Introduction

- Very nice paper.

- Historically low mortgage rates in the US has lead many homeowners to refinance their mortgages. The 30 mortgage rate in 1990 was 10.5% and by June 2003 it was at 5.2%. By the end of 2003 up to 40-45% of the $7 trillion single-family mortgages were finance in 2003 alone.

- Mortgages with prepayment options are generally concave.
  - There are limits on the gains as the mortgage rate decreases as the option becomes valuable.
  - The duration of a pool of passthrough mortgages decreases as borrowers prepay.

- If investors hedge this risk there may be an increase in interest rate volatility or fixed income implied volatilities.
A Tale of Three Theories

• Classical theory. Treasury markets are frictionless and complete and hedging activity has no effect. Predicts no affect on either options or realized volatility.

• Dynamic Hedging. As interest rates drop and borrowers prepay their mortgages, a dynamic hedger will buy long dated bonds to increase the duration of the portfolio. Prediction: An Increase in Refinancing will increase realized volatility in either the TBill or Swap markets. Related to the portfolio insurance literature.

• Static Hedging. As borrowers refinance and new mortgages are issued, lenders grant and embedded option to borrowers. This embedded option may be hedged using over-the-counter interest ratio options such as swaptions. If the supply of these contracts is inelastic then prices/implied volatility will increase as demand increases with interest rate drops. Prediction: Increased implied volatility but no increase in physical volatility.
**Main Contributions**

- Provides an extensive and very careful analysis of the theoretical relationship between prepayment and volatility.
  
  ★ MBS generally have negative convexity.
  
  ★ after 1998 investors started using Swaps rather than Treasuries (decrease in supply and flight to quality).
  
  ★ Many market participants contribute to hedging demands (including the GSE’s, hedge funds, mortgage servicers).

- Empirical contribution: demonstrates using a VAR model that increases in measures of mortgage repayment Granger cause increases in implied volatility.

- Modeling: develops a string market model in which factor volatilities depend on the level of mortgage refinancing.

- Show that model implied volatility from this model forecast future returns better than LSS.
My Questions

• Implied Volatility:

  ★ There are multiple potential causes of the increase in implied volatility:
    ★ 1998: repurchasing wave vs LTCM or questionable data.
    ★ 2001: repurchasing wave vs 9/11.

• The relationship between actual swap and treasury volatility could be explored in greater detail. If investors hedge dynamically then we would expect an increase in TBill or Swap volatility.

  ★ This predicts that volatility will increase as yields decrease while we typically model a positive level effect. At a minimum this model predicts an extremely nonlinear relationship.
  ★ This relationship is addressed indirectly using implied volatilities that come from a range of string market models.
The different behavior implied by the Implied Volatility and Realized Volatility theory could be further explored. At present this is only assessed indirectly.

In particularly, does higher refinancing result in higher realized volatility. This will be a somewhat vexing problem.

Perhaps the easiest approach would be to replicate the implied volatility VARs using realized volatility.
Realized Volatility Plots
This figure displays the Mortgage Bankers Association (MBA) refinancing index and the average Black's volatility of the swaptions with three months to maturity (VOL). The index is based on the number of applications for mortgage refinancing. The index is calculated every week and is based on the weekly survey of the MBA. The index is seasonally adjusted. This figure shows a series of spikes in refinancing activity. These spikes are refinancing waves caused by a drop in the mortgage rate to levels substantially below the current average coupon of the mortgage universe. The spikes in mortgage refinancing are generally accompanied by spikes in interest-rate volatility.
**Realized Volatility**

- Regress $\hat{\sigma}_t^2$ on a constant and two dummy variables Last quarter of 1998 and Q4:2001-Q4:2003.

<table>
<thead>
<tr>
<th>Data</th>
<th>1998</th>
<th>2001</th>
<th>$\log \hat{\sigma}_{t-1}^2$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBill</td>
<td>1.1893</td>
<td>2.9203</td>
<td>2.9263</td>
<td>0.1162</td>
</tr>
<tr>
<td>Swap Data</td>
<td>0.2838</td>
<td>3.1593</td>
<td>4.2782</td>
<td>0.1674</td>
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</tbody>
</table>

- Stochastic Volatility model.

- Regime Switching-GARCH-LEVEL model.
SV Model on 5 Yr Swap Data 1988-2004

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>-0.0374</td>
<td>-0.0376</td>
<td>-0.0578</td>
</tr>
<tr>
<td></td>
<td>( 0.0181)</td>
<td>( 0.0185)</td>
<td>( 0.0266)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.9934</td>
<td>0.9934</td>
<td>0.9899</td>
</tr>
<tr>
<td></td>
<td>( 0.0032)</td>
<td>( 0.0032)</td>
<td>( 0.0046)</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>0.0030</td>
<td>0.0030</td>
<td>0.0037</td>
</tr>
<tr>
<td></td>
<td>( 0.0014)</td>
<td>( 0.0014)</td>
<td>( 0.0018)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0096</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 0.1720)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>0.0030</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 0.0126)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>0.0063</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 0.0043)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>-9387.0484</td>
<td>-9387.0468</td>
<td>-9385.4919</td>
</tr>
</tbody>
</table>

$x_t = \log \text{Var}_{t-1}(y_t - y_{t-1}) = \omega + \phi x_{t-1} + 2 \log y_{t-1} + \delta_{1998,1998} + \delta_{2001,2001} + \eta_t$
Figure 3: Smoothed Probability of High-Volatility State $P(S_t = 1|\psi_T)$

Notes: The four panels contain the time series of the smoothed probabilities that the level factor is in the high-volatility regime at time $t$ according to the Regime Switching (RS) NO GARCH-NO LEVEL model, the RS-LEVEL model, the RS-GARCH model, and the RS-GARCH-LEVEL model. The smoothed probability is based on the entire sample: $P(S_t = 1|\psi_T)$. 
String Market Model

- LSS Model: the forward rates evolve as

\[ dF_{i,t} = \alpha_i F_{i,t} + \sigma_i F_{i,t} dZ_{i,t} \]

of alternatively

\[ dD_t = r Ddt + J^{-1} \sigma F_t dZ_t. \]

- The instantaneous correlation matrix of \( dF_{i,t}/F_{i,t} \) is given by \( H = U \Lambda U^\top \).

- The instantaneous covariance matrix of log-discount bond prices is given by \( \Sigma = U \Psi U' \) where \( \Psi_t \) is a diagonal matrix of eigenvalues and is fitted to a cross-section of swaption implied vols.

- This model has constant correlations (captured by the Eigenvectors in \( U \)) which are estimated historically using the forward rates and held fixed.
String Market Model

- LSS find that four factors are needed, i.e. only the first four diagonal elements are $\Psi_t$ are nonzero. The interpretation is standard: Level, Slope, Curvature and the fourth factor affects the short end of the curve. Interestingly, this fourth factor explained the aberrant behavior around the LTCM crisis.

- There is an inconsistent treatment of volatility.
  - When calculating option prices by simulation the eigenvalues are held fixed (homoskedasticity).
  - BUT: the eigenvalues must vary through time to fit the model from period to period (heteroskedasticity).
Extensions

- Generalizing the time-varying homoskedasticity model has potential added value in two dimensions: 1) more flexible functional form with more degrees of freedom to match IV 2) internally consistent volatility dynamics.

- Duarte presents two extensions:
  - CEV: $\Psi^{(i,i)}_t = \lambda_i y_t^{\beta_i}$.
  - Refinancing: $\Psi^{(i,i)}_t = \lambda_i CPR_t^{\beta_i}$.

- Finds that only three factors are needed. How does this relate to LSS 4 factor model?
Extensions

- One concern is overfitting which may show up as parameter instability or poor out-of-sample performance.
  - This is examined by studying out-of-sample forecasts with 1 and 3 month horizons.
  - Duarte compares the out-of-sample forecasts of realized volatility based on fitted implied volatilities from the various model.
  - The string model with refinancing out-performs the other models.
  - The CEV specification has the same number of parameters as the model with refinancing and has the same number of parameters. Acts as a control against overfitting.
  - It would be nice to compare also the fit when the parameters are estimated once and held fixed through the entire sample. Overcomes scale differences problem.
Extensions

* Conditional heteroskedasticity is driven in both models by observable variables. These models are a nice counterpoint to Han’s (2005) stochastic volatility model, where the factor variances evolve as latent processes:

\[ d\lambda_{i,t} = \kappa_i(\theta_i - \lambda_{i,t})dt + \sigma_i\sqrt{\lambda_{i,t}}dW_{it} \]

where the \( E(dW_{it}dZ_{jt}) = 0 \) (unspanned stochastic volatility). All parameters are held fixed through time. The model performs well.