Modelling Oil Prices in a Dynamic General Equilibrium Model with Free Entry*

[VERY PRELIMINARY]

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Abstract

In this paper, we first set up a model that combines the New Trade Theory initiated by Krugman (1980) with the New Open Economy Macroeconomics introduced by Obstfeld and Rogoff (1995). We then inquire into the cause of soaring oil prices, and their transmission and implications on welfare as well as stabilization policy with such a dynamic general equilibrium model. Both standard contemporaneous shocks and expectation shock, namely anxiety about future oil demand and supply, are also examined. Our conclusion is that for a monetary authority to stabilize the economy it is of great importance for the authority to identify correctly the shocks that cause oil price hikes.

JEL Classification: E52; F10; F41

Keywords: Oil Price; Dynamic General Equilibrium; Free Entry; Endogenous Variety

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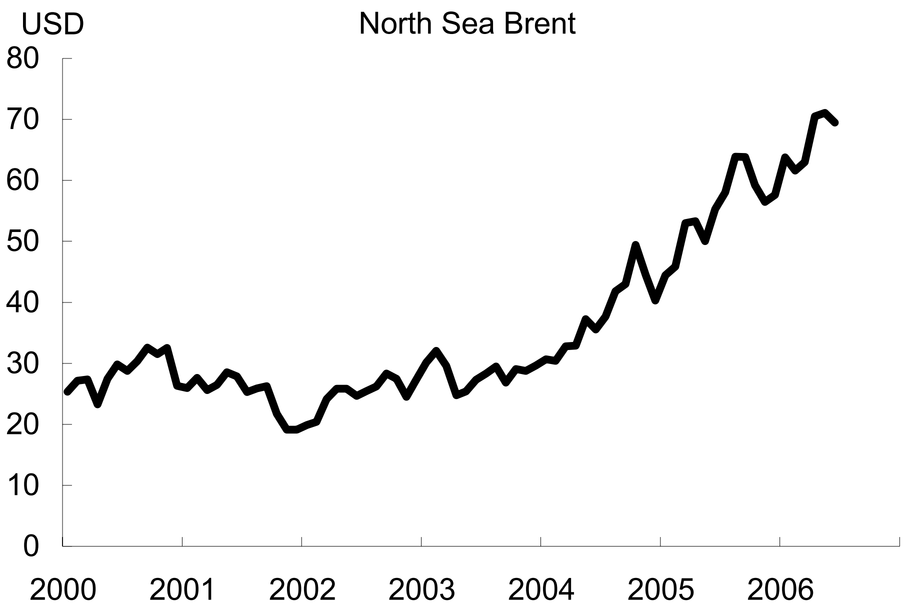

Figure 1: North Sea Brent

1 Introduction

One notable feature of recent world economic developments is soaring oil prices. As shown in Figure 1, the price of North Sea Brent is now more than twice as much as that of three years ago. Yet, the cause of this hike and therefore its transmission mechanism are not very obvious. Some argue that the increasing presence of developing countries in the world economy is the cause of oil price hikes. Particularly, the recent rapid economic growth among the BRICs, namely Brazil, Russia, China, and India, must have resulted in an increase in the demand for raw materials. Since those countries have very large populations, it seems quite possible that oil prices are skyrocketing due to the massive increase in the demand for crude materials. Higher demand for oil in the future is also expected in line with this argument. On the other hand, some point out that expected geographical risks in the Middle East will constrain future oil production. It seems possible that not all but some of the recent soaring oil prices is reflecting those supply-side developments and therefore market speculation. It is not very easy to find a definitive answer. Indeed, in the recent press release by OPEC on 1 June 2006, "141st (Extraordinary) Meeting of the OPEC Conference," it is written that "The Conference also noted that, similarly, world crude oil prices continued to remain high and volatile as a consequence of abiding concern over the lack of effective global oil refining capacity, in the short and medium term, coupled with anxiety about the ability of oil producers to meet anticipated, future oil demand. This price volatility is being exacerbated by geopolitical developments and speculation in the oil futures markets."

Even if we know that recent oil price developments are due either to demand side or to supply side, there exist various candidates for demand and supply factors. For example, as for demand factors, oil price may increase thanks to increased working population ignited by moving from rural to urban areas, changes in preference, and technology growth in the final or intermediate goods sector. For the policy making institutions, it is of great importance to acknowledge the source of economic fluctuations, including oil price developments, so that they can conduct proper stabilization policy. We will clarify the causes of soaring oil prices as well as their theoretical transmission and their implications on economic welfare since the oil price is not exogenous variables in a multi-country dynamic general equilibrium model

So far, there have been various studies concerning the effects of changes in oil price, but very few exist to inquire into the possible cause of it in detail in a dynamic general equilibrium model, where oil prices are not exogenous variables. In this paper, we will show the theoretical mechanism of soaring oil prices, its transmission and welfare implications, that have not been fully analyzed in detail as endogenous mechanisms. For this purpose, we construct a dynamic general equilibrium model with free entry. The reason why we employ the recently developed dynamic general equilibrium model based on the recent developments in the New Trade Theory as in Melitz (2003) than the standard New Open Economy Model with fixed entry is, as written in Bergin and Corsetti (2006), "Firstly, we see strong empirical evidence that entry dynamics comove with the business cycle, a stylized fact that will be discussed below. Secondly, entry has the potential to serve as an amplification and propagation mechanism for real shocks, and to affect the transmission mechanism for monetary policy. Thirdly entry may have notable welfare effects, to the degree that households derive utility from greater variety, or to the degree that the entry of new firms raises competition in a market." Besides the above, since the aim of this paper is theoretically to inquire into the possible scenario of soaring oil prices in detail, a model with plausible mechanism for current oil prices hike is most desirable. We believe that for the scenario of future increase in oil demand, the endogenous variety model is the most useful since it can analyze the effect of increased population on oil demand as well as the home market effect. Furthermore, we can examine various shocks as decreased entry fixed cost for introducing foreign direct investment in the developing economies, which also results in soaring oil prices. This is the main reason for employing the endogenous variety model in this paper.

This paper is organized as follows. In Section 2, we introduce our model for the analysis on oil price developments, namely the dynamic general equilibrium model with free entry. Then, Section 3 shows the impulse response analysis against various kinds of shocks that affect oil prices. One contribution of this paper is to show impulse responses not only against the usual contemporaneous shocks but also expectation shock on future technology or demand condition. We will show how we can express the realistic scenario, "anxiety about the ability of oil producers to meet anticipated, future oil demand" in a dynamic general equilibrium model by employing the expectation shock introduced by Beaudry and Portier (2004) Jaimovich and Rebelo (2005) and Christiano, Motto and Rostagno (2006). Finally, Section 4 summarizes the findings in this paper.

2 The Model

The model used in this paper is based on recent literature, which combines the New Open Economy Macroeconomics initiated by Obstfeld and Rogoff (1995) and the New Trade Theory advocated by Krugman (1980), for example, Ghironi and Melitz (2005), Bilbiie, Ghironi and Melitz (2005), Corsetti, Martin and Pesenti (2005), and Bergin and Corsetti (2006). Since heterogeneous technology level among firms is not considered, our model can be interpreted as a dynamic extension of Corsetti, Martin and Pesenti (2005) or a multi-country extension of Bilbiie, Ghironi and Melitz (2005) and Bergin and Corsetti (2006). Furthermore, our model incorporates nominal rigidities in price and wage settings, and dynamic adjustment costs and dynamic entry/exit decision with time to build constraints. Although the model incorporates free entry condition for endogenous variety as a new dynamic feature, dynamic parts are mostly based on the Global Economy Model (GEM) by Laxton and Pesenti (2002).

The model is a two-country (economy) model, which consists of home and foreign countries. Agents in each country are households, firms, and the monetary authority as a sole institution in the government. Households maximize their welfare from consumption of final goods C and leisure after differentiated labor supply l to domestic firms. The number of households in domestic country is L, while that of in foreign country is L^*, where superscript $*$ denotes the variables in foreign countries. Both are exogenous. They own domestic firms and therefore receive profits as a dividend.[1]

The goods market is monopolistically competitive. Each firm produces differentiated products. The number of domestic goods is n, while that of foreign goods is n^*. Unlike other standard papers in new open economy macroeconomics, they are endogenously determined in this paper, which is its main contribution. When entering the market, each firm needs to incur fixed entry costs. Therefore, entry occurs when the net present value of future profits exceeds entry cost. This eventually determines the macro production level as well as the number of goods, namely variety. On the exit side, a certain proportion δ of firms exits each period.

The production structure of this model can be well understood from the concept chart as shown in Figure 2. Oil production using labor and exogenously supplied land takes place only in the foreign country. Intermediate goods are

[1] We can easily extend our model to incorporate capital stock by assuming that households rent capital to firms. For the simplicity of analysis, however, we abstract capital formation in this paper. The conclusions in this paper will not be affected by incorporation of capital.

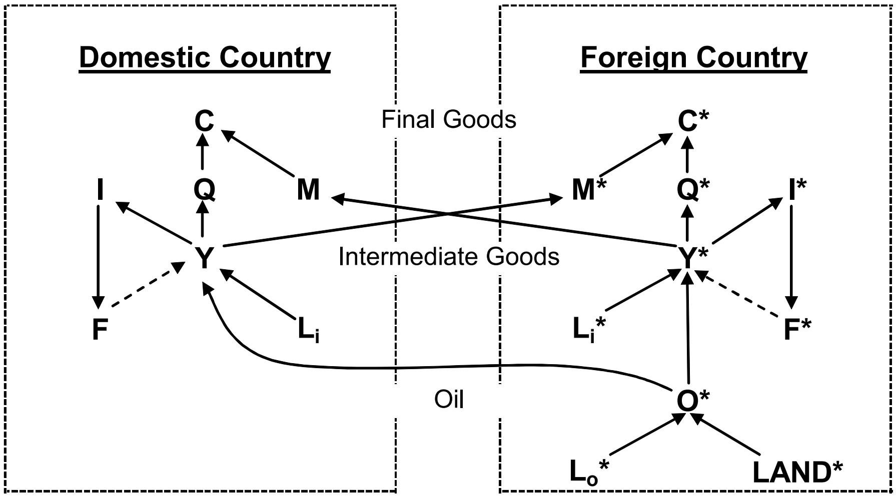

Figure 2: Production Structure

produced using oil and labor. They are used as intermediate inputs in final goods production[2] either in the domestic or foreign country or as a fixed cost to enter the market, namely a cost to start up a firm.[3] Final goods produced as such are all consumed

Below, we first derive structural equations from firms' and then households' optimization behavior. In this model, $j \in [0, L_t]$ denotes the index of domestic households and $j^* \in [0, L_t^*]$ the index of foreign households, while $h \in [0, n_t]$ the index of domestic firms and $f \in [0, n_t^*]$ the index of foreign firms.

2.1 Firms

2.1.1 Final goods production

The final goods consumed by household j, namely $C_t(j)$, are produced by following CES technology using a basket of home goods $Q_t(j)$ and a basket of foreign goods $M_t(j)$:

$$C_t(j) = \left[\nu^{\frac{1}{\varepsilon}} Q_t(j)^{1-\frac{1}{\varepsilon}} + (1-\nu)^{\frac{1}{\varepsilon}} M_t(j)^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

[2] As showin in Bilbiie, Ghironi and Melitz (2005), considering empirical problems associated with increasing returns to specialization and a C.E.S. production function, it may be better to model that the household consumes the basket of goods defined over a continuum of goods. Neigher specification, however, makes a difference in simulations conducted in this paper.

[3] This cost is considered to be an investment although there is no endogenous capital formation. Therefore, we use the notation I in Figure 1.

where ε denotes the elasticity of substitution between home goods and imported goods, and ν is the home bias parameter. By minimizing total expenditure defined as the sum of $P_{Q,t}Q_t(j)$ and $P_{M,t}M_t(j)$, where $P_{Q,t}$ is the aggregate price index for domestic goods and $P_{M,t}$ is that for foreign goods, subject to equation (1), we can obtain demand for $Q_t(j)$ and $M_t(j)$:

$$Q_t(j) = \nu \left(\frac{P_{Q,t}}{P_t} \right)^{-\varepsilon} C_t(j), \tag{2}$$

and

$$M_t(j) = (1-\nu) \left(\frac{P_{M,t}}{P_t} \right)^{-\varepsilon} C_t(j), \tag{3}$$

and the utility-based consumer price index P_t as a Lagrange multiplier on the constraint:

$$P_t = \left[\nu P_{Q,t}^{1-\varepsilon} + (1-\nu) P_{M,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

Furthermore, baskets for home and foreign goods are also expressed as the CES aggregator of each good provided by different firms indexed by h and f:

$$Q_t(j) \equiv A_{Q,t} \left[\int_0^{n_t} Q_t(h,j)^{1-\frac{1}{\theta}} \, dh \right]^{\frac{\theta}{\theta-1}}, \tag{4}$$

and

$$M_t(j) \equiv A_{Q,t}^* \left[\int_0^{n_t^*} M_t(f,j)^{1-\frac{1}{\theta^*}} \, df \right]^{\frac{\theta^*}{\theta^*-1}}, \tag{5}$$

where $\theta\,(\theta^*) > 1$ denotes the elasticity of substitution among intermediate home (foreign) goods and $A_{Q,t}$ and $A_{Q,t}^*$ determine degree of taste for variety and take the forms of:

$$A_{Q,t} \equiv (n_t)^{\gamma - \frac{\theta}{\theta-1}},$$

and

$$A_{Q,t}^* \equiv (n_t^*)^{\gamma^* - \frac{\theta^*}{\theta^*-1}},$$

where γ denotes the degree of taste for good variety. As shown in Benassy (1996), $1-\gamma$ denotes the marginal utility (productivity) gain for increasing a given amount of consumption on a basket that includes one additional good variety. If $\gamma = \frac{\theta}{\theta-1}$ ($\gamma^* = \frac{\theta^*}{\theta^*-1}$), equations (4) and (5) collapse to standard Dixit-Stiglitz aggregator. As analyzed in Corsetti, Martin and Pesenti (1995) and Bergin and Corsetti (2006), effects through taste for variety itself are very intriguing. Furthermore, as emphasized in Ghironi and Melitz (2005) and Bilbiie, Ghironi and Melitz (2005), we can obtain realistic price developments by using average price instead of utility-based measure even with some taste for variety. In this paper, however, our focus is solely on the oil price determination process and its transmission. Therefore, we abstract taste for variety by setting γ and γ^* equal unity.

Each household takes the prices of the home differentiated goods $p_t(h)$ as given and minimizes the total expenditure expressed as $\int_0^{n_t} p_t(h)\, Q_t(h,j)\, dh$ subject to equation (4). The cost-minimizing price of one unit of the home goods basket, $P_{Q,t}$, obtained from this optimization problem is:

$$P_{Q,t} = \frac{1}{A_{Q,t}} \left[\int_0^{n_t} p(h)^{1-\theta}\, dh \right]^{\frac{1}{1-\theta}}.$$

Similarly, that of the Foreign goods basket, $P_{M,t}$ is defined as:

$$P_{M,t} = \frac{1}{A^*_{Q,t}} \left[\int_0^{n^*_t} p(f)^{1-\theta^*}\, df \right]^{\frac{1}{1-\theta^*}}.$$

As is the case of final goods production, we can obtain the demand for domestic demand for each domestically produced good:

$$\begin{aligned} Q_t(h,j) &= A_{Q,t}^{\theta-1} \left(\frac{p_t(h)}{P_{Q,t}} \right)^{-\theta} Q_t(j) \\ &= \nu A_{Q,t}^{\theta-1} \left(\frac{p_t(h)}{P_{Q,t}} \right)^{-\theta} \left(\frac{P_{Q,t}}{P_t} \right)^{-\varepsilon} C_t(j), \end{aligned}$$

as well as that for each foreign good:

$$\begin{aligned} M_t(f,j) &= \left(A^*_{Q,t} \right)^{\theta^*-1} \left(\frac{p_t(f)}{P_{M,t}} \right)^{-\theta^*} M_t(j) \\ & (1-\nu) \left(A^*_{Q,t} \right)^{\theta^*-1} \left(\frac{p_t(f)}{P_{M,t}} \right)^{-\theta^*} \left(\frac{P_{M,t}}{P_t} \right)^{-\varepsilon} C_t(j), \end{aligned}$$

The same derivation holds in the foreign country. Therefore, we can derive:

$$Q^*_t(f,j^*) = \nu^* A_{Q,t}^{\theta-1} \left(\frac{p^*_t(f)}{P^*_{Q,t}} \right)^{-\theta} \left(\frac{P^*_{Q,t}}{P^*_t} \right)^{-\varepsilon} C^*_t(j^*),$$

and

$$M^*_t(h,j^*) = (1-\nu^*) \left(A^*_{Q,t} \right)^{\theta^*-1} \left(\frac{p^*_t(h)}{P_{M,t}} \right)^{-\theta^*} \left(\frac{P^*_{M,t}}{P^*_t} \right)^{-\varepsilon^*} C^*_t(j^*).$$

2.1.2 Intermediate goods production

Production technology Intermediate goods are produced in a monopolistically competitive market. Production Y_t requires two factors, labor l^I_t and oil O_t.[4] Each domestic firm h has a CES production function:

$$Y_t(h) = Z_t \left[(1-\alpha)^{\frac{1}{\xi}} l^I_t(h)^{1-\frac{1}{\xi}} + \alpha^{\frac{1}{\xi}} O_t(h)^{1-\frac{1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \tag{6}$$

[4] We can incorporate capital in our standard model. However, to focus on the oil price developments, we abstract capital formation in this paper.

where ξ is the elasticity of substitution and Z_t is the Hicks-neutral technology,[5] and tries to minimize the total cost as the sum of $\frac{W_t}{P_t} l_t^I(h)$ and $\frac{P_{O,t}}{P_t} O_t(h)$, where $\frac{W_t}{P_t}$ denotes the real wage and $\frac{P_t^o}{P_t}$ is the real oil price, subject to equation (6). We can then derive the factor demands for labor and capital:

$$l_t^I(h) = (1-\alpha)\left(\frac{W_t}{P_t \Psi_t Z_t}\right)^{-\xi} \frac{Y_t(h)}{Z_t}, \tag{7}$$

$$O_t(h) = \alpha \left(\frac{P_{O,t}}{P_t \Psi_t Z_t}\right)^{-\xi} \frac{Y_t(h)}{Z_t}. \tag{8}$$

where Ψ_t is the Lagrange multiplier to equation (6), the marginal cost of producing one unit of intermediate goods. Ψ_t can be computed by substituting equations (7) and (8) into equation (6):

$$\Psi_t = \frac{1}{Z_t}\left[(1-\alpha)\left(\frac{W_t}{P_t}\right)^{1-\xi} + \alpha\left(\frac{P_{O,t}}{P_t}\right)^{1-\xi}\right]^{\frac{1}{1-\xi}}. \tag{9}$$

In addition to this production cost, firms incur sunk entry costs of $f_{E,t}$ unit of final goods prior to entry. From equations (7) and (8), we can derive factor demands required for entry:

$$l_{E,t} = (1-\alpha)\left(\frac{W_t}{\Psi_t Z_t}\right)^{-\xi} \frac{f_{E,t}}{Z_t},$$

$$O_{E,t} = \alpha\left(\frac{P_{O,t}}{\Psi_t Z_t}\right)^{-\xi} \frac{f_{E,t}}{Z_t}.$$

Thus, aggregate factor demands for labor and oil by firm h are expressed as $l_{E,t} + l_t^I(h)$ and $O_{E,t} + O_t(h)$ respectively.

Price setting Each incumbent firm h must set two prices, $p_t(h)$ in the home market and $p_t^*(h)$ in the foreign market so that the present discounted value of profit is maximized. Each firm h takes into account the demand in home $Q_t(h,j)$ as well as foreign market $M_t^*(h,j^*)$. We assume that there are sluggish price adjustment costs measured in terms of total profits, namely the Rotemberg (1982) type adjustment cost. Thus the maximization problem of each incumbent firm h to set its prices is expressed as follows:

$$\max_{p_\tau(h), p_\tau^*(h)} \mathrm{E}_t \sum_{\tau=t}^{\infty} (1-\delta)^{\tau-t} D_{t,\tau}(j)\, \Pi_\tau(h),$$

[5] At the moment, we have not incorporated trend growth into this model.

where $D_{t,i}(j)$ is the stochastic discount factor between t and i, and the profit $\Pi_t(h)$ is defined as:

$$\Pi_t(h) \equiv [p_t(h) - \Psi_t] \int_0^{L_t} Q_t(h,j)\, dj \, [1 - \Gamma_{Q,t}(h)] \qquad (10)$$
$$+ [\mathcal{E}_t p_t^*(h) - \Psi_t(1+\tau)] \int_0^{L_t^*} M_t^*(h,j^*)\, dj^* \left[1 - \Gamma_{M,t}^*(h)\right].$$

$\mathcal{E}_t$ is home currency per unit of foreign currency,[6] τ is the transportation cost of foreign goods, and $\Gamma_{Q,t}(h)$ and $\Gamma_{M,t}^*(h)$ are the Rotemberg (1982) type adjustment costs defined as:

$$\Gamma_{Q,t}(h) \equiv \frac{\phi_Q}{2} \left[\frac{p_t(h)/p_{t-1}(h)}{P_{Q,t-1}/P_{Q,t-2}} - 1\right]^2,$$

$$\Gamma_{M,t}^*(h) \equiv \frac{\phi_M^*}{2} \left[\frac{p_t^*(h)/p_{t-1}^*(h)}{P_{M,t-1}^*/P_{M,t-2}^*} - 1\right]^2,$$

where ϕ_Q and ϕ_M^* define the size of adjustment costs. By solving for the first order condition with respect to $p_t(h)$, we can obtain a price-setting relation for domestically consumed goods:

$$\begin{aligned}
0 = {} & [1 - \Gamma_{Q,t}(n)] [p_t(h)(1-\theta) + \theta\Psi_t] \\
& - [p_t(h) - \Psi_t] \frac{\phi_Q p_t(h)/p_{t-1}(h)}{P_{Q,t-1}/P_{Q,t-2}} \left[\frac{p_t(h)/p_{t-1}(h)}{P_{Q,t-1}/P_{Q,t-2}} - 1\right] \\
& + \mathrm{E}_t (1-\delta_D) D_{t,t+1} [p_{t+1}(h) - \Psi_{t+1}] \\
& \times \frac{\int_0^{L_{t+1}} Q_{t+1}(h,j)\, dj}{\int_0^{L_t} Q_t(h,j)\, dj} \frac{\phi_Q p_{t+1}(h)/p_t(h)}{P_{Q,t}/P_{Q,t-1}} \left[\frac{p_{t+1}(h)/p_t(h)}{P_{Q,t}/P_{Q,t-1}} - 1\right].
\end{aligned}$$

Similarly, with respect to $p_t^*(h)$, the price-setting equation for exported goods

$$\begin{aligned}
0 = {} & \left[1 - \Gamma_{M,t}^*(h)\right] [\mathcal{E}_t p_t^*(h)(1-\theta) + \theta\Psi_t(1+\tau_t)] \\
& - [\mathcal{E}_t p_t^*(h) - \Psi_t(1+\tau_t)] \frac{\phi_M^* p_t^*(h)/p_{t-1}^*(h)}{P_{M,t-1}^*/P_{M,t-2}^*} \left[\frac{p_t^*(h)/p_{t-1}^*(h)}{P_{M,t-1}^*/P_{M,t-2}^*} - 1\right] \\
& + \mathrm{E}_t (1-\delta_D) D_{t,t+1} \left[\mathcal{E}_{t+1} p_{t+1}^*(h) - \Psi_{t+1}(1+\tau_t)\right] \\
& \times \frac{\int_0^{L_{t+1}^*} M_{t+1}^*(h,j^*)\, dj^*}{\int_0^{L_t^*} M_t^*(h,j^*)\, dj^*} \frac{\phi_M^* p_{t+1}^*(h)/p_t^*(h)}{P_{M,t}^*/P_{M,t-1}^*} \left[\frac{p_{t+1}^*(h)/p_t^*(h)}{P_{M,t}^*/P_{M,t-1}^*} - 1\right].
\end{aligned}$$

Under the flexible price equilibrium, where ϕ_Q and ϕ_M^* are zero, firms set prices that reflect the markup $\theta/(\theta-1)$ over marginal cost. Therefore, prices in the steady state become:

$$p_t(h) = \frac{\theta}{\theta - 1}\Psi_t, \qquad (11)$$

[6] Therefore, the intermediate goods firms conduct producer currency pricing.

and

$$\mathcal{E}_t p_t^* (h) = \frac{\theta^*}{\theta^* - 1} \Psi_t (1+\tau).$$

With these equations, if $\theta = \theta^*$, we can derive the following relationship:

$$\mathcal{E}_t p_t^* (h) = p_t (h) (1+\tau).$$

This implies that the law of one price does not hold due to the transportation cost, even if prices are fully flexible and no difference exists in price markups in two countries.

Free entry and value of firms We model firms' entry/exit decision following Ghironi and Melitz (2005). Merit of entry is dependent on the net present value of profit after entry, namely $\{\Pi_\tau (h)\}_{\tau=t+1}^{\infty}$ discounted by stochastic discount factor[7] since firms are eventually owned by households. At the same time, firm h faces such a shock that it needs to exit the market with constant positive probability δ. Such exiting probability also needs to be considered when discounting future profits. Expected profit $\varpi_t (h)$ of firm h at t is now expressed as follows:

$$\varpi_t (h) = \mathrm{E}_t \sum_{\tau=t}^{\infty} (1-\delta)^{\tau-t} D_{t,\tau} (j) \Pi_\tau (h).$$

Firms enter the market until the sunk cost becomes equal to the expected profit. Hence, free entry condition is obtained as:

$$\varpi_t(h) = f_{E,t} \Psi_t. \tag{12}$$

A firm that enters the market at t can only start producing for profit at $t+1$ due to time to build constraints. At the beginning of t, there already exist n_t firms and during t, $n_{E,t}$ new firms entry. As a result, we get $n'_t \equiv n_t + n_{E,t}$ in period t. At the same time, production takes place with only n_t firms. However, a shock then comes that makes δ firms exit from the market at the end of each period. Therefore, at the end of period t as well as at the beginning of period $t+1$, the number of firms is $(1-\delta)(n_t + n_{E,t})$. This also defines the number of firms that distribute profits to households, since profits are assumed to be distributed at the beginning of each period. $\delta n_{E,t}$ firms that enter at t exit market without any production. This can be understood from Figure 2 below.

Forward iteration of the equation for share holdings and absence of speculative bubbles yield the asset price solution:

$$\varpi_t (h) = (1-\delta) D_{t,t+1} (j) [\Pi_{t+1} (h) + \varpi_{t+1} (h)].$$

In the steady state,

$$\varpi (h) = \frac{(1-\delta)D(j)\Pi(h)}{1-(1-\delta)D(j)}.$$

[7] $D_{t,t+1}(j)$ is defined below when households' optimization behavior is explained.

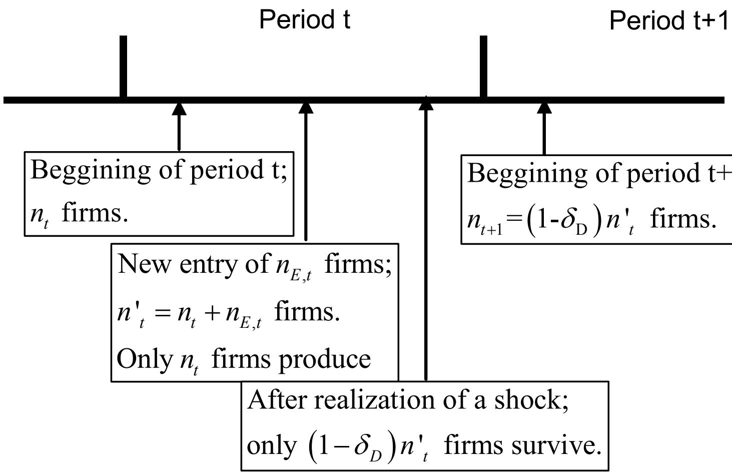

Figure 3: Timing of Entry and Exit

From free entry condition in equation (12), we can derive:

$$f_E \Psi = \frac{(1-\delta)D(j)\Pi(h)}{1-(1-\delta)D(j)}. \tag{13}$$

The corporate profit in equation (10) becomes in the steady state with equation (11):

$$\Pi(h) = \frac{\Psi}{\theta - 1} Y(h). \tag{14}$$

since

$$\mathcal{E}_t p_t^* (h) = p_t (h) (1 + \tau),$$

and the resource constraint of the product of firm h:

$$Y(h) = \int_0^{L_t} Q_t (h, j)\, dj + (1+\tau) \int_0^{L_t^*} M_t^* (h, j^*)\, dj^*.$$

By plugging equation (14) into (13), we can obtain:

$$Y(h) = (\theta - 1) f_E Z \frac{1-(1-\delta) D(j)}{(1-\delta) D(j)}.$$

In the steady state, the production level of each firm is determined by degree of economics of scale $f_E Z$ and product differentiation θ in addition to the discount rate $\frac{1-(1-\delta)D(j)}{(1-\delta)D(j)}$.

2.1.3 Oil production

Oil is produced in a perfectly competitive market. To simplify arguments, we assume the existence of a continuum of firms in the unit mass in the rest of the world

$$O_t^*(s) = Z_t^{O*}\left[(1-\alpha^{O*})^{\frac{1}{\xi^O}} l_t^{O*}(s)^{1-\frac{1}{\xi^{O*}}} + \alpha^{*\frac{1}{\xi^O}} LAND_t^{*1-\frac{1}{\xi^{O*}}}\right]^{\frac{\xi^{O*}}{\xi^{O*}-1}}. \quad (15)$$

2.2 Household

A consumer j receives utility from goods consumption $C(j)$ and disutility from labor supply $l(j)$. Consumer j maximizes the lifetime expected utility as follows:

$$\mathrm{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\left\{U_\tau\left[C_\tau(j)\right] - V_\tau\left[l_\tau(j)\right]\right\}, \quad (16)$$

where,

$$U_t\left[C_\tau(j)\right] = Z_{U,t}\frac{(1-b_C)^\sigma\left[C_t(j)-b_C C_{t-1}(j)\right]^{1-\sigma}-1}{1-\sigma}, \quad (17)$$

$$V_t\left[l_\tau(j)\right] = Z_{V,t}\frac{(1-b_l)^{-\zeta}\left[l_t(j)-b_l l_{t-1}(j)\right]^{1+\zeta}}{1+\zeta}. \quad (18)$$

b_C and b_l are habit formation parameters in consumption and labor supply respectively. σ and ς determine the intertemporal elasticity of substitution in consumption and labor supply.

2.2.1 Budget constraint

Since we incorporate the dynamic entry behavior of firms, the number of firms is determined endogenously. Household income depends on the number of new entry and incumbent firms. Firm's operating profit $\Pi_t(h)$ is paid to households as a dividend income. Furthermore, households recognize the value of share position, some of which are carried into the next period. That dividend income – the value of selling and purchasing shares – also relies on the number of firms.

During period t, the representative home household buys x_{t+1} share in a mutual fund of $n'_t \equiv n_t + n_{E,t}$ home firms (those already operating at time t and the new entrants). Only $n_{t+1} = (1-\delta)n'_t$ firms produce and pay dividends at time $t+1$. As explained above, the dynamics of n_t are already defined as follows:

$$n'_t \equiv n_t + n_{E,t}, \quad (19)$$

$$n_{t+1} = (1-\delta)n'_t,$$

and therefore

$$n_{t+1} = (1-\delta)(n_t + n_{E,t}).$$

Reflecting these dynamics, the budget constraint of the household j now becomes:

$$\begin{aligned} & \mathcal{E}_t B_{F,t+1}(j) + B_{H,t+1}(j) + x_{t+1}(j) \int_0^{n_t'} \varpi_t(h)\, dh \\ \leq\ & (1+i^*)\left[1-\Gamma_{B,t}(j)\right] \mathcal{E}_t B_{F,t}(j) + (1+i_t) B_{H,t}(j) \\ & + W_t(j)\, l_t(j) \left[1-\Gamma_{W,t}(j)\right] \\ & + x_t(j) \int_0^{n_t} \left[\Pi_t(h) + \varpi_t(h)\right] dh \\ & - P_t C_t(j). \end{aligned} \tag{20}$$

Several adjustment costs are assumed in this budget constraint. First, following the GEM, we assume that each household faces the following cost when trading bond in foreign currency:

$$\Gamma_{B,t}(j) = \phi_{B1} \frac{\exp\left[\phi_{B2} \frac{\mathcal{E}_t B^*_{F,t}(j)}{P_t} - Z_{B0}\right] - 1}{\exp\left[\phi_{B2} \frac{\mathcal{E}_t B^*_{F,t}(j)}{P_t} - Z_{B0}\right] + 1} + Z_{B,t}. \tag{21}$$

Since household j is the monopolistic supplier of differentiated labor supply, it has wage setting power. Similar to price setting by firms, wage adjustment cost is a Rotemberg type, as below:

$$\Gamma_{W,t}(j) = \frac{\phi_W}{2} \left[\frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2}} - 1\right]^2. \tag{22}$$

The consumer's optimization problem is to maximize equation (16) subject to equations (17) to (22) with respect to $C_t(j)$, $W_t(j)$, $B_{H,t+1}(j)$, $B_{F,t+1}(j)$, and x_{t+1}.

2.2.2 Euler equation: bonds

The Euler equation below is obtained by differentiating the objective with respect to home $B_{H,t+1}$ and foreign bond holding $B_{F,t+1}$.

$$1 = (1+i_t)\, E_t D_{t,t+1}(j) = (1+i_t^*)\left[1-\Gamma_{B,t+1}(j)\right] \mathrm{E}_t \left[D_{t,t+1}(j) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right],$$

where $D_{t,\tau}(j)$ is the stochastic discount factor:

$$D_{t,\tau}(j) \equiv \beta^{\tau-t} \frac{P_t U'\left[C_\tau(j)\right]}{P_\tau U'\left[C_t(j)\right]}.$$

2.2.3 Euler equation: shares

Concerning stock share, the first order condition below is obtained:

$$\varpi_t(h) = (1-\delta)\, \mathrm{E}_t D_{t,t+1}(j) \left[\Pi_{t+1}(h) + \varpi_{t+1}(h)\right].$$

This and the above equation equates returns from holding of $B_{H,t+1}$, $B_{F,t+1}$, and x_{t+1}.

2.2.4 Wage equation (labor supply)

Wage setting by households is expressed as:

$$\begin{aligned}\psi_t \frac{V_t'(j)}{U_t'(j)} \frac{P_t}{W_t(j)} &= (\psi_t - 1)\left[1 - \Gamma_{W,t}(j)\right] + \left[W_t(j)\frac{\partial \Gamma_{W,t}(j)}{\partial W_t(j)}\right] \\ &\quad + \mathrm{E}_t\left[D_{t,t+1}(j)\frac{l_{t+1}(j)}{l_t(j)} W_{t+1}(j)\frac{\partial \Gamma_{W,t}(j)}{\partial W_t(j)}\right],\end{aligned}$$

where

$$V_t'(j) = Z_{V,t}\left[\frac{l_t(j) - b_l l_{t-1}(j)}{1 - b_l}\right]^{\zeta}.$$

Each household j supplies differentiated labor $l(h,j)$ at wage $W(j)$. We assume that firm h has CES aggregator of differentiated labor$l(j)$:

$$l_t(h) = A_{l,t}\left[\int_0^{L_t} l_t(h,j)^{1-\frac{1}{\psi_t}}\, dj\right]^{\frac{\psi_t}{\psi_t - 1}},$$

where

$$A_{l,t} \equiv L_t^{\gamma_l - \frac{\psi_t}{1-\psi_t}}.$$

Cost minimization implies that firm h's demand for labor input $l(h,j)$ is a function of the relative wage:

$$l_t(h,j) = A_{l,t}^{\psi_t - 1}\left[\frac{W_t(j)}{W_t}\right]^{-\psi_t} l_t(h),$$

where $W(j)$ is the nominal wage paid to home labor input j and the wage index W is defined as:

$$W_t = \frac{1}{A_{l,t}}\left[\int_0^{L_t} W_t(j)^{1-\psi_t} dj\right]^{\frac{1}{1-\psi_t}}.$$

2.2.5 Other

We can define the holdings of net foreign asset as follows:

$$F_t(j) = (1 + i^*)\left[1 - \Gamma_{B,t}(j)\right]\mathcal{E}_t B_{F,t}(j).$$

Then, and exchange rate:

$$\begin{aligned}\mathrm{E}_t D_{t,t+1} L_{t+1} F_{t+1}(j) &= L_t F_t(j) + (1 + i_{t-1}^*)\Gamma_{B,t-1}(j)\mathcal{E}_t L_t B_{F,t}(j) \\ &\quad + \mathcal{E}_t P_{M,t}^* L_t^* M_t^*(j^*) - P_{M,t} L_t M_t(j) \\ &\quad - P_{O_M,t}\left[\int_0^{n_t} O_{M,t}(h) dh + \int_0^{n_{E,t}} O_{M,t}(e) de\right].\end{aligned}$$

2.3 Government

There is no tax collection. Therefore, only the central bank exists for stabilization to facilitate households' consumption smoothing motives and works as a nominal anchor to obtain a unique rational expectation equilibrium. The monetary authority follows this simple instrument rule:

$$\begin{aligned}(1+i_{t+1})^4-1 &= \omega_i\left[(1+i_t)^4-1\right]+(1-\omega_i)\left[(1+\mathrm{E}_t i_{t+1})^4-1\right]\\ &\quad +\omega_1 E_t\left(\frac{P_{t+\tau}}{P_{t+\tau-4}}-\Pi_{t+\tau}\right).\end{aligned}$$

2.4 Market clearing

2.4.1 Industry equilibrium

To close the model, we need to clear goods markets globally, which requires total revenue equal to world expenditure:

$$\begin{aligned}\int_0^{n_t}\left[p_t(h)\int_0^{L_t}Q_t(h,j)\,dj+\mathcal{E}_t p_t^*(h)\int_0^{L_t^*}M_t^*(h,j^*)\,dj^*\right]dh &= \mu_t\int_0^{L_t}E_t(j)\,dj\\ &\quad +(1-\mu_t^*)\,\mathcal{E}_t\int_0^{L_t^*}E_t^*(j^*)\,dj^*.\end{aligned}$$

LHS is total industry revenue in the home and foreign markets, while the RHS is the expenditure for the industry in each country. $E_t(j)$ denotes the total spending by consumer j., which is defined as:

$$E_t(j)=P_tC_t(j). \tag{23}$$

$\mu_{Q,t}$ is home consumer j's expenditure share of home country goods, and $\mu^*_{Q,t}$ is foreign consumer j^*'s expenditure share of foreign country goods, which are defined by using equations (2) and (3) as:

$$\mu_t=\frac{P_{Q,t}Q_t(j)}{E_t(j)}=\frac{P_{Q,t}Q_t(j)}{P_tA_t(j)}=\nu\left(\frac{P_{Q,t}}{P_t}\right)^{1-\epsilon_{QM}},$$

$$\mu_t^*=\frac{P^*_{Q,t}Q^*_t(j)}{E^*_t(j^*)}=\frac{P^*_{Q,t}Q^*_t(j)}{P^*_tA^*_t(j^*)}=\nu^*\left(\frac{P^*_{Q,t}}{P^*_t}\right)^{1-\epsilon^*_{QM}}.$$

Furthermore, this relationship can be transformed so as to derive the number of firms in each industry:

$$n_t=\frac{\mu_Q\int_0^{L_t}E_t(j)dj+(1-\mu_Q^*)L_t^*\mathcal{E}_tE_t^*(j^*)}{p_t(h)L_tQ_t(h,j)+\mathcal{E}_tp_t^*(h)L_t^*M_t^*(h,j^*)}.$$

2.4.2 General equilibrium

In a general equilibrium, spending for goods in each period must the equal factor income of labor, plus net income by bonds plus investment income excluding the cost of investing in new firms. Thus,

$$\begin{aligned}\int_0^{L_t} E_t(j)\,dj &= \int_0^{L_t} (1+i_t^*)\left[1-\Gamma_{B,t}(j)\right]\mathcal{E}_t B_{F,t}(j)\,dj + \int_0^{L_t}(1+i_t)\,B_{H,t}(j)\,dj \\ &\quad -\mathcal{E}_t\int_0^{L_t} B_{F,t+1}(j)\,dj - \int_0^{L_t} B_{H,t+1}(j)\,dj + \int_0^{L_t} W_t(j)\,l_t(j)\left[1-\Gamma_{W,t}(j)\right]dj \\ &\quad +\int_0^{n_t}\Pi_t(h)\,dh - \int_0^{n_{E,t}} \varpi_t(h)\,dh.\end{aligned} \tag{24}$$

In addition, all goods, factors and bonds markets satisfy following market clearing conditions:

$$Y_t(h) = \int_0^{L_t} Q_t^D(h,j)dj + (1+\tau)\int_0^{L_t^*} M_t^{*D}(h^*,j)dj.$$

The factor market clearing conditions can be written as:

$$l_t(j)dj = \int_0^{n_t} l_t^I(h,j)dh + \int_0^1 l_t^O(s,j)\,ds + n_{E,t}\int_0^{n_{E,t}} l_{E,t}(e,j)\,de,$$

These equations imply that labor and capital market clearing conditions are required so that the total supply of the factor should be equal to be the factor demand for production and entry. Market clearing in the bond market requires:

$$\int_0^{L_t} B_{H,t}(j)dj = 0,$$

$$\int_0^{L_t} B_{F,t}(j)dj + \int_0^{L_t^*} B_{F,t}^*(j^*)dj^* = 0.$$

The market clearing condition for the raw materials, namely oil in this paper is:

$$O_t^*(s^*) = n_t^* O_{Q,t}^*(f) + n_{E,t}^* O_{Q,t}^*(e^*) + (1+\tau_t^*)\left[n_t O_{M,t}(h) + n_{E,t} O_{M,t}(e)\right].$$

The model is solved under the assumption of symmetric equilibrium. Details are shown in the appendix.

2.5 Calibration

In this subsection, we show the calibration of major parameters. In the simulations below, symmetry between the home and foreign country is assumed. Basically, parameters are set following previous researches using the GEM. Table 1 shows the values of major parameters.

Table 1
Parameter Values

Parameter	Value	Description and Definitions
ε	1.5	Elasticity of substitution between domestic and imported goods
ζ	3.0	Inverse of Frisch elasticity
ψ	6.00	$\psi/(\psi-1)$ is Wage markup
θ	6.00	$\theta/(\theta-1)$ is Price markup
ϕ_Q	400	Adjustment cost for price setting in domestic market
ϕ_M	400	Adjustment cost for price setting in export market
ϕ_W	400	Adjustment cost for wage setting
γ	1	Degree of taste for variety in goods
γ_l	1	Degree of taste for variety in labor
α	0.3	Scale parameter that determines law materials share
α^O	0.8	Scale parameter that determines land's share in raw material production
ξ	0.75	Elasticity of Substitution between raw material and labor
ξ^O	0.75	Elasticity of Substitution between land and labor
β	$1.03^{-0.25}$	Discount rate
δ	0.025	Firm exit shock
ν	0.5	Home bias parameter
ϕ_{B1}	0.05	Transaction-cost parameter in the bond market
ϕ_{B2}	0.1	Transaction-cost parameter in the bond market
b_c	0.83	Habit persistence parameter in consumption
b_l	0.0	Habit persistence parameter in labor
$1/\sigma$	0.80	*Intertemporal* elasticity of substitution

The parameters that determine the nominal rigidity (ϕ_Q, ϕ_M, ϕ_W), elasticity of substitution in production of intermediate goods and raw materials (ξ, ξ^O) and transaction cost parameters in the bond market (ϕ_{B1}, ϕ_{B2}) are set following Laxton and Pesenti (2003). Elasticity of substitution between domestic and imported goods (ε) are set 1.5 according to Smets and Wouters (2002) and Chari, Kehoe and McGrattan (2002). And we set Inverse of Frisch elasticity (ζ), habit persistence parameter in consumption (b_c) and intertemporal elasticity of substitution ($1/\sigma$) following Julliard,Karam Laxton and Pesenti (2005).

In steady state, we set the size of domestic country (L) and foreign countries (L^*), 0.05 and 1.00 respectively. Furthermore, we assume that there is no home bias.[8]

[8] Therefore, ratio of imports over aggregate output becomes large. Since the aim of this paper is to understand the effects through soaring oil price rather theoretically than empirically. We eliminate the effects of home bias so that we can understand spill-over effects from oil price hikes. There, however, exist almost no qualitative differences between in the case with and without home bias.

3 Various Scenarios for Soaring Oil Price

In this section, we reproduce soaring oil prices as impulse responses against shocks in a dynamic general equilibrium model, where both supply and demand shocks are examined.[9] We show only responses of major variables determined in the model. One is the variable comparable to the GDP concept. In a symmetric equilibrium, budget constraints in equation (24) with equation (23) are written as:

$$
\begin{aligned}
& L_t P_t C_t + n_{E,t} \varpi_t (h) + \left[\mathcal{E}_t L_t B_{F,t+1} - L_t (1 + i_t^*) \mathcal{E}_t B_{F,t} \right] \\
& + \left[L_t (1 + i_t^*) \Gamma_{B,t} \mathcal{E}_t B_{F,t} + \Gamma_{W,t} L_t W_t l_t \right] \\
= \; & L_t W_t l_t + n_t \Pi_t .
\end{aligned}
$$

This is the macro budget constraint. On the LHS of the equation, the first term is macro consumption, the second is mutual fund purchase for newly established firms and considered to be investment, the third is net exports, and the fourth is adjustment costs on trading bonds and labor. On the other hand, on the RHS, the first term is labor income while the second is profit generated from investment. We define the total of variables on the RHS, namely therefore the LHS as well, "aggregate income," which will be shown in the impulse responses analysis below.

At the same time, we also show the responses of natural rate of interest, namely the Vicksellian rate both in domestic and foreign countries in order to understand implications of soaring oil prices on the monetary policy stance. Unlike the case with the small open economy, computing the natural rate of interest is not very trivial with the multi-country model. For analytical simplicity, in this paper, we compute it by assuming that there is no nominal price and wage rigidity all over the world. Therefore, we can define this is the natural rate that is considered to be faced by the international organization.[10]

First, we show the impulse responses on soaring oil prices against standard contemporaneous shocks, then those against expectation shock, such as a perception about future technology and demand conditions.

[9] To be exact, distinction about supply and demand shocks are innocuous in a dynamic general equilibrium model, especially in open economies. For example, technology shock in the rest of the world acts like a demand shock to the domestic country.

[10] On the other hand, we can compute the natural rate of interest which the central banks face. It is, however, a bit tricky to be computed since the central bank should takes what happened in the rest of the world as exogenous event that cannot be controlled by itself. Therefore, we need to compute it as follows. First, we compute impulse responses against a certain shock in multi countries. Then, by keeping the impulse responses in the foreign country as exogenous variables, we simulate the model where nominal price and wage rigidities are eliminated in the domestic country with the same shock. We have found almost no significant differences in the natural rate of interest computed this way from that in this paper.

3.1 Standard shocks

We conduct five simulations against (1) decreased labor disutility, (2) increased working population, (3) increased technology, (4) reduced fixed cost, and (5) reduced oil production technology. All shocks are contemporaneous and occur in the rest of the world. (1) and (2) aim at capturing the situation where demand for oil is increasing due to the expanding economic activities in emerging economies. (3) and (4) are similar experiments but increased oil demand in the rest of the world is due to supply side improvement, unlike the demand improvement in (1) and (2). Shocks in (1) to (4), however, are considered to be demand shocks for the domestic economy. (5) is not a reality as of now. This scenario is examined to compare the realistic scenario examined in the expectation shock simulation, "world crude oil prices continued to remain high and volatile as a consequence of abiding concern over the lack of effective global oil refining capacity, in the short and medium term, coupled with anxiety about the ability of oil producers to meet anticipated future oil demand." (2) and (4) are unique simulation only available with the dynamic general equilibrium model with free entry. We will see how similar simulations can result in different transmission as well as welfare implication.

3.1.1 Decreased labor disutility

Decreased relative labor disutility is a very popular exercise as a demand shock. A positive shock is given to $Z_{U,t}$ in the rest of the world version of equation (17).[11] Impulse responses are shown in Figure 4. The domestic country is the small oil importing country whose share of world production is about 5 per cent.[12] Bold lines show responses of domestic variables while thick lines are those of variables in the rest of the world. Decreased labor disutility in the rest of the world naturally results in the higher labor supply and consumption in the foreign countries. Therefore, aggregate income as well as inflation rates become higher. Naturally, due to the elasticity of substitution between labor and oil, demand for oil becomes higher in the intermediate goods sector. This results in higher oil price inflation. On the other hand, to meet the higher demand in the foreign country, more labor is required to produce more tradeables in the domestic country with less domestic consumption. Yet, its degree is higher for the rest of the world. Therefore, welfare is better off in the domestic country against soaring oil prices caused by decreased disutility of labor in the rest of the world and welfare is worse off in the rest of the world where disutility of labor is decreased Investments and number of firms decrease although they show a small increase right after the shocks since more should spend on consumption to satisfy substitution between labor and consumption.

As for monetary policy implications, in both countries, the natural rate of interest becomes higher. Therefore, with no changes in nominal interest rates,

[11] Almost equivalent results can be obtained by adding a negative shock to $Z_{V,t}$ in the rest of the world version of equation (18).

[12] The parameters, however, are not set according to the Japanese stylized facts.

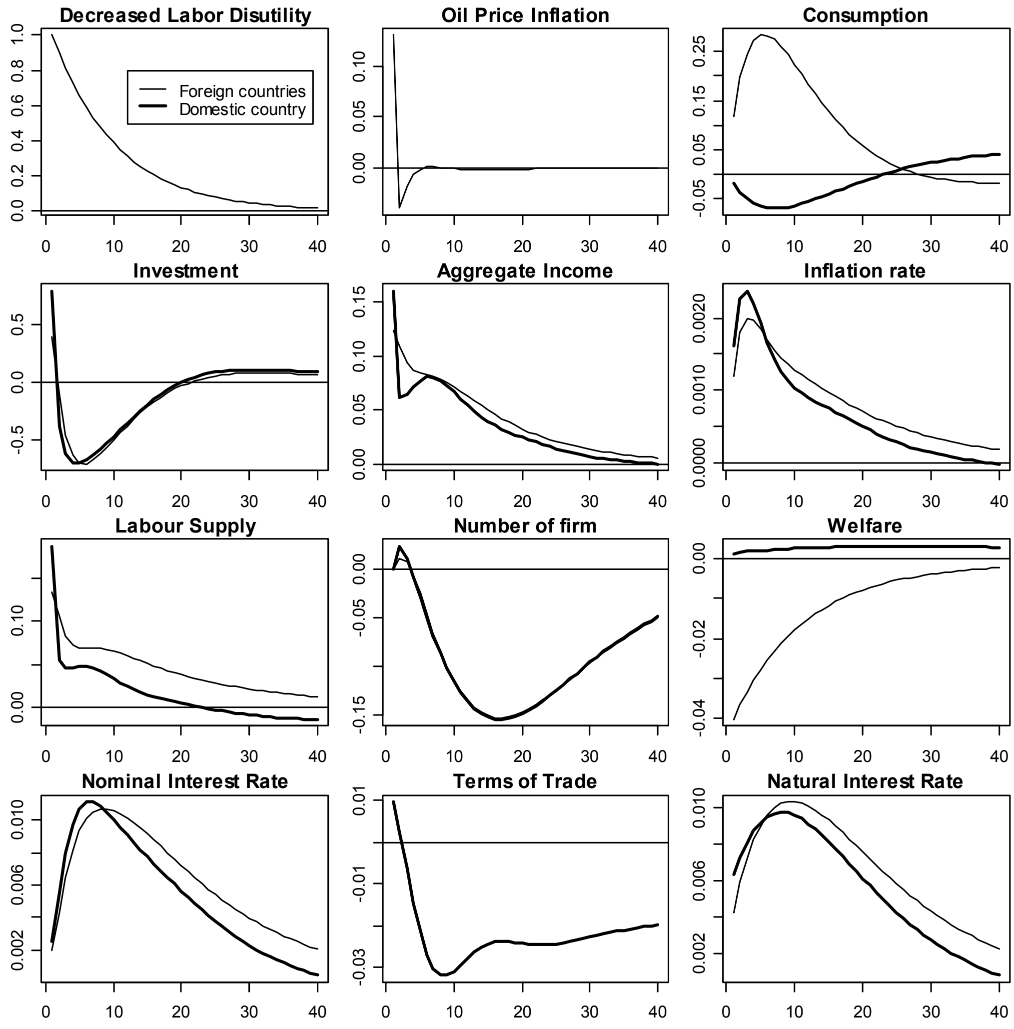

Figure 4: Decreased Labor Disutility

monetary policy loosens across the world.

3.1.2 Increased working population

This is a unique experiment that cannot be examined in the standard dynamic general equilibrium model with fixed varieties. With the fixed variety, an increase in population only results in a proportionate increase in all detrended variables by population. On the other hand, with the endogenous variety model, an increase in working population increases the labor supply so that the number of firms, which equals the number of goods since each firm produces its own goods, becomes larger. Although we have not taken taste for variety into account in this paper, we can analyze the effects of increased population on other economic variables through the creation of firms. Furthermore, since there is a trade cost for exporting τ, the country that experiences larger population can enjoy benefits from increased variety and firms more than the other countries can. This is the so-called "Home Market Effect."

There may be suspicion that increased working population is an inappropriate simulation for current soaring oil prices via increased oil demand in emerging economies. We believe that this is, for example, what is happening in China behind the expanding domestic economy. It is supported by massive population shifts from rural areas to the industrial cities, as Japan experienced in the 1950 and 1960s. Although people in the rural areas are also naturally conducting economic activities, they are considered home production. Therefore, increased working population shifts from rural areas to industrial cities are naturally considered to be the most appropriate simulation for the current increase in domestic demand in the BRIC countries.

Figure 5 shows the responses when working population is increased by 1 per cent in the rest of the world. Although the degree of oil price hikes is quite similar to Figure 4, the mechanism behind it is quite different. Since the resource constraint in equation (24) is expanded in this case, consumption as well as investment on new firms' establishment are raised. On the other hand, in the domestic country, responses are very similar to the above case since nothing has happened in the domestic country.

As a result, the contrast in welfare between domestic and foreign countries become more distinct. Because there is no home bias in final goods production, per capita consumption and therefore welfare decrease in the rest of the world.[13] As more imported goods are required to produce final goods, there exist very large positive spillover from the foreign countries to the domestic country. These developments result in improving terms of trade and welfare in the small oil importing country.

At the same time, thanks to the above-mentioned improvements in demand conditions, natural rate of interest becomes much higher in the foreign countries.

[13] Therefore, with very high home bias, results here are reversed.

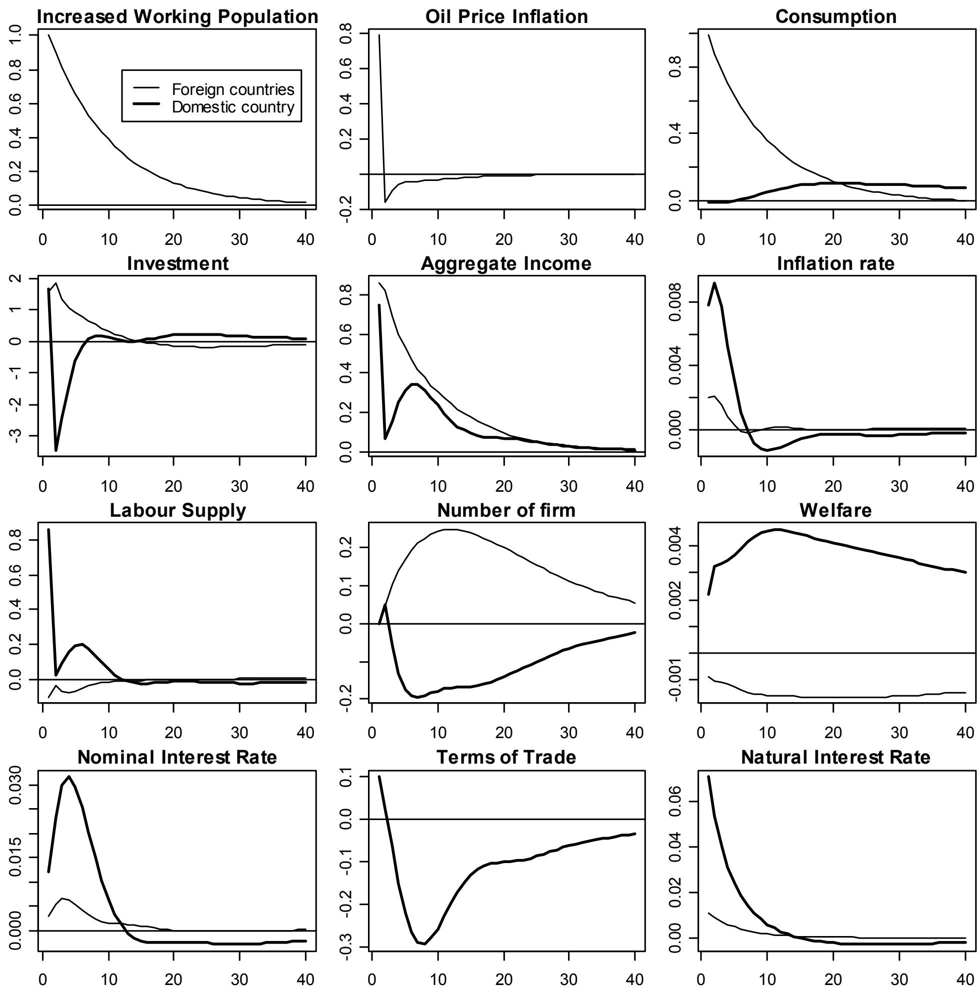

Figure 5: Increased Working Population

3.1.3 Increased technology

Increased technology in the rest of the world is a candidate factor for current soaring oil prices. This is indeed a supply shock to the rest of the world, but works like a demand shock as a better external economic activity to the domestic country. Responses against this shock are shown in Figure 6. In the foreign country, labor supply is increased thanks to the positive technology shock[14]. This results in higher consumption as well as investment thanks to the expanded budget constraints. Although oil prices are increased due to higher demand, inflation rates in aggregate are reduced due to lower marginal costs. On the other hand, in the domestic country, inflation rates are higher thanks to improved external demand conditions. In the medium run, more labor inputs are needed to satisfy the demand from foreign countries. Therefore, welfare becomes worse in the domestic country and naturally better in the rest of the world.

Reflecting those developments in supply and demand conditions, the natural rate of interest is higher in the domestic country but lower in the rest of the world. Qualitatively, a stabilization policy on inflation rates does a good job in neutralizing the effects from exogenous shocks.

3.1.4 Reduced fixed cost

Reduced fixed cost is naturally interpreted as an improved technology since fewer goods are needed to establish a firm, namely for production.[15] Similarly to the case with increased working population, this is a unique simulation to the endogenous variety model. Figure 7 shows the responses against reduced fixed cost in the rest of the world.

This simulation is again on improvement technology, but significant differences exist. When the fixed cost is reduced, more resources are directed to establish new firms. Therefore, consumption decreases in the rest of the world. Even though consumption becomes lower, inflation rates become higher thanks to the higher employment for creating new firms. Oil prices are also raised due to the increase in demand to establish new firms. On the other hand, worsening terms of trade reduce domestic consumption and goods varieties in the domestic country. In the rest of the world, more resources are directed to establish new firms. Hence, consumption and imports from the small oil importing country are decreased. These developments result in the worsening terms of trade in the domestic country. As a result, welfare is reduced in the domestic country while improved in the rest of the world.

Reflecting those economic conditions, the natural interest rate is higher in the rest of the world but becomes temporally lower in the domestic country.

[14] Although sticky price is incorporated, labor is increased after the positive technology shock. This is somewhat contrary to the results in Gali (1999) and Gali and Rabanal (2004). Positive responses of hours against technolgy shock are, however, produced with sticky wages in Altig, Christiano, Eichenbaum and Linde (2005) and with habit formation in consumption in Vigfusson (2004).

[15] In this paper, however, taste for variety is not considered.

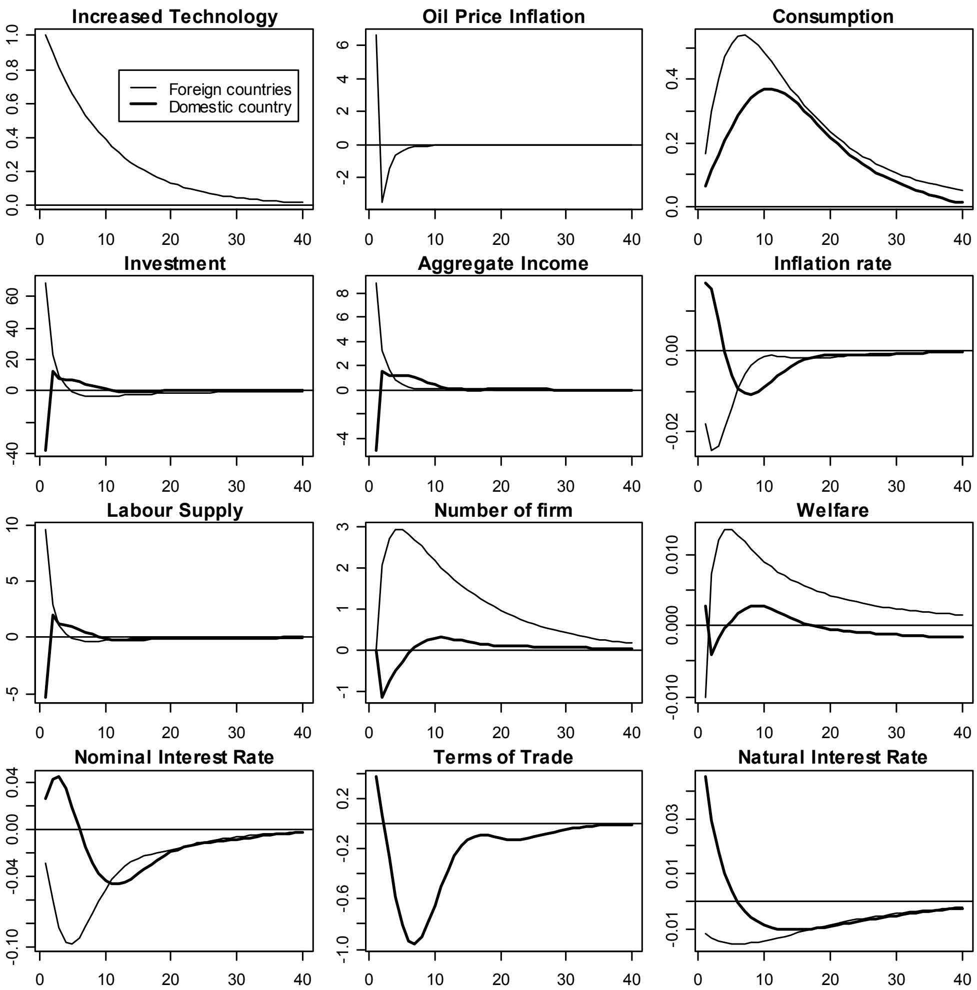

Figure 6: Increased Technology

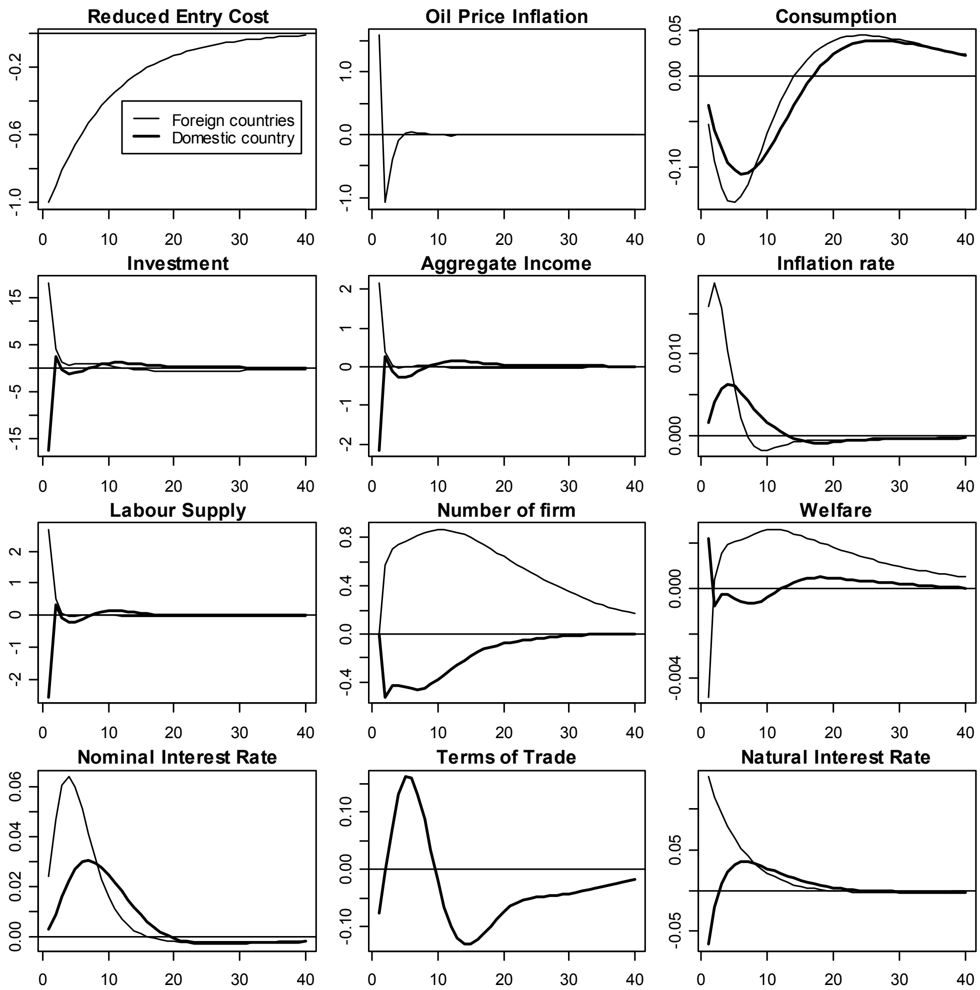

Figure 7: Reduced Entry Cost

3.1.5 Reduced oil-producing technology

Reduced oil-producing technology is not the reality for the current soaring oil prices since no clear evidence of damaged oil plant technology has been reported. Impulse responses here are, however, very useful to understand truly supply side effects on oil prices and the contrast between the expectation shock about future deterioration of oil production, which will be examined in the next subsection. Responses to such a shock are shown in Figure 8.

Negative technology in oil production naturally increases oil prices. This would result in lower consumption, investment, labor supply and welfare in both countries. On monetary policy implication, the natural rate of interest becomes much lower in the domestic country, where a more expansionary monetary policy is needed.

3.2 Expectation shocks

Consider the compelling story on the current oil price hikes that "world crude oil prices continued to remain high and volatile as a consequence of abiding concern over the lack of effective global oil refining capacity, in the short and medium term, coupled with anxiety about the ability of oil producers to meet anticipated, future oil demand." We need to test whether expectation about future demand improvements as well as anxiety on the future oil-producing condition would result in what we observe from the data of major economic variables recently.

In this subsection, we first show how we can draw impulse responses against such expectation shocks. Then, impulse responses against future expectation for larger working population in the rest of the world, as well as that for deteriorating oil producing technology, are demonstrated.

3.2.1 How to simulate expectation shock

First, we will explain a general solution of the rational expectation model. We then show how to incorporate expectation shock.[16] Generally, a rational expectation model can be represented as[17]

$$\begin{aligned} \alpha_0 \mathrm{E}_t \left(Z_{t+1} - Z^*\right) + \alpha_1 \left(Z_t - Z^*\right) + \alpha_2 \left(Z_{t-1} - Z^*\right) & \\ + \beta_0 \left(S_{t+1} - S^*\right) + \beta_1 \left(S_t - S^*\right) &= 0. \end{aligned} \tag{25}$$

The solution that we would like to obtain is

$$Z_t = Z^* + A\left(Z_{t-1} - Z^*\right) + B\left(S_t - S^*\right), \tag{26}$$

and

$$S_t = S^* + P\left(S_{t-1} - S^*\right) + C\varepsilon_t. \tag{27}$$

[16] The contents in this subsection are based on Christiano, Motto and Rostagno (2006).

[17] This is just a difference version of Christiano (2000). Since in our model, there are variables whose steady state value is zero, we use difference rather than log-difference.

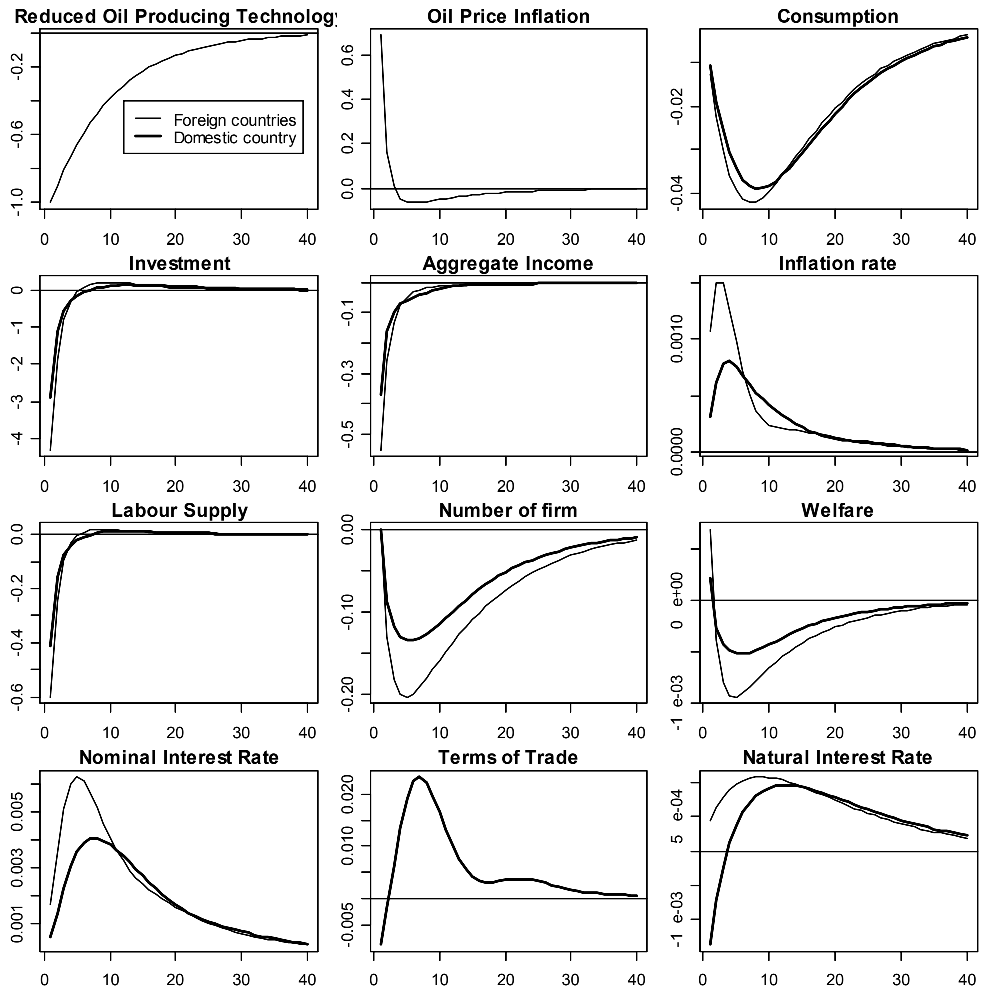

Figure 8: Reduced Oil-Producing Technology

By substituting, equations (26) and (27) into (25), we can obtain:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 = 0,$$

and

$$(\beta_0 + \alpha_0 B)P + (\beta_1 + \alpha_1 B + \alpha_0 AB). \tag{28}$$

Matrix A and B in solutions in equations (26) and (27) are computed by solving the above equations. Especially whether we can obtain unique A is dependent on the usual Blanchard and Kahn (1980) condition.

Simulation with expectation shock can be materialized by making adjustment to β_0 and β_1 so that we can obtain a new B matrix. For simplicity of argument, let us consider a very simple technology shock process s, which is comparable to equation (27),[18] as follows:

$$(s_t - s) = \rho\,(s_{t-1} - s) + \widehat{\varepsilon}_{t-p} + \widehat{\xi}_t.$$

With this shock process, we can express a news shock for higher future productivity. As a simple example, here we suppose a situation that we receive a news that "productivity is raised in period 2" today, but it turns out to be false when period 2 actually comes.[19] The above equation is represented as follows in canonical form as :

$$\begin{pmatrix} s_t - s \\ \widehat{\varepsilon}_t \\ \widehat{\varepsilon}_{t-1} \end{pmatrix} = \begin{pmatrix} \rho & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s_{t-1} - s \\ \widehat{\varepsilon}_{t-1} \\ 0 \end{pmatrix} + \begin{pmatrix} \widehat{\xi}_t \\ \widehat{\varepsilon}_t \\ 0 \end{pmatrix} \tag{29}$$

If we add a news shock, , at period zero,

$$\begin{pmatrix} s_0 - s \\ \widehat{\varepsilon}_0 \\ \widehat{\varepsilon}_{-1} \end{pmatrix} = \begin{pmatrix} \rho & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s_{-1} - s \\ \widehat{\varepsilon}_{-1} \\ \widehat{\varepsilon}_{-2} \end{pmatrix} + \begin{pmatrix} 0 \\ \widehat{\varepsilon}_0 \\ 0 \end{pmatrix}.$$

ε_0 will not affect $\widehat{s}_0$ and $\widehat{s}_1$, but shock on technology at period 2 expected in period zero is now:

$$\mathrm{E}_0 \begin{pmatrix} s_2 - s \\ \widehat{\varepsilon}_2 \\ \widehat{\varepsilon}_1 \end{pmatrix} = \begin{pmatrix} \rho & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^2 \begin{pmatrix} s_0 - s \\ \widehat{\varepsilon}_0 \\ \widehat{\varepsilon}_{-1} \end{pmatrix} = \begin{pmatrix} \rho^2 & 1 & \rho \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s_0 - s \\ \widehat{\varepsilon}_0 \\ \widehat{\varepsilon}_{-1} \end{pmatrix}.$$

Hence,

$$\mathrm{E}_0\,(s_2 - s) = \widehat{\varepsilon}_0.$$

Therefore, the shock on technology at period 2 expected in period zero indeed becomes ε_0. If the expectation is actually materialized, the simulation is conducted using appropriate S vector as above. On the other hand, once period 2

[18] For simplicity, $s = 1$.

[19] In simulations below, we also show the case when the initial guess turns out to be true.

comes, such a positive shock does not happen actually. ε_0 is offset by ξ_2, since $\xi_2 = -\varepsilon_0$. This is depicted as:

$$\begin{pmatrix} s_2 - s \\ \widehat{\varepsilon}_2 \\ \widehat{\varepsilon}_1 \end{pmatrix} = \begin{pmatrix} \rho & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s_1 - s \\ \widehat{\varepsilon}_0 \\ \widehat{\varepsilon}_{-1} \end{pmatrix} + \begin{pmatrix} \widehat{\xi}_2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Thus, we can generate such a shock as at period zero and one, technology shock is expected to happen at period 2, but it turns out to be a bubble expectation in period 2.

This canonical form is exactly equation (27) and shock vector S now incorporates expectational shock terms as $\widehat{\varepsilon}_t$. Therefore, if we appropriately arrange β_0 and β_1 by adding zero vectors to columns corresponding to $\widehat{\varepsilon}_t$ in S and rewrite P, we can compute new B matrix by using equation (28). We can then obtain impulse responses under expectation shock with equations (26) and (27).

3.2.2 Expected increase in working population

Further economic expansion in emerging economies, such as the BRIC countries, are expected. Some analysts have pointed out this is the reason why we are facing the current oil price hikes. Here, we simulate a situation of an increase in economic activities due to increased working population next year. Figure 9 shows responses when such expectation is actually materialized, and Figure 10 demonstrates those when the expectation turns out to be false.[20] By the way, there are no differences in responses up until the date the expected event is supposed to happen. The reason why we show cases when the expectation does not materialize is for our understanding the results from a bubble expectation.

Expecting a future increase in working population, investment level and therefore labor for establishing new firms are raised to prepare for the higher demand in the future, since establishing a new firm incurs adjustment costs in the form of time to build constraint. Oil prices are raised due to the necessity to establish more new firms. Since no significant expansion of resource constraint exists, consumption is reduced up until the expectation is realized. Consumption jumps in Figure 9 because this is the response of total consumption, which is per capita consumption multiplied by working population. Therefore, consumption is smoothed at the individual level. Even in the small oil importing country, labor and investment are increased to prepare future high demand on domestic tradable goods. Reflecting those demand developments, inflation rates rise in both countries. As is the case with the contemporaneous shock on the working population, welfare is greatly improved in the domestic country while it deteriorates in the rest of the world.

[20] As obvious from Figure 10, even with shocks which turns out to be false, economic variables show very volatile movements. This is consistent with the OPEC statement in the introduction.

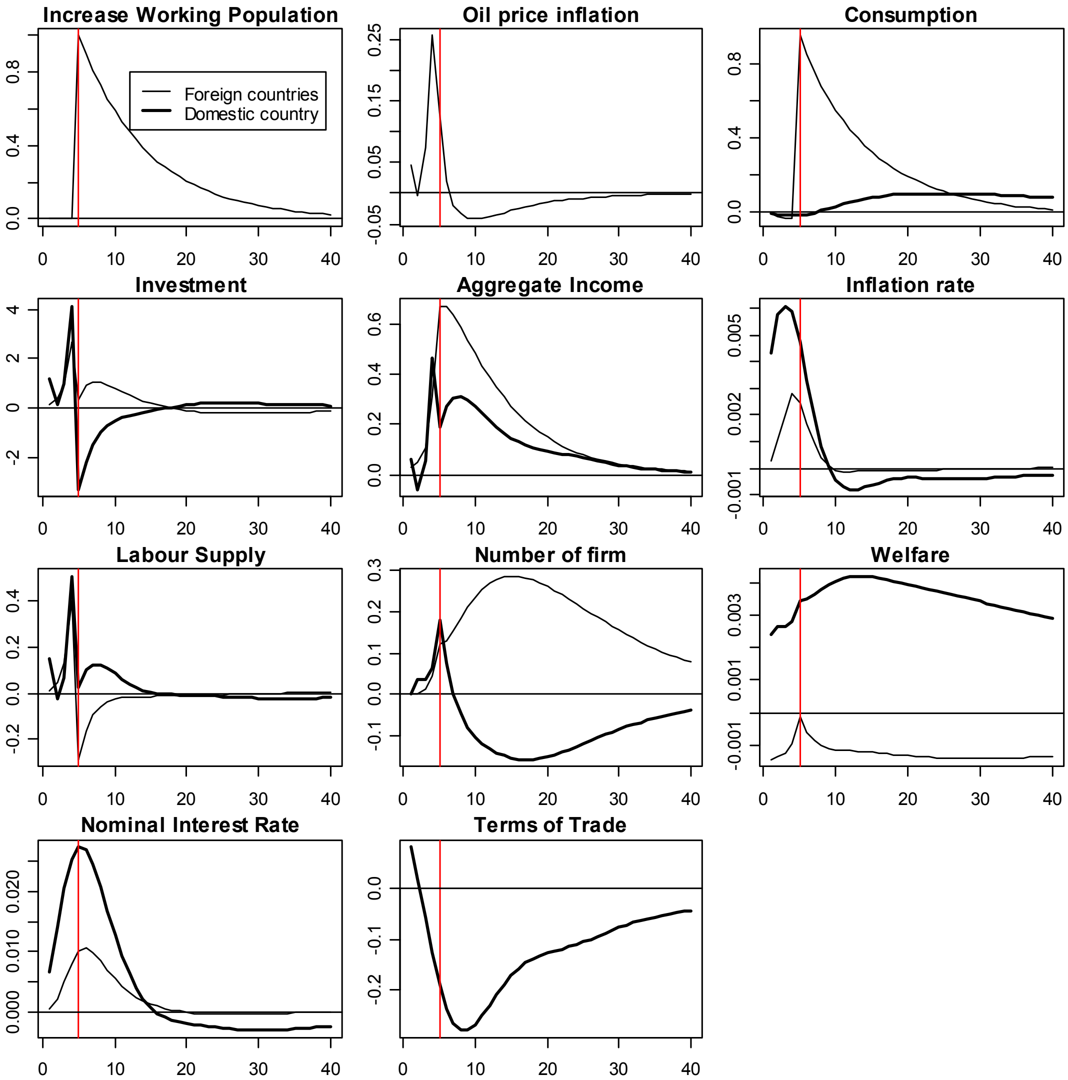

Figure 9: Expected Increase in Working Population (materialized)

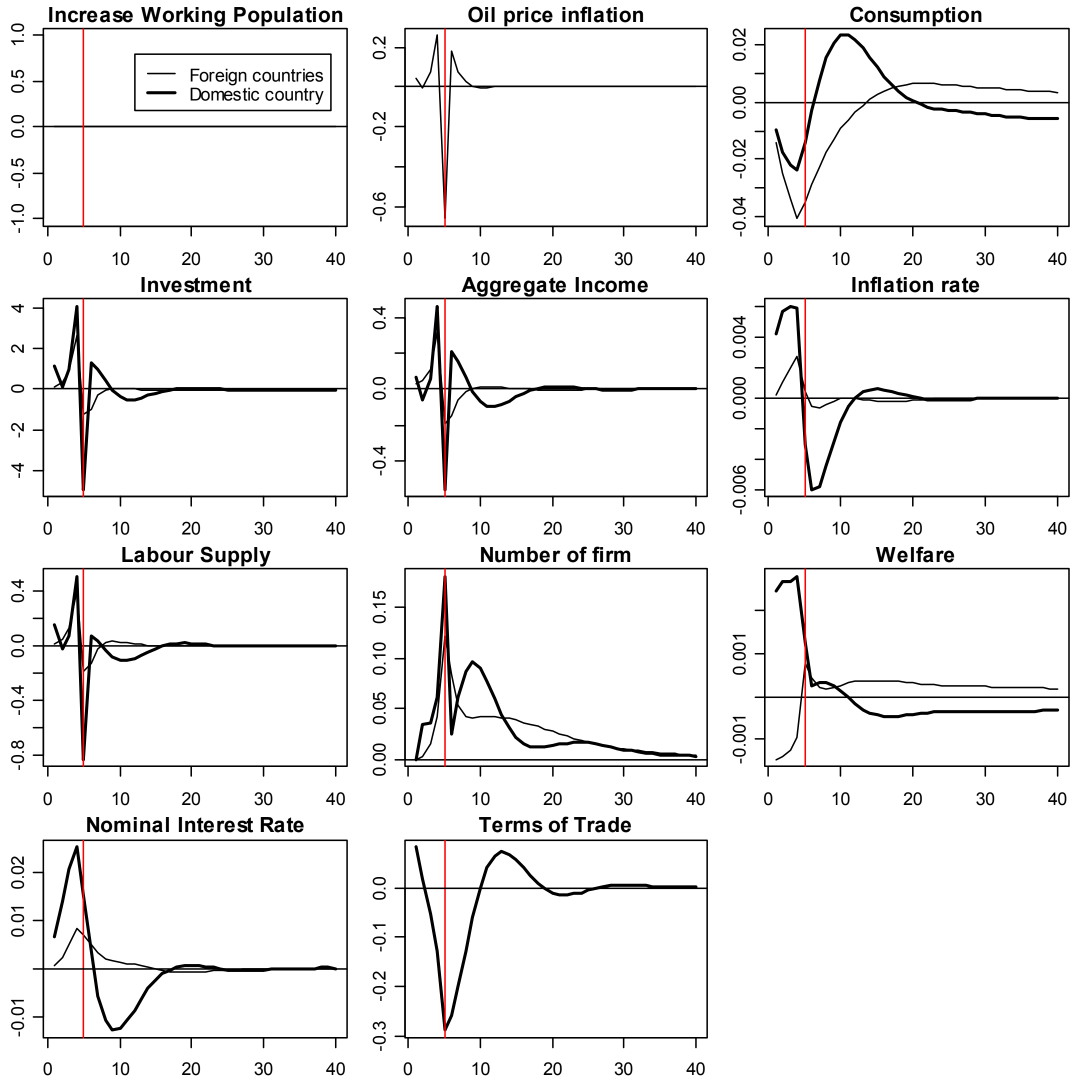

Figure 10: Expected Increase in Working Population (not materialized)

3.2.3 Expected reduction in oil producing technology

As documented in the OPEC statement, the biggest concerns are geopolitical developments and speculation in the oil futures markets. Here, to monitor how such concern affects current as well as future developments of major economic variables, we draw impulse responses when people expect that oil-producing technology is reduced by 1 per cent next year in the rest of the world. Figure 11 shows the case when such expectation is materialized while Figure 12 demonstrates the case when that turns out to be false.

Anticipation about future deteriorating oil-producing technology increases the value of oil and therefore oil prices rise. At the same time, since future lower technology[21] is expected, it is optimal for households to increase the labor supply due to negative wealth effects. Furthermore, consumption is reduced gradually thanks to consumption smoothing to prepare for lower income in the future. As a result, investment is increased to satisfy the resource constraint. It first seems controversial to see the increase in labor and oil prices but a decrease in inflation rates. As was explained above, initially after the news about future lower technology on oil production is received, the labor supply curve shifts outward. This, then, reduces real wages. Therefore, aggregate inflation rates decrease all over the world. To materialize higher inflation across the world after oil price hikes due to anxiety about future oil supply, we need to introduce very strong labor adjustment costs so that the substitution effects for smoothing labor supply dominate. This will keep the labor supply curve from shifting outward.[22]

4 Conclusion

- We have examined several mechanisms which induce soaring oil prices.
- It is of great importance to acknowledge the source of economic fluctuations, including oil price developments, so that they can conduct proper stabilization policy.
- Even with similar magnitude of oil price hikes, effects on inflation, terms of trade and welfare are quite different. Particularly, cases with increased technology, reduced fixed cost, and increased working population are intriguing.
  (1) With increased technology, marginal cost decreases and therefore inflation rates are lowered in the foreign countries. On the other hand, inflation rates rise due to more demand on the domestic goods. Therefore terms of trade improves in the domestic country.
  (2) A reduction in fixed costs induces investment to create new firms.

[21] Technology on oil production works as a standard technology in the aggregate production function.

[22] This is a similar mechanism to what is mentioned on capital formation in Christiano, Motto and Rostagno (2006).

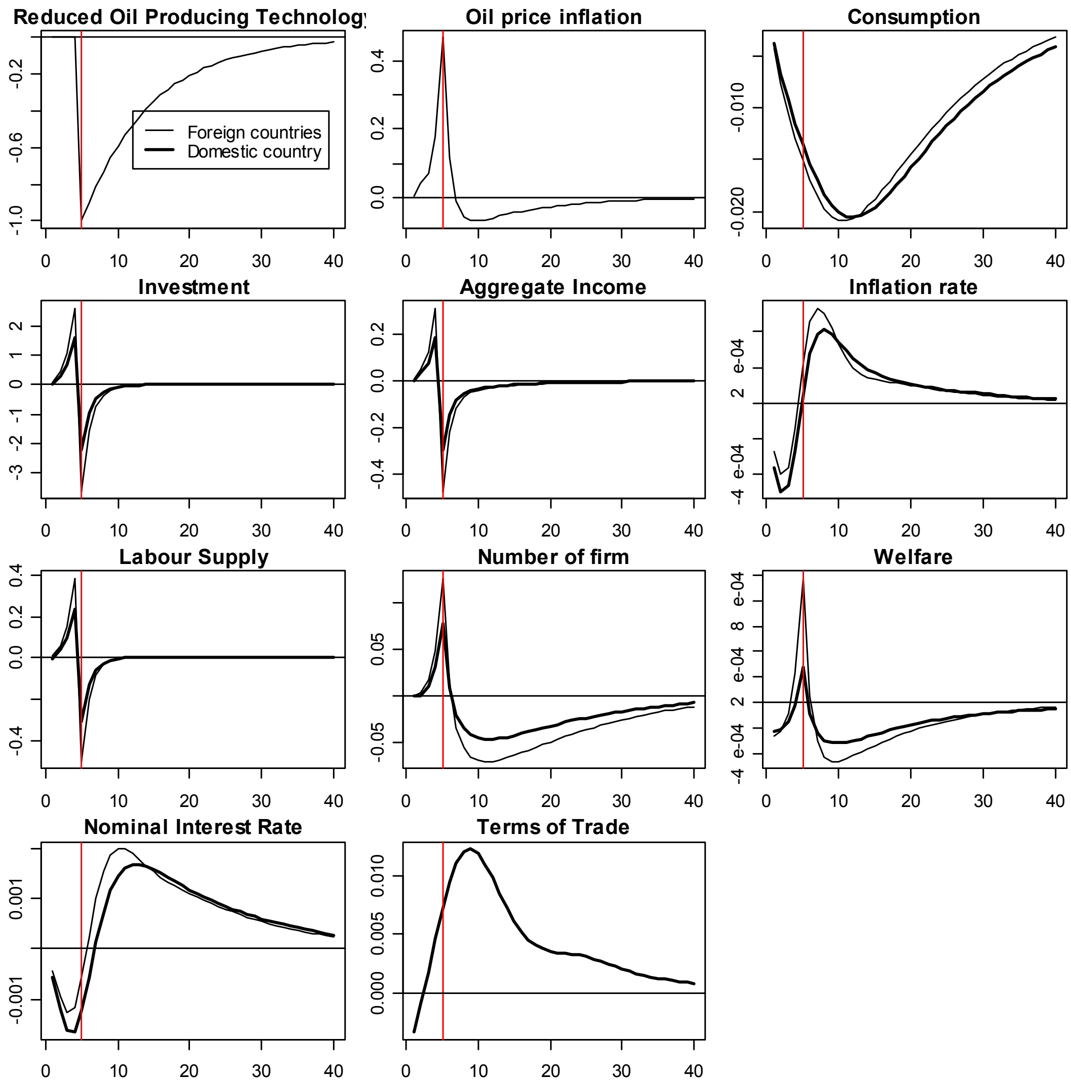

Figure 11: Expected Reduction in Oil-Producing Technology (materialized)

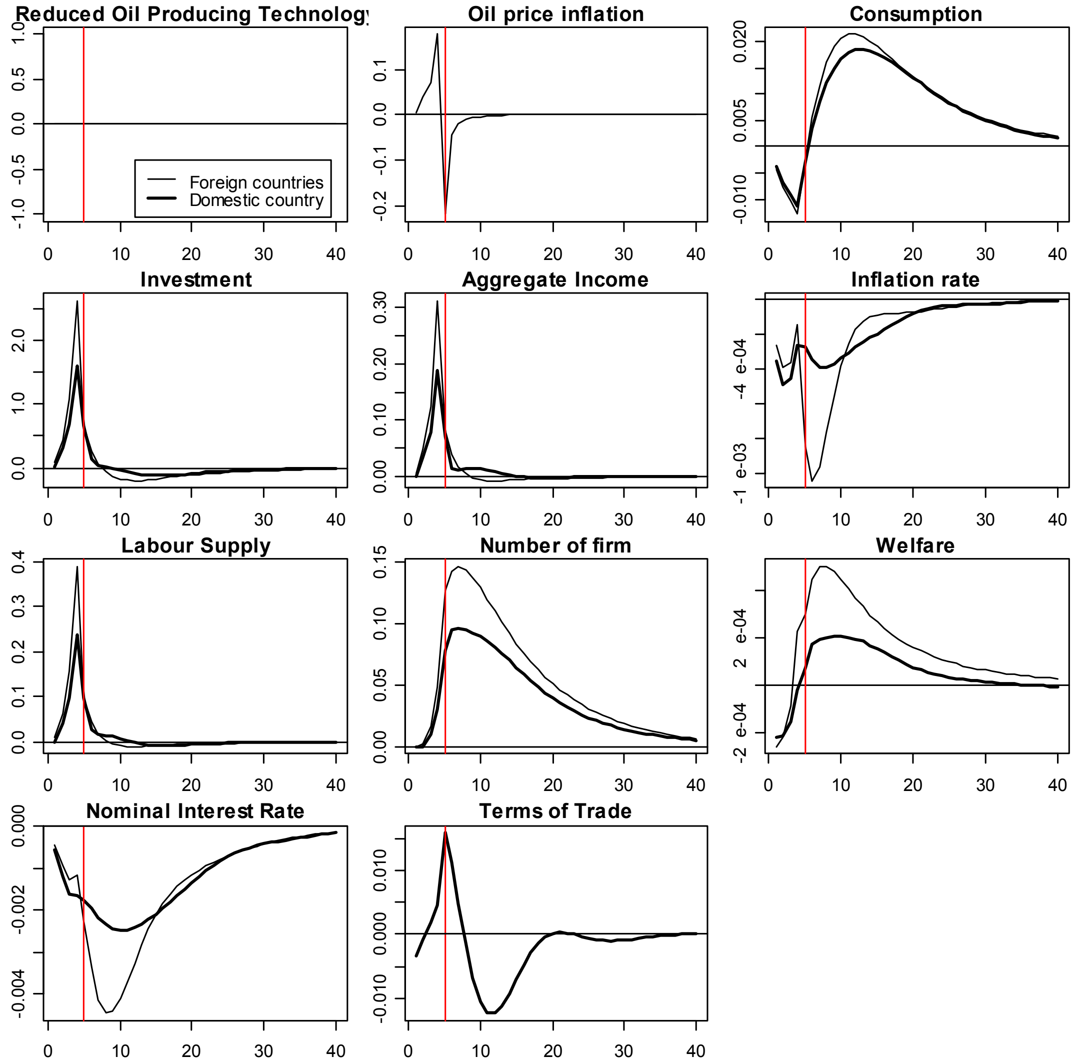

Figure 12: Expected Reduction in Oil-Producing Technology (not materialized)

Since the number of firms cannot be altered immediately, inflation rates rise in the foreign country. On the other hand, more resources are directed to investment in the rest of the world. Therefore, exports from the domestic country decreases with worsening of terms of trade.
(3) With increased working population, marginal cost as well as inflation rates rise in the foreign country similar to the case with reduced fixed cost thanks to the increase in demand. Terms of trade in the domestic country, however, improves since there exists more demand for domestic goods. This is quite contrary to the case with reduced fixed entry cost.

- Following OPEC statement, we simulate with expectation on future increase in oil demand and anxiety for future deterioration of oil producing facility. Both scenarios result in soaring oil prices, but in the latter, aggregate inflation rates across the world decrease. We need to inquire into the role of expectation shock on soaring oil prices.

References

[1] to be added

Appendix

Model Equations

Final Goods Production

$$C_t(j) \qquad C_t(j) = \left[\nu_Q^{\frac{1}{\varepsilon}} Q_t(j)^{1-\frac{1}{\varepsilon}} + (1-\nu_Q)^{\frac{1}{\varepsilon}} M_t(j)^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_t^*(j^*) \qquad C_t^*(j^*) = \left[(\nu_Q^*)^{\frac{1}{\varepsilon^*}} (Q_t^*(j^*))^{1-\frac{1}{\varepsilon^*}} + (1-\nu_Q^*)^{\frac{1}{\varepsilon^*}} (M_t^*(j^*))^{1-\frac{1}{\varepsilon^*}}\right]^{\frac{\varepsilon^*}{\varepsilon^*-1}}$$

Demand for Intermediate Goods

Household's demand for aggregated intermediate goods

$$Q_t(j) \qquad Q_t(j) = \nu_Q \left(\frac{P_{Q,t}}{P_t}\right)^{-\varepsilon} C_t(j)$$

$$Q_t^*(j^*) \qquad Q_t^*(j^*) = \nu_Q^* \left(\frac{P_{Q,t}^*}{P_t^*}\right)^{-\varepsilon^*} C_t^*(j^*)$$

$$M_t(j) \qquad M_t(j) = (1-\nu_Q) \left(\frac{P_{M,t}}{P_t}\right)^{-\varepsilon} C_t(j)$$

$$M_t^*(j^*) \qquad M_t^*(j^*) = (1-\nu_Q^*) \left(\frac{P_{M,t}^*}{P_t^*}\right)^{-\varepsilon^*} C_t^*(j^*)$$

Household's demand for aggregated intermediate goods produced by each firm

$$Q_t(h,j) \qquad Q_t(h,j) = A_{Q,t}^{\theta-1} \left(\frac{p_t(h)/P_t}{P_{Q,t}/P_t}\right)^{-\theta} Q_t(j)$$

$$Q_t^*(f,j^*) \qquad Q_t^*(f,j^*) = (A_{Q,t}^*)^{\theta^*-1} \left(\frac{p_t^*(f)/P_t^*}{P_{Q,t}^*/P_t^*}\right)^{-\theta^*} Q_t^*(j^*)$$

$$M_t(f,j) \qquad M_t(f,j) = \left(A_{Q,t}^*\right)^{\theta^*-1} \left(\frac{p_t(f)/P_t}{P_{M,t}/P_t}\right)^{-\theta^*} M_t(j)$$

$$M_t^*(h,j^*) \qquad M_t^*(h,j^*) = (A_{Q,t})^{\theta-1} \left(\frac{p_t^*(h)/P_t^*}{P_{M,t}^*/P_t^*}\right)^{-\theta} M_t^*(j^*)$$

$$A_{Q,t} \qquad A_{Q,t} \equiv n_t^{\gamma-\frac{\theta}{\theta-1}}$$

$$A_{Q,t}^* \qquad A_{Q,t}^* \equiv (n_t^*)^{\gamma_Q^*-\frac{\theta^*}{\theta^*-1}}$$

Demand for Raw Materials

Incumbents

$$O_{Q,t}(h) \qquad O_{Q,t}(h) = \nu_{O_Q} \left(\frac{P_{O_Q,t}/P_t}{P_{O,t}/P_t}\right)^{-\epsilon_{O_{QM}}} O_t(h)$$

$$O_{Q,t}^*(f) \qquad O_{Q,t}^*(f) = \nu_{O_Q}^* \left(\frac{P_{O_Q,t}^*/P_t^*}{P_{O,t}^*/P_t^*}\right)^{-\epsilon_{O_{QM}}^*} O_t^*(f)$$

$$O_{M,t}(h) \qquad O_{M,t}(h) = (1-\nu_{O_Q}) \left(\frac{P_{O_M,t}/P_t}{P_{O,t}/_tP}\right)^{-\epsilon_{O_{QM}}} O_t(h)$$

$$O_{M,t}^*(f) \qquad O_{M,t}^*(f) = (1-\nu_{O_Q}^*) \left(\frac{P_{O_M,t}^*/P_t^*}{P_{O,t}^*/P_t^*}\right)^{-\epsilon_{O_{QM}}^*} O_t^*(f)$$

Entrants

$$
\begin{array}{ll}
O_{Q,t}(e) & O_{Q,t}(e) = \nu_{O_Q} \left(\frac{P_{O_Q,t}/P_t}{P_{O,t}/P_t} \right)^{-\epsilon_{O_{QM}}} O_t(e) \\
O^*_{Q,t}(e^*) & O^*_{Q,t}(e^*) = \nu^*_{O_Q} \left(\frac{P^*_{O_Q,t}/P^*_t}{P^*_{O,t}/P^*_t} \right)^{-\epsilon^*_{O_{QM}}} O^*_t(e^*) \\
O_{M,t}(e) & O_{M,t}(e) = (1-\nu_{O_Q}) \left(\frac{P_{O_M,t}/P_t}{P_{O,t}/P_t} \right)^{-\epsilon_{O_{QM}}} O_t(e) \\
O^*_{M,t}(e^*) & O^*_{M,t}(e^*) = (1-\nu^*_{O_Q}) \left(\frac{P^*_{O_M,t}/P^*_t}{P^*_{O,t}/P^*_t} \right)^{-\epsilon^*_{O_{QM}}} O^*_t(e^*)
\end{array}
$$

Relative price of intermediate goods

Aggregate price

$$
\begin{array}{ll}
\frac{P_{Q,t}}{P_t} & \frac{P_{Q,t}}{P_t} = \frac{1}{A_{Q,t}} n_t^{\frac{1}{1-\theta}} \frac{p_t(h)}{P_t} \\
\frac{P^*_{Q,t}}{P^*_t} & \frac{P^*_{Q,t}}{P^*_t} = \frac{1}{A^*_{Q,t}} (n^*_t)^{\frac{1}{1-\theta^*}} \frac{p^*_t(f)}{P^*_t} \\
\frac{P_{M,t}}{P_t} & \frac{P_{M,t}}{P_t} = \frac{1}{A^*_{Q,t}} (n^*_t)^{\frac{1}{1-\theta^*}} \frac{p_t(f)}{P_t} \\
\frac{P^*_{M,t}}{P^*_t} & \frac{P^*_{M,t}}{P^*_t} = \frac{1}{A_{Q,t}} n_t^{\frac{1}{1-\theta}} \frac{p^*_t(h)}{P^*_t}
\end{array}
$$

Price Setting

$$
\begin{array}{ll}
\frac{p_t(h)}{P_t} & 0 = (1-\Gamma_{Q,t}(h)) \left[(1-\theta) \frac{p_t(h)}{P_t} + \theta \frac{MC_t(h)}{P_t} \right] - \left[\frac{p_t(h)}{P_t} - \frac{MC_t(h)}{P_t} \right] p_t(h) \Gamma'_{Q,t}(h) \\
 & -E_t \left[(1-\delta_D) D_{t,t+1}(j) \pi_{t+1} \frac{n_{H,t+1} Q_{t+1}(h,j)}{n_{H,t} Q_t(h,j)} \left(\frac{p_{t+1}(h)}{P_{t+1}} - \frac{MC_{t+1}(h)}{P_{t+1}} \right) \Gamma'_{Q,t+1}(h) \right] \\
\frac{p^*_t(h)}{P^*_t} & 0 = (1-\Gamma^*_{M,t}(h)) \left[(1-\theta^*) \frac{p^*_t(h)}{P^*_t} \frac{\mathcal{E}_t P^*_t}{P_t} + \theta^* \frac{MC_t(h)}{P_t} (1+\tau_t) \right] \\
 & - \left[\frac{p^*_t(h)}{P^*_t} (1-\theta^*) \frac{\mathcal{E}_t P^*_t}{P_t} - \theta^* \frac{MC_t(h)}{P_t} (1+\tau_t) \right] p^*_t(h) \Gamma^{*\prime}_{M,t}(h) \\
 & -E_t \Big[(1-\delta_D) D_{t,t+1}(j) \pi_{t+1} \frac{n^*_{H,t+1} M^*_{t+1}(h,j^*)}{n^*_{H,t} M^*_t(h,j^*)} \\
 & \times \left[\frac{p^*_{t+1}(h)}{P^*_{t+1}} \frac{\mathcal{E}_{t+1} P^*_{t+1}}{P_{t+1}} - \frac{MC_{t+1}(h)}{P_{t+1}} (1+\tau_t) \right] p^*_{t+1}(h) \Gamma^{*\prime}_{M,t+1}(h) \Big] \\
\frac{p_t(f)}{P_t} & 0 = (1-\Gamma_{M,t}(f)) \left[\frac{p_t(f)}{P_t} (1-\theta) \frac{P_t}{\mathcal{E}_t P^*_t} + \theta \frac{MC^*_t(f)}{P^*_t} (1+\tau^*_t) \right] \\
 & - \left[\frac{p_t(f)}{P_t} \frac{P_t}{\mathcal{E}_t P^*_t} - \frac{MC^*_t(f)}{P^*_t} (1+\tau^*_t) \right] p_t(f) \Gamma'_{M,t}(f) \\
 & -E_t \Big[(1-\delta^*_D) D_{t,t+1} \frac{n_{H,t+1} M_{t+1}(f,j)}{n_{H,t} M_t(f,j)} \\
 & \times \left[\frac{p_{t+1}(f)}{P_{t+1}} \frac{P_{t+1}}{\mathcal{E}_{t+1} P^*_{t+1}} - \frac{MC^*_{t+1}(f)}{P^*_{t+1}} (1+\tau^*_t) \right] p_{t+1}(f) \Gamma'_{M,t+1}(f) \Big] \\
\frac{p^*_t(f)}{P^*_t} & 0 = (1-\Gamma^*_{Q,t}(f)) \left[(1-\theta^*) \frac{p^*_t(f)}{P^*_t} + \theta^* \frac{MC^*_t(f)}{P^*_t} \right] \\
\text{PH\_RW} & - \left[\frac{p^*_t(f)}{P^*_t} - \frac{MC^*_t(f)}{P^*_t} \right] p^*_t(f) \Gamma^{*\prime}_{Q,t}(f) \\
 & -E_t \left[(1-\delta^*_D) D^*_{t,t+1} \frac{n^*_{H,t+1} Q^*_{t+1}(f,j^*)}{n^*_{H,t} Q^*_t(f,j^*)} \left(\frac{p^*_{t+1}(f)}{P^*_{t+1}} - \frac{MC^*_{t+1}(f)}{P^*_{t+1}} \right) p^*_{t+1}(f) \Gamma^{*\prime}_{Q,t+1}(f) \right]
\end{array}
$$

Relative price of raw materials

$$\frac{P_{OQ,t}}{P_t} \qquad \frac{P_{OQ,t}}{P_t} = \frac{MC_t(s)}{P_t}$$
$$\frac{P^*_{OQ,t}}{P^*_t} \qquad \frac{P^*_{OQ,t}}{P^*_t} = \frac{MC^*_t(s^*)}{P^*_t}$$
$$\frac{P_{OM,t}}{P_t} \qquad \frac{P_{OM,t}}{P_t}\frac{P_t}{\mathcal{E}_t P^*_t} = \frac{MC^*_t(s^*)}{P^*_t}(1+\tau^*_t)$$
$$\frac{P^*_{OM,t}}{P^*_t} \qquad \frac{P^*_{OM,t}}{P^*_t}\frac{\mathcal{E}_t P^*_t}{P_t} = \frac{MC_t(s)}{P_t}(1+\tau_t)$$

Price adjustment costs for differentiated intermediate goods

$$\Gamma_{Q,t}(h) \qquad \Gamma_{Q,t}(h) \equiv \frac{\phi_Q}{2}\left(\frac{\pi_t(h)}{\pi_{Q,t-1}}-1\right)^2$$
$$\Gamma'_{Q,t}(h) \qquad \left[p_t(h)\Gamma'_{Q,t}(h)\right] = \phi_Q \frac{\pi_t(h)}{\pi_{Q,t-1}}\left(\frac{\pi_t(h)}{\pi_{Q,t-1}}-1\right)$$
$$\Gamma'_{Q,t+1}(h) \qquad \left[D_{t,t+1}p_{t+1}(h)\Gamma'_{Q,t+1}(h)\right] = -D_{t,t+1}\phi_Q \frac{\pi_t(h)^2}{\pi_{Q,t-1}}\left(\frac{\pi_t(h)}{\pi_{Q,t-1}}-1\right)$$
$$\Gamma^*_{Q,t}(f) \qquad \Gamma^*_{Q,t}(f) \equiv \frac{\phi^*_Q}{2}\left(\frac{\pi^*_t(f)}{\pi^*_{Q,t-1}}-1\right)^2$$
$$\Gamma^{*\prime}_{Q,t}(f) \qquad \left[P^*_{Q,t}\Gamma^{*\prime}_{Q,t}(f)\right] = +\phi^*_Q \frac{\pi^*_t(f)}{\pi^*_{Q,t-1}}\left(\frac{\pi^*_t(f)}{\pi^*_{Q,t-1}}-1\right)$$
$$\Gamma^{*\prime}_{Q,t+1}(f) \qquad \left[D^*_{t,t+1}P^*_{Q,t+1}\Gamma^{*\prime}_{Q,t+1}(f)\right] = -D^*_{t,t+1}\phi^*_Q \frac{\pi^*_t(h)^2}{\pi^*_{Q,t-1}}\left(\frac{\pi^*_t(h)}{\pi^*_{Q,t-1}}-1\right)$$
$$\Gamma_{M,t}(f) \qquad \Gamma_{M,t}(f) \equiv \frac{\phi_M}{2}\left(\frac{\pi_t(f)}{\pi_{M,t-1}}-1\right)^2$$
$$\Gamma'_{M,t}(f) \qquad \left[P_{M,t}\Gamma'_{M,t}(f)\right] = \phi_M \frac{\pi_t(f)}{\pi_{M,t-1}}\left(\frac{\pi_t(f)}{\pi_{M,t-1}}-1\right)$$
$$\Gamma'_{M,t+1}(f) \qquad \left[D_{t,t+1}P_{M,t+1}\Gamma'_{M,t+1}(f)\right] = -D_{t,t+1}\phi_M \frac{\pi_t(h)^2}{\pi_{M,t-1}}\left(\frac{\pi_t(h)}{\pi_{M,t-1}}-1\right)$$
$$\Gamma^*_{M,t}(h) \qquad \Gamma^*_{M,t}(h) \equiv \frac{\phi^*_M}{2}\left(\frac{\pi^*_t(h)}{\pi^*_{M,t-1}}-1\right)^2$$
$$\Gamma^{*\prime}_{M,t}(h) \qquad \left[p^*_t(h)\Gamma'_{M,t}(h)\right] = \phi^*_M \frac{\pi^*_t(h)}{\pi^*_{M,t-1}}\left(\frac{\pi^*_t(h)}{\pi^*_{M,t-1}}-1\right)$$
$$\Gamma^{*\prime}_{M,t+1}(h) \qquad \left[D^*_{t,t+1}P^*_{M,t+1}\Gamma^{*\prime}_{M,t+1}(h)\right] = -D^*_{t,t+1}\phi^*_M \frac{\pi^*_t(h)^2}{\pi^*_{M,t-1}}\left(\frac{\pi^*_t(h)}{\pi^*_{M,t-1}}-1\right)$$

Marginal costs

Intermediate goods producers

$$\frac{W_t}{P_t} \qquad \frac{MC_t(h)}{P_t} = \frac{1}{Z_{Q,t}}\left[(1-\gamma_Q)\left(\frac{W_t}{P_t}\right)^{1-\xi} + \gamma_Q\left(\frac{P_{O,t}}{P_t}\right)^{1-\xi}\right]^{\frac{1}{1-\xi}}$$
$$\frac{W^*_t}{P^*_t} \qquad \frac{MC^*_t(f)}{P^*_t} = \frac{1}{Z^*_{Q,t}}\left[(1-\gamma^*_Q)\left(\frac{W^*_t}{P^*_t}\right)^{1-\xi^*} + \gamma^*_Q\left(\frac{P^*_{O,t}}{P^*_t}\right)^{1-\xi^*}\right]^{\frac{1}{1-\xi^*}}$$

Raw material goods producers

$$\frac{MC_t(s)}{P_t} \qquad \frac{MC_t(s)}{P_t} = \frac{1}{Z_{s,t}}\left[(1-\gamma_s)\left(\frac{W_t}{P_t}\right)^{1-\xi_s} + \gamma_s\left(\frac{P_{L,t}}{P_t}\right)^{1-\xi_s}\right]^{\frac{1}{1-\xi_s}}$$
$$\frac{MC^*_t(s^*)}{P^*_t} \qquad \frac{MC^*_t(s^*)}{P^*_t} = \frac{1}{Z^*_{s,t}}\left[(1-\gamma^*_s)\left(\frac{W^*_t}{P^*_t}\right)^{1-\xi^*_s} + \gamma^*_s\left(\frac{P^*_{L,t}}{P^*_t}\right)^{1-\xi^*_s}\right]^{\frac{1}{1-\xi^*_s}}$$

Labor market

Taste for the variety of labor input

$$
\begin{array}{ll}
A_{l,t} & A_{l,t} = L_t^{\gamma_l - \frac{\psi}{\psi-1}} \\
A^*_{l,t} & A^*_{l,t} = (L^*_t)^{\gamma^*_l - \frac{\psi^*}{\psi^*-1}}
\end{array}
$$

Aggregate wage

$$
\begin{array}{ll}
\frac{W_t(j)}{P_t} & \frac{W_t}{P_t} = \frac{1}{A_{l,t}} L_t^{\frac{1}{1-\psi}} \frac{W_t(j)}{P_t} \\
\frac{W^*_t(j^*)}{P^*_t} & \frac{W_t*}{P^*_t} = \frac{1}{A^*_{l,t}} (L^*_t)^{\frac{1}{1-\psi^*}} \frac{W^*_t(j^*)}{P^*_t}
\end{array}
$$

Wage for each household

$$
\begin{array}{ll}
V'_t(j) & \psi \frac{V'_t(j)}{U'_t(j)} \frac{P_t}{W_t(j)} = (\psi - 1)(1 - \Gamma_{W,t}(j)) + W_t(j)\Gamma'_{W,t}(j) \\
 & +E_t \left[D_{t,t+1} \frac{l_{t+1}(j)}{l_t(j)} W_{t+1}(j) \Gamma'_{W,t+1}(j) \right] \\
V^{*\prime}_t(j^*) & \psi^* \frac{V^{*\prime}_t(j^*)}{U^{*\prime}_t(j^*)} \frac{P^*_t}{W^*_t(j^*)} = (\psi^* - 1) \left(1 - \Gamma^*_{W,t}(j^*)\right) + W^*_t(j^*)\Gamma^{*\prime}_{W,t}(j) \\
 & +E_t \left[D^*_{t,t+1} \frac{l^*_{t+1}(j^*)}{l^*_t(j^*)} W^*_{t+1}(j^*) \Gamma^{*\prime}_{W,t+1}(j) \right]
\end{array}
$$

Utility functions

$$
\begin{array}{ll}
U'_t(j) & U'_t(j) = Z_{U,t} \left(\frac{C_t(j) - b_C C_{t-1}(j)}{1-b_C} \right)^{-\sigma} \\
U^{*\prime}_t(j) & U^{*\prime}_t(j^*) = Z^*_{U,t} \left(\frac{C^*_t(j^*) - b^*_C C^*_{t-1}(j^*)}{1-b^*_C} \right)^{-\sigma^*} \\
l_t(j) & V'_t(j) = Z_{L,t} \left(\frac{l_t(j) - b_l l_{t-1}(j)}{1-b_l} \right)^{\zeta} \\
l^*_t(j^*) & V^{*\prime}_t(j^*) = Z^*_{V,t} \left(\frac{l^*_t(j^*) - b^*_l l^*_{t-1}(j^*)}{1-b^*_l} \right)^{\zeta^*}
\end{array}
$$

Wage adjustment costs

$$
\begin{array}{ll}
\Gamma_{W,t}(j) & \Gamma_{W,t}(j) = \frac{\phi_W}{2} \left(\frac{\pi_{W,t}(j)}{\pi_{W,t-1}} - 1 \right)^2 \\
\Gamma'_{W,t}(j) & \left[W_t(j) \Gamma'_{W,t}(j) \right] = \phi_W \frac{\pi_{W,t}(j)}{\pi_{W,t-1}} \left(\frac{\pi_{W,t}(j)}{\pi_{W,t-1}} - 1 \right) \\
\Gamma'_{W,t+1}(j) & \left[D_{t,t+1} W_{t+1}(j) \Gamma'_{W,t+1}(j) \right] = \\
 & -D_{t,t+1} \phi_W \frac{\pi_{W,t+1}(j)^2}{\pi_{W,t}} \left(\frac{\pi_{W,t+1}(j)}{\pi_{W,t}} - 1 \right) \\
\Gamma^*_{W,t}(j^*) & \Gamma^*_{W,t}(j^*) = \frac{\phi^*_W}{2} \left(\frac{\pi^*_{W,t}(j)}{\pi^*_{W,t-1}} - 1 \right)^2 \\
\Gamma^{*\prime}_{W,t}(j^*) & \left[W^*_t(j) \Gamma^{*\prime}_{W,t}(j) \right] = \phi^*_W \frac{\pi^*_{W,t}(j)}{\pi^*_{W,t-1}} \left(\frac{\pi^*_{W,t}(j)}{\pi^*_{W,t-1}} - 1 \right) \\
\Gamma^{*\prime}_{W,t+1}(j^*) & \left[D^*_{t,t+1} W^*_{t+1}(j) \Gamma^{*\prime}_{W,t+1}(j) \right] = \\
 & -D^*_{t,t+1} \left[\phi^*_W \frac{\pi^*_{W,t+1}(j)^2}{\pi^*_{W,t}} \left(\frac{\pi^*_{W,t+1}(j)}{\pi^*_{W,t}} - 1 \right) \right]
\end{array}
$$

Demand for labor input by an intermediate goods producer

Incumbents

$$
\begin{array}{ll}
l_t(h,j) & l_t(h,j) = A_{l,t}^{\psi-1}\left(\frac{W_t(j)/P_t}{W_t/P_t}\right)^{-\psi} l_t(h) \\
l_t(h) & l_t(h) = (1-\gamma_Q)\left(\frac{W_t/P_t}{Z_{Q,t}MC_t(h)/P_t}\right)^{-\xi}\left(\frac{Y_t(h)}{Z_{Q,t}}\right) \\
l_t^*(f,j^*) & l_t^*(f,j^*) = (A_{l,t}^*)^{\psi^*-1}\left(\frac{W_t^*(j^*)/P_t^*}{W_t^*/P_t^*}\right)^{-\psi^*} l_t^*(f) \\
l_t^*(f) & l_t^*(f) = (1-\gamma_Q^*)\left(\frac{W_t^*/P_t^*}{Z_{Q,t}^*MC_t^*(f)/P_t^*}\right)^{-\xi^*}\left(\frac{Y_t^*(f)}{Z_{Q,t}^*}\right)
\end{array}
$$

Entrants

$$
\begin{array}{ll}
l_t(e,j) & l_t(e,j) = A_{l,t}^{\psi-1}\left(\frac{W_t(j)/P_t}{W_t/P_t}\right)^{-\psi} l_t(e) \\
l_t(e) & l_t(e) = (1-\gamma_Q)\left(\frac{W_t/P_t}{Z_{Q,t}MC_t(h)/P_t}\right)^{-\xi}\left(\frac{f_{E,t}}{Z_{Q,t}}\right) \\
l_t^*(e^*,j^*) & l_t^*(e^*,j^*) = (A_{l,t}^*)^{\psi^*-1}\left(\frac{W_t^*(j^*)/P_t^*}{W_t^*/P_t^*}\right)^{-\psi^*} l_t^*(e^*) \\
l_t^*(e^*) & l_t^*(e^*) = (1-\gamma_Q^*)\left(\frac{W_t^*/P_t^*}{Z_{Q,t}^*MC_t^*(f)/P_t^*}\right)^{-\xi^*}\left(\frac{f_{E,t}^*}{Z_{Q,t}^*}\right)
\end{array}
$$

Demand for labor input by a raw-material producer

$$
\begin{array}{ll}
l_t(s,j) & l_t(s,j) = A_{l,t}^{\psi-1}\left(\frac{W_t(j)/P_t}{W_t/P_t}\right)^{-\psi} l(s) \\
l_t(s) & l_t(s) = (1-\gamma_O)\left(\frac{W_t/P_t}{Z_{s,t}MC_t(s)/P_t}\right)^{-\xi}\left(\frac{O_t(s)}{Z_{s,t}}\right) \\
l_t^*(s^*,j^*) & l_t^*(s^*,j^*) = (A_{l,t}^*)^{\psi^*-1}\left(\frac{W_t^*(j^*)/P_t^*}{W_t^*/P_t^*}\right)^{-\psi^*} l_t^*(s^*) \\
l_t^*(s^*) & l_t^*(s^*) = (1-\gamma_O^*)\left(\frac{W_t^*/P_t^*}{Z_{s,t}^*MC_t^*(s)/P_t^*}\right)^{-\xi^*}\left(\frac{O_t^*(s)}{Z_{s,t}^*}\right)
\end{array}
$$

Demand for raw materials by an intermediate goods producer

Incumbents

$$
\begin{array}{ll}
O_t(h) & O_t(h) = \gamma_O\left(\frac{P_{O,t}/P_t}{Z_{Q,t}MC_t(h)/P_t}\right)^{-\xi}\left(\frac{Y_t(h)}{Z_{Q,t}}\right) \\
O_t^*(f) & O_t^*(f) = \gamma_O^*\left(\frac{P_{O,t}^*/P_t^*}{Z_{Q,t}^*MC_t^*(f)/P_t^*}\right)^{-\xi^*}\left(\frac{Y_t^*(f)}{Z_{Q,t}^*}\right)
\end{array}
$$

Entrants

$$
\begin{array}{ll}
O_t(e) & O_t(e) = \gamma_O\left(\frac{P_{O,t}/P_t}{Z_{Q,t}MC_t(h)/P_t}\right)^{-\xi}\left(\frac{f_{E,t}}{Z_{Q,t}}\right) \\
O_t^*(e^*) & O_t^*(e^*) = \gamma_O^*\left(\frac{P_{O,t}^*/P_t^*}{Z_{Q,t}^*MC_t^*(f)/P_t^*}\right)^{-\xi^*}\left(\frac{f_{E,t}^*}{Z_{Q,t}^*}\right)
\end{array}
$$

Demand for raw materials by a raw-material producer

$$\frac{P_{L,t}}{P_t} \quad L_t(s) = \gamma_s \left(\frac{P_{L,t}/P_t}{Z_{s,t} MC_t(s)/P_t}\right)^{-\xi} \left(\frac{O_t(s)}{Z_{Q,t}}\right)$$
$$\frac{P^*_{L,t}}{P^*_t} \quad L^*_t(s^*) = \gamma^*_s \left(\frac{P^*_{L,t}/P^*_t}{Z^*_{s,t} MC^*_t(s^*)/P^*_t}\right)^{-\xi^*} \left(\frac{O^*_t(s)}{Z^*_{s,t}}\right)$$

Aggregate price of raw material

$$\frac{P_{O,t}}{P_t} \quad \frac{P_{O,t}}{P_t} = \left[\alpha_O \left(\frac{P_{OQ,t}}{P_t}\right)^{1-\xi_O} + (1-\alpha_O)\left(\frac{P_{OM,t}}{P_t}\right)^{1-\xi_O}\right]^{\frac{1}{1-\xi_O}}$$
$$\frac{P^*_{O,t}}{P^*_t} \quad \frac{P^*_{O,t}}{P^*_t} = \left[\alpha^*_O \left(\frac{P^*_{OQ,t}}{P^*_t}\right)^{1-\xi^*_{O,t}} + (1-\alpha^*_O)\left(\frac{P^*_{OM,t}}{P^*_t}\right)^{1-\xi^*_{O,t}}\right]^{\frac{1}{1-\xi^*_O}}$$

Profits and share prices

Euler equations of share

$$\frac{\varpi_t(h)}{P_t} \quad \frac{\varpi_t(h)}{P_t} = (1-\delta_D) D_{t,t+1}(j)\pi_{t+1}\left[\frac{\Pi_{t+1}(h)}{P_{t+1}} + \frac{\varpi_{t+1}(h)}{P_{t+1}}\right]$$
$$\frac{\varpi^*_t(f)}{P^*_t} \quad \frac{\varpi^*_t(f)}{P^*_t} = (1-\delta^*_D) D^*_{t,t+1}(j^*)\pi^*_{t+1}\left[\frac{\Pi^*_{t+1}(f)}{P^*_{t+1}} + \frac{\varpi^*_{t+1}(f)}{P^*_{t+1}}\right]$$

Free entry conditions

$$\frac{MC_t(h)}{P_t} \quad \frac{\varpi_t(h)}{P_t} = f_{E,t}\frac{MC_t(h)}{P_t}$$
$$\frac{MC^*_t(f)}{P^*_t} \quad \frac{\varpi^*_t(h)}{P^*_t} = f^*_{E,t}\frac{MC^*_t(f)}{P^*_t}$$

Profits

$$\frac{\Pi_t(h)}{P_t} \quad \frac{\Pi_t(h)}{P_t} = \left(\frac{p_t(h)}{P_t} - \frac{MC_t(h)}{P_t}\right) n_{H,t} Q_t(h,j)][1-\Gamma_{Q,t}(h)]$$
$$+\left[\frac{p^*_t(h)}{P^*_t}\frac{\mathcal{E}_t P^*_t}{P_t} - \frac{MC_t(h)}{P_t}(1+\tau_t)\right] n^*_{H,t} M^*_t(h,j^*)[1-\Gamma^*_{M,t}(h)]$$
$$\frac{\Pi^*_t(f)}{P^*_t} \quad \frac{\Pi^*_t(f)}{P^*_t} = \left(\frac{p^*_t(f)}{P^*_t} - \frac{MC^*_t(f)}{P^*_t}\right) n^*_{H,t} Q_t(f,j)][1-\Gamma^*_{Q,t}(f)]$$
$$+\left[\frac{p_t(f)}{P_t}\frac{P_t}{\mathcal{E}_t P^*_t} - \frac{MC^*_t(f)}{P_t}(1+\tau^*_t)\right] n_{H,t} M_t(f,j)[1-\Gamma_{M,t}(f)]$$

Dynamics of the number of firms

$$n_t \quad n_t = (1-\delta)(n_{t-1} + n_{E,t-1})$$
$$n^*_t \quad n^*_t = (1-\delta^*)(n^*_{t-1} + n^*_{E,t-1})$$

Exchange rates

$$\frac{\mathcal{E}_t P^*_t}{P_t} \quad E_t D_{t,t+1} L_{t+1}\frac{F_{t+1}(j)}{P_{t+1}}\pi_{t+1} = L_t\frac{F_t(j)}{P_t} + (1+i^*_{t-1})\Gamma_{B,t-1}(j)\frac{\mathcal{E}_t P^*_t}{P_t} L_t \frac{B_{F,t}(j)}{P_{t-1}}\frac{1}{\pi_t}$$
$$+\frac{\mathcal{E}_t P^*_t}{P_t}\frac{P^*_{M,t}}{P^*_t} L^*_t M^*_t(j^*) - \frac{P_{M,t}}{P_t} L_t M_t(j) - \frac{P_{O_M,t}}{P_t}\left[n_t O_{M,t}(h) + n_{E,t} O_{M,t}(e)\right]$$

Financial assets

$\frac{F_t(j)}{P_t}$	$\frac{F_t(j)}{P_t} = (1+i_t^*)(1-\Gamma_{B,t}(j))\frac{1}{\pi_t^*}\frac{\mathcal{E}_t P_t^*}{P_t}\frac{B_{F,t}(j)}{P_{t-1}^*}$

Government

i_t	$(1+i_{t+1})^4 - 1 = \omega_1\left[(1+i_{t-1})^4 - 1\right] + (1-\omega_1)\left[(1+i_{t,neut})^4 - 1\right]$
	$+\omega_2 E_t\left(\pi_{t+\tau} - \pi_{tar}\right)$
i_t^*	$(1+i_{t+1}^*)^4 - 1 = \omega_1^*\left[(1+i_{t-1}^*)^4 - 1\right] + (1-\omega_1^*)\left[(1+i_{t,neut}^*)^4 - 1\right]$
	$+\omega_2^* E_t\left(\pi_{t+\tau}^* - \pi_{tar}^*\right)$
$i_{t,neut}$	$i_{t,neut} = \pi_{t+3}^{0.25}/\beta$
$i_{t,neut}^*$	$i_{t,neut}^* = \left(\pi_{t+3}^*\right)^{0.25}/\beta^*$

Bond market

$D_{t,t+1}$	$1 = (1+i_t)E_t D_{t,t+1}$
$D_{t,t+1}^*$	$1 = (1+i_t^*)E_t D_{t,t+1}^*$
$\Gamma_{B,t}(j)$	$1 = (1+i_t^*)(1-\Gamma_{B,t+1}(j))E_t\left(D_{t,t+1}\frac{\mathcal{E}_{t+1}P_{t+1}^*}{P_{t+1}}\frac{P_t}{\mathcal{E}_t P_t^*}\frac{\pi_{t+1}}{\pi_{t+1}^*}\right)$
$\frac{B_{F,t+1}(j)}{P_t^*}$	$\Gamma_{B,t}(j) = \phi_{B1}\frac{\exp\left(\phi_{B2}\frac{\mathcal{E}_t P_t^*}{P_t}\frac{B_{F,t+1}(j)}{P_t^*} - Z_{B0,t}\right) - 1}{\exp\left(\phi_{B2}\frac{\mathcal{E}_t P_t^*}{P_t}\frac{B_{F,t+1}(j)}{P_t^*} - Z_{B0,t}\right) + 1} + Z_{B,t}$
$\frac{B_{F,t+1}^*(j)}{P_t}$	$\frac{B_{F,t+1}^*(j^*)}{P_t} = -\frac{L_t}{L_t^*}\frac{B_{F,t+1}(j)}{P_t^*}$

Market clearing

Labor market

$n_{E,t}$	$l_t(j) = n_t l_t(h,j) + n_{E,t} l_t(e,j) + l_t(s,j)$
$n_{E,t}^*$	$l_t^*(j^*) = n_t^* l_t^*(f,j^*) + n_{E,t}^* l(e^* j^*) + l_t^*(s^*,j^*)$

Raw materials market

$O_t(s)$	$O_t(s) = n_t O_{Q,t}(h) + n_{E,t} O_{Q,t}(e) + (1+\tau_t)\left[n_t^* O_{M,t}^*(f) + n_{E,t}^* O_{M,t}^*(e^*)\right]$
$O_t^*(s^*)$	$O_t^*(s^*) = n_t^* O_{Q,t}^*(f) + n_{E,t}^* O_{Q,t}^*(e^*) + (1+\tau_t^*)\left[n_t O_{M,t}(h) + n_{E,t} O_{M,t}(e)\right]$

Intermediate goods market

$Y_t(h)$	$Y_t(h) = L_t Q_t(h,j) + (1+\tau_t) L_t^* M_t^*(h,j^*)$
$Y_t^*(f)$	$Y_t^*(f) = L_t^* Q_t(f,j^*) + (1+\tau_t^*) L_t^* M_t(f,j)$

Inflation rates

Final goods

$\pi_t \quad D_{t,t+1} \equiv \beta \frac{U'_{t+1}}{\pi_{t+1} U'_t}$
$\pi^*_t \quad D^*_{t,t+1} \equiv \beta \frac{U^{*\prime}_{t+1}}{\pi^*_{t+1} U^{*\prime}_t}$

Aggregate intermediate goods

$\pi_{Q,t} \quad \pi_{Q,t} = \frac{P_{Q,t}/P_t}{P_{Q,t-1}/P_{t-1}} \pi_t$
$\pi^*_{Q,t} \quad \pi^*_{Q,t} = \frac{P^*_{Q,t}/P^*_t}{P^*_{Q,t-1}/P^*_{t-1}} \pi^*_t$
$\pi_{M,t} \quad \pi_{M,t} = \frac{P_{M,t}/P_t}{P_{M,t-1}/P_{t-1}} \pi_t$
$\pi^*_{M,t} \quad \pi^*_{M,t} = \frac{P^*_{M,t}/P^*_t}{P^*_{M,t-1}/P^*_{t-1}} \pi^*_t$

A intermediate good by each firm

$\pi_t(h) \quad \pi_t(h) = \frac{p_t(h)/P_t}{p_{t-1}(h)/P_{t-1}} \pi_t$
$\pi^*_t(f) \quad \pi^*_t(f) = \frac{p^*_t(f)/P^*_t}{p^*_{t-1}(f)/P^*_{t-1}} \pi^*_t$
$\pi_t(f) \quad \pi_t(f) = \frac{p_t(f)/P_t}{P_{t-1}(f)/P_{t-1}} \pi_t$
$\pi^*_t(h) \quad \pi^*_t(h) = \frac{p^*_t(h)/P^*_t}{p^*_{t-1}(h)/P^*_{t-1}} \pi^*_t$