The Causal Effect of Mortgage Refinancing on Interest-Rate Volatility: Empirical Evidence and Theoretical Implications

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Abstract

This paper investigates the effects of mortgage-backed securities (MBS) hedging activity on interest-rate volatility and proposes a model that takes these effects into account. An empirical examination suggests that the inclusion of information about MBS considerably improves model performance in pricing interest-rate options and in forecasting future interest-rate volatility. The empirical results are consistent with the hypothesis that MBS hedging affects both the interest-rate volatility implied by options and the actual interest-rate volatility. The results also indicate that the inclusion of information about the MBS universe may result in models that better describe the price of fixed-income securities.

JEL Codes: G12, G13, G14, G21

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The effect of MBS-hedging activity on the volatility of interest rates has been a topic of strong interest among practitioners and policy makers in the last few years. (See for instance Greenspan, 2005a.) The large size of the MBS market combined with record home ownership levels imply that a better understanding if there is a relationship between MBS-hedging activity and interest-rate volatility may have deep and broad consequences.

At least three different theories explain the possible relationship between MBS-hedging activity and interest-rate volatility. The first theory is based on the hypothesis that the fixed-income market is perfect and complete without MBS, and implies that there is no relationship between MBS-hedging activity and interest-rate volatility. The second theory asserts that the dynamic hedging activity of MBS hedgers on the swap and Treasury markets increases the volatility of interest rates. The third theory assumes that interest-rate option markets are imperfect and that the surge in demand for interest-rate options in a refinancing wave should therefore increase the volatility implied by interest-rate options such as swaptions[1]. This paper empirically analyzes these three theories.

The first theory, which we will call the "classic theory," is based on traditional MBS-pricing models. These models assume that mortgage-backed securities are derivatives of the Treasury term structure. (See for instance Schwartz and Torous, 1989.) In these models, as in the Black and Scholes model, the activity of derivative hedgers does not have any effect on the prices of the underlying asset or its derivatives. These models suppose that Treasury markets are frictionless and complete. As a result, the hedging of MBS investors does not have any effect on the price of other fixed-income securities.

The second theory, which we will call the "actual volatility effect," is based on the effect that MBS dynamic hedging is held to have on the Treasury or swap markets. Suppose, for example, that a mortgage investor holds a portfolio of MBS and hedges the portfolio duration risk completely with a short position in Treasury bonds. If interest rates drop, the MBS duration decreases due to a higher probability of refinancing. As a result, the investor will have a portfolio with negative duration. To adjust its duration back to zero, the investor must buy Treasury bonds. If, on the other hand, interest rates rally, the

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[1] Swaptions are options to enter into a plain-vanilla fixed versus floating swap at a certain future date and at a certain fixed rate. For instance, a payer in a three into seven at-the-money swaption will have the right (not the obligation) to be the fixed payer in a seven-year swap, three years after the issuance of the swaption. Here, the time-to-maturity of the swaption is three years and the tenor of the swaption is seven years. The swaption is at-the-money and hence the agreed upon swap rate is the relevant forward swap rate at the swaption creation.

mortgage duration increases and the MBS investor must short additional Treasury bonds in order to adjust the duration of the portfolio. Notice that provided that bond prices are affected by flows in the Treasury market, the MBS-hedging flows (buying bonds when bond prices are going up and selling bonds when bond prices are going down) will have the effect of reinforcing both the initial movement of bond prices and their volatility.

The actual volatility effect is similar to that described in the portfolio insurance literature. Analogously to MBS hedgers, portfolio insurers following a dynamic replication strategy will sell stocks when stock prices go down and buy stocks when prices go up. The portfolio insurance literature describes this hedging activity and provides theoretical models in an incomplete market setting wherein the portfolio insurers' hedging increases the volatility of stock prices[2]. In these models, the demand for the underlying security is downward-sloped and the underlying security prices are therefore affected by the flows generated by portfolio insurers.

The third theory is based on the effect of the static hedging activity of MBS investors on the interest-rate options market, which we are calling herein the "implied volatility effect." MBS investors buy portfolios of loans with embedded call options that allow homeowners to prepay. A MBS investor may therefore statically hedge the prepayment options with over-the-counter interest-rate options such as swaptions. Due to the hedging activity of MBS investors, intense mortgage refinancing activity results in a surge in the demand for at-the-money interest-rate options. That is, when interest rates drop, homeowners exercise their deep-in-the-money prepayment options and take new mortgages with new at-the-money prepayment options. These new mortgages are hedged by MBS investors with new at-the-money interest-rate options. As a result, if the supply of options is not perfectly elastic, a surge in the demand for options caused by an increase in mortgage refinancing will increase the implied volatility of swaptions.

The implied volatility effect is similar to that described in the limits to arbitrage literature in the stock options market. The implied volatility effect is analogous to the relationship between shocks in the demand for S&P 500 options and their implied volatility. In both cases, market imperfections coupled with increases in the demand for options result in increases in the options' implied volatility. That is, market imperfections pre-

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[2]See for instance Grossman (1988), Gennotte and Leland (1990) and Brunnermeier (2001).

clude option market makers to hedge perfectly, and thus options market makers charge higher prices for carrying larger imbalanced inventories of options. As a result, the supply of options is not perfectly elastic and implied volatility increases with rightward shocks to options demand[3].

Note that these three theories have distinct implications. The implied volatility effect states that increases in mortgage refinancing should not affect the actual volatility of interest rates, but it should affect the swaptions' implied volatility because of the surge in demand for swaptions during a refinancing wave. The actual volatility effect implies that increases in mortgage refinancing should increase both actual and implied interest-rate volatility, because increases in refinancing activity make the duration of mortgages more sensitive to interest changes and MBS dynamic hedging flows are therefore larger during periods of high refinancing activity. The classic theory implies that hedging activity does not have any effect on the volatility of the underlying securities and refinancing should therefore not have any effect on the volatility of interest rates.

To differentiate between the classic theory and the other two effects, a vector autoregressive (VAR) system is estimated. The results of the VAR indicate that increases in refinancing activity forecasts increases in interest-rate volatility even after controlling for the level and slope of the term structure. The results are in agreement with the results in Perli and Sack (2003), even though their econometric framework is different from the one used herein. The results of the VAR are evidence against the classic theory.

To differentiate between actual and implied volatility effects, this paper proposes and calibrates a term-structure model that incorporates information about MBS prepayments. This paper is the first to propose and empirically examine a term-structure model that incorporates mortgage prepayment information. The proposed term-structure model with mortgage refinancing effects is called the MRE model and it is an extension of the Longstaff, Santa-Clara and Schwartz (2001) model or LSS model.

The MRE model is a non-arbitrage model based on empirical relationships justified with the presence of limits to arbitrage. The MRE model is a reduced-form model in the sense that it abstracts from the possible causes for the relationship between interest-rate

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[3]See Froot and O'Connell (1999), Bollen and Whaley (2004) and Gârleanu, Pedersen and Poteshman (2005).

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volatility and mortgage refinancing and takes this relationship as a given. The MRE model therefore does not explain the reasons for the possible relationship between refinancing and interest-rate volatility. The MRE model, however, is flexible to price fixed-income derivatives, including swaptions with different tenors and times-to-maturity. The flexibility of the MRE model makes it a useful tool with which to analyze how, or whether mortgage refinancing affects the prices of interest-rate derivatives with different maturities and payoffs, thereby ultimately providing a deeper understanding of the effects of mortgage refinancing on interest-rate derivatives.

To differentiate between actual and implied volatility effects, the MRE model is used to forecast future actual interest-rate volatility. If refinancing affects only the implied volatility of swaptions and not the actual volatility of interest rates, the inclusion of mortgage effects in a swaption pricing model will improve the model's ability to fit swaption prices, but not the model's ability to forecast the future actual volatility of interest rates. If, on the other hand, refinancing equally affects both the actual and the implied volatility, then the implied volatility calculated by the model with refinancing effects should be an unbiased forecast of the actual future volatility of interest rates. The empirical analysis of the MRE model indicates that the inclusion of refinancing effects on the swaption pricing model improves the model's ability to forecast future interest-rate volatility, implying that mortgage refinancing affects the actual volatility of interest rates. The volatilities implied by the MRE model, however, are not unbiased forecasts of the actual interest-rate volatility. Consequently, the implied volatility effect cannot be completely discarded.

The remainder of this paper is organized as follows: Section 1 describes different types of MBS and MBS investors. Section 2 describes the data used in this paper. Section 3 presents a VAR examination of the empirical relationship between the implied volatility of short-term swaptions, the yield curve, and mortgage refinancing. Section 4 presents all of the calibrated term-structure models. Section 5 presents in-sample and out-of-sample comparisons of the calibrated models. Section 6 concludes.

1 Types of MBS and MBS investors

The residential MBS may be divided between agency and non-agency MBS. The agency sector consists of MBS created through the securitization of residential mortgages by

government-sponsored enterprises (GSEs) such as Fannie Mae and Freddie Mac and the agency Ginnie Mae. The majority of the securitized residential mortgages in the US are securitized into agency MBS. Indeed, Table 1 displays data from Inside Mortgage Finance (2004) on the amount of outstanding agency and non-agency MBS since 1994. Table 1 shows that since 1994, more than 80% of all securitized residential mortgages in the US are securitized into agency MBS.

The main risks of the agency MBS are interest-rate risk (duration risk) and prepayment risk. Credit risk is usually not an issue in agency MBS because, in exchange for a guarantee fee, GSE itself guarantees that the cash flow payments will be made. In addition, mortgages are over collateralized loans and the mortgages securitized by Ginnie-Mae have the full credit guaranty of the United States government. Prepayment risk, on the other hand, is considerable in MBS because residential mortgages allow borrowers to prepay their mortgages, thereby creating uncertainty regarding the timing of the cash flows of mortgage-backed securities.

The prepayment risk is different for different types of MBS. MBS may be divided in two types regarding the distribution of cash flows to investors. The first type is a passthrough, which is a mortgage-backed security that passes all of the interest and principal cash flows of a pool of mortgages (after servicing and guarantee fees) to investors. Table 1 shows that 70% of the total outstanding amount of agency MBS is composed of passthroughs. The prepayment risk of a passthrough is the same as the prepayment risk of the underlying pool of mortgages. The second type of MBS is a collateralized mortgage obligation (CMO), the cash flows of which are derived from passthroughs and are distributed to different investors according to pre-specified rules. Because different CMOs have different cash flow distribution rules, they are subject to differing prepayment risks and hence there are CMOs with smaller exposure to prepayment risk than that of passthroughs. CMOs, however, do not change the total prepayment risk of the pool of mortgages underlying the CMO classes. See Fabozzi and Modigliani (1992) on this point.

The prepayment options embedded in passthroughs generate the negative convexity of these securities. Indeed, a passthrough price is usually a concave function of the level of interest rates. Since borrowers can refinance their mortgages when interest rates drop, the upside potential of a passthrough is limited. The price of the passthrough therefore

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gets closer to a constant when interest rates drop, creating the negative convexity of this security[4]. Because of its negative convexity, the duration risk of a passthrough is dynamically hedged by buying bonds when bond prices rally and selling bonds when bond prices drop, or analogously, by receiving fixed rate in interest-rate swaps when swap rates drop and paying fixed rate in interest-rate swaps when swap rates rally.

To understand the hedging flows generated by a MBS investor, assume that an investor gets a long position on a passthrough with notional amount n_{MBS} and hedges the duration risk with $n_{Tsy,0}$ Treasury notes. Take the yield of the Treasury note as proxy for the interest-rate level and assume that the initial yield is y_0. Hence $n_{Tsy,0}$ is chosen to make the derivative of the portfolio price with respect to the Treasury yield equal to zero at y_0 (or the initial duration of the portfolio equal to zero). Suppose that the yield of the note instantaneously moves from y_0 to y_1, and consequently the hedge needs to be readjusted to drive the duration of the portfolio back to zero. That is, the MBS investor has to trade in the Treasury notes in order to rebalance its portfolio. The amount of the Treasury note necessary to readjust the duration of the portfolio is given by the following expression derived in the Appendix:

$$n_{Tsy,1} - n_{Tsy,0} \approx -\frac{[n_{MBS} \times P''_{MBS}(y_0) + n_{Tsy,0} \times P''_{Tsy}(y_0)]}{P'_{Tsy}(y_1)} \times (y_1 - y_0) \qquad (1)$$

In Equation 1, $n_{Tsy,1} - n_{Tsy,0}$ is the notional amount that needs to be traded on the notes to readjust the duration of the portfolio to zero. The prices of the passthrough and of the Treasury note are P_{MBS} and P_{Tsy} respectively. Because $P''_{MBS}(y_0)$ is usually negative, the term between brackets in the formula above is normally negative, which implies that the hedging flows have the opposite sign of the change in rates. Therefore when the Treasury yield goes up, $(y_1 - y_0)$ is positive and $n_{Tsy,1} - n_{Tsy,0}$ is negative, which implies that the duration is adjusted by short selling additional notes. On the other hand, when the Treasury yield goes down, $(y_1 - y_0)$ is negative and $n_{Tsy,1} - n_{Tsy,0}$ is positive and thus the duration is adjusted by buying notes. Also observe that even if the duration target of the hedged portfolio were not zero, the size of the hedging flows would be given

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[4]If the coupon of a passthrough is much smaller than the current interest rate then the passthrough price can be a convex function of the level of interest rates. For plots of passthrough prices as functions of the level of interest rates, see Boudoukh, Whitelaw, Richardson, and Stanton (1997) and page 329 of Sundaresan (2002).

by Equation 1. (See Appendix for proof.) Consequently, as long as the convexity of the hedged portfolio is negative, the hedging flows on the Treasury notes are to buy notes when the note price goes up and sell notes when the note price goes down.

Recall that the actual volatility effect is the increase in interest-rate volatility due to the dynamic hedging activity of MBS investors on the Treasury or swap markets. Equation 1 clarifies that the actual volatility effect is based on the assumption that the convexity of the marginal mortgage hedger portfolio is negative. To verify this assumption, it would be necessary to have information about the convexity of the marginal hedger portfolio, which is not available. The universe of MBS however has negative convexity and hence, as long as the marginal hedger portfolio is a representative piece of the MBS universe, it is likely that the marginal hedger portfolio has negative convexity. For example, in a daily sample of 16,757 Bloomberg option-adjusted convexities of Ginnie Mae passthroughs with coupons between 5% and 9.5% from November 1996 to February 2005, around 96% of the option-adjusted convexities are negative.

Naturally, the negative convexity of the MBS universe is not sufficient to establish a link between interest-rate volatility and the MBS-hedging flows. In fact, if the MBS-hedging flows of MBS are small in relation to the liquidity provision on the hedging instrument market, then it would be unlikely that any channel between MBS-hedging activity and interest-rate volatility would exist. In order to infer the possible relative size of the MBS-related hedging flows, Table 1 displays data on the amount of interest-bearing marketable Treasury securities outstanding. The data on the amount of Treasury securities outstanding are from various issues of the Federal Reserve Bulletin. Note that the amount of MBS outstanding is quite large. For instance, between 1994 and 1997, the amount of MBS outstanding was close to the amount of Treasury notes outstanding, while between 2000 and 2003, the amount of MBS outstanding was larger than that of marketable Treasury securities. Table 1 also displays estimates from Inside Mortgage Finance (2004) of the holdings of MBS by three types of investors that are commonly assumed to be hedgers: Hedge Funds, MBS dealers and the GSEs.

The growth and size of the GSEs portfolios are impressive. The GSEs hold more than 15% of the total amount of MBS since 1998. GSEs are required to manage their interest-rate exposure and do so by issuing debt and using a series of fixed-income products such as

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Treasury securities, swaps and swaptions. Indeed, as an attempt to understand the impact of the dealers' concentration on the over-the-counter interest-rate options markets, staff of the Federal Reserve System conducted interviews with seven leading bank and non-bank over-the-counter derivative dealers during the summer of 2004. (See Federal Reserve, 2005 and Greenspan, 2005b.) The dealers indicated that "Fannie Mae and Freddie Mac together account for more than half of options demand when measured in terms of the sensitivity of the instruments to changes in interest rate volatility (rather than notional amounts)". Naturally, the GSEs' MBS portfolios were smaller in 1994, indicating that the MBS-hedging demand from the GSEs was not as large in the mid-nineties.

The estimates displayed in Table 1 indicate that MBS dealers had around 5.8% of the outstanding MBS in 1994 and, opposed to the GSEs, the portfolios of MBS dealers have decreased between 1994 and 2003. Dealers typically manage the duration of their portfolio and they are among the set of investors whose hedging activity may drive interest-rate volatility. Fernald, Keane, and Mosser (1994) estimates that the size of the dealers' inventory of passthroughs and CMOs was more than $50 billion in the 1993-1994 period, while the size of the new five- to ten-year Treasury supplies, for example, was around $45 billion a quarter during 1993. As such, Fernald, Keane, and Mosser (1994) argues that the size of the MBS dealers' hedging demand was large enough to possibly influence some of the term-structure movements in the 1993-1994 period.

Hedge funds are another class of MBS investors which usually dynamically hedge their portfolios. Hedge funds' fixed-income strategies have been described in Lowenstein (2000) and in Duarte, Longstaff and Yu (2005). These strategies usually involve the use of dynamic hedging. Hedge funds' MBS holding composed up to 8.9% of the MBS universe in 1994. Note however that the estimate of hedge funds MBS holdings also include other types of investors whose MBS holdings are not available, and hence the estimates for hedge fund holdings in Table 1 are the upper-bounds for actual holdings, which could in fact be much smaller. Perold (1999), however, indicates that the well-known hedge fund Long-Term Capital Management (LTCM) alone had positions of up to $20 billion dollars in market value of passthroughs and CMOs between 1994 and 1997, which suggests that the participation of hedge funds in the MBS market was not trivial in the mid-nineties.

In the same way that the relative importance of the hedge funds, MBS dealers and

the GSEs on the MBS market changed between 1994 and 2003, the hedge instruments also changed. For instance, Fernald, Keane, and Mosser (1994) indicate that MBS dealers mostly likely used on-the-run Treasury notes for duration hedge in 1993-1994. Moreover, Goodman and Ho (1998) indicate that the GSEs started relying more on swap based products in their hedging activity around 1997, while prior to 1997 the GSEs appear to have relied more on their own callable debt and Treasuries as hedging instruments. The switch from Treasury-based to swap-based hedging could also be driven by the change in benchmark in the fixed-income market. Fleming (2000), for example, indicates that due to a decrease in the supply of Treasuries and the flight-to-quality at the end of 1998, fixed-income hedgers started relying more on swaps in order to hedge their portfolio duration. Consequently, it appears that the hedging instrument of the marginal MBS hedger switched from Treasury to swaps during the sample period.

MBS hedgers such as hedge funds and MBS dealers invest in CMOs as well as in passthroughs and CMOs account for around 30% of the outstanding MBS. Consequently, it is important to understand whether CMOs have an impact on the total hedging flow generated by MBS hedgers. Unfortunately, it is not clear whether CMOs would increase or decrease the total hedging activity of MBS investors. On the one hand, it is possible that CMOs decrease the total amount of hedging because they allow a multitude of duration exposures appropriate for many different types of investors; on the other, it might also be the case that CMOs increase the total amount of MBS-hedging activity because the creation of a CMO with stable duration comes at the expense of creating another CMO with unstable duration.

To understand how the creation of CMOs might increase the total amount of MBS-hedging activity, assume that two CMO classes (CMO_1 and CMO_2) are backed by the cash flows of a passthrough. In this case, the sum of the second derivatives of the CMO prices with respect to interest-rate level satisfy the equation:

$$n_{Passthrough}P''_{Passthrough} = n_{CMO_1}P''_{CMO_1} + n_{CMO_2}P''_{CMO_2} \qquad (2)$$

Assume that CMO_1 resembles a non-callable bond with slightly positive convexity. In this case, Equation 2 and the usual negative convexity of passthroughs implies that CMO_2

is highly negatively convex. Assume that CMO_1 is bought by an investor that does not dynamically hedge, for example a small commercial bank, while CMO_2 is bought by an investor that normally dynamically hedges, for instance, a hedge fund[5]. If these assumptions hold true, the creation of the CMOs could increase hedging activity because the dynamic hedge of CMO_2 may have to be adjusted more often than the underlying passthrough.

In addition to investors that normally hedge, such as MBS dealers, the GSEs and hedge funds, the use of hedging by institutions in the mortgage related business such as mortgage originators and servicers is also substantial. Federal Reserve (2005) points out that over-the-counter interest-rate derivative dealers indicate that mortgage servicers[6] are the second most important source of demand for over-the-counter interest-rate options. A mortgage servicer performs the administrative tasks of servicing the pool of mortgages in exchange for a fee, which is a fixed percentage of the outstanding balance of the mortgage pool and hence servicing rights are subject to prepayment risk. See Goodman and Ho (2004) for a description of the hedging activity of mortgage servicers and originators.

In summary, the possibility of a link between MBS hedging and interest-rate volatility from 1994 to 2003 cannot be dismissed based on the relative holdings of MBS investors and on the existence of CMOs. As a result, the relationship between MBS-hedging activity and interest-rate volatility has to be studied by means of indirect evidence–that is by studying the relationship between proxies of MBS-hedging activity and interest-rate volatility. Ideally, any study trying to establish a link between interest-rate volatility and MBS hedging should be based on a time series of the trading activity of MBS hedgers. Unfortunately, this kind of data is not available. As a consequence in order to investigate the relationship between MBS-hedging activity and interest-rate volatility, this paper assumes that the refinancing activity of the mortgage universe is a proxy for both the negative convexity of the marginal mortgage hedger portfolio (dynamic hedging in the actual volatility effect) and the demand for swaptions during period of high refinancing activity (static hedging in the implied volatility effect). This paper then analyzes the relationship between interest-rate volatility and refinancing activity.

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[5]In this example, CMO_2 is the so called "toxic waste". Gabaix, Krishnamurthy and Vigneron (2004) note that usually the success of CMOs creation depends on finding investors willing to buy the "toxic waste" piece. Investors with expertise in dynamic hedging, such as hedge funds are natural buyers of the "toxic waste" piece.

[6]Large commercial banks are examples of servicers. Inside Mortgage Finance (2004) indicates that four of the five largest mortgage servicers were among the largest commercial banks in US in 2004.

2 Description of data

In the remainder of this paper, six kinds of data are used: Libor+swap term-structure data; constant maturity Treasury yields (CMT) data; swaption implied volatilities data; data on the outstanding amounts, prepayment speeds and weighted-average coupons of Ginnie Mae, Fannie Mae as well as Freddie Mac mortgage pools; the rate on 30-year-fixed-rate mortgages; and data on the Mortgage Bankers Association (MBA) refinancing index. The MBA refinancing index is from Bloomberg (ticker: MBASAREFI Index). The data on the mortgage pools are also from Bloomberg. The Libor+swap rates, the swaption volatilities and the mortgage rates are from Lehman Brothers. The CMT data are from the Federal Reserve Board.

The CMT data are daily from April 8, 1994 to August 29, 2003. The CMT rates have two, three, four, five, seven and ten years to maturity. There are 2,351 observations for each maturity. The rate on 30-year-fixed-rate mortgage is used as a proxy for the current mortgage rate (MR_t). The mortgage rate data are weekly (Friday) from January 31, 1992 to August 29, 2003, which is a total of 605 observations.

The Libor rates are the six-month and one-year Libor. The swap rates are the plain vanilla fixed versus floating swap rates with two, three, four, five, seven and ten years to maturity. The Libor/swap rates are the daily closing from July 24, 1987 to August 29, 2003. There are 4,153 observations for each maturity. These rates are used to estimate the zero-coupon, continuously-compounded yields with a procedure similar to the one used by Longstaff, Santa-Clara and Schwartz (2001) and Driessen, Klaassen and Melenberg (2003). As in Longstaff, Santa-Clara and Schwartz (2001), the one-year and the six-month discount rates are directly estimated from the six-month and one-year Libor rates. As in Driessen, Klaassen and Melenberg (2003), the discount rates for maturities between one and a half and ten years are estimated by assuming that the price of a zero-coupon bond with maturity T at time t is $\exp(\sum_{i=1}^{3} \omega_{i,t}(T-t) + \sum_{j=1}^{2} \theta_{j,t} \max(0, (T-t-2 \times j)))$, where the parameters $\omega_{i,t}$, $\theta_{j,t}$ are estimated by least squares from the swap rates observed at time t.

By market convention, the swaption prices are displayed as volatilities of the Black (1976) model, and the dollar prices of the swaptions are calculated by Black's formula.

The swaption data are composed of a time series of 40 at-the-money swaption volatilities with time-to-maturity and tenor given by: three and six months, one, two and three years into one, two, three, four, five and seven years (30 swaptions); and four and five years into one, two, three, four and five years (ten swaptions). The data used for the swaptions with time-to-maturity equal to three months are the weekly Friday closing from April 8, 1994 to August 29, 2003, a total of 491 observations. The data used for the other swaptions are monthly (taken on the last Friday of each month) from January 31, 1997 to August 29, 2003, which is a total of 80 observations.

The data on the generic mortgage pools are from Bloomberg (CPH pages). The mortgage pools are composed by 30-year-fixed-rate mortgages securitized by Ginnie Mae, Fannie Mae and Freddie Mac. Ginnie Mae and Freddie Mac pools data are on two types of pools: Ginnie I (Bloomberg ticker: GNSF), Ginnie II (Bloomberg ticker: G2SF), Freddie Mac Gold (ticker: FGLMC) and Freddie Mac Non-Gold (ticker: FHLMC). The pools selected have coupons between 4% and 15%, equally spaced by 0.5%. The pools with coupons ending in 0.25% or 0.75% were not selected because they have much smaller outstanding amounts. The available pools from Ginnie I have coupons between 4.5% and 15%, the Ginnie II pools have coupons between 4% and 14%, the Freddie Mac Non-Gold pools have coupons between 5.5% and 15%, the Freddie Mac Gold pools have coupons between 4% and 13% and the Fannie Mae pools have coupons between 4% and 15%. The data are monthly from December 1, 1996 to August 1, 2003 with a total of 8,342 observations. The sum of the total outstanding amount of the available pools is on average 95% of the agency passthrough outstanding amount in Table 1, indicating that the selected pools indeed represent a significant part of the mortgage universe. Each monthly observation on the mortgage pools is composed by the mortgage ticker; the coupon; the total outstanding amount at the beginning of the month; the weighted-average coupon[7]; and the prepayment speed observed in the previous month.

The prepayment speed of a mortgage pool is usually measured by its single monthly mortality rate (SMM) or by its constant prepayment rate (CPR). If a mortgage pool has total balance MB_{t-1} at the end of the month $t-1$, and its scheduled principal payment

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[7]The weighted-average coupon of a pool is different from the coupon paid to investors due to servicer and guarantee-enhancement fees. The difference is usually around 50 basis points.

at month t is SP_t, then the total amount prepaid at month t is $SMM_t \times (MB_{t-1} - SP_t)$. The CPR is an annual prepayment rate and is given by:

$$CPR = 1 - (1 - SMM)^{12} \tag{3}$$

The generic pools data are used to calculated monthly proxies for the mortgage universe weighted-average coupon (WAC) and prepayment speed (CPR). The WAC of the mortgage universe at the beginning of each month is calculated by taking the averages of the weighted-average coupons of the agency pools weighted by their outstanding amount. Analogously, the prepayment speed of the mortgage universe during each month is calculated by taking the averages of the $CPRs$ of each agency pool weighted by their outstanding amount. The WAC and the CPR database has a total of 81 monthly observations from December 1, 1996 to August 1, 2003.

The Mortgage Bankers Association (MBA) refinancing index is used as a weekly measure of refinancing activity. The MBA refinancing index is based on the number of applications to refinance existing mortgages received during one week. The index is published every Friday as part of the MBA weekly mortgage application survey, which generates a comprehensive overview of the activity in the mortgage markets. In 2004, the MBA survey covered around 50% of all retail U.S. mortgage applications (see Mortgage Bankers Association, 2004). The MBA index is a broad measure of refinancing activity based on applications for all kinds of residential mortgages, not only on the applications for the mortgages that are securitized into agency MBS. The index used in this paper is seasonally adjusted. The MBA index is available as of January 5, 1990 and its value was one hundred on March 16, 1990. The period used herein is from April 8, 1994 to August 29, 2003 (491 observations). Figure 1 displays the time series of the MBA refinancing index. An examination of Figure 1 reveals that the refinancing index time series is characterized by many spikes between 1994 and 2003. These spikes are refinancing waves: that is, periods of high refinancing activity caused by a decrease in the mortgage rate to a level substantially below the average coupon of the mortgage universe.

Both the MBA index and the weighted-average CPR of the agency pools are proxies of refinancing activity of the entire mortgage universe. The weighted-average CPR is a

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measure of prepayments based on agency pools. The MBA index, on the other hand, is a measure of refinancing activity based on the entire mortgage universe. These two measures therefore differ because prepayments may be caused by a range of factors other than refinancing such as homeowners' mobility and homeowners' default and because the MBA index considers all the mortgage universe while the weighted-average CPR is a measure based only on agency MBS. However, the MBA index and the weighted-average CPR should be highly correlated because refinancing is by far the single most important cause of prepayments and the agency MBS compose a large part of the securitized mortgage universe. To show the properties of these two proxies of refinancing activity, Figure 2a displays the time series of the weighted-average CPR and of the monthly average of the MBA index. Note that changes in the MBA index anticipate changes in the weighted-average CPR. The time lag between these series is unsurprising due to the fact that there is a delay between the application for mortgage refinancing and the actual prepayment of a mortgage. See, for instance, Richard and Roll (1989) for further details on this delay. As Figure 2a suggests, the correlation between the weighted-average CPR in one month and the average MBA index in the previous month is quite high at 0.92. In addition, the correlation between the changes in the CPR in one month and the changes in average MBA index in the previous month is also high at 0.72.

3   A VAR analysis of mortgage refinancing and implied volatility

Figure 1 shows that periods of high refinancing activity are characterized by relatively high interest-rate volatility, clearly indicating a positive correlation between interest-rate volatility (VOL) and refinancing activity. The questions that arise are whether increases in VOL are causing increases in refinancing or vice-versa and whether the relationship between interest-rate levels and volatility can account for the relationship between VOL and refinancing activity. After all, it is well known that refinancing is caused by interest-rate decreases and hence a researcher interested in explaining VOL could potentially model a simple decreasing relationship between interest-rate levels and volatility without having to worry about mortgage refinancing. To address these questions, a VAR analysis is performed.

The estimated VAR system provides an analysis of the relative importance of refinancing in explaining interest-rate volatility after controlling for the level and slope of the term structure. The VAR system is clearly misspecified since there is no linear mapping among the variables in the VAR system. The VAR system nevertheless is a simple way to study the relationship between refinancing and interest-rate volatility. (See Duffie and Singleton, 1997 for an example of a similar VAR exercise.)

The variables in the VAR are the first differences of the MBA refinancing index divided by 10,000 (MBAREFI); the six-month Libor rate (LIBOR6); the difference between the five-year zero-coupon rate and the six-month Libor (SLOPE); and the average Black's volatility of the swaptions with three months to maturity (VOL). The division of the MBA refinancing index is done for scaling purposes and is innocuous. Because all the variables in this system are very close to non-stationary, the VAR is estimated on first differences. The refinancing index is the proxy used for the level of mortgage refinancing. The six-month Libor is a proxy for the level of interest rates. The difference between the five-year zero-coupon rate and the six-month Libor is a proxy for the slope of the term structure. LIBOR6 and SLOPE are included in the VAR to control for the effect of term-structure movements on swaption volatilities. The average volatility of three-month swaptions is a proxy for the current level of interest-rate volatility.

As previously mentioned, it is likely that in the mid-nineties the hedging activity of MBS investors was performed with Treasuries, whereas from approximately 1998 until the end of the sample period, swaps and swaptions became the likely hedging instruments of the largest MBS hedgers. This change in hedging instrument could potentially represent a problem for the choice of variables in the VAR, since the proxies for interest level, term-structure slope and interest-rate volatility are Libor/swap based, and swaps likely became the principal MBS-hedging instrument only around 1998. As a consequence, the swap based proxies may not be appropriate for the early part of the sample. On the other hand, Treasury-based variables are not appropriate for the latest part of the sample.

The use of changes in Libor/swap rates and swaption volatilities in the VAR is justifiable, however, because of the very high correlation between changes in Treasury yields and changes in swap rates. Table 2 displays estimates of the correlation between daily changes in swap rates and daily changes in CMT yields for different periods. The corre-

lation estimated between April 1994 and December 1998 is in fact very close to one. The correlation between the daily squared-changes (a proxy for volatility) is also very high in this period. In contrast, note that after 1998, the correlation between these changes decreases slightly. The high correlations in Table 2 indicate that changes in swap rates and in swaption volatilities are good proxies for the changes in rates and volatilities of Treasury notes, which were the likely hedging instrument in the early sample period.

The VAR is fitted with seven lags. The number of lags is chosen by sequential likelihood ratio tests at the 5% significance level. Formally, let $y_t = [\text{MBAREFI}_t \ \text{LIBOR6}_t \ \text{SLOPE}_t \ \text{VOL}_t]'$ and $\Delta y_{t+1} = y_{t+1} - y_t$ be the weekly change on y. The estimated VAR is:

$$\Delta y_t = \mu + \sum_{i=1}^{7} C_i \times \Delta y_{t-i} + \varepsilon_t \tag{4}$$

The adjusted $R^2 s$ of the OLS regressions in this VAR are 22.1%, 5.8%, 7.3%, and 12.7% respectively. The VAR is estimated with weekly data from April 8, 1994 to August 29, 2003 with 483 observations in the OLS regressions. Standard errors are estimated with standard maximum likelihood estimation.

To evaluate the importance of the variables in the VAR in explaining subsequent changes in VOL, Wald tests are performed. The results of these tests suggest that changes in the SLOPE and in MBAREFI do have significant power in forecasting changes in VOL. Changes on the level of interest rates however, do not have any power to predict changes in VOL at the usual significance levels. The Wald test statistics for the exclusion of all the lags of the explanatory variables in the VAR system are displayed in the first panel of Table 3. The p-values in the first panel of Table 3 indicate that at usual significance levels, MBAREFI Granger causes interest-rate volatility.

A variance decomposition of the changes in VOL in the VAR system is also performed. The first panel of Table 4 displays the relative amount of the variance of the error due to an impulse in the explanatory variable from forecasting changes in VOL n weeks ahead. The results of the variance decomposition reveal that shocks in refinancing activity explain approximately 2% of the error in forecasting changes in VOL in the short term and approximately 9% in the long term.

In order to better understand the direction of the effect of shocks on MBAREFI, LI-

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BOR6 and SLOPE onto VOL, impulse response functions are displayed in Figures 3a to d. These response functions represent the effect on the variable VOL of a positive and orthogonalized shock on a variable of magnitude equal to the standard deviation of its own residual. The dotted lines represent two standard deviations around the mean-estimated response. The functions are plotted with a time horizon of 51 weeks. The standard deviations of the impulse response functions and of the variance decomposition are estimated with 10,000 Monte Carlo runs which are based on the MLE asymptotic distribution of the estimated parameters. The variance decomposition and the impulse response depend on the order of the variables in the system (see Hamilton, 1994). If MBAREFI is made the third variable in the system instead of the first, there is no qualitative difference in the results of the impulse response or in the variance decomposition.

The impulse response function shows that an increase in mortgage refinancing in the VAR significantly increases VOL only for a few weeks, after which the effects die out. The length of the effect might be a consequence of the time lag between an application for a mortgage and the time at which it is securitized. As previously described, the MBA refinancing index measures the number of applications for mortgage refinancing and there are several weeks between the time of the mortgage application and the time of the mortgage origination and another few weeks from the mortgage origination to the mortgage securitization. Furthermore, a mortgage application may not result in a mortgage origination for several reasons such as credit concerns.

The impulse response functions also show that the effect of shocks on SLOPE and LIBOR6 into VOL are consistent with the hypothesis that refinancing activity causes VOL. An increase in the long-term interest rates caused by an increase in LIBOR6 or by an increase in SLOPE decreases both mortgage refinancing activity and the average short-term swaption volatility, VOL. This is consistent with the directions of the impulse responses in Figures 3a to d.

The results of the VAR displayed in the first panel of Tables 3 and 4 and in Figure 3 are consistent with the actual and the implied volatility effects. There are however a series of possible alternative explanations that may prevent us from arriving at this conclusion: first, it is possible that the unusually strong refinancing activity between 2001 and 2003 is driving the results of the VAR. (See also Chang, McManus and Ramagopal, 2005 on this

point.) To address this possibility, the same VAR is also estimated using data through December 2000. The results are qualitatively similar to those displayed in the first panel of Tables 3 and 4 and in Figure 3, and they are in the second panel of Tables 3 and 4 and in Figure 4. Second, the Granger causality test could simply be picking up the dependence of the refinancing decision on the subsequently realized changes in interest-rate volatility. If homeowners use expected future interest-rate volatility in their refinancing decision, MBAREFI could then potentially forecast VOL due to the dependency of the refinancing decision on the expected volatility of interest rates. Note however that if homeowners were in fact optimally using the expected volatility in their refinancing decisions, higher MBAREFI would then be associated with smaller future VOL[8], which is the opposite of the result displayed in the impulse response functions. In addition, it is possible that homeowners do not optimally refinance, in which case the dependence of the refinancing decision on VOL in the VAR is not a concern. Whether homeowners optimally exercise their prepayment options is a subject of debate in the prepayment literature. For instance, Stanton (1995) provides empirical evidence showing that homeowners do not act optimally in their refinancing decisions. Moreover, a series of prepayment models abstract from the assumption of optimal prepayment behavior.

In order to better understand the direction of the effect of shocks on LIBOR6, SLOPE and VOL onto MBAREFI, impulse response functions are displayed in Figures 3e to h and in Figures 4e to h. These response functions represent the effect on the variable MBAREFI of a positive and orthogonalized shock on a variable of magnitude equal to the standard deviation of its own residual. The impulse response functions show that the effect of shocks on SLOPE and LIBOR6 into MBAREFI are consistent with the standard prediction that increases in long-term rates decrease refinancing activity. The impulse response of VOL onto MBAREFI, on the other hand, does not agree with options pricing theory, since increases in VOL seem to be related to subsequent increases in refinancing. In addition, the Granger causality tests in Table 3 indicate that VOL forecasts refinancing activity, hence the effects of VOL in refinancing are not only opposite to those predicted by standard options theory, but are also significant. One possible way to explain these results is that swaption market participants anticipate increases in refinancing activity

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[8] See Giliberto and Thibodeau (1989) and Richard and Roll (1989).

and update the volatility implied by swaptions based on the assumption that refinancing activity increases interest-rate volatility.

In conclusion, the results in this VAR are consistent with actual and implied volatility effects. Nevertheless, as previously mentioned, the VAR is misspecified and the interpretation of the results as evidence that MBS hedging affects interest-rate volatility relies on the assumptions that: First, changes in swap rates and swaption volatilities are proxies for the changes in the hedging instrument rate and volatility during the whole sample period; and second, the MBA refinancing index is a proxy for both the negative convexity of the marginal mortgage hedger portfolio and the demand for swaptions during periods of high refinancing activity.

4   A string model with mortgage refinancing effects

This section implements a string model that takes into account the effect of mortgage refinancing on the implied volatilities of the swaptions. This model allows us to examine how important mortgage effects are in fitting the cross-section of swaption prices (the implied volatility effect) and in forecasting the future actual volatility of interest rates (the actual volatility effect).

A total of three models are calibrated: the Longstaff, Santa-Clara and Schwartz (2001) model (LSS); an extension of the LSS model in which the volatility of the term-structure factors are affected by the yield of the five-year zero bond (the CEV model); and a model with mortgage refinancing effects (the MRE model). The LSS and CEV models are used as benchmarks for models without refinancing effects.

All models are calibrated to end-of-month swaption prices which are taken on the last Friday of each month. The swaptions used have time-to-maturity longer than three months. The data are from January 1997 to August 2003. The beginning calibration date, swaptions tenors and times-to-maturity are based on those in Longstaff, Santa-Clara and Schwartz (2001). For each calibration day, the models' free parameters are set to those which minimize the sum of the 34 relative errors between the model implied swaption prices and the market swaption prices. Swaptions are evaluated with Monte Carlo simulations in all calibrated models. A total of 2000 simulation paths are used to evaluate the swaptions. The Monte Carlo simulations use an antithetic control variate

and an Euler discretization scheme with time interval equal to one month. All calibrations use the same set of generated Brownian motion paths.

4.1 The LSS model

The LSS model is a string term-structure model. (For a detailed description of this model, see Longstaff, Santa-Clara and Schwartz, 2001.) The fundamental variables in this model are the forward rates out to ten years. These rates are represented by $F_i = F(t, T_i, T_i + 1/2), T_i = i/2$ years, and $i = 1, 2, ..., 19$. The forward rate F_i follows a diffusion under the risk-neutral measure represented by the SDE, $dF_i = \alpha_i F_i dt + \sigma_i F_i dZ_i$, where α_i and σ_i are constant and Z_i, $i = 1$ to 19 are possibly correlated Brownian motions. The instantaneous covariance of the changes in the forward rates (dF_i/F_i) is a 19×19 positive definite matrix represented by $\Sigma = U\Psi U'$, where Ψ is a 19×19 diagonal matrix with diagonal given by $[0, ...0, \lambda_N, ..., \lambda_2, \lambda_1]'$. The $\lambda's$ are non-negative constants and they are the variances of the N factors affecting term-structure movements. The matrix U is the eigenvector matrix of the correlation matrix of the log changes in the forward rates.

The matrix U is estimated with weekly term-structure observations from July 24, 1987 to January 17, 1997. The ending date for the estimation of this matrix is the same as the one in Longstaff, Santa-Clara and Schwartz (2001). An examination of the eigenvectors of the three most relevant factors reveals that the most important factors are as in Litterman and Scheinkman (1991), the level, slope and curvature of the term structure.

Even though the model is initially defined in terms of the forward rate, it is implemented with the discount bonds because the implementation of the model with discount bonds is easier than implementing the model with forward rates. Let $D(t, T)$ represent the price at time t of a discount bond with maturity at time T, and D a vector with 19 discount bonds with maturity $T_i = i/2$, $i = 2, ..., 20$. In this model, the discount bonds follow the risk-neutral diffusion $dD = rDdt + J^{-1}\sigma F dZ$, where $\sigma F dZ$ is a vector with the ith element given by $\sigma_i F_i dZ_i$, J^{-1} is the inverse of the Jacobian matrix for the mapping from discount bond prices to forward rates, and r is the short-term interest rate. Note that non-arbitrage implies that the discount bonds have risk-neutral drift, rD. Hence, by working with discount bonds directly, one does not need to calculate the drift of the forward rate, α_i, and it is therefore easier to implement the model with discount bonds

directly.

Swaptions are priced by Monte Carlo simulations in this model. Given the initial values of the 20 relevant discount bonds and the matrix $\Sigma = U\Psi U'$, the diffusion of the discount bonds is simulated and the payoff of the swaptions in each simulation path is determined. The payoff at maturity τ of a payer swaption with notional principal equal to one dollar, an exercise coupon c and tenor $(T - \tau)$ is $\max(0, -V(c, \tau, T))$. The payoff of a receiver swaption is $\max(0, V(c, \tau, T))$. The term $V(c, \tau, T)$ is the value for a fixed-rate receiver in a swap with maturity at time T and with fixed rate c, and is given by $c/2 \times \sum_{i=1}^{2(T-\tau)} D(\tau, \tau + i/2) + D(\tau, T) - 1$. The values of the swaptions implied by the model are the average discounted payoffs along all the simulated paths.

In the simulations, the short rate (r) and the forward rates' covariance matrix are fixed for each six-month period. In each simulation path, at time $t_i = i/2$, $i = 0, ..., 10$, the short rate is set to $-2 \times \ln(D(t_i, t_i + 0.5))$ and the forward rate covariance matrix is set to Σ without the last i^{th} columns and rows. The maximum t_i is five years because since the maximum swaption time-to-maturity is five years in the executed calibrations, there is no need to simulate more than five years ahead.

The calibration of the LSS model entails the calculation of the variances of the term-structure factors $(\lambda_1, ..., \lambda_N)$ that best fits the cross-section of the swaption prices available at the end of each month in the sample. The calibration scheme of the LSS model therefore is analogous to the calculation of implied volatilities in option prices in the sense that it calculates the implied volatilities of the factors affecting term-structure movements. The calibration entails finding the parameters $\lambda_1, ..., \lambda_N$ that minimize the sum of the squared relative swaption pricing errors of the LSS model. As in Longstaff, Santa-Clara and Schwartz (2001), models with different numbers of factors were calibrated. Likelihood ratio tests indicate that the null hypothesis of three latent factors is not rejected in favor of the alternative of four factors. Consequently, the number of factors (N) is set equal to three.

4.2   The MRE model

The proposed model with mortgage refinancing effects is essentially an extension of the LSS string model described in Section 4.1. In this model, the variances of the factors

are functions of the prepayment speed of the mortgage universe. Mathematically, the instantaneous covariance of the changes in the forward rates (dF_i/F_i) is a 19×19 positive definite matrix represented by $\Sigma_t = U\Psi_t U'$, where Ψ_t is a 19×19 diagonal matrix with diagonal given by $[0, ...0, \lambda_N \times CPR_t^{\gamma_N}, ..., \lambda_1 \times CPR_t^{\gamma_1}]'$, N is the number of factors in the model, λ_i, γ_i, $i = 1, ..., N$ are positive constants and CPR_t is the prepayment speed of the mortgage universe calculated by a prepayment model that is estimated herein. The instantaneous variance of the ith factor is $\sigma_i^2(CPR_t) = \lambda_i \times CPR_t^{\gamma_i}$, which implies that the elasticity of the variance of the ith factor to prepayment speed is constant and equal to $\gamma_i = \partial\sigma_i^2(CPR_t)/\partial CPR_t \times CPR_t/\sigma_i^2(CPR_t)$. The LSS model is a special case of the proposed model where $\gamma_i = 0$, for all $i = 1, ..., N$.

Because the MRE model depends on a prepayment model, Section 4.2.1 describes the prepayment model used in the calibration of the MRE model, while Section 4.2.2 gives details on the MRE model and its calibration.

4.2.1 Estimating the prepayment speed of the mortgage universe

Econometric prepayment models estimate the prepayment speed of a mortgage pool as a function of a series of variables that affect prepayments, such as the age of the mortgages in the pool and the incentive to refinance. As noted by Mattey and Wallace (2001), these models use loosely motivated and ad hoc measures of refinancing incentive, which are simplified measures based on optimization-based measures of refinancing incentive[9]. Indeed there are few measures of refinancing incentive in the econometric prepayments literature: for instance, Schwartz and Touros (1989) use the difference between the weighted-average coupon of the mortgage pool and the current mortgage rate, $WAC - MR$; Richard and Roll (1989) use the ratio WAC/MR; LaCour-Little, Marschoun and Maxam (2002) use the log of this ratio, $\ln(WAC/MR)$; and Schwartz and Touros (1993) use the ratio MR/WAC.

Herein, the ratio of the weighted-average coupon of the mortgage universe divided by the mortgage rate (WAC/MR) is used as measure for refinancing incentive for the mortgage universe, where WAC and MR are respectively the proxies for the mortgage universe weighted-average coupon and mortgage rate presented in Section 2. In order

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[9] See Green and Shoven (1986); Richard and Roll (1989); Schwartz and Torous (1989); Hayre and Young (2001); Mattey and Wallace (2001); Westhoff and Srinivasan (2001); and LaCour-Little, Marschoun and Maxam (2002) for some examples of econometric prepayment models. See Stanton (1995), Stanton and Wallace (1998) and Longstaff (2004) for examples of optimization-based prepayment models.

to understand this measure of refinancing incentive note that a mortgage is an annuity with current value A. Thus the prepayment option is analogous to an American option on an annuity with exercise price equal to the current principal balance, P, plus refinancing costs. Consequently A/P is a measure of the moneyness of the prepayment option and a measure of the refinancing incentive. The ratio A/P however has not often been used in the prepayment literature because the computation of A/P is cumbersome and, for longer maturities, A/P is well approximated by the ratio of the mortgage coupon to the mortgage rate. (See Richard and Roll, 1989.) Therefore, since the average maturity of the mortgage universe is quite high (the weighted-average maturity of the mortgage pools in database is close to twenty six years and two months), the ratio of WAC/MR is a measure of the average moneyness of the outstanding prepayment options and a measure of the average refinancing incentive in the mortgage universe.

The prepayment speed of the mortgage universe is assumed to be a non-decreasing function, $f(.)$, of the mortgage universe refinancing incentive, WAC/MR. That is, the prepayment speed of the mortgage universe is:

$$CPR = f(WAC/MR) \tag{5}$$

Equation 5 does not represent the prepayment model that best matches the prepayment speed of individual mortgage pools. In fact, the prepayment speed of a mortgage pool depends on the average age of the mortgages in the pool (or seasoning effect) and on past mortgage rates (or burnout effect). These important effects are not included in Equation 5 because the objective of the prepayment model used is solely to exemplify the use of mortgage information in the term-structure model, and is not expected to pin down all the nuances of prepayments[10]. In addition, while burnout and seasoning effects are important for explaining individual pool prepayments, these effects may be less important for explaining the average prepayment speed of the mortgage universe. Even though the used prepayment model is quite simple, it captures the fundamental non-linear increase in refinancing due to decreases in interest rates and the most important cause of prepayments (refinancing). Theoretically, I do not foresee any problem in using a more

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[10] See Pavlov (2001) for a detailed account of the different reasons for mortgage prepayments.

realistic prepayment model in the MRE model; however, it is not the objective of this paper to add in any way to the extensive literature on prepayments.

The refinancing profile in Equation 5 is estimated by nonparametrically regressing the prepayment of the mortgage universe during the month t on the proxy for the refinancing incentive at time $t - 1$. The delay between the application for mortgage refinancing and the actual prepayment of a mortgage creates uncertainty regarding the mortgage rate that ultimately triggers the refinancing decision. This uncertainty is solved herein as in Richard and Roll (1989), by using the refinancing incentive lagged by one month. Hence, the prepayment speed of the mortgage universe during month t is regressed on the mortgage universe WAC at the beginning of month $t-1$ divided by the average mortgage rates during the month $t - 1$. A total of 80 observations are used in this regression. The nonparametric estimation is done through the method developed by Mukerjee (1988) and Mammen (1991) and extended by Aït-Sahalia and Duarte (2003). In this method, the refinancing speed profile estimated is by construction a non-decreasing function of the level of refinancing incentive. See Appendix for details on this estimation.

The prepayment model estimated fits the actual history of prepayments in the mortgage universe reasonably well. Figure 2b displays the estimated prepayment function and Figure 2a plots the estimated prepayment of the mortgage universe each month in the sample period. Note that the estimated prepayment speeds and the actual prepayment speeds are highly correlated. The RMSE of the prepayment model is 4.5%, while the correlation between the actual prepayment and the model prepayment is 94%.

4.2.2   Calibration of the MRE model

Recall that in the MRE model, the instantaneous variance of the ith factor is $\sigma_i^2(CPR_t) = \lambda_i \times CPR_t^{\gamma_i}$. The calibration of the MRE model entails the calculation of the parameters $\lambda_i, \gamma_i, i = 1, ..., N$ that best fit the cross-section of the swaption prices in the sample that are available at the end of each month. The calibration entails finding the parameters that minimize the relative pricing errors of the model. The number of factors (N) in the calibrated model is set equal to three.

As with the LSS model, the short rate (r) and the dimension of the forward rates' covariance matrix are fixed for each six-month period in the Monte Carlo simulation. In

each simulation path, at time $t_i = i/2$, $i = 0, ..., 10$, the short rate is set to $-2 \times \ln(D(t_i, t_i + 0.5))$ and the dimension of the forward rate covariance matrix is set to $(19 - i) \times (19 - i)$.

Note that Σ_t is the covariance matrix of forward rates with constant time-to-maturity. To price swaptions at any given date, however one needs the covariance matrix of the forward rates with constant maturity time rather than constant time-to-maturity. Note that every six months (at time $t_i = i/2$ in the simulation path), all of the forward rates relevant to pricing the given swaptions have time-to-maturity multiples of six months, and hence have covariance matrices equal to Σ_t without the last i^{th} columns and rows. At in-between dates however, the relevant covariance matrix is different from a submatrix of Σ_t. To calculate the covariance matrix of the forward rates at in-between dates, Han (2004) analyzes a series of interpolation schemes of the matrix Σ_t. He concludes that the estimation results are not affected by the interpolation scheme. Based on this conclusion, I assume that the covariance matrix of the relevant forward rates at in-between dates is equal to $\Sigma_t = U \Psi_t U'$ without the last i^{th} columns and rows.

The instantaneous covariance matrix, Σ_t, of the forward rates changes is assumed to have the same eigenvector matrix U as the unconditional covariance matrix of the changes in the forward rates. This assumption is the same as in Han (2004) and in Jarrow, Li and Zhao (2004) and it implies that the eigenvector matrix U used in the calibration of the MRE model is the same as the one used in the LSS model. Mortgage refinancing could have implications for how shocks to the term-structure factors affect the forward rates with different maturities; in practice, however, the calibration of the MRE model and the comparison between the calibrated models would be complicated if the eigenvector matrix U were allowed to change across models. This simplifying assumption is also convenient because it implies that the calibrated models match the common principal components interpretation of the factors driving the term structure as being the level, slope and curvature of the term structure.

In contrast to the LSS model, each simulation path in this model is composed not only by the simulated discount function, but also by the simulated mortgage rate (MR) and the simulated weighted-average coupon of the mortgage universe (WAC). Given the WAC_t and MR_t at simulation time t, the current mortgage prepayment speed (CPR_t) is calculated by the estimated prepayment function. The current CPR_t implies a covariance

matrix for the forward rates ($\Sigma_t = U\Psi_t U'$) which is used to simulate the discount curve in the following simulation period. Based on this new simulated discount curve, MR_{t+1} and WAC_{t+1} are calculated.

The mortgage rate in the simulation period $t+1$ (MR_{t+1}) is calculated from the mortgage rate in period t and the changes in the simulated five-year continuously-compounded yield. Note that only at time $t_i = i/2$ in the simulation paths is the five-year discount yield directly available. At in-between dates, the five-year yield is calculated by linear interpolation of the two yields with maturities closest to five years. The mortgage rate at period $t+1$ is set equal to MR_t plus a linear function of the changes on the five-year yield. The coefficients of this linear function are estimated through OLS regression of the monthly changes on mortgage rates onto the monthly changes on the five-year continuously compounded zero-coupon yield. This regression is estimated with data from January 31, 1992 through August 29, 2003. The results of this estimation are in Table 5. The regression has an adjusted R^2 of 90%. Naturally, there are other ways of simulating the paths of the mortgage rates. On the other hand, the high R^2 of the estimated regression indicates that these changes in regressors would cause small improvements in the calibration of the MRE model at most.

The weighted-average coupon of the mortgage universe at simulation time $t+1$ is calculated with the simulated CPR_t and the WAC_t with the expression:

$$WAC_{t+1} = (1 - SMM_t) \times WAC_t + SMM_t \times MR_t \tag{6}$$

where SMM_t is calculated through Equation 3. There are three assumptions supporting this iteration process for the WAC: first, the WAC of the mortgage universe is assumed to be constant without prepayments; second, mortgage prepayments are assumed not to affect the balance of the mortgage universe; and third, the refinancing speed is assumed to be the same across coupons. (See Appendix for proof.) The mortgage refinancing simulated is unrealistic in the sense that refinancing does not change the balance of the mortgage universe, and mortgages with different coupons are assumed to have the same prepayment speed. On the other hand, there is no theoretical problem in using a more realistic refinancing procedure, other than adding unnecessary complications that will

detract from the main innovation in the MRE model, which is the inclusion of mortgage refinancing in a term-structure model.

The MRE model extends the LSS string model in two ways. First, since the prepayment speed of the MBS depends on the mortgage rate and coupon, the MRE model is calibrated to information about the mortgage universe as well as to the current term structure. Second, because MBS prepayment speed is a non-linear function of the level of interest rates, the relationship between interest-rate level and variance in the MRE model is non-linear. Non-linear relationships between interest-rate volatility and level are not uncommon in the term-structure literature. Indeed, with the objective of improving the empirical properties of term-structure models, a series of researchers developed term-structure models where the interest rate process is highly non-linear. See for instance Aït-Sahalia (1996), Duarte (2004) and Stanton (1997). The difference in the MRE model is that its non-linear relationships are economically motivated by the connection between the level of mortgage refinancing and interest-rate volatility.

In a general equilibrium framework, mortgage rates, swaption volatilities and discount prices are jointly determined. In the simulated model on the other hand, the initial mortgage rates are exogenous to the model and the simulated changes on mortgage rates depend only on the simulated changes in the five-year yield. The MRE model therefore cannot be used to specify the current mortgage rate because the determination of the mortgage rate should take into account interest-rate volatility, instead the simulated model uses the current mortgage rate to specify interest-rate volatility. That MBS-pricing models are unable to correctly specify the current mortgage rate is typical however, and this limitation of the MRE model is therefore typically shared by MBS-pricing models. Model prices typically differ from the observed market prices, and hence MBS-pricing models do not usually match the price of the passthrough priced at par. Since the current mortgage rate is the coupon of passthrough priced at par plus the servicing and guarantee fees, the MBS-pricing models typically do not correctly specify the current mortgage rate[11].

Even though the model does not jointly specify mortgage rates and swaption volatilities, it is nonetheless arbitrage-free. One way to recognize this is to realize that this model

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[11]Model prices are computed by taking the average of the discounted cashflows of MBS under different interest-rate scenarios. In order to make model and market prices equal, a spread is added to the interest rates generated in each scenario. This spread is called option-adjusted spread (OAS). See Gabaix, Krishnamurthy and Vigneron (2004) on this point.

is equivalent to an arbitrage-free model in which the interest-rate volatility is a non-linear function of the five-year yield. This non-linear relationship between interest-rate volatility and the five-year yield depends on the current mortgage rate and on the current mortgage universe coupon, and it is economically motivated by the connection between refinancing and interest-rate volatility.

4.3   The CEV model

It is possible that the empirical performance of the model with refinancing effects is generated by characteristics of the model that are not related to mortgage refinancing. The MRE model has twice as many parameters as the LSS model, and in addition, it allows for the dependence of the volatility of the term-structure factors to the five-year yield in the form of dependence to the speed of prepayments.

A second benchmark is calibrated in order to address this possibility. This second benchmark has the same number of parameters as the model with refinancing effects and allows for the dependence of volatility of the factors with respect to the five-year yield. In this benchmark, the instantaneous variance of the factors are functions of the five-year yield. The instantaneous covariance of the changes in the forward rates (dF_i/F_i) in this benchmark model is $\Sigma_t = U\Psi_t U'$, where Ψ_t is a diagonal matrix with diagonal given by $[0, ...0, \lambda_N \times y_t^{\beta_N}, ..., \lambda_1 \times y_t^{\beta_1}]'$. The parameters λ_i and β_i, $i = 1, ..., N$ are constants and y_t is the yield of the five-year discount bond. The definition of Ψ_t implies that the instantaneous variance of the ith factor is $\sigma_i^2(y_t) = \lambda_i \times y_t^{\beta_i}$, this model is therefore herein called the constant elasticity of variance (CEV).

The calibration of the CEV model is analogous to the calibration of the model with refinancing effects. The parameters $\lambda's$ are positive and no restrictions are imposed on the parameters $\beta's$ in the calibration of the CEV model. Any restriction on the parameters $\beta's$ could worsen the empirical performance of the CEV model, which could bias the results in favor of the MRE model. As in the other calibrated models, the matrix U is the eigenvector matrix of the correlation matrix of the log changes in the forward rates and the number of factors is equal to three.

5 Models' performance in forecasting volatility and fitting swaption prices

This section compares the ability of the calibrated models to fit the cross-section of swaption prices and the time series behavior of interest-rate volatility. The comparison between these models sheds some light on the presence of actual and implied volatility effects.

5.1 Fitting swaption prices

The results of two likelihood ratio tests analogous to the ones in Longstaff, Santa-Clara and Schwartz (2001) are displayed in the first panel of Table 6. Here, the test statistics are given by the difference in the logs of the sum of the mean-squared errors multiplied by the number of swaptions in the sample (34×80), and they are distributed as chi-square with 3×80 degrees of freedom, χ^2_{240}. The null hypothesis in the first test in this panel is that $\beta_i = 0$, and the alternative is that $\beta_i \neq 0$, $i = 1, ..., 3$. The null hypothesis of the LSS model is rejected in favor of the CEV model at usual significance levels. The null hypothesis of the second test in this panel is that $\gamma_i = 0$, and the alternative is that $\gamma_i \neq 0$, $i = 1, 2$ and 3. The second test in this panel indicates that the null hypothesis of the LSS model is rejected in favor of the MRE model at usual significance levels.

A Diebold and Mariano (1995) test indicates that the MRE model fits the cross-section of the swaption prices better than the CEV model. The second panel of Table 6 presents the results of a test analogous to the ones in Jarrow, Li and Zhao (2004). The MRE and the CEV models are non-nested, and hence the likelihood ratio test does not apply. Let $SSE_{CEV}(t)$ represent the sum of the squared relative pricing errors of the CEV model at date (t), $SSE_{MRE}(t)$ represent the sum of the squared relative pricing errors of the MRE model at date (t), and $d(t)$ be the difference between these sums of squared errors, $d(t) = SSE_{CEV}(t) - SSE_{MRE}(t)$. In the Diebold and Mariano (1995) test, the null hypothesis is $E[SSE_{CEV}] = E[SSE_{MRE}]$ and the test statistic is

$$S = \frac{\overline{d}}{\sqrt{2\pi \widehat{f}_d / T}} \tag{7}$$

where $\overline{d}$ is the sample mean of the differences, $\overline{d} = 1/T \times \sum_{t=1}^{T} d(t)$, and $\widehat{f}_d$ is an estimate of the spectral density of the differences at frequency zero. Newey and West (1987) estimator

with numbers of lags equal to twenty is used to estimate $2\pi f_d$. The results are robust to changes in the number of lags. Under technical conditions, S is asymptotically standard normally distributed.

In addition to the performed tests, the Akaike Information Criteria (AIC) is also used to examine the performance of each calibrated model. The AIC indicates that the MRE model is the preferred one among the three calibrated models. The AIC of a model is given by $-2/M \times \ln L(\hat{\theta}) + 2p/M$, where M is the sample size (34×80), $\ln L(\hat{\theta})$ is the log-likelihood function evaluated at the estimated parameters, and p is the number of parameters in each model. In the LSS model, p is 3×80, while in the estimated MRE and CEV models, p is equal to 6×80. As noted by Longstaff, Santa-Clara and Schwartz (2001), the calibration procedures are analogous to the estimation of a non-linear least squares regression, and hence the AIC in the estimated models is given by $\ln(2 \times \pi) + 1 + \ln(MSE) - 2p/M$, where MSE is the mean of the squared relative pricing errors of the estimated model. As illustrated in the third panel of Table 6, the AIC of the model with refinancing effects is the smallest one and consequently the MRE model is the preferred model. (See Amemiya, 1985.)

Table 7 presents statistics on the calibrated parameters of each model. All of the calibrated models have unstable parameters, which is a usual consequence of backing out the structural parameters of the observed month-end option prices[12]. The parameter instability in the calibration procedure is troublesome because it raises the suspicion that the superior performance of the model with mortgage refinancing might be attributable to overfitting. The calibration of the CEV model partially addresses this suspicion because the CEV model has the same number of free parameters as the MRE model. On the other hand, the parameter variability of the MRE model might be larger than the parameter variability of the CEV model. A simple comparison between the parameter variability of these models is clearly not appropriate since the models have different parametric specifications and the parameters are therefore in different scales.

Out-of-sample comparisons of the calibrated models is therefore performed in order to further address the possibility of overfitting. The out-of-sample analyses have two time

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[12]See Amin and Ng (1997), and Bakshi, Cao and Chen (1997) and Longstaff, Santa-Clara and Schwartz (2001).

horizons. One out-of-sample analysis consists of backing out the model parameter values from the previous month swaption prices and using these parameters as an input to price swaptions at the current month. The other out-of-sample analysis consists of backing out the model parameter values from the swaption prices three months prior to the current month and using these parameters to price the swaptions at the current month. Both time horizons are used in order to better examine the effect of the time variability of the calibrated parameters on the performance of the models. In fact, if the performance of the MRE model were driven by a larger variability in its calibrated parameters, the MRE out-of-sample performance would deteriorate as the out-of-sample time horizon increased.

The model with mortgage refinancing effects performs better than the benchmark models in the out-of-sample analyses. The relative and absolute mean out-of-sample pricing errors are displayed in Table 8. The absolute pricing errors displayed are in Black's implied volatilities by swaption-display convention. Table 8 shows that the model with refinancing effects has the smallest relative out-of-sample one-month relative errors in 27 of the 34 cases. In addition, it shows that the improvement caused by the inclusion of refinancing effects is independent of the time horizon of the out-of-sample analysis, indicating that the MRE improvement is not due to a large variation in its calibrated parameters.

The MRE model performs better than the CEV or the LSS model in terms of fitting swaption prices, particularly in periods of high refinancing activity. Figure 5 plots the RMSEs of the calibrated models. The plot of the LSS RMSEs is qualitatively similar to the one in Longstaff, Santa-Clara and Schwartz (2001) in the sense that it has spikes in late 1997, early and late 1998, and mid 1999. On average, the RMSE of the LSS model is 8.83%. The average RMSE of the CEV model is 7.72%. The average RMSE of the model with mortgage refinancing effects is 5.30%. The RMSE of the MRE model is smaller than the RMSE of the CEV model 52 out of 80 days. Note that in periods of very high refinancing activity such as early 1998, late 1998 and the period between January 2001 and August 2003, the RMSEs of the LSS and CEV models are much larger than the RMSEs of the model with refinancing effects. Indeed, the RMSEs of the LSS and CEV models in the refinancing wave of January 1998 are 5.8% and 5.6% respectively, while the RMSE of the MRE model is 3.2%. The average RMSE of the MRE model in the high refinancing

period between January 2001 and August 2003 is 8.1%, while the average RMSE of the CEV model is 13.5% and of the LSS model is 15.9%.

The reason for the superior performance of the model with mortgage refinancing effects during periods of high refinancing activity is exemplified in Figure 6, which plots the mean Black's swaption volatilities as functions of the time-to-maturity of the swaptions (or the term structure of swaption volatilities). Note that in periods of high refinancing activity (Figure 6a to 6c), the term structure of swaption volatilities is downward-sloped, while in periods of low refinancing activity (Figure 6d), the volatility term structure is practically flat. Note as well that the term structure of swaption volatilities is steeper during 2001-2003 than in the other high refinancing periods in the sample. This might be a consequence of the increasing size of the MBS market and of the portfolios of MBS hedgers shown in Table 1. (See also Feldhütter and Lando, 2005 on this point.) Figure 6 also indicates that the MRE model can capture the changes in the term structure of swaption volatilities better than the CEV and the LSS models.

The movements in the term structure of swaption volatilities provide a direct theoretical link between mortgage refinancing and swaption volatilities. During mortgage refinancing, there is an increase in the instantaneous interest-rate volatility. This increase is mean-reverted because as mortgages are refinanced, the coupon of the mortgage universe, the speed of refinancing, and the volatility of interest rates all decrease. The cross-section of swaption prices reveals this expected movement in interest-rate volatility. The MRE model can capture this movement while the CEV model cannot, even though the CEV model can, by construction, capture the negative correlation between interest rates and interest-rate volatility that is implied by the MRE model.

A particularly interesting period when the MRE model performs better than the LSS and the CEV models is the second half of 1998. This period is characterized by the LTCM hedge fund fall down. In August of 1998, Russia defaulted on its debt causing a drop in the Treasury rates and large losses for LTCM. During September of 1998, the fund losses mounted and Treasury yields dropped even further. On September 24th, news broke that the fund had been bailed out by a consortium of banks. By October 5th, Treasury rates dropped to their lowest of the period and mortgage rates followed, deepening the 1998 refinancing wave. By January 1999, refinancing activity was at the same level as during

the end of August 1998 and the autumn 1998 refinancing wave was finished. The model with mortgage refinancing effects fits swaption prices better than the LSS and the MRE models during the second half of 1998, and particularly during the end of October 1998. (See Figure 5.) Between August and October 1998, the average RMSEs of the LSS and CEV models are 11.3% and 10.6% while the average RMSE of the MRE is 5.5%. This superior performance is due to the fact that the MRE model has the flexibility to fit the actual term structure of swaption volatilities in this period, which was downward-sloped. (See Figure 6b.)

Because, the second half of 1998 is such an abnormal period, the superior performance of the MRE model needs to be carefully interpreted. One interpretation is that MBS-hedging activity caused the actual term structure of swaption volatilities to be downward-sloped. Another possibility is that the quality of the swaption quotes for this period is poor due to the lack of liquidity in the swaption markets. Indeed, Longstaff, Santa-Clara and Schwartz (2001) conducted a series of interviews with swaption traders who experienced this period. The traders indicated that the liquidity of the swaption markets in this period was less than usual. Alternatively, it is possible that swaption traders interpreted the events of the fall of 1998 as temporary and hence the implied volatility of short-term swaptions increased more than the volatility of long-term swaptions. Ultimately, to distinguish among these explanations, it is necessary to have data on the flows generated by MBS hedgers during this period. Unfortunately, as previously mentioned, this kind of data is not available.

5.2 Forecasting interest-rate volatility

All of the analyses performed so far indicates that mortgage refinancing can explain the variation of interest rates' implied volatility. It is possible, however, that mortgage refinancing is only affecting implied rather than actual interest-rate volatility. If the market imperfections that prevent the supply of swaptions to be perfectly elastic are the only cause of the relationship between interest-rate volatility and mortgage refinancing, the surge in demand for interest-rate options during a refinancing wave would then affect only the implied volatilities of swaptions. In addition, the inclusion of mortgage refinancing effects in the term-structure models would not improve the model's ability to forecast

actual interest-rate volatility. This section therefore analyzes the ability of each calibrated model to forecast actual interest-rate volatility.

The forecasting regressions consist of regressing a proxy for the actual volatility of the five-year yield ($\sigma_{t+\Delta t}^{Actual}$) on the five-year yield volatility implied by the calibrated models ($\sigma_t^{\text{Implied}}$), that is

$$\sigma_{t+\Delta t}^{Actual} = \alpha_0 + \alpha_1 \times \sigma_t^{\text{Implied}} + \varepsilon_{t+\Delta t} \tag{8}$$

The five-year yield is used as a benchmark to analyze the ability to forecast interest-rate volatility because the five-year yield is computed at every point of the simulation paths generated in Section 4. In the forecasting regressions, the actual realized volatility of the five-year yield between time t and time $t + \Delta t$ is estimated from a daily time series of the five-year yield. The implied volatility of the five-year yield is calculated with the same 2,000 simulation paths and calibrated parameters as those in Section 4. The implied volatility of the five-year yield between t and $t + \Delta t$ is the standard deviation of the simulated five-year yields at time $t + \Delta t$ calculated across all the simulation paths. Table 9 presents the results of Regression 8. The forecasting horizons are one, three, six and twelve months.

The results in Table 9 indicate that the MRE model produces interest-rate volatility forecasts with the largest R^2s at all forecasting horizons, which indicates that mortgage refinancing indeed helps to explain actual interest-rate volatility. The forecasts generated by the MRE are biased, however, which is an indication of the presence of the implied volatility effect. Note that in all the MRE forecasting regressions, the null hypothesis that $\alpha_0 = 0$ and $\alpha_1 = 1$ is rejected and hence the volatility of the five-year yield implied by the MRE model is a biased forecast of the actual volatility. The fact that implied volatility is a biased forecast of the actual volatility is a stylized fact in the equity options literature; see Canina and Figlewski (1993). There are some explanations for this stylized fact, among which is the possibility that market imperfections make the perfect dynamic replication of options impossible, and hence market-makers charge a premium for taking the risk of not perfectly replicating the options. A word of caution, however: The presence of market imperfections is only one of the many possible explanations for the documented bias in the forecasting regressions. A series of well-known problems might also affect the results

of the forecasting regressions displayed in this section, and could potentially explain the bias of the MRE model forecasts as well. (See Poon and Granger, 2003 for a review.) For instance, any proxy for actual interest-rate volatility is subject to error, which might affect the results of the forecasting regression. In addition, the calibrated MRE model has misspecification risk, since the actual functional relation between mortgage refinancing and interest-rate volatility is not necessarily equal to the one assumed in the MRE model.

6   Conclusion

This paper identifies two possible transmission channels between the mortgage market and the volatility of interest rates. The first is a direct channel related to the hedging activity of MBS investors on the swap or Treasury markets, which is the actual volatility effect. The second is the implied volatility effect, which is related to the hedging activity of MBS investors in the swaption market. The evidence provided in this paper indicates that both of these effects are possibly present in the relationship between mortgage refinancing and volatility of interest rates.

Mortgage refinancing helps considerably in explaining swaption prices and in forecasting the actual future volatility of interest rates. A series of in-sample and out-of-sample formal statistical tests indicate that refinancing seems to affect the volatility of the factors driving the term structure. The calibration of the models to swaption prices indicates that the model with refinancing effects outperforms the models without refinancing effects particularly during periods of high refinancing activity.

There are nevertheless a series of issues that complicate the interpretation of the results as strong evidence in favor of the actual and implied volatility effects. First, the actual flows generated by MBS hedgers in the swap or Treasury markets cannot be observed. Hence, even though Federal Reserve (2005) indicates that these flows have been large in the last few years, the empirical evidence provided herein is indirect. Second, the fixed-income markets suffered some structural changes in the sample period, for instance, there appears to have been a shift from hedging based on Treasury securities to hedging based on swaps. And third, the composition of those making up the majority of mortgage investors significantly changed during the nineties.

A Appendix

A.1 Proof of Equation 1

Assume a hedged portfolio with price, $\Pi = n_{MBS} \times P_{MBS}/100 + n_{Hedge,0} \times P_{Hedge}/100$, where n_{MBS} is the principal amount of a MBS in the portfolio, P_{MBS} is the price of the MBS, and $n_{Hedge,0}$ is the notional amount of the fixed-income instrument used to hedge the duration of this portfolio. This instrument could be an interest-rate swap or a Treasury note where P_{Hedge} is the price of the hedging instrument. The amount of the hedging instrument $(n_{Hedge,0})$ is chosen to make the derivative of the price of the portfolio with respect to the yield of the hedging instrument at the current yield level (y_0) equal to a constant (c); that is $\Pi^{'}(y_0) = c = n_{MBS} \times P^{'}_{MBS}(y_0)/100 + n_{Hedge,0} \times P^{'}_{Hedge}(y_0)/100$. Without loss of generality, the yield of the hedge instrument (the current swap rate or the yield of the note) is taken as a proxy for the level of interest rates. The constant (c) could be zero in the case of a zero-duration target, or different from zero were this portfolio holder willing to take some duration risk.

Assume that the level of interest rates move from y_0 to y_1, and hence $\Pi^{'}$ moves to $\Pi^{'}(y_1) = n_{MBS} \times P^{'}_{MBS}(y_1)/100 + n_{Hedge,0} \times P^{'}_{Hedge}(y_1)/100$, which is different from c. In order to rebalance the portfolio, the portfolio holder investor will have to trade in the notes in such way that $\Pi^{'}$ becomes equal to the constant c again, that is $c = n_{MBS} \times P^{'}_{MBS}(y_1)/100 + n_{Hedge,1} \times P^{'}_{Hedge}(y_1)/100$. The amount that is needed to be traded in order to rebalance the portfolio is

$$(n_{Hedge,1} - n_{Hedge,0}) = 100 \times (c - \Pi^{'}(y_1))/P^{'}_{Hedge}(y_1) \tag{9}$$

Plugging the first order Taylor expansion of $\Pi^{'}$ around y_0 in the expression above:

$$n_{Hedge,1} - n_{Hedge,0} \approx -\frac{[n_{MBS} \times P^{''}_{MBS}(y_0) + n_{Hedge,0} \times P^{''}_{Hedge}(y_0)]}{P^{'}_{Hedge}(y_1)} \times (y_1 - y_0) \tag{10}$$

The term within brackets in the equation above is negative under fairly general conditions. For example, assume that a hedger has a long position in a passthrough, the hedge instrument is a Treasury note or a swap, and the hedger wishes a portfolio with interest-rate risk smaller than the interest-rate risk of a passthrough. In this case, the

term between brackets in Equation 10 will be normally smaller than zero. To see this, note that $P'_{Hedge}(y_1)$ and $P'_{MBS}(y)$ are negative, $P''_{Hedge}(y_0)$ is positive and $P''_{MBS}(y)$ is normally negative. In addition, the assumption that the hedger has a long position in a passthrough implies that n_{MBS} is positive. Moreover, the assumption that the hedger wants a portfolio with smaller interest-rate risk than the interest-rate risk of the passthrough implies that the absolute value of the targeted delta of the portfolio (c) is smaller than the absolute value of the delta of the position in the passthrough; that is, $|c| < |n_{MBS} \times P'_{MBS}(y_0)|$. As a consequence, the investor has to short notes in order to hedge; that is $n_{Hedge,0} = (c - n_{MBS} \times P'_{MBS}(y_0))/P'_{Hedge}(y_0) < 0$. As a result the term inside brackets in the equation above is negative.

A.2   Proof of Equation 6

Let WAC_t^i be the WAC of the ith pool in the mortgage universe. If prepayments do not affect the balance of the mortgage universe, then by definition, WAC_{t+1} is given by:

$$\frac{\sum_i((MB_{t-1}^i - SP_t^i) \times (1 - SMM_t^i) \times WAC_t^i)}{\sum_i(MB_{t-1}^i - SP_t^i)} + \frac{\sum_i(SMM_t^i \times (MB_{t-1}^i - SP_t^i) \times MR_t)}{\sum_i(MB_{t-1}^i - SP_t^i)} \tag{11}$$

Where SP_t^i is the scheduled principal payment at time t and MB_{t-1}^i is the total balance at the end of the month $t-1$ of ith pool in the mortgage universe. If SMM_t^i is the same across all coupons, then the second term in this expression is $SMM_t \times MR_t$, and if the WAC of the mortgage universe remains constant without prepayments, the first term of this expression is $(1 - SMM_t) \times WAC_t$.

A.3   Estimation of the refinancing profile of the mortgage universe

The method used to estimate the prepayment function is a two-step procedure. The first step is a constrained least squares regression, and the second step is a Nadaraya-Watson kernel regression. The constrained least squares regression consists in finding the values m_i, $i = 1, ..., 80$ that are closer in the least squares sense to the observed prepayments (CPR_i), and satisfying a monotonicity restriction. Without loss of generality, assume that the observations on the refinancing incentive WAC/MR have been ordered, that is $(WAC/MR)_i \geqslant (WAC/MR)_j$, for $i > j$, $i, j \in \{1, ..., 80\}$. The constrained least squares

regression problem is therefore:

$$\min_{m_i, i=1,...,80} \sum_{i=1}^{80} (m_i - CPR_i)^2 \tag{12}$$

subject to $m_i - m_j \geqslant 0$ $i > j$, i, $j \in \{1, ..., 80\}$. The second step of the estimation is a Nadaraya-Watson kernel regression, which is given by:

$$\hat{f}(\frac{WAC}{MR}) = \frac{\sum_{i=1}^{80} K_h(\frac{WAC}{MR} - (\frac{WAC}{MR})_i) \times m_i}{\sum_{i=1}^{80} K_h(\frac{WAC}{MR} - (\frac{WAC}{MR})_i)} \tag{13}$$

The used Kernel, $K(.)$, is normal and the bandwidth, h, is chosen by cross-validation. The bandwidth value is 2.471×10^{-2}.

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Table 1: Some statistics on the mortgage-backed and Treasury securities

Outstanding Mortgage-Backed Securities

	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Agency Passthroughs	863	972	1,150	1,221	1,375	1,582	1,775	1,962	2,163	2,409
Agency CMOs	579	582	541	580	607	665	664	808	926	955
Non-Agency MBS	206	224	256	311	405	455	500	591	692	843
Total	1,648	1,779	1,947	2,112	2,387	2,702	2,939	3,362	3,781	4,207

Outstanding Marketable U.S. Treasury Debt

	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Bills	734	761	777	715	691	737	647	811	889	929
Notes	1,867	2,010	2,112	2,106	1,961	1,785	1,557	1,414	1,581	1,906
Bonds	510	521	555	587	621	644	627	603	589	564
Total	3,111	3,292	3,444	3,408	3,273	3,166	2,831	2,828	3,059	3,399

MBS Holdings by MBS Hedgers (% of total)

	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
MBS Dealers	5.8	5.1	5.7	5.3	3.9	2.5	1.3	1.5	0.9	0.8
Fannie Mae and Freddie Mac	4.6	7.6	9.9	11.5	15.3	18.2	20.0	21.9	29.3	29.3
Hedge Funds and Other Investors	8.9	12.9	9.0	6.9	8.0	11.3	10.9	4.6	6.2	6.7
Total	19.3	25.6	24.6	23.7	27.2	32.0	32.2	28.0	36.4	36.8

This table presents general statistics on mortgage-backed and Treasury securities. The outstanding amounts are in billions of dollars. The agency-MBS data include only the mortgages securitized by Ginnie Mae, Freddie Mac and Fannie Mae. The data on non-agency MBS include mortgage-related, asset-backed securities such as those collateralized by home equity loans. The outstanding U.S. Treasury amounts include only interest-bearing marketable Treasury securities. The holdings of "Hedge Funds and Other Investors" include all types of investors whose MBS holdings are not available, such as hedge funds. It is therefore an upper-bound estimate of MBS holdings of hedge funds. The data on the amounts of Treasury securities are from several issues of the Federal Reserve Bulletin. The data on the MBS are from Inside Mortgage Finance (2004).

Table 2: Correlations between changes in CMT yields and in swap rates

Changes

Sample Period	# Observations	Swaps and CMT Years-to-Maturity				
		Two	Three	Five	Seven	Ten
April 1994 - December 1998	1,185	0.95	0.96	0.97	0.97	0.98
January 1999 - August 2003	1,166	0.94	0.93	0.95	0.95	0.94
April 1994 - August 2003	2,351	0.95	0.95	0.96	0.96	0.96

Squared-Changes

Sample Period	# Observations	Swaps and CMT Years-to-Maturity				
		Two	Three	Five	Seven	Ten
April 1994 - December 1998	1,185	0.93	0.95	0.96	0.96	0.97
January 1999 - August 2003	1,166	0.89	0.85	0.87	0.89	0.89
April 1994 - August 2003	2,351	0.85	0.84	0.91	0.92	0.93

The first panel of this table presents the correlation between changes in swap rates and changes in constant maturity Treasury rates (CMT) with the same time-to-maturity. The second panel presents the correlation between the squared-changes of the swap rates and the squared-changes of the CMT rates. The data are daily and the correlations are estimated for different sample periods.

Table 3: Pairwise Granger causality tests

Sample Period: April 1994 to August 2003

Dependent	Variable Excluded			
Variable	ΔMBAREFI	ΔLIBOR6	ΔSLOPE	ΔVOL
ΔMBAREFI		22.07 (0.00)	56.77(0.00)	21.53(0.00)
ΔLIBOR6	12.40 (0.09)		6.52 (0.48)	11.54 (0.12)
ΔSLOPE	13.16 (0.07)	16.01 (0.03)		14.84 (0.04)
ΔVOL	44.58 (0.00)	11.41 (0.12)	16.38 (0.02)	

Sample Period: April 1994 to December 2000

Dependent	Variable Excluded			
Variable	ΔMBAREFI	ΔLIBOR6	ΔSLOPE	ΔVOL
ΔMBAREFI		15.52 (0.03)	22.83(0.00)	27.64(0.00)
ΔLIBOR6	5.23 (0.63)		7.04 (0.42)	8.05 (0.33)
ΔSLOPE	6.46 (0.49)	3.50 (0.84)		19.37 (0.00)
ΔVOL	69.18 (0.00)	6.71 (0.46)	13.44 (0.06)	

This table presents the results of the Granger causality tests. The results of these tests indicate that the refinancing activity forecasts the volatility of interest rates. The Wald test statistics are asymptotically distributed as chi-square with seven degrees of freedom, χ^2_7 and they are displayed in this table with p-values in parenthesis. The null hypothesis is that the variable excluded does not forecast the dependent variable. The first panel of this table shows the results for the VAR estimated with 487 weekly observations between April 8, 1994 to August 29, 2003, and the second panel shows the results for the VAR estimated with 344 observations through December 29, 2000. The VARs are estimated on the first differences of the variables because all of the variables above are very close to unit root processes. The Wald tests are based on the standard MLE of the covariance matrix of the estimated coefficients. These VARs are estimated with seven lags.

Table 4: Variance decomposition in the VAR system

Sample Period: April 1994 to August 2003

Weeks	Explanatory Variable			
Ahead (n)	ΔMBAREFI	ΔLIBOR6	ΔSLOPE	ΔVOL
1	2.37 (1.39)	6.26 (2.10)	2.04 (1.24)	89.33 (2.66)
4	3.48 (1.59)	6.24 (1.95)	4.53 (1.72)	85.74 (2.81)
7	6.09 (2.04)	7.00 (2.08)	4.74 (1.84)	82.17 (3.09)
51	8.59 (2.36)	7.24 (2.09)	5.21 (1.91)	78.95 (3.33)

Sample Period: April 1994 to December 2000

Weeks	Explanatory Variable			
Ahead (n)	ΔMBAREFI	ΔLIBOR6	ΔSLOPE	ΔVOL
1	3.44 (1.96)	0.70 (0.94)	0.52 (0.87)	95.34 (2.28)
4	8.36 (2.96)	2.55 (1.76)	1.64 (1.51)	87.45 (3.52)
7	15.42 (3.59)	3.45 (2.10)	1.92 (1.67)	79.21 (4.04)
51	20.98 (4.26)	3.60 (2.02)	2.44 (1.76)	72.98 (4.64)

This table presents the variance decomposition of the first difference in the volatility of interest rates (VOL) n weeks ahead. Standard errors are in parenthesis and are estimated with 10,000 simulation runs. The first panel of this table shows the results for the VAR estimated with 487 observations between April 8, 1994 to August 29, 2003, and the second panel shows the results for the VAR estimated with 344 observations through December 29, 2000. The VARs are estimated with seven lags.

Table 5: Regression of changes in mortgage rates onto changes in five-year yields

# Observations	139
R^2	0.90
α	-1.6×10^{-5}
	(-0.24)
β	0.77
	(34.38)

This table displays the results of the regression of the changes in mortgage rates on the changes on the five-year zero coupon bond estimated from the Libor/swap rates. That is, $\Delta MR_t = \alpha + \beta \times \Delta y_t^{5-year}$. The sample is monthly from January 1992 to August 2003. T-statistics are between parenthesis.

Table 6: Comparison of RMSEs of each model

Likelihood Ratio Tests

H_0	H_A	Test Statistics	p-value
LSS	CEV		
$\beta_i = 0$, $i = 1$ to 3	$\beta_i \neq 0$, $i = 1$ to 3	850	0.00
LSS	MRE		
$\gamma_i = 0$, $i = 1$ to 3	$\gamma_i \neq 0$, $i = 1$ to 3	3,442	0.00

Diebold and Mariano Test

H_0	H_A	Test Statistics	p-value
$E[SSE_{CEV}] = E[SSE_{MRE}]$	$E[SSE_{MRE}] < E[SSE_{CEV}]$	-1.85	0.03

Akaike Information Criteria

LSS	-1.37
MRE	-2.46
CEV	-1.50

The first panel of this table presents the results of two likelihood ratio tests, the second panel the results of one Diebold and Mariano (1995) test and the third panel displays the Akaike information criteria. The columns denoted by H_0 and H_A contain the null and the alternative hypotheses respectively. The test statistic of the two likelihood ratio tests is the difference between the log of the sum of the mean squared-errors multiplied by the number of swaptions. These two tests have test statistic distributed as chi-square with 240 degrees of freedom. The Diebold and Mariano (1995) is used because the MRE and the CEV models are non-nested. The null hypothesis of the Diebold and Mariano (1995) test is that the MRE and CEV models have the same mean sum of the relative squared errors (SSE). Under technical conditions, the Diebold and Mariano (1995) test statistic is asymptotically standard normally distributed. The third panel displays the AIC of each calibrated model. The AIC indicates that the MRE model is the preferred one.

Table 7: Models' calibrated parameters

Model	Parameter	Mean	Standard Deviation	Min	Max
LSS	λ_1	0.416	0.133	0.248	0.832
	λ_2	1.312	1.310	0.128	4.842
	λ_3	0.127	0.155	0.015	0.593
CEV	λ_1	0.050	0.080	0.002	0.593
	λ_2	10.32	26.31	0.040	117.3
	λ_3	0.004	0.006	0.001	0.039
	β_1	-1.002	0.475	-1.755	0.305
	β_2	0.174	0.996	-1.286	2.507
	β_3	-1.284	0.254	-1.740	-0.618
MRE	λ_1	30.53	43.11	0.238	144.9
	λ_2	11.83	16.17	0.001	87.68
	λ_3	6.147	9.606	0.025	53.90
	γ_1	1.546	1.364	0	4.560
	γ_2	1.017	0.987	0	3.257
	γ_3	1.503	1.149	0	3.459

This table displays statistics on the calibrated parameters. The models are calibrated to end-of-month swaption prices. A total of 34 swaptions with different tenors and times-to-maturity are used in this calibration procedure. The models' parameters are chosen to minimize the square-root of the mean relative squared pricing error.

Table 8: Out-of-sample pricing errors for each calibrated model

		One Month ahead												Three Months ahead											
		Relative Errors						Absolute Errors						Relative Errors						Absolute Errors					
		Tenor $(T-\tau)$						Tenor $(T-\tau)$						Tenor $(T-\tau)$						Tenor $(T-\tau)$					
τ		1	2	3	4	5	7	1	2	3	4	5	7	1	2	3	4	5	7	1	2	3	4	5	7
0.5	MRE	-18	-3	-2	-1	0	4	-6	-1	0	0	0	1	-18	-2	-2	-1	1	5	-5	-1	0	0	1	2
	CEV	-23	-8	-10	-10	-8	-5	-8	-3	-3	-3	-2	-1	-23	-8	-10	-10	-8	-5	-8	-4	-4	-3	-3	-2
	LSS	-23	-8	-13	-13	-10	-6	-8	-4	-4	-4	-3	-2	-24	-9	-14	-14	-10	-7	-9	-4	-5	-5	-3	-2
1.0	MRE	-4	0	0	1	1	2	-2	0	0	0	0	1	-3	0	0	1	2	3	-2	0	0	0	1	1
	CEV	-5	-3	-6	-5	-3	-3	-3	-2	-2	-2	-1	-1	-5	-3	-6	-5	-3	-3	-3	-2	-2	-2	-1	-1
	LSS	-4	-4	-9	-8	-4	-4	-2	-2	-3	-2	-1	-1	-5	-5	-10	-8	-5	-5	-3	-2	-3	-3	-2	-2
2.0	MRE	0	0	-2	-3	-3	1	0	0	0	-1	-1	-1	0	0	-2	-3	-2	-2	0	0	0	-1	0	0
	CEV	2	0	-3	-4	-3	-5	0	0	-1	-1	-1	-1	2	0	-3	-4	-3	-5	0	0	-1	-1	-1	-1
	LSS	2	-1	-5	-5	-3	-6	0	-1	-1	-1	-1	-1	1	-3	-6	-6	-4	-7	0	-1	-2	-1	-1	-2
3.0	MRE	1	-1	-5	-5	-4	1	0	0	-1	-1	-1	0	1	-2	-5	-5	-4	1	0	0	-1	-1	-1	0
	CEV	4	1	-3	-3	-2	0	1	0	0	-1	0	0	4	1	-3	-3	-2	0	1	0	-1	-1	-1	0
	LSS	3	0	-3	-3	-1	0	1	0	-1	-1	0	0	2	-1	-4	-4	-2	-1	0	0	-1	-1	0	0
4.0	MRE	1	1	2	1	1		0	0	0	0	0		1	2	2	1	1		0	0	0	0	0	
	CEV	8	7	5	4	3		2	2	1	1	1		8	7	5	4	3		1	1	1	1	0	
	LSS	9	8	6	5	4		2	2	1	1	1		7	7	5	4	3		1	1	1	1	1	
5.0	MRE	4	6	5	4	6		1	1	1	1	1		4	6	5	4	7		1	1	1	1	1	
	CEV	9	10	7	6	7		2	2	1	1	1		9	10	7	6	7		2	2	1	1	1	
	LSS	11	12	9	6	8		2	2	2	1	2		10	10	8	5	7		2	1	1	1	1	

This table displays the means of the relative and absolute out-of-sample errors of each calibrated model for each swaption available in the sample. The relative error is $(model\_price - market\_price)/(market\_price)$. The absolute errors are the Black's volatility errors. The out-of-sample analysis consists of backing out the model parameters from the previous month swaption prices or from the swaption prices three months prior the current month and using these parameters to price swaptions at the current month.

Table 9: Forecasting interest-rate volatility

	LSS	CEV	MRE	LSS	CEV	MRE
	$\Delta t = 1$ month			$\Delta t = 3$ months		
R^2	6%	20%	23%	9%	26%	34%
α_0	9.64×10^{-4}	-8.80×10^{-4}	1.11×10^{-3}	2.11×10^{-3}	-2.62×10^{-4}	1.84×10^{-3}
	(1.17)	(-1.14)	(3.01)	(1.68)	(-0.20)	(2.74)
α_1	0.709	1.369	0.532	0.626	1.121	0.584
	(2.23)	(4.68)	(4.36)	(2.29)	(4.05)	(4.62)
p-value	0.14	0.32	0.00	0.07	0.27	0.00
# Obs.	79	79	79	77	77	77
	$\Delta t = 6$ months			$\Delta t = 12$ months		
R^2	17%	28%	36%	20%	27%	37%
α_0	2.19×10^{-3}	7.06×10^{-4}	2.54×10^{-3}	4.05×10^{-3}	2.55×10^{-3}	2.76×10^{-3}
	(1.47)	(0.45)	(2.72)	(2.39)	(1.43)	(2.15)
α_1	0.692	0.892	0.576	0.570	0.716	0.667
	(3.12)	(4.11)	(5.27)	(3.11)	(3.81)	(6.32)
p-value	0.34	0.87	0.00	0.06	0.32	0.00
# Obs.	74	74	74	68	68	68

This table displays the results of the forecasting regression $\sigma_{t+\Delta t}^{Actual} = \alpha_0 + \alpha_1 \times \sigma_t^{\text{Implied}} + \varepsilon_{t+\Delta t}$, where $\sigma_{t+\Delta t}^{Actual}$ is the volatility of the five-year yield between t and Δt estimated from the daily changes on the five-year discount yield and $\sigma_t^{\text{Implied}}$ is the volatility of the five-year yield between t and $t + \Delta t$ implied by a swaption pricing model at time t. Standard errors are corrected for autocorrelation on the residuals by Newey and West (1987) estimator. The p-values are for the Wald test with null hypothesis that $\alpha_0 = 0$ and $\alpha_1 = 1$. T-statistics are in parenthesis. The T-statistics are for the null hypothesis that $\alpha_i = 0$, $i = 0, 1$. The results indicate that the MRE model outperforms the benchmarks in forecasting future interest-rate volatility. The MRE forecasts are, however, biased in the sense that the null hypothesis that $\alpha_0 = 0$ and $\alpha_1 = 1$ is rejected in all regressions with implied volatilities generated by the MRE model.

Figure 1 - MBA refinancing index and interest-rate volatility
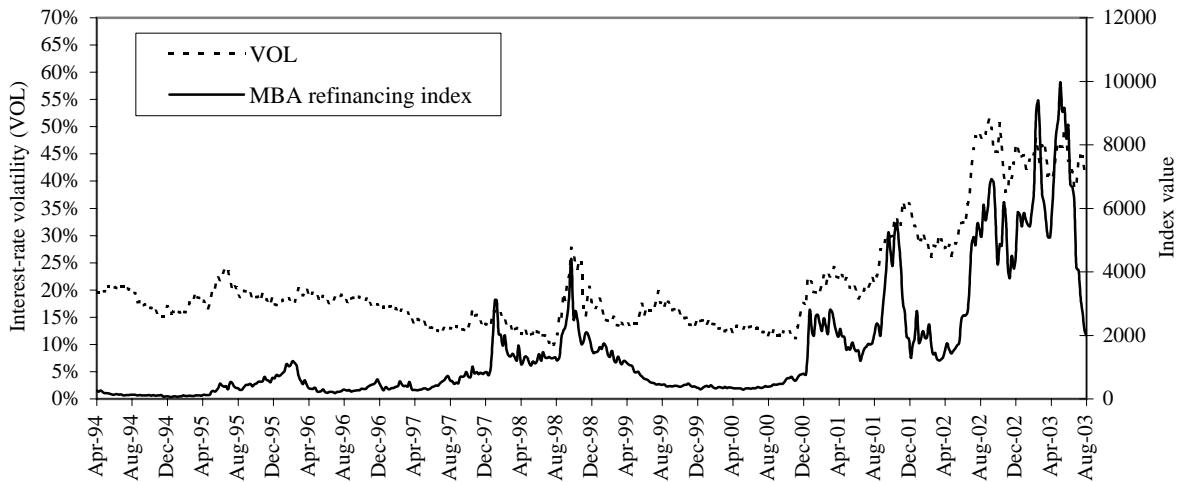
This figure displays the Mortgage Bankers Association (MBA) refinancing index and the average Black's volatility of the swaptions with three months to maturity (VOL). The index is based on the number of applications for mortgage refinancing. The index is calculated every week and is based on the weekly survey of the MBA. The index is seasonally adjusted. This figure shows a series of spikes in refinancing activity. These spikes are refinancing waves caused by a drop in the mortgage rate to levels substantially below the current average coupon of the mortgage universe. The spikes in mortgage refinancing are generally accompanied by spikes in interest-rate volatility.

Figure 2a - MBA refinancing index and prepayment speed of the mortgage universe
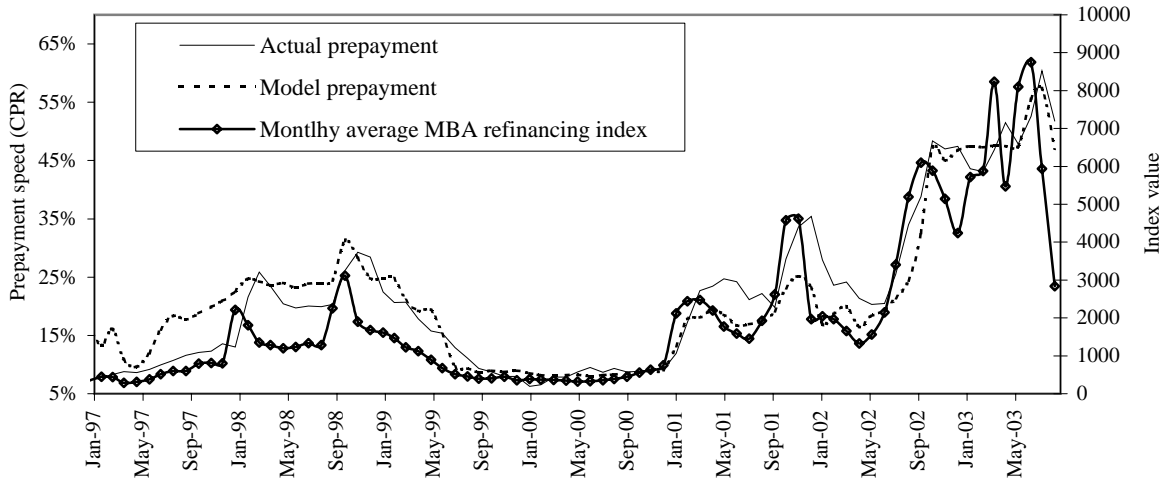
Figure 2b - Prepayment speed as function of refinancing incentive
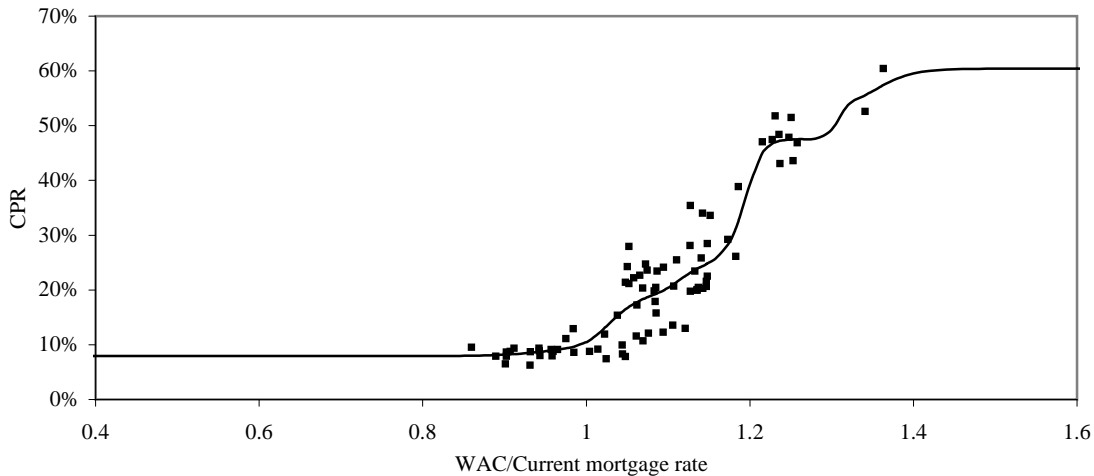
Figure 2a displays the time series of the proxy of the CPR of the mortgage universe, the CPR estimated with the prepayment model described in Section 4.2.1, and the monthly average of the MBA refinancing index. Note that these series trend together and the MBA refinancing index anticipates the CPR in the mortgage universe. This is unsurprising because there is a delay between the application for refinancing and the actual prepayment of a mortgage. Figure 2b displays the average prepayment speed of the MBS universe as function of refinancing incentive. The refinancing incentive is defined as the proxy of the weighted-average coupon (WAC) of the mortgage universe divided by the proxy of the mortgage rate. The prepayment model is non-parametrically estimated with data between January 1997 and August 2003. Each dot in this figure represents one observation.
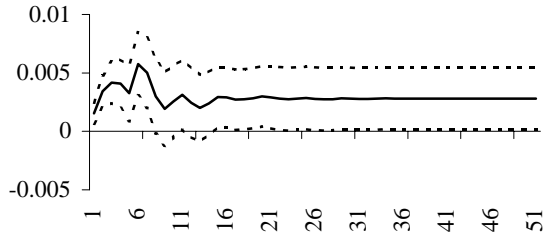
Figure 3a - Response of VOL to MBAREFI
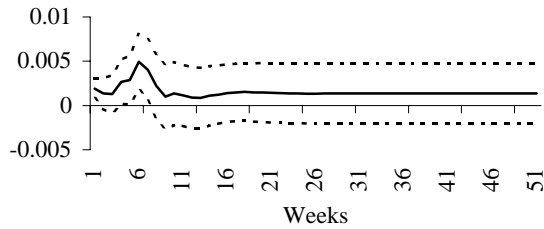
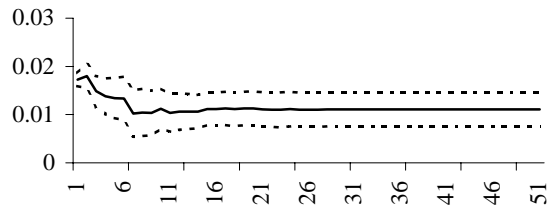Figure 3e - Response of MBAREFI to MBAREFI
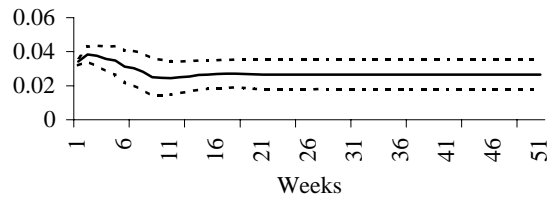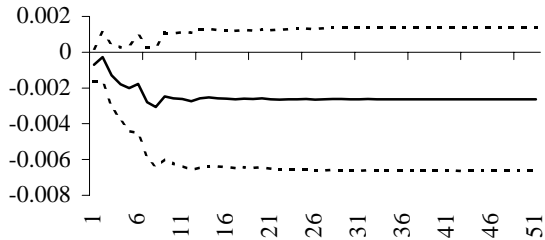
Figure 3b - Response of VOL to LIBOR6
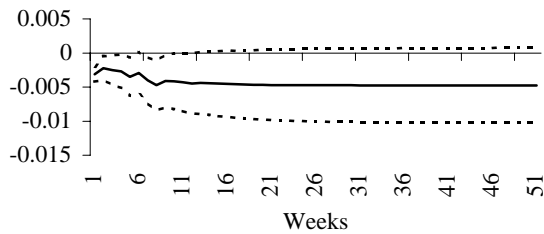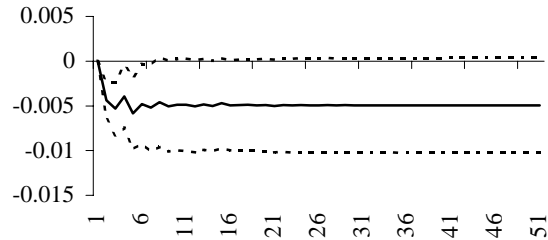
Figure 3f - Response of MBAREFI to LIBOR6
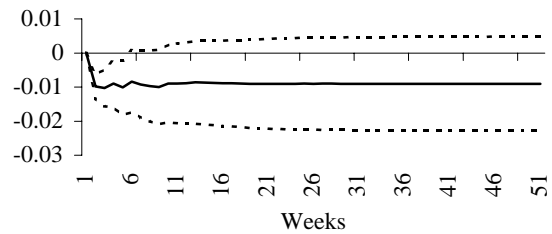
Figure 3c - Response of VOL to SLOPE
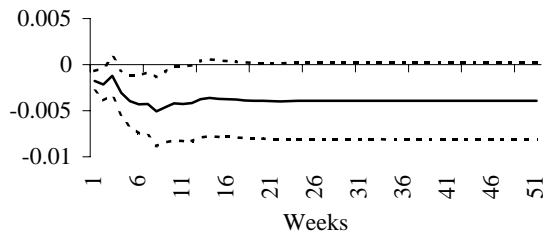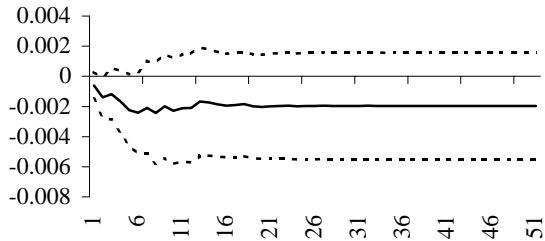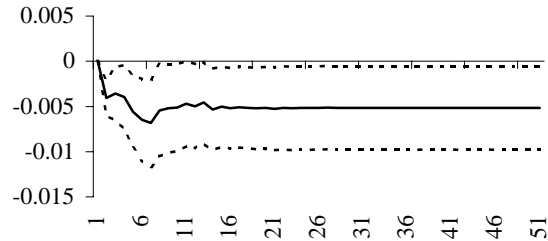
Figure 3g - Response of MBAREFI to SLOPE
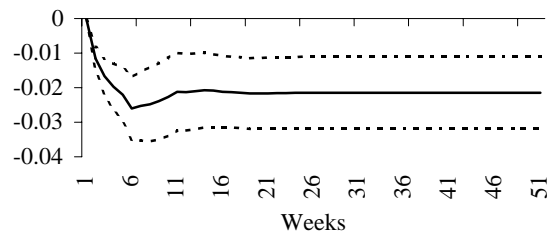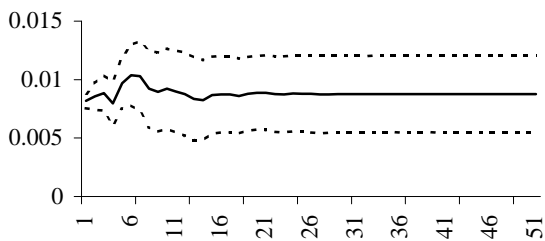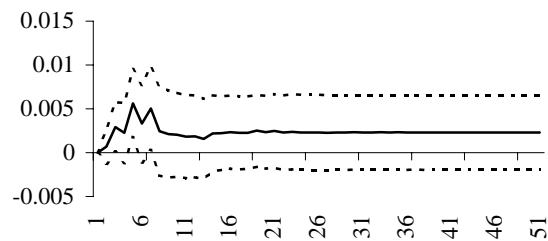
Figure 3d - Response of VOL to VOL
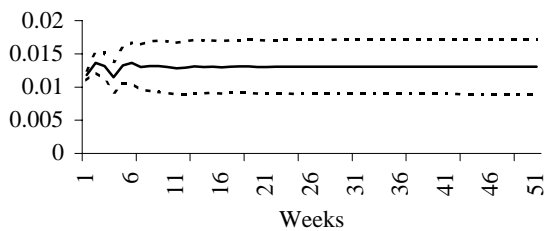
Figure 3h - Response of MBAREFI to VOL
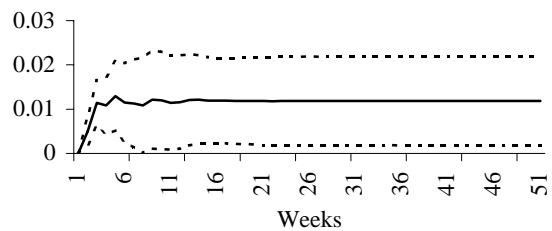
These figures plot the cumulative impulse response functions of VOL and MBAREFI in the estimated VAR. The VAR is estimated on the first differences of four variables: the mortgage refinancing activity (MBAREFI); the six-month Libor (LIBOR6); the difference between the five-year discount yield and the six-month Libor (SLOPE); and the average implied volatility of short-term swaptions (VOL). Figures 3a to 3d plot the response on the variable VOL to a shock in each variable. Figures 3e to 3h plot the response on the variable MBAREFI to a shock in each variable. The shock in each variable is equal to one standard deviation in its ortogonalized innovation. The dashed lines represent two standard deviations estimated by 10,000 Monte Carlo runs. The sample period is from April 1994 to August 2003.

Figure 4a - Response of VOL to MBAREFI

Figure 4e - Response of MBAREFI to MBAREFI

Figure 4b - Response of VOL to LIBOR6

Figure 4f - Response of MBAREFI to LIBOR6

Figure 4c - Response of VOL to SLOPE

Figure 4g - Response of MBAREFI to SLOPE

Figure 4d - Response of VOL to VOL

Figure 4h - Response of MBAREFI to VOL

These figures plot the cumulative impulse response functions of VOL and MBAREFI in the estimated VAR. The VAR is estimated on the first differences of four variables: the mortgage refinancing activity (MBAREFI); the six-month Libor (LIBOR6); the difference between the five-year discount yield and the six-month Libor (SLOPE); and the average implied volatility of short-term swaptions (VOL). Figures 4a to 4d plot the response on the variable VOL to a shock in each variable. Figures 4e to 4h plot the response on the variable MBAREFI to a shock in each variable. The shock in each variable is equal to one standard deviation in its ortogonalized innovation. The dashed lines represent two standard deviations estimated by 10,000 Monte Carlo runs. The sample period is from April 1994 to December 2000.

Figure 5 - RMSE of each calibrated model
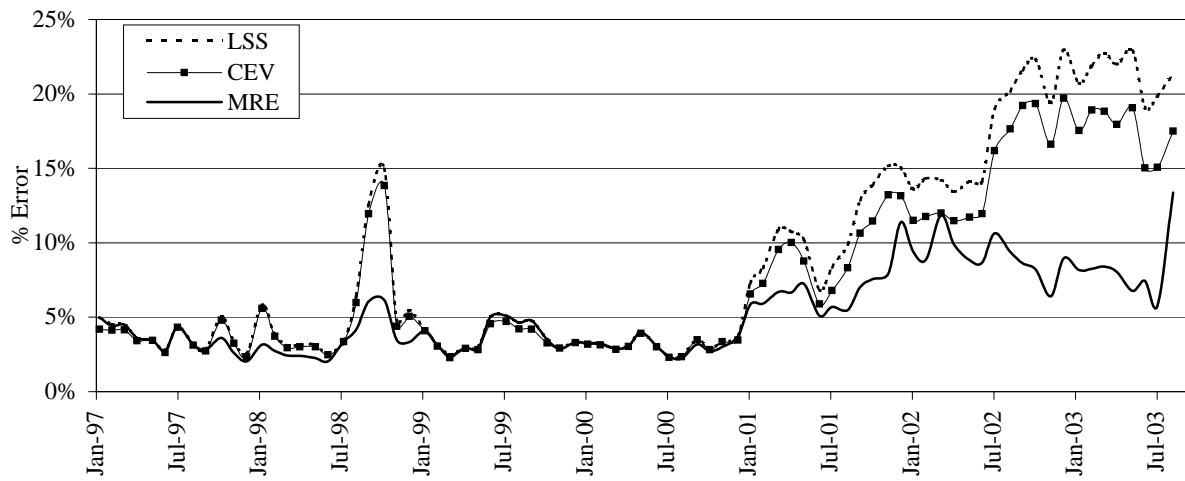
This figure displays the RMSEs of three calibrated models: The LSS model, the CEV model and the model with mortgage refinancing effects (MRE). The models are calibrated monthly to 34 swaption prices. The difference between the performance of the models is specially high in periods of high mortgage refinancing activity. See, for instance, early 1998, late 1998 and the period between 2001 and 2003.
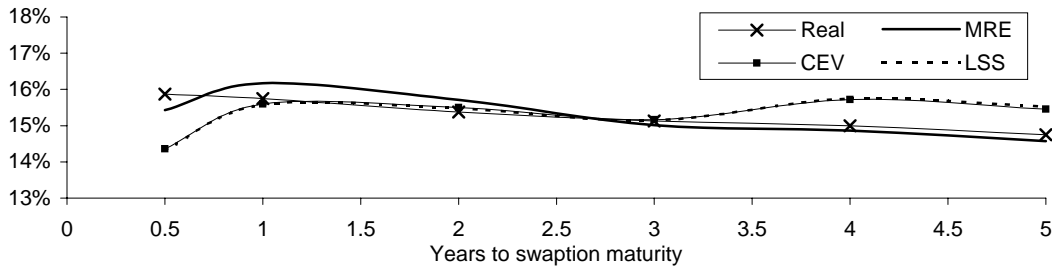
Figure 6a - Average implied volatility in January 1998

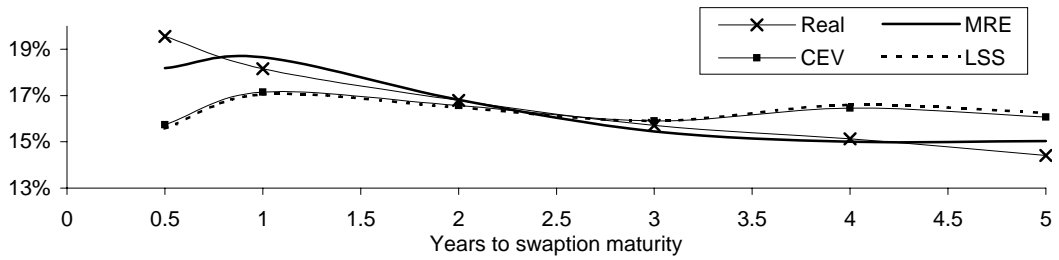Figure 6b - Average implied volatility between August 1998 and October 1998

Figure 6c - Average implied volatility between January 2001 and August 2003
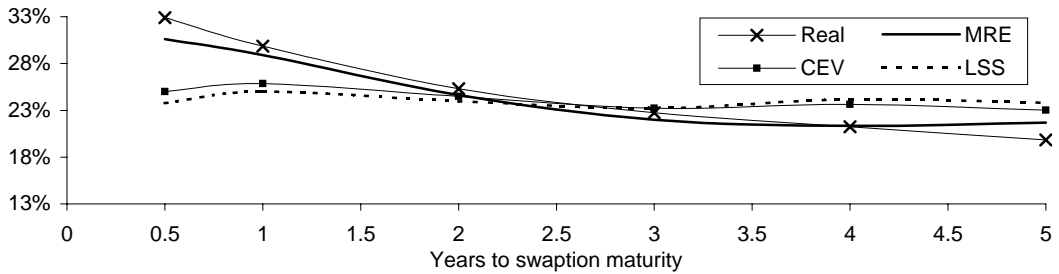

Figure 6d - Average implied volatility in periods of low refinancing activity
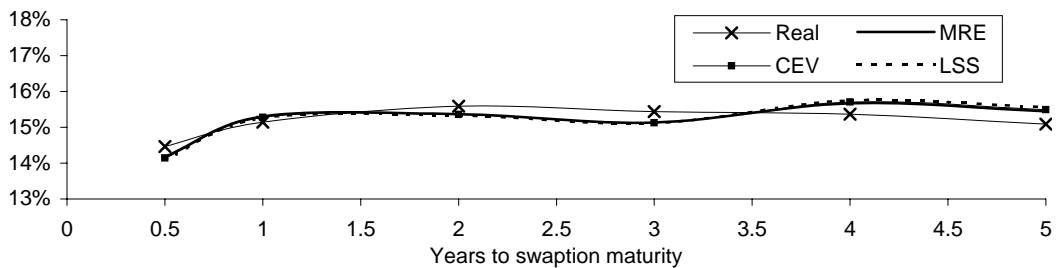
Figure 6a to 6c present the average of the Black's volatility of swaptions by time to maturity in periods when the refinancing activity is high and the models without refinancing effects have a RMSE greater than five percent. Figure 6d presents the term structure of swaption volatilities in the other periods. The lines denoted by "Real" are the Black's volatility of the actual swaption prices. The other lines are the Black's volatilities of the swaption prices calculated by each model. The term structure of swaption volatilities is downward-sloped in periods of high refinancing and practically flat in periods of low refinancing activity. The MRE model adapts well to the change in the term structure of swaption volatilities.