

# Welfare Effects of Commodity Price and Exchange Rate Volatilities in a Multi-Sector SOE Model\*

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Preliminary and incomplete version. Comments are very welcomed.

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## Abstract

This paper develops a multi-sector New Keynesian model for a small open economy, which includes commodity, tradable, non-tradable, and import sectors. Wages and prices are sticky in each sector à la Calvo-Yun style. Labour and capital are not perfectly mobile across sectors. Commodity output, whose price is exogenously set in foreign currency, is divided between exports and home uses as direct inputs in the production of tradable and non-tradable goods. We estimate structural parameters of monetary policy, wages and prices stickiness, capital adjustment costs, and exogenous processes shocks using Canadian and U.S. data and a maximum likelihood procedure. The model is then used to evaluate the effects of commodity price shocks on exchange rate volatility and on the welfare employing second-order solution method. The main results show that commodity shocks are one of the most important sources of real exchange rate variations. Welfare effects of exchange rate variability are much smaller with a flexible exchange rate regime, but they are considerably large when adopting a fixed exchange rate regime.

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## 1. Introduction

Exchange rates and commodity prices are among the most volatile variables in an open-economy macroeconomics.<sup>1</sup> Much theoretical research has been devoted to understanding the causes of exchange rate volatility and to explaining its macroeconomic effects. Other empirical studies have, however, examined the role of commodity price fluctuations in explaining exchange rate movements and volatility.

For Canada, Amano and van Norden (1993) find a long run-relationship between the real exchange rate and real commodity prices, splits into energy and non-energy components. The appreciation of the Canadian dollar usually coincides with rising real commodity prices. We note that the recent increases in real commodity prices, which have occurred since 2001, have led to rapid and significant appreciations in the Canadian exchange rates. The data also show that the Canadian real exchange rate is highly correlated with real commodity prices.<sup>2</sup>

The purpose of this paper is to quantitatively examine the contribution of real commodity price fluctuations in exchange rate volatility and to assess the welfare effects of exchange rate volatility under alternative exchange rate regimes. This work is related to previous studies that analyze exchange rate regimes and their real effects. Macklem, Osakwe, Pioro, and Schembri (2000) examine the economic effects of alternative exchange rate regimes in Canada, focusing on the role of terms of trade shocks. They find that a flexible exchange rate regime isolates the Canadian economy from external shocks. Kollmann (2005) analyzes the effects of pegged and floating exchange rates in a two-country model. Obstfeld and Rogoff (2000) and Devereux and Engel (2003) compare the welfare effects of pegs and floats, using standard sticky price models, but generate insufficient exchange rate volatility. Bergin (2004) presents a quantitative investigation of the welfare effects of exchange rate variability in a two-country model. He finds that the effects of exchange rate volatility appear to be small for most studied economies.

We consider a multi-sector New Keynesian model of a small open economy that consists of monop-

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<sup>1</sup>Using HP-filtered Canadian series, Table 6, hereafter, shows that real exchange rate and commodity prices volatilities are 3.8% and 7.35% for the period 1981–2005, while output volatility is 1.44%.

<sup>2</sup>Table 6 reports that the correlation between the HP-filtered series of the real exchange rate and real commodity price is -0.62 for the period 1981–2005, meaning that an increase in real commodity prices implies an appreciation of the real exchange rate.

olistically competitive households; three production sectors (commodity, tradables, and non-tradables), import sector, and a government (a central bank). The economy is disturbed by nine shocks: six domestic shocks—commodity price, natural resource, tradable-sector technology, non-tradable-sector technology, government spending, and monetary policy—and three foreign shocks— world interest rates, inflation, and output.

We assume that labour and capital are not perfectly mobile between production sectors, where nominal wages are sticky and it is costly to adjust capital stocks. We also introduce nominal price rigidity in tradable, non-tradable, and import sectors. Nominal wage and price rigidities are modelled à la Calvo-Yun style contracts and solved using Schmitt-Grohé and Uribe's (2004a) non-linear recursive procedure. Commodity output, which is either exported abroad or used as inputs in the production of tradable and non-tradable goods, is produced using capital, labour, and a natural resource factor. Commodity prices are exogenously set in world markets and denominated in foreign currency (the U.S. dollar). The central bank conducts its monetary policy by following a standard Taylor-type rule.

The model's structural parameters are either calibrated using common values or estimated using a maximum-likelihood procedure with a Kalman filter. We estimate two versions of the model: (1) under the assumption of local currency pricing (LCP) and (2) price-to-market (PTM). To estimate the non-calibrated parameters, we solve the models by taking a log-linear approximation of the equilibrium systems around deterministic steady-state values. We estimate only the parameters that do not affect the steady-state equilibrium, for the complexity of its solution. We estimate, therefore, only monetary policy, capital-adjustment costs, nominal wage rigidities, nominal price rigidities, and exogenous shock process parameters. The estimates mainly indicate that (1) price and wage rigidities are higher in all sectors, (2) real commodity price shocks are highly persistent and volatile, and (3) commodity price shocks significantly contribute to exchange rate volatility.

To calculate the welfare effects of commodity prices and real exchange rate variability for alternative flexible and fixed exchange rate regimes, we use a second-order approximation procedure to solve the model. Then, welfare measures are calculated as an unconditional expectation of utility function in deterministic steady-state values. The main results show that the presence of the risk, related to model's

structural shocks, has negative effects on all examined variables. These negative effects are much higher when the exchange rate is fixed. In the estimated LCP model, the overall welfare effect with the flexible exchange rate regime, measured by consumption compensation, is about -0.31%, divided into the level effect, -0.29%, and the variance effect, -0.016%, while it is about -9% with the fixed exchange rate.

Thus, commodity price and exchange rate volatilities lead to small welfare effects of uncertainty for the economy with flexible exchange rate regime. However, these welfare effects are considerably large when adopting a fixed exchange rate regime. These results are similar to those found in Bergin (2004) using a two-country model.

This paper is organized as follows. Section 2 presents the salient features of the model. Section 3 describes the data and the calibration procedures. Section 4 reports and discusses the estimation and simulation results. Section 5 measures and discusses the welfare effects of commodity and exchange rate volatilities. Section 6 offers some conclusions.

## **2. The Model**

We consider a small open economy with a continuum of households, a perfectly competitive commodity-producing firm, a continuum of tradable and non-tradable intermediate-goods producing firms, a continuum of intermediate-foreign-goods importers, and a central bank. Households are monopolistically competitive in the labour market, and there is monopolistic competition in intermediate goods markets. Domestic- and imported-intermediate goods are used by a perfectly competitive firm to produce a final good that is divided between consumption and investments. Nominal wages, domestic- and imported-intermediate-goods prices are sticky à la Calvo-Yun style contracts.

### **2.1 Households**

The economy is populated by a continuum of households indexed by  $j$  ( $j \in [0,1]$ ). Each household  $j$  has preferences defined over consumption,  $c_{jt}$ , real balances,  $M_{jt}/P_t$ , and labour effort  $h_{jt}$ . Preferences are

described by the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{jt}, M_{jt}/P_t, h_{jt}),$$

where  $E_t$  denotes the mathematical expectations operator conditional on information available at  $t$ ,  $\beta \in (0, 1)$  is a subjective discount factor, and  $u(\cdot)$  is a utility function, which is assumed to be strictly concave, strictly increasing in  $c_{jt}$  and  $M_{jt}/P_t$ , and strictly decreasing in  $h_{jt}$ .

The single-period utility function is specified as

$$u(\cdot) = \frac{c_{jt}^{1-\tau}}{1-\tau} + \frac{b(M_{jt}/P_t)_t^{1-\nu}}{1-\nu} - h_{jt}^{\frac{\varsigma}{1+\varsigma}}, \quad (1)$$

where  $h_{jt} = h_{T,jt}^{\frac{1+\varsigma}{\varsigma}} + h_{N,jt}^{\frac{1+\varsigma}{\varsigma}} + h_{X,jt}^{\frac{1+\varsigma}{\varsigma}}$ ;  $h_{T,jt}$ ,  $h_{N,jt}$ , and  $h_{X,jt}$ , represent hours worked by household  $j$  in tradable, non-tradable, and commodity sectors, respectively; the preference parameters,  $\tau$ ,  $\nu$ , and  $\varsigma$  are strictly positive.

It is assumed that household  $j$  is a monopoly supplier of differentiated labour services to the three production sectors indexed by  $i (= N, T, X)$ . The household  $j$  sells these services to a representative competitive firm that transforms them into aggregate labour inputs supplied to each sector  $i$  using the following technology:

$$H_{i,t} = \left( \int_0^1 h_{i,jt}^{\frac{\vartheta-1}{\vartheta}} dj \right)^{\frac{\vartheta}{\vartheta-1}}, \quad i = N, T, X, \quad (2)$$

where  $H_{N,t}$ ,  $H_{T,t}$ , and  $H_{X,t}$  denote the labour supply to non-tradable, tradable, and commodity sectors, respectively; and  $\vartheta > 1$  is the constant elasticity of substitution in the labour market.

The demand curve for each type of labour in the sector  $i$  is given by

$$h_{i,jt} = \left( \frac{W_{i,jt}}{W_{i,t}} \right)^{-\vartheta} H_{i,t}, \quad (3)$$

where  $W_{i,jt}$  is the nominal wage of household  $j$  in the sector  $i$ , and  $W_{i,t}$  is the nominal wage index in sector  $i$ , which satisfies:

$$W_{i,t} = \left( \int_0^1 (W_{i,jt})^{1-\vartheta} dj \right)^{\frac{1}{1-\vartheta}}. \quad (4)$$

Household  $j$  takes  $H_{i,t}$  and  $W_{i,t}$  as given and considers them to be beyond its control.

It is assumed that households have access to incomplete international financial markets, in which they can buy or sell bonds denominated in foreign currency.

Household  $j$  enters period  $t$  with  $k_{i,jt}$  units of capital in sector  $i$ ,  $M_{jt-1}$  units of nominal balances,  $B_{jt-1}$  units of domestic treasury bonds, and  $B_{jt-1}^*$  units of foreign bonds denominated in foreign currency. During period  $t$ , the household supplies labour and capital to firms in all production sectors and receives total factor payment  $\sum_{i=N,T,X} (Q_{i,t}k_{i,jt} + W_{i,jt}h_{i,jt})$ , where  $Q_{i,t}$  is the nominal rental rate of capital in sector  $i$ , and receives factor payment of natural resources,  $\varpi_j P_{L,t}L_t$ , where  $P_{L,t}$  is the price of the natural resource input  $L_t$  and  $\varpi_j$  is household  $j$ 's share of natural resource payments.<sup>3</sup> Furthermore, household  $j$  receives a lump-sum transfer from the central bank  $T_{jt}$ , and dividend payments from intermediate goods producing firms  $D_{jt}$ . The household uses some of its funds to purchase the final good at the nominal price  $P_t$ , which it then divides between consumption and investment in each production sector. Wage stickiness is introduced through Calvo-Yun style nominal wage contracts.

The budget constraint of the household  $j$  is given by:

$$\begin{aligned} & P_t(c_{jt} + i_{jt}) + M_{jt} - M_{jt-1} + B_{jt}/R_t + e_t B_{jt}^*/(\kappa_t R_t^*) \\ & \leq \sum_{i=N,T,X} (Q_{i,t}k_{i,jt} + W_{i,jt}h_{i,jt}) + B_{jt-1} + e_t B_{jt-1}^* + \varpi_j P_{L,t}L_t + D_{jt}, \end{aligned} \quad (5)$$

where  $i_t = i_{N,t} + i_{T,t} + i_{X,t}$  is total investment in non-tradable, tradable, and commodity production sectors, respectively; and  $D_{jt} = D_{N,jt} + D_{T,jt} + D_{F,jt}$  is the total profit from intermediate-goods producers and importers.

The capital stock in sector  $i$  evolves according to:

$$k_{i,jt+1} = (1 - \delta)k_{i,jt} + i_{i,jt} - \Psi(k_{i,jt+1}, k_{i,jt}), \quad (6)$$

where  $\delta \in (0, 1)$  is the capital depreciation rate common to all sectors, and  $\Psi(\cdot) = \frac{\psi_i}{2} \left( \frac{k_{i,jt+1}}{k_{i,jt}} - 1 \right)^2 k_{i,jt}$  is the sector  $i$ 's capital-adjustment cost function that satisfies  $\Psi(0) = 0$ ,  $\Psi'(\cdot) > 0$  and  $\Psi''(\cdot) < 0$ .

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<sup>3</sup>Note that,  $\int_0^1 \varpi_j dj = 1$ .

The foreign bond return rate,  $\kappa_t R_t^*$ , depends on the world interest rate  $R_t^*$  and a country-specific risk premium,  $\kappa_t$ . The world interest rate evolves exogenously according to

$$\log(R_t^*) = (1 - \rho_{R^*}) \log(R^*) + \rho_{R^*} \log(R_{t-1}^*) + \varepsilon_{R^*,t}, \quad (7)$$

where  $R^*$  is a steady-state equilibrium value,  $\rho_{R^*} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{R^*,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{R^*}$ .

The country-specific risk premium is increasing in the foreign-debt-to-GDP ratio. It is given by

$$\kappa_t = \exp\left(-\chi \frac{e_t \tilde{B}_t^*/P_t^*}{P_t Y_t}\right), \quad (8)$$

where  $Y_t$  is total real GDP of the domestic economy,  $\tilde{B}_t^*$  is the total level of indebtedness of the economy, and  $P_t^*$  is foreign price index. The introduction of this risk premium ensures that the model has a unique steady state. It is assumed that the world inflation,  $\pi_t^* = P_t^*/P_{t-1}^*$ , evolves according to:

$$\log(\pi_t^*) = (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log(\pi_{t-1}^*) + \varepsilon_{\pi^*,t},$$

where  $\pi^*$  is a steady-state equilibrium value,  $\rho_{\pi^*} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{\pi^*,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{\pi^*}$ .

The household  $j$  chooses  $c_{jt}, m_{jt}, k_{i,jt+1} (i = N, T, X), B_{jt}, B_{jt}^*$  to maximize its lifetime utility, subject to the budget constraint, (5), and (6).

Household  $j$ 's first-order conditions, expressed in real terms, are:

$$c_{jt}^{-\tau} = \lambda_{jt}; \quad (9)$$

$$b m_{jt}^{-v} = \lambda_{jt} - \beta E_t \left[ \frac{\lambda_{jt+1}}{\pi_{t+1}} \right]; \quad (10)$$

$$\beta E_t \left[ \frac{\lambda_{jt+1}}{\lambda_{jt}} \left( q_{i,t+1} + 1 - \delta + \psi_i \left( \frac{k_{i,jt+2}}{k_{i,jt+1}} - 1 \right) \frac{k_{i,jt+2}}{k_{i,jt+1}} \right) \right] = \psi_i \left( \frac{k_{i,jt+1}}{k_{i,jt}} - 1 \right) + 1; \quad (11)$$

$$\frac{\lambda_{jt}}{R_t} = \beta E_t \left[ \frac{\lambda_{jt+1}}{\pi_{t+1}} \right]; \quad (12)$$

$$\frac{\lambda_{jt}}{\kappa_t R_t^*} = \beta E_t \left[ \frac{\lambda_{jt+1} s_{t+1}}{s_t \pi_{t+1}^*} \right], \quad (13)$$

in addition to the budget constraint, equation (5), where  $\lambda_{jt}$  is the Lagrangian multiplier associated with the budget constraint;  $m_{jt} = M_{jt}/P_t$ ,  $q_{i,t} = Q_{i,t}/P_t$ , and  $s_t = e_t P_t^*/P_t$  denote household  $j$ 's real balances, real capital return in sector  $i$ , and real exchange rate, respectively.

Furthermore, there are three first-order conditions for setting the nominal wages in sector  $i$ ,  $W_{i,jt}$ , when the household  $j$  is allowed to revise its nominal wages. This happens with probability  $(1 - \varphi_i)$  for the sector  $i$ , at the beginning of each period  $t$ .

Therefore, the household  $j$  sets  $\widetilde{W}_{i,jt}$  that maximizes the flow of its expected utility, so that

$$\max_{\widetilde{W}_{i,jt}} E_0 \left[ \sum_{l=0}^{\infty} (\beta\phi)^l \left\{ u(c_{jt+l}, h_{i,jt+l}) + \lambda_{jt+l} \pi^l \widetilde{W}_{i,jt} h_{i,jt+l} / P_{t+l} \right\} \right],$$

subject to (2). See Appendix B for wage setting details.

The first-order condition derived for  $\widetilde{W}_{i,jt}$  is

$$E_0 \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{jt+l} \left( \frac{\widetilde{W}_{i,jt}}{W_{i,t+l}} \right)^{-\vartheta} H_{i,t+l} \left\{ \zeta_{i,t+l} - \frac{\vartheta - 1}{\vartheta} \frac{\widetilde{W}_{i,jt}}{P_t} \frac{\pi^l P_t}{P_{t+l}} \right\} \right] = 0, \quad (14)$$

where  $\zeta_{i,t} = -\frac{\partial U / \partial h_{i,jt}}{\partial U / \partial c_{jt}}$  is the marginal rate of substitution between consumption and labour type  $i$ .

Dividing equation (14) by  $P_t$  and rearranging yield:

$$\widetilde{w}_{i,jt} = \frac{\vartheta}{\vartheta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{jt+l} \zeta_{i,t+l} (\widetilde{w}_{i,jt} / w_{i,jt+l})^{-\theta} H_{i,t+l} \prod_{k=1}^l \pi_{t+k}^{\theta}}{E_t \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{jt+l} (\widetilde{w}_{i,jt} / w_{i,jt+l})^{-\theta} H_{i,t+l} \prod_{k=1}^l \pi_{t+k}^{\theta-1}},$$

where  $\widetilde{w}_{i,jt} = \widetilde{W}_{i,jt} / P_t$  and  $w_{i,jt} = W_{i,jt} / P_t$ .

The nominal wage index in sector  $i$  evolves over time according to the following recursive equation:

$$(W_{i,t})^{1-\vartheta} = \varphi_i (\pi W_{i,t-1})^{1-\vartheta} + (1 - \varphi_i) (\widetilde{W}_{i,t})^{1-\vartheta}, \quad (15)$$

where  $\widetilde{W}_{i,jt}$  is the average wage of those workers who are allowed to revise their wage at period  $t$  in sector  $i$ .

Dividing (15) by  $P_t$  yields

$$(w_{i,t})^{1-\vartheta} = \varphi \left( \frac{\pi w_{i,t-1}}{\pi_t} \right)^{1-\vartheta} + (1 - \varphi_i) (\widetilde{w}_{i,t})^{1-\vartheta}, \quad (16)$$

Equations (2.1) and (16) allows us to derive the New Keynesian Phillips curve.



## 2.2 Aggregator

There is an aggregator that acts in a perfectly competitive market and uses a fraction of tradable (manufactured) output,  $Y_{T,t}^d$ , non-tradable output,  $Y_{N,t}$ , and imports,  $Y_{F,t}$  to produce a final good  $Z_t$  according to the following CES technology:

$$Z_t = \left[ \omega_T^\nu \left( Y_{T,t}^d \right)^\nu + \omega_N^\nu Y_{N,t}^\nu + \omega_F^\nu Y_{F,t}^\nu \right]^{\frac{1}{\nu}}, \quad (17)$$

where  $\omega_T + \omega_N + \omega_F = 1$ ;  $\omega_T$ ,  $\omega_N$ , and  $\omega_F$  denote share of tradable, non-tradable, and imported goods in the final good, respectively; and  $\nu > 0$  is the elasticity of substitution between domestic and imported goods used in the production of the final good. It is also the price elasticity of domestic and imported goods of demand functions.

Inputs in (17) are produced with a continuum of differentiated goods using the following CES technology:

$$Y_{\iota,t} = \left( \int_0^1 (Y_{\iota,jt})^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad \text{for } \iota = F, N, T. \quad (18)$$

where  $\theta > 1$  is the constant elasticity of substitution between the intermediate goods. The demand function for domestic tradable-, non-tradable- and imported-intermediate goods  $\iota (= F, N, T)$  are:

$$Y_{\iota,jt} = \left( \frac{P_{\iota,jt}}{P_{\iota,t}} \right)^{-\theta} Y_{\iota,t}, \quad (19)$$

Domestic tradable, non-tradable, and imported goods prices satisfy

$$P_{\iota,t} = \left( \int_0^1 (P_{\iota,jt})^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (20)$$

Given  $P_t$ ,  $P_{T,t}$ ,  $P_{N,t}$ , and  $P_{F,t}$ , the aggregator chooses  $Y_{T,t}^d$ ,  $Y_{N,t}$ , and  $Y_{F,t}$  to maximize its profit. Its maximization problem is

$$\max_{\{Y_{N,t}, Y_{T,t}^d, Y_{F,t}\}} P_t Z_t - P_{N,t} Y_{N,t} - P_{T,t} Y_{T,t}^d - P_{F,t} Y_{F,t}, \quad (21)$$

subject to (17). Profit maximization implies the following demand functions for non-tradable, tradable, and imported goods:

$$Y_{N,t} = \omega_N \left( \frac{P_{N,t}}{P_t} \right)^{-\nu} Z_t, \quad Y_{T,t}^d = \omega_T \left( \frac{P_{T,t}}{P_t} \right)^{-\nu} Z_t, \quad \text{and} \quad Y_{F,t} = \omega_F \left( \frac{P_{F,t}}{P_t} \right)^{-\nu} Z_t. \quad (22)$$

Thus, as the relative prices of domestic and imported goods rise, the demand for domestic and imported goods decreases.

The zero-profit condition implies that the final-good price level, which is the consumer-price index (CPI), is linked to tradable-, non-tradable- and imported-goods prices through:

$$P_t = \left[ \omega_T (P_{T,t})^{1-\nu} + \omega_N P_{N,t}^{1-\nu} + \omega_F P_{F,t}^{1-\nu} \right]^{1/(1-\nu)}. \quad (23)$$

The final good is divided between consumption,  $c_t$ , private investment,  $i_t$ , and government spending,  $g_t$ , so that  $Z_t = c_t + i_t + g_t$ .

### 2.3 Commodity sector

Production in the commodity sector is modelled to capture the importance of natural resources in the Canadian economy. In this sector, there is a representative firm that produces commodity goods  $Y_{X,t}$  using capital,  $K_{X,t}(= \int_0^1 k_{X,jt} dj)$ , labour,  $H_{X,t}$ , and a natural-resource factor,  $L_t$ . The production function is the following Cobb-Douglas technology

$$Y_{X,t} \leq (K_{X,t})^{\alpha_X} (H_{X,t})^{\gamma_X} (L_t)^{\eta_X}, \quad \alpha_X, \gamma_X, \eta_X \in (0, 1), \quad (24)$$

and  $\eta_X = 1 - \alpha_X - \gamma_X$ ;  $\alpha_X$ ,  $\gamma_X$ , and  $\eta_X$  are shares of capital, labor, and natural resources in the production of commodities, respectively. It is assumed that the supply of  $L_t$  evolves exogenously according to

$$\log(L_t) = (1 - \rho_L) \log(L) + \rho_L \log(L_{t-1}) + \varepsilon_{L,t}, \quad (25)$$

where  $L$  is a steady-state value,  $\rho_L \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{L,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_L$ .

We assume that commodity output,  $Y_{X,t}$ , is divided between exports and domestic uses as direct inputs in tradable and non-tradable sectors.

Nominal commodity price,  $P_{X,t}^*$ , is determined exogenously in the world markets and denominated in the U.S. dollar. Multiplying  $P_{X,t}^*$  by the nominal exchange rate  $e_t$  yields the commodity producer's revenues in terms of domestic currency. The commodity-producing firm takes commodity prices and the

nominal exchange rate,  $e_t$ , as given. Therefore, given  $e_t$ , the nominal exchange rate,  $P_{X,t}^*$ ,  $Q_{X,t}$ ,  $W_{X,t}$ , and  $P_{L,t}$ , the price of the natural-resource factor, the commodity-producing firm chooses  $K_{X,t}$ ,  $H_{X,t}$ , and  $L_t$  that maximize its real profit flows. Its maximization problem is<sup>4</sup>

$$\max_{K_{X,t}, H_{X,t}, L_t} \left\{ \frac{e_t P_{X,t}^*}{P_t} Y_{X,t} - \frac{Q_{X,t}}{P_t} K_{X,t} - \frac{W_{X,t}}{P_t} H_{X,t} - \frac{P_{L,t}}{P_t} L_t \right\},$$

subject to the production technology, equation (24).

The first-order conditions, in real terms, derived from the commodity producer's optimization problem are:

$$\frac{\alpha_X Y_{X,t}}{K_{X,t}} = \frac{q_{X,t}}{s_t p_{X,t}^*}; \quad (26)$$

$$\frac{\gamma_X Y_{X,t}}{H_{X,t}} = \frac{w_{X,t}}{s_t p_{X,t}^*}; \quad (27)$$

$$\frac{\eta_X Y_{X,t}}{L_t} = \frac{p_{L,t}}{s_t p_{X,t}^*}; \quad (28)$$

$$Y_{X,t} = (K_{X,t})^{\alpha_X} (H_{X,t})^{\gamma_X} (L_t)^{\eta_X}, \quad (29)$$

where  $s_t = e_t P_t^*/P_t$  is the real exchange rate,  $p_{X,t}^* = P_{X,t}^*/P_t^*$  is real commodity prices, while  $q_{X,t} = Q_{X,t}/P_t$ ,  $w_{X,t} = W_{X,t}/P_t$ , and  $p_{L,t} = P_{L,t}/P_t$  are real capital returns, real wages and real natural resource prices in the commodity sector, respectively.

These first-order conditions give the optimal choice of inputs that maximize commodity producer's profits. The demand for  $K_{X,t}$ ,  $H_{X,t}$ , and  $L_t$  are given by equations (26)–(28), respectively. These equations stipulate that the marginal cost of each input must be equal to its marginal productivity. Because the economy is small, the demand for domestic exports and their prices are completely determined in the world markets and domestic exports are only a negligible fraction in the rest of the world spending.

We assume that the real commodity price,  $p_{X,t}^*$ , exogenously evolves according to

$$\log(p_{X,t}^*) = (1 - \rho_{pX}) \log(p_X^*) + \rho_{pX} \log(p_{X,t-1}^*) + \varepsilon_{pX,t}, \quad (30)$$

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<sup>4</sup>This profit maximization problem is static because there is no real or nominal frictions in commodity sector.

where  $p_X^*$  is the steady-state value of real commodity price,  $\rho_{PX} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{PX,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{PX}$ .

## 2.4 Tradable-intermediate-goods sector

Tradable-intermediate goods sector is indexed by  $T$ . There are a continuum of tradable-intermediate goods producing firms indexed by  $j \in [0, 1]$ . Firm  $j$  produces its intermediate-tradable good using the capital  $K_{T,jt} (= \int_0^1 k_{T,jt} dj)$ , labour,  $H_{T,jt}$ , and commodity input  $Y_{XT,jt}$ . Its production function is given by,

$$Y_{T,jt} \leq A_{T,t} (K_{T,jt})^{\alpha_T} (H_{T,jt})^{\gamma_T} (Y_{XT,jt})^{\eta_T}, \quad \alpha_T, \gamma_T, \eta_T \in (0, 1). \quad (31)$$

where  $A_{T,t}$  is a technology shock in the tradable sector. It evolves exogenously according to

$$\log(A_{T,t}) = (1 - \rho_{AT}) \log(A_T) + \rho_{AT} \log(A_{T,t-1}) + \varepsilon_{T,t}^A, \quad (32)$$

where  $A_T$  is the steady-state value of tradable technology shock,  $\rho_{AT} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{AT,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{AT}$ .

Domestic manufactured goods are divided between domestic use,  $Y_{T,jt}^d$ , and exports,  $Y_{T,jt}^e$ , so that  $Y_{T,jt} = Y_{T,jt}^d + Y_{T,jt}^e$ . The foreign demand function for domestic manufactured-goods exports, under the assumption of PTM, is given by:<sup>5</sup>

$$Y_{T,t}^e = \varkappa \left( \frac{P_{T,t}^*}{P_t^*} \right)^{-\nu} Y_t^*,$$

where  $Y_t^*$  is foreign output. The elasticity of demand for domestic manufactured-goods by foreigners is  $-\nu$ , and  $\varkappa > 0$  is a parameter determining the fraction of domestic manufactured-goods exports in foreign spending. The economy is small, so domestic manufactured-goods exports form an insignificant fraction of foreign expenditures, and have a negligible weight in the foreign price index.

<sup>5</sup>Under the LCP assumption,  $P_{T,t} = e_t P_{T,t}^*$ . Therefore, the foreign demand function for domestic manufactured-goods exports becomes  $Y_{T,t}^e = \varkappa \left( \frac{e_t P_{T,t}}{P_t^*} \right)^{-\nu} Y_t^*$ .

The foreign output is exogenous and evolves according to

$$\log(Y_t^*) = (1 - \rho_{Y^*}) \log(Y^*) + \rho_{Y^*} \log(Y_{t-1}^*) + \varepsilon_{Y^*,t}, \quad (33)$$

where  $Y^*$  is the steady-state value,  $\rho_{Y^*} \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{Y^*,t}$  is uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_{Y^*}$ .

The profit function for period  $t + l$ ,  $D_{T,jt+l}$ , is given by<sup>6</sup>

$$\begin{aligned} D_{T,jt+l} = & \pi^l \tilde{P}_{T,jt} Y_{T,jt+l}^d + \pi^{*l} e_{t+l} \tilde{P}_{T,jt}^* Y_{T,jt+l}^e - Q_{T,t+l} K_{T,jt+l} \\ & - W_{T,t+l} H_{T,jt+l} - e_{t+l} P_{X,t+l}^* Y_{XT,jt+l}. \end{aligned} \quad (34)$$

Given  $Q_{T,t}$ ,  $W_{T,t}$ , and  $P_t$ , the intermediate-goods producer  $j$  chooses  $K_{T,jt}$ ,  $H_{T,jt}$ , and  $Y_{XT,jt}$  that maximize its profit. The firm is allowed to revise its prices in domestic and foreign markets with probability  $(1 - \phi_T)$  for  $l$  period. Therefore, it sets the prices  $\tilde{P}_{T,jt}$  and  $\tilde{P}_{T,jt}^*$  that maximize the expected discounted flow of its profits. Under the assumption of PTM, the tradable (manufactured) goods producer's maximization problem is:

$$\max_{K_{T,jt}, H_{T,jt}, Y_{XT,jt}, \tilde{P}_{T,jt}, \tilde{P}_{T,jt}^*} E_0 \left[ \sum_{t=0}^{\infty} (\beta \phi_T)^l \lambda_{t+l} D_{T,jt+l} / P_{t+l} \right], \quad (35)$$

subject to (31), (34), and the following demand functions:

$$Y_{T,jt+l}^d = \left( \frac{\tilde{P}_{T,jt}}{P_{T,t+l}} \right)^{-\theta} Y_{T,t+l}^d, \quad \text{and} \quad Y_{T,jt+l}^e = \left( \frac{\tilde{P}_{T,jt}^*}{P_{T,t+l}^*} \right)^{-\theta} Y_{T,t+l}^e. \quad (36)$$

where the producer's discount factor is given by the stochastic process  $(\beta^l \lambda_{t+l})$ ;  $\lambda_{t+l}$  denotes the marginal utility of consumption in period  $t + l$ .

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<sup>6</sup>Under the assumption of local currency pricing,  $P_{T,jt} = e_t P_{T,jt}^*$ . In this case, the firm  $j$  sets only the price  $P_{T,jt}$  and its profit function is given by  $D_{T,jt+l} = \pi^l \tilde{P}_{T,jt} Y_{T,jt+l} - Q_{T,t+l} K_{T,jt+l} - W_{T,t+l} H_{T,jt+l} - e_{t+l} P_{X,t+l}^* Y_{XT,jt+l}$ .

The first-order conditions are:

$$\begin{aligned}
q_{T,t} &= \alpha_T \frac{Y_{T,jt} \xi_{T,t}}{K_{T,jt}}; \\
w_{T,t} &= \gamma_T \frac{Y_{T,jt} \xi_{T,t}}{H_{T,jt}}; \\
s_t p_{X,t}^* &= \eta_T \frac{Y_{T,jt} \xi_{T,t}}{Y_{XT,jt}}; \\
Y_{T,jt} &= A_{T,t} (K_{T,jt})^{\alpha_T} (H_{T,jt})^{\gamma_T} (Y_{XT,jt})^{\eta_T}; \\
\tilde{p}_{T,jt} &= \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_T)^l \lambda_{t+l} \xi_{T,t+l} (\tilde{p}_{T,jt} / p_{T,jt+l})^{-\theta} Y_{T,t+l}^d \prod_{k=1}^l \pi_{t+k}^{\theta}}{E_t \sum_{l=0}^{\infty} (\beta \phi_T)^l \lambda_{t+l} (\tilde{p}_{T,jt} / p_{T,jt+l})^{-\theta} Y_{T,t+l}^d \prod_{k=1}^l \pi_{t+k}^{\theta-1}}; \\
\tilde{p}_{T,jt}^* &= \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_T^*)^l \lambda_{t+l} s_{t+l} \xi_{T,t+l} (\tilde{p}_{T,jt}^* / p_{T,jt+l}^*)^{-\theta} Y_{T,t+l}^e \prod_{k=1}^l \pi_{t+k}^{*\theta}}{E_t \sum_{l=0}^{\infty} (\beta \phi_T^*)^l \lambda_{t+l} s_{t+l} (\tilde{p}_{T,jt}^* / p_{T,jt+l}^*)^{-\theta} Y_{T,t+l}^e \prod_{k=1}^l \pi_{t+k}^{*\theta-1}},
\end{aligned}$$

where  $\xi_{T,t}$  is the real marginal cost of the manufactured-goods producing firm,  $q_{T,jt} = Q_{T,jt} / P_t$ ,  $w_{T,jt} = W_{T,jt} / P_t$ ,  $\tilde{p}_{T,jt} = \tilde{P}_{T,jt} / P_t$ , and  $\tilde{p}_{T,jt}^* = \tilde{P}_{T,jt}^* / P_t^*$ .

The aggregate manufactured-intermediate-goods price for domestic use is

$$(P_{T,t})^{1-\theta} = \phi_T (\pi P_{T,t-1})^{1-\theta} + (1 - \phi_T) \left( \tilde{P}_{T,t}^d \right)^{1-\theta}, \quad (37)$$

while the price for the exports is

$$(P_{T,t}^*)^{1-\theta} = \phi_T (\pi^* P_{T,t-1}^*)^{1-\theta} + (1 - \phi_T) \left( \tilde{P}_{T,t}^T \right)^{1-\theta}. \quad (38)$$

Dividing equations (37) and (38) by  $P_t$  and  $P_t^*$ , respectively, yields

$$(p_{T,t})^{1-\theta} = \phi_T \left( \frac{\pi p_{T,t-1}}{\pi_t} \right)^{1-\theta} + (1 - \phi_T) (\tilde{p}_{T,t})^{1-\theta}, \quad (39)$$

and

$$(p_{T,t}^*)^{1-\theta} = \phi_T^* \left( \frac{\pi^* p_{T,t-1}^*}{\pi_t^*} \right)^{1-\theta} + (1 - \phi_T^*) (\tilde{p}_{T,t}^*)^{1-\theta}. \quad (40)$$

## 2.5 Non-tradable-intermediate-goods sector

The non-tradable-intermediate goods sector is indexed by  $N$ . There are a continuum of non-tradable-intermediate-goods producing firms indexed by  $j \in [0, 1]$ . Firm  $j$  produces its intermediate goods using capital,  $K_{N,jt} (= \int_0^1 k_{N,jt} dj)$ , labour,  $H_{N,jt}$ , and commodity input,  $Y_{XN,jt}$ . Its production function is given by,

$$Y_{N,jt} \leq A_{N,t} (K_{N,jt})^{\alpha_N} (H_{N,jt})^{\gamma_N} (Y_{XN,jt})^{\eta_N}, \quad \alpha_N, \gamma_N, \eta_N \in (0, 1), \quad (41)$$

where  $A_{N,t}$  is a non-tradable technology shock that evolves exogenously following an AR(1) process:

$$\log(A_{N,t}) = (1 - \rho_{AN}) \log(A_N) + \rho_{AN} \log(A_{N,t-1}) + \varepsilon_{AN,t}. \quad (42)$$

The firm  $j$  chooses  $K_{N,jt}$ ,  $H_{N,jt}$ ,  $Y_{XN,jt}$ , and sets the price  $\tilde{P}_{N,jt}$  that maximizes the expected discounted flow of its profits. Its maximization problem is:

$$\max_{\{K_{N,jt}, H_{N,jt}, Y_{XN,jt}, \tilde{P}_{N,jt}\}} E_0 \left[ \sum_{t=0}^{\infty} (\beta\phi)^l \lambda_{t+l} D_{N,jt+l} / P_{t+l} \right], \quad (43)$$

subject to (41) and the following demand function:

$$Y_{N,jt+l} = \left( \frac{\tilde{P}_{N,jt}}{P_{N,t+l}} \right)^{-\theta} Y_{N,t+l}, \quad (44)$$

where the profit function is

$$D_{N,jt+l} = \pi^l \tilde{P}_{N,jt} Y_{N,t+l} - Q_{N,t+l} K_{N,t+l} - W_{N,t+l} Y_{N,t+l} - e_{t+l} P_{X,t+l}^* Y_{XN,jt+l}. \quad (45)$$

The producer's discount factor is given by the stochastic process  $(\beta^l \lambda_{t+l})$ , where  $\lambda_{t+l}$  denotes the marginal utility of consumption in period  $t+l$ .

The first-order conditions in real terms are:

$$q_{N,t} = \frac{\alpha_N Y_{N,jt} \xi_{N,t}}{K_{N,jt}}; \quad (46)$$

$$w_{N,t} = \frac{\gamma_N Y_{N,jt} \xi_{N,t}}{H_{N,jt}}; \quad (47)$$

$$s_t p_{X,t}^* = \frac{\eta_N Y_{N,jt} \xi_{N,t}}{Y_{XN,jt}}; \quad (48)$$

$$Y_{N,jt} = A_{N,t} (K_{N,jt})^{\alpha_N} (H_{N,jt})^{\gamma_N} (Y_{XN,jt})^{\eta_N}, \quad (49)$$

where  $\xi_{N,t}$  is the real marginal cost of the firm producing non-tradable good,  $q_{N,jt} = Q_{N,jt}/P_t$ , and  $w_{N,jt} = W_{N,jt}/P_t$ .

The firm that is allowed to revise its price, which happens with the probability  $(1 - \phi_N)$ , chooses  $\tilde{P}_{N,jt}$ , so that

$$\tilde{p}_{N,jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_N)^l \lambda_{t+l} \xi_{N,t+l} (\tilde{p}_{N,jt}/p_{N,jt+l})^{-\theta} Y_{N,t+l} \prod_{k=1}^l \pi_{t+k}^{\theta}}{E_t \sum_{l=0}^{\infty} (\beta \phi_N)^l \lambda_{t+l} (\tilde{p}_{N,jt}/p_{N,jt+l})^{-\theta} Y_{N,t+l} \prod_{k=1}^l \pi_{t+k}^{\theta-1}},$$

where  $\tilde{p}_{N,jt} = \tilde{P}_{N,jt}/P_t$ .

The aggregate manufactured-intermediate-goods price for domestic use is

$$(P_{N,t})^{1-\theta} = \phi_N (\pi P_{N,t-1})^{1-\theta} + (1 - \phi_N) (\tilde{P}_{N,t})^{1-\theta}. \quad (50)$$

Dividing equation (50) by  $P_t$  yields

$$(p_{N,t})^{1-\theta} = \phi_N \left( \frac{\pi p_{N,t-1}}{\pi_t} \right)^{1-\theta} + (1 - \phi_N) (\tilde{p}_{N,t})^{1-\theta}.$$

## 2.6 Imported-intermediate-goods sector

There are a continuum of domestic importers, indexed by  $j \in [0, 1]$ , that import a homogeneous intermediate good produced abroad for the foreign price  $P_t^*$ . Each importer uses its imported good to produce a differentiated good,  $Y_{F,jt}$ , that it sells in a domestic monopolistically-competitive market to produce the imported-composite good,  $Y_{F,t}$ . Importers can only change their prices when they receive a random signal. The constant probability of receiving such a signal is also  $(1 - \phi_F)$ .

When an importer  $j$  is allowed to change its price, it sets the price  $\tilde{P}_{F,jt}$  that maximizes its weighted expected profits, given the price of the imported-composite output  $P_{F,t}$ , the nominal exchange rate  $e_t$ , and the foreign price level  $P_t^*$ . The maximization problem is:

$$\max_{\{\tilde{P}_{F,jt}\}} E_0 \left[ \sum_{t=0}^{\infty} (\beta \phi)^l \lambda_{t+l} D_{F,jt+l}/P_{t+l} \right], \quad (51)$$

subject to

$$Y_{F,jt+l} = \left( \frac{\tilde{P}_{F,jt}}{P_{F,t+l}} \right)^{-\theta} Y_{F,t+l}, \quad (52)$$



where the nominal profit function is

$$D_{F,jt+l} = \left( \pi^l \tilde{P}_{F,jt} - e_{t+l} P_{t+l}^* \right) Y_{F,jt+l}. \quad (53)$$

In period  $t$ , the importer's nominal marginal cost is  $e_t P_t^*$ , so its real marginal cost is equal to the real exchange rate,  $s_t = e_t P_t^* / P_t$ . The importer's discount factor is given by the stochastic process  $(\beta^l \lambda_{t+l})$ .

The first-order condition of this optimization problem is:

$$\tilde{p}_{F,jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_F)^l \lambda_{t+l} s_{t+l} (\tilde{p}_{F,jt} / p_{F,jt+l})^{-\theta} Y_{F,t+l} \prod_{k=1}^l \pi_{t+k}^{\theta}}{E_t \sum_{l=0}^{\infty} (\beta \phi_F)^l \lambda_{t+l} (\tilde{p}_{F,jt} / p_{F,jt+l})^{-\theta} Y_{F,t+l} \prod_{k=1}^l \pi_{t+k}^{\theta-1}}. \quad (54)$$

The aggregate import price is

$$(P_{F,t})^{1-\theta} = \phi_F (\pi P_{F,t-1})^{1-\theta} + (1 - \phi_F) (\tilde{P}_{F,t})^{1-\theta}. \quad (55)$$

Dividing equations (55) by  $P_t$  yields

$$(p_{F,t})^{1-\theta} = \phi_F \left( \frac{\pi p_{F,t-1}}{\pi_t} \right)^{1-\theta} + (1 - \phi_F) (\tilde{p}_{F,t})^{1-\theta}. \quad (56)$$

## 2.7 Government

The government's budget is given by:

$$P_t g_t + T_t + B_{t-1} = M_t - M_{t-1} + B_t / R_t. \quad (57)$$

The government spending,  $g_t$ , evolves exogenously according to

$$\log(g_t) = (1 - \rho_g) \log(g) + \rho_g \log(g_{t-1}) + \varepsilon_{gt}, \quad (58)$$

where  $g$  is a steady-state value,  $\rho_g \in (-1, 1)$  is an autoregressive coefficient, and  $\varepsilon_{gt}$  is an uncorrelated and normally distributed innovation with zero mean and standard deviations  $\sigma_g$ .

It is assumed that the central bank manages the short-term nominal interest rate,  $R_t$ , to mainly respond to deviations of the domestic CPI inflation rate,  $\pi_t$ , and output,  $Y_t$ . Thus, the monetary policy rule evolves according to:

$$\log \left( \frac{R_t}{R} \right) = \varrho_R \log \left( \frac{R_{t-1}}{R} \right) + \varrho_{\pi} \log \left( \frac{\pi_t}{\pi} \right) + \varrho_Y \log \left( \frac{Y_t}{Y} \right) + \varrho_s \log \left( \frac{s_t}{s} \right) + \varepsilon_{Rt}. \quad (59)$$

where  $\varrho_R$  is a smoothing-term parameter, while  $\varrho_\pi$ ,  $\varrho_y$ , and  $\varrho_s$  are the coefficients measuring the responses of the central bank to inflation, output, and real exchange rate variations, respectively; and  $\varepsilon_{Rt}$  is uncorrelated and normally distributed monetary policy shock with zero mean and standard deviations  $\sigma_R$ .

When considering the exchange rate regime, we set  $\varrho_s$  equal to 10. This means that the central bank aggressively intervenes to offset real exchange rate movements.

## 2.8 Symmetric equilibrium

In a symmetric equilibrium, all households and intermediate goods-producing firms make identical decisions, so that  $c_{jt} = c_t$ ,  $m_{jt} = m_t$ ,  $b_{jt} = b_t$ ,  $b_{jt}^* = b_t^*$ ,  $h_{i,jt} = h_{i,t}$ ,  $w_{i,jt} = w_{i,t}$ ,  $\tilde{w}_{i,jt} = \tilde{w}_{i,t}$ ,  $k_{i,jt} = k_{i,t}$ ,  $Y_{\iota,jt} = Y_{\iota,t}$ ,  $p_{\iota,jt} = p_{\iota,t}$ ,  $\tilde{p}_{\iota,jt} = \tilde{p}_{\iota,t}$ ,  $p_{T,jt}^* = p_{T,t}^*$ , and  $\tilde{p}_{T,jt}^* = \tilde{p}_{T,t}^*$ , for  $j \in [0, 1]$ ,  $i = N, Y, X$ , and  $\iota = F, N, T$ . Furthermore, the market-clearing conditions  $M_t = M_{t-1} + T_t + P_t g_t$  and  $B_t = 0$  must hold for all  $t \geq 0$ .

To estimate the non-calibrated structural parameters, we solve the model by taking a log-linear approximation of the equilibrium system around its deterministic steady-state values. Using Blanchard and Kahn's (1980) procedure yields a state-space solution of the form:

$$\begin{aligned}\widehat{\mathbf{s}}_{t+1} &= \Phi_1 \widehat{\mathbf{s}}_t + \Phi_2 \widehat{\boldsymbol{\varepsilon}}_{t+1}, \\ \widehat{\mathbf{d}}_t &= \Phi_1 \widehat{\mathbf{s}}_t,\end{aligned}$$

where  $\widehat{\mathbf{s}}_t$  is a vector of state variables that includes predetermined and exogenous variables;  $\widehat{\mathbf{d}}_t$  is a vector of control variables; and the vector  $\widehat{\boldsymbol{\varepsilon}}_t$  contains the model's shocks.

## 3. Calibration and Data

The non estimated parameters are calibrated to capture the salient features of the Canadian economy. The subjective discount rate,  $\beta$ , is set equal to 0.9902, which implies an annual real interest rate of 4% in the steady-state equilibrium. The parameter  $\tau$ , which measures the degree of risk aversion, is given a standard value of 2, so the intertemporal elasticity of substitution is equal to 0.5. The parameter  $\varsigma$  is set

equal to 1, as in Bouakez et al. (2005). The capital depreciation rate,  $\delta$ , is set equal to 0.025, the same in all three sectors. The shares of capital, labour, and natural resources in the production of commodities,  $\alpha_X$ ,  $\gamma_X$ , and  $\eta_X$ , are assigned values 0.41, 0.39, and 0.2, respectively. On the other hand, the shares of capital, labour, and commodity input in production of tradable (non-tradable) goods,  $\alpha_T(\alpha_N)$ ,  $\gamma_T(\gamma_N)$ , and  $\eta_T(\eta_N)$ , are set equal to 0.26(0.28), 0.63 (0.66), and 0.11(0.06), respectively. All the share values are taken from Macklem et al. (2000) which calculated them from Canadian input-output tables.<sup>7</sup>

The parameters  $\theta$  and  $\vartheta$ , which measure the degree of monopoly power in intermediate-goods and labour markets, are set equal to 6 and 10, respectively. These values are commonly used in the literature. The calibrations of the parameters  $\omega_F$ ,  $\omega_N$ , and  $\omega_T$ , that measure the share of imports, tradable and non-tradable goods in the final good  $Z_t$ , are based on average ratios observed in the data for the period 1981Q1 to 2005Q4. Therefore, we set  $\omega_F$ ,  $\omega_N$ , and  $\omega_T$  equal to 0.3, 0.18, and 0.52, respectively. The parameter  $\nu$ , which captures the elasticity of demand for imports and domestic goods, is set equal to 1.2, based on previous estimates by Dib (2003) and Ambler et al. (2004). The parameter  $\varkappa$  is a normalization that ensures the ratio of manufactured exports to GDP is equal to the one observed in the data. Therefore,  $\varkappa$  is set equal to 0.23. The parameter  $\chi$  is calibrated to match foreign-debt-to-GDP ratio that is equal to 0.3. This calibration gives an average annual risk premium of about 80 basis points.

It is assumed that households allocate 33% of their available time to market activities. Following Macklem et al. (2000), we assume that the shares of employment in non-tradable, tradable, and commodity sectors are 0.64, 0.21, and 0.15, respectively. Consequently, in the steady-state equilibrium,  $H_N$ ,  $H_T$ , and  $H_X$  are set equal to 0.21, 0.07, and 0.05, respectively. The steady-state stock of natural resources,  $L$ , is assigned a value of 0.014 to match the share of commodity production in Canadian GDP. The steady-state value of government spending,  $g$ , is calibrated so that the ratio  $g/Y = 0.226$  matches the one observed in the data. The steady-state level of the exogenous variables,  $A_N$ ,  $A_T$ ,  $p_X^*$ , and  $Y^*$  are simply set equal to unity. Table 1 reports the calibration values.

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<sup>7</sup>Therefore,  $\eta_X$ , the share of natural resources in production of commodities, is equal to  $1 - \alpha_X - \gamma_X$ . Similarly, the shares of commodity inputs in production of tradable and non-tradable goods,  $\eta_T$  and  $\eta_N$ , are equal to  $1 - \alpha_T - \gamma_T$  and  $1 - \alpha_N - \gamma_N$ , respectively.

The remaining parameters are estimated using the maximum likelihood procedure: a Kalman filter is applied to a model's state-space form to generate series of innovations, which are then used to evaluate the likelihood function for the sample. Since the model is driven by nine shocks, the structural parameters embedded in  $\Phi_1$ ,  $\Phi_2$ , and  $\Phi_3$  are estimated using data for nine series: commodities, tradable goods, non-tradable goods, real commodity price, nominal interest rate, government spending, real exchange rate, foreign inflation, and foreign output.

Commodities are measured by the total real production in primary industries (agriculture, fishing, forestry, and mining) and resource processing, which includes pulp and paper, wood products, primary metals, and petroleum and coal refining. The non-tradables are in real terms and include construction; transportation and storage; communications, insurance, finance, and real estate; community and personal services; and utilities. The tradables are measured by the total real production in manufactured sectors. Real commodity price is measured by deflating the nominal commodity prices (including energy and non-energy commodities) by the U.S. GDP deflator. Nominal interest rate is measured by the rate on Canadian three-month treasury bills. Government spending is measured by total real government purchases of goods and services. The real exchange rate is measured by multiplying the nominal U.S./CAN exchange rate by the ratio of U.S. to Canadian prices. Foreign inflation is measured by changes in the U.S. GDP implicit price deflator. Finally, foreign output is measured by U.S. real GDP per capita. The series for commodities, tradables, non-tradables, and government spending are expressed real and per capita terms using the Canadian population aged 15 and over.

The model implies that all variables are stationary and fluctuate around constant means; however, the series described above are non-stationary, with the exception of the foreign inflation rate. Thus, before estimating the model, we render them stationary by using the HP-filter. Using quarterly Canadian and U.S. data from 1981Q1 to 2005Q4, we estimate the non-calibrated structural parameters of different versions of the multi-sector model.<sup>8</sup> Figure 1 plots the nine HP-filtered series used in the estimation of the model.

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<sup>8</sup>The sample starts at 1981Q1 because of the availability of the data (Data on commodities, tradables, and non-tradables are available only since 1981.)

## 4. Empirical Results

### 4.1 Estimation results

Table 2 reports the maximum-likelihood estimates of the remaining structural parameters of two versions of the above-described model: LCP and PTM models.<sup>9</sup> In LCP model, the estimated values of the parameters  $\phi_i$ , for  $i = F, N, T$ , which determines the degrees of nominal price rigidity are about 0.5 in the non-tradable sector and 0.7 in the import and tradable sectors. These estimates indicate some degree of heterogeneity across sectors. Nevertheless, in the PTM model, the estimates of these parameters are very similar and estimated at slightly larger than 0.7. In both models, the estimates of nominal wage stickiness parameters,  $\varphi_i$ , for  $i = N, T, X$ , are statistically significant. They range from 0.5 in the tradable sector to 0.83 in the commodity sector. We note that the hypothesis of equal nominal rigidity across non-tradable and commodity sectors is not rejected at conventional confidence levels.

Next, the estimates of the monetary policy parameters are reported. They are all positive and statistically significant. The estimated values of the interest rate smoothing coefficient,  $\rho_R$ , are 0.73 and 0.87 in the LCP and PTM models, respectively.<sup>10</sup> The estimates of  $\rho_\pi$  and  $\rho_Y$ , the coefficients that measure responses of monetary policy to deviations in inflation and output, are about 0.30 in the LCP model and about 0.22 in the PTM model. The estimated values of,  $\sigma_R$ , the standard deviation of the monetary policy shock, are about 0.006 in both models, which is very close to the estimated values in Dib (2003).

The estimates of capital-adjustment cost parameters in tradable, non-tradable, and commodity sectors,  $\psi_T$ ,  $\psi_N$ , and  $\psi_X$ , range between 17 and 25 in the LCP model and between 10 and 17 in the PTM model. These values are comparable to estimated values by previous studies.<sup>11</sup> Note that their standard errors are much higher in the LCP model, which suggest that the estimates of  $\psi_T$ ,  $\psi_N$ , and  $\psi_X$  may be statistically similar in both models and across sectors.

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<sup>9</sup>Because of difficulties in calculating steady-state equilibrium values, we have estimated only the parameters that do not affect the steady-state ratios: wage and prices stickiness parameters, monetary policy parameters, and autoregressive coefficients and standard deviations of the exogenous processes.

<sup>10</sup>Using a Bayesian estimation method, Rebei and Ortega (2006) estimate the smoothing coefficient at 0.46, which is much lower in present estimation, but for the period 1972Q1–2003Q3.

<sup>11</sup>Rebei and Ortega (2006) estimate the capital-adjustment cost parameter, which they assume to be common to all sectors, to be about 10.

Commodity price shocks,  $p_{PX}^*$ , appear to be persistent and highly volatile, with autoregressive coefficients,  $\rho_{PX}$ , at least equal to 0.9 and 0.8 in the LCP and PTM models, respectively; while the estimated values of their standard deviations,  $\sigma_{PX}$ , are about 0.03. Natural resource shocks,  $L_t$ , are also very persistent and highly volatile, with values of their autoregressive coefficient,  $\rho_L$ , and standard errors,  $\sigma_L$ , estimated at 0.9 and 0.05, respectively. The remaining domestic and foreign shocks—technologies, government spending, the world interest rate, world inflation, and world output—are persistent and volatile, with estimated values of their autoregressive coefficients and standard deviations similar to common findings in previous studies (for example Ambler et al. 2004, Rebei and Ortega 2006).

## 4.2 Impulse responses

Figures 1 and 2 show impulse responses of some key macroeconomic variables to a 1% shock to commodity prices and natural resources (land) generated by the estimated LCP model. These shocks represent an increase in real commodity prices and a positive increase in the natural resource factor due to, for example, to a favorable weather or a new mining discovery. Each response is expressed as the percentage deviation of a variable from its steady-state level.

Figure 1 plots the impulse responses to a 1% positive commodity price shock. This shock is an exogenous increase in commodity prices in the world markets. Overall, following an increase in commodity prices, the real exchange rate immediately appreciates before returning in a few quarters to its steady-state value. The exogenous increase in commodity prices and the appreciation of the real exchange rate induces the commodity-producing firm to increase its output and, therefore, its exports; it leads, however, tradable and non-tradable goods-producing firms to reduce their demand for commodity inputs used in their production technologies as the prices of commodity inputs increase. The demand for commodity inputs, therefore, falls sharply by more than 1% after the shock and persists for many quarters. Since capital and labour are not perfect substitutes to commodity inputs in the production of tradable and non-tradable goods, outputs in these two sectors slightly decrease. Even though the negative effects of commodity prices increases in tradable and non-tradable sectors, overall output (GDP in this economy) increases, due to rising commodity production.

On the other hand, the nominal interest rate slightly increases as a response of monetary policy to the increase in the total GDP and inflation rates. We also note a progressive increase in the real wage and a sharp jump of labour in the commodity sector. Foreign debt stock decreases after a positive commodity price shock, which is equivalent to a positive effect on the current account. Surprisingly, the present model generates an unexpected behavior of consumption, as it negatively reacts to a positive commodity price shock. This negative response might be the result of the decreases in the tradable and non-tradable outputs that enter as major inputs in the production of the final good, which is then used for consumption.<sup>12</sup>

Figure 2 plots the impulse responses for a positive natural resource factor shock. Overall, this shock has expected, but moderate, effects on the model's key macroeconomic variables. It results first in an increase in production, exports, hours, and real wages in the commodity sector. It leads also to a slight appreciation of the domestic currency and to a marginal increase in the nominal interest rate. We note that this shock induces a significant decrease in the foreign debt stock and therefore to an amelioration in the current account position. However, it has a very marginal negative effects on the production in tradable and non-tradable sectors because of the increase of commodity input prices associated with the appreciation of the real exchange rate.

### **4.3 Variance decomposition**

In this subsection, we examine the forecast-error variance decomposition of the real exchange rate generated in the estimated LCP and PTM models. This decomposition enables us to calculate the proportion of real exchange rate volatility explained by each of the model's structural shocks. The decomposition results are reported in Table 3 for two scenarios: (1) the estimated model with all shocks and (2) the estimated model without commodity price shocks.

Panel A shows that, for the estimated LCP model with all shocks, world interest rate and commodity price shocks account for 49% and 36% of the real exchange rate variations in a one-quarter-ahead hori-

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<sup>12</sup>The negative response of consumption may be corrected by using a CES production technology in tradable and non-tradable sectors with a lower elasticity of substitution ( $< 0.5$ ). I leave this for future work.

zon, respectively. Monetary policy shocks account for about 10% of these variations. Nevertheless, when commodity price shocks are turned off, with  $\sigma_{PX} = 0$ , the contribution of world interest rate shocks is larger than 75% of short-term real exchange rate variations. Monetary policy shocks, however, account for about 14% in one-quarter-ahead horizon.

Panel B shows that, for the estimated PTM model with all shocks, world interest rate and monetary policy shocks contribute mostly to real exchange rate fluctuation in the short term, with proportions of 55% and 23%, respectively. Commodity price shocks still contribute significantly to real exchange rate variations in a one-quarter-ahead horizon, with a proportion of 8.5%.

Thus, the variance decompositions show that commodity price shocks have a significant role in explaining the short-term fluctuations of the real exchange rate. It is the second source of real exchange rate fluctuations in the estimated LCP model, and contribute significantly to its volatility. Note that natural resource shocks (land) contribute modestly into real exchange rate fluctuations. They account for only about 3% of total exchange rate volatility in both estimated models.

## 5. The welfare measure

To analyze welfare effects, we solve the model using a second-order approximation around deterministic steady-state values. Then, welfare measures are calculated as the unconditional expectation of utility in deterministic steady state.<sup>13</sup> It allows us to compare two alternative steady states: stochastic and deterministic steady states.<sup>14</sup> Thus, we first calculate a second-order Taylor expansion of the single-period utility function that is given by:

$$u(\cdot) = \frac{c_t^{1-\tau}}{1-\tau} - h_t^{\frac{\varsigma}{1+\varsigma}}.$$

The second-order Taylor expansion of  $u(\cdot)$  around deterministic steady-state values of its arguments

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<sup>13</sup>An alternative approach is to use conditional expectations of utility.

<sup>14</sup>In stochastic steady state, there is a risk related to the shocks in the model, while in deterministic steady state the shock are set equal to zero, so there is no risk.



yields:

$$E(u_t) = \frac{\bar{c}^{1-\tau}}{1-\tau} - \bar{h}^{\frac{\varsigma}{1+\varsigma}} + \bar{c}^{1-\tau} E(\hat{c}_t) - \frac{\tau}{2} \bar{c}^{1-\tau} E(\hat{c}_t^2) \quad (60)$$

$$- \frac{\varsigma}{1+\varsigma} \bar{h}^{\frac{\varsigma}{1+\varsigma}} E(\hat{h}_t) + \frac{\varsigma}{2(1+\varsigma)^2} \bar{h}^{\frac{\varsigma}{1+\varsigma}} E(\hat{h}_t^2), \quad (61)$$

where  $\hat{c}_t$  and  $\hat{h}_t$  are the log deviations of  $c_t$  and  $h_t$  from their deterministic steady state values, while  $E(\hat{c}_t^2)$  and  $E(\hat{h}_t^2)$  are their variances, respectively.

We decompose the total welfare effect into a level effect, which influences the expected mean of variables by implying a permanent shift in steady state of consumption, and into a variance effect which implies a permanent shift in steady state consumption associated with the effects of the shocks on the variables' variances. Let  $\zeta_m$  denote the level effect. Therefore, the level effect is defined as:

$$E_t u((1 + \zeta_m)c_t, h_t) = \frac{((1 + \zeta_m)\bar{c})^{1-\tau}}{1-\tau} - \bar{h}^{\frac{\varsigma}{1+\varsigma}} \quad (62)$$

$$= \frac{\bar{c}^{1-\tau}}{1-\tau} + \bar{c}^{1-\tau} E(\hat{c}_t) - \frac{\varsigma}{1+\varsigma} \bar{h}^{\frac{\varsigma}{1+\varsigma}} E(\hat{h}_t). \quad (63)$$

Solving for  $\zeta_m$  yields:

$$\zeta_m = \left[ 1 + (1 - \tau)E(\hat{c}_t) - \frac{\varsigma(1 - \tau)}{1 + \varsigma} \frac{\bar{h}^{\frac{\varsigma}{1+\varsigma}}}{\bar{c}^{1-\tau}} E(\hat{h}_t) \right]^{\frac{1}{1-\tau}} - 1. \quad (64)$$

Similarly, let  $\zeta_v$  denotes that denotes the variance effect. Therefore,

$$E_t u((1 + \zeta_v)c_t, h_t) = \frac{((1 + \zeta_v)\bar{c})^{1-\tau}}{1-\tau} - \bar{h}^{\frac{\varsigma}{1+\varsigma}} \quad (65)$$

$$= \frac{\bar{c}^{1-\tau}}{1-\tau} - \frac{\tau}{2} \bar{c}^{1-\tau} E(\hat{c}_t^2) + \frac{\varsigma}{2(1+\varsigma)^2} \bar{h}^{\frac{\varsigma}{1+\varsigma}} E(\hat{h}_t^2). \quad (66)$$

Solving for  $\zeta_v$  yields:

$$\zeta_v = \left[ 1 - \frac{\tau(1 - \tau)}{2} E(\hat{c}_t^2) - \frac{\varsigma(1 - \tau)}{2(1 + \varsigma)^2} \frac{\bar{h}^{\frac{\varsigma}{1+\varsigma}}}{\bar{c}^{1-\tau}} E(\hat{h}_t^2) \right]^{\frac{1}{1-\tau}} - 1. \quad (67)$$

Table 4 reports the results from the two estimated versions of the model: LCP and PTM. The results are for three experiments: (1) the benchmark model with a flexible exchange rate simulated with all

shocks; (2) the benchmark model with a flexible exchange rate simulated without commodity price shock; and (3) the benchmark model with a fixed exchange rate simulated with all shocks. The results shown are standard deviations, stochastic steady state deviations, and welfare effects decomposed into level and variance effects of deviations of variables from their levels in the deterministic steady state, reported for consumption, hours worked, the real exchange rate, and output.

Panel A shows firstly that the volatility of the variables is much lower in the estimated LCP model than in the PTM model. This is mainly explained by differences in the estimated values of the parameters. Secondly, the volatility of the variables is much lower in both flexible exchange rate models compared to the one generated under the assumption of a fixed exchange rate regime. Thirdly, commodity price shocks increase exchange rate volatility by 20% in the estimated LCP model, and by about 5% in the estimated PTM model.

Panel B reports stochastic steady state deviations as a percentage of levels in the deterministic steady state. The presence of risk has negative effects on all examined variables. These negative effects are much higher when the exchange rate is fixed. Note that the fall in consumption is higher in the estimated LCP than PTM model. The decrease in stochastic mean of the real exchange rate is between -0.34% and -0.46% in the estimated LCP and PTM models without commodity shocks.

Panel C shows the total and decomposed welfare effects expressed as a percentage of deterministic steady state of consumption. First, for the estimated LCP model, the overall welfare effect is -0.312% in the estimated benchmark LCP model, divided into the level effect, -0.296%, and the variance effect, -0.016%. Commodity price shocks reduce overall welfare by only -0.016%. Nevertheless, under a fixed exchange rate regime, the overall welfare decreases by -8.9%, decomposed into the level effect, -8.7 %, and the variance effect, -0.16%. The decrease in overall welfare is very marginal in the estimated PTM model when the exchange rate is flexible; however, it decreases by -1.60 % when the real exchange rate is fixed. Therefore, most of the welfare lost comes from the level effect, and commodity price shocks marginally affect the welfare in the flexible exchange rate models.

Table 5 reports the welfare analysis results from the PTM model calibrated with the estimated values in the LCP model. The results indicate that the welfare effects of commodity prices and exchange rate

volatilities are very similar in the LCP and PTM models calibrated with the same parameter values. This is in contradiction with previously found results by Devereux and Engel (2003) .

## **6. Conclusion**

In this paper, we develop a multi-sector New Keynesian model of a small open economy. Then, we use the model to examine the role of commodity price fluctuations in explaining exchange rate volatility and to assess the welfare effects of real exchange rate variability. A fraction of the model's structural parameters are estimated. Using a second-order procedure, we resolve the model to calculate welfare measures. A general result indicates that the welfare effects of commodity and exchange rate variabilities are small in the economies with a flexible exchange rate; whereas they are significantly higher with a fixed exchange regime.

While these results are interesting, some sensitivity analysis is required to check their robustness. Furthermore, future work requires some extensions of the model by using the CES technology in a production function that allows lower elasticities of substitution between different inputs. It will also be interesting to consider the case of an optimal monetary policy response to exchange rate fluctuations.

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**Table 1: Calibrated Parameters**

Parameters	Values	Parameters	Values
$\beta$	0.99	$\alpha_X$	0.41
$\tau$	2	$\gamma_X$	0.39
$\nu$	1	$\eta_X$	0.20
$\varsigma$	1	$\alpha_N$	0.28
$\delta$	0.025	$\gamma_N$	0.66
$\theta$	6	$\eta_N$	0.06
$\vartheta$	10	$\alpha_T$	0.28
$\nu$	1.2	$\gamma_T$	0.63
$\omega_F$	0.3	$\eta_T$	0.11
$\omega_N$	0.18	$\varkappa$	0.23
$\omega_T$	0.52	$\chi$	0.0058
$b$	0.021	$g/Y$	0.226

**Table 2: Maximum likelihood estimates: Sample 1981Q1–2005Q4**

Parameters	LCP Model		PTM Model	
	Estimates	St. Errors	Estimates	St. Errors
$\phi_T$	0.6773	0.0032	0.7181	0.0731
$\phi_N$	0.5142	0.0119	0.7222	0.0181
$\phi_F$	0.7050	0.0452	0.7015	0.1021
$\varphi_T$	0.5162	0.0267	0.4958	0.1073
$\varphi_N$	0.8051	0.0069	0.7118	0.2608
$\varphi_X$	0.8353	0.0230	0.7634	0.1811
$\varrho_R$	0.7299	0.0586	0.8703	0.0884
$\varrho_\pi$	0.2952	0.0369	0.2209	0.0718
$\varrho_Y$	0.0781	0.0037	0.0980	0.0095
$\sigma_R$	0.0058	0.0006	0.0066	0.0008
$\psi_T$	21.379	3.6940	17.4042	2.9639
$\psi_N$	24.999	12.069	9.7909	0.6126
$\psi_X$	17.063	5.7816	14.5430	1.2097
$\rho_{AT}$	0.8685	0.0518	0.9109	0.0136
$\sigma_{AT}$	0.0140	0.0012	0.0108	0.0011
$\rho_{AN}$	0.9404	0.0387	0.8922	0.0305
$\sigma_{AN}$	0.0090	0.0010	0.0099	0.0010
$\rho_{PX}$	0.9451	0.0203	0.8005	0.0843
$\sigma_{PX}$	0.0273	0.0028	0.0312	0.0033
$\rho_L$	0.8807	0.0714	0.9199	0.0809
$\sigma_L$	0.0594	0.0062	0.0485	0.0051
$\rho_g$	0.7492	0.0789	0.8439	0.0884
$\sigma_g$	0.0085	0.0009	0.0092	0.0009
$\rho_{R^*}$	0.8359	0.0280	0.8992	0.0178
$\sigma_{R^*}$	0.0057	0.0006	0.0042	0.0004
$\rho_{\pi^*}$	0.6996	0.0734	0.8331	0.1047
$\sigma_{\pi^*}$	0.0033	0.0003	0.0030	0.0003
$\rho_{Y^*}$	0.9080	0.0936	0.8733	0.0884
$\sigma_{Y^*}$	0.059	0.0006	0.0076	0.0008
<i>L.L.</i>	2501		2569	

**Table 3: Forecast-error variance decomposition of the real exchange rate**

	Percentage owing to:				
	Comm. prices	Land	Policy	Wor. Int.	Others
<i>A. LCP Model</i>					
All the shocks	35.64	2.11	8.88	48.84	4.53
No com. price shock, $\sigma_{PX} = 0$	0.00	3.28	13.79	75.89	7.04
<i>B. PTM Model</i>					
All the shocks	8.50	2.83	23.46	55.16	10.50
No com. price shock, $\sigma_{PX} = 0$	0.00	3.09	25.63	60.29	11.00



**Table 4: Welfare Effects**

	Det. S.S.	Estimated LCP Model			Estimated PTM Model		
		Flexible <sup>a</sup>	Flex.: $\sigma_{PX} = 0$	Fixed <sup>b</sup>	Flexible <sup>a</sup>	Flex. $\sigma_{PX} = 0$	Fixed <sup>b</sup>
<i>A. Standard deviations<sup>c</sup> (percent)</i>							
$c_t$		0.5660	0.5175	1.6521	0.7728	0.7625	1.3951
$h_t$		0.3197	0.2898	0.3966	0.3394	0.3342	0.3115
$s_t$		3.2296	2.5908	0.1055	3.2902	3.1472	0.0774
$Y_t$		0.8886	0.7747	1.8677	1.1743	1.1109	1.3895
<i>B. Stochastic steady state deviations (percent)</i>							
$c_t$	0.3994	-0.1454	-0.1501	-4.2242	-0.0188	-0.0234	-0.6801
$h_t$	0.0515	-0.0774	-0.0687	-1.2160	-0.0643	-0.0596	-0.2799
$s_t$	0.6642	-0.4135	-0.3417	-0.4605	-0.4614	-0.4239	-0.5381
$Y_t$	0.6756	-0.2857	-0.2761	-5.1053	-0.1168	-0.1148	-0.7034
$u_t$	-2.7307	-0.7805	-0.8228	-24.2130	-0.0573	-0.0948	-3.9418
<i>C. Welfare effects as percentage of deterministic steady state of consumption.</i>							
$\zeta_m$		-0.2964	-0.3156	-8.7411	0.0007	-0.0149	-1.4867
$\zeta_v$		-0.0157	-0.0132	-0.1641	-0.0325	-0.0317	-0.1177
Overall		-0.3122	-0.3288	-8.9053	-0.0318	-0.0465	-1.6045

<sup>a</sup> Under flexible exchange rate regime  $\varrho_s$ , in the monetary policy rule, is set equal 0.

<sup>a</sup> Under fixed exchange rate regime  $\varrho_s$ , in the monetary policy rule, is set equal 10.

<sup>c</sup> Notes: Standard errors are for variables in level (not in log). The standard errors of variables in log is simply the deterministic steady-state values divided by the standard deviations of variables in level.

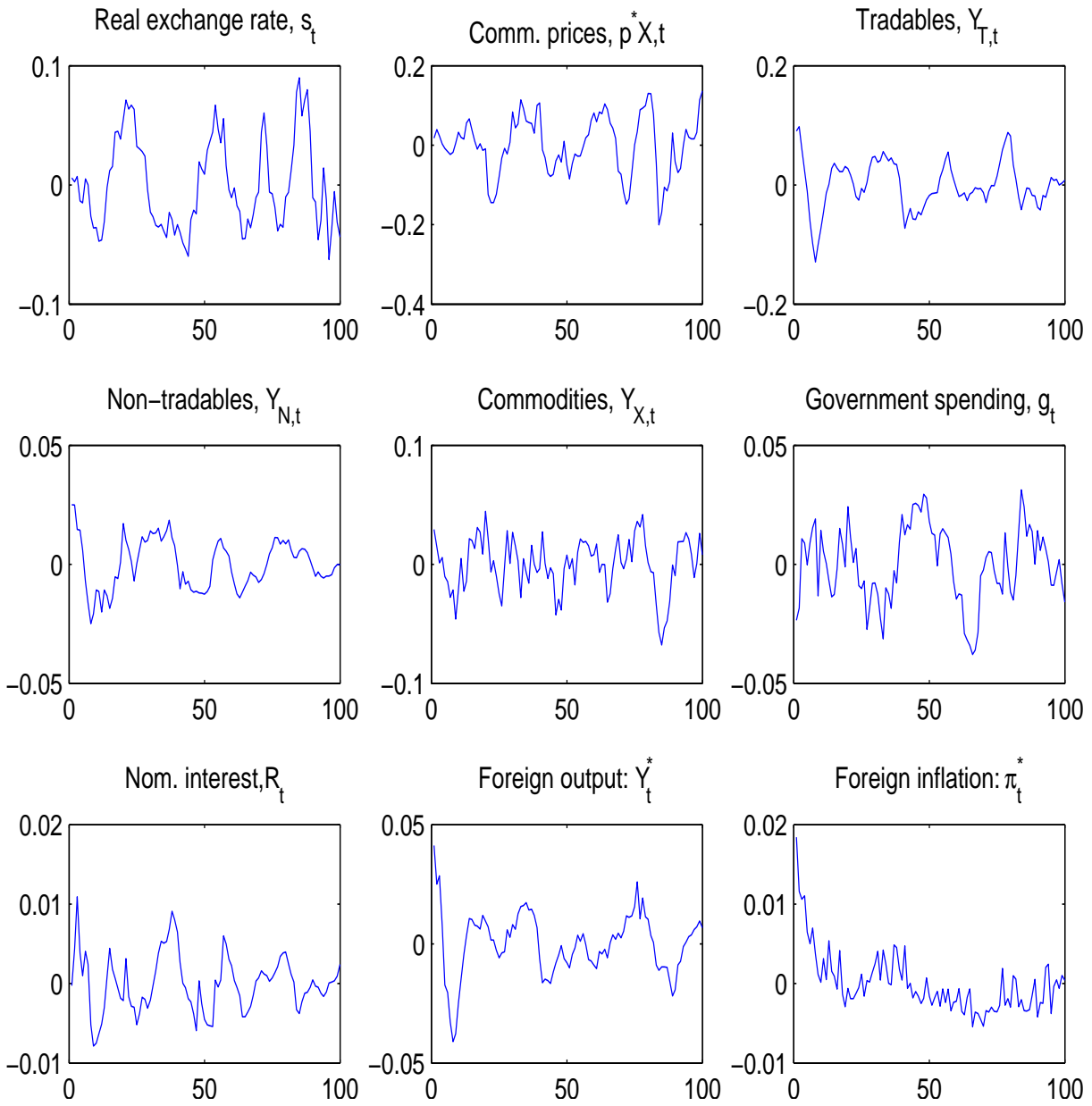
**Table 5: PTM with LCP estimates**

	Flexible	Flex. $\sigma_{PX} = 0$	Fixed
<i>A. Standard deviations (percent)</i>			
$c_t$	0.5662	0.5177	1.6520
$h_t$	0.3188	0.2888	0.3961
$s_t$	3.2321	2.5937	0.1055
$Y_t$	0.8895	0.7759	1.8676
<i>B. Stochastic steady state deviations</i>			
$c_t$	-0.1451	-0.1498	-4.2213
$h_t$	-0.0775	-0.0687	-1.2152
$s_t$	-0.4128	-0.3411	-0.4602
$Y_t$	-0.2850	-0.2755	-5.1016
$u_t$	-0.7781	-0.8208	-24.1964
<i>C. Welfare effects</i>			
$\zeta_m$	-0.2955	-0.3149	-8.7356
$\zeta_m$	-0.0158	-0.0132	-0.1641
Overall	-0.3112	-0.3281	-8.8998

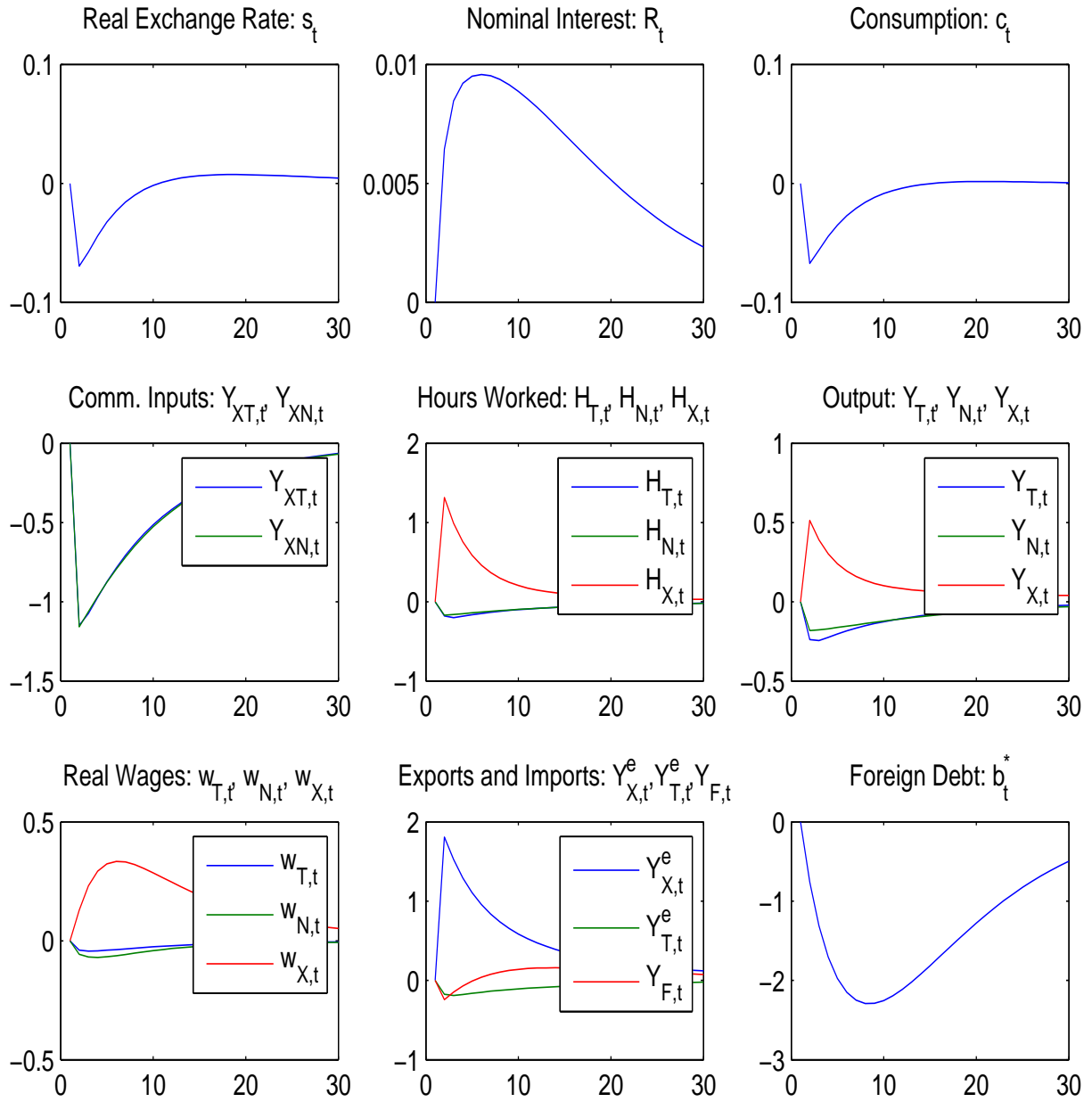
**Table 6: Data: Sample 1981Q1–2005Q4**

<i>A. Standard deviations</i>			
$s_t$	3.80	$Y_t$	1.44
$p_{X,t}^*$	7.35	$g_t$	1.60
$Y_{T,t}$	4.09	$R_t$	0.36
$Y_{N,t}$	1.04	$Y_t^*$	1.32
$Y_{X,t}$	2.25	$\pi_t^*$	0.38
<i>B. Correlations of exchange rate</i>			
$s_t$	1	$Y_t$	0.11
$p_{X,t}^*$	-0.62	$g_t$	0.33
$Y_{T,t}$	0.05	$R_t$	-0.12
$Y_{N,t}$	0.18	$Y_t^*$	-0.01
$Y_{X,t}$	-0.13	$\pi_t^*$	-0.18

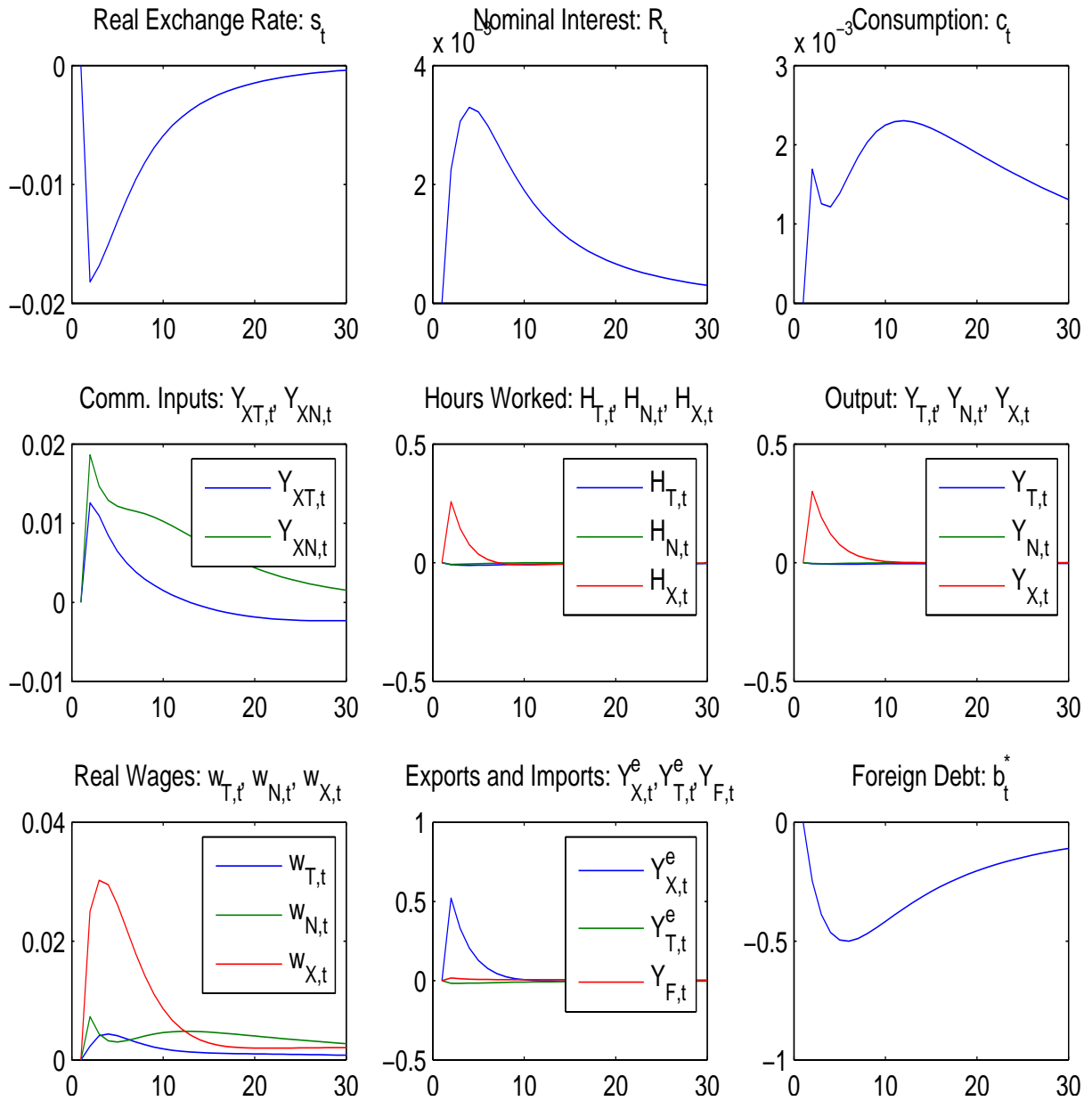
**Figure 1: HP-filtered data**



**Figure 2: The effects of a 1% positive commodity price shock ( $p_{X,t}$ )**



**Figure 3: The effects of a 1% positive natural resource shock ( $L_t$ )**



## Appendix A. First-order conditions

### A.1 Households

$$c_{jt}^{-\tau} = \lambda_{jt}; \quad (\text{A.1})$$

$$b \left( \frac{M_{jt}}{P_t} \right)^{-\nu} = \lambda_{jt} - \beta E_t \left[ \frac{\lambda_{jt+1} P_t}{P_{t+1}} \right]; \quad (\text{A.2})$$

$$\psi_i \left( \frac{k_{i,jt+1}}{k_{i,jt}} - 1 \right) + 1 = \beta E_t \left[ \frac{\lambda_{jt+1}}{\lambda_{jt}} \left( \frac{Q_{i,t+1}}{P_{t+1}} + 1 - \delta + \psi_i \left( \frac{k_{i,jt+2}}{k_{i,jt+1}} - 1 \right) \frac{k_{i,jt+2}}{k_{i,jt+1}} \right) \right]; \quad (\text{A.3})$$

$$\frac{\lambda_{jt}}{R_t} = \beta E_t \left[ \frac{\lambda_{jt+1} P_t}{P_{t+1}} \right]; \quad (\text{A.4})$$

$$\frac{\lambda_{jt}}{\kappa_t R_t^*} = \beta E_t \left[ \frac{\lambda_{jt+1} e_{t+1} P_t}{e_t P_{t+1}} \right], \quad (\text{A.5})$$

$$\zeta_{i,jt} = (h_{i,jt}/h_t)^{\frac{1}{\eta}} / \lambda_{jt}; \text{ for } i = N, T, X. \quad (\text{A.6})$$

$$E_t \left[ \sum_{l=0}^{\infty} (\beta \varphi_i)^l \lambda_{jt+l} \left( \frac{\widetilde{W}_{i,jt}}{\widetilde{W}_{i,t+l}} \right)^{-\vartheta} H_{i,t+l} \left\{ \zeta_{i,t+l} - \frac{\vartheta - 1}{\vartheta} \frac{\widetilde{W}_{i,jt}}{P_t} \frac{\pi^l P_t}{P_{t+l}} \right\} \right] = 0; \quad (\text{A.7})$$

### A.2 Commodity sector

$$\frac{Q_{X,t}}{P_t} \frac{e_t P_{X,t}^*}{P_t} = \frac{\alpha_X Y_{X,t}}{K_{X,t}}; \quad (\text{A.8})$$

$$\frac{W_{X,t}}{P_t} = \frac{e_t P_{K,t}^* \gamma_X Y_{X,t}}{P_t H_{X,t}}; \quad (\text{A.9})$$

$$\frac{P_{L,t}}{P_t} = \frac{e_t P_{X,t}^* \eta_X Y_{X,t}}{P_t L_t}; \quad (\text{A.10})$$

$$Y_{X,t} = (K_{X,t})^{\alpha_X} (H_{X,t})^{\gamma_X} (L_t)^{\eta_X} \quad (\text{A.11})$$

### A.3 Tradable-Intermediate-goods sector

$$\frac{Q_{T,t}}{P_t} = \frac{\alpha_T Y_{T,jt} \xi_{T,t}}{K_{T,jt}}, \quad (\text{A.12})$$

$$\frac{W_{T,t}}{P_t} = \frac{\gamma_T Y_{T,jt} \xi_{T,t}}{H_{T,jt}}, \quad (\text{A.13})$$

$$\frac{e_t P_{X,t}^*}{P_t} = \frac{\eta_T Y_{T,jt} \xi_{T,t}}{Y_{XT,jt}}, \quad (\text{A.14})$$

$$Y_{T,jt} = (K_{T,jt})^{\alpha_T} (H_{T,jt})^{\gamma_T} (Y_{XT,jt})^{\eta_T}; \quad (\text{A.15})$$

$$\tilde{P}_{T,jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_T)^l \lambda_{t+l} \xi_{T,t+l} \left( \tilde{P}_{T,jt} / P_{T,jt+l} \right)^{-\theta} Y_{T,jt+l}^d}{E_t \sum_{l=0}^{\infty} (\beta \phi_T)^l \lambda_{t+l} \left( \tilde{P}_{T,jt} / P_{T,jt+l} \right)^{-\theta} Y_{T,jt+l}^d \pi^l / P_{t+l}}; \quad (\text{A.16})$$

$$\tilde{P}_{T,jt}^* = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_T)^l \lambda_{t+l} \xi_{T,t+l} \left( \tilde{P}_{T,jt}^* / P_{T,jt+l}^* \right)^{-\theta} Y_{T,jt+l}^e}{E_t \sum_{l=0}^{\infty} (\beta \phi_T)^l \lambda_{t+l} s_{t+l} \left( \tilde{P}_{T,jt}^* / P_{T,jt+l}^* \right)^{-\theta} Y_{T,jt+l}^e \pi^{*l} / P_{t+l}^*}. \quad (\text{A.17})$$

### A.4 Non-tradable-intermediate-goods sector

$$\frac{Q_{N,t}}{P_t} = \frac{\alpha_N Y_{N,jt} \xi_{N,t}}{K_{N,jt}}; \quad (\text{A.18})$$

$$\frac{W_{N,t}}{P_t} = \frac{\gamma_N Y_{N,jt} \xi_{N,t}}{H_{N,jt}}; \quad (\text{A.19})$$

$$\frac{e_t P_{X,t}^*}{P_t} = \frac{\eta_N Y_{N,jt} \xi_{N,t}}{Y_{XN,jt}}; \quad (\text{A.20})$$

$$Y_{N,jt} = (K_{N,jt})^{\alpha_N} (H_{N,jt})^{\gamma_N} (Y_{XN,jt})^{\eta_N}; \quad (\text{A.21})$$

$$\tilde{P}_{N,jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_N)^l \lambda_{t+l} \xi_{N,t+l} \left( \tilde{P}_{N,jt} / P_{N,jt+l} \right)^{-\theta} Y_{N,jt+l}^d}{E_t \sum_{l=0}^{\infty} (\beta \phi_N)^l \lambda_{t+l} \left( \tilde{P}_{N,jt} / P_{N,jt+l} \right)^{-\theta} Y_{N,jt+l}^d \pi^l / P_{t+l}}. \quad (\text{A.22})$$

## A.5 Imported-intermediate-goods sector

$$\tilde{P}_{F,jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi_F)^l \lambda_{t+l} e_{t+l} P_{t+l}^* \left( \tilde{P}_{F,jt} / P_{F,jt+l} \right)^{-\theta} Y_{F,t+l}}{E_t \sum_{l=0}^{\infty} (\beta \phi_F)^l \lambda_{t+l} \left( \tilde{P}_{F,jt} / P_{F,jt+l} \right)^{-\theta} Y_{F,t+l} \pi^l / P_{t+l}}. \quad (\text{A.23})$$

## Appendix B. Wage Setting

The household chooses  $\tilde{W}_{i,jt}$  to solve:

$$E_t \left[ \sum_{l=0}^{\infty} (\beta \varphi)^l \left\{ u(c_{jt+l}, h_{i,jt+l}) + \lambda_{t+l} \pi^l \tilde{W}_{i,jt} h_{i,jt+l} / P_{t+l} \right\} \right], \quad (\text{B.1})$$

where  $h_{i,jt+l} = \left( \frac{\tilde{W}_{i,jt}}{W_{i,t+l}} \right)^{-\vartheta} H_{i,t+l}$ .

Thus,

$$E_t \left[ \sum_{l=0}^{\infty} (\beta \varphi_i)^l \left\{ u(c_{jt+l}, h_{i,jt+l}) + \lambda_{t+l} \pi^l \left( \frac{\tilde{W}_{i,jt}}{W_{i,t+l}} \right)^{1-\vartheta} \frac{W_{i,t+l}}{P_{t+l}} H_{i,t+l} \right\} \right], \quad (\text{B.2})$$

The FOC:

$$E_t \left[ \sum_{l=0}^{\infty} (\beta \varphi_i)^l \left\{ \frac{\partial u_{jt+l}}{\partial h_{i,jt+l}} \frac{\partial h_{i,jt+l}}{\partial \tilde{W}_{i,jt}} + (1 - \vartheta) \lambda_{t+l} \pi^l \left( \frac{\tilde{W}_{i,jt}}{W_{i,t+l}} \right)^{-\vartheta} \frac{H_{i,t+l}}{P_{t+l}} \right\} \right] = 0, \quad (\text{B.3})$$

with  $\frac{\partial h_{i,jt+l}}{\partial \tilde{W}_{i,jt}} = -\vartheta \left( \frac{\tilde{W}_{i,jt}}{W_{i,t+l}} \right)^{-\vartheta-1} \frac{H_{i,t+l}}{W_{i,t+l}}$ .

Let  $\zeta_{i,jt+l} = -\frac{\partial u_{jt+l}}{\partial h_{i,t+l}} / \frac{\partial u_{jt+l}}{\partial c_{jt+l}}$  denote the marginal rate of substitution between  $c_{jt+l}$  and  $h_{i,jt+l}$ ; and  $\lambda_{jt+l} = \frac{\partial u_{jt+l}}{\partial c_{jt+l}}$ . Therefore,

$$E_t \left[ \sum_{l=0}^{\infty} (\beta \varphi_i)^l \lambda_{t+l} \left( \frac{\tilde{W}_{i,jt}}{W_{i,t+l}} \right)^{-\vartheta} H_{i,t+l} \left\{ \zeta_{i,t+l} - \frac{\vartheta - 1}{\vartheta} \frac{\tilde{W}_{i,jt}}{P_t} \frac{\pi^l P_t}{P_{t+l}} \right\} \right] = 0, \quad (\text{B.4})$$

which may be rewritten as:

$$E_t \left[ \sum_{l=0}^{\infty} (\beta \varphi_i)^l \lambda_{jt+l} H_{i,jt+l} \left\{ \zeta_{i,jt+l} - \frac{\vartheta - 1}{\vartheta} \tilde{w}_{i,jt} \prod_{k=1}^l \frac{\pi^k}{\pi_{t+k}} \right\} \right] = 0, \quad (\text{B.5})$$



where  $\tilde{w}_{i,jt} = \frac{\tilde{W}_{i,jt}}{P_t}$  is the real contracted wage.

In symmetric equilibrium all households are identical, so  $\tilde{w}_{i,jt} = \tilde{w}_{i,t}$ ,  $h_{i,jt} = h_{i,t}$ ,  $\lambda_{i,jt} = \lambda_{i,t}$ ,  $\zeta_{i,jt} = \zeta_{i,t}$

$$\begin{aligned} \text{Let } f_t^1 &= \sum_{l=0}^{\infty} (\beta\varphi)^l \lambda_{t+l} H_{i,t+l} \zeta_{i,t+l} (w_{i,t+l})^\vartheta \prod_{k=1}^l \pi^l \pi_{t+k}^\vartheta; \\ &= \lambda_t h_{i,t} \zeta_{i,t} w_{i,t}^\vartheta + \beta\varphi f_{t+1}^1; \end{aligned}$$

$$\begin{aligned} \text{and let } f_t^2 &= \sum_{l=0}^{\infty} (\beta\varphi)^l \lambda_{t+l} H_{i,t+l} (w_{i,t+l})^\vartheta \prod_{k=1}^l \pi^l \pi_{t+k}^{\vartheta-1}, \\ &= \lambda_t h_{i,t} + \beta\varphi w_{i,t}^\vartheta \pi_t^\vartheta f_{t+1}^2; \end{aligned}$$

so we can rewrite equation (B.5) as

$$f_t^1 = \frac{\vartheta-1}{\vartheta} \tilde{w}_{i,t} f_t^2.$$

Note that price setting in the intermediate-goods sectors is similar to the wage setting.

## Appendix C. Non-linear equilibrium

$$c_t^{-\tau} = \lambda_t; \quad (\text{C.1})$$

$$bm_t^{-\nu} = \lambda_t - \beta E_t (\lambda_{t+1}/\pi_{t+1}); \quad (\text{C.2})$$

$$\frac{\lambda_t}{R_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} \right]; \quad (\text{C.3})$$

$$\frac{\lambda_t}{\kappa_t R_t^*} = \beta E_t \left[ \frac{\lambda_{t+1} s_{t+1}}{s_t \pi_{t+1}^*} \right]; \quad (\text{C.4})$$

$$\kappa_t = \exp(-\chi s_t \theta_t^*/Y_t); \quad (\text{C.5})$$

$$\zeta_{i,t} = (h_{i,t}/h_t)^{\frac{1}{\eta}} / \lambda_t; \text{ for } i = N, T, X \quad (\text{C.6})$$

$$f_{i,t}^1 = \lambda_t h_{i,t} \xi_{i,t} + \beta \varphi f_{i,t+1}^1; \quad (\text{C.7})$$

$$f_{i,t}^2 = \lambda_t h_{i,t} + \beta \varphi \pi f_{i,t+1}^2 / \pi_{t+1}; \quad (\text{C.8})$$

$$f_{i,t}^1 = \frac{\vartheta - 1}{\vartheta} \tilde{w}_{i,t} f_{i,t}^2; \quad (\text{C.9})$$

$$1 = \phi \left( \frac{w_{i,t-1}}{\pi_t} \right)^{1-\vartheta} + (1-\phi) \tilde{w}_{i,t}^{1-\vartheta}; \quad (\text{C.10})$$

$$\psi_i \left( \frac{k_{i,t+1}}{k_{i,t}} - 1 \right) + 1 = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( q_{i,t+1} + 1 - \delta + \psi_i \left( \frac{k_{i,t+2}}{k_{i,t+1}} - 1 \right) \frac{k_{i,t+2}}{k_{i,t+1}} \right) \right]; \quad (\text{C.11})$$

$$Y_{X,t} = (K_{X,t})^{\alpha_X} (H_{X,t})^{\gamma_X} (L_t)^{\eta_X} \quad (\text{C.12})$$

$$\frac{\alpha_X Y_{X,t}}{K_{X,t}} = \frac{q_{X,t}}{s_t p_{X,t}^*}; \quad (\text{C.13})$$

$$\frac{\gamma_X Y_{X,t}}{H_{X,t}} = \frac{w_{X,t}}{s_t p_{X,t}^*}; \quad (\text{C.14})$$

$$\frac{\eta_X Y_{X,t}}{L_t} = \frac{p_{L,t}}{s_t p_{X,t}^*}; \quad (\text{C.15})$$

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_i} H_{i,t}^{\gamma_i} Y_{X_{i,t}}^{\eta_i}; i = N, T \quad (\text{C.16})$$

$$q_{i,t} = \alpha_i Y_{i,t} \xi_{i,t} / K_{i,t}; \quad (\text{C.17})$$

$$w_{i,t} = \gamma_i Y_{i,t} \xi_{i,t} / H_{i,t}; \quad (\text{C.18})$$

$$stP_{xt}^* = \eta_i Y_{i,t} \xi_{i,t} / Y_{X_{i,t}}; \quad (\text{C.19})$$

$$x_{i,t}^1 = \lambda_t h_{i,t} \xi_{i,t} + \beta \phi x_{i,t+1}^1; \quad (\text{C.20})$$

$$x_{i,t}^2 = \lambda_t h_{i,t} + \beta \phi \pi x_{i,t+1}^2 / \pi_{t+1}; \quad (\text{C.21})$$

$$x_{i,t}^1 = \frac{\theta - 1}{\theta} \tilde{w}_{i,t} x_{i,t}^2; \quad (\text{C.22})$$

$$1 = \phi \left( \frac{p_{i,t-1}}{\pi_t} \right)^{1-\theta} + (1 - \phi) \tilde{p}_{i,t}^{1-\theta}; \quad (\text{C.23})$$

$$Z_t = \left[ \omega_T^{\frac{1}{\nu}} \left( Y_{T,t}^d \right)^{\frac{\nu-1}{\nu}} + \omega_N^{\frac{1}{\nu}} Y_{N,t}^{\frac{\nu-1}{\nu}} + \omega_F^{\frac{1}{\nu}} Y_{F,t}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}; \quad (\text{C.24})$$

$$1 = \omega_T p_{T,t}^{1-\nu} + \omega_N p_{N,t}^{1-\nu} + \omega_F p_{F,t}^{1-\nu}; \quad (\text{C.25})$$

$$Y_{N,t} = \omega_N (p_{N,t})^{-\nu} Z_t; \quad (\text{C.26})$$

$$Y_{T,t}^d = \omega_T (p_{T,t})^{-\nu} Z_t; \quad (\text{C.27})$$

$$Y_{F,t} = \omega_F (p_{F,t})^{-\nu} Z_t; \quad (\text{C.28})$$

$$Y_{T,t}^e = \varkappa (p_{T,t}^*)^{-\nu} Y_t^*; \quad (\text{C.29})$$

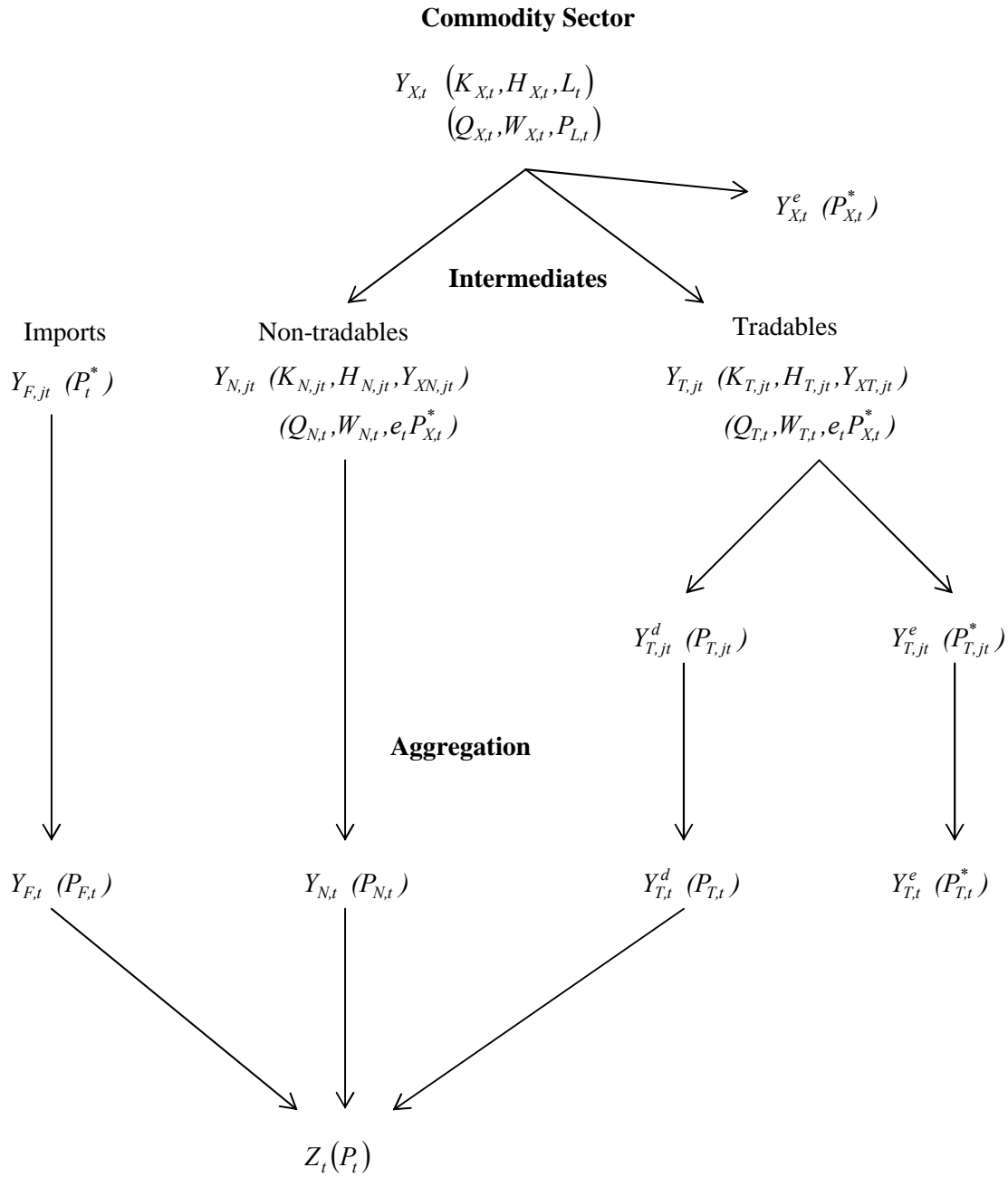
$$Y_{T,t} = Y_{T,t}^d + Y_{T,t}^e; \quad (\text{C.30})$$

$$Y_t = p_{N,t} Y_{N,t} + p_{T,t} Y_{T,t}^d + stP_{T,t}^* Y_{T,t}^e + stP_{X,t}^* (Y_{X,t} - Y_{XN,t} - Y_{XT,t}); \quad (\text{C.31})$$

$$\frac{b_t^*}{\kappa_t R_t^*} = \frac{b_{t-1}^*}{\pi_t^*} + p_{X,t}^* (Y_{X,t} - Y_{XN,t} - Y_{XT,t}) + p_{T,t}^* Y_{X,t}^* - Y_{F,t}; \quad (\text{C.32})$$

$$\log \left( \frac{R_t}{R} \right) = \varrho_R \log \left( \frac{R_{t-1}}{R} \right) + \varrho_\pi \log \left( \frac{\pi_t}{\pi} \right) + \varrho_y \log \left( \frac{y_t}{y} \right) + \varrho_s \log \left( \frac{s_t}{s} \right) + \varepsilon_{Rt}. \quad (\text{C.33})$$

Figure 1: Production Structure of the Model



$$Z_t = c_t + g_t + i_{N,t} + i_{T,t} + i_{X,t}$$

$$K_{i,t+1} = (1 - \delta)K_{i,t} + i_{i,t}, \quad i = N, T, X$$