Oil Price Shocks, Monetary Policy Rules and Welfare*

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Abstract

Sudden and protracted oil-price increases are generally accompanied by economic contractions and high inflation. How should monetary policy react to oil-price shocks in order to minimize their adverse macroeconomic effects? We build a DSGE model characterized by two oil-importing countries and one oil-exporting country. Oil-importing countries use oil for consumption and as input in production. The oil-exporting country produces only oil and consumes imported goods. We calibrate the model and evaluate the performance of simple Taylor-type interest rate rules when the economy is hit by oil-price shocks, on the basis of a micro-founded welfare metric. We search for rules that i) maximize welfare to a second order of approximation, ii) satisfy the zero-lower-bound for the nominal interest rate and iii) produce either a Nash or a cooperative equilibrium. Under complete international financial markets, we find that the optimal interest rate rule is inertial, it reacts strongly and positively to headline inflation and to output deviations from the non-stochastic steady state level, while it reacts negatively to oil-price inflation.

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*The views expressed in this paper are those of the authors and do not necessarily reflect the views of the ECB or the ESCB.

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1 Introduction

Sudden and protracted oil-price increases are generally accompanied by economic contractions and high inflation, as documented, e.g. by Hamilton (1983) and Hamilton (1996). The oil crises of the '70s provide one major example of the consequences of oil-price turbulence for the macroeconomy. Not surprisingly, the surge in oil-prices experienced since 2003 has raised concerns among analysts and policy makers. However, the macroeconomic effects could be different from those observed in the past. One reason is that most advanced economies experienced a steady reduction in oil dependence over the last decade. Another reason is that both demand and supply pressures have played a major role in the recent episode, as opposed to the dominant supply component of the oil-price shock experienced in the '70s.

Some authors have attributed the adverse effects of the oil-price increase in the '70s to the inappropriate response of monetary policy (see, e.g., Bernanke et al. (1997) and Barsky and Kilian (2002)) In this paper, we build a dynamic stochastic general equilibrium (DSGE) model that can capture the main features of the recent episode and ask how should monetary policy react to oil price shocks. More specifically, we evaluate the welfare consequences of oil price shocks when the central bank commits to a feedback interest rate rule. We compare the performance of optimized simple rules that react to different macroeconomic variables (such as the lagged interest rate, real GDP, inflation and energy price inflation) according to a micro-founded welfare metric. We consider two alternative specifications concerning the international financial market: a complete market structure conducive to consumption-risk sharing across countries, and a non-contingent bonds structure. Under complete international financial markets, we find that the optimal interest rate rule is inertial, it reacts strongly and positively to headline inflation and to output deviations from the non-stochastic steady state level, while it reacts negatively to oil-price inflation. As headline inflation contains an oil-price inflation component, our rule amounts to a form of core price inflation, although the weight given to oil-prices is larger than the share of oil in total households' consumption. Under incomplete markets [. . . to be completed.]

Our framework is a three-country DSGE model, which is characterized by two oil-importing countries (the US and the euro area), and one oil exporting country (or a block of net oil-exporting countries).

The two oil-importing countries are identical in structure. They are inhabited by infinitely-lived households consuming a basket of domestically produced goods, imported goods and oil. Each household supplies a differentiated type of labor to a union that combines the different types and offers this aggregate labor input to firms. Nominal wage contracts can be renegotiated only at random intervals of time and in a staggered fashion, although wages that cannot be changed are indexed to lagged and trend inflation. Firms produce differentiated goods using labor and capital and set prices optimally at random intervals, with prices also being indexed to lagged and trend inflation. Oil enters production in two ways. First, capital can only be used in production if it is combined with oil (as in Kim and Loungani (1992)). Second, oil is needed to vary capital utilization and hence to produce capital services (as in Finn (1995, 1996)). The fiscal authorities are constrained by a debt-to-GDP criterion. They can finance public expenditures by issuing debt certificates and by levying taxes on labor, consumption, oil and lump-sum taxes. The central banks credibly commit to a feedback nominal interest rate rule that responds to measures of the lagged interest rate, inflation, output and oil prices.

The oil-exporting country is modeled in a stylized way. It is inhabited by a representative household consuming a basket of goods that can only be imported from the rest of the world. Firms produce oil, which is exported and not used for internal consumption, and set prices optimally at each instant of time. Government expenditures are set to zero, so that the fiscal authority has no need to raise taxes or to issue debt. Finally, the monetary authority is committed to a credible peg of the currency to the US dollar.

A few features of our model are worth notice. First, we introduce oil in an otherwise standard medium-scale model (such as the one presented in Smets and Wouters (2003) or Christiano et al. (2005)), where a number of frictions and economic disturbances are shown to be important to replicate some empirical stylized facts. The reason for choosing such an environment is that we aim at performing a welfare analysis based on a model that closely mimic the data. In this regard, our approach is similar to the one followed by Schmitt-Grohé and Uribe (2004a).[1]

Second, our model can simulate oil-price shocks that are either exogenous or endogenous to the world economy. Since oil enters both consumption and production in the oil-importing countries, demand and supply shocks in those countries generate endogenous volatility in the price of oil. On the contrary, a productivity shock in the oil-exporting country acts as an exogenous supply shock in the world economy. In the welfare analysis, we can modify the importance of the exogenous supply-driven oil-price shock relative to other sources of economic disturbances and check the implications for the optimal monetary policy reaction.

Third, we consider the effects of fiscal policy on the propagation of oil-price shocks to the macroeconomy. In particular, we aim at capturing the asymmetric role of energy taxes in different countries. By imposing taxes on labor, consumption and energy, our model is able to measure the impact of fiscal asymmetries on the propagation of oil-price shocks and on the desirable monetary policy response.

Finally, we consider two alternative cases of international asset trade: a complete market structure and a non-contingent bonds structure. The reason is that the degree of completeness in international financial market can have important implications for the welfare analysis of monetary policy in the face of oil-price shocks. For instance, the existence of international trade in state-contingent assets implies that any domestic idiosyncratic risk not offset by a sub-optimal monetary policy can be insured. Thus, the welfare loss associated to a sub-optimal rule might be larger under incomplete markets.

Our paper relates to different strands of the literature. One strand discusses the transmission of oil-price shocks and their macroeconomic effects (see e.g. Hamilton (1983, 1996), Bernanke et al. (1997), Barsky and Kilian (2002), Dotsey and Reid (1992) and Jimènez-Rodrìguez and Sanchèz (2004) for an analysis based on VARs, or Backus and Crucini (2000) and de Walque et al. (2005) for an analysis based on calibrated or estimated DSGE models). A second strand addresses the role of monetary policy in stabilizing or amplifying the macroeconomic effects of oil-price shocks, as done e.g. by Bernanke et al. (1997), Barsky and Kilian (2002) Kamps and Pierdzioch (2002) and Leduc and Sill (2004). Our paper differs from these contributions in the way we capture the interaction between energy and the macroeconomy, and in the use of a micro-founded welfare metric to compare systematically different policy responses to

[1] Examples of policy analysis based on more stylized models are provided by Leduc and Sill (2004) and Kamps and Pierdzioch (2002).

oil price shocks. A further strand of the literature addresses optimal monetary and/or fiscal policy in models with steady state distortions, such as in Benigno and Woodford (2004b), Benigno and Woodford (2004a), Schmitt-Grohé and Uribe (2004b) and Schmitt-Grohé and Uribe (2004a, 2005). Our model carries out a similar analysis in a medium scale three-country open-economy model with an explicit role for oil. Finally, a number of contributions propose alternative algorithms to implement second-order solutions to non-linear rational expectations models, such as Schmitt-Grohé and Uribe (2004b) and Lombardo and Sutherland (2005). Our paper follows the solution method to the second-order approximation of the model proposed by Lombardo and Sutherland (2005).

The paper proceeds as follows. In section 2, we describe the main building blocks of the model. In section 3, we present the details of the calibration exercise and the empirical fit of the model with the data. In section 4, we describe the effects of an exogenous oil-price shock in our model. In section 5, we present the welfare analysis and discuss our main results. Finally, in section 6, we conclude and indicate plans for future research.

2 The model

Our world economy consists of two oil-importing countries and one oil-exporting country. We label the domestic oil-importing countries as the euro area (EA), the foreign oil-importing country as the United States (US), and the oil-producing country as the block of net oil-exporting countries (O). The world population has measure $1 + \vartheta$. A fraction b lives in country EA, a fraction $1 - b$ lives in country US and the remaining fraction ϑ lives in country O.

Countries EA and US share the same preferences, production technologies and trade structures, and have access to the same set of domestic and cross-border financial assets. Differences between the two economies are reflected in the value of various structural parameters.

In each of the two oil-importing countries, households maximize the discounted sum of expected future utilities, defined over a consumption basket, real money balances and leisure. Consumption displays habit persistence. Households consume a basket of goods made of domestically produced final goods, imported final goods and imported oil. Labor is differentiated over households, which implies some market power in the wage setting decision and allows the introduction of sticky wages à la Calvo. We allow for wage indexation to an index of past inflation and trend inflation. Households rent labor and capital services to firms, and they decide the level of investment taking into account the costs of adjusting the capital stock.

Households also allocate nominal wealth among nominal money balances, one-period nominal government bonds, one-period internationally traded risk-free nominal assets denominated in foreign currency and a portfolio of state-contingent assets denominated in domestic currency. We analyze two polar specifications on the international capital markets and evaluate the importance of this assumption for the optimal response of monetary policy to oil shocks. Under the first specification (complete markets) a full set of state-contingent one-period nominal assets denominated in domestic currency is traded across countries. Under the second specification (incomplete markets) only non-state-contingent one-period nominal bonds denominated in foreign currency can be traded internationally. State-contingent assets are traded only domestically.[2]

[2]The existence of complete domestic markets guarantees equality of the households' marginal income, despite

A perfectly competitive intermediate sector produces an input for the production of domestic final-goods by assembling together the existing stock of capital with imported energy (à la Kim and Loungani (1992) and Backus and Crucini (2000)). We call this product energy-loaded capital.

A continuum of imperfectly competitive firms produce differentiated goods using labor and energy-loaded capital, taking factor prices as given. However, they can react to changes in the rental cost of capital by adjusting the intensity of capital utilization. Varying capital utilization is costly in terms of energy (à la Finn (1995)). Firms producing differentiated goods cannot reset their prices optimally at any instant of time. Price setting occurs as in Calvo (1983) except that prices which cannot be changed are linked to an index of past and steady state inflation.

In order to finance public expenditures, fiscal authorities levy taxes on labor, consumption, and oil, and issue debt certificates (denominated in local currency). The governments can also make use of a lump-sum tax (transfer) to finance its deficit. On the contrary, seigniorage is not used to finance the budget but it is rebated lump-sum to the households. We impose a debt-to-GDP criterion in steady state.

Monetary policy is characterized by a feedback interest rate rule. We consider a class of simple rules that respond to the lagged interest rate, inflation, output and its growth rate, oil prices and oil price inflation.

We model country O in a stylized way. Households consume a basket of goods, which includes only goods imported from EA and US. They also invest in the internationally traded bonds (denominated in US dollars), under incomplete markets, or in Arrow-Debreu securities (denominated in domestic currency), under complete markets. Firms only produce oil, which is exported and not used for internal consumption. Although they operate in a monopolistically competitive market, firms can set prices optimally at each instant of time. Government expenditures are set to zero, so that the fiscal authority has no need to raise taxes or to issue debt. Finally, the monetary authorities implement a peg of the currency to the US dollar.

We assume that the three economies are hit by a set of shocks. In particular, each of the oil-importing country faces a preference shock, a productivity shock, a government expenditure shock, a monetary policy shock, a labor supply shock, and an investment shock. The oil-exporting country only faces a technological shock that affects its productivity in extracting oil. All exogenous stochastic processes are AR(1).

2.1 The EA and US economies

In this section, we describe in some details the EA economy. The US economy is modeled in a symmetric way. Unless differently specified, we denote EA variables without a country label and US variables with a star.

2.1.1 Households

There is a continuum of households denoted by j,with $j \in [0, b]$. Each household j can allocate its beginning of period nominal wealth to nominal money balances, one-period nominal

the presence of staggered wage setting.

government bonds, one-period internationally traded risk-free nominal assets denominated in foreign currency and a portfolio of state-contingent assets denominated in domestic currency. Households also receive payments of the factors they rent to domestic firms, profits from owing shares of the domestic firms, and transfers from the government. They use this income to finance consumption and investment expenditures. Their budget constraints are given by

$$\frac{B_{j,t}}{P_t}+\frac{S_tF_{j,t}}{P_t}+E_tQ_{t,t+1}\frac{A_{j,t+1}}{P_t}+\frac{M_{j,t+1}}{P_t}+C_{j,e,t}+\frac{P_t^I}{P_t}I_{j,t}\left[1+\Phi\left(\varepsilon_t^I\frac{I_{i,t}}{I_{i,t-1}}\right)\right] \tag{1}$$
$$=\frac{M_{j,t}}{P_t}+\frac{R_{t-1}B_{j,t-1}}{P_t}+\frac{A_{j,t}}{P_t}+\frac{(1-\tau_{l,t})W_{j,t}}{P_t}l_{j,t}+\frac{R_{K,t}}{P_t}K_{j,t}+\frac{\Pi_{j,t}}{P_t}+\frac{S_tR_{t-1}^*\mathcal{P}\left(F_{t-1}\right)}{P_t}F_{j,t-1}+T_t$$

where $C_{j,e,t}$ denotes consumption of the final consumption good, P_t is the consumer price index, $l_{j,t}$ is labor, $M_{j,t+1}$ is end-of-period money, $\Pi_{j,t}$ are total profits that accrue to the household by owning shares of domestic firms, $B_{j,t}$ are domestically traded government bonds issued in domestic currency and paying with certainty $R_tB_{j,t}$ units of currency in period $t+1$, $A_{j,t+1}$ denote a state-contingent bond paying one unit of domestic currency in period $t+1$, $Q_{t,t+1}$ is the price of such bond, $F_{j,t}$ denotes an internationally traded one-period nominal bond in foreign currency delivering a return of $R_t^*\mathcal{P}\left(F_t\right)F_{j,t}$ units of foreign currency in period $t+1$, $\mathcal{P}\left(F_t\right)$ is an interest rate premium that depends on the aggregate net foreign-asset position, T_t is a real government transfer to the households, S_t is the nominal exchange rate (in domestic currency per unit of foreign currency) and $\tau_{l,t}$ is a labor income tax.

The function $\Phi\left(\cdot\right)$ denotes the adjustment costs incurred for each unit of investment, where $\Phi'\left(\cdot\right)>0, \Phi''\left(\cdot\right)>0$ and $\Phi\left(\cdot\right)=\Phi'\left(\cdot\right)=0$ in steady state, ε_t^I is a shock to the investment technology.[3] Thus, $I_{j,e,t}\left[1+\Phi\left(\cdot\right)\right]$ is the amount of investment devoted to next period capital stock and $P_{I,t}$ is the price of the investment good. The household's capital stock follows the law of motion

$$K_{j,t+1}=\left(1-\delta\right)K_{j,t}+I_{j,t}, \tag{2}$$

where δ is the constant depreciation rate of capital.

Households maximize the discounted sum of expected future utilities, defined over the final consumption good, real money balances and leisure

$$\max\ E_t\sum_{t=s}^{\infty}\beta^{t-s}\varepsilon_t^C\left\{\frac{\left(C_{j,e,t}-hC_{j,e,t-1}\right)^{1-\gamma}}{1-\gamma}+\frac{1}{1-\phi}\left(\frac{M_{j,t+1}}{P_t}\right)^{1-\phi}-\zeta_t\frac{l_{j,t}^{1+\varsigma}}{1+\varsigma}\right\}, \tag{3}$$

where ε_t^C is a preference shock and ζ_t is a labor supply shock. The final consumption good $C_{j,e,t}$ is a CES aggregator of a basket of final goods, $C_{j,t}$, and of the demand of energy by domestic households, $e_{j,h,t}$. Neglecting the household index, we can write it as

$$C_{e,t}=\left[\left(1-o\right)^{\frac{1}{\xi}}C_t^{\frac{\xi-1}{\xi}}+o^{\frac{1}{\xi}}\left(e_{h,t}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}, \tag{4}$$

[3] In the numerical solution of the model, we adopt the functional form $\Phi\left(\varepsilon_t^I\frac{I_{j,t}}{I_{j,t-1}}\right)=e^{\phi\,\epsilon\left(\frac{II}{II(-1)}-1\right)}+e^{-\phi\,\epsilon\left(\frac{II}{II(-1)}-1\right)}-2$.

where o is the share of oil in the energy-loaded consumption basket and $\xi > 0$ is (minus) the elasticity of substitution. The bundle C_t is also a CES aggregator of a bundle of domestically produced goods, $C_{H,t}$, and of foreign produced final goods, $C_{F,t}$, i.e.

$$C_t = \left[n^{\frac{1}{\chi}} C_{H,t}^{\frac{\chi-1}{\chi}} + (1-n)^{\frac{1}{\chi}} C_{F,t}^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}}, \tag{5}$$

where n measures the degree of home bias in consumption and $\chi > 0$ is (minus) the elasticity of substitution. The bundles of domestic and imported consumption goods are given respectively by

$$C_{H,t} = \left\{ b^{-\frac{1}{\theta}} \int_0^b c(z)_t^{\frac{\theta-1}{\theta}} dz \right\}^{\frac{\theta}{\theta-1}}, \quad C_{F,t} = \left\{ (1-b)^{-\frac{1}{\theta}} \int_b^1 (c(z)_t^*)^{\frac{\theta-1}{\theta}} dz \right\}^{\frac{\theta}{\theta-1}}, \tag{6}$$

where $\theta > 1$ is the elasticity of substitution. Finally, investment goods I_t are produced by combining domestic and imported consumption goods in the same way as for the consumption bundle (5). It follows that the price index associated to consumption C_t and investment I_t are identical, i.e. $P_t^c = P_t^I$.

Households maximize their preferences (3) subject to the budget constraint (1), the law of motion for capital (2), the expressions for the baskets (4), (5), (6), and a transversality conditions. Our assumption that households can hold a set of state-contingent securities (traded across countries in the case of internationally complete markets or just domestically in the case of incomplete markets) ensures them against household-specific variations in labor income due the presence of wage setting rigidities. Since the marginal utility of wealth is identical for all households, the optimality conditions are also identical across households. Up to the choice of labor supply, the optimality conditions are given by the households' first-order conditions, reported in Appendix A, together with the demand functions

$$C_t = (1-o) \left(\frac{(1+\tau_{c,t}) P_t^c}{P_t} \right)^{-\xi} C_{e,t} \tag{7a}$$

$$I_t = (1-o) \left(\frac{P_t^c}{P_t} \right)^{-\xi} C_{e,t} \tag{7b}$$

$$e_{h,t} = o \left(\frac{(1+\tau_{e,t})(S_t P_t^e + \varpi)}{P_t} \right)^{-\xi} C_{e,t} \tag{7c}$$

$$C_{H,t} = n \left(\frac{P_t^H}{P_t^c} \right)^{-\chi} C_t, \quad C_{F,t} = (1-n) \left(\frac{S_t P_t^F}{P_t^c} \right)^{-\chi} C_t$$
$$c(z)_t = \frac{1}{b} \left(\frac{p(z)_t}{P_t^H} \right)^{-\theta} C_{H,t}, \quad c(z)_t^* = \frac{1}{1-b} \left(\frac{p(z)_t^*}{P_t^F} \right)^{-\theta} C_{F,t}$$

$$I_{H,t} = n \left(\frac{P_t^H}{P_t^c} \right)^{-\chi} I_t, \quad I_{F,t} = (1-n) \left(\frac{S_t P_t^F}{P_t^c} \right)^{-\chi} I_t$$
$$I(z)_t = \frac{1}{b} \left(\frac{p(z)_t}{P_t^H} \right)^{-\theta} I_{H,t}, \quad I(z)_t^* = \frac{1}{1-b} \left(\frac{p(z)_t^*}{P_t^F} \right)^{-\theta} I_{F,t}.$$

Here $\tau_{c,t}$ denotes a value-added tax on final goods, $\tau_{e,t}$ a value-added tax on final purchases of energy, ϖ an excise tax on energy, P_t^H is the producer-price index in the EA, P_t^F is the producer-price index in the US, P_t^c is the domestic *core* consumer price index, P_t^e is the price of energy (in US dollars), $p(z)_t$ is the price in EA of the differentiated good z, and $p(z)_t^*$ is the price of the differentiated good z in dollars. We assume that the law of one price holds, implying that $p(z)_t = S_t p(z)_t^*$. Notice, from (7a) and (7b), that investment purchases are not subject to VAT taxes.

The absence of arbitrage opportunities requires the existence of a discount factor $Q_{t,t+1}$ such that the price of any portfolio of financial assets with random value A_{t+1} in the following period is given by $E_t\left[Q_{t,t+1}A_{t+1}\right]$. Optimality requires that the riskless nominal interest rate solves the equation $R_t^{-1} = E_t\left[Q_{t,t+1}\right]$.

Substituting the demand functions (7a) and (7c) into the CES function (4), we obtain the expression for the consumer price index

$$P_t = \left[(1-o)\left[(1+\tau_{c,t})P_t^c\right]^{1-\xi} + o\left[(1+\tau_{e,t})\left(S_t P_t^e + \varpi\right)\right]^{1-\xi}\right]^{\frac{1}{1-\xi}}.$$

Similarly, we obtain the following expressions for the core consumer price index and for the EA and US producer-price index,

$$P_t^c = \left[n\left(P_t^H\right)^{1-\chi} + (1-n)\left(S_t P_t^F\right)^{1-\chi}\right]^{\frac{1}{1-\chi}},$$

$$P_t^H = \left[\frac{1}{b}\int_0^b \left(p_{z,t}\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}, \qquad P_t^F = \left[\frac{1}{1-b}\int_b^1 \left(p_{z,t}^*\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}.$$

Wage setting Each household supplies its labor services to a trade union. Labor services are only partially substitutable so that households have wage-setting power. Each household in each period has a probability $(1-\xi_w) \geq 0$ of re-optimizing the posted wage. When the wage is not re-optimized, the household updates the last period wage according to an index of last period inflation and steady-state inflation.

The total labor supplied by the union to firms at time s, L_s, is an aggregator of each differentiated labor type j,

$$L_s = \left[b^{-\frac{1}{\omega}}\int_0^b l_{j,s}^{\frac{\omega-1}{\omega}} dj\right]^{\frac{\omega}{\omega-1}}.$$

where $\omega > 1$ is the elasticity of substitution between labor types.

An agent setting the wage optimally at time t faces the following demand for labor at time s,

$$l_{j,s} = \frac{1}{b}\left(\frac{W_t\left(\pi^{s-t}\right)^{1-\kappa}\left(\pi_{t-1|s-1}\right)^{\kappa}}{\overline{W}_s}\right)^{-\omega} L_s \tag{12}$$

where $\bar{W}_s$ is the aggregate nominal wage, W_t is the optimal nominal wage set at time t, $\pi_{t-1|s-1} \equiv \frac{P_{s-1}}{P_{t-1}}$ is the gross cumulative inflation rate between period $t-1$ and $s-1$, π is the steady-state inflation rate and $\kappa \in (0,1)$ is the degree of indexation to past-quarter inflation relative to total indexation.

Each household chooses the optimal wage by maximizing the discounted stream of future utilities subject to the budget constraint, taking into account the probability of being able to renegotiate optimally her wage at time s, conditional on t being the last time the wage was set optimally. Denote as λ_s^c the marginal utility of income.Using the problem's first-order conditions, together with equation (12), we obtain the following condition for the optimal wage

$$(\tilde{w}_t)^{1+\omega\varsigma}\,\bar{w}_t = \frac{\omega}{(\omega-1)}\left(\frac{\pi_t}{\pi}\right)^{-\kappa(1+\omega\varsigma)}\frac{\widetilde{\mathcal{W}}_{1,t}}{\widetilde{\mathcal{W}}_{2,t}},$$

and

$$\widetilde{\mathcal{W}}_{1,t} = \gamma_w \varepsilon_t^C \zeta_t \left(\frac{L_s}{b}\right)^{\varsigma+1}\frac{\pi_t^{\omega\kappa(\varsigma+1)}}{\pi^{\omega\varsigma\kappa}} + \beta\xi_w\left[\frac{E_t\left(\pi_{t+1}\right)}{\pi}\right]^{\omega(1-\kappa)(\varsigma+1)}\left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right)^{\omega(\varsigma+1)}\widetilde{\mathcal{W}}_{1,t+1},$$

$$\widetilde{\mathcal{W}}_{2,t} = \gamma_w \lambda_t^c (1-\tau_t^l)\left(\frac{L_s}{b}\right)\frac{\pi_t^{-\kappa(1-\omega)}}{\pi^{-\kappa}} + \beta\xi_w\left[\frac{E_t\left(\pi_{t+1}\right)}{\pi}\right]^{-(1-\kappa)(1-\omega)}\left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right)^{\omega}\widetilde{\mathcal{W}}_{2,t+1},$$

where $\tilde{w}_t \equiv \frac{W_t}{\bar{W}_t}$ is the optimal wage set by the household relative to the contemporaneous aggregate nominal wage and where $\gamma_w \equiv (1-\beta\xi_w)/\pi^{\omega\kappa}$. The domestic real wage index can then be written as

$$\bar{w}_t = \left[(1-\xi_w)\tilde{w}_t^{1-\omega} + \xi_w\left(\frac{(\pi)^{1-\kappa}\left(\pi_{t-1}\right)^{\kappa}}{\pi_t}\bar{w}_{t-1}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}}.$$

In order to measure welfare, we need to express the aggregate supply of differentiated labor. Using equation (12), we obtain

$$\frac{1}{b}\int_0^b l_{j,s}^{1+\varsigma} = \left(\frac{1}{b}L_s\right)^{1+\varsigma}\widehat{w}_s^*,$$

where $\widehat{w}_s^\star$ denote a wage dispersion wedge. The wedge, derived in Appendix B, is given by

$$\widehat{w}_t^* = (1-\xi_w)\tilde{w}_t^{-\omega(1+\varsigma)} + \xi_w\left(\frac{\pi_t}{\pi^{(1-\kappa)}\pi_{t-1}^{\kappa}}\right)^{\omega(1+\varsigma)}\left(\frac{\bar{w}_t}{\bar{w}_{t-1}}\right)^{\omega(1+\varsigma)}\widehat{w}_{t-1}^*.$$

2.1.2 Premia and UIP

Combining the household's first-order condition with respect to government bonds (equation (28c) in Appendix A) with the corresponding equation derived for the US economy, given by

$$\lambda_t^{c,*} = \beta E_t \lambda_{t+1}^{c,*} R_t^* \mathcal{P}\left(F_t^*\right)\frac{P_t^*}{P_{t+1}^*}, \tag{15}$$

and equating the return of the internationally traded bond to that of the domestic risk-free bond, we obtain the uncovered interest parity (UIP) condition

$$R_t = R_t^* \mathcal{P}\left(F_t\right)\frac{E_t S_{t+1}}{S_t}.$$

2.1.3 The energy-loaded capital sector

Each oil-importing country has a competitive sector producing energy-loaded capital and a monopolistically competitive intermediate goods sector. As in Kim and Loungani (1992) and Backus and Crucini (2000), the existing capital stock must be combined with oil in order to be used in the production of differentiated goods. This is done by a competitive sector that rents capital from households, purchases energy from the oil-exporting country and assembles the two to sell a capital-energy bundle to firms. We assume an equal number of firms as households in the two oil-importing countries.

The energy-loaded capital sector solves the following problem

$$\min_{K_t, e_{p,t}} R_t^K K_t + \left(S_t P_t^e + \varpi\right) e_{p,t} \tag{16a}$$

$$\text{s.t. } K_{e,t} = \left[\varphi^{\frac{1}{\eta}} K_t^{\frac{\eta-1}{\eta}} + (1-\varphi)^{\frac{1}{\eta}} e_{p,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{16b}$$

where K_t denotes the physical capital stock, $e_{p,t}$ the demand of energy used in production, R_t^K the rental cost of capital, $\eta > 0$ is (minus) the elasticity of the demand for oil with respect to its relative price and $\varphi \in (0,1)$ determines the weight of capital in the capital aggregation function. Equation (16b) describes the technology used to produce energy-loaded capital using capital and energy as inputs.

Using the optimality conditions, we can express the demand by the representative producer of energy-loaded capital as

$$K_t = \varphi \left(\frac{R_t^K}{P_t^{Ke}}\right)^{-\eta} K_{e,t} \tag{17}$$

$$e_{p,t} = (1-\varphi) \left(\frac{S_t P_t^e + \varpi}{P_t^{Ke}}\right)^{-\eta} K_{e,t}, \tag{18}$$

where P_t^{Ke} is the Lagrangian associated with the constraint (16b). Replacing these conditions in equation (16b), we obtain the deflator of the energy-loaded capital,

$$P_t^{Ke} = \left[\varphi \left(R_t^K\right)^{1-\eta} + (1-\varphi)\left(S_t P_t^e + \varpi\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

2.1.4 Final goods sector

Firms that produce differentiated tradable goods operate in monopolistically competitive markets and set prices only at random intervals of time à la Calvo (1983). In each country there are as many final goods producers as households. In the following, we first describe the cost minimization problem of the firm and, second, the price setting problem.

Each firm produces using labor and energy-loaded capital services according to the production function

$$y_t = \epsilon_t L_t^{1-\alpha} \left(u_t K_{e,t}\right)^{\alpha}.$$

where we have omitted the firm-specific sub-index. Here u_t denotes the degree of utilization of the energy-loaded capital and ϵ_t an exogenous stationary technology shock. We follow Finn

(1995) and Leduc and Sill (2004) by assuming that capital utilization is costly in terms of energy. The total amount of energy used directly in production, $e_{u,t}$, satisfies the following relationship

$$e_{u,t} = a(u_t) K_{e,t}. \tag{19}$$

In particular, we assume that $a(u_t) \equiv \xi_e \left(\frac{u_t^\nu}{u^\nu}\right)$.

The firm chooses the capital-energy bundle, its degree of utilization and its demand for labor, L_t, by solving the following problem

$$\min_{L_t, K_{e.t}, u_t} \overline{W}_t L_t + P_t^{Ke} K_{e,t} + (S_t P_t^e + \varpi) e_{u,t} \tag{20}$$

$$\text{s.t. } y_t = \epsilon_t L_t^{1-\alpha} (u_t K_{e,t})^\alpha \tag{21}$$

$$e_{u,t} = a(u_t) K_{e,t}. \tag{22}$$

Using the first order conditions, we obtain an expression for marginal costs mc_t,

$$mc_t = \frac{1}{\epsilon_t} \frac{\overline{W}_t^{1-\alpha} \left(P_t^{Ke} + (S_t P_t^e + \varpi) a(u_t)\right)^\alpha u_t^{-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^\alpha}$$

and one for the degree of capital utilization as a function of the relative cost of energy-loaded capital and the price of energy,

$$\frac{u_t}{u} = \left(\frac{P_t^{Ke}}{(S_t P_t^e + \varpi)(\nu - 1)\xi_e}\right)^{\frac{1}{\nu}},$$

where u denotes the steady state level of utilization.

Firms producing final goods set prices only at random intervals of time. In each quarter a fraction ξ_p of firms sells goods at the price posted in the previous quarter, after updating it in part to the inflation observed in the past quarter and in part to trend inflation. The remaining firms are able to post the optimal price. Firms are owned by the domestic households. Therefore, each firm chooses the optimal price in order to maximize the expected discounted dividends accruing to households

$$\max_{p_t} E_t \sum_{s=t}^{\infty} (\xi_p)^{s-t} \bar{R}_{s,t} \left[\frac{p_t \left(\pi^{s-t}\right)^{1-\psi} \left(\pi_{t-1|s-1}\right)^\psi}{P_s} y_s - \frac{TC_s}{P_s}\right]$$

where TC denotes total costs of production and $\bar{R}_{s,t} \equiv \beta^{s-t} \frac{\lambda_s^c}{\lambda_t^c}$ is the household nominal discount factor between period s and $t \leq s$.

The solution to this problem yields a relative price

$$\bar{p}(i)_t = \mu_f \frac{\pi^\psi \tilde{\mathbb{Q}}_{1,t}}{\pi_t^\psi \tilde{\mathbb{Q}}_{2,t}}$$

where

$$\tilde{\mathbb{Q}}_{1,t} = \gamma_\psi \overline{mc}_t \left(\pi_t^\psi \bar{P}_t^H\right)^\theta C_t^W + \xi_p E_t \overline{R}_{t+1,t} \left[\frac{\pi_{t+1}}{\pi}\right]^{\theta(1-\psi)} \tilde{\mathbb{Q}}_{1,t+1},$$

$$\tilde{\mathbb{Q}}_{2,t} = \gamma_\psi \frac{\pi_t^{\psi(\theta-1)}}{\pi^{-\psi}} \left(\bar{P}_t^H\right)^\theta C_t^W + \xi_p E_t \bar{R}_{t+1,t} \left(\frac{\pi_{t+1}}{\pi}\right)^{(\theta-1)(1-\psi)} \tilde{\mathbb{Q}}_{2,t+1},$$

$\mu_f = \frac{\theta}{\theta-1}$ is the firm's mark-up and $\gamma_\psi \equiv \frac{(1-\xi_p\beta)}{\pi^{\theta\psi}}$. Here C_t^W denotes the demand for EA goods for consumption, investment and government expenditures from residents of countries EA, US and O respectively, i.e. $C_t^W = C_{H,t} + I_{H,t} + G_{H,t} + C_{H,t}^* + I_{H,t}^* + G_{H,t}^* + C_{H,t}^o$.

The producer price index in terms of the domestic CPI can be written as

$$\bar{P}_t^H \equiv \frac{P_t^H}{P_t} = \left[(1-\xi_p)\left(\bar{p}_t\right)^{1-\theta} + \xi_p \left(\frac{(\pi)^{1-\psi}\left(\pi_{t-1}\right)^\psi}{\pi_t} \bar{P}_{t-1}^H \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

2.1.5 Monetary policy

We assume that the central bank commits itself to a feedback interest rate rule. We study the performance of a simple class of linear feed-back rules, of the form

$$\begin{aligned} R_t = {} & \lambda_R R_{t-1} + (1-\lambda_R) \left[\lambda_\pi \left(\frac{\pi_t}{\pi} - 1\right) + \lambda_Y \left(\frac{Y_t}{Y} - 1\right) + \lambda_{\Delta Y} \left(\frac{Y_t}{Y_{t-1}} - 1\right) \right. \\ & \left. + \lambda_{\Delta Pe} \left(\frac{P_{e,t}}{P_{e,t-1}} - 1\right) + \lambda_{Pe} \left(\frac{P_{e,t}}{P_e} - 1\right) + 1 \right] + \varepsilon_t^R \end{aligned} \tag{23}$$

where variables without a subscript denote steady-state values. Here Y_t denotes real GDP at current prices.

2.1.6 Fiscal policy

Governments are assumed to purchase a basket of goods with the same composition as the one purchased by the households.[4] Governments' expenditures do not contribute directly to households´ welfare. In order to finance expenditures, the fiscal authorities can levy lump-sum taxes on households (T), taxes on domestic labor income (τ^l), taxes on domestic purchases of final goods (τ^c) and taxes on oil purchases. The latter taxes are of two types: a value added tax (τ^e) and an excise tax (ϖ). Furthermore, governments can issue bonds denominated in domestic currency.

The constraint of the government (in per capita terms) is therefore given by

$$\begin{aligned} B_t^G = {} & \tau_t^l \overline{W}_t L_t + \tau^c \left(P_t^H C_{H,t} + S_t P_t^F C_{F,t}\right) + \tau^e \left(S_t P_t^e + \varpi\right)\left(e_{h,t} + e_{G,t}\right) + \\ & \varpi \left(e_{u,t} + e_{h,t} + e_{G,t} + e_{p,t}\right) - P_t G_t + R_{t-1} B_{t-1}^G + P_t T. \end{aligned} \tag{24}$$

2.2 The Oil-exporting country

The O country is populated by a fraction ϑ of the world population. The representative household produces and exports oil and consumes exclusively goods imported from the US and

[4]Given the parameter values used in the calibration, this assumption amounts to a strong home bias in government spending.

the EA. Government expenditures are set to zero, so that the fiscal authority has no need to raise taxes or issue debt. The monetary authorities implement an irrevocable (one-to-one) peg of the O currency with the dollar.

Each household produces one particular type of oil in monopolistic competition with the other households. As prices are flexible and all households face the same technology, equilibrium prices and quantity will be identical across agents. We denote with the superscript o the variables of the representative agent in the O country. The problem of the household is given by

$$\max E_t \beta^{s-t} \sum_{t=s}^{\infty} \left\{ \frac{C_t^{o1-\gamma}}{1-\gamma} + \frac{1}{1-\phi} \left(\frac{M_{t+1}^o}{P_t^o} \right)^{1-\phi} - \varphi_o \frac{(l_t^o)^{\zeta_o+1}}{\zeta_o+1} \right\} \tag{25a}$$

subject to

$$\frac{F_t^o}{P_t^o} + \frac{E_t Q_{t,t+1}^o A_{t+1}^o}{S_t P_t^o} + C_t^o + \frac{M_{t+1}^o}{P_t^o} = \frac{M_t^o}{P_t^o} + \frac{R_{t-1}^* \mathcal{P}\left(F_{t-1}^o\right)}{P_t^o} F_{t-1}^o + \frac{A_t^o}{P_t^o} + \frac{P_t^e}{P_t^o} e_t^o + T_t^o$$
$$e_t^o = \nu_t^e \, (l_t^o)^{\iota} \tag{25b}$$

where e_t^o is total supply of oil, T_t^o amounts to seigniorage transfers, ν_t^e is a productivity shock, and $\iota \in (0,1)$ measures the degree of return to labor in production.[5] First-order conditions, demand functions and price indexes (reported in Appendix A) can be derived similarly to the case of the representative household of each oil-importing countries.

Since households can adjust the price of their oil product et each instant in time, and given the absence of asymmetry among households, the optimal price is given by

$$P_t^e = \mu_o P_t^o \frac{\varphi_o}{\nu_t^e \, \iota} \left(\frac{e_t^o}{\nu_t^e} \right)^{\frac{\zeta_o - \iota + 1}{\iota}},$$

where μ_o denotes the mark-up.

2.3 Market clearing conditions

The following conditions ensure clearing in the EA country for the following markets (corresponding conditions hold for the US country).

- Energy-loaded capital:

$$\int_0^b K_t^e = \int_0^b mc_t \alpha \frac{y(z)_t}{P_t^{Ke}} dz.$$

- Goods: Recall that total demand for firm i good is given by

$$y\left(z\right)_t = \left(\frac{p(z)_t}{P_t^H} \right)^{-\theta} C_t^W.$$

[5] This amounts to assuming that there is a firm-specific factor, e.g. oil wells, that is constant in the short run.

Now, aggregating over firms (in per-capita terms), we can equate aggregate supply Y_t^s to aggregate demand to obtain

$$Y_t^s = \frac{1}{b}\int_0^b y(z)_s dz = \left(\bar{P}_t^H\right)^{\theta} C_s^W \frac{1}{b}\int_0^b \left(\bar{p}(z)_t\right)^{-\theta} dz = \left(\bar{P}_t^H\right)^{\theta} C_t^W \widehat{P}_t^H, \tag{26}$$

where $\widehat{P}_t^H$ denotes a price dispersion wedge which can be derived similarly to the wage dispersion wedge and is given by

$$\widehat{P}_t^H = (1-\xi_p)\,\bar{p}_t^{-\theta} + \xi_p \left(\frac{(\pi)^{1-\psi}(\pi_{t-1})^{\psi}}{\pi}\right)^{-\theta} \widehat{P}_{t-1}^H.$$

- Labor: equating demand and supply, we get

$$L_t = \left[b^{-\frac{1}{\omega}}\int_0^b l_{j,t}^{\frac{\omega-1}{\omega}} dj\right]^{\frac{\omega}{\omega-1}} = mc_t(1-\alpha)\frac{1}{b}\int_0^b \frac{y(z)_t}{\overline{W}_t} dz.$$

- Domestic asset market: State-contingent bonds (if domestically traded) must be in zero net supply. Also, market clearing for government bonds implies that $B_t^G = -\int_0^b B_{i,t} di$.

The following conditions ensure clearing for the following markets in country O.

- Energy:

$$e_t^o = \int_0^b \left(e_{h,t} + e_{G,t} + e_{p,t} + e_{u,t}\right) + \int_b^1 \left(e_{ht}^* + e_{G,t}^* + e_{p,t}^* + e_{u,t}^*\right),$$

  where $e_{G,t}$ denotes the demand for energy of the government.

- Labor:

$$l_t^o = \left(\frac{e_t^o}{\nu_t^e}\right)^{\frac{1}{\iota}}.$$

Finally, the following condition ensures clearing for the internationally traded assets.

- Internationally traded assets: the sum of the net foreign asset position of the three countries (obtained by integrating the budget constraint of the households in each country) has to add up to zero.

3 Calibration of the model

In order to calibrate the model we solve it to a first order of approximation. We compute variances and cross correlations on the basis of simulated series after using the Hodrik-Prescott filter for quarterly data.[6]

[6] We solve and simulate the model using DYNARE. For computational convenience we then HP filter the series using a separate code. Furthermore we comupte the variance decomposition using SYMBSOLVE (available from the authors on request).

We set the stochastic properties of the shocks and the parameter values of the model in order to match three criteria: i) replicate the volatility and correlations of some relevant macroeconomic variables, such as output, consumption, labor, investment, the net trade and the terms of trade (defined as the price of imported goods over the price of exported goods); ii) reproduce the oil intensity in production and consumption observed in the data; iii) generate a contribution of the oil-price shocks to the overall variance of GDP as obtained in related empirical work (e.g., Dotsey and Reid (1992) Jiménez-Rodríguez and Sánchez (2004), de Walque et al. (2005)).

Our benchmark calibration exercise is performed using the incomplete markets version of the model. We obtain similar calibration results (not reported) under complete markets. We adopt the following procedure: first, we evaluate the model according to the three criteria reported above using available estimates of the parameter values (e.g. using the values reported in de Walque et al. (2005)); second, we marginally modify the stochastic properties of the shocks and parameter values in order to improve the match of sample moments and to replicate the observed contribution of the oil price shock to the variance of GDP and inflation.

The list of all parameter values is given in Appendix C. Here we briefly comment on some of these parameters.

3.1 Parameters and stochastic processes

3.1.1 Size and preferences

We set the size of the EA relative to the US at 75% and the size of the oil-exporting block relative to the US at 20%. The parameter measuring consumption habit for the EA and US is close to the values used in the related literature (0.7 for EA and 0.45 for US), although it is in the low range for the US.[7] The elasticity of labor supply is set at 0.4, symmetrically for US and EA. This number is lower than the values used in the real business cycle literature, but closer to the values found in models with wage stickiness. It is also similar to the posterior mode estimated by de Walque et al. (2005).[8] Finally, we set the elasticity of inter-temporal substitution in consumption at 2.5 in the EA and 2 in the US. These values are also close to those estimated by de Walque et al. (2005).

3.1.2 Oil shares

The steady-state shares of oil in consumption and production are measured as the ratio between the value of energy expenditures and the value of total consumption and nominal GDP, respectively. In the model, we treat energy as oil and natural gas. Concerning the share of energy in consumption, the total weight of gas, fuel and car fuels in the euro-area HICP amounts to 6.2

[7]For instance, Christiano et al. (2005) set it at 0.65 for the US. The posterior mode reported in de Walque et al. (2005) is between 0.72 and 0.74 for the EA (depending on the model specification) and between 0.71 and 0.72 for the US.

[8]The posterior mode reported in de Walque et al. (2005) is between 0.42 and 0.53 for the EA (depending on the model specification) and between 0.34 and 0.37 for the US.

while the same weight in the US CPI is 5.6.[9] Concerning the share of energy in production, it is between 1% and 1.5% in the EA. Moreover, energy intensity in 2003 was about 30% higher in the US than in the EA.[10] Hence, we take the share of oil in production to be below 2% in the US.

We set the elasticities of substitution in the CES aggregators of oil at the values used by Backus and Crucini (2000) and the other relevant parameters of the model to replicate the evidence reported above. The model produce the following shares.[11]

Table 1: Oil shares

	Consumption	Production
EA	6.03%	1.56%
US	6.89%	1.96%

3.1.3 Production of final goods

The share of labor in production is set to 0.64 for both the oil-importing countries. The degree of capital utilization is normalized to unity in the steady state. In order to match the share of energy in production, the cost of capital utilization in terms of energy is assumed to be very elastic to the level of utilization. The elasticity is set to 28 in the EA and to 20 in the US.

3.1.4 Price and wage setting

The elasticity of the demand for final goods is assumed to be the same in both country (implying profit margins of 20%). As for the Calvo-probability of not adjusting the price in a particular quarter, we set them to 0.8 for the EA and 0.6 for the US, as from the estimates of de Walque et al. (2005). As for wages, the mark-up over marginal disutility of labor is 1.5. The Calvo-probability of not readjusting wages is set to 0.83 for the EA and 0.73 for the US. We assume instead an equal degree of price and wage indexation in both the EA and US (0.4 for prices and 0.7 for wages).

3.1.5 Capital and investment

The adjustment cost of investment is set to 7 for EA and 5 for US, close to the estimates obtained by de Walque et al. (2005). The capital depreciation rate is 0.025.

[9]Source: ECB calculations. Notice that the total weight of oil and gas in the euro area HICP cannot be compared to the weight of the same aggregate in the US CPI, as the basket underlying the latter index includes a larger number of items. A comparable basket would produce a weight larger than 5.6 for the US.

[10]Source: ECB calculations.

[11]Part of the value of oil and gas used in consumption and production has a domestic value-added component. For example, in the US, the Energy Information Administration for the year 1999 attributes to refining and distribution costs and profits about 26% of the final price of petrol. In the EA this share would probably be somewhat smaller due to a larger share of taxes. To this extent our calibration might overstate the share of oil in final consumption.

3.1.6 Taxes and fiscal policy

The labor income tax rate and the VAT and sales taxes are taken from Coenen et al. (2005). As for the taxes on oil, we distinguish between VAT or sales taxes and excise taxes. The former are assumed to be the same as the VAT taxes on other final consumption goods.[12] Excise taxes are measured in units of domestic currency per unit of energy. Transforming these values in relative terms requires fixing a baseline price of oil. For example, if the price of oil is taken to be the 1999 price (about $17.5 per barrel) the Federal and State taxes in the US would amount to about 36% of the retail price of regular grade gasoline. At 2000 prices (about $28.4 per barrel) the tax burden is only 28%.[13] We set the excise tax at 20% of the steady-state price of oil for the US, corresponding to the case when the price of oil is at about $30 per barrel. At this prevailing price, the excise tax on oil would be at 70% for the EA.

3.1.7 International capital market and aggregate demand elasticity

The calibration of the model is not particularly sensitive to the assumption concerning the international financial market. The same set of parameters delivers satisfactory results both under complete markets and under incomplete markets.

Under the assumption of incomplete markets, our benchmark case assumes an interest rate premium on foreign liabilities such that a 1% increase in net foreign liabilities relative to steady-state GDP, ceteris paribus, increases the domestic interest rate by about 2 basis points.

A key parameter in an open economy model is the elasticity of substitution between imported goods and domestically produced goods. We set this parameter to a rather low number, 0.7 both for the US and the EA, as limited substitutability contributes to produce a positive cross-country correlation of output and a negative correlation of net-export and output.

3.1.8 Oil-production

The calibration of the oil-exporting country is key for the model to replicate the shares of oil in consumption and production of the oil-exporting countries, as well as to reproduce the volatility of the price of oil and the contribution of oil-shocks to the forecast-error variance decomposition.

We set the partial elasticity of the price of oil to its demand equal to 9. Given the small share of oil in global demand this is a relatively small number. The production function is further scaled by a productivity coefficient of one thousand and a mark-up of 10. These numbers are needed to generate a price of oil relative to the CPI that is compatible with the observed shares of oil in consumption and GDP. Their extreme value reflect the highly simplified structure of the oil-exporting country.

[12]Not all the States in the US impose sales taxes on oil derivatives. As a consequence, the sale tax on oil for the US might be overestimated.

[13]Source: Energy Information Administration (2001).

3.1.9 Monetary policy

For the calibration of our model we choose the parameters reported in Table 2 for the interest rate rule (23).

Table 2: Interest rules parameters

Country	λ_R	λ_π	$\lambda_{\Delta y}$	λ_y	λ_{Pe}	$\lambda_{\Delta Pe}$
EA	0.9	1.72	0.2	0.1	0.0	0.0
US	0.87	1.7	0.2	0.13	0.0	0.0

These values are taken from de Walque et al. (2005), although their rule differs from ours in two important respects. First, their rule implies that the interest rate responds to aggregate demand while our rule respond to domestic value added (at current prices). Second, in their rule the interest rate responds to deviations of the target variables from their value in the flexible price equilibrium, while in our rule the policy instrument responds to deviations of the target variables from their value in the non-stochastic steady state. Notice that the stochastic properties of our model remain virtually unchanged if we alternatively use the rule estimated by Christiano et al. (2006) for the euro area and the US in a closed economy setting.[14]

3.1.10 Stochastic processes

We choose to add to the model a large number of stochastic disturbances for two reasons. First, we want to base our normative analysis on a model that provides a reasonable fit with the data. Second, we aim at capturing the main channels linking oil prices to the macroeconomy but we also aim at matching the relative importance of the oil price shock relative to other relevant economic disturbances. In particular, we try to obtain a share of oil-shocks in the total forecast-error variance that is of the same magnitude of the one obtained in empirical studies.

3.2 Data and empirical fit

The sample period we consider in the calibration is 1987:1 to 2002:4. We start the sample just after the oil-price collapse of 1986 as this marked the end of the high oil-price period started 13 years earlier. We neglect most recent observations (2003:1 onwards) as they coincide with the sharp escalation in oil-prices.[15]

[14] We take the following values from Christiano et al. (2006)

Country	λ_R	λ_π	$\lambda_{\Delta y}$	λ_y	λ_{Pe}	$\lambda_{\Delta Pe}$
EA	0.88	1.63	0.0	0.18	0.0	0.0
US	0.76	1.63	0.0	0.31	0.0	0.0

[15] According to Backus and Crucini (2000), the pattern of co-movements and volatility in major macroeconomic variables during the period of high oil-price volatility (what they call the OPEC period) is considerably different from the pattern observed before and after that period. Further below we will discuss the implications for monetary policy of different degrees of oil-price volatility.

We follow the related literature (e.g. Backus and Crucini (2000)) and HP-filter the data in per capita real terms[16]. Nevertheless, some correlations are strongly affected by the filtering while for others the HP-filter looks rather inappropriate (e.g. for oil prices). The latter problem is particularly severe when periods of persistent deviations from the historical mean are included (e.g. the 1970s).

In our calibration, we have aimed at obtaining shares of the oil-shock in the total forecast-error variance of GDP that are close to those reported in the literature. The existing literature finds a rather wide range of values. For example, Dotsey and Reid (1992) argue for a 5 to 6% contribution of oil-price shocks to the (12-quarter horizon) variance of US real GNP. Jimènez-Rodrìguez and Sanchèz (2004) estimate larger values for the (12-quarter) share of variance induced by oil-price shocks, i.e. 7.5% for the EA and 10.9% for the US. de Walque et al. (2005) report smaller values in the order of 1.0% (EA) and 1.5% (US) at 10-quarter horizon. In our calibration we obtain a share of oil-price shocks in the variance of the forecast error of real GDP of 1.9% (EA) and 4.37% (US) at 12-quarter horizon.[17]

Table 3 compares a selection of simulated moments with the empirical counterparts. The standard deviations and cross-correlations of most of the macroeconomic variables described by the model are broadly in line with those observed in the data. In particular, the model is able to reproduce qualitatively the evidence of a correlation of output that is greater than that of consumption and of investment. We reproduce almost exactly the sample evidence on the cross-country output correlation while our model predicts a correlation of consumption that is too low and of investment that is too large.

As for the correlation between the real exchange rate (denoted as RER) and the consumption differential, in the data this correlation is typically small and often negative, implying a low degree of risk sharing. In our sample, the HP-filtered series only produce a very small, though positive, correlation. If we don't filter the real exchange rate the correlation remains very small, though in this case negative. Quantitatively, our model is not far from our sample evidence.

Another important stylized fact in the international macroeconomics literature is the negative correlation between net-exports relative to GDP (denoted as NX) and GDP and between the terms of trade (denoted as tot) and GDP. Backus and Crucini (2000) argue that oil price shocks help in explain this negative correlation. In their model, oil price increases lead to a fall in the relative price of the domestic goods and to lower output, while productivity shocks lead to a fall in the relative price of the domestic goods but to an increase in output. Our model is able to reproduce the sign of the correlation and is also quantitatively close to the data as for the EA. For the US the model predicts a correlation that is twice as large as that in the data.

The model fails to match the correlation of investment and net export for the US. Altogether, the model has limited ability in predicting the correlation of consumption and investment with net export.

As for the standard deviations, the major failure of the model concerns the terms of trade

[16]Data are HP-filtered using the longer sample 1970:1 to the latest available observations (2005:2 for most US series and 2003:4 for most EA series). Most of the US series are taken from the St. Louis Fed data set (FRED) and from Global Insight. Most of the EA series are taken from the data set constructed by Fagan et al. (2001). The data set used in the paper is available from the authors on request.

[17]The model-based real GDP is computed at steady-state prices. The contribution of oil-shocks to the real GDP at current prices is more than three times as large.

and inflation (too volatile in the model). The standard deviation of the (real) price of oil (denoted as Pe) is smaller than in the data both for the US and the EA. Notice that increases in the variance of the oil-price could be obtained at the cost of further increasing the volatility of the terms of trade.

The correlation between the price of oil and net export, and between the terms of trade and the real exchange rate is of the correct sign and magnitude. On the contrary, the correlation of the real exchange rate and US net export is of the wrong sign. Nevertheless, if we compute the empirical measure without filtering the data (or filtering only net export) the sign of the correlation becomes negative (and of the order of magnitude of -0.10).

Table 3: Empirical moments

	US	**model**	**EA**	**model**
Standard deviations in ppt				
GDP	1.02	1.42	0.93	1.22
C	.90	1.43	0.80	0.53
I	4.90	4.00	2.6	2.56
NX	0.37	0.47	0.46	0.32
tot	1.4	5.5	1.2	3.32
RER	7.0	1.44	7.0	1.44
π	1.2	3.1	1.05	2.18
R	1.16	1.32	0.99	0.87
Pe	17.1	11.34	19.0	11.5
x-correlations				
(GDP, NX)	-0.39	-0.71	-0.69	-0.57
(C, NX)	-0.54	-0.20	-0.75	-0.072
(I, NX)	-0.51	0.27	-0.79	-0.11
(Pe, NX)	-0.25	-0.75	-0.41	-0.49
(tot, NX)	-0.51	-0.71	-0.38	-0.45
(Pe, tot)	0.80	0.98	0.76	0.88
(RER, NX)	0.08	-0.2	-0.15	-0.14
(REX, Pe)	0.05	0.09	0.43	0.21
Cross-country correlations				
	data	**model**		
(C, C^*)	0.14	0.081		
(I, I^*)	0.17	0.39		
(GDP, GDP^*)	0.29	0.27		
(R, R^*)	0.56	0.57		
(π, π^*)	0.39	0.47		
$((C - C^*), RER)$	0.007[18]	-0.028		

[18]Notice that if measure the correlation between the consumption differential and the non-filtered RER obtain a correlation of about -0.06.

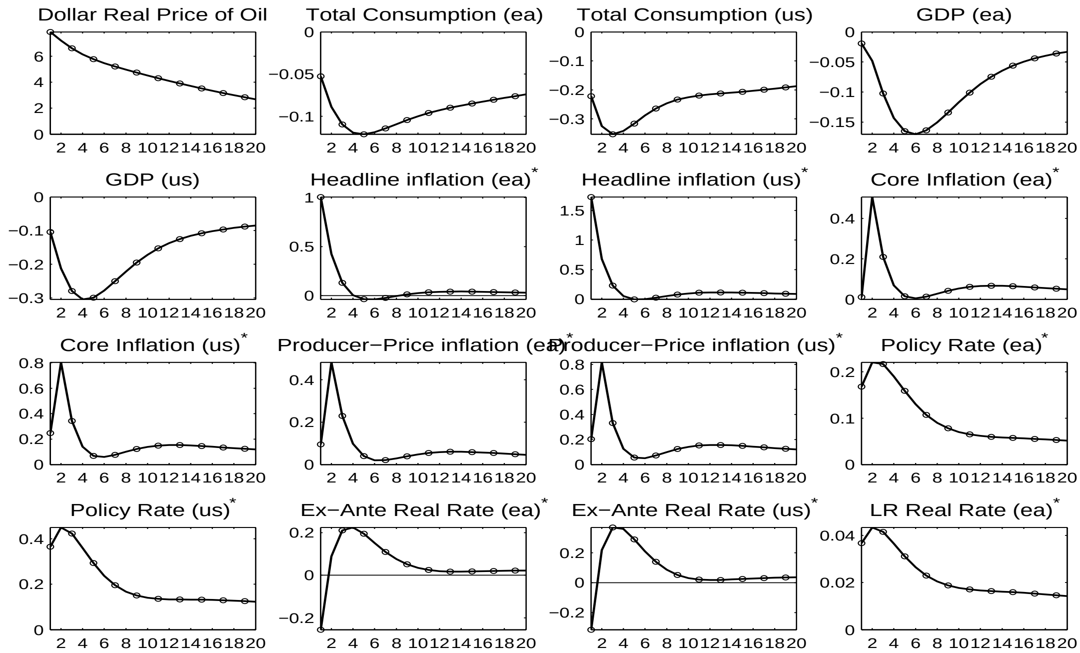

Figure 1: Response to one standard deviation innovation to the productivity of the oil sector.

4 The effect of an exogenous oil-price increase

We model the oil-price shock as a negative transitory one-standard-deviation innovation to the level of productivity of the oil-producing sector.

Figures 1 and 2 show the impulse-response diagrams of a selection of variables after the shock, which generates on impact a 7.8% increase in the real price of oil (i.e. the ratio of the nominal price of oil to the US CPI). The shock is persistent (auto-correlation coefficient of 0.95) so that the half-life of the oil-price change is about 12 quarters.

A robust finding in the VAR literature is that transitory oil-price shocks drive up inflation (CPI as well as GDP-deflator) and down real GDP.[19] Bernanke et al. (2004) using a five-variable quarterly VAR find that a 10% increase in the price of oil implies a change of GDP of about -0.7% (at the trough four quarters after the innovation).[20] Bernanke et al. (1997), using a seven-variable monthly VAR, find that a 10% increase in the price of oil produce a fall in GDP of around -0.25% 24 months after the shock. As for prices, the monthly VAR used in Bernanke et al. (1997) yields an increase in the CPI of about 0.2% two years after the oil-price shock. The quarterly VAR used in Bernanke et al. (2004) produces a milder increase (about 0.1%).

[19]See for example Bernanke, Gertler and Watson et al. (1997, 2004), Hamilton and Herrera (2004) and Jiménez-Rodríguez and Sànchez (2004).

[20]The oil-price shock is constructed as in Hamilton (1996), i.e. it is the amount by which the log oil price in month t exceeds its maximum value over the previous 12 months; if oil prices are lower than they have been at some point during the past year, no oil shock is said to have occurred.

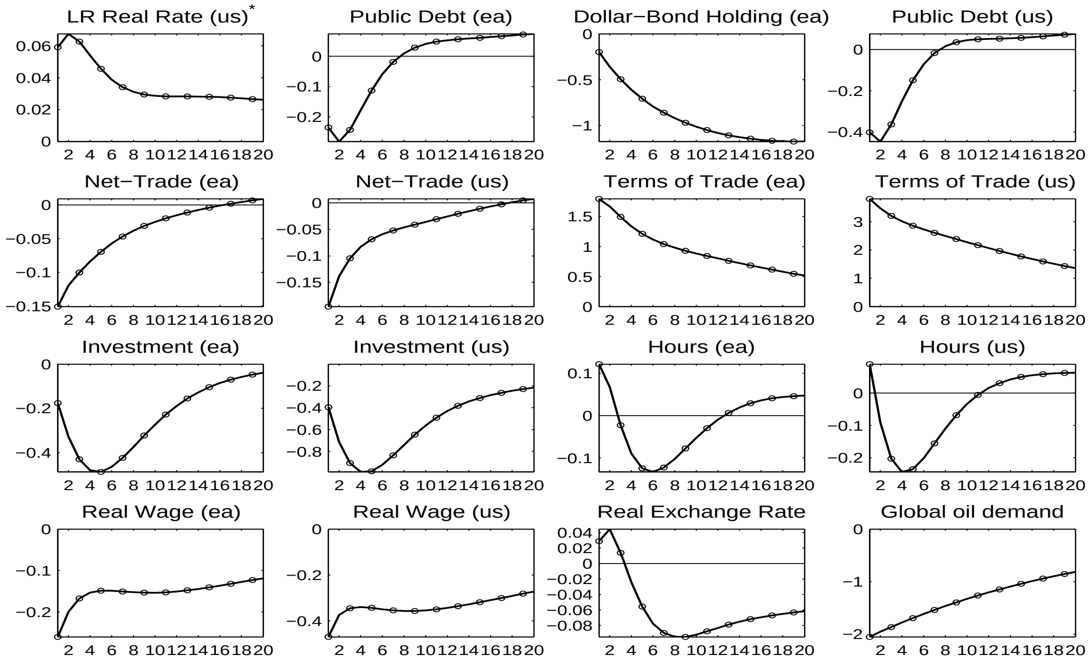

Figure 2: Response to one standard deviation innovation to the productivity of the oil sector.

In our model, the oil price shock brings about an increase in headline inflation as well as in core inflation and producer-price inflation. For the euro area, a 10% increase in the dollar real price of oil increases the CPI by about 0.47% after 2 years. For the US, this number is about 0.88%.[21] Although the oil-price shock is relatively persistent, the effect on inflation is short lived. This result is not uncommon in the related literature. Indeed, some researchers have dismissed the hypothesis that oil-price shocks caused the Great Stagflation of the 1970s on the basis of this abrupt reaction of inflation to the oil-price shock (Barsky and Kilian (2002)). Concerning the reaction of real output, this latter falls reaching the trough after about 6 (4) quarters at -0.23% (-0.38%) in the EA (US). The effect on real output falls on the lower range of values suggested by the literature. The marked hump-shape of GDP is driven by the persistence in consumption habit and by the presence of investment adjustment costs. Not surprisingly, the dynamics of consumption (Figure 1) and investment (Figure 2) is very similar to the dynamics of GDP. Hours increase initially, as firms substitute hours for costly capital utilization, then fall for several quarters and increase again before converging back to the long run steady state. The real wage falls as well, but not sufficiently to avoid the reduction in employment.

The nominal policy rate increases by about 28 (56) basis points by the second quarter in the euro area (US). Except for a small initial hump, the policy rates decrease smoothly thereafter (half life of about 7 quarters). Bernanke et al. (1997) estimate a value of about 81 bp after 6

[21] In order to compare the impulse responses of our model to the existing empirical literature, we report the values that correspond to a shock that increases by 10% in the price of oil. A rigorous comparison with the VAR literature should also take into account the various measures and definitions used in the different studies.

months for the US.

In our model, a measure of the overall monetary policy stance can be gauged by looking at the long run real rate of interest (here the implicit yield of a ten-year risk-free bond). The long run real rate increases by about 4 (6) basis points in the EA (US).

As a consequence of higher revenues from taxes on oil, public debt improves in the first year after the shock both in the EA and the US, returning to benchmark and improving after 6-7 quarters. The long-run debt-to-GDP criterion imposes that any improvement in the public finances be offset by a subsequent budget deficit. This is accomplished by an adjustment in public spending.

As for the open economy variables, the net foreign asset position, net-trade and the terms of trade (price of import over price of export) all worsen.[22]

5 Monetary policy and welfare

On the basis of our calibrated model we study the welfare properties of alternative simple interest rate rules when the economy is hitted by three shocks only: the oil-price shock, and the US and EA technology and government spending shocks.[23] We search for the rule that maximizes the households' welfare under the assumption that the central banks can commit to the announced interest rate rule.

The optimal policy analysis is carried out on a second-order approximation of the model obtained using the method described in Lombardo and Sutherland (2006). This latter produces an analytical representation of the second-order expansion of the dynamic system so that searching for the optimal parameters does not involve repeated approximations of the non-linear model.[24]

5.1 The welfare measure

Each central bank aims at choosing the interest rate rule that maximizes the households' welfare within the class of simple rules we specify below. In aggregating utility across households, we neglect the real-balance component of welfare.[25] Aggregate welfare can then be expressed as

[22]In order to gather a sense of the effects of an oil-shock in our data set, we ran a number of VARs for a selection of variables under alternative Cholesky orderings. We did these experiments both jointly for the EA and US as well as for each block separately for the period 1970q1:2005q2. The robust finding of these casual experiments are a deterioration of the terms of trade and an increase in inflation. Furthermore, we observed that the trade balance tends to worsen in the short run but improves after about one year (more for the EA). The policy rates increase and output tends to fall with a hump at between one and two years. We leave a less casual empirical investigation of these relationships to future work.

[23]For computational reasons, at this stage of our analysis we had to reduce the size of the model. We decided to reduce the number of shocks and limit ourself to the analysis under three types of shocks only. We leave to future work the extension to a larger set of shocks.

[24]We compute an analytical representation of the second-order expansion using SYMBSOLVE. The solution has been successfully tested against the solution produced by DYNARE for particular parametrizations.

[25]In the related literature we find several similar treatments of aggregate welfare (e.g. Woodford (2003)). Schmitt-Grohè and Uribe (2004), on the contrary, emphasise the monetary welfare costs as they investigate the steady-state optimal inflation target. We don't address this issue in our model.

$$\mathbb{W}_0 = E_0 \sum_{t=1}^{\infty} \beta^{t-1} \varepsilon_t^C \left\{ \frac{\left(C_{e,t} - hC_{e,t-1}\right)^{1-\gamma}}{1-\gamma} - \zeta_t \frac{\int_0^b l_{i,t}^{1+\varsigma} \mathrm{d}i}{1+\varsigma} \right\} \tag{27}$$

where the stream of welfare is evaluated conditional on the information held by the central bank at the steady state (i.e. the hypothetical situation at time 0). We evaluate this welfare measure to the second order of accuracy.

5.2 Optimality criteria

Our choice of a desirable monetary policy rule satisfies four criteria:[26]

1. The rule must be simple and operational, i.e. it must react to aggregate macroeconomic variables that are easily and timely available to the policy maker. We restrict our attention to linear feed-back rules.
2. The rule must satisfy a zero lower-bound condition. In particular, we discard all those rules that, with high probability, lead to a negative nominal interest rate. Our lower-bound condition is defined by $\log(R_0) > 2\sigma_R$.
3. Among the rules satisfying criteria 1. and 2., we choose the rule that maximizes welfare, as measured in terms of a second order approximation of equation (27).
4. The choice of policy parameters must constitute either a Nash equilibrium or a cooperative equilibrium.

The fourth criterion is dictated by the fact that the choice of policy parameters by a given central bank is influenced by the choice made by the other central bank. We assume that each central bank responds optimally to the choice made by the other central bank. Under cooperation we assume that international transfers are made between EA and US in order to compensate the country that is worse off under the chosen policies.

5.3 Results

Different assumptions concerning the international asset market have important consequences for the welfare effects of monetary policy. For instance, the existence of international trade in state-contingent assets implies that any idiosyncratic risk not offset by a sub-optimal monetary policy can be insured. Therefore, the welfare loss associated to a sub-optimal rule might be larger under incomplete markets.

We analyze two specifications of the model: the complete markets case and the case of trade limited to riskless bonds. Although complete financial markets (in the sense of Arrow-Debreu) do not provide a realistic description of international asset trade, they offer an interesting conceptual benchmark by allowing for complete risk-sharing among countries. The case of trade in riskless bonds lies at the opposite extreme. Nevertheless it allows us to trace the dynamics of the net foreign asset position easily, while allowing for some form of consumption smoothing across countries.

[26]The first three criteria are similar to those used in Schmitt-Grohé and Uribe (2005).

5.3.1 Complete markets

The search of the optimal policy parameters is computationally intensive.[27] Hence, we need to limit our search over the parameter values of the policy rules. In all cases, we set $\lambda_{\Delta y} = 0$. For the remaining parameters, we adopt the following procedure.

1. Unconstrained case: we search over a wide range of response parameters for headline inflation (λ_π) and for either the oil-price inflation or the oil-price level ($\lambda_{\Delta Pe}$ or λ_{Pe}), for three different degrees of inertia, i.e. $\lambda_R = \{0, 0.5, 0.95\}$, and for three different responses to output, i.e. $\lambda_Y = \{0, 0.5, 1.98\}$.[28]

2. Constrained case: we set the response to oil price and to oil-price inflation to zero ($\lambda_{Pe} = \lambda_{\Delta Pe} = 0$) and we search over a wide range of response parameters for headline inflation and for output (λ_π and λ_y), for three different degrees of inertia, i.e. $\lambda_R = \{0, 0.5, 0.95\}$.

The optimal parameters in the unconstrained case are reported in Table 4, while the optimal parameters under the constraint of no response to oil prices and to oil-price inflation are reported in Table 4. Finally, Table 6 reports the welfare losses under the various policy rules in units of steady-state consumption.[29] In Table 6 the flexible price case provides a way to gauge the cost of nominal rigidities under the selected optimal policy rule.[30]

[27]One single evaluation of the welfare function to the second order of approximation takes between 5 and 6 seconds on a PC with a 3GHz processor. We search over 4 policy parameters at a time, using a grid search.

[28]We start the search with rather coarse grids spanning wide ranges. We then refine the search reducing the grid steps.

[29]This measure can be thought of as the maximum price (in percentages of steady-state consumption) that a household is willing to pay in order to live in a non-stochastic world under the given policy rule.

[30]Under flexible prices and wages the central bank is assumed to maintain inflation at the steady-state level.

Table 4: Optimal parameters (unconstrained)

Country	λ_R	λ_π	$\lambda_{\Delta y}$	λ_y	λ_{Pe}	$\lambda_{\Delta Pe}$	$\frac{\lambda_{\Delta Pe}}{\lambda_\pi}$
EA	0.95	27.2	0	1.98	0	-2.4	-0.088
US	0.95	20.8	0	1.98	0	-2.6	-0.125

Table 5: Optimal parameters (constrained)

Country	λ_R	λ_π	$\lambda_{\Delta y}$	λ_y	λ_{Pe}	$\lambda_{\Delta Pe}$
EA	0.95	16.8	0	1.5	0	0
US	0.95	18.8	0	2.5	0	0

Table 6: Welfare losses

	EA	US
Optimal rule	0.128%	0.246%
Constrained Optimum	0.136%	0.252%
Benchmark rule	0.156%	0.279%
Flexible prices	0.074%	0.181%

The main results emerging from these exercises can be summarized as follows.

- The optimal linear rule shows a very marked degree of inertia. Lower degree of inertia, other things equal, lead to violations of the zero-lower bound on the nominal policy rate.

- The optimal rule calls for a negative response to the oil-price inflation. By responding negatively to energy-price inflation the central bank is able to respond more strongly to headline inflation without hitting the zero lower-bound for the nominal interest rate. As headline inflation contains an energy-price inflation component, our rule amounts to a form of core price inflation, although the weight given to oil-prices is larger than the share of oil in total households' consumption.

- Responding to deviations of the real oil-price from its steady-state value does not improve on the simple inertial Taylor rule where the policy instrument reacts only to oil-price inflation.

- The optimal responses to inflation are large. This suggests that the marginal cost of reducing inflation in our model is not particularly high relatively to the marginal benefit.

- The magnitude of the welfare losses is nonetheless limited. In particular, the optimal rule under the constraint of zero response to oil-prices (inflation or level) is about 0.008% and 0.006% worse than the optimal rule, for EA and US respectively. Using the 2003 value of US GDP per capita (at PPP), our model implies that the constrained optimum costs about \$2.45 more per person and per year than the unconstrained optimum, while the cost of the unconstrained optimum relative to the flexible price equilibrium is about \$24.05. These relatively small numbers are not unusual in the related literature.[31]

It should be noted that the optimal policy rule does not induce improvements in all the determinants of welfare. For example, while the average consumption increases, labor effort is also expected to increase. Consumption is more volatile under the optimal rule, while effort is less volatile. Finally, the volatility of inflation is lower under the optimal rule and the expected wage dispersion across labor-types is also lower.

To check whether the optimal rule is qualitatively affected by the presence of the exogenous oil-price shock, we simulate two polar cases: i) oil-price shocks are set to zero; ii) only oil-price shocks hit the economy.

Table 7 reports the parameters of the optimal rule when oil-price shocks are absent. Table 8 reports the parameters when oil-prices are the only source of stochastic volatility in the economy.[32]

Table 7: Optimal parameters (no oil-price shocks)

Country	λ_R	λ_π	$\lambda_{\Delta y}$	λ_y	λ_{Pe}	$\lambda_{\Delta Pe}$	$\frac{\lambda_{\Delta Pe}}{\lambda_\pi}$
EA	0.95	29.7	0	1.98	0	1.23	0.0414
US	0.95	27.2	0	1.98	0	2.72	0.1

Table 8: Optimal parameters (oil-price shocks only)

Country	λ_R	λ_π	$\lambda_{\Delta y}$	λ_y	λ_{Pe}	$\lambda_{\Delta Pe}$	$\frac{\lambda_{\Delta Pe}}{\lambda_\pi}$
EA	0.95	12.4	0	1.98	0	-0.99	-0.08
US	0.95	12.4	0	1.98	0	-1.73	-0.14

When the economy is not hit by exogenous oil-price shocks, the central bank should strongly respond to headline inflation but also positively react to oil-price inflation. On the other hand, when exogenous oil-price shocks are the only source of volatility in the economy, the central bank should react less aggressively to headline inflation but partially accommodate increases in oil-price inflation. The intuition is that the presence of exogenous oil-price shocks introduce a trade-off between inflation and output and thus a higher cost of reducing inflation.

Interestingly, the optimal weight to be given to oil-price inflation is very similar when all shocks are considered and when only oil-price shocks are hitting the economy (the cases reported in Table 4 and Table 8). These weights can be calculated as the ratio $\lambda_{\Delta Pe}/\lambda_\pi$. In both exercises, they are around 8-9% for the EA and 12-14% for the US. Notice that these

[31] For example, Schmitt-Grohè and Uribe (2005) in a medium scale DSGE model for the US, report a welfare cost of using a simple interest rate rule (relative to the Ramsey-optimal rule) of about \$9.1 per US citizen per year (at 2003 prices). Their simple, non-optimized rule is $R_t = 1.5\,(\pi_t - \pi)$.

[32] We have not checked for the optimal response to the oil-price level under these two polar cases.

numbers are larger than the weight of oil and gas in the HICP and in the US CPI, reflecting the role of oil in production captured in our model.

The optimal rule is designed to maximize welfare on average, i.e. to affect means and variances of the variables relevant for welfare. Nevertheless, it is of some interest to look at the adjustment path of the economy implied by the optimal rule. Figure 5.3.1 and 5.3.1 show the impulse responses produced under the optimal rule against the benchmark model. The negative coefficient on oil-price inflation implies that, on impact, the short-run nominal interest rate falls. In our model the short-run nominal rate has important implications only to the extent that it affects the revenue from holding financial assets.[33] Therefore, the government partially benefits from the fall in short term real rates. Consumption and investment (to a first order of approximation) are affected by the short-run rates only to the extent that these bring about variations in the long-run real rate of interest.[34] Furthermore, consumption habit and adjustment costs of investment imply that the long-run rate of interest determines the rate of change of consumption and investment rather than their level: an increase in the long run rate implies a fall in the growth rate of consumption and (for a given long run productivity of capital) a fall in investment. Overall, to a first order of approximation the optimal rule does not bring about a clear pattern in the dynamics of consumption and investment. The same can be said for output and hours.

More evident is the effect that the optimal rule exerts on the dynamics of inflation: except on impact, all measure of inflation approach the long-run equilibrium more quickly.

[33]To a first order of approximation this reduces to the interest paid on government debt. To a second order of approximation (not relevant for the impulse responses) short term rates are relevant only for their covariance with the net foreign asset positions.

[34]In the last panel of Figure 5.3.1 the real return on the implicit 10-year bond slightly decreases for the EA. The longer the maturity of the assets the larger (more positive) is the reaction of their return.

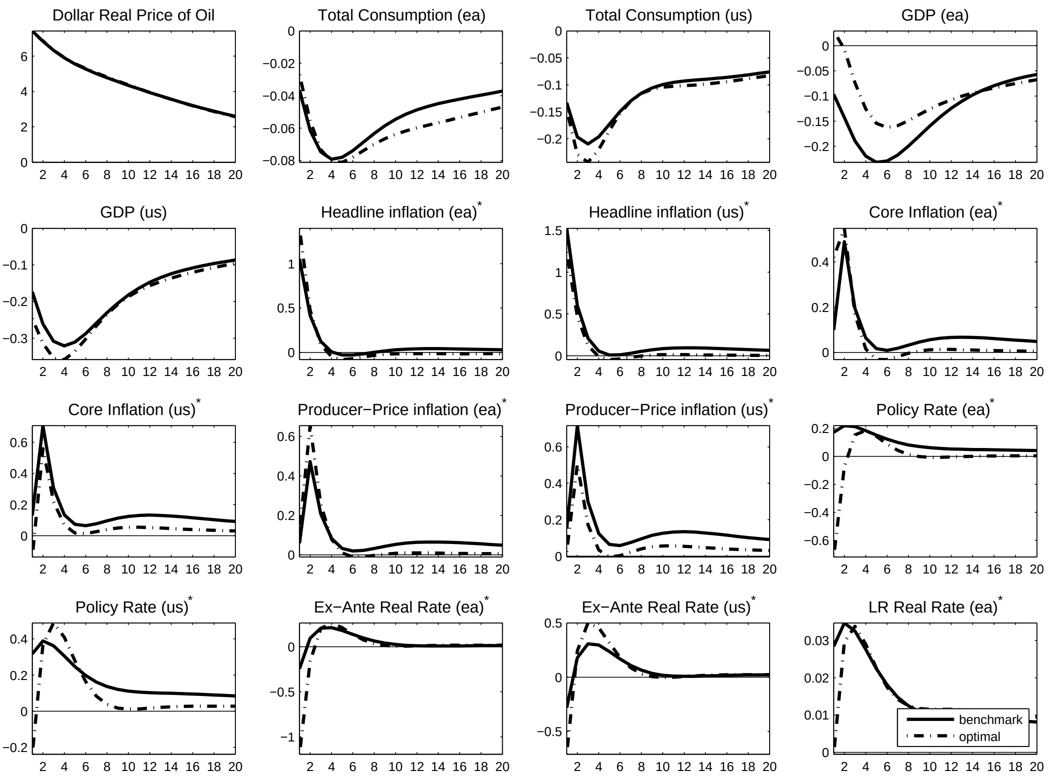

Figure 3: Optimal rule vs. benchmark under complete markets

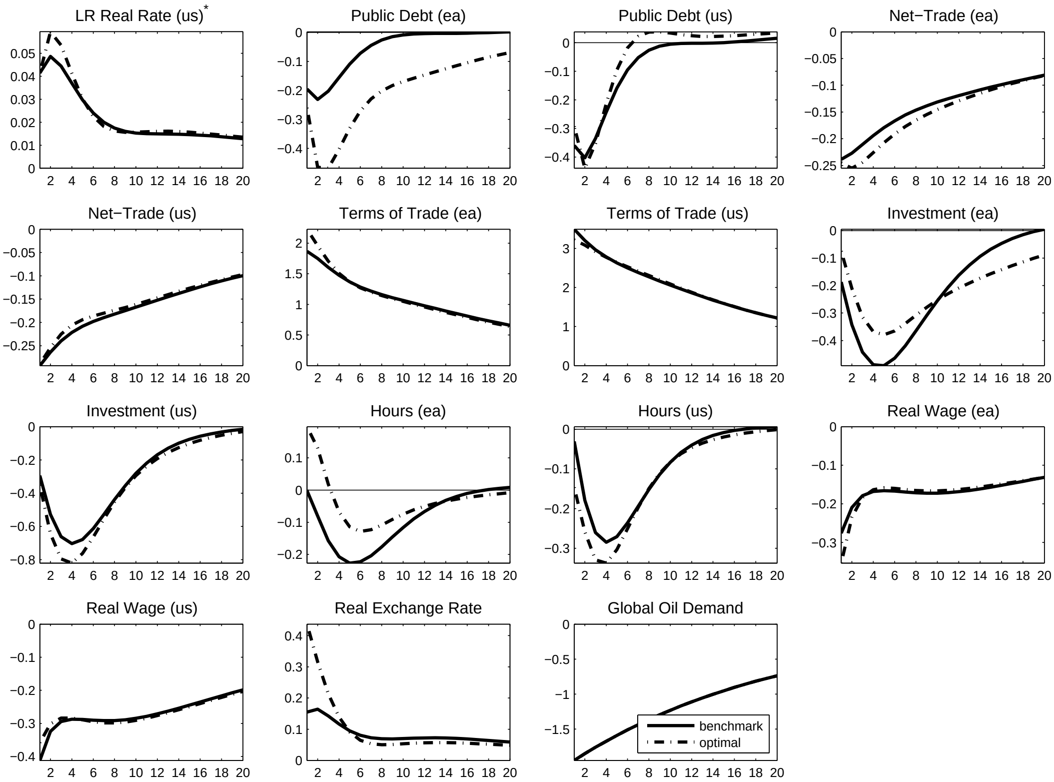

Figure 4: Optimal rule vs. benchmark under complete markets

5.3.2 Incomplete markets

[TO BE COMPLETED]

6 Conclusion

We have built a three-country open-economy model with an explicit role for energy in consumption and production. The calibrated model is able to replicate the volatility and correlations of most macroeconomic variables and to generate a contribution of the oil-price shocks to the overall variance of GDP close to the one obtained in related empirical work.

We have used the model to analyze the welfare properties of alternative simple interest rate rules under two polar cases concerning the international financial market: complete markets and markets with trade limited to risk-free bonds. In the normative part of our analysis we have focused on three type of transitory shocks: an exogenous oil-price shocks, technology shocks and government spending shocks.

Under the assumption of complete markets, we find that the optimal interest rate rule is inertial, reacts strongly and positively to headline inflation and to output deviations from the non-stochastic steady-state level, while it reacts negatively to oil-price inflation. We also find that when the economy is not hit by exogenous oil-price shocks, the central bank should respond even more strongly to headline inflation but should also react positively to oil-price inflation. The intuition for the different results is that the presence of exogenous oil-price shocks introduces a trade-off between inflation and output, thus imposing a higher cost of reducing inflation. As headline inflation contains an energy-price inflation component, our optimal simple rule in the presence of exogenous price shocks amounts to a form of core price inflation. Nonetheless, the weight given to oil-prices is larger than the share of oil in total households' consumption, reflecting the role of oil in production arising in our model.

Under the assumption of incomplete markets, [to be completed ...]

The model can be extended in a number of directions. First, it would be interesting to introduce a distribution sector that imports crude oil, refines it and distribute it to the domestic market. This would allow to disentangle the domestic and foreign component of the oil used in consumption and production, with implications for the effects of oil price changes on the consumer price index and on aggregate output of the oil-importing countries. Moreover, the introduction of a distribution sector would amplify the expectational channel operating in our model. In the current setup, expectations of future oil-price shocks have negligible effects on the current price of oil. However, a distribution sector would react to the news by purchasing crude oil and by building up inventories, to take advantage of the profit opportunities linked to the higher future expected oil-price. This would lead to a larger impact on the current price of oil.

The second extension relates to the fiscal side of the model. Large asymmetries in the level of energy taxes among importing-countries have important implications for the effects of an oil-price shock on inflation and output. The model provides a useful tool to design optimal simple fiscal rules or to study alternative fiscal scenarios, such as the reduction of energy taxes to a symmetric level in US and EA or a temporary reduction of energy taxes during periods of oil-price turbulence.

Appendix

A. Household problems

In the EA the household's first-order conditions are given by

$$\varepsilon_t^C \left(C_{e,t} - hC_{e,t-1}\right)^{-\gamma} = \varepsilon_{C,t+1} h\beta \left(C_{e,t+1} - hC_{e,t}\right)^{-\gamma}) + \lambda_t^c \tag{28a}$$

$$\lambda_t^c = R_t \beta E_t \lambda_{t+1}^c \frac{P_t}{P_{t+1}} \tag{28b}$$

$$S_t \lambda_t^c = \beta E_t S_{t+1} \lambda_{t+1}^c R_t^* \mathcal{P}\left(F_t\right) \frac{P_t}{P_{t+1}} \tag{28c}$$

$$\varepsilon_t^C \left(\frac{M_{t+1}}{P_t}\right)^{-\phi} = \lambda_t^c - \beta \lambda_{t+1}^c \frac{P_t}{P_{t+1}} \tag{28d}$$

$$\lambda_t^I = \beta E_t \left[\lambda_{t+1}^c \frac{R_{t+1}^K}{P_{t+1}} + \lambda_{t+1}^I (1-\delta)\right] \tag{28e}$$

$$\lambda_t^c \frac{P_{c,t}}{P_t}\left[1 + \Phi(\varepsilon_{I,t}\frac{I_t}{I_{t-1}}) + \Phi'(\varepsilon_{I,t}\frac{I_t}{I_{t-1}})\varepsilon_{I,t}\frac{I_t}{I_{t-1}}\right] = \beta E_t \lambda_{t+1}^c \frac{P_{c,t+1}}{P_{t+1}} \Phi'(\varepsilon_{I,t+1}\frac{I_{t+1}}{I_t})\varepsilon_{I,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 + \lambda_t^I, \tag{28f}$$

where λ_t^c and λ_t^I are the Lagrangian multipliers associated to the budget constraint (1) and to the capital accumulation equation (2), respectively. Similar conditions can be derived for US.

In the O country, the first order conditions of the household optimization problem are:

$$l_t^{1+\zeta_o-\iota} = \frac{1}{\varphi_o} \iota \lambda_t^{c,o} \frac{P_t^{e,o}}{P_t^o}$$
$$\left(C_t^o\right)^{-\gamma} = \lambda_t^{c,o}$$

and the demand functions for the components of the consumption basket are given by

$$C_{,t}^o = \overline{n}\left(\frac{P^H}{S_t P_t^o}\right)^{-\chi} C_t^o, \quad C_{*,t}^o = (1-\overline{n})\left(\frac{P_t^F}{P_t^o}\right)^{-\chi} C_t^o$$
$$c(z)_{,t}^o = \frac{1}{b}\left(\frac{p(z)_t}{P_t^H}\right)^{-\theta} C_{,t}^o, \quad c(z)_{*,t}^o = \frac{1}{1-b}\left(\frac{p(z)_t^*}{P_t^F}\right)^{-\theta} C_{*,t}^o.$$

B. The wage dispersion wedge

In order to measure welfare, we need to aggregate differentiated labor supplies. Recall that an agent j faces at time t a labor demand that depends on the time j when the wage setting decision was taken,

$$l_{j,t} = \frac{1}{b}\left(\frac{\tilde{w}_j \bar{w}_j \pi_j}{\bar{w}_t \pi_t}\right)^{-\omega} L_t$$

We define a cohort z of agents who have set the wage optimally at the same time z. Clearly, agents belonging to the same cohort face the same demand for labor. Aggregating across agents, we get

$$\frac{1}{b}\int_0^b l_{j,s}^{1+\varsigma} dj = \left(\frac{1}{\bar{w}_t}\right)^{-\omega(1+\varsigma)} \left(\frac{1}{b}L_t\right)^{1+\varsigma} \frac{1}{b}\int_0^b \left(\frac{\tilde{w}_j \bar{w}_j}{\bar{w}_t}\frac{\pi_j}{\pi_t}\right)^{-\omega(1+\varsigma)} dj$$
$$= \left(\frac{1}{\bar{w}_t}\right)^{-\omega(1+\varsigma)} \left(\frac{1}{b}L_t\right)^{1+\varsigma} w_t^{\star}$$

where

$$w_t^{\star} = \sum_{z=0}^{t} \left(\bar{w}_{t-z}\tilde{w}_{t-z}\frac{\pi_{t-z}}{\pi_t}\right)^{-\omega(1+\varsigma)}$$
$$= \left[\begin{array}{c} (1-\xi_w)\left(\bar{w}_t\tilde{w}_t\right)^{-\omega(1+\varsigma)} + \xi_w (1-\xi_w)\left(\bar{w}_{t-1}\tilde{w}_{t-1}\frac{\pi_{t-1}}{\pi_t}\right)^{-\omega(1+\varsigma)} \\ +\xi_w^2 (1-\xi_w)\left(\tilde{w}_{t-2}\bar{w}_{t-2}\frac{\pi_{t-1}\pi_{t-2}}{\pi_t\pi_{t-1}}\right)^{-\omega(1+\varsigma)} + ... \end{array}\right].$$

Define $\widehat{w}_{t-z,t} = \bar{w}_{t-z}\tilde{w}_{t-z}\frac{\pi_{t-z}}{\pi_t}$. Then,

$$w_t^{\star} = (1-\xi_w)\sum_{z=0}^{t} \xi_w^{t-z}\widehat{w}_{t-z,t}^{-\omega(1+\varsigma)}$$
$$= (1-\xi_w)\,\widehat{w}_{t,t}^{-\omega(1+\varsigma)} + (1-\xi_w)\left(\frac{\pi_{t-1}}{\pi_t}\right)\sum_{z=1}^{t} \xi_w^{t-z}\widehat{w}_{t-z,t}^{-\omega(1+\varsigma)}$$
$$= (1-\xi_w)\,\widehat{w}_{t,t}^{-\omega(1+\varsigma)} + (1-\xi_w)\left(\frac{\pi_{t-1}}{\pi_t}\right) w_{t-1}^{\star}$$

where the last term derives from the fact that starting the summation one period earlier would entail indexing one period more.

Notice that the steady state value of $\left(\frac{1}{\bar{w}_s}\right)^{-\omega(1+\varsigma)}$ could be very large and this could lead to accuracy problems. We find it convenient to define $\widehat{w}_t^{\star} \equiv \frac{w_t^{\star}}{(\bar{w}_t)^{-\omega(1+\varsigma)}}$. It follows that

$$\widehat{w}_t^{\star} = (1-\xi_w)\left(\tilde{w}_t\right)^{-\omega(1+\varsigma)} + \xi_w\left(\frac{\pi_{t-1}}{\pi_t}\frac{\bar{w}_{t-1}}{\bar{w}_t}\right)^{-\omega(1+\varsigma)} \widehat{w}_{t-1}^{\star}$$

and hence

$$\frac{1}{b}\int_0^b l_{j,t}^{1+\varsigma} = \left(\frac{1}{b}L_t\right)^{1+\varsigma} \widehat{w}_t^{\star}.$$

C. Parameter values

Stochastic processes

Autocorrelation coefficient	**EA**	**US**	**O**
Oil-price shock			0.95
Government spending shock (EA)	0.90	0.90	
Investment shock (EA)	0.90	0.90	
Labour supply shock (EA)	0.80	0.80	
Monetary policy shock	0.00	0.00	
Preference shock	0.84	0.74	
Productivity shock	0.90	0.90	
Standard Deviation	**EA**	**US**	**O**
Government spending shock	0.014	0.022	
Innovation to investment shock	0.008	0.0198	
Labour supply shock	0.06	0.07	
Oil-price innovation			0.2
Innovation to consumers' preferences	0.018	0.036	
Monetary policy innovation	0.002	0.002	
Productivity innovation	0.0053	0.0053	

Structural parameters

Parameter	EA	US	O
Calvo probability of not-resetting prices	0.8	0.6	
Calvo probability of not-resetting wages	0.83	0.76	
Constant in cost of utilization function	0.001167	0.003254	
Consumption bias in country O for US goods			0.57
Correlation of US and EA productivity shocks	0.044	0.044	
Debt-to-GDP criterion for public debt (quarterly)	2.84	1.92	
Degree of habit persistence	0.7	0.45	0.2
Indexation of prices to past inflation	0.4	0.4	
Indexation of wages to past inflation	0.7	0.7	
Degree of risk aversion	2.5	2	4
Degree of substitution capital-oil (firms)	0.09	0.09	
Degree of substitution domestic goods-imports	0.7	0.7	
Degree of substitution goods-oil for households	0.09	0.09	
Depreciation rate of capital	0.025	0.025	
Elasticity of interest rate premium w.r. to external debt	0.01	0.01	0.01
Investment adjustment cost parameter	1.322876	1.118034	
Elasticity of utilization-cost w.r. to utilization	28	20	
Excise tax on oil	0.7	0.2	
Gross inflation rate (annual)	1.02	1.02	
Home bias	0.9	0.85	
Inverse of elasticity of labour supply	2.5	2.5	10
Labour tax	0.24	10	
Mark-up in oil-producing sector			10
Mark-up of final-goods producers	1.2	1.2	
Mark-up of wage setting households	1.5	1.5	
Share of oil in energy-loaded capital aggregator	0.0010	0.0013	
Productivity scaling factor in production function of oil			1000
Reaction of public spending to deviation from debt criterion	0.1	0.1	
Share of capital in production	0.36	0.36	
Share of labour in production of oil			0.1
Share of oil in consumption aggregator	0.045	0.085	
Size of EA relative to US	0.43		
Size of oil-exporting country relative to total	0.11		
Subjective discount factor	0.99	0.99	0.99
VAT on final goods and oil	0.183	0.077	
Weight of labour in preferences	3	3	0.001

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