Financial Development and Macroeconomic Stability

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VERY PRELIMINARY AND INCOMPLETE

Abstract

The volatility of the US business cycle has declined during the last two decades. During the same period the financial structure of firms has become more volatile. In this paper we develop a model in which financial factors play a central role in generating economic recessions. Financial markers innovations allow for greater financial flexibility and lead to a more volatile financial structure of firms but a lower volatility of output.

1 Introduction

Recent studies have emphasized two features of the U.S. business cycle: asymmetry and declining volatility. The first feature is that macroeconomic downturns are characterized by sharp downfall and quick recoveries, while expansions are more gradual and long lasting. The second feature is the declined volatility of the US business cycle in the last 20 years. The asymmetry and milder volatility is observed in all major macroeconomic variables including consumption, investment and employment. In this paper we provide a financial explanation for these business cycle features. Our theory is motivated by the observation of two empirical regularities about the dynamics of the financial structure of firms. First, the value of debt in the business sector drastically drops during a recession. This suggests that recessions are periods in which firms must restructure their financial position. If firms cannot compensate the debt reduction with new equity, it must cut investments and this generates sharper recessions.

The second regularity is that the change in debt and the issue of new equity in the business sector has become more volatile during the last two decades. Because the change in debt and the issue of new equity is negatively correlated, these findings suggest that firms have become more flexible in the choice of their financial structure, perhaps as a consequence of innovations in financial markets. This greater flexibility is, in our theoretical framework, the driving force for the milder business cycle.

We develop a model in which firms finance investments with equity and debt. Debt contracts are not fully enforceable and the ability to borrow is limited by a non-default constraint, which depends on the expected lifetime profitability of the firm. Because the firm looses the market rents after defaulting, the incentive to repay is determined by the loss of these rents which change over the business cycle. In this respect our model is similar to Kiyotaki & Moore (1997). However, our model differs in one important dimension: we allow firms to issue new equity in addition to reinvesting profits. This extra margin plays a central role in our paper. More specifically, it is the greater flexibility in issuing new equity that generates a milder business cycle.

2 Real and financial cycles in the U.S.

The top panel of Figure 1 plots the real nonfarm business output. Each of the five major recessions—outlined by the shaded areas—are characterized by sharp output fall after a phase of relatively stable expansion. The output falls are followed by quick recoveries. In general, expansionary phases are relatively smoother and longer than recessions. This pattern is also observed for the multifactor productivity as shown in the bottom panel of Figure 1. Data on multifactor productivity is released annually by the Bureau of Labor Statistics. To construct the graph in quarterly frequency, we have attributed the yearly value to each sub-quarter. The asymmetry of the business cycle has been emphasized in several empirical studies but there is not an established





Figure 1: Output and multifactor productivity in nonfarm business sector.

Another important pattern shown by Figure 1 is the reduction in output volatility during the last 20 years. Also this fact has received considerable attention in business cycle studies but the sources of the lower volatility are still under investigation.

The goal of this paper is to study the role of financial market frictions in generating the business cycle asymmetry and the lower volatility. Our study is motivated by some stylized facts about the dynamics of the financial structure of firms shown in Figure 2. Data is from the Flow of Funds. The top panel of Figure 2 plots the credit market liabilities, as a fraction of GDP, in the nonfarm business sector. This variable includes only liabilities that are directly related to credit markets instruments. It does not include, for instance, tax liabilities. We refer to this variable as 'outstanding debt'. The shaded areas outline five of the major recessions experienced by the U.S. economy as we did in Figure 1.



Figure 2: Financial structure in the nonfarm business sector.

There are two important patterns to emphasize. The first pattern is that the outstanding debt of nonfarm business companies has increased during the last 50 years. In the early fifties this ratio was only 35 percent while in 2005 it has reached the level of about 85 percent.

The second pattern is the increased volatility of debt. While the debtoutput ratio has been growing at a relatively stable phase during the fifties and sixties, in the last three decades it has displayed large oscillations around the trend, with three major picks: 1975, 1991, 2001. Each of the three picks coincides with a major recession. After the recession, the debt exposure declines drastically. During this period (last three decades of the sample) there are other two major recessions: at the end of the 1970s and at the beginning of the 1980s. These two recessions, however, took place when the debt exposure was already low, relative to the trend, which may explain why the drop in outstanding debt has been relatively small.

The cyclical pattern of debt suggests that recessions lead firms to restructure their financial exposure and the magnitude of the restructuring is particularly severe when the debt exposure is high.

Debt is only one of the sources of financing for the firm. Another important source is equity. The bottom panel of Figure 2 plots the change in the value of debt and the issue of new equities. Both variables are in percentage of GDP.

Also this figure outlines two important facts. The first is that both variables have become more volatile during the last two decades. The second fact is that the issue of new equity is strongly negatively correlated with the change in the value of debt. The correlation coefficient is -0.6. Therefore, the greater volatility of debt has been accompanied by a greater volatility of firm's equity. This suggests that firms have become more flexible in the two main sources of financing: debt and equity.

In summary, the main facts outlined in Figures 1 and 2 are the following:

- 1. Business cycle asymmetry and lower volatility. Downturns are characterized by sharp falls in output and productivity followed by rapid recoveries while booms are more gradual and long-lasting. The magnitude of recessions has declined during the last two decades.
- 2. Greater debt exposure and more volatile leverage. Business leverage shows an upward trend during the last fifty years and it has become more volatile during the last three decades. The leverage picks right before or in the middle of a major recession.
- 3. *Pro-cyclicality and greater volatility of equity issue*. The issue of new equity increases during recessions and decreases during expansions. The cyclical movements have increased during the last two decades.
- 4. Greater substitution between debt and equity. Changes in the value of

debt is negatively correlated with the issue of new equity. The substitution has increased during the last two decades.

The first set of facts have been emphasized in several empirical papers and are well-known. The others are less known and explored in the macro literature. In the next section we present a model with financial market frictions that is consistent with these facts.

3 Changes in the financial sector

Collateralization: Some recent financial market developments have made it easier for firms to pledge their assets to lenders, that is, to relax their collateral constraint. As a major financial innovation, Asset Backed Securities (ABS) created through the process of securitization have become an effective way of debt collateralization. Securitization began in the late 70s as a way to finance residential mortgages. By the second half of the 80s, securitization was used for automobiles, manufactured housing and equipment leasing, as well as for credit cards. Growth has been fast since then and ABS are today an important component of firm financing. According to the The Bond Market Association (2004), ABS issuance overtook the issuance of long term corporate bonds in the third quarter of 2004.

Cost of issuing equity: Kim, Palia, & Saunders (2003) provide some evidence about the behavior of underwriting cost for new equity issues. They show that underwriting spreads for seasoned equity offerings have been on average decreasing during the period 1980-2000. Making the simple comparison between the mean spreads in 1980 and 2000 shows that spreads have decreased by 20 percent over this period.

One of the changes that has contributed to lower the cost of new issues is the ability to make 'shelf' offerings under Rule 415. This was introduced in 1983. Under a shelf offering a firm can issue at short notice, up to a given limit, during a period of 2 years. The study by Bhagat, Marr, & Thompson (1985) found that this additional flexibility had allowed firms to lower offering costs by 13 percent in syndicated issues and 51 percent in non-syndicated issues.

4 Model

Production and market structure: There is a continuum of intermediate goods x_i , indexed in the [0, 1] interval. Intermediate goods are used as inputs in the production of final goods Y according to:

$$Y = \left(\int_0^1 x_i^\eta \mathrm{d}i\right)^{\frac{1}{\eta}}$$

Each intermediate good is controlled by a single firm that acts as a monopolist. Given the production technology for the final goods, the inverse demand function of intermediate good i is:

$$p_i = Y^{1-\eta} x_i^{\eta-1}$$

where p_i is the price in units of the final good and $1/(1-\eta)$ is the elasticity of demand.

The intermediate good is produced with the inputs of capital, k_i , and labor, l_i , according to:

$$x_i = F(k_i, l_i) = A(k_i^{\theta} l_i^{1-\theta})^{\nu}$$

where A is for the moment a constant and ν determines the return to scale. We allows for increasing returns, that is, $\nu > 1$. This captures, in simple form, the presence of fix factors and variable capacity utilization. Capital depreciates at rate δ .

Given w the wage rate, the resources of a firm i after production and the payment of wages can be written as:

$$(1-\delta)k_i + Y^{1-\eta}F(k_i, l_i)^{\eta} - wl_i$$

where the term $Y^{1-\eta}F(k_i, l_i)^{\eta}$ is the monopoly revenue $p_i x_i$ after substituting the demand and production functions.

We assume that $\eta\nu < 1$ so that the revenue function is strictly concave in the output of the firm. In a symmetric equilibrium, $k_i = K$ and $l_i = L$ for all firms, and therefore, $Y = F(K, L) = A(K^{\theta}L^{1-\theta})^{\nu}$. This implies that in aggregate the revenue function is homogenous of degree ν .

Before proceeding, it will be convenient to derive the optimal input of labor. This solves the first order condition $\eta Y^{1-\eta}F(k_i, l_i)^{\eta-1}F_l(k_i, l_i) = w$. This condition defines the input of labor as a function of Y, w and k_i , that is, $l(Y, w, k_i)$. We can then express the firm's resources (net of wages) as a function of these three variables, that is:

$$\pi(Y, w, k_i) = (1 - \delta)k_i + Y^{1 - \eta} F(k_i, l(Y, w, k_i))^{\eta} - wl(Y, w, k_i)$$

Firms and finance: There is a continuum of firms. Each firm produces an intermediate input x_i and acts as a monopolist in that particular market. The firm retains the control of its market with probability p. Because there is a continuum of firms, the law of large numbers implies that in each period a fraction 1 - p of firms lose their markets. These markets are acquired by new firms. The probability p is stochastic and follows a first order Markov process with transition probability $\Gamma(p'/p)$. The change in p is the only source of uncertainty in the model. It is the change in p that affects the market valuation of a firm, and therefore, changes in asset prices.

Firms maximize the expected discounted value of dividends, that is,

$$E_0 \sum_{t=0}^{\infty} \alpha_t d_{i,t}$$

where α_t is the firm's discount factor *t*-periods ahead. This is derived from the maximization of the shareholders' welfare as specified below.

Firms finance the input of capital with equity and debt. The issue of new equities is costly. Given $n_{i,t}$ the new equities, the issuing cost is $\kappa n_{i,t}^2$. The convexity of this cost is consistent with the work of Hansen & Torregrosa (1992) and Altinkilic & Hansen (2000). These studies show that underwriting fees paid by corporations for seasoned equity offers display increasing marginal cost in the size of the offering. A broader definition of the cost of new equity issues would also include other considerations that are likely to depend on the size of the issue.¹

Given the issuing cost, the budget constraint of the firm is:

$$\pi(Y_t, w_t, k_{i,t}) + R_t b_{i,t+1} + n_{i,t} - \kappa n_{i,t}^2 = b_{i,t} + d_{i,t} + k_{i,t+1}$$

The firm starts with resources $\pi(Y_t, w_t, k_{i,t})$ to which it adds the funds raised with the new debt, $R_t b_{i,t+1}$, and the new equity (net of the issuing cost). The

¹For instance, to the extent that seasoned equity offerings are interpreted as signalling managers and shareholders the belief that shares are currently overvalued, increasing marginal cost seems a reasonable conjecture. Alternatively, to the extent that larger issues are more risky for the issuer, the cost might be increasing in the size of the issue.

total resources are then used to pay back the previous debt, to distribute dividends and to finance the new capital.

In order to create a peaking order in the choice of equity and debt, we assume that interest payments are tax deductable from the earnings of the firm. Given r_t the interest rate and τ the tax rate, the effective cost of debt is $r_t(1-\tau)$. Hence, the present value of one unit of debt is $R_t = 1/[1+r_t(1-\tau)]$. We adopt this simple specification of the tax system to limit the notational complexity of the model.

Because new equities are costly, the firm would not raise equities when it distributes dividends. Therefore, choosing a positive value of $n_{i,t}$ is equivalent to have negative dividends. We can then rewrite the budget constraint compactly as follows:

$$\pi(Y_t, w_t, k_{i,t}) + R_t b_{i,t+1} = b_{i,t} + \varphi(d_{i,t}) + k_{i,t+1}$$

where the function φ is defined as:

$$\varphi(d_{i,t}) = \begin{cases} d_{i,t} & \text{if } d_{i,t} \ge 0\\ \\ d_{i,t} + \kappa \cdot d_{i,t}^2 & \text{if } d_{i,t} < 0 \end{cases}$$

If dividends are positive, the resources of the firm are reduced by $d_{i,t}$. If the dividends are negative, implying that the firm raises new equities, the increase in the firm's resources is net of the cost $\kappa d_{i,t}^2$.

The parameter κ captures the degree of market incompleteness. When $\kappa = 0$ the economy is equivalent to a frictionless set-up. In the polar case in which $\kappa = \infty$, dividends cannot be negative and the only way to increase equities is by reinvesting profits. With $\kappa = \infty$ our framework is similar to other models such as Kiyotaki & Moore (1997). We will see later, however, that the ability to issue new equity changes the dynamic properties of the model and will be key to capture the stylized facts described earlier.

The ability to borrow is bounded by the limited enforceability of debt contracts. Once the firm has received the loan, it can repudiate the debt and distribute part of the firm's resources to its shareholders. Let ϕ be the fraction of the debt that can be recovered by the lender if the firm defaults. Therefore, the value of defaulting is $k_{i,t+1} - \phi b_{i,t+1}$. We are assuming that the firm looses the market for the intermediate good after defaulting.

Incentive-compatibility requires,

$$E_t \sum_{j=1}^{\infty} \alpha_{t+j} d_{i,t+j} \ge k_{i,t+1} - \phi b_{i,t+1}$$

that is, the value of the firm can not be smaller than the value of defaulting. The important feature of the model is that the borrowing limit is endogenous and depends on the valuation of the firm. More specifically, the borrowing limit is relaxed when the value of the firm (the left-hand-side term) increases. The survival probability p plays a crucial role in the determination of this value because it affects the discount factor α_i .

The last assumption we make is that there is no cost to transfer funds from one firm to the other. This implies that, when a firm is liquidated and a new firm is created, the funds of the exiting firm can be transfer without any cost to the new firm. This assumption greatly simplifies the analysis because it insures that all firms are alike (representative firm). The alternative would be to assume that new firms are also subject to the cost of issuing new equity. In this case they would experience a transition period and there would be a heterogeneous distribution of firms which, however, should not change the main properties of the model.

Households: We close the model by specifying the household sector which is populated by a continuum of homogeneous consumers with lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$$

where c_t is consumption, h_t is labor and β is the intertemporal discount rate. Households are the owners (shareholders) of the firms. In addition to equity they also own non-contingent bonds. The households' budget constraint is:

$$w_t h_t + b_t + s_t (\bar{d}_t + p_t q_t) = \frac{b_{t+1}}{1 + r_t} + s_{t+1} q_t + c_t$$

where w_t and r_t are the wage and interest rates, b_t is the bond (inclusive of the interest), s_t the equity shares, \bar{d}_t the dividend payments from each share and q_t is the market price of equity for surviving firms. Each household owns a diversified portfolio. The payments of this portfolio, \bar{d}_t , include the liquidation value of exiting firms. Because only a fraction p_t survives to the next period, the price q_t is multiplied by this probability.

The first order conditions with respect to labor, h_t , next period bonds, b_{t+1} , and next period shares, s_{t+1} , are:

$$w_t U_c(c_t, h_t) + U_h(c_t, h_t) = 0$$
(1)

$$U_c(c_t, h_t) - \beta(1+r_t)EU_c(c_{t+1}, h_{t+1}) = 0$$
(2)

$$U_c(c_t, h_t)q_t - \beta E(\bar{d}_{t+1} + p_t q_{t+1})U_c(c_{t+1}, h_{t+1}) = 0$$
(3)

These are standard optimizing conditions for the household's problem. The first two conditions are important for determining the supply of labor and the risk-free interest rate. The last condition determines the market price of equity. After re-arranging and using forward substitution, the price of equity can be written as:

$$q_{t} = E_{t} \left(\sum_{j=1}^{\infty} \frac{(\prod_{\iota=0}^{j-2} p_{t+\iota}) \cdot \beta^{j} \cdot U_{c}(c_{t+j}, h_{t+j})}{U_{c}(c_{t}, h_{t})} \right) \bar{d}_{t+j}$$

Firms' optimization is consistent with households' optimization. Therefore, conditional on survival, the discount factor used by firms is:

$$\alpha_{t+j} = \frac{(\prod_{\iota=0}^{j-2} p_{t+\iota}) \cdot \beta^j \cdot U_c(c_{t+j}, h_{t+j})}{U_c(c_t, h_t)}$$

This expression makes clear that a persistent fall in the survival probability p affects negatively the market value of shares.

5 Recursive problem and equilibrium

We write the maximization problem of the firm recursively. Let $\alpha(\mathbf{s}, \mathbf{s}') = \beta U_c(c', h')/U_c(c, h)$ be the one-period ahead discount factor. Conditional on survival, the firm chooses d, k', b' to maximize:

$$V(\mathbf{s};k,b) = \max_{d,k',b'} \left\{ d + E\alpha(\mathbf{s},\mathbf{s}')\overline{V}(\mathbf{s}';k',b') \right\}$$
(4)

subject to:

$$\pi(Y, w, k) + Rb' - b - \varphi(d) - k' = 0$$
$$E\alpha(\mathbf{s}, \mathbf{s}')\overline{V}(\mathbf{s}'; k', b') \ge k' - \phi \cdot b'$$

The function $V(\mathbf{s}; k, b)$ is the value of the firm conditional on survival while the function $\overline{V}(\mathbf{s}; k, b)$ is the value before knowing the survival shock. This is given by:

$$\overline{V}(\mathbf{s};k,b) = p \cdot V(\mathbf{s};k,b) + (1-p) \cdot \left[\pi(Y,w,k) - b\right]$$
(5)

Both functions depend on the aggregate states \mathbf{s} . These are given by the survival probability p, the aggregate capital K, and the aggregate bonds B. The optimization problem is subject to two constraints: the budget constraint and the enforcement constraint. The firm takes as given the wage rate w, the interest rate r (and R), the aggregate production Y, the discount factor $\alpha(\mathbf{s}, \mathbf{s}')$ and the law of motion for the aggregate states, $\mathbf{s}' = H(\mathbf{s})$.

The first order conditions are:

$$(1+\mu)E\alpha(\mathbf{s},\mathbf{s}')\overline{V}_{k'}(\mathbf{s}';k',b') - \frac{1}{\varphi'(d)} - \mu = 0$$
(6)

$$(1+\mu)E\alpha(\mathbf{s},\mathbf{s}')\overline{V}_{b'}(\mathbf{s}';k',b') + \frac{R}{\varphi'(d)} + \phi\mu = 0$$
(7)

and the envelope conditions are:

$$\overline{V}_k(\mathbf{s};k,b) = \pi_k(Y,w,k) \left[\frac{p}{\varphi'(d)} + 1 - p\right]$$
(8)

$$\overline{V}_b(\mathbf{s};k,b) = -\left[\frac{p}{\varphi'(d)} + 1 - p\right]$$
(9)

where μ is the Lagrange multiplier for the non-default constraint and the term $\varphi'(d)$ is 1 if the firm pays dividends and smaller than 1 if the firm issues new equities. See the appendix for the detailed derivation of the first order and envelope conditions.

We can now provide the definition of a recursive general equilibrium. The sufficient set of aggregate states are given by the survival probability p, the aggregate capital K, and the aggregate bonds B.

Definition 5.1 (Recursive equilibrium) A recursive competitive equilibrium is defined as a set of functions for (i) households' policies $c(\mathbf{s})$, $h(\mathbf{s})$; (ii) firms' policies $d(\mathbf{s}; k, b)$, $k(\mathbf{s}; k, b)$ and $b(\mathbf{s}; k, b)$; (iii) firms' value function $V(\mathbf{s}; k, b)$; (iv) aggregate prices $w(\mathbf{s})$ and $r(\mathbf{s})$. Such that: (i) household's

policies satisfy the optimality conditions (1)-(2); (ii) firms' policy are optimal and $V(\mathbf{s}; k, b)$ satisfies the Bellman's equation (4); (iii) the wage and interest rates are the equilibrium clearing price in the labor and bond markets; (iv) the law of motion $H(\mathbf{s})$ is consistent with individual decisions and the stochastic process for p.

6 Dynamic Properties

To illustrate some of the properties of the model, it will be convenient to consider the simple case in which the economy is affected by an unexpected and permanent change in the survival probability. Suppose that the economy is in a steady state equilibrium with a non-stochastic \bar{p} . At time t, the survival probability switches unexpectedly and permanently to either $p^L < \bar{p}$ or $p^H > \bar{p}$. There is no uncertainty after switching. We have the following property:

Property 1 The no-default constraint is always binding.

This property can be shown as follows. Substituting the envelope condition (9) and using $\alpha(\mathbf{s}, \mathbf{s}') = 1/(1+r)$, equation (7) can be written as:

$$(1+\mu)\left[\frac{p}{\varphi'(d')} + 1 - p\right]\varphi'(d) = (1+r)R + (1+r)\phi\mu$$
(10)

Let's observe first that $\varphi'(d) \leq \varphi'(d') \leq 1$. In fact, if the firm pays dividends in the current period, the persistence of the shock implies that the firm will also pay dividends in the next period. Therefore, $\varphi'(d) = \varphi'(d') = 1$. Let's consider now the case in which the firm does not pay dividends in the current period, and therefore, $\varphi'(d) < 1$. By retaining earnings the firm increases the next period equity. Therefore, the marginal value of equity cannot be higher in the next period, that is, $\varphi'(d) \leq \varphi'(d')$. Because (1+r)R > 1, the fact that $\varphi'(d) \leq \varphi'(d') \leq 1$ implies that condition (10) is satisfied only if $\mu > 0$.

This result has a simple intuition. The firm would like to reduce equity by paying dividends and finance capital with debt. The no-default constraint, however, will prevent the firm from excessive borrowing. This result does not necessarily hold when future values of p are stochastic. In general, however, it will hold if the tax rate τ is sufficiently large and p highly persistent.

Property 2 If $\kappa = 0$, aggregate investment and output are not affected by changes in p.

To show this, suppose that current and future values of output Y, wages w, hours h and households' consumption c, are not affected by p. Because $\alpha = \beta U_c(c', h')/U_c(c, h)$ and $\beta(1+r)EU_c(c', h')/U_c(c, h) = 1$, the constancy of consumption implies that the discount factor and the interest rate are constant and equal to $\alpha = \beta$ and $r = 1/\beta - 1$. Taking into account that $\varphi'(d)$ is always equal to 1 when $\kappa = 0$, condition (6) can be written as:

$$\pi_k(Y', w', k') = 1 + r \tag{11}$$

The constancy of r then implies that k' must also be constant. This confirms our guess that output, wages, hours and consumption are not affect by changes in p.

The economy with $\kappa = 0$ is equivalent to the complete market economy in which production is only affected by shocks to productivity (which we have ignored so far). When the ability to borrow falls, the firm simply replaces debt with equity and keeps investment at the optimal level. When κ is positive, however, new equity are costly and the firm replaces the debt only partially. Therefore, our asset prices shocks affect the real sector of the economy only if there are financial market frictions. The following properties characterizes the economy with financial market frictions, that is, $\kappa > 0$.

Property 3 The response of capital is asymmetric. It does not change after a permanent switch to p^H but it temporarily falls after a switch to p^L .

To show this, let's compare the case in which the survival probability increases to the case in which it falls.

Stock market boom: Suppose that there is a permanent switch to p^H . Because of the higher survival, the value of the firm increases. If the firm does not distribute dividends, the non-default constraint would not be binding. But we have seen above that this constraint is always binding. Therefore, the firm must distribute dividends in both the current and future periods. This implies that $\varphi'(d) = \varphi'(d') = 1$ and condition (6) can be written as in (11). The optimality condition before the shock takes the same form because in the steady state the firm pays dividends and $\varphi'(d) = 1$. However, the interest rate r may differ. We want to show now that r is the same before and after the shock. We show this by showing that all the optimality conditions are satisfied if r does not change.

Suppose that current and future output Y, wages w, hours h, and households' consumption c, are not affected by the change in p. Because $\alpha = \beta U_c(c',h')/U_c(c,h)$ and $\beta(1+r)EU_c(c',h')/U_c(c,h) = 1$, the constancy of consumption and labor implies that the discount factor and the interest rate are constant and equal to $\alpha = \beta$ and $r = 1/\beta - 1$. Therefore, the interest rate r does not change. Taking into account that $\varphi'(d) = 1$, equations (7) and (9) imply that $1 + \mu = (1+r)R + (1+r)\phi\mu$. Because r stays constant, then condition (11) implies that the capital does not change, that is, k' = k. This confirms our guess that output, wages, hours and consumption are not affected by an increase in p.

Stock market crash: Let's consider now the case of a permanent switch to p^L . This leads to a fall in the value of the firm. If the default constraint was binding (as established above), then b and k must decrease if the cut in dividends is not sufficient to maintain the same input of capital. This will be the case when the fall in the value of the firm is large. To see this, consider the no-default constraint:

$$\alpha \overline{V}(\mathbf{s}';k',b') \ge k' - \phi b'$$

This constraint was satisfied with the equality sign in the previous period when $\overline{V}(\mathbf{s}'; k', b')$ was higher. Suppose that, after the fall in $\overline{V}(\mathbf{s}'; k', b')$, the firm maintains the same capital, k' = k, and the same debt, b' = b. Because the constraint was satisfied with the equality sign in the previous period, the constraint will be violated in the current period, unless b' is reduced. The reduction in b' can be compensated by a reduction in d. If the fall in pis large, this requires negative values of d, that is, the issue of new equity. However, the convexity of the issuing cost imposes a limit to the use of this margin. Once the firm has reached this limit, it must reduce investment. The reduction in investment induces a change in all other variables including consumption, hours worked, interest rate and wages.

To summarize, there is an asymmetry in the response of the economy to large changes in asset prices. After an asset price drop the firm cannot reduce the dividends below zero and the increase in equity cannot fully compensate the reduction in debt. As a result, the firm must cut investments. After an asset price boom, instead, the firm simply replaces equity with debt (by distributing more dividends).

It is important to emphasize that the asymmetry described above and more in general the impact of p on the real sector of the economy, derive from the particular structure of the cost of issuing equity: the firm incurs a cost when it increases equity but there is no cost in reducing equity.

7 Quantitative properties

In this section we use a parameterized version of the model to show how the economy responds to shocks and how the responses change with innovations in the financial system.

Parameterization: Assuming that a period is a quarter, we assign the following parameter values. The discount rate is set to $\beta = 0.99$. The utility function takes the form $U(c_t, h_t) = \ln(c_t) - Bh_t$ with the parameter B such that the steady state value of h is 0.33.

The production function is parameterized as follows. We normalize A to 1 and set the return to scale parameter $\nu = 1.45$. The parameter θ is assigned the value of 0.37 so that the capital income share is 0.4. Capital depreciates at rate $\delta = 0.02$ and the fraction of debt that can be recovered in case of default is $\phi = 0.25$. The elasticity parameter is set to $\eta = 0.655$ so that the return to scale in the revenues of the firm is $\nu \eta = 0.95$. The implied elasticity of demand for the firm is 2.9.

The probability of market survival can take two values— $p^L = 0.91$ and $p^H = 0.99$ —with transition probabilities:

$$\Gamma(p'/p) = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

Finally, the cost parameter for new equities is set to $\kappa = 1$ and the tax rate to $\tau = 0.3$. The full set of parameter values are reported in Table 1.

Response to shocks: Suppose that the probability of market survival has been $p^H = 0.99$ for a long period of time and the economy has converged to the long-term equilibrium. Starting from this equilibrium, the market

Table 1: Parameter values.

Discount factor	$\beta = 0.99$
Utility parameter	B = 2.35
Production technology	$A=1, \ \theta=0.368, \ \nu=1.45, \ \delta=0.02$
Elasticity parameter	$\eta = 0.655$
Market survival	$p^L = 0.91, \ p^H = 0.99, \ \Gamma(p/p) = 0.9$
Recoverable debt	$\phi = 0.25$
Cost of new equities	$\kappa = 1.0$
Tax rate	au = 0.3

survival drops to the lower level $p^L = 0.91$ and stays at this level for several periods. The top panel of Figure 3 plots the response of output and measured TFP.

The computation of TFP requires some explanation. The aggregate production function in the model is $Y = A(K^{\theta}L^{1-\theta})^{\nu}$, where A is constant, and therefore, the actual TFP is constant. However, following the standard accounting procedure, we compute the TFP assuming that the aggregate production function takes the standard Cobb-Douglas form, that is, $Y = \hat{A}K^{\hat{\theta}}L^{1-\hat{\theta}}$, where $\hat{\theta} \neq \theta$ is the capital income share. The variable \hat{A} is what we identify as measured TFP. Because this representation of the production function ignores the increasing returns, the variable \hat{A} is not constant. In particular, we have that this variable is determined by:

$$\hat{A} = A \frac{\left(K^{\theta} L^{1-\theta}\right)^{\nu}}{K^{\hat{\theta}} L^{1-\hat{\theta}}}$$

which in general increases with the scale of production.

As can be seen from the top panel of Figure 3, the drop in p generates a large fall in measured TFP and output. This is in contrast to the case in which debt contracts are fully enforceable or the cost of issuing new equities is zero. In this case the output would not be affected. On the other hand, an increase in p does not have any macroeconomic effect as shown in the second panel of Figure 3. In this case output does not change even if debt contracts are not enforceable and new equities are costly.

Financial innovations: Within our model, innovations in the financial system are captured by changes in the fraction of recoverable debt ϕ and the



Figure 3: Macroeconomic dynamics after a shock.

cost to issue new shares κ . The first parameter captures the greater ability of firms to increase their leverage. The second parameter captures the lower cost and greater flexibility in using equity as a source of financing.

The top panel of Figure 4 plots the response of output to a drop in p for economies that differ only in the parameter ϕ . The sensitivity of aggregate output (and TFP, although not reported) increases with the increase in the parameter ϕ . The reason is because with a higher leverage firms have to restructure their debt more after an asset price shock. As a result, the impact on investment and on the real economy is larger. This finding suggests that it is not the greater ability to borrow that have contributed to the lower volatility of output in the US economy. If any, the effect would have been in



Figure 4: Financial development and macroeconomic dynamics.

the opposite direction.

The bottom panel of Figure 4 plots the response of output to a drop in p for economies that differ only in the parameter κ (they have the same ϕ). The sensitivity of aggregate output now drastically falls when the parameter κ is smaller, that is, when the cost of new equities is lower. Therefore, financial innovations that make equity finance more accessible reduces the impact of an asset price shock and the economy experiences milder recessions.

The result that the drop in κ can generate a lower volatility of output, while this cannot be generated by a reduction in ϕ , shows the importance of allowing for the issue of new shares. This is a feature that differentiates our model from other models with financial markets frictions. As discussed



Figure 5: Financial development and macroeconomic dynamics.

earlier, there are other models with financial markets frictions where movements in asset prices are an important transmission mechanism. An example is Kiyotaki & Moore (1997). Our model would be similar to Kiyotaki and Moore' model if we do not allow for the issue of new equity (only retaining earnings). This is obtained by setting $\kappa = \infty$. But we have seen that in this model the type of innovation that allow for a greater leverage would lead to a greater variability of output.

Figure 5 shows the output response for economies that differ in both dimensions. The first economy, which we identify as less financially developed, has a lower ϕ and a higher κ . The second economy, which we identify as more financially developed, has a higher ϕ and a lower κ . As can be seen from the figure, the more developed economy displays a lower sensitivity of output.

8 Conclusion

To be written.

Appendix: First order conditions

Consider the optimization problem (4) and let λ and μ be the Lagrange multipliers attached to the two constraints. Taking derivatives we get:

$$\begin{aligned} d : & 1 - \lambda \varphi'(d) = 0 \\ k' : & (1 + \mu) E \alpha(\mathbf{s}, \mathbf{s}') \overline{V}_k(\mathbf{s}'; k', b') - \lambda - \phi \mu = 0 \\ b' : & (1 + \mu) E \alpha(\mathbf{s}, \mathbf{s}') \overline{V}_b(\mathbf{s}'; k', b') + \lambda R = 0 \end{aligned}$$

The envelope conditions are:

$$V_k(\mathbf{s}; k, b) = \lambda \pi_k(Y, w, k)$$

$$\overline{V}_k(\mathbf{s}; k, b) = pV_k(\mathbf{s}; k, b) + (1 - p)\pi_k(Y, w, k)$$

$$V_b(\mathbf{s}; k, b) = -\lambda$$

$$\overline{V}_b(\mathbf{s}; k, b) = pV_b(\mathbf{s}; k, b) - (1 - p)$$

Using the first condition to eliminate λ , we get the first order conditions (6)-(7) and the envelope conditions (8)-(9).

Solution strategy

Consider the following equations:

$$wU_c(c,h) + U_h(c,h) = 0$$
 (12)

$$U_c(c,h) - \beta(1+r)EU_c(c',h') = 0$$
(13)

$$wh + b - \frac{b'}{1+r} + d - c = 0 \tag{14}$$

$$(1+\mu)E\tilde{\alpha}(\mathbf{s}')\overline{V}_k(\mathbf{s}';k',b') - \left[\frac{1}{\varphi'(d)} + \phi\mu\right]U_c(c,h) = 0 \qquad (15)$$

$$(1+\mu)E\tilde{\alpha}(\mathbf{s}')\overline{V}_b(\mathbf{s}';k',b') + \frac{RU_c(c,h)}{\varphi'(d)} = 0$$
(16)

$$E\tilde{\alpha}(\mathbf{s}')\overline{V}(\mathbf{s}';k',b') \ge \phi \cdot k' \cdot U_c(c,h)$$
(17)

$$\pi(Y, w, k) - b + Rb' - k' - \varphi(d) = 0 \tag{18}$$

$$l(Y, w, k) - h = 0 (19)$$

Equations (12)-(14) are the first order conditions for the households plus their budget constraint. Equations (15)-(17) are the first order conditions and the enforcement constraint for firms. In writing these conditions we have used the new function $\tilde{\alpha}(\mathbf{s}') = \beta U_c(c', h')$. Using this function, the discount factor can be written as $\alpha(\mathbf{s}, \mathbf{s}') = \tilde{\alpha}(\mathbf{s}')/U_c(c, h)$. This transformation facilitates the computational procedure because the function $\tilde{\alpha}(\mathbf{s}')$ is fully determined by the next period states, while the function $\alpha(\mathbf{s}, \mathbf{s}')$ depends also on the current states. Equation (18) is the budget constraint for the firm and the last equation is the equilibrium condition in the labor market: the demand of labor, l(Y, w, k), must be equal to the supply from households, h.

The computational procedure is based on the following observation: If we knew the terms $E\tilde{\alpha}(\mathbf{s}')\overline{V}(\mathbf{s}';k',b')$, $E\tilde{\alpha}(\mathbf{s}')\overline{V}_k(\mathbf{s}';k',b')$, $E\tilde{\alpha}(\mathbf{s}')\overline{V}_b(\mathbf{s}';k',b')$, and $E\beta U_c(c',h')$, we could solve the eight conditions (12)-(19) for the eight unknowns $c, h, w, r, \mu, d, b', k'$. The numerical procedure, then, is based on the approximation of these functions, that is, $\tilde{\alpha}(\mathbf{s}')\overline{V}(\mathbf{s};k,b)$, $\tilde{\alpha}(\mathbf{s}')\overline{V}_k(\mathbf{s};k,b)$, $\tilde{\alpha}(\mathbf{s}')\overline{V}_b(\mathbf{s};k,b)$ and $\beta EU_c(c,h)$.

We create a two-dimensional grid for k and b. Then, for each value of the shock p, we guess the values of the four functions at each grid point. The grid points are joined with bilinear functions so that the approximated functions are continuous. At this point we solve for the eight variables at each grid point and update the initial guesses until convergence.

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