Affine-Quadratic Term Structure Models: Towards an Understanding of Jumps in Interest Rates

Discussion by
Peter Christoffersen
McGill University, CIRANO, CIREQ
Motivation and Contribution

• Motivating Research Questions:
  – What causes jumps in interest rates?
  – What determines the arrival rate of jumps?

• Contribution
  – Develop a class of affine-quadratic jump-diffusion term structure models
  – Model jump intensity as a stochastic variable depending on short rate and stochastic volatility.

• Incorporating Macro information in model building / estimation?
Affine vs Quadratic

• Cheng and Scaillet (2002-2006) develop an LQJD class of models and show that it can be embedded into the affine class using an augmented state vector.

• Thus a low-order quadratic model can be viewed as a high-order affine model with restrictions on the factors.

\[ \mu_S = \alpha_1 + \beta_1^T X_1 + \gamma_1^T X_2 + X_2^T \Phi_1 X_2, \]

\[ \sigma_S^T \sigma_S = \alpha_2 + \beta_2^T X_1 + \gamma_2^T X_2 + X_2^T \Phi_2 X_2, \]

• This is not necessarily a bad thing but I think discussing it would help the reader understand the models better.

• Compare quadratic with higher order affine.
Affine vs Non-Affine

• Ahn and Gao (RFS, 1999)
  – Considers model with nonlinear drift and diffusion term (no SV). Finds superior fit from nonlinearities.

• Andersen, Benzoni and Lund (2004)
  – Compares affine and non-affine models with SV and Jumps.

• Compare with non-affine models to assess the constraints in the lin-quad setup.
Show the Realized Vols

• Paper uses high-frequency intra-day data in the GMM estimation. I would love to see the affine SV specifications informally verified using the daily realized volatility (RV) measures.

• I don’t have RV data for the T-bill rate but consider the following S&P500 example

• Upshot: dLog(V) appears much better behaved than d(V).
Heston (1993)

• In the canonical affine SV model

\[ dS = \mu S dt + \sqrt{V} S dw^S \]

\[ dV = \kappa(\theta - V) dt + \sigma \sqrt{V} dw^V \]

• Which implies

\[ d\sqrt{V} = \mu(V) dt + \frac{1}{2} \sigma dw^V \]
Properties of \( d(RV^{1/2}) \)
Properties of dLog(RV)
Which SV Specification?

• Affine SV assumes
  \[ dr = (\mu_r^* + \kappa_{rr} r + \kappa_{rv} v) dt + \sqrt{v} dZ_r^*, \]
  \[ dv = (\mu_v^* + \kappa_{vr} r + \kappa_{vv} v) dt + \sigma_v \sqrt{v} dZ_v^*, \]

• Quadratic SV assumes
  \[ d\sqrt{v} = \left( \mu_v^* + \kappa_{vv} \sqrt{v} + \kappa_{v\lambda} \sqrt{\lambda^*} \right) dt + \sigma_v dZ_v^*, \]

• Drift versus Diffusion term.
• \implies\text{ Harder to compare models. Convince reader that this doesn’t matter.}
GMM vs AMLE

• GMM delivers estimates but implies some arbitrariness due to choice of moments.
• GMM generally does not deliver filtration of latent factors.
• Bates (RFS, 2006) suggests an attractive approximate MLE methodology which delivers estimates and filtration.
• Requires model which can be transformed to an affine model. Uses characteristic function. Available here!
Diagnostics Needed

- The only model diagnostic given is

<table>
<thead>
<tr>
<th></th>
<th>SV</th>
<th>SVJ</th>
<th>SVJT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$</td>
<td>34.99</td>
<td>22.76</td>
<td>9.23</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>11</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>p-value</td>
<td>0.02%</td>
<td>0.19%</td>
<td>2.64%</td>
</tr>
</tbody>
</table>

- I would like to see evidence on the fit of the various 18 moments applied, see e.g. Andersen, Benzoni and Lund (2004). T-tests on average scores.
- It would help me understand the model properties.
- Which moments does the quadratic model help fit better than the affine model?
More Diagnostics Needed

- Do MC simulation from model and compute moment confidence bands from simulation and compare with the empirical moments in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Mean*</th>
<th>StDev</th>
<th>Skew</th>
<th>Kurt</th>
<th>Min</th>
<th>Max</th>
<th>Autocorrelations of Monthly Series</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\rho_1$ $\rho_2$ $\rho_3$ $\rho_4$ $\rho_5$</td>
</tr>
<tr>
<td>Panel A: Summary statistics of daily interest rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R3M</td>
<td>5.597</td>
<td>1.780</td>
<td>0.406</td>
<td>0.029</td>
<td>1.55</td>
<td>10.67</td>
<td>0.98 0.96 0.94 0.92 0.89</td>
</tr>
<tr>
<td>R6M</td>
<td>5.708</td>
<td>1.789</td>
<td>0.429</td>
<td>0.182</td>
<td>1.59</td>
<td>10.77</td>
<td>0.98 0.96 0.94 0.91 0.89</td>
</tr>
<tr>
<td>R1Y</td>
<td>6.190</td>
<td>1.980</td>
<td>0.585</td>
<td>0.429</td>
<td>1.93</td>
<td>12.34</td>
<td>0.98 0.96 0.93 0.91 0.88</td>
</tr>
<tr>
<td>R2Y</td>
<td>6.635</td>
<td>1.989</td>
<td>0.772</td>
<td>0.678</td>
<td>2.32</td>
<td>13.17</td>
<td>0.98 0.95 0.93 0.90 0.87</td>
</tr>
<tr>
<td>R3Y</td>
<td>6.834</td>
<td>1.990</td>
<td>0.877</td>
<td>0.775</td>
<td>2.70</td>
<td>13.49</td>
<td>0.98 0.95 0.92 0.89 0.86</td>
</tr>
<tr>
<td>R5Y</td>
<td>7.120</td>
<td>1.956</td>
<td>1.031</td>
<td>0.991</td>
<td>3.47</td>
<td>13.84</td>
<td>0.97 0.95 0.92 0.89 0.85</td>
</tr>
<tr>
<td>R7Y</td>
<td>7.341</td>
<td>1.938</td>
<td>1.060</td>
<td>0.970</td>
<td>3.95</td>
<td>13.95</td>
<td>0.97 0.95 0.92 0.89 0.85</td>
</tr>
<tr>
<td>R10Y</td>
<td>7.435</td>
<td>1.932</td>
<td>1.026</td>
<td>0.872</td>
<td>4.16</td>
<td>13.99</td>
<td>0.97 0.95 0.92 0.89 0.85</td>
</tr>
<tr>
<td>R30Y</td>
<td>7.672</td>
<td>1.817</td>
<td>1.044</td>
<td>0.931</td>
<td>4.70</td>
<td>13.94</td>
<td>0.97 0.95 0.92 0.89 0.86</td>
</tr>
</tbody>
</table>
More Diagnostics Needed Still

• Duffee (JF, 2002) finds that affine models don’t do better than a random walk for forecasting the yield curve.
• What are the in-sample and out-of-sample bond pricing or yield errors in the quadratic model?
• Are the forecast errors related to observables e.g. the yield curve slope as in Duffee?
Benchmarks

• I would think that in a rich and mature literature such as this it is necessary to compare a new model to some established benchmarks:
  – Two factor SV model not enough.
  – Any quadratic model. E.g. Duffee’s essentially affine.
  – Ahn and Gao’s nonlinear model.

• The macro interpretations could also be compared with existing “macro models” e.g., Ang and Piazzesi, and Bibkov and Chernov.
Parameter Significance

• Quite a few of the parameters in Table 3 are not significant.
• How much worse would the fit of the model be if these were set to zero?
• How would the restricted model fare out of sample?
• Is the mean positive jump significantly different from the mean negative jump?
Summary

• Adding diagnostics would be very helpful
• Comparing with existing three-factor models would be helpful.
• Statistical versus economic performance?
• Show me what exactly it is that the quadratic models have to offer empirically?
• Use macro data in estimation.