Affine-Quadratic Term Structure Models: Towards an Understanding of Jumps in Interest Rates

Discussion by

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Motivation and Contribution

- Motivating Research Questions:
  – What causes jumps in interest rates?
  – What determines the arrival rate of jumps?
- Contribution
  – Develop a class of affine-quadratic jump-diffusion term structure models
  – Model jump intensity as a stochastic variable depending on short rate and stochastic volatility.
- Incorporating Macro information in model building / estimation?

Affine vs Quadratic

- Cheng and Scaillet (2002-2006) develop an LQJD class of models and show that it can be embedded into the affine class using an augmented state vector.
- Thus a low-order quadratic model can be viewed as a high-order affine model with restrictions on the factors.

$$\mu_S = \alpha_1 + \beta_1^\top X_1 + \gamma_1^\top X_2 + X_2^\top \Phi_1 X_2,$$

$$\sigma_S^\top \sigma_S = \alpha_2 + \beta_2^\top X_1 + \gamma_2^\top X_2 + X_2^\top \Phi_2 X_2,$$

- This is not necessarily a bad thing but I think discussing it would help the reader understand the models better.
- Compare quadratic with higher order affine.

Affine vs Non-Affine

- Ahn and Gao (RFS, 1999)
  - Considers model with nonlinear drift and diffusion term (no SV). Finds superior fit from nonlinearities.
- Andersen, Benzoni and Lund (2004)
  - Compares affine and non-affine models with SV and Jumps.
- Compare with non-affine models to assess the constraints in the lin-quad setup.

Show the Realized Vols

- Paper uses high-frequency intra-day data in the GMM estimation. I would love to see the affine SV specifications informally verified using the daily realized volatility (RV) measures.
- I don’t have RV data for the T-bill rate but consider the following S&P500 example
- Upshot: dLog(V) appears much better behaved than d(V).

Heston (1993)

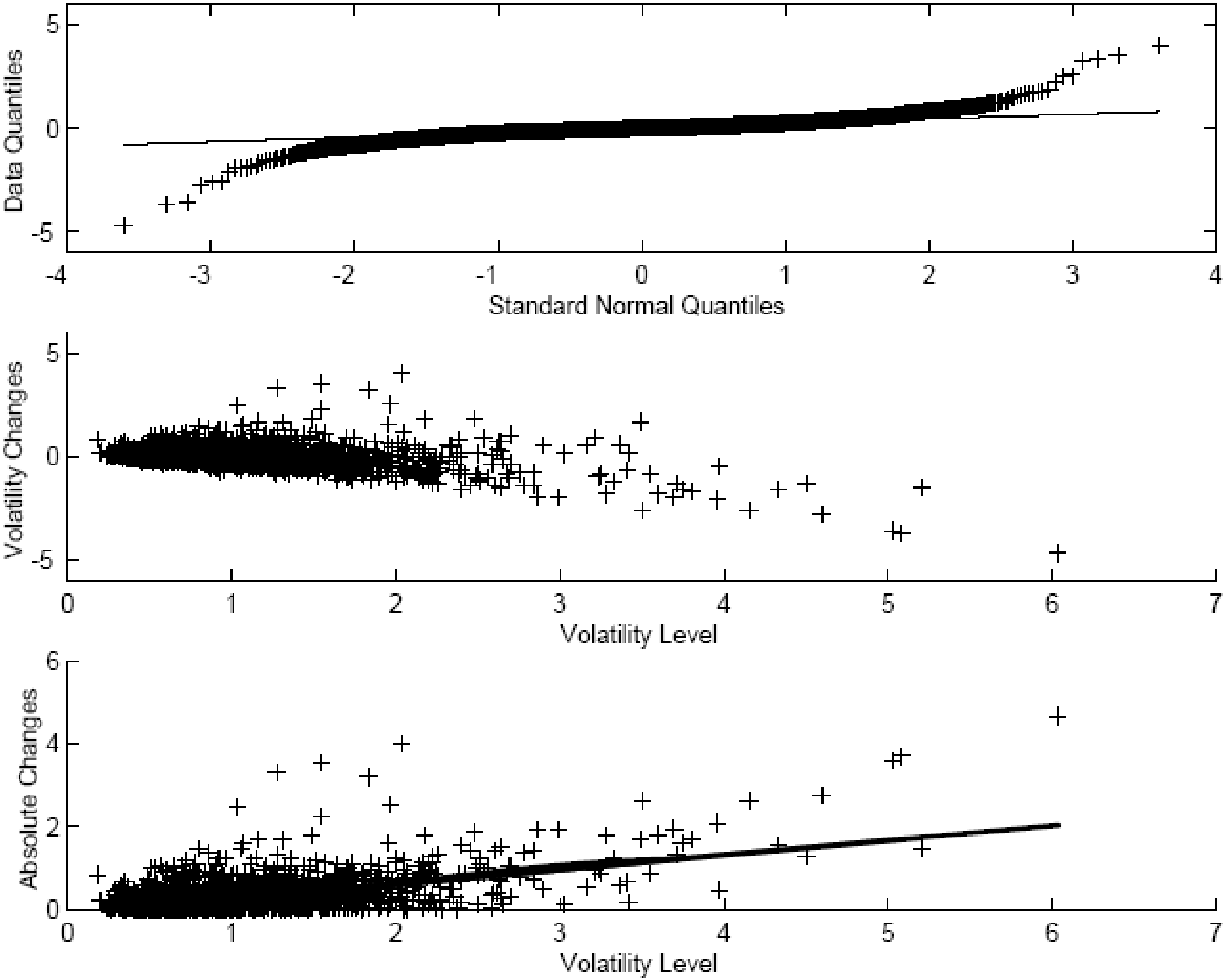
- In the canonical affine SV model

$$\begin{aligned} dS &= \mu S dt + \sqrt{V} S dw^S \\ dV &= \kappa(\theta - V)dt + \sigma\sqrt{V} dw^V \end{aligned}$$

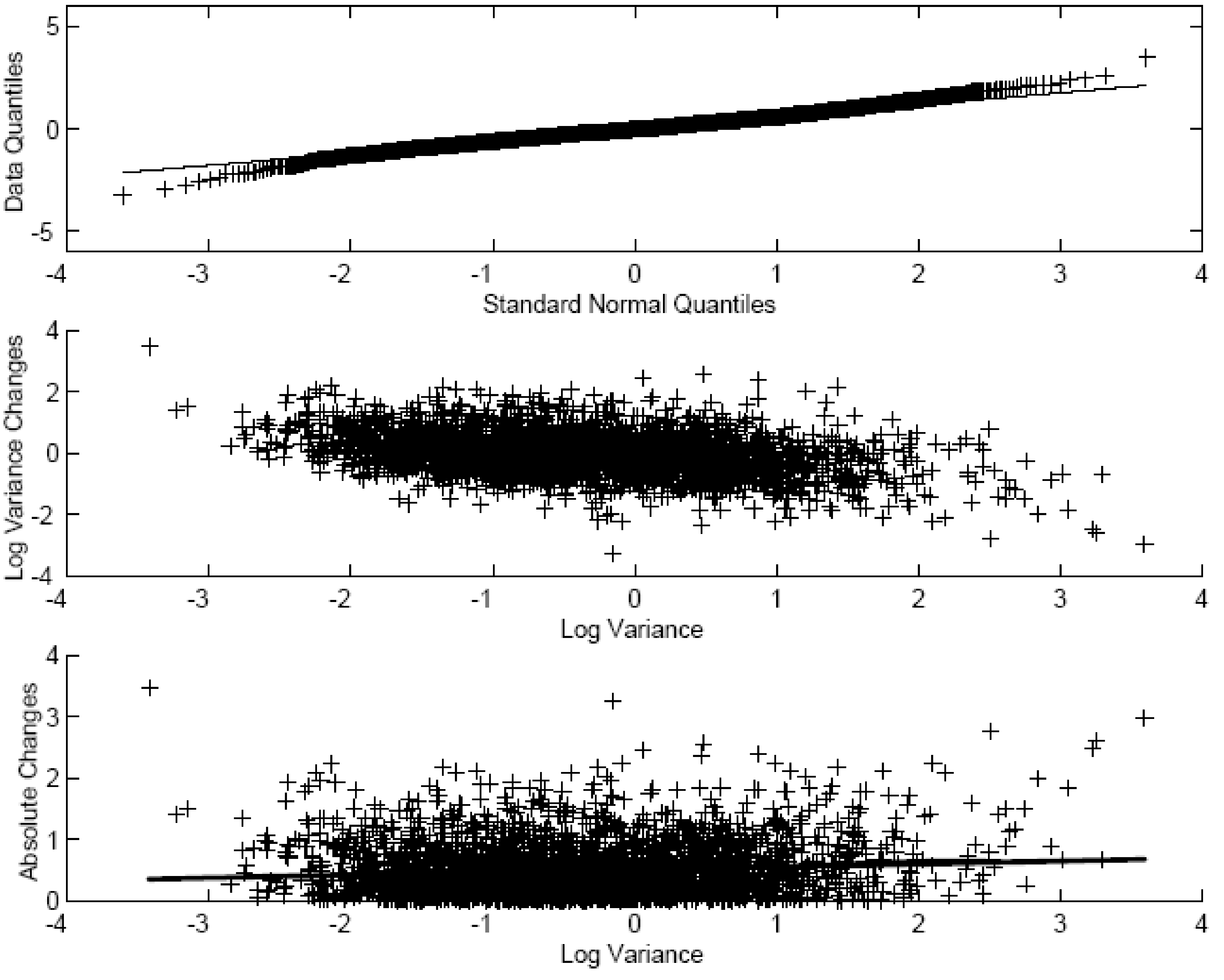
- Which implies

$$d\sqrt{V} = \mu(V)dt + \frac{1}{2}\sigma dw^V$$

Properties of $d(RV^{1/2})$



Properties of dLog(RV)



Which SV Specification?

- Affine SV assumes

$$
\begin{aligned}
dr &= (\mu_r^* + \kappa_{rr}^* r + \kappa_{rv}^* v)\, dt + \sqrt{v} dZ_r^*, \\
dv &= (\mu_v^* + \kappa_{vr}^* r + \kappa_{vv}^* v)\, dt + \sigma_v \sqrt{v} dZ_v^*,
\end{aligned}
$$

- Quadratic SV assumes

$$
d\sqrt{v} = \left(\mu_v^* + \kappa_{vv}^* \sqrt{v} + \kappa_{v\lambda}^* \sqrt{\lambda^*}\right) dt + \sigma_v dZ_v^*,
$$

- Drift versus Diffusion term.
- => Harder to compare models. Convince reader that this doesn't matter.

GMM vs AMLE

- GMM delivers estimates but implies some arbitrariness due to choice of moments.
- GMM generally does not deliver filtration of latent factors.
- Bates (RFS, 2006) suggests an attractive approximate MLE methodology which delivers estimates and filtration.
- Requires model which can be transformed to an affine model. Uses characteristic function. Available here!

Diagnostics Needed

- The only model diagnostic given is

	SV	SVJ	SVJT
χ^2	34.99	22.76	9.23
d.o.f.	11	7	3
p-value	0.02%	0.19%	2.64%

- I would like to see evidence on the fit of the various 18 moments applied, see e.g. Andersen, Benzoni and Lund (2004). T-tests on average scores.
- It would help me understand the model properties.
- Which moments does the quadratic model help fit better than the affine model?

More Diagnostics Needed

- Do MC simulation from model and compute moment confidence bands from simulation and compare with the empirical moments in Table 1.

	Mean*	StDev	Skew	Kurt	Min	Max	Autocorrelations of Monthly Series				
							ρ_1	ρ_2	ρ_3	ρ_4	ρ_5
Panel A: Summary statistics of daily interest rates											
R3M	5.597	1.780	0.406	0.029	1.55	10.67	0.98	0.96	0.94	0.92	0.89
R6M	5.708	1.789	0.429	0.182	1.59	10.77	0.98	0.96	0.94	0.91	0.89
R1Y	6.190	1.980	0.585	0.429	1.93	12.34	0.98	0.96	0.93	0.91	0.88
R2Y	6.635	1.989	0.772	0.678	2.32	13.17	0.98	0.95	0.93	0.90	0.87
R3Y	6.834	1.990	0.877	0.775	2.70	13.49	0.98	0.95	0.92	0.89	0.86
R5Y	7.120	1.956	1.031	0.991	3.47	13.84	0.97	0.95	0.92	0.89	0.85
R7Y	7.341	1.938	1.060	0.970	3.95	13.95	0.97	0.95	0.92	0.89	0.85
R10Y	7.435	1.932	1.026	0.872	4.16	13.99	0.97	0.95	0.92	0.89	0.85
R30Y	7.672	1.817	1.044	0.931	4.70	13.94	0.97	0.95	0.92	0.89	0.86

More Diagnostics Needed Still

- Duffee (JF, 2002) finds that affine models don’t do better than a random walk for forecasting the yield curve.
- What are the in-sample and out-of-sample bond pricing or yield errors in the quadratic model?
- Are the forecast errors related to observables e.g. the yield curve slope as in Duffee?

Benchmarks

- I would think that in a rich and mature literature such as this it is necessary to compare a new model to some established benchmarks:
  - Two factor SV model not enough.
  - Any quadratic model. E.g. Duffee's essentially affine.
  - Ahn and Gao's nonlinear model.
- The macro interpretations could also be compared with existing "macro models" e,g, Ang and Piazzesi, and Bibkov and Chernov.

Parameter Significance

- Quite a few of the parameters in Table 3 are not significant.
- How much worse would the fit of the model be if these were set to zero?
- How would the restricted model fare out of sample?
- Is the mean positive jump significantly different from the mean negative jump?

Summary

- Adding diagnostics would be very helpful
- Comparing with existing three-factor models would be helpful.
- Statistical versus economic performance?
- Show me what exactly it is that the quadratic models have to offer empirically?
- Use macro data in estimation.