

Firm Life-Cycle Dynamics and Productivity

Danny Leung Césaire A. Meh Yaz Terajima *

Bank of Canada

PRELIMINARY AND INCOMPLETE

May 17, 2006

Abstract

Total factor productivity differs greatly across countries, in particular between the U.S and Canada. We construct a general equilibrium model of firm dynamics and technological adoption to provide some rationale for this difference. We calibrate the model to Canada and use it to determine whether and how much the differences in the determinants of firm size can account for the differences in the firm size distributions and the productivity gap. These determinants are taxation, the access to finance and the costs of technological adoption. Our preliminary analysis suggests that these determinants, in particular, the costs of adopting a more advanced technology can play an important role in determining the productivity level.

Keywords: Firm size dynamics, financial constraints, productivity.

*Bank of Canada, 3 West, 234 Wellington, Ottawa, Ontario, K1A 0G9 Canada. *E-mail address:* dleung@bankofcanada.ca (Leung); cmeh@bankofcanada.ca (Meh); yterajima@bankofcanada.ca (Terajima). The views expressed in this paper are those of the authors. No responsibility should be attributed to the Bank of Canada.

1 Introduction

Total factor productivity (TFP) differs greatly across countries (Hall and Jones (1997), and Klenow and Rodriguez-Clare (2004)). This is true between rich countries and poor countries, but also among more developed nations with the U.S. being the leader. Parente and Prescott (1999, 2000) argue that differences in institutions are the source of these differences in TFP across countries. What is difficult to understand is the large differences in TFP among developed countries whose basic institutions (property rights, minimal levels of corruption, independent judiciary) are well-established. This paper addresses this issue by examining the sources of the difference in TFP between Canada and the United States. Understanding the sources of this difference in TFP between these two countries will provide insights into explaining the heterogeneity in TFP among other developed countries.

Since the mid-1980s, Canada's TFP growth has been weak compared to the United States. Canada's level of TFP relative to the United States dropped from 96.4 to 83.4% between 1985 and 2005. Over much of the same period, ICT investment has been lagging and average firm size has fallen relative to the United States. Employment per firm in Canada declined from 64% of that in the United States in 1988 to 54% in 1999, and ICT investment as a fraction of GDP in the United States has been more than twice as large as in Canada. The decline in firm size and the lower rate of ICT investment are related as Sharpe (2005) has shown that smaller firms are less likely to adopt ICT than larger firms.

This paper presents a model of firm life-cycle dynamics that is used to examine how much the differences in the determinants of firm size and technology adoption between Canada and the United States contribute to the observed difference in TFP between the two countries.¹ Specifically, we construct a model where a firm is started up through the occupational choice of a household, is grown and then sold via a initial public offering to the publicly-traded business sector. A business sale is modeled in the spirit of Holmes and Schmitz (1990, 1995), and Chari, Golosov and Tsyvinski (2004). The firm size distribution endogenously arises in the equilibrium and aggregate TFP is obtained by summing across individual firms.

The adoption of new technology increases TFP at the firm level.² This adoption is costly and, in this sense, our theory is similar to Parente and Prescott (1994).³ The cost of tech-

¹Examining firm life-cycle dynamics is important for understanding the divergence in TFP because it has been shown by OECD (2001) that TFP growth within each firm contributes most to aggregate TFP growth.

²As argued by Brynjolfsson and Hitt (2000), ICT adoption facilitates TFP enhancing changes in the organization of production. The increase in TFP subsequently leads to increases in the scale of the firm as the marginal product of each input rises.

³In particular, Astebro (2002) argues that small firms are slower to adopt new technologies because there

nology adoption inhibits firm growth because of capital market imperfections. There are several papers that study firm dynamics and productivity under capital market imperfections including Cooley and Quadrini (1999), Jermann and Quadrini (2003), Kochlerlakota (2001), and Erosa and Hidalgo-Cabrillana (2005). There are also papers that examine the impact of institutions on firm size and aggregate TFP, such as Restuccia and Rogerson (2003) and Guner, Ventura and Yi (2005). Our theory of firm dynamics differs from them in three main dimensions. First, all the firms in our model are endogenously created by a household in the model economy. Second, the technological adoption is endogenous. Third, a private business owned by a single household can be sold to the publicly-traded sector, where the firm will be owned by all the households in the economy. We argue these dimensions are important in analyzing the effects of determinants of firm size on the aggregate productivity.

The rest of the paper is organized as follows. Section 2 discusses the empirical observations on productivity and firm size for Canada and the United States. The model is introduced in Section 3, and in Section 4 we lay out our calibration exercise. Finally in Section 5, we discuss our preliminary results. Section 6 concludes.

2 Empirical Observations in Canada and the United States

In this section, the data on productivity levels, productivity growth, average firm size, and firm size distribution in Canada and the United States are presented first. Afterwards, an experiment that illustrates the potential impact of the changes in the average firm size on the changes in the productivity level gap between the two countries is provided. The possible determinants of both productivity and firm size are then discussed in turn.

2.1 Productivity Levels and Growth

Between 1961 and 1985, business sector total factor productivity (TFP) growth was stronger in Canada than in the United States. The 1.7% average annual growth experienced in Canada was 0.3 percentage points higher than in the United States. This brought the level of Canadian TFP to 96.4% of that of the United States in 1985.⁴ After 1985, Canadian TFP growth

exists non-capital fixed costs, such as the costs of information acquisition.

⁴Relative levels of TFP are obtained by taking the official TFP growth rates from Statistics Canada and the Bureau of Labor Statistics, and applying them to the relative level of TFP in 1999 from Rao, Tang and Wang. (2004). They find that Canada's level of TFP was 86% of that of the United States.

Table 1: Employment per Firm, Canada

	<19	20-99	100-499	500+	Total
1988	3.2	40.7	203.7	2593.3	13.0
1989	3.0	38.5	192.9	2364.7	12.3
1990	2.9	37.7	187.2	2304.6	11.8
1991	3.0	39.6	196.0	2049.3	12.1
1992	3.0	39.5	194.9	2071.4	11.7
1993	3.0	39.5	195.9	2066.6	11.7
1994	3.0	39.6	193.2	1965.4	11.8
1995	3.1	39.6	194.1	1963.9	12.1
1996	3.1	39.6	195.2	1970.8	12.1
1997	3.0	40.0	193.9	2038.5	12.3
1998	3.0	40.3	192.8	2011.7	12.4
1999	3.0	40.4	192.6	2000.8	12.3

weakened substantially. The 0.4% average annual TFP growth in Canada over the 1985-2005 period was 0.7 percentage point lower than in the United States. As a result, by 2004, the level of TFP in Canada had fallen to 83.4% of that of the United States. The evolution of relative business sector labour productivity followed a similar pattern. It rose from 79.4% to 90.0% between 1961 and 1985, but fell back to 73.5% by 2005. These gaps in TFP and labour productivity are not due to differences in industry composition as Rao, Tang and Wang (2004) show that both TFP and labour productivity were lower in Canada than in the United States for a number of industries.

2.2 Firm Size

It is commonly perceived that Canada has smaller firms than in the United States. Data on the average level of employment per firm by employment size of firm is shown in Tables 1 and 2.⁵ In all the years (1988-1999) in which the data are available for both countries, firms in the United States on average employed at least seven more employees than firms in Canada. This implies that the average Canadian firm was at most only 64% of the size of

⁵The U.S. data are from the Statistics of U.S. Businesses, while the Canadian data are from Statistics Canada's Longitudinal Employment Analysis Program. In both data, the definition of a firm is an enterprise, employment is obtained using information from payroll data, and only firms with paid employees are included. There are methodological differences in the manner in which employment is constructed, and these are documented in the appendix. Firms in cropping, livestock and government industries are omitted in the data for both countries. The U.S. data also omits other more specific industries such as the postal service, rail transportation and private households.

Table 2: Employment per Firm, United States

	<19	20-99	100-499	500+	Total
1988	4.8	39.1	191.3	3113.7	20.3
1989	4.8	39.1	192.1	3047.1	20.9
1990	4.8	39.0	192.2	3088.0	21.0
1991	4.8	39.0	192.3	3098.3	20.9
1992	4.8	39.0	192.4	3089.1	20.9
1993	4.8	39.1	193.3	3039.0	21.0
1994	4.7	39.1	192.7	3048.2	21.1
1995	4.8	39.2	192.3	3086.1	21.4
1996	4.7	39.1	192.4	3138.6	21.5
1997	4.7	39.2	192.2	3156.5	21.8
1998	4.7	39.2	192.5	3239.3	22.2
1999	4.7	39.3	192.2	3284.1	22.6

the average U.S. firm. A large part of this difference was accounted for by differences in the average employment per firm in the 1 to 19 and 500+ employment size classes. Canadian firms in the 1-19 and 500+ size classes employed at least 1.6 and 520 fewer workers, respectively, than similar firms in the United States. In percentage terms, Canadian firms in the 1-19 class were at most only 63% of the size of U.S. firms in the same category, and Canadian firms in the 500+ class were at most only 83% of the size of U.S. firms in the same category.⁶ The Canada-U.S. difference in average firm size cannot be attributed to differences in industrial composition, as Table 3 shows that employment per firm was larger in the United States than in Canada for all the broad industry groups except for mining, oil and gas extraction.⁷

Looking at changes in employment per firm over time, it is observed that while employment per firm was rising over time in the U.S., it was falling in Canada. Employment per firm in Canada as fraction of employment per firm in the United States fell from 64% in 1988 to 54% in 1999. This change was predominately the result of changes in the 100-499 and 500+ categories, but mostly 500+ category. Here, employment per firm fell by nearly 600 employees in Canada, but rose by 170 employees in the United States. Once again, industrial composition

⁶The Canada-U.S. difference in the average employment per firm was also partly explained by the fact that Canada has more small firms. Table A1 in the appendix shows that, in 1988, 92.9% of Canadian firms employed 1 to 19 employees, compared to 89.1 % for the United States. In the other larger size categories, the United States had relatively more firms.

⁷For the United States, the industry and aggregate data are not comparable because a firm can be counted in more than one industry. Therefore, the weighted sum of Canada-U.S. industry differences do not have to add up to the aggregate difference. Furthermore, comparable industry-level data is available only up to 1997 because of the change from the SIC to the NAICS industrial classification system.

Table 3: Canadian Employment per Firm as Fraction of United States by Industry (%)

	1988	1997
Ag. Srv., Forestry, Fishing	61.7	60.3
Mining	141.2	103.9
Construction	57.5	56.6
Manufacturing	64.0	60.0
Trans., comm., and utilities	71.6	66.8
Wholesale trade	74.2	72.1
Retail trade	56.4	58.0
FIRE	78.8	79.4
Other services	84.8	69.6

is not a major factor in the observed changes over time as employment per firm in the U.S. relative to Canada has increased in all industries except retail trade and FIRE.

2.3 Potential Impact of Firm Size on Aggregate Productivity

Data on employment per firm and the firm size distribution can be combined to give employment shares by firm size, a more direct indicator of the relative importance of small firms in Canada and a more direct link to aggregate labour productivity. Table 4 shows that while Canada had more workers employed in firms of size 1-19 in the entire 1988-1999 period, the Canadian employment shares in 1988 were only marginally different from the U.S. employment shares. However, between 1988 and 1999, the employment share in the 500+ category rose by 4.2 percentage points in the United States, and fell by 4.7 percentage points in Canada.

If data on value added per worker by firm size were available, it would be possible to decompose the change in aggregate productivity into the part due to increases in productivity within the firm size categories and the part due to changes in the employment shares. This is because aggregate productivity is the weighted sum of productivity in each firm size category, where the weights are the employment shares. Unfortunately, data on value added per worker by firm size that is representative of all firms in the economy is not readily available.⁸ Gross output (total sales) per employee for different firm size classes is available in the United States from the 2002 U.S. Survey of Business Owners (SBO).⁹ Table 5 shows that as expected, the ratio of total sales to total employees in firms with 500 or more employees is significantly

⁸Studies that look at productivity by size generally focus on specific sectors like manufacturing or retail trade. Furthermore, plants or establishments are the unit of analysis and not firms.

⁹See the appendix for more details on this survey.

Table 4: Employment Shares

	Canada				US			
	<19	20-99	100-499	500+	<19	20-99	100-499	500+
1988	0.228	0.188	0.148	0.436	0.209	0.192	0.145	0.455
1989	0.228	0.191	0.150	0.431	0.203	0.189	0.146	0.461
1990	0.232	0.189	0.148	0.432	0.202	0.190	0.145	0.463
1991	0.229	0.206	0.166	0.399	0.203	0.186	0.142	0.469
1992	0.236	0.207	0.162	0.394	0.202	0.184	0.143	0.470
1993	0.237	0.209	0.164	0.390	0.201	0.184	0.146	0.469
1994	0.239	0.210	0.167	0.384	0.199	0.183	0.146	0.473
1995	0.233	0.209	0.167	0.390	0.195	0.184	0.146	0.475
1996	0.233	0.210	0.167	0.390	0.195	0.182	0.143	0.480
1997	0.229	0.211	0.170	0.390	0.191	0.182	0.146	0.482
1998	0.225	0.212	0.170	0.393	0.188	0.179	0.143	0.491
1999	0.224	0.213	0.174	0.389	0.184	0.178	0.141	0.497

Table 5: Sales per Employee in the United States, 2002

	<19	20-99	100-499	500+	Total
Total	142.9	147.7	159.6	239.6	197.3
Privately-held	140.3	144.0	150.6	134.8	145.2
Publicly-traded	183.1	169.4	177.1	263.6	249.5

Table 6: Sales per Employee in the United States by Industry, 2002

	Large/Small
Forestry, fishing, hunting and ag. services	0.62
Mining	3.37
Utilities	1.67
Construction	1.48
Manufacturing	2.53
Wholesale trade	2.39
Retail trade	1.12
Transportation and warehousing	0.89
Information	2.02
Finance and insurance	3.39
Real estate, rental and leasing	1.33
Professional, scientific and technical	1.10
Management of companies	0.10
Administrative, support and waste management	0.56
Education	1.28
Health care and social assistance	0.90
Arts, entertainment and recreation	0.89
Accommodation and food	1.11
Other services	1.10

higher than in smaller firm size categories.¹⁰ At 239.6 thousand dollars per employee, firms in the 500+ category are 1.5 times as productivity as firms in the 100-499 category and 1.7 times as productive as firms with 1 to 19 employees.¹¹

The finding that larger firms are more productive than smaller firms is not driven a few capital intensive industries dominated by large firms. As shown in Table 6 , firms with more than 500 employees are more productive than firms with less than 20 employees over a broad range of industries, including ones in the service sector. Industries where large firms are more productive than small firms account for 84% of the total GDP of the industries covered by the SBO.¹²

¹⁰Other statistics, such as the firm size distribution, employment shares and sales shares from the 2002 SBO are presented in Tables A2-A4.

¹¹This statistic likely overstates the average performance of firms with 1 to 19 employees. The sum of sales over the sum of employees in a firm size class overstates the average sales per employee in a firm size class if sales per employee is rising in the number of employees. Assuming sales per employee is rising more quickly at small firm sizes than at large firm sizes, the overstatement is larger in the small firm size classes.

¹²The numbers from the SBO are not too different from the relative productivity between small and large plants in the manufacturing sector that have been previously documented. See Baldwin, Jarmin and Tang (2002), for example.

If the level of productivity in the United States is normalized to 1 in 1999, and if we take 1.7 as the relative productivity between firms in the 500+ category and firms in the <500 category in 1999, it is possible to infer the productivity level in each firm size category.

$$1 = \left(\frac{Y}{L}\right)_{1999}^{US} = \left(\frac{L_1}{L}\right)_{1999}^{US} x + \left(\frac{L_2}{L}\right)_{1999}^{US} 1.7x,$$

where 1 and 2 denote the <500 and 500+ employment size categories respectively, Y is output, and L is employment. The part of the Canada-U.S. productivity gap in 1999 that can be accounted for by the difference in the employment shares in 1999 is:

$$\left(\frac{L_1}{L}\right)_{1999}^{Can} x + \left(\frac{L_2}{L}\right)_{1999}^{Can} 1.7x - \left(\frac{Y}{L}\right)_{1999}^{US}.$$

Furthermore, the part of the change in the Canada-U.S. productivity gap that can be accounted for by the change in the employment shares is:¹³

$$\left(\Delta\frac{L_1}{L}\right)^{Can} x + \left(\Delta\frac{L_2}{L}\right)^{Can} 1.7x - \left[\left(\Delta\frac{L_1}{L}\right)^{US} x + \left(\Delta\frac{L_2}{L}\right)^{US} 1.7x\right].$$

Canadian labour productivity was 82% of U.S. labour productivity in 1999 and 86.6% of U.S. labour productivity in 1988. Using the calculations above, it is shown that 31% of the gap in 1999 can be accounted for by differences in employment shares in 1999, and 102% of the change in the gap can be accounted for by the changes in the employment shares.

The point of the exercise is not to provide a precise decomposition or to argue that changes in the employment shares are the main cause of changes in aggregate productivity, but to show, through an accounting framework, that changes in the employment shares of the magnitude observed can potentially be related to substantial changes in productivity levels. The economic factors that possibly influence both the average size of the firm and aggregate productivity are discussed in the following section.

¹³Alternatively, we could allow Canada to have its own x by first solving for the productivity level in each firm size category that matches Canada's aggregate productivity in 1999, and then applying the decomposition below. In that case, more than 100% of the change in the productivity gap can be accounted for by changes in the employment shares as well.

Table 7: Investment to GDP Ratio by Type of Investment

	Total	M&E	ICT	Computers	Software	Communications
Canada						
1961-1984	12.8	6.4	1.0	0.4	—	0.6
1985-2004	11.6	6.8	1.7	0.6	0.6	0.5
US						
1961-1984	11.0	6.9	1.5	0.3	0.2	1.2
1985-2004	11.0	7.9	3.5	0.8	1.1	1.6

2.4 Determinants of Firm Size and Productivity

It has been pointed that one possible reason why Canadian productivity has declined relative to the United States is that Canada lags behind the U.S. in investment in machinery and equipment (M&E) and in particular information and communications technology (ICT)¹⁴ In this subsection, the evolution of investment in Canada and the United States is presented. The relationship between investment, firm size and productivity are then discussed. Finally, the major changes in the Canadian and U.S. economies that affected all three are reviewed.

2.4.1 *Investment, Productivity and Firm Size*

Table 7 shows that prior to 1985, Canada had an advantage in the investment to GDP ratio and was trailing the United States slightly in the M&E investment to GDP ratio.¹⁵ The average annual investment to GDP ratio was 12.8% in Canada, 1.8 percentage points higher than in the U.S, while the average annual M&E investment to GDP ratio was 6.4% in Canada, 0.5 percentage points lower than in the U.S. After 1985, the gap in the investment to GDP ratio narrowed to just 0.6 percentage points in favour of Canada, and the gap in the M&E investment to GDP ratio widened to 1.1 percentage points in favour of the United States. The entire gap in the M&E investment to GDP ratio after 1985 can be accounted for by the gap in the ICT investment to GDP ratio. ICT investment to GDP ratio was 1.7% in Canada and 3.5% in the United States; a gap of 1.8 percentage points. Table 7 also shows that Canada lagged behind the United States in all components of ICT.

This weakness in ICT investment is important because a number of paper have shown the importance of ICT to recent labour productivity growth in both Canada and the United States. For example, growth accounting exercises have shown that ICT capital deepening -

¹⁴See for example, Crawford (2002), Macklem (2003) and Sharpe (2005).

¹⁵See the appendix for details on the source of these data.

increases in the ICT to labour ratio - has been important for labour productivity growth in both countries.¹⁶ Growth accounting, however, likely underestimates the true impact of ICT on labour productivity. This is because ICT investment affects labour productivity not only via ICT capital deepening, but also through TFP. It has been argued that ICT investments facilitate improvements in the organization of the workplace that are picked up by TFP.¹⁷

Firm size is related to investment because it has been shown that small firms invest less in ICT and are slower to adopt new technologies than large firms.¹⁸ Sharpe (2005) presents Canadian evidence showing that large firms are more likely to use basic ICT, such as PCs, e-mail, and the internet, and are also the predominate users of advanced ICT applications, such as having a web-site, on-line sales, and purchasing on-line.¹⁹ North American evidence on firm size and ICT investment is lacking, but there is a study by Fabiani et al. (2005) on Italian manufacturing firms in 2000. They show that firm size is a crucial determinant of ICT expenditure. Firms with more than 500 employees spent 1095 euros per worker on ICT, nearly twice as much firms in the 250 to 499 employee size category (551 euros) and more than twice as much as firms with 50 to 99 employees (452 euros).

Sharpe (2005) argues that one of the reasons why small firms are less likely to invest and adopt ICT is that small firms might be less aware of the benefits of ICT. Related to this is Astebro (2002) argument that small firms are slower to adopt because there are fixed costs associated with the adoption of new technologies. Astebro (2002) tests four potential explanations why small plants are slower to adopt and finds support for only non-capital cost spreading. He argues that the average fixed costs of information acquisition before adoption are smaller for larger establishments.

2.4.2 Tax Policy

Each year, the World Economic Forum carries out an Executive Opinion Survey of business executive and entrepreneurs. The results of their 2004 survey are reported in Porter et al. (2004). When asked what are the five most problematic factors for doing business in Canada,

¹⁶For examples, see Khan and Santos (2002), Oliner and Sichel (2002), Van Ark (2003), and Ho, Rao and Tang (2004).

¹⁷Examples of such reorganizations include increased outsourcing, introduction of “just-in-time” production, improvements in inventory management, flexible work arrangements, and decentralized decision making. See Bresnahan, Brynjolfsson and Hitt (2002), Brynjolfsson and Hitt (2000) and Leung (2004) for further discussion and empirical evidence.

¹⁸Many of the most recent studies are cited in Sharpe (2005).

¹⁹See Baldwin and Sabourin (1998) and Baldwin et al. (1999) for more Canadian evidence, and Astebro (2002), Dunne (1994) and Karshnas and Stoneman (1993) for evidence from the United States.

Table 8: Effective Corporate and Personal Tax Rates (%)

	Corporate	Personal
Canada		
1961-1984	36.5	14.7
1985-2004	37.1	21.4
US		
1961-1984	38.3	17.6
1985-2004	33.1	20.5

over 25% of respondents cited tax rates, 15% cited tax regulations, and approximately 13% cited access to financing. In contrast, approximately 18% of respondents in the U.S. cited tax regulations, 16% cited inefficient bureaucracy, and 15% cited inadequately trained workforce. Only 14% responded that tax rates were among the five most problematic, and only 5% responded that access to financing was a problem. These results suggest that there are likely significant differences in tax policies and access to financing between the two countries. In this section, data on the evolution of corporate and personal tax structure is presented. In the following section, the financing environment is examined.

Prior to 1986, combined federal and state/provincial statutory corporate income tax rates facing large companies were similar in Canada and the United States. The U.S. Tax Reform Act of 1986, however, cut the statutory tax rates by more than 10 percentage points. Rates in Canada were also lowered, but not by as much and at slower pace. Only recently have statutory rates in Canada become lower than in the United States. However, corporate income taxes are not the only form of taxation faced by firms in Canada. Both federal and provincial government levy a tax on capital assets as well.²⁰ Capital taxes were introduced in the province of Quebec in 1947. Other provinces, British Columbia, Saskatchewan, Manitoba, Ontario, and Newfoundland, introduced their capital taxes in 1982. Alberta, New Brunswick, and Nova Scotia followed suit in the late 1980s and early 1990s. Furthermore, as a temporary measure to reduce the deficit, the federal government introduced Large Financial Institutions Capital tax on banks and other financial institution in 1985. This temporary measure was later made permanent, and followed in 1989 by capital tax with a broader base called the Large Corporate Tax.²¹ Table 8 shows that the reduction statutory tax rates in the United

²⁰Several states in the United States also have capital taxes, but they are minor in comparison to the capital taxes in Canada.

²¹Each provincial and federal capital tax differs with respect to the tax rate paid and how much capital is exempt.

States and the introduction of capital taxes in Canada had a significant impact in the effective corporate tax rate (corporate taxes as a fraction of corporate profits) over time. Prior to 1985, corporate taxes as a percentage of corporate profits at 38.3% in the United States was higher than in Canada at 36.5%. In the post-1985, corporate taxes as a percentage of corporate profits rose in Canada to 37.1% and fell in the United States to 33.1%.²²

The evolution of personal income taxes follows a similar pattern. Personal income taxes including contributions to social assistance programs as a fraction of personal income was 14.7% in Canada in the pre-1985 period, compared to 17.6% in the United States. It rose in the United States to 20.5% in the post-1985 period, but it rose even higher in Canada to 21.4%.

A number of papers have examined the effect of business taxes on investment. Hall and Jorgenson (1967) find that the three post-World War II tax reforms in the United States changed the level of investment. Cummins, Hasset and Hubbard (1994) examine the corporate tax reforms in the post-1962 era and come to similar conclusions. The effect of personal income taxes on the sales growth of small firms, employment and their investment decisions have been studied in Carroll et al. (1998a, 1998b, 2000). Using U.S. administrative tax data before and after the Tax Reform Act of 1986, they find that raising the marginal tax personal income tax rate by 10 per cent, lowers the growth of receipts of sole proprietorships (unincorporated businesses) by 8.4 per cent, and lowers the mean probability of hiring employees by 12 per cent. Furthermore, a 5 percentage point increase in the marginal tax rate reduces the proportion of the number of businesses that make new capital investments by 10.4 per cent and decreases the mean investment expenditures by 9.9 percent. The findings of each of these papers suggest investment and hence productivity growth is affected by changes in corporate and personal income tax rates.

2.4.3 Financing Environment: Credit Market

There is some evidence to suggest financing environment may be more adverse in Canada than in the United States. An Ipsos-Reid poll of 1200 Canadian and American small and medium sized enterprises, done for Royal Bank (2002), finds that one of the few Canada-U.S. distinction was that a modestly higher fraction of respondents in Canada cite access to financing as a barrier to growth. In that poll, both the cost of borrowing and access to equity

²²Without provincial capital taxes, the average effective corporate tax rate would be 32.9% in Canada in the post-1985, lower than in the United States. In the pre-1985 period, the gap between the average effective corporate tax rates with and without provincial capital taxes was small.

Table 9: Real Prime and T-Bill Rates (%)

	Prime	T-Bill
Canada		
1962-1984	3.1	1.5
1985-2004	5.1	3.6
US		
1962-1984	2.9	1.0
1985-2004	4.7	1.7

markets was cited as a barrier.

A variety of interest rates could be used as indicators of how the cost of borrowing has changed over time. Table 9 presents two possible measures, the real prime - the prime rate deflated by the consumer price index - and the real t-bill rate. The evolution of both rates suggest that the cost of borrowing has risen in Canada more than in the United States. The Canada-U.S. prime rate gap doubled from 0.2 percentage points in the pre-1985 period to 0.4 percentage points post-1985, and the Canada-U.S. t-bill rate gap more than doubled from 0.5 percentage points to 1.9 percentage points.²³ A more appropriate measure of the effective cost of borrowing would be to take interest paid on borrowing and divide it by the amount of borrowing. For the 1994-1999 time period, there was also a positive Canada-U.S. gap in this measure as well. The effective cost of borrowing was 5.0% in Canada and 4.7% in the United States.²⁴

There is much empirical evidence on how credit constraints and firm size interact to have significant aggregate implications. Using cross-industry, cross-country data Beck et al. (2004) find that financial developments have a disproportionate effect on the growth of industries that are more dependent on small firms. Ghosal and Loungani (1996) find that increases in the uncertainty about future profits lowers investment in industries dominated by small firms, but has an negligible effect on other industries.

²³McKenzie and Thompson (1997) argue that higher real interest rates in Canada caused the cost of capital to be generally higher than in United States during the 1971-1996 period, but more substantially so in 1984-1996 period.

²⁴The data are from Financial and Taxation Statistics for Canada, and the IRS for the United States. We are in the process of getting these data for other years.

2.4.4 *Financing Environment: Equity Market*

Initial access to equity financing is hindered by high IPO costs. The costs to an IPO comes in two forms: direct costs and underpricing. Direct costs consists of legal and accounting expenses, a variety of fees, and primarily the underwriter's commission - the difference between the offer price and the price per share received by the company. Underpricing is the difference between the underwriter's offer price of the new shares and the market value of the shares shortly after the start of trading. Using data for the majority of IPOs between 1997 and 1999, Kooli and Suret (2003) find significant differences between Canada and the United States in IPO costs. Total direct costs are about 13.3% of the size of the issue in Canada and 10.4% of the size of the issue in the United States. This advantage for the United States in terms of lower direct costs is offset by higher underpricing. Underpricing is 37.8% in the United States and 34.9% in Canada. Accounting for direct costs and underpricing leaves the total issue costs nearly identical at 48.2% for both countries.

The degree of underpricing is generally measured using the difference between the first closing market price and the offering price. Given the volatility in stock markets, this measure of underpricing is likely a noisy measure of the difference between the "true" value of the stock and the offering price. Differences in underpricing between two countries might be better estimated by examining the market valuation of firms over a longer period of time. Using data from Compustat from 1989-2000, King and Segal (2005) show that the shares of Canadian firms trade at a discount in comparison to U.S. firms, even after taking into account company and market specific factors. King and Segal (2005) examine the relative discount using four different measures: market to book value, Tobin's q, price of a share over earnings, and enterprise value over operating income before depreciation. The average discount across these four measures is 13%. That is to say, shares of Canadian firms trade at a 13% discount relative to comparable U.S. firms. Adding to this the 2.8 percentage point difference in the direct costs of a IPO, leads to a 15.8 percentage point difference in the costs of an IPO between the two countries.

Less access to equity financing should ultimately lead to a smaller publicly-traded sector in Canada. Sales by privately-held and publicly-traded companies is available from the 2002 U.S. SBO. The publicly-traded sector -including publicly-held, foreign-owned, and not-for-profit firms, accounted for 60.9% of total sales in the United States. Data for the Canadian corporate sector in 2002 shows that the publicly-traded sector accounts for 52.4% of sales.²⁵

²⁵The number for Canada was provided by Statistics Canada's Industrial Organization and Finance Division and is calculated from corporate income tax returns. To make the data broadly consistent with the SBO, firms

If data for the unincorporated sector were available, the percentage for Canada would be even lower.

Better access to equity markets and a larger publicly traded sector in the United States could also be related to the Canada-U.S. productivity gap. As shown in Table 5, the difference in sales per employee between publicly-traded and privately-held firms is as large as the gap between firms with 500+ employees and 1 to 19 employees. Part of this is due to the fact that publicly-traded firms are the majority of firms with 500+ employees and privately-held firms form the majority of firms with 1 to 19 employees, but even within size classes, publicly-traded firms have a higher total sales to employee ratio.

2.4.5 *Other determinants*

Market size may also be a determinant of both productivity and firm size. Firms in a small closed economy would not be able to exploit scale economies as well as firms in a large economy. Furthermore, a closed economy tends to foster complacency and higher degrees of x-inefficiency.²⁶ Changes in market size through population growth or trade liberalization would effectively increase the market available to a firm located in a small economy, expose it to greater competition, and give it more opportunity and motivation to learn of best practices in the larger economy.

Market size might be a potential determinant of why Canada has smaller firms and lower productivity throughout the 1961-2004 period, but changes in the market size cannot explain declines in productivity and firm size relative to the United States. First, between 1985 and 2004, the Canadian population has increased slightly relative to the United States. Canada's population relative to the United States increased from 10.8% to 10.9%.²⁷ Second, trade liberalization through the FTA and NAFTA arguably increased the effective market size faced by Canada firms more than U.S. firms. Thus determinants other than market size explain the decline in relative productivity and firm size in Canada.

with less than a 1000 dollars in receipts, and firms in government, crop production and animal production were excluded.

²⁶See Baldwin and Gu (2004), and Winters (2004) for further discussion.

²⁷The U.S. data are from the U.S. Statistical Abstract, and the Canadian data are from Statistics Canada. The U.S. population count includes both military personnel overseas and individuals in institutions.

3 Model economy

3.1 Household Sector

3.1.1 Preferences

In this economy there is a continuum of measure one of households that maximize the expected lifetime utility:

$$E \sum_{t=0}^{\infty} \beta^t U(c_t), \quad U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \quad (1)$$

where c_t is consumption at time t , β is the intertemporal discount factor, and σ is the coefficient of relative risk aversion.

3.1.2 Endowment and occupation

In each period, if an individual is a worker, she is endowed with $\varepsilon \in \{\varepsilon_1, \dots, \varepsilon_N\}$ efficiency units of labour. This labor endowment is stochastic and follows a first-order Markov process with a transition probability $\Gamma(\varepsilon', \varepsilon)$. A new worker who had another occupation in the previous period draws ε from a distribution $\bar{\Gamma}(\varepsilon)$. Individuals can choose between three occupations in the next period: workers, entrepreneurs, and sellers. Households have to pay a fixed cost of $\bar{e}_1 > 0$ units of output, if it decides to start a new business. Whereas workers supply their units of labour inelastically to the market in return for a common fixed wage rate, w , entrepreneurs (managers or business owners) use their entire labour unit to manage a single firm and receive (stochastic) profits from operating that business. A manager can also choose to sell his business to risk-neutral financial intermediaries in the next period. Such a manager becomes a business seller.²⁸ A seller transits through the worker stage before he has a chance to start a new business. When the agent chooses the next period's occupation (worker, manager or seller), he must stay with that occupation for at least one period.

Each household can run at most one business at a time. To start a new business, the manager has to sell the current business and then becomes a worker. Only worker can start a new business.

The household not running a business can receive an entrepreneurial idea, θ , drawn at the beginning of each period from the set $\{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$. The entrepreneurial idea, θ , is a random variable with a probability distribution denoted by $\Delta(\theta'|\theta)$, where Δ is increasing in θ . A new worker draws θ from the distribution $\Delta^{sw}(\theta)$.

²⁸There will also be a failed business exit. This type of exit is also assumed to go through the seller stage.

In this version of the paper, we assume that, every period, each worker can draw an entrepreneurial idea $\theta \in \{0, 1\}$ from the distribution $\Delta(\theta'|\theta)$, where $\theta = 0$ means no idea or no business opportunity was drawn and $\theta = 1$ means that a good idea with a business opportunity was drawn.

3.2 Business Sector

In this section, we describe a life-cycle of a firm starting from the business start-up by a household. Once the business is started, it becomes a privately-held firm run by the household. The household has the choice of growing the firm, or selling the firm to the intermediary. If sold, since the intermediary is owned by the public, the firm becomes publicly-held/publicly-traded. Also, at any time during the business operation, a firm can draw a bad shock and exit the business sector. We call this a firm life-cycle.²⁹

3.2.1 Privately-held business

Each privately-held business is run by a household whose occupation is a manager. The production function operated by a manager with an index j is given by the following decreasing return production function:

$$f(z_j, k, n) = z_j (k^\alpha n^{1-\alpha})^\nu, \quad (2)$$

where $0 < \nu < 1$ determines the degree of returns to scale, k is the capital input, n is the labour input, z_j is an idiosyncratic productivity shock that is observed at the beginning of the period before the choice of k and n .

The stochastic process for z is conditioned by j such that there is a separate process given j . We interpret j to be the technology that represents the scale of production and it is an endogenous outcome of the choice made by the business owner. It takes discrete values, $j \in \{1, \dots, J\}$. A higher j implies a higher average productivity level, $\bar{z}_j < \bar{z}_{j+1}$.³⁰ In the first period when a business is started, the business has $j = 1$. Every period, a manager with a current scale j can choose to advance it for the next period to $j + 1$, provided that the firm is currently operating and provided he makes a positive investment \bar{e}_{j+1} for the improvement

²⁹It is interesting to note that the model presented here, as in Chari, Golosov and Tsyvinski (2004), is a micro-founded theory of the aggregate production technology often used in various macro literature.

³⁰Given the decreasing returns to scale assumption in the production function, this implies $n_j^* < n_{j+1}^*$ as well as $k_j^* < k_{j+1}^*$ at $z_j = \bar{z}_j$ and $z_{j+1} = \bar{z}_{j+1}$, respectively. The superscript * implies the optimal choice.

in scale.

The determination of the scale of production j can come from many sources. It could be the organizational structure necessary to manage a larger organization, or it could be the information technology that facilitates the communication between employees to allow the firm to grow larger. Hence, the size of this investment could depend on many things such as the direct cost of adopting a new organizational structure or the new technology, and the barriers to technology adoption in the country in which the firm is located.

The idiosyncratic productivity shock z is drawn from the set $\mathcal{Z}_j = \{z_{1,j}, \dots, z_{N,j}\}$ and follows a first-order Markov process with a transition probability $Q_j(z', z)$. The set from which the shock, z , takes values, as well as its probability distribution, depends on the implemented technology, j . The mean productivity shock is higher for a more advanced technology level $j + 1$ than a less advanced technology j . Moreover, the transition probability is increasing in j , in the sense of first-order stochastic dominance. The first element of the set, \mathcal{Z}_j , is assumed to be a bad shock and is highly persistent; i.e., $z = 0$ and $Q_j(0, 0) = 1$ for all j . As a result, if a manager receives it, he will exit from entrepreneurship and become a seller with $z = 0$. For notational simplicity later on we assume that a worker is running a technology $j = 0$.

The productivity shock $z \in \mathcal{Z}_j$ of a new business manager—ie., a worker that becomes a manager—is drawn from $H^{we}(z|\theta)$ which is increasing in θ in the sense of first order stochastic dominance. If the household was running a technology level $j = 1, \dots, J - 1$ in period $t - 1$ and moves to a new technology level $j + 1$ in period t , it will receive a productivity shock z from a probability distribution $H_{j+1}(z)$ defined on \mathcal{Z}_{j+1} .

3.2.2 *Publicly-traded business sector*

When a business is sold, it will operate in the publicly-traded business sector. The ownership of the sold business shift from the manager household to the intermediary who buys the business. A direct consequence of this is that the publicly traded sector is endogenous. Given the focus of the paper, we will not model this sector in detail. Therefore, we assume, for simplicity, that (i) this sector does not face financial constraints which implies that firms will operate at the optimal scale, and (ii) the adoption of technologies is stochastic and persistent. The production function is the same as the one operated in the privately held business sector with (j, z) following jointly an exogenous transition matrix $\Omega(j', z'|j, z)$. Firms operating in this sector fails exogenously with some probability.

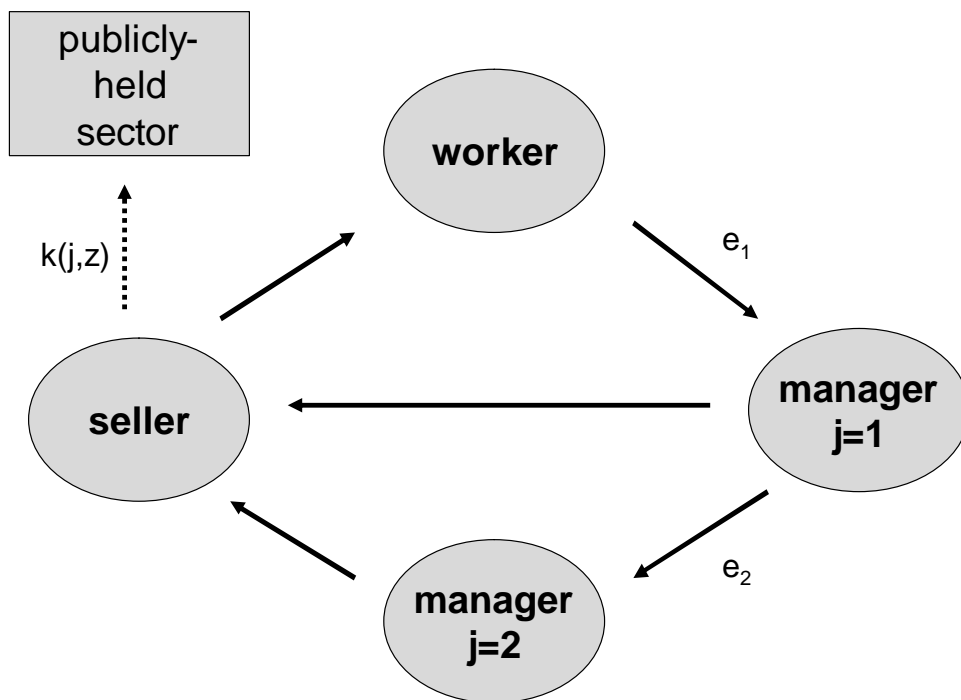


Figure 1: Occupation Dynamics and Business Life-cycle, $J=2$

3.3 Intermediation sector

In the model economy, intermediaries collect deposits from households with positive balances (by paying the interest rate, r) and lends those funds to other households and the publicly-traded sector. There is a positive proportional cost, ϕ , per unit of funds intermediated to households undertaking entrepreneurial activities as well as loans made to the publicly-traded sector. Given the large number of banks behaving competitively, bank profits are zero. This assumption implies that the lending rate equals $r_L = r + \phi$ for loans to both the publicly-traded sector and the household sector.

In this paper, loans are provided only for business investment purposes. Thus, we assume that entrepreneurs cannot use the funds borrowed from intermediaries for consumption purposes. We also assume that there is no intertemporal borrowing, that is, capital markets are incomplete, and therefore, individuals have to save to self-insure against uninsurable idiosyncratic business risk ($a' \geq 0$).

Financial intermediaries also buy businesses from entrepreneurs. To determine the price of firms we will follow the approach of Chari, Golosov and Tsyvinski (2004). The price is determined such that expected profits for financial intermediaries are equal to zero. The price function, $p(j, z)$, of a business with a technology level j and productivity shock $z \in \mathcal{Z}_j$ is defined recursively as follows:

$$p(j, z) = \lambda\pi(z) + \left(\frac{1}{1+r}\right) \sum_{j', z'} p(j', z')\Omega(j', z'|j, z), \quad (3)$$

where $0 < \lambda < 1$ which means that entrepreneurs are more efficient at managing business than financial intermediaries, $\pi(z)$ is the profit function of running a business with productivity shock z in the absence of constraints.

3.4 Government

The government is assumed to levy taxes on individuals' incomes $T(\cdot)$. Tax revenues are in turn used to finance government consumption, G . Agents' incomes subject to taxation are the sum of wage, capital, and/or entrepreneurial income, capital gain income from business sale. The government operates under a balanced budget each period.

3.5 Timing of events

The timing of events during a period is as follows, and is identical across all periods.

Beginning of period

1. Individuals (entrepreneurs and business sellers) observe the current shock z , and workers receive the labor endowment ε and the entrepreneurial idea θ ;
2. after observing the productivity shock, z , managers of both publicly-traded and privately-held firms rent capital, k , hire labor efficiency units, n , and then produce;
3. workers, entrepreneurs, business sellers make consumption and saving decisions;
4. workers, entrepreneurs, and business sellers make the next period's occupation decisions; managers make the next period's technological adoption decisions.

End of period

Individuals observe their current shocks, ε , θ and z . In particular, workers receive a labor endowment ε and an entrepreneurial idea $\theta \in \{\theta_L, \theta_H\}$, and entrepreneurs and business sellers receive an idiosyncratic productivity shock z . Once the information is revealed, entrepreneurs decide how much of their own funds to invest in their own businesses, how much capital to borrow from financial intermediaries, and how many units of labor efficiency to hire. Production in the privately-held business and the publicly-traded sectors then takes place. At the end of the period, factor payments are made and entrepreneurs receive entrepreneurial income and business sale income. Entrepreneurial income is the residual of output after payments of wages, interest on capital borrowed from financial intermediaries, and depreciation. Consumers then pay taxes, $T(\cdot)$, on their income and consume. Once the savings decision is made, individuals choose their next period's occupation. This choice of occupation depends on savings and the current realization of the technological shock. The risk from entrepreneurial activities comes from two margins. First, the decisions on the next period's occupation and technological adoption are made currently without knowing the realization of the next period's productivity shock, and from the fact that after observing the shock the agent cannot immediately go back to being worker or choose a different level of technology. Second, all exits from a business go through the seller stage where there is no wage earnings. The second assumption makes a potential business failure particularly risky since there will be two periods of no business or wage income.

3.5.1 Business profit

In an environment without any constraints the input demands by the firm will be given by the solution to the profit maximization problem:

$$\pi(z) = \max_{k,n} \left\{ f(z, k, n) - wn - (r + \delta)k \right\}, \quad (4)$$

$$(5)$$

where w is the wage and r is the interest rate and δ is the depreciation rate in the economy. We call these input demands k^u and n^u . Let us define a labor demand function for given k , z , and w . This function is derived from the first-order condition with respect to labour.

$$n(z, w, k) = \left[\frac{(\nu - \nu\alpha)zk^{\nu\alpha}}{w} \right]^{\frac{1}{1-(\nu-\nu\alpha)}}. \quad (6)$$

The optimal size of the business in terms of capital, k^u , is given by the following function:

$$k^u(z) = \left[\left(\frac{r + \delta}{\alpha\nu z} \right) \left(\frac{\alpha w}{(1 - \alpha)(r + \delta)} \right)^{\nu(1-\alpha)} \right]^{\frac{1}{\nu-1}}. \quad (7)$$

Combining equations (6) and (7) we obtain n^u .

However, as explained above, firms will not always be able to operate at the unconstrained profit-maximizing level because of the existence of the borrowing constraint and intermediation costs. Because of this cost and the borrowing constraint, the level of households assets, will affect the determination of the capital and labour demands. As a result, in general the (realized) labor and capital inputs depend (for fixed inputs prices) both on the productivity shock, z , and the level of assets a of the household that operates the firm. We denote constrained demands as $k(z, a)$ and $n(z, a)$. The current profit of an entrepreneur with net worth a and productivity shock z is given by:

$$\pi(z, a) = \max_{k,n} \left\{ f(z, k, n) - wn - (r + \delta)k - \phi \max\{0, k - a\} \right\}, \quad (8)$$

s.t

$$k \leq (1 + \gamma)a, \quad (9)$$

where equation (9) states that business managers can borrow up to a fraction $1 + \gamma$ of their net worth, a .

The capital demand function, $k(z, a)$:

$$k(z, a) = \begin{cases} (1 + \gamma)a & \text{if } (1 + \gamma)a < k^c(z), \\ k^c(z) & \text{if } a \leq k^c(z) \leq (1 + \gamma)a, \\ a & \text{if } k^c(z) < a < k^u(z), \\ k^u(z) & \text{if } k^u(z) \leq a, \end{cases} \quad (10)$$

where $k^c(\cdot)$ is the capital demand of constrained entrepreneurs that borrow $k - a > 0$ at an interest rate $r_L = r + \phi$ and is derived from (8) as follows:

$$k^c(z) = \left[\left(\frac{r + \phi + \delta}{\alpha \nu z} \right) \left(\frac{\alpha w}{(1 - \alpha)(r + \phi + \delta)} \right)^{\nu(1 - \alpha)} \right]^{\frac{1}{\nu - 1}}. \quad (11)$$

Given $k(z, a)$, the labor demand $n(z, a)$ is determined by using equation (6).

3.6 The individual's decision problem

In this paper, we consider only stationary equilibria in which the distribution of agents over individual states is constant and prices do not change over time. At the beginning of each period, the state of an agent in the model includes the current occupation; the net amount of asset holdings, a ; the TFP shock, z , observed at the beginning of the period if the household operates a business; the labor endowment, ε , and the entrepreneurial idea, θ , if the household is currently a worker; and the technology level, j .

To simplify the description of the model, define $V^w(a, \varepsilon, \theta)$ to be the value function of a worker whose current period entrepreneurial idea is θ and beginning-of-period net asset holdings are a . Similarly, define $V^m(z, a, j)$ as the value function of a manager whose beginning-of-period productivity shock is z , technology level is j and net asset holdings are a . We also denote $V^s(z, a, j)$ a the value function of a seller whose beginning-of-period productivity shock is z , technology level is j and net asset holdings are a .

3.6.1 The worker's problem

$$V^w(a, \varepsilon, \theta) = \max_{\{a', m'\}} \left\{ u(c) + \beta \max \left\{ \sum_{\varepsilon', \theta'} \Gamma(\varepsilon' | \varepsilon) \Delta(\theta' | \theta) V^w(a', \varepsilon', \theta'), \sum_{z' \in \mathcal{Z}_1} H_1^{we}(z' | \theta) V^m(1, z', a', \theta) \right\} \right\},$$

subject to

$$\begin{aligned} c &= w\varepsilon + (1+r)a - a' - e(m') - T(w + ra - e(m')), & (12) \\ m' &\in \{W, M\}, \\ c, a' &\geq 0. \end{aligned}$$

Workers choose a non-negative consumption, c , and the next period's risk-free asset holding, a' , which is restricted to be non-negative. Workers also choose the next period's occupation, m' . If the worker decides to become an entrepreneur ($m' = M$) he pays a cost $e(m') = \bar{e}_1$ and zero otherwise. At any point in time, a worker's resources come from the return on the asset holding, a , and labour unit.

3.6.2 The manager's problem

A manager's problem is described below in a dynamic programming language:

$$\begin{aligned} V^m(j, z, a) &= \max_{\{a', m', e'\}} \left\{ u(c) + \beta I(e' > 0) \max \left\{ \sum_{z' \in \mathcal{Z}_{j+1}} H_{j+1}(z') V^m(j+1, z', a'), \right. \right. \\ &\quad \left. \left. \sum_{z' \in \mathcal{Z}_{j+1}} H_{j+1}(z') V^s(j+1, z', a') \right\} + \right. & (13) \end{aligned}$$

$$\left. \beta(1 - I(e' > 0)) \max \left\{ \sum_{z' \in \mathcal{Z}_j} Q_j(z', z) V^m(j, z', a'), \right. \right. & (14)$$

$$\left. \left. \sum_{z' \in \mathcal{Z}_j} Q_j(z', z) V^s(j, z', a') \right\} \right\},$$

subject to

$$c = (1+r)a + \pi(z, a) - a' - e' - T(ra + \pi(z, a) - e'), & (15)$$

$$m' \in \{M, S\}, & (16)$$

$$e' \in \{0, \bar{e}_{j+1}\}, & (17)$$

$$a', c \geq 0. & (18)$$

Managers choose a non-negative amount of consumption, c , and the next period's risk-free asset holding, a' , which is also restricted to be non-negative. The non-negativity constraint on a' implies that there is no intertemporal borrowing; consequently, people's assets must be positive to finance their consumption and to take advantage of entrepreneurial opportunities.

In addition, entrepreneurs can choose to advance the next period's level of technology of the business. If a positive amount $e' = \bar{e}_{j+1} > 0$ is invested, then manager advances his technology from j to $j + 1$. If he does not invest ($e' = 0$) then he will keep the level of technology at the current level j . When a new level technology is chosen for the next period, the next period's productivity shock $z' \in \mathcal{Z}_{j+1}$ is drawn from the distribution $H_{j+1}(\cdot)$. To capture the exogenous exit from business, it is convenient to assume that when $z = 0$, there is no investment in improving the technology, that is, $e' = 0$. Given the perfect persistence of this realization of z , the manager becomes a seller with $z = 0$ in the next period.³¹

Once the manager makes his decision of whether to adopt a new technology, he also chooses the next period's occupation m' which is either becoming a business seller $m' = S$ or a continuing to a business manager $m' = M$.

The function, π , in the budget constraint is the business profit defined in equation (8) and $T(\cdot)$ is the tax function. The taxable income is given by capital income plus business income, net of the cost of adopting a new technology.

3.6.3 The business seller's problem

A business seller's problem is described below in a dynamic programming language:

$$V^s(j, z, a) = \max_{a'} \left\{ u(c) + \beta \sum_{\varepsilon', \theta'} \bar{\Gamma}(\varepsilon') \Delta^{sw}(\theta') V^w(a', \varepsilon', \theta') \right\}, \quad (19)$$

subject to

$$c = (1 + r)a + p(j, z) - a' - \kappa(z) - T(ra + p(j, z) - \kappa(z)), \quad (20)$$

$$a', c \geq 0. \quad (21)$$

Business sellers choose a non-negative consumption, c , and the next period's risk-free asset holding, a' , which is restricted to be non-negative. After selling the business to financial intermediaries, the business seller becomes a worker in the next period. At any point in time,

³¹A consequence of this assumption is that household spends at least two periods (first as business seller and then as a worker) before starting a new business.

a business seller's resources come from the return on the asset holding, a , and the capital gain, $p(j, z)$, from selling a business with a technology level, j , and a productivity shock, z . A business sale requires a fixed cost, $\kappa(z)$, where $\kappa(0) = 0$ and positive otherwise. The taxable income of a business seller is the capital gain plus return on asset net of the cost of business sale.

3.7 Characterization of the equilibrium

Law of motion of the measure of publicly-held firms: The size of the publicly-held business sector is endogenous and it is determined by the entrance of newly sold privately-held firms and exit of existing publicly-held firms. Let us denote $\Phi_t(j, z)$ the measure of publicly-held businesses with a technology level j and a productivity shock z at time t .

$$\Phi_{t+1}(j', z') = \sum_{a, j, z} \Omega(j', z' | j, z) \mu_t^s(j, z, a) + \sum_{j, z} \Omega(j', z' | j, z) \Phi_t(j, z) \quad (22)$$

Measure of business sellers: The measure of business sellers is determined by managers selling their business. For all $j = 1, \dots, J$ with $\mu_t^m(0, \cdot) = 0$.

$$\begin{aligned} \mu_{t+1}^s(j, S_z, S_a)' &= \sum_{z' \in S_z} \sum_{a' \in S_a} \sum_{a, z} I(a^m(j, z, a) = a' \text{ and } m^m(j, z, a, \theta) = S \text{ and } e^m(j, z, a, \theta) = 0) \\ &\quad Q_j(z', z) \mu_t^m(j, \theta, z, a) + \\ &\quad \sum_{z' \in S_z} \sum_{a' \in S_a} \sum_{a, z} \end{aligned} \quad (23)$$

$$\begin{aligned} &I(a^m(j-1, z, a, \theta) = a' \text{ and } m^m(j-1, z, a, \theta) = S \text{ and } e^m(j-1, z, a, \theta) > 0) \\ &H_j(z') \mu_t^m(j-1, \theta, z, a) \end{aligned} \quad (24)$$

The first part in the right hand side of equation 24 is the measure of managers that decide not to stay at the level j and the second part is those decide to advance from $j-1$ to j .

Measure of business managers: The measure of business managers is given by those becoming managers and those continuing to be business managers.

For $j = 2, \dots, J$, the of managers is defined as follows:

$$\begin{aligned}
\mu_{t+1}^m(j, S_z, S_a) &= \sum_{z' \in S_z} \sum_{a' \in S_a} \sum_{z, a} I(a^m(j, z, a) = a' \text{ and } m^m(j, z, a) = M \text{ and } e^m(j, z, a) = 0) \\
&\quad Q_j(z', z) \mu_t^m(j, z, a) + \\
&\quad \sum_{\eta' \in S_\eta} \sum_{a' \in S_a} \sum_{z, a} I(a^m(j-1, z, a) = a' \text{ and } m^m(j-1, z, a) = M \text{ and } e^m(j-1, z, a) > 0) \\
&\quad H_j(z') \mu_t^m(j-1, z, a)
\end{aligned} \tag{25}$$

The measure of managers running a business with the first level of technology, $j = 1$, is given by the following expression:

$$\begin{aligned}
\mu_{t+1}^m(1, S_z, S_a) &= \sum_{z' \in S_z} \sum_{a' \in S_a} \sum_{a, \varepsilon, \theta} I(a^w(a, \varepsilon, \theta) = a' \text{ and } m^w(a, \varepsilon, \theta) = M) \\
&\quad H_1^{we}(z'|\theta) \mu_t^w(\varepsilon, \theta, a) + \\
&\quad \sum_{z' \in S_z} \sum_{a' \in S_a} \sum_{z, a} I(a^m(1, z, a) = a' \text{ and } m^m(1, z, a) = M \text{ and } e^m(1, z, a) = 0) \\
&\quad Q_1(z', z) \mu_t^m(1, z, a)
\end{aligned} \tag{26}$$

Measure of workers: The measure is determined by workers that decide to stay workers and the sellers who become workers.

$$\begin{aligned}
\mu_{t+1}^w(S_\varepsilon, S_\theta, S_a) &= \sum_{a' \in S_a} \sum_{\varepsilon' \in S_\varepsilon} \sum_{\theta' \in S_\theta} \sum_{a, \varepsilon, \theta} I(a^w(a, \varepsilon, \theta) = a' \text{ and } m^w(a, \varepsilon, \theta) = W) \\
&\quad \Gamma(\varepsilon'|\varepsilon) \Delta(\theta'|\theta) \mu_t^w(\varepsilon, \theta, a) + \\
&\quad \sum_{a' \in S_a} \sum_{\varepsilon' \in S_\varepsilon} \sum_{\theta' \in S_\theta} \sum_{j, z, a} I(a^s(j, z, a) = a') \overline{\Gamma(\varepsilon')} \Delta^{sw}(\theta') \mu_t^s(j, z, a)
\end{aligned} \tag{27}$$

4 Calibration

In this section, our calibration exercise is described. We focus on the aggregate TFP as the measure of the aggregate productivity. There are five groups of parameters to be calibrated. These are the parameters concerning (i) household's preferences, labor endowment and business ideas, (ii) production technology, (iii) cost of technology adoption, (iv) financial constraints, and (v) taxation. The general calibration strategy that we will use is the fol-

lowing. We assume that the first two groups of parameters to be common between the two countries, and vary the remaining three. The quantitative experiment that we consider is the following. Suppose in Canada the cost of technology adoption, the financial constraints and taxation became the same as those in the United States. Then, what will be the change in the aggregate productivity?

In conducting the experiment, the parameters will be calibrated to the selected aggregate moments from the latest data. After calibrating the model to the economy for Canada, we use it as the baseline economy. We will then feed in the U.S. values of the parameters from group (iii), (iv) and (v) to the model, and derive the results in terms of the changes in the aggregate productivity and the firm size distribution by comparing the baseline economy and the hypothetical economy with the U.S. values of the parameters in consideration. This calibration exercise will give us the effects of the changes in the cost of technology adoption, the financial constraints and taxation on the aggregate productivity through the change in the firm size distribution.

For this version of the paper, the baseline calibration is done in the partial equilibrium environment where the interest rate is exogenous and the wage rate is endogenous in the equilibrium. This assumption is made due to the fact that Canada is a small open economy where the flow of physical capital is relatively freer than that of labour. The interest rate set through out this calibration exercise is 0.035. The model is solved for the stationary equilibrium. The model period is assumed to be one year.

4.1 Household's Preferences, Labour Endowment and Business Ideas

As discussed in the model section, we assume the flow utility function to be $U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. The risk aversion parameter σ is assumed to be 1.5 and the discount factor β is set to be 0.94.

A worker is stochastically endowed with efficiency units of labor ε . We assume ε to take two values $\varepsilon \in \{0.5, 1.5\}$. The transition probability function $\Gamma(\varepsilon', \varepsilon)$ is assumed to be a symmetric matrix,

$$\begin{bmatrix} \Gamma(1, 1) & \Gamma(2, 1) \\ \Gamma(1, 2) & \Gamma(2, 2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}.$$

A worker also draws θ every period from $\Delta(\theta', \theta)$. We interpret θ to represent a business idea. When θ is high the probability of successfully starting a business is high. In our calibration, we take the extreme case of this assumption and assume that θ takes only two values, $\theta \in \{0, 1\}$,

where $\theta = 0$ implies that there is no option of investing in a business and $\theta = 1$ implies there is an opportunity to invest in a business and draw an idiosyncratic productivity z from the distribution $H_1(z)$. Given that θ takes two values, $\Delta(\theta', \theta)$ can be represented in a 2-by-2 matrix. We assume the following transition probabilities.

$$\begin{bmatrix} \Delta(1, 1) & \Delta(2, 1) \\ \Delta(1, 2) & \Delta(2, 2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \\ 0.4 & 0.6 \end{bmatrix}.$$

4.2 Production Technology

As discussed, there are two distinct production sectors, the privately-held and the publicly-traded sectors. The calibration for the parameters dictating each sector is described.

4.2.1 Privately-Held Sector

There are several parameter to be calibrated for the production technology, which include ν for the profit share of the revenue, α for the after-profit share of capital³², δ for the depreciation rate, and the parameters concerning the stochastic processes of z_j 's. In addition, we have to pin down the fixed costs e_j 's for technology adoption. We follow Restuccia and Rogerson (2004) by assuming the firm makes a 10% profit and set $\nu = 0.9$. α is set to $\frac{1}{3}$ to have the one-third of the remaining share of the revenue go to capital and the two-thirds to labor. The depreciation rate δ is assumed to be 0.08.

For the stochastic process of z_j 's, for each given j we assume that the positive values of z follow an AR(1) process such that

$$\begin{aligned} \ln(z'_j) &= (1 - \rho_j)\ln(\bar{z}_j) + \rho_j\ln(z_j) + \epsilon_j, \text{ and} \\ \epsilon_j &\sim N(0, \sigma_j^2) \end{aligned}$$

Thus, we have here three parameters, \bar{z}_j , ρ_j and σ_j , for each j . We assume $J = 2$ making the total number of parameters for these stochastic processes to be six. For a lack of good data to calibrate these parameters, we make an assumption that the persistence ρ_j to be the same for all j 's and set $\rho_j = 0.9$. The standard deviation σ_j is assumed to be higher for $j = 1$ than for $j = 2$ with $\sigma_1 = 0.15$ and $\sigma_2 = 0.05$, respectively. This is motivated by the observation that smaller firms face a more volatile income process. We pin down the \bar{z}_j 's by matching the relative optimal employment size of the business for $j = 1$ and $j = 2$ to the employment

³²These will be the shares given no financial constraints.

ratio observed in Canadian data of the average employment size for the firms with less than 100 employees and for those with more than 100 employees. The observed employment ratio is 94.2 in 1999. From equations (6) and (7), we can derive the following relationship between the ratio of n 's and that of z 's. For any z_1 and z_2 and the corresponding unconstrained profit maximizing choices of n_1 and n_2 , we have

$$\frac{n_1}{n_2} = \left(\frac{z_1}{z_2} \right)^{\frac{1}{1-\nu}}.$$

Given the above logic, this implies the ratio of \bar{z}_j 's to be 1.57. We normalize \bar{z}_1 to be 1 so that $\bar{z}_2 = 1.57$.

In constructing the Q_j 's, we discretize these AR(1) processes using Tauchan and Hussey (1991) methodology. In addition, we add $z_j = 0$ for each j as one of the values z_j can take and assume an exogenous transition probability to this state to be $Q_1(z'_1 = 0|z_1) = 0.3$ and $Q_2(z'_2 = 0|z_2) = 0.1$. As discussed in the model section, we assume this bad state to be perfectly persistent with $Q_j(z'_j = 0|z_j = 0) = 1$ for all j 's. These values will generate the exit rate around the number documented by Quadrini (2000) of 24.3%. We assume H_j 's to be the ergodic distribution of respective Q_j 's.

Finally, the fixed cost parameters e_j 's for the technology adoption have to be pinned down. These parameters are calibrated so that the relative number of firms in each j matches the same statistics in the data for the firms with less than 100 employees and the others with more than 100 employees.

4.2.2 Publicly-Traded Sector

When a privately-held business is sold, it becomes a publicly-traded firm. We assume that publicly-traded firms do not face any financial frictions such that it can always produce at the optimal size. Given this assumption, it is sufficient to know the distribution of firms over j 's and z 's in order to determine their demand for capital and labor, and their output. In the stationary equilibrium, this distribution is given by the invariant distribution $\Phi^*(j, z)$ from equation (22). Since the distribution of the entering businesses into the publicly-traded sector, $\mu^s(j, z, a)$, is endogenously generated in the equilibrium, we only need to calibrate $\Omega(j', z'|j, z)$. For this, we assume that the transition of z_j given j follows the same process as in the privately-held sector. For the current experiment, we assume there is no transition

between j 's in this sector such that

$$\begin{bmatrix} \Omega(1, \cdot | 1, \cdot) & \Omega(2, \cdot | 1, \cdot) \\ \Omega(1, \cdot | 2, \cdot) & \Omega(2, \cdot | 2, \cdot) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

4.3 Cost of Technology Adoption

The cost, e_1 and e_2 , where e_1 is the start-up cost and e_2 is the cost of technology adoption, will be calibrated together with the equilibrium calculation for Canada. We think of these cost as the total cost including distortions coming from different institutional environment. e_2 is of our particular interest. We calibrate this parameter together with others for the baseline economy to match the selected moments from the model outcome and the data. For the experiment with the U.S. parameter values, we calibrate e_2 to match the U.S. employment share for $j = 2$ firms.

4.4 Financial Constraints

There are two types of financial constraints that we consider here, the cost of the access to the equity market $\kappa(j, z)$ and the cost of borrowing. The cost of borrowing consists of two parameters, the intermediation cost ϕ of borrowing and the collateral constraint γ . For $\kappa(j, z)$, we assume the following form.

$$\kappa(j, z) = \bar{\kappa}_j + \kappa \cdot p(j, z)$$

$\bar{\kappa}_j$ is the j specific fixed cost and κ is the proportional cost to the price of the business. $\bar{\kappa}_j$'s will be calibrated to match a moment from the model outcome to that of the data. For κ , we assign a value of 0.13 and 0.10 for Canada and the US, respectively. These values represent the direct cost of the IPO based on the empirical study by Kooli and Suret (2002). For the U.S. value of $\bar{\kappa}_j$, we decrease the calibrated Canadian values by 15% such that $\bar{\kappa}_j^{US} = 0.85 \cdot \bar{\kappa}_j^{CA}$. This increase represent the difference in the long-term valuation of the publicly-traded firms in Canada and the U.S. based the study by King and Segal (2006).

We set the value of ϕ to be 0.053 for Canada and 0.05 for the U.S. The value for the U.S. is a standard number used in several studies. The Canadian value was obtained by calculating the effective interest rate for business loans in two countries and apply the relative difference to the value of the U.S. intermediation cost. We first calculated the aggregate business loan interest payment over the aggregate business loan for each country. We then took the ratio of the Canadian number to the U.S.'s. We obtained the value of 1.064 for this ratio. Hence,

$0.053 \simeq 1.064 \cdot 0.05$.

For γ , we assign a value of 0.5 for the U.S. taken from Evans and Jovanovic (1989). This makes the absolute borrowing limit in the model of 1.5 times that of the household asset. We assign a value that is 10% lower for Canada, 1.35. This implies that γ for Canada is 0.35.

4.5 Taxation

For this version, we consider a proportional tax rate for the personal income. We assign a tax rate of 21.93% and 20.75% for Canada and the U.S., respectively. These rates were obtained by taking the ratio of the aggregate personal income tax receipts to the aggregate personal income for each country over the period of 1990 to 1999.

4.6 Quantitative Method

The procedure of solving for the general equilibrium is described here. In solving for the stationary equilibrium of our model, we follow the procedure described here. We discretize the household asset space in addition to discretizing the idiosyncratic productivity shock realizations, z 's. Given these we solve for the equilibrium according to the following steps.

1. Guess the wage rate, r and w , respectively.
2. Given r and w , solve for the price of business sale, $p(j, z)$.
3. Given r , w and $p(j, z)$, solve for the value functions, V^w , V^m and V^s , by the value function iteration.
4. Given the decision rules obtained from the value function iteration, simulate an economy with a large sample of households for 1000 periods.
5. Take statistics from the last 100 periods to represent the stationary equilibrium and check if the capital and the labour markets clear.
6. If both markets are not cleared, update the guess of r and w and repeat the process from step 2.

Table 10: Calibrated Parameter Values

Parameter	Value
e_1	0.000
e_2	9.599
$\bar{\kappa}_1$	0.369
$\bar{\kappa}_2$	21.912

Table 11: Calibration Results

Moment	Canada Data	Model
% Business owners	9.5%	7.3%
% Sellers among business owner	3-16%	15.7%
Publicly-held share of output	52.4%	46.6%
Small firm share of employment	43.7%	42.6%

4.7 Matched Moments

The rest of the parameters $\{e_1, e_2, \bar{\kappa}_1, \bar{\kappa}_2\}$ will be calibrated by targeting the four moments, the fraction of households who are business owners, the fraction of sellers among business owners, the fraction of the output produced by the publicly-held sector, and the fraction of $j = 1$ employment. Table 10 shows the parameter values obtained in this calibration.

Table 11 show the moments from the data and the model equilibrium.

The data source for the fraction of households who are business owners is the 1999 Financial Security Survey in Canada. The only available data we are aware for the fraction of business sellers are from the United States. The range in the table is reported by Holmes and Schmitz (1995) from the 1982 Characteristics of Business Owners in the United States. We assume this range to be similar in Canada. The fraction of the output produced by the publicly-held sector is obtained from the SBO for the U.S. and from a custom tabulation from Statistics Canada for Canada. The fraction of $j = 1$ employment is the fraction of employment in firms with less than 100 employees from the SUSB for the U.S. and the LEAP for Canada.

With the preliminary calibration so far, the calibration result shows that our model can replicate all the data moments fairly well. However, this baseline calibration is still a work-

Table 12: Change in Productivity (%)

	e_2	κ	ϕ	γ	τ	All	US-Canada Gap
Aggregate TFP	5.98	0.91	0.19	-0.52	0.70	7.45	16.33
Average TFP	1.85	-0.10	0.05	0.30	0.14	1.96	—
Output per Capita	0.75	0.67	0.00	0.15	0.15	2.54	20.93
Wage Rate	1.26	0.64	0.15	0.27	0.34	2.96	20.91

Table 13: Firm Distributions (%)

	Baseline	e_2	κ	ϕ	γ	τ	All
Business Owners	7.3	7.6	7.2	7.3	7.3	7.3	7.7
Business Sold	15.7	14.9	16.2	15.8	15.5	15.7	15.1
Public Sector Output	46.6	44.5	53.6	47.6	43.5	45.9	46.6
$j = 2$ Firm Output	57.4	60.5	58.7	57.7	58.6	58.1	62.5
$j = 2$ Firms	24.9	27.6	24.8	25.0	25.3	25.1	27.8

in-progress at this point and will be improved.

5 Quantitative Results

Table 12 and 13 show our main results in terms of the changes in the aggregate productivity and the changes in the firm distribution, respectively.³³

Although our focus is on the aggregate TFP, in Table 12, we report the percentage changes in four measurements of the aggregate productivity, the average TFP, the aggregate TFP, the

³³Before interpreting these results, the readers should be reminded that the calibration stage of this paper is still preliminary, and these results should not be interpreted as the final quantitative ones. These results could change as we improve the calibration.

output per household and the wage rate when we change the parameter values as described in the previous section. The second to the last column is the results from changing all the parameters simultaneously. The last column in the table shows the average percentage differences between the U.S. and Canada for the period of 1990-1999.³⁴ The first row in the table reports the aggregate TFP which is the aggregation of all the individual firm TFPs. This measure takes into account the extensive margin that comes with the number of business owners in the economy. The second measure of the productivity is the TFP per firm. This measure depends solely on the distribution of firms across j 's. However, the average TFP ignores the productivity of the economy which comes from the number of business owners, i.e., the extensive margin of the occupational choice. That is to say, the average TFP is independent of how many businesses exist in the economy. The third measure is the output per capita. The last measure is the wage rate. Even though the interest rate is fixed in these experiments, the wage rate is endogenous, and thus it reflects the change in the demand for labour. As all workers receives the same wage rate in our economy, this is a measure of the labour productivity also. Table 13 shows several dimensions of the changes in the firm distributions. These two tables should be analyzed together.

Our results show that out of the total change in the aggregate TFP, a large part is coming from the change in the cost of adopting technology, e_2 . This result is pretty intuitive. As the cost of technology adoption goes down, the number of firms who adopt a higher technology and the attractiveness of owning a business increases. As observed in Table 13 in that the fraction of $j = 2$ firms' output as well as the fraction of $j = 2$ firms increases relative to the baseline case when the cost is lowered.

We observe a negative change in the aggregate TFP when the liquidity constraint loosened by an increase in parameter γ . This seems to be the result of a smaller publicly-held sector as observed in the decreases in the business sold and the fraction of the publicly-held sector output in Table 13. As the existence of the liquidity constraint is one of the differences between the privately-held and the publicly-held sectors, when this difference diminishes, the business sale to the publicly-held sector becomes less attractive. Since the number of business sales go down as a result, the publicly-held sector shrank affecting the aggregate TFP negatively.

When all the parameters are changed simultaneously, overall change in the aggregate TFP seems to capture about 46% ($\simeq 7.45/16.33$) of the observed Canada-US TFP gap.

³⁴The US-Canada output per capita percentage difference is the average over 1990 to 1999 and taken from the International Financial Statistics, IMF. The PPP of 1.2 from OECD was applied for the calculation. The labour productivity number is calculated by applying the growth rate numbers to the level estimates for 1999 from Rao, Tang and Wang (2004) for the period of 1990 to 1999.

Similar to the aggregate TFP change, the cost of adopting technology seems to play a large role in the change in the average TFP. The decrease in the cost of accessing the equity market seem to have a negative effect on the average TFP. From Table 13, this is the result of a decrease in the number of $j = 2$ firms. Even though the decrease in the cost leads to more publicly-held firms, this happens more through the increase in the sale of $j = 1$ firms. A decrease in the costs lowers the business owner's incentive to move to $j = 2$ before selling their business. As a result, the relative number of $j = 1$ firms increased and the average TFP lowered.

In terms of the change in the output per capita, all determinants of firm size except ϕ seem to be contributing to the total change of 2.54%. The most notable changes is coming from the cost of technology adoption. This result is likely coming from the increase in the relative size of the business owners together with a higher fraction of the publicly-held sector output as seen in Table 13.

In regards to the total change in the labour productivity of 2.96%, all determinants are contributing to it. The main increase are coming again from the change in the cost of technology adoption as well as the tax rate. The main difference between the effects from these two margins is that the fraction of $j = 1$ firms is higher with the change in the tax rate.

6 Conclusion

To be added.

Appendix A. Data Sources

Firm Size Distribution and Employment per Firm

Data for the firm counts and employment comes from the Statistics of U.S. Small Business (SUSB) and Statistics Canada's Longitudinal Analysis Program (LEAP). These data are for the universe of firms so are not subject to sampling variability. In both countries, firms are enterprises that can own or control more than one establishment, firm counts are obtained from business registers, employment counts are derived from payroll data, and there is no distinction between made between part and full-time employees. In addition, only firm with paid employees are included. Self-employed individuals with do not have employees working for them are not included. The U.S. omits crop and animal production, rail transportation, postal service, pension, health and vacation funds, trusts, estates, agency account, private households, and public administration. Data for Canada for the total economy is taken from Kanagarajah (2005). Industry-level data from *Employment Dynamics*, another Statistics Canada product (Catalogue No. 61F0020XCB) is used to remove crop and animal production, and public administration from the aggregate data. Data for the other minor industries excluded in the U.S. data are not available for reasons of confidentiality. The small numbers in these industries likely do not have much of an effect on the aggregate. There are some methodological differences in the way the employment counts are obtained. In the U.S., payroll data in pay period including March 12 is used to determine employment counts. In Canada, the annual earnings on all T4s (issued by the firm to each employee detailing annual earnings of each employee of the firm for tax purposes) of the firm are summed to obtain the firm's payroll. The payroll is then divided by the average annual earnings of a typical worker (from the Survey of Employment Payroll and Hours (SEPH) - an establishment survey generating numbers similar to that of the Current Employment Statistics in the United States) in the firm's industry, province and employment size class. The resulting average labour unit is conceptually identical to the employment measure in SEPH. In the SUSB, there are instances where a firm has zero employees. In the United States, firms might have an annual payroll and thus included in the counts, but no employees around March 12. These firms include those that exited before that period or entered after that period. In the firm size distribution statistic presented here, the firms with zero employees are distributed across size classes according to birth and death rates by firm size from the SUSB. In the calculation, employment per firm by firm size, firms with zero employee are omitted. This implicitly assumes that firms with zero employees have the same average number of employees as incumbents in the size class they

eventually enter into or exit from.

2002 U.S. Survey of Business Owners

The data from the Survey of Business Owners (SBO) is partly from administrative and partly from survey data. The IRS provides certain classification and measurement data, sales and employment data are from the 2002 Economic Census, and firm ownership characteristics, such as whether the business is privately or publicly held, are from the survey. The 2002 SBO excludes a few more small industries than the SUSB and excludes firms with receipts less than \$1000, but has the advantage of covering firms with no paid employees. The 2002 SBO was mailed to approximately 2.3 million businesses. The 2002 SBO covers more of the economy than in 1997. New in 2002 are data on FIRE, information and health care industries. Numbers from the SBO presented in this paper are for firms with paid employees only. Like the SUSB, there are firms in the paid employee category that have zero employees. In this paper, these firms are excluded from the calculation of sales per employee and employment shares, but included in the firm size distribution and sales shares.

Corporate and Personal Income Tax Rates

Canadian corporate profits and taxes are from National Income and Expenditure Accounts (Table 380-0014). U.S. corporate profits and taxes are from National Income and Products Accounts (Table 6.17A-D and Table 6.18A-D). Canadian personal income and taxes are from National Income and Expenditure Accounts (Table 380-0004). U.S. personal income and taxes are from National Income and Products Accounts (Table 2.1).

Prime Rates and CPI

Canadian and U.S. prime lending rates are taken from *International Financial Statistics*. Real prime rates are calculated by deflating them by the all-items Consumer Price Index (Table 326-0001 for Canada, and from the Bureau of Labor Statistics for the United States.).

Aggregate Labour and TFP Productivity

Canadian labour productivity growth rates are taken from the Canadian Productivity Accounts (Table 383-0008 for 1981-2005 and Table 383-0003 for 1962-1980). Canadian TFP growth rates are also from the Canadian Productivity Accounts (Table 383-0019 for 1981-2004 and Table 383-0001 for 1962-1980). U.S. labour and TFP growth rates are from the Bureau of Labor Statistics.

Investment and GDP

Canadian private nonresidential investment, investment in M&E and nominal total economy GDP at market prices are from Fixed Capital Flows and Stocks and the National Income and Expenditure Accounts (Table 031-0002 and Table 380-0002). Canadian private investment in ICT and its components come from custom tabulations from Statistics Canada. The data for 1981 to 2004 are taken from Sharpe (2005) and the data for 1961-1980 are taken from Khan and Santos (2002). The data from Khan and Santos are for the non-farm business sector, and not the business sector as in Sharpe. However, the numbers in the overlapping years are quite similar. To obtain the series presented in the paper, the numbers from Khan and Santos were rescaled to match the numbers in Sharpe for 1981, the first overlapping year. U.S. private nonresidential investment of all types and nominal total economy GDP at market prices are from the National Income and Products Accounts (Table 5.3.5 and Table 1.1.5). Software investment is available in Canada only from 1981, so for comparison purposes software investment was removed from U.S. ICT investment.

Table A1. Firm Size Distribution

	Canada				US			
	<19	20-99	100-499	500+	<19	20-99	100-499	500+
1983	0.937	0.052	0.009	0.002	—	—	—	—
1984	0.935	0.054	0.009	0.002	—	—	—	—
1985	0.934	0.055	0.009	0.002	—	—	—	—
1986	0.932	0.057	0.009	0.002	—	—	—	—
1987	0.930	0.058	0.009	0.002	—	—	—	—
1988	0.929	0.060	0.009	0.002	0.891	0.092	0.014	0.003
1989	0.927	0.061	0.010	0.002	0.889	0.094	0.014	0.003
1990	0.929	0.059	0.010	0.002	0.888	0.095	0.014	0.003
1991	0.925	0.063	0.010	0.002	0.890	0.093	0.014	0.003
1992	0.927	0.061	0.010	0.002	0.891	0.092	0.014	0.003
1993	0.926	0.062	0.010	0.002	0.891	0.091	0.014	0.003
1994	0.925	0.063	0.010	0.002	0.892	0.091	0.014	0.003
1995	0.924	0.064	0.010	0.002	0.890	0.093	0.015	0.003
1996	0.923	0.064	0.011	0.002	0.890	0.092	0.014	0.003
1997	0.922	0.065	0.011	0.002	0.888	0.094	0.015	0.003
1998	0.921	0.065	0.011	0.002	0.888	0.094	0.015	0.003
1999	0.922	0.065	0.011	0.002	0.887	0.095	0.015	0.003
2000	—	—	—	—	0.885	0.097	0.016	0.003
2001	—	—	—	—	0.884	0.097	0.016	0.003
2002	—	—	—	—	0.887	0.095	0.015	0.003

Table A2. Firm Size Distribution in the United States, 2002

	1-19	20-99	100-499	500+	
Public	0.046	0.011	0.005	0.002	0.064
Private	0.848	0.076	0.010	0.001	0.936
Total	0.894	0.088	0.015	0.003	1.000

Source: U.S. Survey of Business Owners.

Table A3. Share of Sales in the United States, 2002

	1-19	20-99	100-499	500+	
Public	0.010	0.022	0.044	0.555	0.632
Private	0.117	0.107	0.072	0.065	0.368
Total	0.127	0.128	0.115	0.620	1.000

Source: U.S. Survey of Business Owners.

Table A4. Employment Shares in the United States, 2002

	1-19	20-99	100-499	500+	
Public	0.011	0.025	0.049	0.415	0.500
Private	0.165	0.146	0.094	0.095	0.500
Total	0.176	0.171	0.142	0.510	1.000

Source: U.S. Survey of Business Owners.

Appendix B. Definition of a stationary equilibrium

A stationary equilibrium for a given set of policy arrangements, $\{T(\cdot), G\}$, is a collection of value functions for workers, business managers, and business sellers, $\{V^w(a, \theta), V^m(j, z, a), V^s(j, z, a)\}$; policy functions for workers, business managers, and business sellers, $(a^w, e^w, c^w, m^w), (a^m, e^m, c^m, m^m), (a^s, c^s)$; invariant distributions of workers, managers and sellers, $\mu^w(a, \theta), \mu^m(j, z, a), \mu^s(j, z, a)$; measure of firms managed by financial intermediaries, $\Phi(j, z)$; and prices, $(w, r, p(j, z))$, such that:

1. For given prices, V^w , V^m and V^s satisfy workers', managers' and sellers' problems (12), (13), and (19) respectively. (a^w, e^w, c^w) , (a^m, c^m, e^m, m^m) , and (a^s, c^s) are optimal decision rules.
2. Privately held business, publicly traded business and intermediation sectors make zero profits and prices are competitive:
3. Capital and labor markets clear:

$$\begin{aligned} \sum_{j,z,a} k(j, z, a) \mu^m(j, z, a) + \sum_{j,z} k(j, z) \Phi(j, z) = \\ \sum_{\theta,j,z,a} a \mu^m(j, z, a) + \sum_{\theta,j,z,a} a \mu^s(j, z, a) + \sum_{\theta,a} a \mu^w(\theta, a) \end{aligned} \quad (28)$$

$$\sum_{\theta,j,\eta,a} n(j, \eta, a, \theta) \Lambda(\theta) \mu^m(j, \theta, \eta, a) + \sum_{j,\eta} n(j, \eta) \Phi(j, \eta) = \sum_{\theta,a} \Lambda(\theta) \mu^w(\theta, a) \quad (29)$$

4. The government budget is balanced:

$$\begin{aligned} G = \sum_{\theta,j,\eta,a} T^m(\theta, j, \eta, a) \Lambda(\theta) \mu^m(j, \theta, \eta, a) + \sum_{\theta,j,\eta,a} T^s(\theta, j, \eta, a) \Lambda(\theta) \mu^s(j, \theta, \eta, a) + \\ \sum_{\theta,a} T^w(\theta, a) \Lambda(\theta) \mu^w(\theta, a) \end{aligned} \quad (30)$$

5. Invariant distributions, $\{\mu^w, \mu^m, \mu^s\}$ are consistent with individuals' optimal behaviour.
6. Invariant distributions, Φ are consistent the optimal behaviour of individuals and financial intermediaries.

References

- [1] Astebro, T. 2002. "Noncapital Investment Costs and the Adoption of CAD and CNC in U.S. Metalworking Industries," *Rand Journal of Economics*, 33(4): 672-688.
- [2] Baldwin, J. and J. Chowhan. 2003. "The Impact on Self-Employment on Labour Productivity Growth: A Canada-U.S. Comparison," Economic Analysis Research Paper No. 16, Statistic Canada.
- [3] Baldwin, J., R. Jarmin, and J. Tang. 2002. "A Trend to Smaller Producers in Manufacturing: A Canada/U.S. Comparison," Economic Analysis Research Paper No. 3, Statistics Canada.
- [4] Baldwin, J. and W. Gu. 2003. "Participation on Export Markets and Productivity Performance in Canadian Manufacturing," *Canadian Journal of Economics*, 36(3): 634-657.
- [5] Baldwin, J. and W. Gu. 2004. "Trade Liberalization: Export-market Participation, Productivity Growth and Innovation," Economic Analysis Research Paper No. 027, Statistics Canada.
- [6] Baldwin, J., J.-P. Maynard, M. Tanguay, F. Wong, B. Yan. 2005. "A Comparison of Canadian and U.S. Productivity Levels: An Exploration of Measurement Issues." Statistics Canada, Economic Analysis Research Paper No. 28.
- [7] Baldwin, J. and D. Sabourin. 1998. "Technology Adoption: A Comparison Between Canada and the United States," Statistics Canada, Analytical Studies Research Paper Series No. 11.
- [8] Baldwin, J., E. Rama and D. Sabourin. 1999. "Growth of Advanced Technology Use in Canadian Manufacturing During the 1990s," Statistics Canada.
- [9] Bartlesman, E., and M. Doms. 2000. "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 38(September): 569-594.
- [10] Beck, T., A. Demircuc-Kunt, L. Laeven, and R. Levine. 2004. "Finance, Firm Size and Growth," NBER Working Paper No. 10983.
- [11] Breshnahan, T., E. Brynjolfsson and L. Hitt. 2002. "Information Technology, Workplace Organization, and the Demand for Skilled Labor: Firm-Level Evidence," *Quarterly Journal of Economics*, 117(1): 339-376.

- [12] Brynjolfsson, E. and L. Hitt. 2000. "Beyond Computation: Information Technology, Organizational Transformation and Business Performance," *Journal of Economic Perspectives*, 11(4): 23-48.
- [13] Cooley, T., and V. Quadrini. 2001. "Financial Markets and Firm Dynamics," *American Economic Review*, 91(5): 1286-1310.
- [14] Carroll, R., D. Holtz-Eakin, M. Rider and H. Rosen. 2000. "Personal Income Taxes and the Growth of Small Firms," mimeo.
- [15] Carroll, R., D. Holtz-Eakin, M. Rider and H. Rosen. 1998a. "Entrepreneurs, Income Taxes and Investment," NBER Working Paper No. 6374.
- [16] Carroll, R., D. Holtz-Eakin, M. Rider and H. Rosen. 1998b. "Income Taxes and Entrepreneurs Use of Labor," Metropolitan Studies Program Series, Occasional Paper No. 193. Center for Policy Research, Syracuse University.
- [17] Caves, R. 1998. "Industrial Organization and New Findings on the Turnover and Mobility of Firms," *Journal of Economic Literature*, 36(December): 1947-1982.
- [18] Chari, V.V., M. Golosov, and A. Tsyvinski. 2002. "Business Start-ups, the Locking-in Effect, and Capital Gains Taxation," Manuscript, Federal Reserve Bank of Minneapolis.
- [19] Covas, F., W. den Haan. 2005. "The Role of Debt and Equity Financing over the Business Cycle," Manuscript, Bank of Canada.
- [20] Crawford, A. 2002. "Trends in Productivity Growth," *Bank of Canada Review*, (Spring): 19-32.
- [21] Cummins, J., K. Hasset, and G. Hubbard. 1994. "A Reconsideration of Investment Behavior Using Tax Reforms as Natural Experiments," *Brookings Papers on Economic Activity* (2):1-59.
- [22] Dunne, T. 1994. "Plant Age and Technology Use in the U.S. Manufacturing Industries," *Rand Journal of Economics*, 25: 488-99.
- [23] Erosa, A., and A. Hidalgo-Cabrillana. 2005. "On Capital Market Imperfections as a Source of Low TFP and Economic Rents," manuscript.

- [24] Evans, D., and B. Jovanovic. 1989. "An Estimated Model of Entrepreneurial Choice Under Liquidity Constraints," *Journal of Political Economy*, 98(3): 657-674.
- [25] Fuss, M. and L. Waverman. 2005. "Canada's Productivity Dilemma: The Role of Computers and Telecom," Bell Canada's submission to the Telecommunications Policy Review Panel, Appendix E-1.
- [26] Ghosal, V. and P. Loungani. 1996. "Firm Size and the Impact of Profit-Margin Uncertainty on Investment: Do Financing Constraints Play a Role?" International Finance Discussion Paper No. 557, Board of Governors of the Federal Reserve System.
- [27] Guner, N., G. Ventura, X. Yi. 2005. "Macroeconomic Implications of Size-Dependent Policies," manuscript.
- [28] Hall, R., and C. Jones. 1999. "Why Do Some Countries Produce So Much More Output per Worker than Others?" *Quarterly Journal of Economics*, 114(1): 83-116.
- [29] Hall, R., and D. Jorgenson. 1967. "Tax Policy and Investment Behaviour," *American Economic Review*, 57(3): 391-414.
- [30] Ho, M., S. Rao and J. Tang. 2004. "Sources of Output Growth in Canadian and U.S. Industries in the Information Age," in Dale Jorgenson (ed.) Economic Growth in Canada and the U.S. in the Information Age, Industry Canada Research Monograph.
- [31] Holmes, T., and J. Schmitz. 1990. "A Theory of Entrepreneurship and Its Application to the Study of Business Transfers," *Journal of Political Economy*, 92(2): 265-294.
- [32] Holmes, T., and J. Schmitz. 1995. "On the Turnover of Business Firms and Business Managers," *Journal of Political Economy*, 103(5): 1005-1038.
- [33] Jermann, U. and V. Quadrini. 2003. "Stock Market Boom and the Productivity Gains of the 1990s," Manuscript, University of Pennsylvania.
- [34] Kanagarajah, S. 2005. "Business Dynamics in Canada." Statistics Canada, Catalogue No. 61-534-XIE.
- [35] Karshenas, M. and Stoneman, P. 1993. "Rank, Stock and Order and Epidemic Effect in the Diffusion of New Process Technologies: An Empirical Model," *Rand Journal of Economics*, 24: 503-28.

- [36] Khan, H. and M. Santos. 2002. "Contributions to ICT Use to Output and Labour-Productivity Growth in Canada," Bank of Canada Working Paper No. 2002-07.
- [37] King, M. and D. Segal. 2003. "Valuation of Canadian-vs. U.S. -Listed Equity: Is There a Discount?" Bank of Canada Working Paper No. 2003-6.
- [38] Kooli, M., and J-M. Suret. 2003. "How Cost-Effective are Canadian IPO Markets?" *Canadian Investment Review* Winter: 20-28.
- [39] Leung, D. 2004. "The Effect of Adjustment Costs and Organizational Change on Productivity in Canada: Evidence from Aggregate Data," *International Productivity Monitor*, Fall (9):52-61.
- [40] Loughran, T., and J. Ritter. 2004. "Why has IPO Underpricing Changed Over Time?" *Financial Management*, 33 (3): 5-37.
- [41] Macklem, T. 2003. "Future Productivity Growth in Canada: Comparing to the United States," *International Productivity Monitor*, Fall (7):50-57.
- [42] McKenzie, K., and A. Thompson. 1997. "Taxes, the Cost of Capital, and Investment: A Comparison of Canada and the United States," Working Paper 97-3, Technical Committee on Business Taxation, Department of Finance.
- [43] OECD. 2001. "Productivity and Firm Dynamics: Evidence From Microdata," *OECD Economic Outlook*, (69): 209-223.
- [44] Parente, S., and E. Prescott. 1994. "Barriers to Technology Adoption and Development," *Journal of Political Economy*, 102(2): 298-321.
- [45] Parente, S., and E. Prescott. 1999. "Monopoly Rights: A Barrier to Riches," *American Economic Review*, 85(5): 1216-1233.
- [46] Porter, M., K. Schwab, X. Salai-i-Martin, A. Lopez-Claros. 2004. The Global Competitiveness Report 2004-2005. World Economic Forum, Geneva, Palgrave MacMillan.
- [47] Quadrini, V. 2000. "Entrepreneurship, Savings and Social Mobility," *Review of Economic Dynamics*, 3 (1): 1-40.

- [48] Oliner, S. and D. Sichel. 2002. "Information Technology and Productivity: Where Are We Now and Where are We Going?" *Federal Reserve Bank of Atlanta Economic Review*, 87 (Summer): 15-44.
- [49] Rao, S., J. Tang, and W. Wang. 2004. "Measuring the Canada-U.S. Productivity Gap: Industry Dimensions," *International Productivity Monitor*, Fall (9):3-14.
- [50] Restuccia, D. and R. Rogerson. 2004. "Policy Distortions and Aggregate Productivity with Heterogeneous Plants," 2004 Meetings Papers 69, Society of Economic Dynamics.
- [51] Royal Bank. 2002. "The Path to Prosperity: Canada's Small and Medium Sized Enterprises," www.rbc.com/newsroom.
- [52] Tauchen, G., and R. Hussey. 1991. "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, 57(5): 1091-1120.
- [53] Sharpe, A. 2005. "What Explains the Canada-US ICT Investment Intensity Gap?" Centre for the Study of Living Standards Research Report No. 6.
- [54] Treffer, D. 2004. "The Long and Short of the Canada-U.S. Free Trade Agreement," *American Economic Review*, 94 (4):870-895.
- [55] Van Ark., B., R. Inklaar and R. McGuckin. 2002. "Changing Gear: Productivity, ICT and Services: Europe and the United States," GGDC Working Paper GD-60.
- [56] Winters, L. 2004. "Trade Liberalisation and Economic Performance: An Overview," *Economic Journal*, 144 (February): F4-F21