HERE.

In region $\bar{A}_1$

$$V_e^1(m) = \int_0^\infty\int_0^B W_e(m-p, R_e)dF_e(R_e|R_i)dF_i(R_i)$$

is the expected outcome of a meeting where $p \le m$. In $\bar{A}_2$,

$$V_e^2(m) = \int_0^\infty\int_B^H W_e(0, R_e)dF_e(R_e|R_i)dF_i(R_i)$$

is the expected outcome when $p > m$, but they trade at m rather than putting the deal on hold. Finally, in $\bar{A}_3$,

$$V_e^3(m) = \int_0^\infty \int_H^\infty \overline{W}_e dF_e(R_e|R_i) dF_i(R_i)$$

is the expected return to putting a deal on hold.

Simplifying, we get

$$\begin{aligned}
\frac{V_e(m)}{\beta} &= W_e(m,0) + \alpha_e \theta \int_0^\infty \int_0^B R_e \; dF_e(R_e|R_i) dF_i(R_i) \\
&+ \alpha_e \int_0^\infty \int_B^H (R_e - \phi m) \; dF_e(R_e|R_i) dF_i(R_i) \\
&+ \alpha_e \gamma \theta \int_0^\infty \int_H^\infty R_e \; dF_e(R_e|R_i) dF_i(R_i).
\end{aligned}$$

The rest of the analysis is similar to the baseline model. Taking the derivative, e.g., we get

$$\frac{\partial V_e}{\partial m} = \beta \phi \left[1 + l(z)\right],$$

where for any z

$$\begin{aligned}
l(z) &\equiv (1-\gamma)\alpha_e \int_0^\infty \tfrac{(1-\gamma)}{\gamma^2(1-\theta)^2} \; z \; F_e' \left[\tfrac{z}{\gamma(1-\theta)} | R_i\right] dF_i(R_i) \qquad (20) \\
&\quad -\alpha_e \int_0^\infty \left\{ F_e\left[\tfrac{z}{\gamma(1-\theta)} | R_i\right] - F_e\left[\tfrac{z}{1-\theta} | R_i\right] \right\} dF_i(R_i)
\end{aligned}$$

as long as $\gamma > 0$ and $\theta < 1$.

6.4 Ideas as Intermediate Inputs

To Be Added.

7 Conclusion

To Be Added.

Appendix A: Derivation of (10)-(15)

First combine the expressions for $V_e^j(m')$, $j = 1, ...5$, to write (9) as

$$\begin{aligned} V_e(\hat{m}) &= (1-\alpha_e)\beta W_e'(\hat{m},0) + \alpha_e\beta \int_{A_1} W_e'(\hat{m},0) \\ &\quad + \alpha_e\beta \int_{A_2} W_e'(\hat{m}-p', R_e) + \alpha_e\beta \int_{A_3} W_e'(0, R_e) + \alpha_e\beta \int_{A_4 \cup A_5} \overline{W}_e' \end{aligned}$$

where $\int_{A_j}(\cdot)$ denotes the integral over region A_j, and it is understood that $\int(\cdot) = \int\int(\cdot)dF_e(R_e|R_i)dF_i(R_i)$. Using the linearity of W_e', we can simplify this to

$$\begin{aligned} V_e(\hat{m}) &= \beta W_e'(\hat{m},0) + \alpha_e\beta \int_{A_2} (R_e - \phi' p') \\ &\quad + \alpha_e\beta \int_{A_3} (R_E - \phi'\hat{m}) + \alpha_e\beta \int_{A_4 \cup A_5} \gamma(R_e - \phi' p'). \end{aligned}$$

Inserting p', we have

$$\begin{aligned} V_e(\hat{m}) &= \beta W_e'(\hat{m},0) + \alpha_e\beta\theta \int_{A_2} (R_e - R_i) \\ &\quad + \alpha_e\beta \int_{A_3} (R_e - \phi'\hat{m}) + \gamma\alpha_e\beta\theta \int_{A_4 \cup A_5} (R_e - R_i) \end{aligned}$$

Inserting the limits for the integrals over the various regions A_j yields (10).

We now show how to differentiate this to get the expressions for $Z(\cdot)$ for the various cases. When $\gamma > 0$ and $\theta < 1$, by Leibniz Rule, the derivatives of the integrals in the different regions are:

$$\frac{\partial}{\partial \hat{m}} \int_{A_2} (\cdot) = \phi' \int_0^{\phi'\hat{m}} \frac{\left(\phi'\hat{m} - R_i\right)}{(1-\theta)^2} F_e'[B(R_i)|R_i] dF_i(R_i)$$

$$\begin{aligned} \frac{\partial}{\partial \hat{m}} \int_{A_3} (\cdot) &= \phi' \int_0^{\phi'\hat{m}} \frac{(\phi'\hat{m} - R_i)(1-\gamma+\theta\gamma)}{\gamma^2(1-\theta)^2} F_e'[H(R_i)|R_i] dF_i(R_i) \\ & -\phi' \int_0^{\phi'\hat{m}} \frac{\theta(\phi'\hat{m} - R_i)}{(1-\theta)^2} F_e'[B(R_i)|R_i] dF_i(R_i) \\ & -\phi' \int_0^{\phi'\hat{m}} \int_{B(R_i)}^{H(R_i)} dF_e(R_e|R_i) dF_i(R_i) \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial m} \int_{A_4} (\cdot) &= \phi' F_i'(\phi'\hat{m}) \int_{\phi'\hat{m}}^{\infty} (R_e - \phi'\hat{m}) dF_e(R_e|\phi'\hat{m}) \\ & -\phi' \int_0^{\phi'\hat{m}} \frac{\phi'\hat{m} - R_i}{\gamma^2(1-\theta)^2} F_e'[H(R_i)|R_i] dF_i(R_i) \end{aligned}$$

$$\frac{\partial}{\partial \hat{m}} \int_{A_5} (\cdot) = -\phi' F_i'(\phi'\hat{m}) \int_{\phi'\hat{m}}^{\infty} (R_e - \phi'\hat{m}) dF_e(R_e|\phi'\hat{m})$$

Substituting these into $\partial V/\partial \hat{m}$ and simplifying yields (13).

In the case $\gamma = 0$, still maintaining $\theta < 1$, the results similar except that region A_4 vanishes, and where the above derivative in region A_3 is not correct because $H(R_i) = \infty$. In this case, the correct derivative in A_3 is:

$$\begin{aligned}\frac{\partial}{\partial \hat{m}} \int_{A_3} (\cdot) &= \phi' F_i'(\phi'\hat{m}) \int_{B(R_i)}^{\infty} (R_e - \phi'\hat{m}) dF_e(R_e|\phi'\hat{m}) \\ &- \int_0^{\phi'\hat{m}} \frac{\theta(\phi'\hat{m} - R_i)}{(1-\theta)^2} F_e'[B(R_i)|R_i] dF_i(R_i) \\ &- \phi' \frac{\theta(\phi'\hat{m} - R_i)}{(1-\theta)^2} \int_0^{\phi'\hat{m}} \int_{B(R_i)}^{\infty} dF_e(R_e|R_i) dF_i(R_i)\end{aligned}$$

Following the procedure that previously gave us (13) now leads to (14).

When $\theta = 1$, $B(R_i) = H(R_i) = \infty$ both become vertical at z, and A_3 as well as A_4 vanish. In this case

$$\frac{\partial}{\partial \hat{m}} \int_{A_2} (\cdot) = \phi' F_i'(R_i) \int_0^{\infty} \left(R_e - \phi'\hat{m}\right) dF_e(R_e|z)$$

$$\frac{\partial}{\partial \hat{m}} \int_{A_5} (\cdot) = -\phi' F_i'(R_i) \int_0^{\infty} \left(R_e - \phi'\hat{m}\right) dF_e(R_e|z)$$

Following the same procedure now leads to (15).

Appendix B: Existence

Here we derive some properties of $l(z)$ and use them to show that a monetary steady state exists iff i_n is not too big, assuming for simplicity a continuous joint density for (R_i, R_e) and $ER_j < \infty$. We claim first that $\lim_{z\to\infty} l(z) = 0$. Consider the generic case $\gamma > 0$ and $\theta < 1$, and begin by rewriting (13) as $l(z) = \alpha_e \sum_j I_j(z)$, where

$$\begin{aligned}
I_1(z) &\equiv \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \int_0^z zF_e' \left[H(R_i)|R_i\right] dF_i(R_i) \\
I_2(z) &\equiv -\tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \int_0^z R_i F_e' \left[H(R_i)|R_i\right] dF_i(R_i) \\
I_3(z) &\equiv -\int_0^z F_e \left[H(R_i)|R_i\right] dF_i(R_i) \\
I_4(z) &\equiv \int_0^z F_e \left[B(R_i)|R_i\right] dF_i(R_i).
\end{aligned}$$

We claim each $I_j(z) \to 0$ as $z \to \infty$.

Consider $I_1(z)$, and suppose that $\int_0^\infty zF_e' \left[H(R_i)|R_i\right] dF_i(R_i) \nrightarrow 0$ as $z \to \infty$. Making a change of variable using $R_e = \frac{z-R_i(1-\gamma+\theta\gamma)}{\gamma(1-\theta)} = H(R_i) = H$, this is equivalent to

$$\int_0^\infty \left[\gamma(1-\theta)H + R_i(1-\gamma+\gamma\theta)\right] F_e'(H|R_i) dF_i(R_i) \nrightarrow 0 \text{ as } H \to \infty.$$

Integrating with respect to H over $(0, \infty)$, this implies

$$\begin{aligned}
\infty &= \int_0^\infty \int_0^\infty \left[\gamma(1-\theta)H + R_i(1-\gamma+\gamma\theta)\right] F_e'(H|R_i) dF_i(R_i) dH \\
&= \gamma(1-\theta) \int_0^\infty \int_0^\infty H F_e'(H|R_i) dF_i(R_i) dH \\
&\quad + (1-\gamma+\gamma\theta) \int_0^\infty \int_0^\infty R_i F_e'(H|R_i) dF_i(R_i) dH
\end{aligned}$$

But this implies either $ER_e = \infty$ or $ER_i = \infty$, a contradiction. Hence $I_1(z) \to 0$ as $z \to \infty$. Similar arguments can be used to show $I_j(z) \to 0$ as $z \to \infty$, $j = 2, ...4$. Hence $\ell(z) \to 0$ as $z \to \infty$ when $\gamma > 0$ and $\theta < 1$.

The same basic approach works when $\gamma = 0$ and $\theta < 1$. Rewrite (14) as $\ell(z) = \sum_j I_j(z)$ where now $I_1(z) \equiv F_i'(z) \int_z^\infty R_e dF_e(R_e|z)$ and so on. For example, consider $I_1(z)$, and suppose $F_i'(z) \int_0^\infty R_e dF_e(R_e|z) \nrightarrow 0$. Integrating with respect to z, this implies the contradiction

$$ER_e = \int_0^\infty F_i'(z) \int_0^\infty R_e dF_e(R_e|z) dz = \infty.$$

Similar arguments show $I_j(z) \to 0$ as $z \to \infty$, $j = 2, ...4$. Hence $\ell(z) \to 0$ as $z \to \infty$ when $\gamma = 0$. The same basic approach works for $\theta = 1$. However, to ease the presentation somewhat, for the rest of the discussion we focus on the generic case $\gamma > 0$ and $\theta < 1$ and leave other cases as exercises.

The next thing we prove is that, in the generic case, $\ell(\underline{R}) = 0$ and $\ell(z) > 0$ for some z in the neighborhood of $\underline{R}$, where $\underline{R} = \inf\{R|F_i'(R)F_e'(R|R) > 0\}$. For this we assume that $\underline{R} < \infty$, since otherwise the DM shuts down. For the first result, notice that

$$\begin{aligned}
\ell(\underline{R}) &= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \int_0^{\underline{R}} (\underline{R} - R_i) F_e'[H(R_i)|R_i] dF_i(R_i) \\
&\quad - \int_0^{\underline{R}} \{F_e[H(R_i)|R_i] - F_e[B(R_i)|R_i]\} dF_i(R_i) \\
&= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} (\underline{R} - \underline{R}) F_e'[H(\underline{R})|\underline{R}] F_i'(\underline{R}) \\
&\quad - \{F_e[H(\underline{R})|\underline{R}] - F_e[B(\underline{R})|\underline{R}]\} F_i'(\underline{R}) = 0,
\end{aligned}$$

because $H(\underline{R}) = B(\underline{R}) = \underline{R}$ when $z = \underline{R}$.

Now consider

$$
\begin{aligned}
\ell'(\underline{R}) &= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \int_0^{\underline{R}} \left\{ F_e'[H(R_i)|R_i] + \tfrac{\underline{R}-R_i}{\gamma(1-\theta)} F_e''[H(R_i)|R_i] \right\} dF_i(R_i) \\
&\quad - \tfrac{1}{\gamma(1-\theta)} \int_0^{\underline{R}} \left\{ F_e'[H(R_i)|R_i] - \gamma F_e'[B(R_i)|R_i] \right\} dF_i(R_i) \\
&= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \left\{ F_e'(\underline{R}|\underline{R}) + \tfrac{\underline{R}-\underline{R}}{\gamma(1-\theta)} F_e''(\underline{R}|\underline{R}) \right\} F_i'(\underline{R}) \\
&\quad - \tfrac{1}{\gamma(1-\theta)} \left\{ F_e'(\underline{R}|\underline{R}) - \gamma F_e'(\underline{R}|\underline{R}) \right\} F_i'(\underline{R}) \\
&= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} F_e'(\underline{R}|\underline{R}) F_i'(\underline{R}) \left[1 - \gamma(1-\theta) \right].
\end{aligned}
$$

By definition of $\underline{R}$, $l'(\underline{R}+\varepsilon) > 0$ for some $\varepsilon > 0$. Hence, $\ell(z) > 0$ for some z near $\underline{R}$. The combination of the results in this Appendix, $\ell(z) > 0$ for z near $\underline{R}$ and $\lim_{z \to \infty} \ell(z) = 0$, tells us that for small i_n there always exists a solution to $\ell(z) = i_n$ with $\ell'(z) < 0$, and for big i_n there does not.

Appendix C: Lotteries

First, we verify that agents never use lotteries in the CM. Assume e pays p' to i in exchange for a lottery that gives e the idea with probability μ' (the possibility that e pays a random amount is easily ruled out as in Berentsen et al. 2002). The payoff to e is $\mu' W_e'(\hat{m}_e - p', R_e) + (1-\mu') W_e'(\hat{m}_e - p', 0)$ and the payoff to i is $\mu' W_i'(p', 0) + (1-\mu') W_i'(p', R_i)$, while the threat points are as before. Using the linearity of W_j', the analogue of (5) is:

$$
\max_{p', \mu'} \left(\mu' R_e - \phi' p' \right)^{\theta} \left(\phi' p' - \mu' R_i \right)^{1-\theta}
$$

Maximizing wrt p', we get $\phi' p' = \mu' \left[\theta R_i + (1-\theta) R_e \right]$. Using this, we can reduce the derivative wrt μ' to

$$(1-\theta)(R_e - R_i)(\mu' R_e - \phi' p').$$

As long as $R_e > R_i$ and $\mu' R_e > \phi' p'$, both of which are necessary for trade, this is strictly positive for all $\mu' > 0$. Hence, for a maximum $\mu' = 1$.

Returning to the DM, the next claim to verify is that profitable deals are never put on hold when we have lotteries. The usual calculation indicates that i puts the deal on hold iff $R_e > H(R_i)$, except that with lotteries we have $H(R_i) = \frac{\phi' \hat{m} - R_i \mu(1-\gamma+\theta\gamma)}{\mu\gamma(1-\theta)}$. Substituting μ from the bargaining solution into H, it is easy to show $R_e > R_i$ implies $R_e < H(R_i)$, establishing the claim.

Next we verify $\partial\Omega/\partial R_j < 0$, $j = i, e$. Considering i (the other case is symmetric), straightforward algebra yields $\partial\Omega/\partial R_i \simeq -c_1 R_e^2 - c_2 R_i R_e - c_3 R_i^2$, where $\simeq$ means "equal in sign" and c_1, c_2 and c_3 are functions of (θ, γ). One can show c_1, c_2 and c_3 are positive, the only tricky case being c_1, which is a complicated polynomial in θ and γ. Consider minimizing c_1 over (θ, γ). First we checked that $c_1 > 0$ on the boundary of $[0,1]^2$, then we checked that it is positive at every possible critical point in $[0,1]^2$. This establishes the claim.

Next we verify $\mu < 1$ iff $R_e > B(R_i)$. This actually follows easily from inspection of Figure 4. Suppose we fix R_i and increase R_e starting at $R_e = R_i$. Then we switch from $\mu = 1$ to $\mu < 1$ at some point, say $\tilde{R}_e = \tilde{R}_e(R_i)$. Since this is the same point at which switch from $p = [\theta R_i + (1-\theta)R_e] / \phi' < \hat{m}$ to $[\theta R_i + (1-\theta)R_e] / \phi' > \hat{m}$, we conclude that this point is $\tilde{R}_e(R_i) =$

$\frac{\phi\hat{m}-\theta R_i}{1-\theta}$, which tells us that $\tilde{R}_e = B(R_i)$.

Finally, we verify that $\partial\mu/\partial\theta > 0$ and $\partial\mu/\partial\gamma < 0$. The first derivative is simple, the second less so. It is easy to show $\partial\mu/\partial\gamma \simeq \Upsilon$, where

$$\Upsilon \equiv -(1-\gamma+2\gamma\theta)R_i^3+(1-3\gamma+6\gamma\theta)R_eR_i^2+(1+3\gamma-6\gamma\theta)R_e^2R_i-(1-\gamma+2\gamma\theta)R_e^3.$$

Notice that $\gamma = 0$ implies $\Upsilon < 0$. Can Υ ever be positive? Suppose we try to maximize it. Since $\partial\Upsilon/\partial\theta = 2\gamma(R_e - R_i)^3 > 0$, this means, as long as $\gamma > 0$ which is must be if we are to have any hope of $\Upsilon > 0$, we must set $\theta = 1$. Then $\partial\Upsilon/\partial\gamma = (R_e - R_i)^3(2\theta - 1)$, which is also positive given $\theta = 1$, and we must also set $\gamma = 1$. Hence, the unique maximum occurs at $\gamma = \theta = 1$, where $\Upsilon = -2R_i(R_e - R_i)^2 < 0$. This completes the argument.

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Graphs

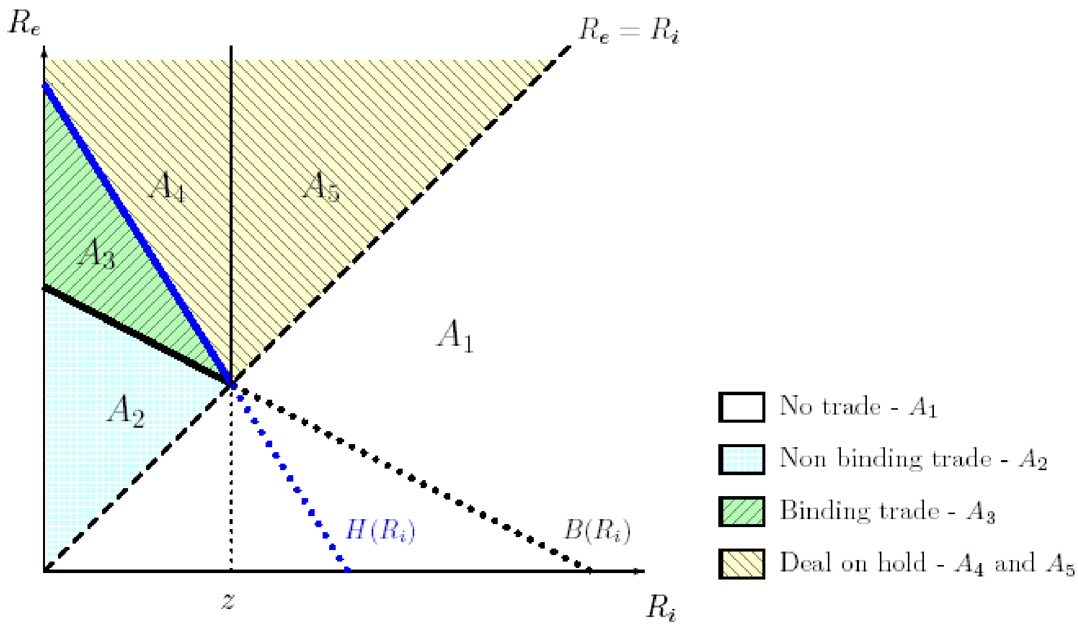

Figure 1: Meeting outcomes for (Ri,Re).

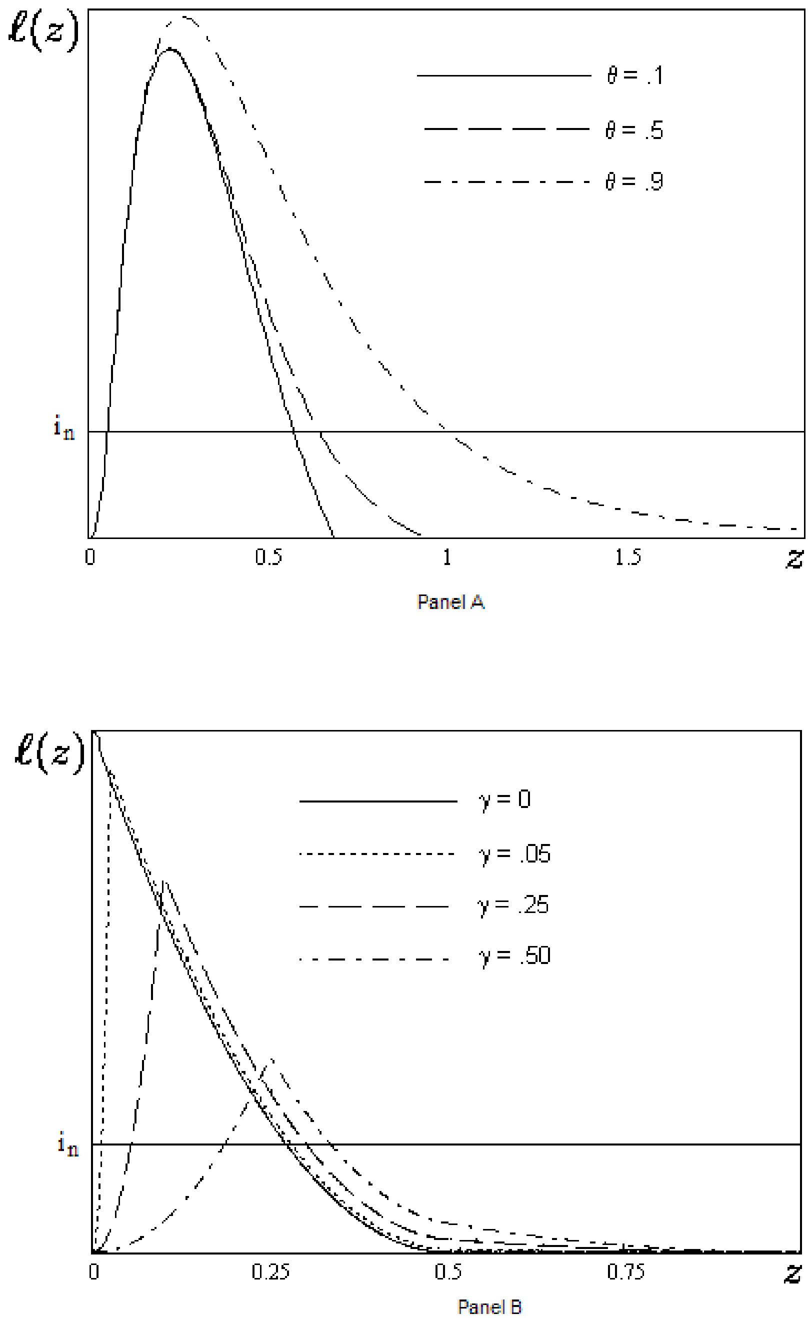

Figure 2: Plots for independent Lognormal and Uniform distribution.

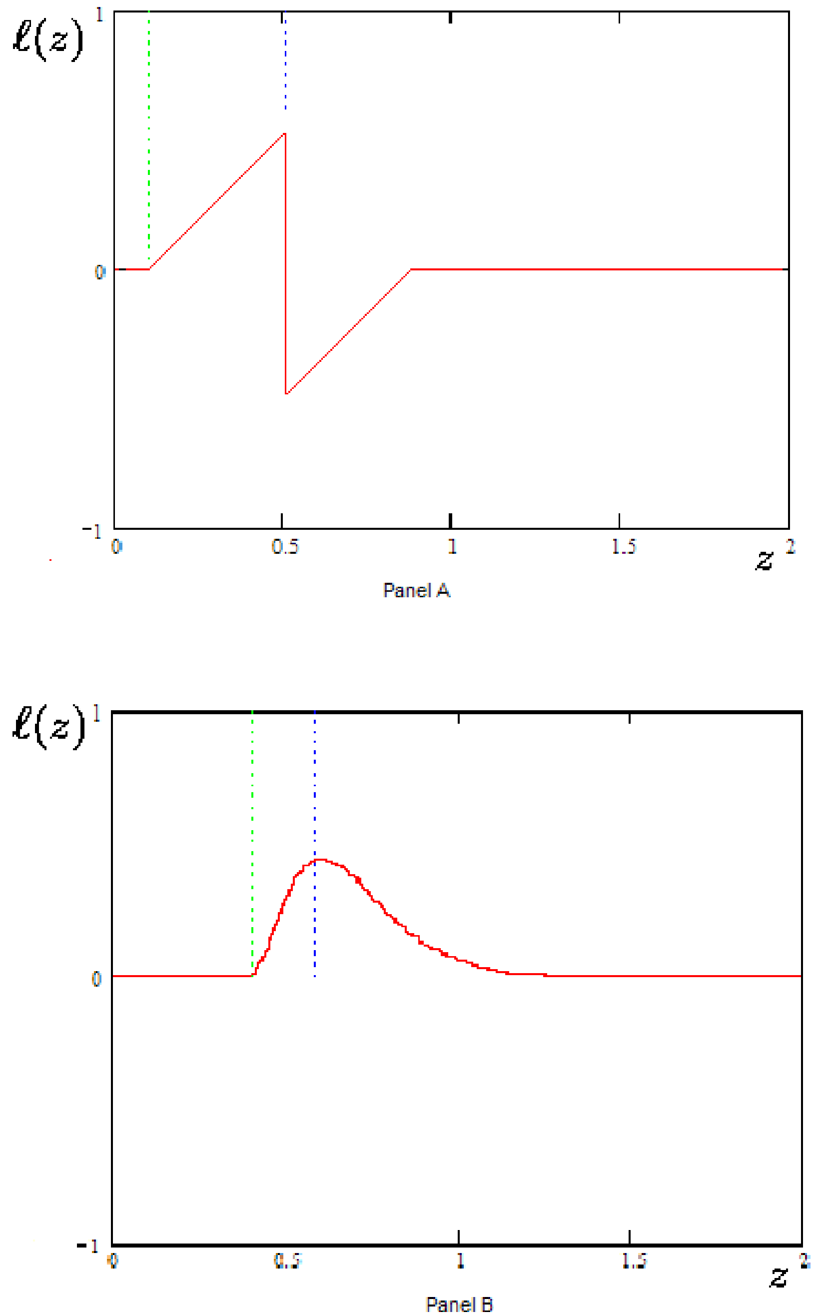

Figure 3: Plots for Re uniform and lognorm and Ri degenerate.

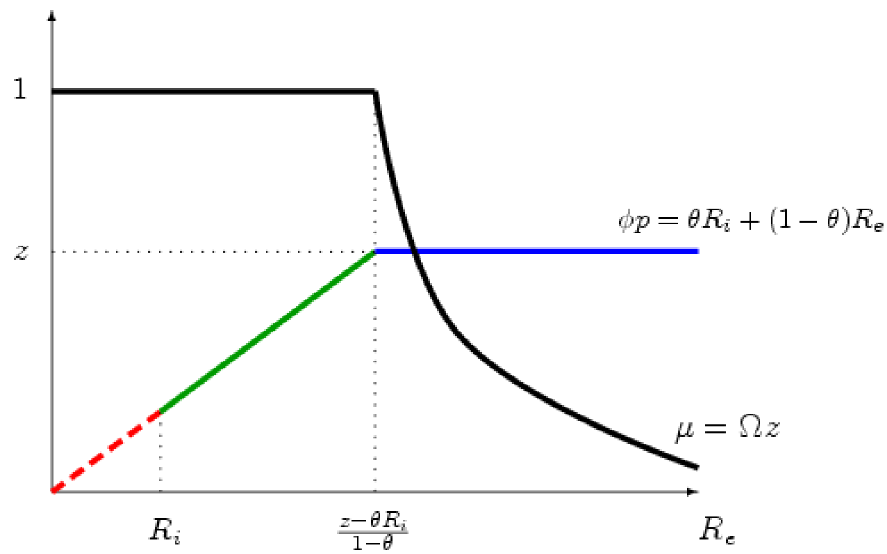

Figure 4: Lottery outcomes given z and Ri.

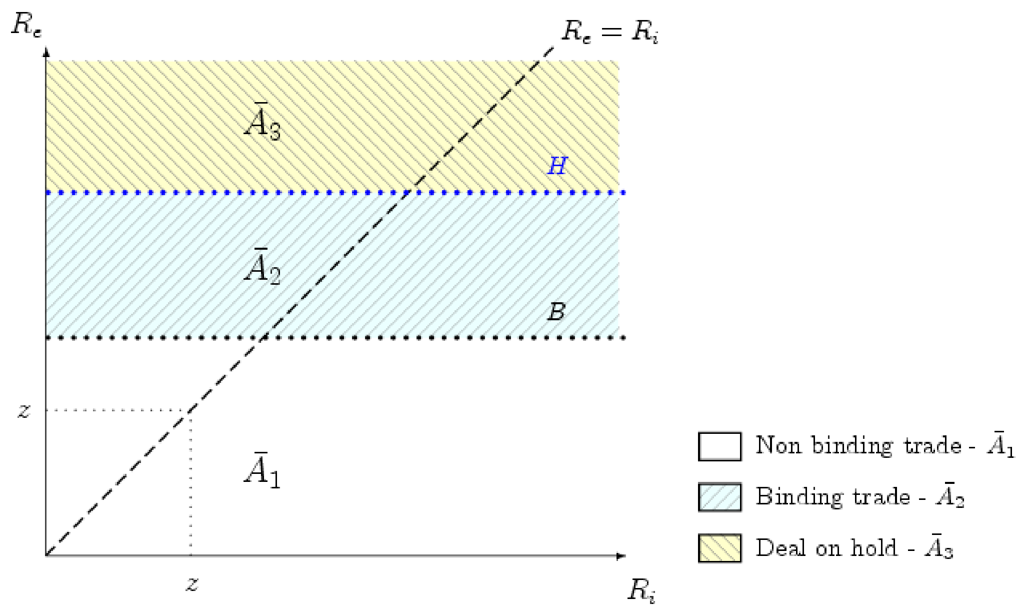

Figure 5: Meeting outcome for Non-rivalrous ideas.