Session 2:
Exchange Rate Determination in a Global Setting

Multilateral Adjustment and the Canadian Dollar

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Introduction

In 2003, the Canadian dollar appreciated by almost 25 per cent in less than twelve months; it was the most rapid rise of the dollar on record. Since the exchange rate directly influences the link between external demand and domestic economic activity, this appreciation posed a serious challenge for monetary policy. In particular, the appropriate monetary policy response depended on the factors underlying this movement. In theory, the monetary policy response to a sustained exchange rate appreciation or depreciation would be muted if this relative price movement were driven largely by real fundamentals, normally identified as shifts in the demand for and supply of Canadian-produced goods and services relative to those produced in the rest of the world.[1] Some monetary accommodation, however, might be useful in this case, because it would facilitate the reallocation of resources between traded and non-traded sectors in response to the exchange rate movement.

Attempting to understand the forces behind the rapid appreciation of the dollar in 2003 is the primary motivation for this paper. It is well known that adequately explaining the behaviour of exchange rates is one of the major

1. If, however, an exchange rate movement were driven by non-fundamental (e.g., speculative) forces, then monetary policy should react to neutralize the effect of these forces in order to shelter the domestic economy from unnecessary movements in the exchange rate.

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puzzles in international economics. For the Can$/US$ bilateral exchange rate, the Bank of Canada has developed an equation based primarily on non-energy commodity prices and lagged interest rate differentials that seems to explain movements in the exchange rate reasonably well.[2] There are episodes, however, such as in 2003, when the equation fares poorly in explaining the rapid appreciation, because there are other factors driving the exchange rate that are not included in the empirical model.

This paper focuses on one potential explanation: namely, that the bilateral exchange rate equation currently in use at the Bank of Canada does not fully allow for multilateral adjustment to US current account and fiscal imbalances. In particular, because the US economy occupies a predominant position in the world economy, when it incurs, for example, a current account deficit that is viewed as unsustainable, then all countries will see the value of their currencies appreciate relative to the US dollar to facilitate global adjustment to the US imbalances. A multilateral adjustment, however, cannot easily be captured by an equation that uses only bilateral differences in macro variables as explanatory variables, because the bilateral current account position is of little relevance in explaining a current account imbalance that is determined by transactions with all other countries. In 2003, the United States was running a current account deficit of roughly 5 per cent of gross domestic product (GDP) that many observers felt was unsustainable at the existing exchange rate levels. Consequently, all major currencies began to appreciate relative to the US dollar. Table 1 shows that the rapid appreciation of the Canadian dollar was comparable to that experienced by other currencies.

To capture situations in which the Canadian dollar is being driven largely by the forces of multilateral adjustment to US imbalances, we need to go beyond the standard bilateral empirical exchange rate model, such as the one developed by the Bank of Canada. Bilateral models are common in the literature, despite the fact that they ignore multilateral influences, because they are relatively easy to estimate and interpret, and in most periods, but not all, provide relatively good explanations of the observed movements in exchange rates. To capture fully, however, the multilateral influences would require an econometric dynamic general-equilibrium (DGE) model, which is much more difficult to estimate and interpret. Moreover, the cost of

2. The original research on the exchange rate equation was done by Amano and van Norden (1993). Their key finding was that separating Canada's terms of trade into energy and non-energy components greatly increased the equation's explanatory power. Subsequent research by Murray, Zelmer, and Antia (2000) extended the model. Ongoing research by Helliwell et al. (2005) focuses on the role of relative productivity as a determinant of exchange rate movements.

Table 1
Nominal appreciation from 2 January 2003 vs. US$ (percentage)

	To 1 October 2003	To 9 January 2004 (CAD Peak)	To 27 September 2004
Canadian dollar (Can$)	16.82	24.07	23.63
Euro	12.96	24.01	18.76
British pound	4.30	15.62	13.14
Japanese yen	8.34	12.53	7.84
Australian dollar	21.31	38.07	26.66
New Zealand dollar	14.42	30.46	27.44

Source: Daily recorded values at 12:00 p.m. EST by the Bank of Canada.

operationalizing such a model may not be worth the potential benefit in terms of additional explanatory power. Thus, our approach in this paper is to extend the bilateral model by adopting a threshold methodology that will allow the specification of the empirical model to change when US macroeconomic imbalances are significant. The rationale for such an approach is that, under normal circumstances, the forces of multilateral adjustment are superseded by bilateral considerations, and only when US imbalances are large does the need for multilateral adjustment dominate.

The key finding in this paper is that in periods when the United States is running a substantial fiscal deficit (i.e., more than 2 per cent of GDP), the specification of the empirical regression model describing the Canadian dollar changes. The result is intuitively appealing, because during episodes in the post-Bretton Woods period when the US fiscal deficit was large, especially on a cyclically adjusted basis, the United States often had a substantial current account deficit. Yet the reverse was, in general, less often true, because current account deficits also occurred during investment booms when there was a fiscal surplus. Hence, US current account deficits generated by increases in government spending or tax cuts appear to have been viewed by the market as less sustainable (perhaps because the foreign borrowing was not used to finance investments that would have generated sufficient returns to service the debt) and thus warranted a substantial multilateral exchange rate adjustment.

The paper is organized into six sections. The next section examines large US external imbalances since the Bretton Woods period and their implications for the adjustment of exchange rates, including the Canadian dollar. Section 2 provides theoretical arguments for a multilateral approach to exchange rate modelling, in general, and the threshold approach, in particular. The empirical framework and data required are explained in section 3, which is followed in section 4 by a description of the estimation procedure and a presentation and interpretation of the empirical results. Concluding remarks are made in the final section.

1 US Imbalances in the Postwar Period and the Canadian Dollar

Since 1960, the United States has experienced two periods of significant external imbalance. Figure A1.1 in Appendix 1 shows that the first imbalance took place over the period 1984 to 1989, when the US current account deficit exceeded 2 per cent of GDP over the entire six-year period, which, at that time, was the largest US deficit recorded in the postwar period. More recently, the US current account went into deficit starting in 1992, but only crossed the 2 per cent of GDP threshold in 1998, and it has continued to increase, with the deficit now approaching 6 per cent of GDP in 2004.

Figure A1.1 displays the US current account and the Can$/US$ exchange rate.[3] The figure shows that the exchange rate and the US current account are most closely correlated during the two periods of significant US external imbalance. In particular, the value of the Canadian dollar and the US current account hit bottom at roughly the same time. This was clearly true in the 1980s, whereas in the most recent period, the US current account has continued to decline, despite the depreciation of the US dollar. In addition, in the earlier period as the Canadian dollar subsequently appreciated relative to the US dollar, the US current account deficit shrank. This pattern is expected to be repeated in the most recent period as well, but it is not clear when the US current account will turn around. The experience of the Canadian dollar is not unique, however, as the movements of currencies of other countries during these two periods are somewhat similar. Figure A1.2 shows the US dollar effective nominal rate (expressed as the US-dollar price of a trade-weighted basket of currencies) and the US current account. Again, the correlation between the effective rate and the current account appears strongest when the US external imbalances are significant, although the correlation is not as strong as with the Canadian dollar exchange rate. In particular, during the 1980s, the effective US nominal rate begins to depreciate slightly before the trough in the US current account. In the most recent period of external imbalance, the path of the effective nominal rate is essentially the same as that of the Canadian dollar.

3. Given that real and nominal exchange rates are highly correlated, Figures A1.1 and A1.2 show only the nominal exchange rates.

Hence, the evidence seems to suggest that when US external imbalances are large, multilateral exchange rate adjustment to these imbalances is driving the movement of bilateral exchange rates. Given that the United States represents approximately 30 per cent of world GDP, it is not unexpected that when it runs a large external deficit, the relative value of the currencies of all other countries (whose currencies float) must change to facilitate the adjustment of the world economy to this imbalance. Indeed, if we compare the simulated values of the nominal exchange rate generated by the Bank of Canada's bilateral exchange rate model (which is discussed in more detail later in the paper) to the actual value of the exchange rate shown in Figure A1.3, we see that the model cannot explain adequately the slope and magnitude of the appreciation of the Canadian dollar in 2003.

To examine these US external imbalances further, it is useful to employ the national income accounting identity; this identity implies that current account imbalances occur when there are fiscal deficits or a deficit of private savings relative to domestic investment.[4] Figure A1.4 plots the US current account deficit and the US federal government fiscal deficit. Also shown is the difference between private savings and investment, which is calculated as the residual. Figure A1.4 shows that over the periods 1984–88, and 2002–04, the large US external balances coincided with large fiscal deficits. When this simultaneous occurrence was observed in the 1980s, it was labelled the "twin-deficits" phenomenon, and the argument was made that the significant reductions in taxes and the concomitant increase in military spending during the Reagan administration caused both the fiscal and current accounts deficits over this period.[5] While there was much public debate over this causal argument at the time, the standard non-Ricardian, open-economy model would predict that a temporary increase in domestic (government) spending would increase the current account deficit. This increase in demand would fall partly on traded goods, but largely on non-traded goods and services. Thus, resources would be shifted out of the traded goods sector to meet this demand. As a result, there would be less domestic supply of traded goods to satisfy the increase in demand, and a current account deficit would ensue. Domestic interest rates would rise and foreign borrowing would be used to finance the higher level of absorption.

The second period of significant external imbalance (i.e., more than 2 per cent of GDP), 1998 to 2004, is different from the earlier period in the 1980s, because the imbalance occurred several years before the fiscal deficits of

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4. Because the relation is an identity and because most of the variables are endogenous, care must be taken when using the identity to make causal statements.

5. Mann (1999, 2002) provides an insightful discussion of the twin-deficits experience in the United States.

2002–04; indeed, the US current account deficit begins when there is a large fiscal surplus. The critical difference is that over this period (1998–2001), the current account deficit is caused by an investment boom and relatively low domestic savings. As a result of this investment-savings gap, foreign capital flowed into the United States in expectation of higher returns owing to the rapid increases in productivity, which were anticipated to continue for the foreseeable future. This expectation of higher productivity growth also increased domestic consumption and reduced savings as US residents intertemporally shifted higher expected future outputs and incomes to the present.

It is also noteworthy that in the three recessionary periods, 1974–75, 1981–82, and 1991–92, there was slowdown in economic activity, and consequently, the fiscal position went into deficit because of lower tax revenues and increased transfers, and the current account deficit declined as imports fell. In these situations, higher fiscal deficits did not coincide with current account deficits, because aggregate investment fell below savings as economic prospects turned negative. To measure the underlying degree of fiscal stimulus, it is sometimes useful to adjust the fiscal position for effects of the business cycle. In Figure A1.5, the cyclically adjusted fiscal position is included in addition to the fiscal and current account positions. This figure shows clearly that a large part of the fiscal deficits observed during these episodes were indeed cyclical.

In summary, we have tried to demonstrate in this section that two large appreciations of the Canadian dollar, and also that of other currencies, have taken place as a result of multilateral adjustments to large US external imbalances, which were caused in part by fiscal imbalances. Thus, incorporating this multilateral adjustment into an empirical model of the Canadian exchange rate may help improve the explanatory power of the traditional bilateral model.

2 Theoretical Considerations

The discussion in the previous section suggests that there are periods in which movements in the Canadian dollar that are not well explained by intercountry differences in Canada-US fundamentals may be accounted for by factors driving the US dollar. Therefore, an analysis of the Canadian dollar based on a bilateral exchange rate equation that does not account for these multilateral effects stemming from the United States may suffer from omitted variable bias. Furthermore, it also suggests that these effects can be modelled as threshold effects given that they likely only emerge as an important determinant of the Canadian dollar in periods where there are significant imbalances. The determinants of a currency like the Canadian dollar might thus be better modelled in the context of a model with two regimes: one in which intercountry differences in Canada-US fundamentals are the main drivers of the bilateral exchange rate and a second in which multilateral effects from the United States kick in to become an additional important determinant.

Although little research has been conducted on modelling multilateral exchange rate effects, a number of multi-country empirical models have been developed that account for these types of factors. In addition, the concept has been discussed in the econometric literature, and empirical evidence suggests the presence of such effects. Furthermore, the inclusion of a multilateral term in a bilateral exchange rate equation—as well as its treatment as a threshold effect—can be motivated theoretically by drawing on related work. This section first discusses the theoretical rationale for multilateral currency effects, then provides an overview of how these effects have been modelled in multi-country empirical models, and concludes by reviewing the empirical evidence.

2.1 Theoretical rationale for multilateral currency effects

The bulk of the empirical literature that examines the macroeconomic determinants of exchange rates is based on a two-country framework where the bilateral exchange rate is viewed as the relative price of the monies of the two countries in question.[6] There are a large number of such models, all of which describe the evolution of the exchange rate as being a function of a different set of fundamentals. Cheung, Chinn, and Pascual (2002) provide a useful overview of the main empirical models used in the 1990s in their update of Meese and Rogoff's (1983) work comparing the forecasting performance of the major models used in the 1970s. As they discuss, these models view the bilateral exchange rate as a function of variables such as prices, money, income, interest rates, productivity differentials, relative price of non-tradables, government debt, terms of trade, and net foreign assets—typically expressed as *intercountry* differences.

In contrast, empirical exchange rate models that incorporate multilateral effects view exchange rates as being interdependent. In this type of framework, a bilateral exchange rate can thus be driven either by intercountry differences in its fundamentals, as noted above, or by fundamentals that are influencing another currency (i.e., multilateral effects). There are a few potential explanations for this latter phenomenon.

6. This type of framework also typically underlies empirical work focusing on the determinants of effective exchange rates where the second country in this case is defined as the rest of the world (expressed in a trade-weighted fashion).

First, news about the fundamentals of the other currency may reveal information about the domestic economy. For instance, news about US fundamentals may reveal information about the Canadian economy. And given the economic significance of the United States, news about US fundamentals may provide information regarding the direction of the world economy, both of which should influence the Canadian dollar.[7] Second, multilateral effects could arise because of trade and financial market linkages across countries. In other words, given that the real and financial sectors of the world economies are linked, so will their respective exchange rates be linked. This is the basic idea behind models of exchange rate determination based on a multi-country framework, which are discussed below.

Finally, the treatment of multilateral factors as threshold effects can be motivated by drawing on an exchange rate model based on informational heterogeneity. Developed by Bacchetta and van Wincoop (2004), this framework was inspired by survey data that suggest that foreign exchange traders change the weight they attach to different macroeconomic indicators over time. Based on this, they develop a "scapegoat" model where foreign exchange traders give a certain fundamental excessive weight during a period of time. The basic mechanism they rely on in the model is confusion in the market as to the true source of exchange rate fluctuations because agents have heterogeneous information. As the market rationally searches for an explanation, it may attribute these fluctuations to some observed macroeconomic indicator; this indicator then becomes the scapegoat and influences trading strategies. This type of model could justify why a variable like the US fiscal deficit might only drive the US dollar once it hits a certain threshold level and becomes a scapegoat variable.

7. The presence of both chartists and fundamentalists in the foreign exchange market may also explain the occurrence of multilateral effects (Frankel and Froot (1988) were the first to develop a foreign exchange rate model with these two types of agents). For example, suppose that chartists apply a trading rule where they buy or sell US dollars relative to all currencies in response to a movement in the US dollar. This movement in the effective US dollar could have occurred as a result of fundamentalists buying or selling the US dollar in response to a change in a US fundamental. The fundamentalists, however, would only trade in a bilateral currency if the change in the US fundamental had a caused a change in the fundamental *relative* to the other country, whereas the chartists would simply apply the trading rule regardless of intercountry differences in fundamentals. Thus, a change in a US fundamental that did not result in a variation in the intercountry difference in that fundamental with respect to Canada could still influence the Canadian dollar.

2.2 Modelling multilateral exchange rate effects

In contrast to the traditional two-country framework that is typically used in the empirical exchange rate literature, models that incorporate multilateral exchange rate effects view exchange rates as being based on a multi-country framework. For example, the Research Department at the International Monetary Fund (IMF) has developed a multi-country approach to estimating equilibrium exchange rates. This work, outlined in Isard and Faruqee (1998) and Faruqee, Isard, and Masson (1999), is based on the macroeconomic balance framework, which defines the equilibrium exchange rate as the level of the real exchange rate that achieves both internal and external balance simultaneously (it is assumed that this occurs in the medium term). In these studies, the equilibrium real exchange rate is defined as the level of the real exchange rate that would equate the cyclically adjusted current account balance, as estimated from a standard trade model, to the equilibrium current account position.[8] This concept of the equilibrium exchange rate is viewed in the medium-term context when all output gaps are assumed to go to zero (and hence economies are in a position of internal, as well as external, balance). Both the cyclically adjusted and equilibrium measures of the current account for each country are estimated in a multi-country setting to capture the interdependent nature of these variables, and hence also of the equilibrium exchange rates (i.e., given that the latter are consistent with the attainment of internal and external balance in all countries in the *world*). Thus, an exchange rate will only be in equilibrium in this model provided that all the other exchange rates in the system are also in equilibrium.

In some earlier work, Buttler and Schips (1987) also compute equilibrium exchange rates based on a multi-country version of the macroeconomic balance approach. In contrast to the IMF studies, they limit the number of currencies in their model to six major currencies plus a composite currency for the rest of the world.[9] In addition, they define equilibrium exchange rates as those that are consistent with a world equilibrium that is achieved when the following conditions hold for all countries in the system: (i) current accounts are in balance, (ii) real incomes are adjusted for changes in the current account, and (iii) other domestic variables remain constant.

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8. The equilibrium current account is estimated using two different methodologies. In the first, the equilibrium current account is defined as the level that stabilizes the net foreign assets to GDP ratio at an appropriate level. The second is based on an estimated model that links saving and investment flows to their structural determinants and views the equilibrium current account position as determined by the cyclically adjusted saving-investment balance, conditional on policies and other exogenous variables.

9. The six major currencies they consider are the US dollar, Japanese yen, German mark, Canadian dollar, pound sterling, and Swiss franc.

The role of multilateral effects has also been examined in the context of target-zone models. Flandreau (1998) extended Krugman's model of a bilateral target zone to a multilateral context where exchange rates depend on all the fundamentals in the system (i.e., each country's fundamentals affect all the exchange rates in the system). In this type of framework, interventions aimed at influencing one given exchange rate turn out to influence other exchange rates as well.

Another approach used in the literature that recognizes the interdependent nature of equilibrium exchange rates is to estimate globally consistent bilateral exchange rates based on a two-country model. Alberola et al. (1999) use such an approach. They estimate bilateral equilibrium exchange rates for several currencies, using a methodology that ensures global consistency between the multilateral and bilateral exchange rates in the system. Based on a simple two-country theoretical framework, they develop an empirical model that describes the evolution of the real exchange rate as a function of two key fundamentals: a relative sectoral price differential index and the level of net foreign assets.[10] After showing the existence of a cointegration relationship between the real exchange rate and these fundamentals in the panel of currencies under study, the authors decompose the real exchange rate into permanent and transitory components; the time-varying permanent component is identified as the equilibrium exchange rate. A series of globally consistent bilateral equilibrium exchange rates is then derived based on the estimates for the effective rates.

2.3 Empirical evidence

Although relatively little empirical research has been conducted on multilateral exchange rate effects, there is evidence suggesting the presence of such effects.[11] For instance, MacDonald and Marsh (2004) extend a

10. In their study, the equilibrium exchange rate is defined as the level of the real exchange rate that is associated with both internal and external balance in the economy. The former occurs when there is no excess demand in the non-tradable sector, whereas the latter is characterized by the achievement of a desired stock of net foreign assets.

11. Another strand of the literature has examined the related question of whether there are volatility spillovers in exchange rates both across markets and across currencies. In seminal papers in this area, Engle, Ito, and Lin (1990) and Ito, Engle, and Lin (1992) found evidence of so-called *meteor-shower effects*, or volatility spillovers across regional markets for the same currency (i.e., the US dollar-Japanese yen exchange rate). Melvin and Melvin (2003) confirmed this result using a richer data set. There is also evidence confirming the presence of cross-currency effects. Indeed, using multivariate stochastic volatility models, Chowdhury and Sarno (2004) found that cross-country volatility spillovers play a role in explaining currency volatility, albeit not as important as that played by lagged own-volatility (so-called *heat-wave effects*).

single-currency exchange rate model based on the joint modelling of exchange rates, prices, and interest rates to a multi-currency setting to capture spillover effects among the three major currencies (i.e., the US dollar, the Japanese yen, and the German mark). They build on a single-currency model developed in previous work (see, for example, MacDonald and Marsh 1997), which is predicated on the Casselian view of purchasing-power parity (PPP). The latter assumes that absolute PPP holds only in the very long run, and hence that there could be deviations in the short, medium, or even long run owing to factors such as capital flows. To capture this, they supplement PPP with an uncovered-interest-rate-parity (UIRP) condition and model the relationship using a vector error-correction model (VECM) with exchange rates, prices, and interest rates.[12] In the multi-currency setting, MacDonald and Marsh model spillovers both in the long-run relationship and in the short-run dynamics. Thus, the error-correction term in each bilateral exchange rate equation will capture the response of the exchange rate to its long-run fundamentals, which include the other bilateral exchange rate in the tri-polar system, as well as prices and interest rates for the three countries. The latter are also used to capture short-run dynamics. The authors find that their tri-polar model generally out-performs the random walk in out-of-sample forecasting exercises.

Hodrick and Vassalou (2002) also find evidence that an exchange rate specification based on a multi-country framework can outperform a specification based on a two-country model. Using a multi-country model of the term structure, they find evidence that the first moment of the German mark-US dollar exchange rate is influenced by third-country factors. However, they find no evidence of spillover effects for the other bilateral exchange rates considered (i.e., those for Japan and the United Kingdom) nor any evidence of spillovers in the second moments of the exchange rates considered. Their results thus suggest that, in some cases, multi-country models that incorporate spillover effects from third countries contain more information than traditional two-country models.

3 Empirical Framework

3.1 A model of the bilateral exchange rate with multilateral effects

As a starting point for our analysis, we use an error-correction model for the bilateral Canada-US real exchange rate that was initially developed by

12. It is worth pointing out that exchange rates, prices (domestic and foreign), and interest rates (domestic and foreign) are used to capture both long-run equilibria and short-run dynamics in their VECM.

Amano and van Norden (1993). This single-equation model is built around a long-run relationship between the real exchange rate, real energy commodity prices, and real non-energy commodity prices.[13] Short-run dynamics are captured by an interest rate differential and a relative public sector indebtedness term. Although parsimonious, this equation has been relatively successful at tracking most of the major movements in the Canadian dollar over the past few decades, has proven to be stable over time, and has outperformed the random walk in out-of-sample forecasting exercises.[14]

The specification for the single-equation model is as follows:

$$\Delta \ln(rfx) = \alpha(\ln(rfx_{t-1}) - \beta - \phi \ln(comtot_{t-1}) - \pi \ln(enetot_{t-1})) + \delta intdif_{t-1} + \gamma \Delta debtdif_{t-1} + \varepsilon_t, \quad (1)$$

where rfx is the real Canada-US dollar exchange rate;[15] $comtot$ is the real non-energy price index;[16] $enetot$ is the real energy price index; $intdif$ is the Canada-US short-term interest rate differential; and $\Delta debtdif$ is the first difference of the Canada-US relative public sector debt. Appendix 2 provides more details on the data. Unit-root tests were conducted on all the series in equation (1) using the Dickey-Fuller Generalized Least Squares (DF-GLS) test developed by Elliott, Rothenberg, and Stock (1996). The results, as well as a description of this test, are provided in Table A3.1 in Appendix 3. The results suggest that rfx, $comtot$, and $enetot$ are non-stationary and that $intdif$ is stationary, as assumed. However, the results also suggest that $\Delta debtdif$ is non-stationary, which is contrary to what is assumed (and to what unit-root tests in the past have found). Thus, this variable cannot be left in the equation as is, given that it is an I(1) variable and its inclusion to capture short-run dynamics would result in an unbalanced equation.

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13. Under certain circumstances, a single-equation approach—as opposed to estimating the entire vector error-correction model—can be justified. Indeed, as discussed by Johansen (1992), estimation and inference based on the single-equation system will be equivalent to that of the full system if there is only one cointegrating vector and all the other cointegrating variables are weakly exogenous with respect to the first variable under consideration (in this case, the real exchange rate). As shown in Tables A3.2 and A3.4 in Appendix 3, cointegration and weak exogeneity tests support this approach.

14. For more information on this equation and its performance over time, see Murray, Zelmer, and Antia (2000).

15. The nominal exchange rate is deflated by the ratio of the GDP deflators for the two countries.

16. The energy and non-energy price indexes are each deflated by the US GDP deflator to convert them into real terms.

As shown in Appendix 3, there is a cointegrating relationship between the real exchange rate, the real non-energy commodity prices, and the real energy prices—rfx, $comtot$, and $enetot$. This is shown in Table A3.2, which reports the Johansen cointegration test statistics (i.e., the trace and Lambda-max statistics) as well as the 5 per cent critical value for these tests, as computed in MacKinnon, Haug, and Michelis (1999). In all cases, the null hypothesis is rejected if the test statistic is larger than the critical value. These results thus support the presence of one cointegrating vector between the real exchange rate, real non-energy commodity prices, and real energy commodity prices. We also tested for the presence of a cointegrating relationship between these three variables and the level of the Canada-US relative public sector debt (which, as shown in Table A3.1, is a non-stationary variable). It does seem more intuitive that this variable should influence the long-run value of the real exchange rate rather than the short-run dynamics. However, previous work had failed to find a long-run cointegrating relationship between these four variables.[17] As shown in Table A3.3, the Johansen cointegration test statistics also support the presence of one cointegrating relationship between the real exchange rate, real non-energy commodity prices, real energy commodity prices, and the level of the Canada-US relative public sector debt.

We make two modifications to this basic framework. First, we remove the $\Delta debtdif$ term from the short-run dynamics and consider two versions of the model: one with the three variables depicted in equation (1) in the cointegrating relationship and another with these three variables plus the level of the Canada-US relative public sector debt in the cointegrating relationship. Second, we add two terms to reflect multilateral exchange rate effects stemming from the United States. As discussed in section 2, the two key variables that reflect US imbalances and that are likely to instigate a multilateral adjustment of the US dollar are the US fiscal deficit and the US current account deficit.

Unit-root tests were also conducted on these two variables and are reported in Table A3.1. As shown, the DF-GLS unit-root test suggests that the fiscal balance to GDP ratio follows a stationary process but that the current account to GDP ratio contains a unit root. The latter is contrary to what one would expect and suggests that the intertemporal budget constraint is violated and that the current account is on an explosive path. Christopoulos and León-Ledesma (2004) also find that traditional unit-root tests for the US current account to GDP ratio suggest that the series is non-stationary, even when the sample is extended back to 1960. However, they argue that these tests suffer from an important loss of power if the dynamics of the series

17. See, for instance, Djoudad and Tessier (2000).

being tested exhibit non-linearities, which they show is the case for the US current account. They address this issue by analyzing the stationarity of the US current account using new econometric tests based on a non-linear adjustment, and find evidence that the US current account to GDP ratio is stationary when this non-linearity is taken into account. Given these results and our priors based on theoretical considerations, we decide to treat the US current account to GDP ratio as a stationary variable in our analysis.

By making these modifications, we obtain the following specifications for the two versions of our model:

$$\Delta \ln(rfx) = \alpha(\ln(rfx_{t-1}) - \beta - \phi \ln(comtot_{t-1}) - \pi \ln(enetot_{t-1})) + \delta intdif_{t-1} + \chi US\_cabal_{t-1} + \lambda US\_fisbal_{t-1} + \varepsilon_t \quad (2a)$$

$$\Delta \ln(rfx) = \alpha(\ln(rfx_{t-1}) - \beta - \phi \ln(comtot_{t-1}) - \pi \ln(enetot_{t-1}) - \eta debtdif_{t-1}) + \delta intdif_{t-1} + \chi US\_cabal_{t-1} + \lambda US\_fisbal_{t-1} + \varepsilon_t, \quad (2b)$$

where the dependent variable and the first four explanatory variables in equation (2b) were explained above; *US_cabal* is the US current account balance as a proportion of GDP; and *US_fisbal* is the US fiscal balance as a proportion of GDP. Equations (2a) and (2b) depict two versions of our bilateral exchange rate equation that incorporate multilateral factors but do not treat them as threshold effects. The next section describes a threshold model of the bilateral exchange rate with multilateral effects.

Before turning to the threshold model, it may be useful to discuss the expected signs on the coefficients in equations (2a) and (2b). First, the energy and non-energy price indexes in the cointegrating vector are proxies for the Canadian terms of trade and should play a role in the determination of the long-run value of the Canada-US dollar exchange rate. Since Canada is a major net exporter of both energy and non-energy commodities, one would expect that an increase in their price would lead to an appreciation of the Canadian dollar.[18] Second, the level of government debt in Canada relative to that in the United States should also play a role in the determination of the Canadian dollar in the longer run. One would expect an increase in this ratio to lead to a depreciation of the Canadian dollar in the

18. In terms of energy commodities, Canada became a major net exporter starting in the early 1990s with the increase in natural gas exports to the United States. Before then, net exports of these commodities were much smaller and sometimes negative.

long run, since higher Canadian government debt will likely lead to both higher domestic and foreign debt, which will eventually necessitate higher net exports to finance this excess absorption. It should be noted that in the short run, the effects on the exchange rate can be ambiguous. Indeed, the stimulative effects of higher government debt could put upward pressure on the currency in the short run, but this could be partially or fully offset by risk considerations if the level of the debt increased beyond the level considered to be sustainable.

Third, the Canada-US short-term interest rate differential term captures the effect of relatively higher interest rates in Canada—as a result of, for instance, relatively tighter monetary policy in Canada—on the Canada-US exchange rate. One would expect that an increase in this variable would lead to an appreciation of the Canadian dollar, as an increase in the rate of return of Canadian dollar-denominated assets should increase the demand for such assets.

Finally, the expected effects of the US current account and fiscal balances on the Canadian dollar are ambiguous. The arguments for the effects of the fiscal balance on the national currency were presented above, and suggest that they depend on both the time horizon and the market's perception as to the sustainability of the level of national debt. Similar arguments can also be made for the current account balance. Thus, if the US government is running fiscal and current account deficits, this could put upward pressure on the US dollar and hence lead to a depreciation of the Canadian dollar, as long as the market perceived the twin deficits to be sustainable. If, however, the market's perception were to change, this could reverse the effect and lead to downward pressure on the US dollar and hence an appreciation of the Canadian dollar. This analysis suggests that the effects of these US variables on the Canadian dollar might be best modelled in a framework with threshold effects. Such an approach is developed in the next section.

3.2 A threshold model of the bilateral exchange rate with multilateral effects

Threshold regression models have a variety of applications in economics and have increased in popularity in recent years. This type of model splits the sample into "regimes" based on the value of an observed variable, the so-called threshold variable. Given that the threshold level of the variable is typically unknown, it needs to be estimated along with the other parameters of the model. Several authors have contributed to developing a theory of estimation and inference of threshold models (also referred to as sample-splitting models) over the past decade or so, including Chan (1993), Hansen (1996, 1999), Caner (2002), and Caner and Hansen (2004).

Our bilateral exchange rate equation with multilateral effects shown above can be transformed into the following threshold model with two regimes:

$$\Delta\ln(rfx) = \alpha_1(\ln(rfx_{t-1}) - \beta_1 - \phi_1\ln(comtot_{t-1}) - \pi_1\ln(enetot_{t-1})) + \delta_1 intdif_{t-1} + \chi_1 US\_cabal_{t-1} + \lambda_1 US\_fisbal_{t-1} + \varepsilon_t, \qquad q_i \le q^* \tag{3a}$$

$$\Delta\ln(rfx) = \alpha_2(\ln(rfx_{t-1}) - \beta_2 - \phi_2\ln(comtot_{t-1}) - \pi_2\ln(enetot_{t-1})) + \delta_2 intdif_{t-1} + \chi_2 US\_cabal_{t-1} + \lambda_2 US\_fisbal_{t-1} + \varepsilon_t, \qquad q_i > q^*, \tag{3b}$$

where q is the threshold variable and q^* is the estimated threshold value. It is worth pointing out that this model allows the regression parameters to vary in the two regimes. We use the first lag of US fiscal balance as a proportion of GDP as our threshold variable to reflect the likelihood of multilateral exchange rate adjustment to a twin-deficits situation. The estimation procedure is discussed in the next section.

4 Estimation Methodology and Results

4.1 Estimation procedure

The parameters in the two single-equation models (equations (1) and (2)) are estimated by the non-linear least-squares method. Such a procedure is necessary given the presence of the long-term relationship between the real exchange rate, the terms-of-trade variables, and the debt differential (i.e., the error-correction terms). On the other hand, the threshold regression model (equation (3)) is estimated using a two-step procedure. The first step involves estimating the threshold parameter, q^*, that splits the sample into two regimes. In the second step, the other parameters associated with each regime are then estimated.

Simplifying Hansen's (1999) notation, we rewrite the threshold regression model (equation 3) in the following form:

$$y_t = \theta_1' X_t + e_t, \qquad q_t \le q^* \tag{4}$$

$$y_t = \theta_2' X_t + e_t, \qquad q_t > q^*, \tag{5}$$

where θ_1 and θ_2 are two parameter vectors associated with regimes 1 and 2, the observed sample is $\{y_t, X_t, q_t\}^T t = 1$, y_t is the dependent variable

$(\Delta \ln(rfx_t))$, X_t is a vector of exogenous variables, q_t is the threshold variable that is also included in X_t, and e_t is a mean-zero disturbance term.[19] The threshold parameter q^*, which is an element of q_t, is unknown and needs to be estimated.

The estimator of q^* minimizes the sum of squared errors from the regression of y_t on X_{qt} where $X_{qt} = X_t$ (if $q_t \le q^*$) or 0 otherwise. The non-linear least-square estimator of q^* minimizes the sum of squared errors, $S_r(q^*)$, as follows:

$$q^* \text{ minimizes } S_r(q^*);\ q^* \text{ is an element of } q_t. \quad (6)$$

Since $S_T(q^*)$ may take *T* distinct values, the estimation of q^* requires *T* evaluations of the function $S_T(q^*)$ (where *T* is the total number of observations).

As mentioned earlier, we use the first lag of the US fiscal balance, US_fisbal_{t-1}, as the threshold variable, q_t. The estimated value of the threshold parameter, q^*, is –2.65. Given the estimate of the threshold q^*, the sample is split into two subsamples, based on the indicators $US\_fisbal_{t-1} > -2.65$ and $US\_fisbal_{t-1} \le -2.65$. There are 71 observations in the first regime and 52 in the second. Figure A4.1 in Appendix 4 plots the evolution of the two multilateral variables—the US current and fiscal account balances—across the two regimes, where the shaded area indicates periods where the US fiscal balance is below its threshold value of –2.65 per cent of GDP. The two vectors of parameters, θ_1 and θ_2, in equations (4) and (5), are estimated using the non-linear least-squares method separately on each subsample.

4.2 Estimation results

Estimation results for equations (1), (2), and (3) are shown in Tables A4.1 and A4.2 in Appendix 4. Table A4.1 depicts the estimates for the bilateral exchange rate model without the debt differential in the cointegrating relationship (version A of the model), whereas Table A4.2 shows the estimates for the model with the debt differential in the cointegrating relationship (version B of the model). The estimates for the model without multilateral effects are presented first, followed by those for the models with multilateral effects (both with and without threshold effects). The first two models are estimated for the entire sample period (i.e., 1973Q1 to 2004Q1),

19. For example, $\theta_i = (\alpha_i \beta_i \varphi_i \pi_i \delta_i \chi_i \lambda_i)$ for $i = 1, 2$ and $X_t = (1 \ln(rfx_{t-1}) \ln(comtot_{t-1}) \ln(enetot_{t-1}) intdif_{t-1} US\_cabal_{t-6} US\_fisbal_{t-3})$.

whereas the threshold model is estimated for the two subsamples defined by the regimes.

The parameter estimates for the two specifications with multilateral effects in Table A4.1 are generally statistically significant at conventional levels and are of the expected sign. The estimated long-run effects suggest that an increase in the non-energy commodity price index leads to an appreciation of the real exchange rate across both models, as expected. The coefficient on the energy commodity price term is only statistically significant in the first regime of the threshold model, and it is positive, suggesting that an increase in energy prices leads to a depreciation in the Canadian dollar. This seemingly counterintuitive result has been noted in previous research and might be explained by the argument that Canada only became a significant net exporter of energy products starting in the early 1990s and that an energy-price increase could raise the demand for US dollar assets in the short run, especially if the price increase comes at a time of heightened global uncertainty (e.g., the oil-price shocks in 1974 and 1979) and US dollar assets are perceived as a safe refuge.[20] An increase in the Canadian short-run interest rate spread results in an appreciation of the Canadian dollar across all specifications, as expected.

The coefficients on the multilateral variables are also statistically significant, suggesting that multilateral effects do play a role in explaining movements in the Canadian dollar. In the model without threshold effects, both the US fiscal balance and the US current account balance are statistically significant. The coefficients suggest that a deterioration of both the US current and fiscal accounts leads to an appreciation of the Canadian dollar. In the threshold model, the current account balance is statistically significant across both regimes, whereas the fiscal balance is only significant in the first regime (i.e., when the threshold value for the fiscal balance is larger than –2.65 per cent of GDP). It should be noted, however, that the current account variable is lagged six quarters and the fiscal account is lagged three quarters, and thus that effects on the real exchange rate appear nine months to a year and a half later. Given the persistence of both these series, this suggests that the exchange rate responds to cumulative changes in these variables.

The results of the model with the Canada-US debt differential in the cointegrating relationship are shown in Table A4.2. They are very similar to the results for the specification without the debt differential. Although the coefficient on the debt-differential term is statistically significant in the

20. See Krugman (1983) for a model that captures the effect of oil prices on the demand for US-dollar assets.

specification without the multilateral effects, it is only significant in the second regime in the threshold model with multilateral effects. In both cases, it is positive, suggesting than an increase in Canadian government debt relative to the United States will tend to depreciate the Canadian dollar, as expected.

The models with multilateral effects also do a good job of explaining variations in the dependent variable. Indeed, the model without threshold effects explains about 27 per cent of the variation in the real exchange rate (compared to 15 per cent for the model without multilateral effects). The figures are similar for the model with the debt differential. The threshold model does much better, explaining 30 per cent of the variation in the dependent variable in the first regime and 35 per cent in the second regime; the corresponding values for the model with the debt differential are 29 and 47 per cent.

We conducted a series of specification and diagnostic tests on our two models with multilateral effects, equations (2) and (3). First, we tested whether the coefficients on the two multilateral variables, *US_cabal* and *US_fisbal*, are equal to zero. We did this by constructing a likelihood-ratio test using the maximum values of the log-likelihood functions for equations (1) and (2), which are reported in Tables A4.1 and A4.2. The likelihood-ratio test rejects the restriction that the coefficients on the US fiscal and current account balance in equation (2) are equal to zero.[21] Second, we tested whether the model's parameters are equal across the two regimes in equation (3), using a Wald test. Evidence that this was the case would suggest that the model with multilateral effects without threshold effects would be a better specification than that with threshold effects. The Wald test rejected the null hypothesis that all of the estimated parameters of the threshold regression model are stable across the two regimes.[22]

Third, we tested for the presence of serial correlation in the residuals. The Durbin-Watson (DW) tests reported in Tables A4.1 and A4.2 are in the inconclusive region (i.e., between the lower and upper bounds for the test

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21. The likelihood-ratio test statistic is equal to 8.72 (calculated as the difference between the log-likelihood values for the restricted and the unrestricted models times –2) for version A of the model and 10.9 for version B of the model, which are both greater than the 5 per cent critical value of 5.99. This test statistic is distributed as a chi-square with degrees of freedom equal to the number of restrictions (which in this case is two).

22. The Wald statistic is distributed as a chi-square random variable with degrees of freedom under the null hypothesis of parameter stability equal to the number of parameters to be tested. The Wald statistic in this case is 12.97 for version A of the model and 16.37 for version B of the model, whereas the 5 per cent critical value with 7 degrees of freedom is 14.07 and the 10 per cent critical value is 12.02.

statistic), suggesting that no conclusion can be drawn from this test regarding the presence of serial correlation in the residuals. Thus, we also investigated this issue using the Lagrange Multiplier (LM) test, which is valid in a wider range of situations than the DW test and allows for autoregressive or moving-average errors of arbitrary order. The LM test (one and two quarters out) suggested the presence of autocorrelation. To correct for this, we added a lagged dependent variable. By adding this variable to all of our specifications, the problem with serial correlation was eliminated (as shown in the LM test results in Table A4.3) and the estimation results were very similar. We concluded from this exercise that the consequences of the serial correlation were minor and decided to work with our specifications without the lagged dependent variable. Finally, we also tested the residuals for evidence of heteroscedasticity, using the autoregressive conditional heteroscedasticity (ARCH) test of first and second order. As shown in Table A4.3, the ARCH tests suggest that the residuals are not characterized by heteroscedasticity.

4.3 Simulations and forecasting performance

Dynamic simulations of the different models are shown in Figures A4.2 and A4.3 in Appendix 4, using parameter estimates for the entire sample (i.e., 1973Q1 to 2004Q1). Figure A4.2 shows version A of the models (i.e., those without the debt differential in the cointegrating relationship), whereas Figure A4.3 shows version B of the models (i.e., those with the debt differential in the cointegrating relationship). All of the models are fairly successful at accounting for broad movements in the Canada-US real exchange rate over the sample period. As shown, the correspondence between the simulated and actual values is quite close. There are, however, episodes of important deviations between the actual and simulated values—particularly in the mid-1980s, in 1998, and in the early part of this decade—but they tend to disappear after a short period of time. There are also differences across the models. Indeed, the simulated values from the two models with multilateral effects (with and without threshold effects) appear to match more closely the actual values than the Amano-van Norden model. In particular, they are both more successful at accounting for the appreciation of the Canadian dollar in the 2003–04 period, especially the threshold model.

Another way of analyzing these dynamic simulations is to decompose them over two periods: the period from 1973 to 2002 (when there was a broad depreciation of the Canadian dollar) and the period from 2003 to 2004 (when the loonie appreciated). This enables us to examine the relative contribution of the different variables in each model in explaining these two broad movements in the Canadian currency. Over the period from 1973 to 2002, the Canada-US real exchange rate depreciated by 29 per cent. Table A4.4 shows the decomposition of the dynamic simulations for each model over this period (for version A of the models only). The first column in the table lists the variables included in the different models with an arrow indicating the direction of the movement in the variable in question over the period 1973 to 2002, whereas the second column depicts whether each movement contributed to appreciating or depreciating the currency. As shown, the decline in commodity prices, the increase in energy prices, and the improvement in the US fiscal balance all tended to put downward pressure on the Canadian dollar. On the other hand, this was partially offset by the increase in the Canada-US short-term interest spread and the deterioration of the US current account, both of which tended to appreciate the Canadian dollar.

Given this, the Amano-van Norden model attributed this depreciation mainly to the fall in commodity prices, with a contribution from rising energy prices, and an offsetting effect from an increasing interest rate spread. The other two models found the same qualitative results, but they also suggest that the improvement in the US fiscal balance over this period made a small contribution to the depreciation, and that there was a large offsetting effect on this depreciation from a deterioration in the US current account balance that tended to appreciate the Canadian dollar.

Over the period 2003Q1 to 2004Q1, the Canadian-US real exchange rate appreciated by 14 per cent. Table A4.5 shows the decomposition of the dynamic simulations for each model over this period (for version A of the models only). As shown, the movements in four variables put upward pressure on the Canadian dollar: the increase in commodity prices and in the interest rate spread, as well as the deterioration in both the US fiscal and current account balances. The only offsetting factor over this period came from the increase in energy prices, which tended to depreciate the Canadian dollar. So in contrast to the previous period, all the explanatory variables over this period, except for energy prices, were moving in such a way as to put upward pressure on the Canadian dollar. Given this, the Amano-van Norden model attributed this appreciation to the increase in both commodity prices and an increasing interest rate spread, with a small offsetting effect from energy prices and a large proportion left unexplained (as shown by the contribution of the error). The other two models suggest that it was the increase in commodity prices and the interest rate spread as well as the deterioration in the US fiscal and current account balances that explained the appreciation, with a small offsetting effect from energy prices; there is still a proportion of the appreciation that is unexplained by these models, but the contribution of the error is much smaller compared with the

Amano-van Norden model. Thus, this decomposition of the dynamic simulations suggests that multilateral adjustment effects helped to explain part of the appreciation of the Canadian dollar in 2003–04.

We also examined the out-of-sample forecasting performance of the different models. To do this, we estimated all the models using dynamic rolling regressions starting with 1973Q1–2002Q4 as the sample period, moving up one quarter each time to generate a new forecast. This enabled us to compare the forecasting performance of the competing models from 2003Q1 to 2004Q1, the period of the recent appreciation of the Canadian dollar. Figures A4.4 and A4.5 in Appendix 4 depict the actual values of the real exchange rate as well as the forecasted values produced by the different models. The models with multilateral effects generally appear to do a better job with the out-of-sample forecasting over the 2003–04 period than the Amano-van Norden equation, with the possible exception of the threshold model with the debt differential in the cointegrating vector (i.e., version B of the model). This is confirmed by the forecasting performance measures reported in Table A4.6. Indeed, the reported values for the root-mean-squared error (RMSE) and Theil coefficient suggest that the models with multilateral effects (both versions A and B) outperform the Amano-van Norden specification over the period considered. Of the two models with multilateral effects, the model without threshold effects has a superior out-of-sample forecasting performance.

Conclusion

Rapid and significant appreciations of floating exchange rates, such as that experienced by the Canadian dollar in 2003 and again in 2004, pose a number of challenges for central banks in formulating the optimal monetary policy response, if any. In particular, how the central bank should react depends critically on the underlying forces behind the appreciation.

In 2003, the evidence, based on the experience of the exchange rates of other countries, indicated that some of the appreciation of the Canadian dollar reflected a multilateral adjustment to the large US external imbalance. The US current account deficit now stands at around 6 per cent of US GDP, and given that the US economy produces 30 per cent of global output, this deficit represents a significant and ongoing adjustment hurdle for the world economy. The adjustment problem is aggravated by the fact that several important US trading partners are not allowing their exchange rates to adjust, so that more of the adjustment burden in the short run is being placed on partners such as Canada, with floating rates.

In this paper, we successfully modified the traditional Bank of Canada bilateral exchange rate equation to allow for episodes of multilateral adjustment to US imbalances. Although the bilateral equation has had success in the past in explaining movements in the Can$/US$ exchange rate—especially over the medium to longer term—by relying on non-energy commodity prices and the interest rate differential, it was unable to explain adequately the magnitude or speed of the appreciation in 2003. In particular, our primary modification was the introduction of a threshold for the US fiscal deficit that allowed the specification of the equation to change between periods in which the deficit is high (greater than 2.65 per cent of GDP) to periods in which it was low. The finding that the US fiscal deficit variable is the empirically appropriate choice for the threshold, rather than the current account deficit, is intuitively appealing, because current account deficits can occur during investment booms (e.g., the latter half of the 1990s), and such external deficits are likely to be viewed as being more sustainable at current exchange rate levels than ones caused by fiscal deficits (the twin-deficits phenomenon). Domestic private investment financed by foreign borrowing is more likely to generate returns to service the debt than is government or private consumption. We also added the US fiscal and current account deficits as explanatory variables and found that the augmented threshold model dominates the traditional model in terms of in-sample explanatory power and out-of-sample forecasting ability.

Although our findings are preliminary, the main conclusion of the paper is that during periods of large US imbalances, fiscal and external, an exchange rate model of the Canadian dollar should allow for multilateral adjustment effects. Under such circumstances, monetary policies at home (and abroad) should not try to impede the adjustment but support domestic demand in order to facilitate the adjustment of resources from traded into non-traded goods.

Our effort represents one of the first attempts to allow for multilateral adjustment in a bilateral exchange rate model. The results of the threshold model suggest that there are significant non-linearities in the determination of the real exchange rate. Future work should concentrate on testing the robustness of these preliminary findings by exploring these non-linearities further with Canadian data and data of other countries.

Appendix 1
US Imbalances in the Postwar Period and the Canadian Dollar: A Graphical Depiction

Figure A1.1
The Canadian dollar and the US current account balance

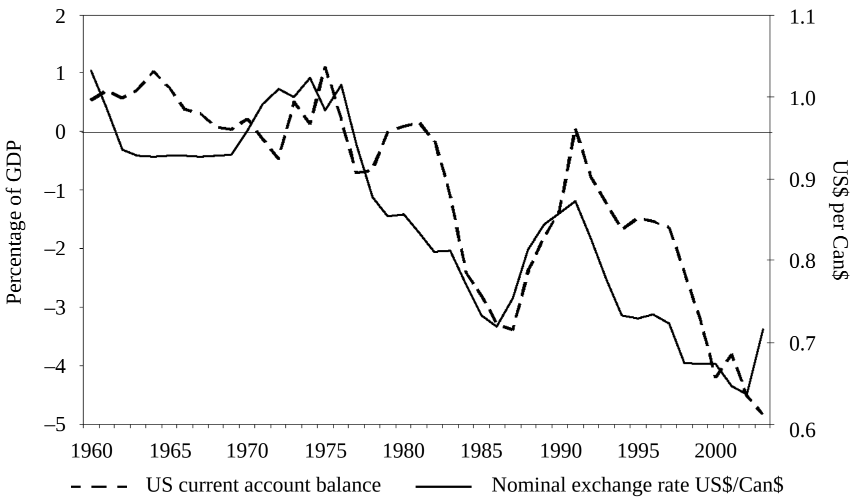


Figure A1.2
The US dollar and the US current account balance

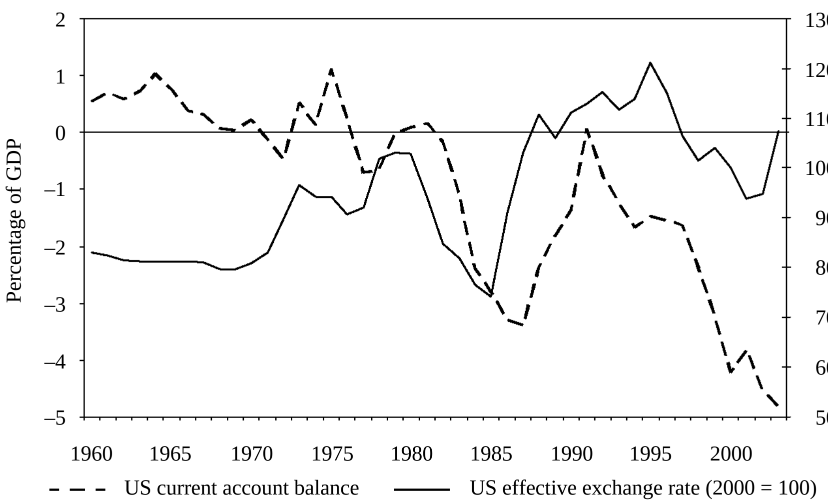


Figure A1.3
The US current account and the Canadian dollar (dynamic simulation and actual values)

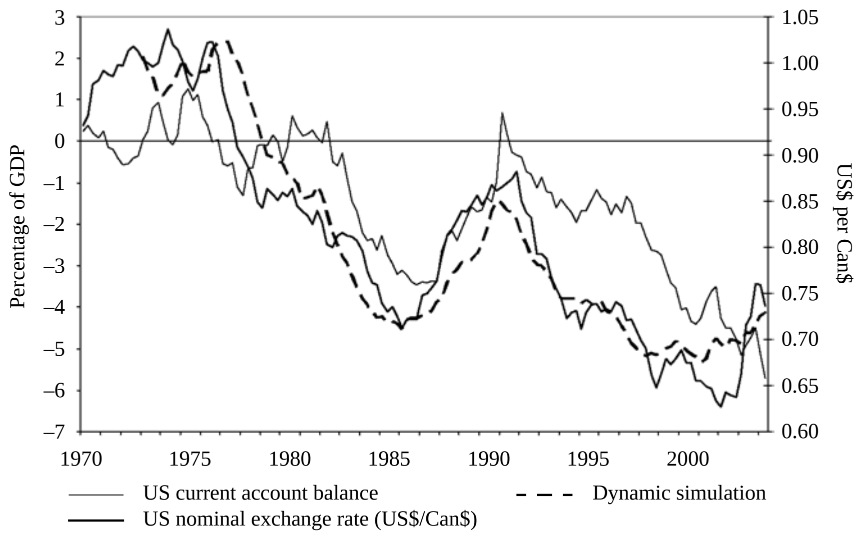


Figure A1.4
The US current and fiscal account balances

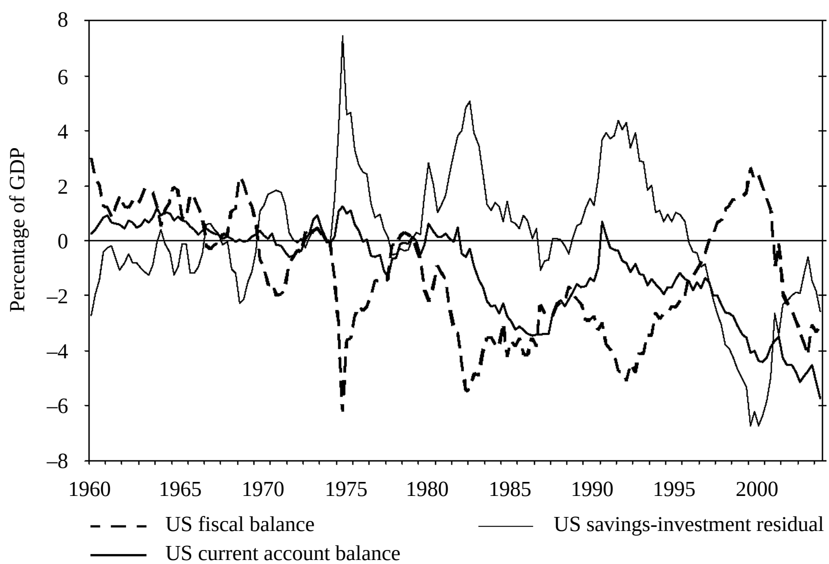
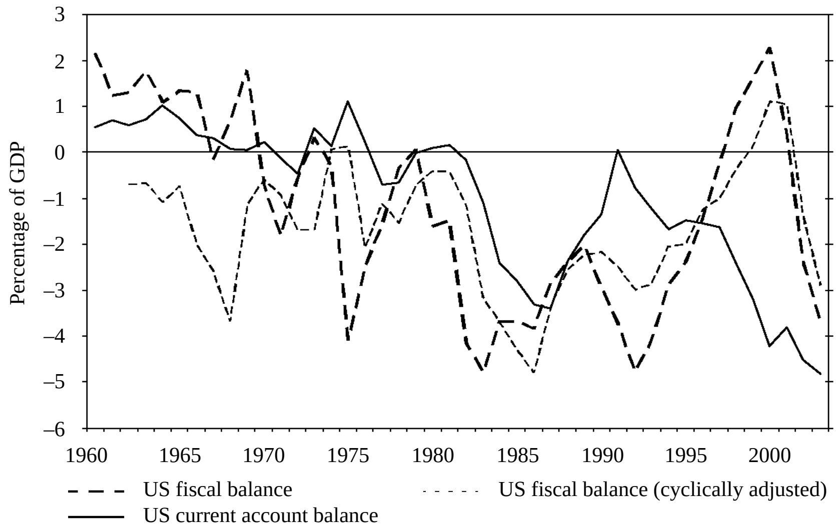


Figure A1.5
The cyclically adjusted US fiscal balance, the US fiscal balance, and the US current account



Appendix 2
Sources and Definitions of Variables

Dependent variable

1. $\Delta \ln(rfx_t)$

(Source: Bank of Canada internal database [for the nominal exchange rate], ratio of Statistics Canada series *v98086* to *v1992067* [for Canadian GDP deflator], and US Department of Commerce—Bureau of Economic Analysis series *jpgdp* [for US GDP deflator])

- log difference in the real quarterly Canada-US bilateral exchange rate constructed using the quarterly average of the noon daily spot rate, deflated by the ratio of the Canadian and US GDP deflators. Both deflators are indexed to 1997 = 1.0.

Explanatory variables

2. $\ln(comtot_{t-1})$

(Source: Bank of Canada [for nominal non-energy commodity price index, US dollar terms])

- log of the real non-energy commodity price index constructed as the nominal non-energy commodity price index deflated by the US GDP deflator.

3. $\ln(enetot_{t-1})$

(Source: Bank of Canada [for nominal energy commodity price index, US dollar terms])

- log of the real energy commodity price index computed as the nominal energy commodity price index deflated by the US GDP deflator.

4. $intdif_{t-1}$

(Source: Statistics Canada series *v122491* [for Canadian rates], and Federal Reserve Board [for US rates])

- short-term interest rate spread constructed as the difference between Canadian three-month prime corporate paper rate and US 90-day AA non-financial commercial paper closing rate.

5. $debtdif_{t-1}$

(Source: Sum of Statistics Canada series *v34422*, *v34460*, *v34584* [for total government debt in Canada], *v498086* [for Canadian GDP], and US

Congressional Budget Office [for total US government debt as a proportion of GDP])

- Canada-US total government debt to GDP ratio.

6. US_cabal_{t-6}

(Source: US Department of Commerce, Bureau of Economic Analysis series *bopcrnt* [for current account balance] and *gdp* [for GDP])

- Balance on US current account as a proportion of GDP.

7. US_fisbal_{t-3}

(Source: US Department of Commerce, Bureau of Economic Analysis series *def@gi*)

- US total government fiscal balance as a proportion of GDP.

Appendix 3
Unit-Root, Cointegration, and Weak Exogeneity Test Results

Table A3.1
DF-GLS unit-root tests
(Sample period: 1973Q1 to 2004Q1)

Variable	Trend	No trend
ln(*rfx*)	–2.38	–1.74
Δ*ln*(*rfx*)	**–2.96**	**–2.48**
ln(*comtot*)	–2.91	–1.45
ln(*enetot*)	–1.64	–0.38
debtdif	–0.83	–1.00
Δ*debtdif*	–1.13	–0.89
intdif	–2.17	**–2.15**
US_cabal	–1.76	–0.22
US_fisbal	–2.38	**–2.19**

Notes:
(i) The Dickey-Fuller Generalized Least Squares (DF-GLS) test is based on Elliott, Rothenberg, and Stock's (1996) modification to the Augmented Dickey-Fuller (ADF) test. Under this test, the variable is first locally detrended/demeaned and then tested for the presence of a unit root in the usual ADF manner. The power of DF-GLS is substantially improved over the original version of ADF, particularly for finite samples. As with the ADF test, the null hypothesis states that the variable contains a unit root.
(ii) The number of lags used in the test was selected based on the Modified Akaike Information Criterion, developed by Ng and Perron (2001).
(iii) Bolded values exceed the 5 per cent critical value.

Table A3.2
Johansen cointegration test results for ln(*rfx*), ln(*comtot*), and ln(*enetot*)
(Sample period: 1973Q1 to 2004Q1)

	No trend			Trend		
No. of cointegrating vectors under H_0	**Trace statistic**	**5% critical value**	***P*-value**	**Trace statistic**	**5% critical value**	***P*-value**
Fewer than 1	18.40	29.80	0.53	42.78	42.92	0.052
Fewer than 2	07.70	15.49	0.50	12.66	25.87	0.760
No. of cointegrating vectors under H_0	**λ-max statistic**	**5% critical value**	***P*-value**	**λ-max statistic**	**5% critical value**	***P*-value**
Fewer than 1	10.72	21.13	0.67	30.12	25.82	0.01
Fewer than 2	05.33	14.26	0.70	09.41	19.39	0.68

Notes:
(i) The values reported under the column labelled "No trend" assume a constant in the cointegration space and a linear deterministic trend in the data.
(ii) The values reported under the column labelled "Trend" assume a constant and a linear deterministic trend in the cointegration space, as well as a linear deterministic trend in the data.
(iii) The number of lags used in the test was selected based on a sequential modified likelihood ratio test.
(iv) The critical values are based on MacKinnon, Haug, and Michelis (1999).

Table A3.3
Johansen cointegration test results for ln(*rfx*), ln(*comtot*), ln(*enetot*), and *debtdif*
(Sample period: 1973Q1 to 2004Q1)

	No trend			Trend		
No. of cointegrating vectors under H_0	**Trace statistic**	**5% critical value**	***P*-value**	**Trace statistic**	**5% critical value**	***P*-value**
Fewer than 1	76.28	47.86	0.00	90.58	63.88	0.00
Fewer than 2	27.29	29.80	0.09	35.72	42.92	0.22
No. of cointegrating vectors under H_0	**λ-max statistic**	**5% critical value**	***P*-value**	**λ-max statistic**	**5% critical value**	***P*-value**
Fewer than 1	48.99	27.58	0.00	54.86	32.12	0.00
Fewer than 2	14.94	21.13	0.29	22.68	25.82	0.12

Notes: See notes for Table A3.2.

Table A3.4
Weak exogeneity tests for ln(*rfx*), ln(*comtot*), and ln(*enetot*)
(Sample period: 1973Q1 to 2004Q1)

LR test statistic (no trend)	***P*-value**	**LR test statistic (trend)**	***P*-value**
3.95	0.14	3.78	0.15

Notes:
(i) The number of lags used in the test was selected based on a sequential modified likelihood-ratio test.
(ii) The likelihood-ratio (LR) test statistic follows a χ^2 (df) distribution.

Appendix 4
Estimation and Forecasting Results

Table A4.1
Estimates for bilateral exchange rate model (version A) with and without multilateral effects
(Sample period: 1973Q1 to 2004Q1)

	No multilateral effects		Multilateral effects		Multilateral effects threshold model			
	All observations				*q > –2.648*		*q = –2.648*	
Variable	**Estimate**	**Std. error**	**Estimate**	**Std. error**	**Estimate**	**Std. error**	**Estimate**	**Std. error**
speed of adj.	–0.1561***	0.0329	–0.1358***	0.0344	–0.2122***	0.0486	–0.1057**	0.0522
constant	1.7680***	0.2415	2.9429***	0.5052	2.4879***	0.2953	3.8033**	1.6974
comtot	–0.3621***	0.0455	–0.5567***	0.0940	–0.4958***	0.0583	–0.6018**	0.2741
enetot	0.0640**	0.0304	0.0376	0.0344	0.0614**	0.0286	–0.0961	0.1033
intdif	–0.6973***	0.1521	–0.5158***	0.1516	–0.4569**	0.1763	–0.7435**	0.2791
US_fisbal			0.0023**	0.0011	0.0030**	0.0015	–0.0007	0.0026
US_cabal			0.0070***	0.0016	0.0051***	0.0019	0.0087***	0.0028
R^2	0.1806		0.3044		0.3575		0.4252	
Adj. R^2	0.1540		0.2684		0.2973		0.3485	
Log-likelihood	327.26		322.90		197.64		135.12	
No. of obs.	128		123		71		52	
D.-Watson	1.17		1.44		1.60		1.39	

Note:
(***), (**), and (*) indicate statistical significance at the 1 per cent, 5 per cent, and 10 per cent levels, respectively.

Table A4.2
Estimates for bilateral exchange rate model (version B) with and without multilateral effects
(Sample period: 1973Q1 to 2004Q1)

	No multilateral effects		Multilateral effects		Multilateral effects threshold model			
	All observations				*q > –2.648*		*q = –2.648*	
Variable	**Estimate**	**Std. error**	**Estimate**	**Std. error**	**Estimate**	**Std. error**	**Estimate**	**Std. error**
speed of adj.	–0.1528***	0.0326	–0.1356***	0.0343	–0.2114***	0.0488	–0.1152**	0.0473
constant	1.0048**	0.4979	2.2141***	0.6915	2.7095***	0.4962	1.8090	2.0930
comtot	–0.2396***	0.0812	–0.4492***	0.1133	–0.5321***	0.0890	0.2071	0.3162
enetot	0.0909**	0.0357	0.0729	0.0451	0.0545*	0.0306	0.2094	0.1282
debtdif	0.0023*	0.0013	0.0020	0.0017	–0.0007	0.0011	0.0150**	0.0074
intdif	–0.6296***	0.1543	–0.4565***	0.1580	–0.5048**	0.1949	–0.6972***	0.2526
US_fisbal			0.0024**	0.0011	0.0027*	0.0016	0.0009	0.0024
US_cabal			0.0066***	0.0016	0.0051***	0.0019	0.0044	0.0028
R^2	0.2057		0.3144		0.3611		0.5409	
Adj. R^2	0.1731		0.2726		0.2901		0.4679	
Log-likelihood	329.24		323.79		197.84		140.96	
No. of obs.	128		123		71		52	
D.-Watson	1.20		1.45		1.64		1.62	

Note:
(***), (**), and (*) indicate statistical significance at the 1 per cent, 5 per cent, and 10 per cent levels, respectively.

Table A4.3
Residual diagnostic tests *p*-values (all observations)

	Multilateral effects		Amano-van Norden	
	Version A	**Version B**	**Version A**	**Version B**
LM(1)	0.85	0.87	0.94	0.95
LM(2)	0.15	0.17	0.41	0.40
ARCH(1)	0.41	0.55	0.17	0.30
ARCH(2)	0.65	0.82	0.19	0.44

Note: The reported values for the LM test are those for the model that includes a lagged dependent variable.

Table A4.4
Decomposition of dynamic simulation
Simulation period: 1973Q1 to 2002Q4
Contribution of each variable to explaining depreciation of real exchange rate over simulation period

Variable (direction of move in variable)	App./dep.	Amano-van Norden	Multilateral	Threshold
Commodity prices (↓)	dep.	91%	137%	128%
Energy prices (↑)	dep.	13%	7%	13%
Interest rate spread (↑)	app.	–20%	–16%	–11%
US fiscal balance (↑)	dep.		5%	3%
US current account (↓)	app.		–52%	–27%
Lags		–2%	–1%	–12%
Error		18%	20%	6%
Total		100%	100%	100%

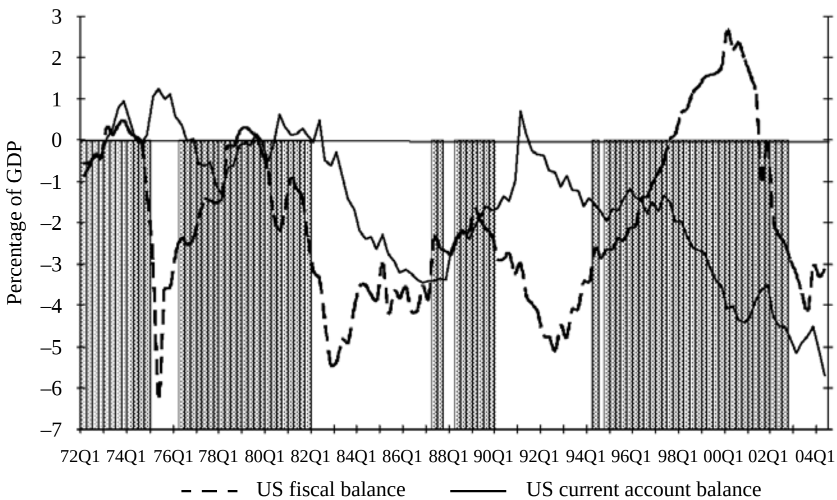
Table A4.5
Decomposition of dynamic simulation
Simulation period: 2003Q1 to 2004Q1
Contribution of each variable to explaining appreciation of real exchange rate over simulation period

Variable (direction of move in variable)	App./dep.	Amano-van Norden	Multilateral	Threshold
Commodity prices (↑)	app.	12%	16%	14%
Energy prices (↑)	dep.	–3%	–2%	4%
Interest rate spread (↑)	app.	7%	6%	8%
US fiscal balance (↓)	app.		5%	–1%
US current account (↓)	app.		11%	13%
Lags of dependent variable		16%	23%	39%
Error		68%	41%	23%
Total		100%	100%	100%

Table A4.6
Out-of-sample forecasting performance (dynamic) using actual values for the explanatory variables
Estimation period: 1973Q1 to 2002Q4
Forecasting period: 2003Q1 to 2004Q1

	Version A		Version B	
Model	**RMSE**	**Theil coefficient**	**RMSE**	**Theil coefficient**
Amano-van Norden	0.046	0.086	0.045	0.084
Multilateral effects	0.036	0.067	0.036	0.067
Threshold model	0.043	0.078	0.044	0.080

Figure A4.1
Multilateral variables in the two regimes



Note: Shaded regions indicate periods of total government fiscal balance less than the –2.648 per cent threshold.

Figure A4.2
Dynamic simulations of competing models
Version A of models

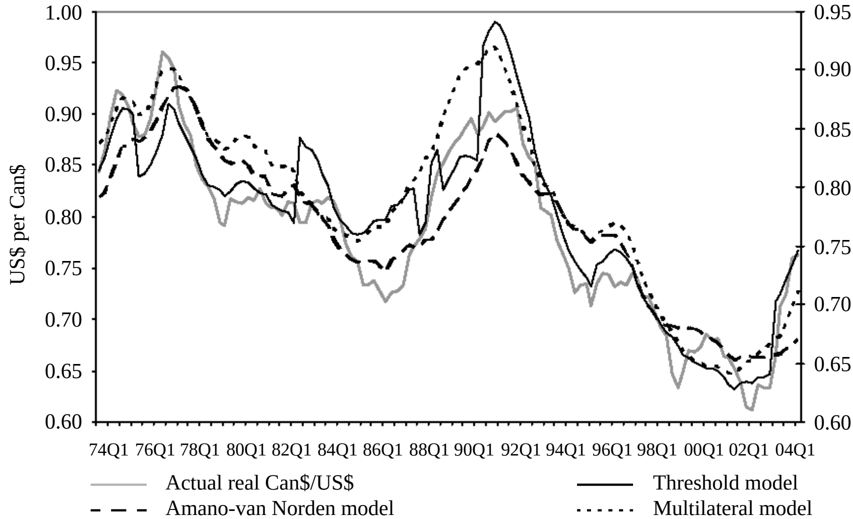


Figure A4.3
Dynamic simulations of competing models
Version B of models

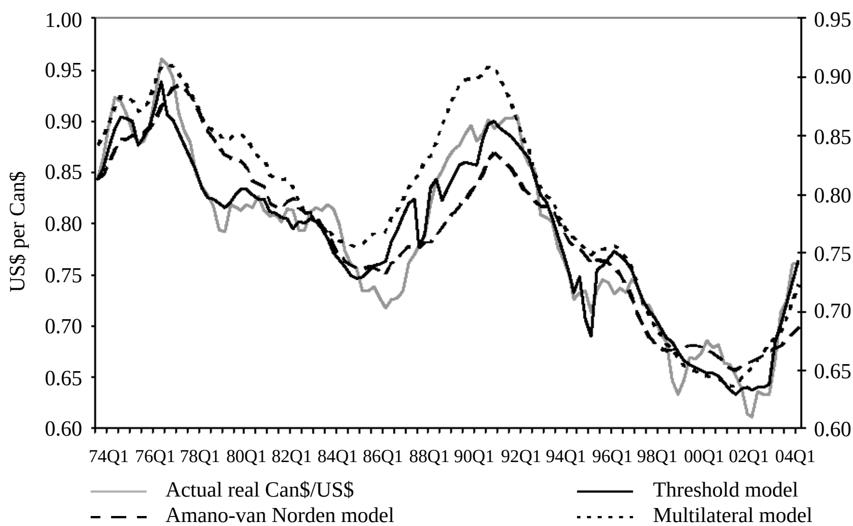


Figure A4.4
Out-of-sample performance of competing models using actual values for explanatory variables
Version A of models
Estimation period: 1973Q1 to 2002Q4
Forecasting period: 2003Q1 to 2004Q1

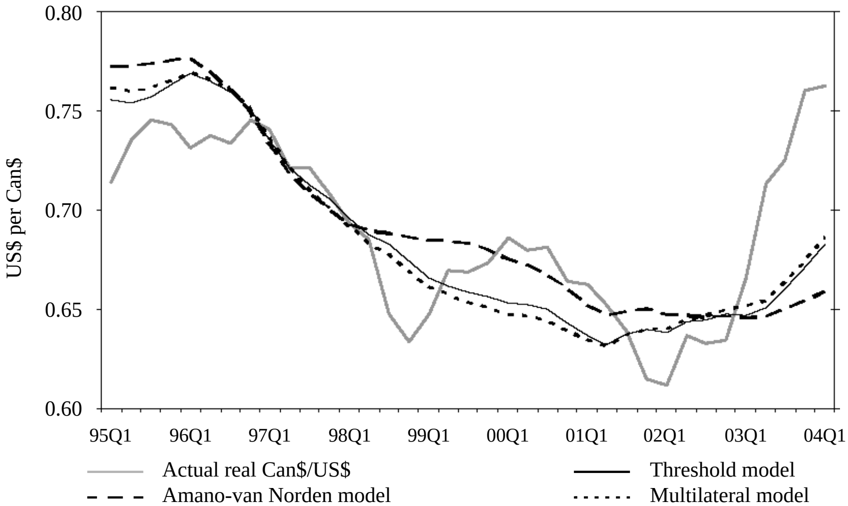
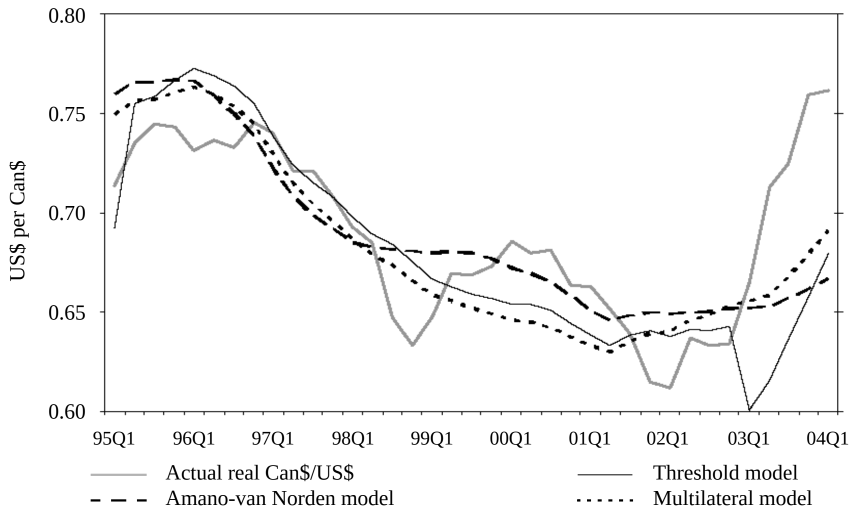


Figure A4.5
Out-of-sample performance of competing models using actual values for explanatory variables
Version B of models
Estimation period: 1973Q1 to 2002Q4
Forecasting period: 2003Q1 to 2004Q1



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Discussion

Paul Masson

Introduction

The major goals of the paper by Bailliu, Dib, and Schembri are to understand the appreciation of the Canadian dollar in 2003 and to predict its level in the future. The heretofore successful Bank of Canada exchange rate equation, termed the Amano-van Norden equation, did not predict the appreciation. This equation is driven mainly by Canada-US interest differentials and non-energy commodity prices.

The authors identify possible culprits—in particular, the omission of the US current account and fiscal deficits. They add these variables to the equation for the bilateral real exchange rate of the Canadian dollar against the US dollar, and in one version of the model, the US fiscal deficit is used as a threshold variable for dividing the sample into two subsamples, with the notion that in the case of a large deficit, the two US variables should have a significant influence in explaining appreciation of the Canadian dollar, which might not necessarily be the case when the deficit is smaller.

The authors argue that multilateral models of exchange rate adjustment justify the inclusion of the US current account and fiscal variables. When the US dollar is depreciating against all (or many) currencies, presumably as a result of large domestic and external imbalances, then naturally enough Canada will participate in the adjustment—in addition to the purely Canadian influences driving the loonie.

The authors' model, however, is not fully fleshed out. They acknowledge that a rigorous multilateral model would involve other variables (and perhaps involve estimating other equations) and be considerably more complicated. In the interest of simplicity and tractability, however, and building on the previous work, they chose to remain with a reduced-form framework.

The Macroeconomic Balance Approach

Even if one takes this tack, it is useful to attempt to relate the reduced form to the variables that would be suggested by a more structural approach. One such model is the macroeconomic balance approach, which views the current account as the bridge between two identities—one that expresses the current account as the difference between domestic saving (in which fiscal deficits appear as dissaving) and domestic investment, and the other as the sum of the trade balance and net investment income from abroad. The first identity can be related to the usual determinants of saving and investment—interest rates, economic growth, and the fiscal position—and the second to price competitiveness and the output gap. With a little manipulation, this model can be used to create a reduced form for the bilateral real exchange rate in terms of US and Canadian variables, since third-country effects wash out under certain conditions.

Let me illustrate this approach in the case that we are considering (see box). Saving depends positively (though the sign is theoretically ambiguous) on the level of interest rates, while investment depends negatively. Saving may also involve a Ricardian offset of fiscal deficits, and investment may depend on actual or expected economic growth, as implied by the accelerator model. It is assumed that the coefficients here are the same across all countries. Looking at the international side, an improvement in the current account needs to involve a real effective exchange rate depreciation, output below potential, or both. (The role of net foreign assets in augmenting net investment income from abroad, which is also a component of the current account, is ignored, but it could easily be included here.) In the current account equation, the coefficients are not assumed to be the same across countries, since they will depend on country size and openness.

These simple equations can be manipulated to give an equation for the real bilateral Canadian dollar, by subtracting the real effective exchange rate of the Canadian dollar from that for the US dollar. Third-country effects will wash out, provided the United States' weight in Canada's effective rate calculation is equal to Canada's weight in the US calculation. This is unlikely to be the case in practice, making the exclusion of third-country effects problematic. Like the authors, however, I will proceed as if they could be excluded.

Even though simple, this multilateral framework suggests some stark differences with respect to the estimating equation in the paper.

The Macroeconomic Balance Approach

Let

I = the investment/GDP ratio,

S = the saving/GDP ratio,

y = the log of output,

$\bar{y}$ = potential output,

r = the interest rate,

F = the fiscal balance, as a ratio to GDP,

C = the current account balance, as a ratio to GDP,

reer = the log of the real effective exchange rate,

p = the log of the domestic price level,

rxr = the log of the real bilateral (US dollar) exchange rate,

e = log of the nominal bilateral (US dollar) exchange rate.

Let us assume the following saving, investment, and net exports equations, for each country i:

$$S_i = a_0 + a_1 r_i - a_2 F_i + u_i$$

$$I_i = b_0 - b_1 + b_2 \Delta y_i + v_i$$

$$C_i \equiv S_i - I_i + F_i$$

$$C_i = c_{0i} + c_{1i}\, reer_i + c_{2i}\, (y_i - \bar{y}_i) + w_i \ . \qquad (1)$$

From the last of these equations, we can derive an equation for each country's real effective exchange rate in terms of its current account and output gap:

$$reer_i = [C_i - c_{0i} - c_{2i}(y_i - \bar{y}_i) - w_i] / c_{1i} . \qquad (2)$$

But the current account, in turn, can be written in terms of the interest rate, fiscal position, and rate of growth:

$$C_i = (a_0 - b_0) + (a_1 - b_1) r_i + (1 - a_2)\, F_i - b_2 \Delta y_i + (u_i - v_i). \qquad (3)$$

In addition, we can express the real effective exchange rate in terms of bilateral real exchange rates:

$$reer_i = p_i + e_i - \sum_{j \neq i} x_{ij} (p_j + e_j) = rxr_i - \sum_{j \neq i} x_{ij} rxr_j ,$$

where x_{ij} = weight of country j in the effective exchange rate calculation for country i (summing to one across all j), and

$$rxr_i = p_i + e_i - p_{US} \text{ and } rxr_{US} = 0 \text{ since } e_{US} \equiv 1 .$$

Let us gather all of the third-country exchange rates into one variable, e_{ROW}.

Now we can express the bilateral rate in terms of the difference between Canadian and US real effective rates:

$$rxr_{CA} = reer_{CA} - reer_{US} + x_{CA,\,US} p_{US} - x_{US,\,CA}(p_{CA} + e_{CA}) \\ + (x_{CA,\,ROW} - x_{US,\,ROW})(p_{ROW} + e_{ROW}) .$$

If $x_{CA,\,US} = x_{US,\,CA} = \bar{x}$, then the third term above goes to zero and the rest simplifies to

$$rxr_{CA} = \frac{1}{1 + \bar{x}} (reer_{CA} - reer_{US}) . \qquad (4)$$

Substituting equations (2) and (3) into equation (4) yields a reduced-form equation for Canada's bilateral real exchange rate:

$$rxr_{CA} = \frac{1}{1 + \bar{x}} \{[(a_0 - b_0 - c_{0,\,CA}) + (a_1 + b_1) r_{CA} + (1 - a_2) F_{CA} \\ - b_2 \Delta y_{CA} + (u_{CA} - v_{CA} - w_{CA}) - c_{2,\,CA}(y_{CA} - \bar{y}_{CA})] / c_{1,\,CA} \\ - [(a_0 - b_0 - c_{0,\,US}) + (a_1 + b_1) r_{US} + (1 - a_2) F_{US} - b_2 \Delta y_{US} \\ + (u_{US} - v_{US} - w_{US}) - c_{2,\,US}(y_{US} - \bar{y}_{US})] / c_{1,\,US} \} ,$$

or, more compactly,

$$rxr_{CA} = d_0 + d_1 r_{CA} + d_2 F_{CA} + d_3 \Delta y_{CA} + d_4 (y_{CA} - \bar{y}_{CA}) \\ + d_5 r_{US} + d_6 F_{US} + d_7 \Delta y_{US} + d_8 (y_{US} - \bar{y}_{US}) + z , \qquad (5)$$

where d_i are composite coefficients and z is a composite error term.

- Current account balances do not appear—either for Canada or for the United States.
- Both countries' fiscal variables appear, not just that for the United States.
- Additional variables are growth and the output gap, for both the United States and for Canada. Growth in Canada appreciates its real effective exchange rate, while output below potential depreciates it, *ceteris paribus*.

Re-estimating the Model

Even if one does not take the restrictions of the model too seriously, and even if one ignores third-country effects (on the basis of the argument made above), the macroeconomic balance approach suggests that there are additional variables that should enter the equation, while excluding the US current account balance in favour of its fundamental determinants. As a first attempt at estimating such a model, I tried adding Canadian fiscal and current account balances to their equation, along with their US counterparts (which are in the Bank of Canada model). They are added to the basic (non-threshold) model, in which the other explanatory variables are the interest differential, energy, and non-energy commodity terms of trade.

Interestingly enough, Canada's fiscal position (lagged three times, like the US variable) was significant—with *t*-values ranging from 2.4 to 4.3—whatever the lag selected for Canada's current account balance. The latter variable was never significant with the expected sign and sometimes had the wrong sign.

What I didn't do (lacking immediately available data) was to include the output gaps or the rate of output growth in the two countries—but this would be an obvious thing to attempt, while dropping the US current account variable in favour of its more basic determinants.

While the use of a more complicated error-correction model, estimation of a cointegrating vector, and the use of the US fiscal balance as a threshold variable are fully consistent with the general macroeconomic balance framework I have described, I have additional concerns about the paper's empirical implementation.

- The sample seems to have been divided in the same way (with the same threshold for the US fiscal balance of –2.64 per cent of GDP), whatever the other estimates of the model. Surely a complete estimation of model B would give a different threshold than that for model A. It is not clear why a two-step approach was used here.

- The test for equality of the coefficients in the two regimes (which apparently rejects the hypothesis) needs to reflect the fact that the classification of the data points is endogenous. After all, we are choosing the subsamples to maximize differences between them. This makes the critical values larger (I believe that Andrews has calculated the correct critical values).

Conclusion

I would note in closing that the out-of-sample predictive power of the threshold model is, in fact, very poor, so that instead of a price of the US dollar of Can$1.30, which we saw early in 2004, the model predicted Can$1.47. There is clearly more work to do. Or perhaps the authors, like many of the rest of us, will accept the consensus in the economics profession that structural exchange rate models are not very effective at explaining short-term exchange rate movements out of sample.