Towards a New Measure of Interest Rate Expectations in Canada: Estimating a Time-Varying Term Premium

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Introduction

A clear view of how financial markets expect interest rates to behave is very useful in the conduct of monetary policy. Information on the market's view of future short-term interest rate movements enables monetary authorities to identify any discrepancies between the path they desire for interest rates over the medium term and the path the market expects. The authorities can then take action to avoid the kind of financial market disturbance that can arise when monetary policy takes what the markets see as an unanticipated turn.[1]

The main objective of this study is to derive a more accurate measure of market expectations about the behaviour of three-month interest rates in

1. See Zelmer (1996) for a review of tactical considerations in the conduct of monetary policy.

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Canada.[2] As a starting point, the expectations hypothesis of the term structure of interest rates (EHTS), if it holds true, provides one measure of market expectations. It offers a practical analytical framework for obtaining from the term structure of interest rates an estimate of how financial markets expect short-term rates to behave. The EHTS is based on an equilibrium relationship between interest rates of different maturities. According to the EHTS, each long-term rate represents the average of short-term rates expected by the market over the long-term instrument's maturity, plus a constant-term premium. The literature, however, does not find unanimous empirical support for EHTS; this is discussed in greater detail in Section 2.

Until very recently, there was little empirical work on the validity of the EHTS as applied to Canadian short-term rates. A recent study conducted at the Bank of Canada by Paquette and Stréliski (1998) tested the validity of the EHTS using rates on forward-rate agreements (FRAs) and rates of 90-day bankers' acceptances (BAs). The study showed that the hypothesis cannot be rejected for most Canadian interest rate maturities. Stréliski found, however, that the relationship between forward and spot rates is unstable across subsamples. In this study, we postulate that this instability is in part the result of a time-varying term premium.

Most studies to date, including Paquette and Stréliski's, use single-equation regression methods to test the EHTS's validity. Single-equation methods have many shortcomings; in particular, they do not make use of all the information available for estimating non-biased parameters. Moreover, single-equation methods ignore the long-term equilibrium aspects of the term structure, because they fail to take into account the time series properties related to cointegration.[3]

The equilibrium relationship between daily forward rates and spot rates used in this study assumes that there is a time-varying term premium. This relationship is thus more general than the EHTS, which assumes a constant-term premium. In order to obtain an accurate measure of the variable component of the term premium associated with various forward rates, we estimate a vector error-correction model (VECM). If we assume that there is a cointegrating relation between spot and forward rates, i.e., that

2. The short-term orientation of monetary policy is often measured in Canada by a three-month interest rate, or by the monetary conditions index, which is composed of the interest rate on 90-day commercial paper and the exchange rate, weighted for Canada's major trading partners. We therefore focus our efforts on measuring expected 90-day interest rates.

3. Boothe (1991) is an example of a recent Canadian study that has attempted to address these questions relating to cointegration. However, this study uses a single-equation cointegration technique, which presupposes that one of the interest rates is weakly exogenous. It also focuses on the long-term segment of the term structure.

these rates are driven by a common stochastic trend, then the VECM technique allows us to calculate a time-varying, but stationary, measure of the term premium. This approach also allows testing of the necessary conditions imposed by the EHTS under the assumptions that the spot and forward interest rates are non-stationary. These conditions are: that spot and forward rates are driven by a common stochastic component; and that the cointegrating vector is such that there exists a one-to-one relation between the spot and forward rates.

In Section 1, we describe the EHTS and present the methodology used in this paper. We then describe the data in Section 2. In Section 3, we formally test the long-run conditions imposed by the EHTS using a cointegration model. This model also allows us to calculate a time-varying component of the term premium, which we then use to obtain a measure of interest rate expectations. In this section, we also examine the statistical properties of the variable component of the term premium. Subsequently, in Section 4, we test the robustness of our cointegration model results. In the last section, we present our conclusions and suggest some avenues for further research.

1 Methodology

1.1 The EHTS

The EHTS stipulates that each long-term rate represents the average of short-term rates expected to occur over the life of a long-term instrument, plus a constant-term premium. Based on the no-arbitrage relation between forward and spot rate markets, the EHTS can be expressed in terms of forward rates as follows:

$$f(n)_t^k = E_t[r(n)_{t+k}] + \theta(n, k), \tag{1}$$

where $f(n)_t^k$ is the forward rate at time t of an n period instrument beginning in k periods, $r(n)_{t+k}$ is the spot rate at time $t+k$ of an n period instrument, E_t is the mathematical expectations operator conditional on information available at time t, and $\theta(n, k)$ is the constant-term premium associated with these rates. If we allow the term premium in equation (1) to vary over time, then (1) becomes an arbitrage relationship between spot and forward rates in which no restrictions are imposed on the term premium time series characteristics, except that it be stationary.[4] This relationship is therefore

4. Ilmanen (1996) and Evans and Lewis (1994) show how the equilibrium relationship between forward and spot rates of equations (1) and (2) can be considered an arbitrage relation.

more general than the EHTS and includes the latter as a particular case. By subtracting the spot rate from both sides of this arbitrage relation, we obtain:

$$f(n)_t^k - r(n)_t = E_t[r(n)_{t+k} - r(n)_t] + \theta^e(n, k)_t, \tag{2}$$

where $f(n)_t^k - r(n)_t$ is the forward premium, $E_t[r(n)_{t+k} - r(n)_t]$ is the expected change in the spot rate, and $\theta^e(n, k)_t$ is the expected time-varying term premium associated with the forward rate. This equilibrium relationship postulates that an upward-sloping yield curve is a reflection of two polar assumptions: that short-term interest rates are expected to rise (the EHTS) and that longer-dated instruments provide investors with a higher return for bearing the risks associated with holding these instruments (i.e., a term premium). By decomposing the expected term premium variable into its constant and time-varying components, $\theta^e(n, k)_t = \alpha + x_t$, and by modifying the ordering of the terms of equation (2), we obtain the following adjusted measure of expected changes in interest rates:

$$E_t[r(n)_{t+k} - r(n)_t] = f(n)_t^k - r(n)_t - \alpha - x_t. \tag{3}$$

Most empirical studies test the EHTS using the following equation:

$$r(n)_{t+k} - r(n)_t = \alpha + \phi(f(n)_t^k - r(n)_t) + \zeta_{t+1}, \tag{4}$$

where $\zeta_{t+1} = \omega_{t+1} + x_t$ includes forecast errors ω_{t+1} plus the time-varying component of the term premium x_t. The parameters estimated are unbiased if ω_{t+1} follows a white noise process and $x_t = 0$.[5] These empirical studies consequently test the EHTS under the assumption that the term premium is constant.

Fama (1984), Hardouvelis (1988), and Roberds, Runkle, and Whiteman (1996) are examples of studies of this kind. They analyze the relationship between U.S. Treasury bill rates of different maturities. Generally speaking, most of these studies find that forward rates provide biased forecasts of future spot rates $(\phi \neq 1)$. According to certain authors, this bias is a result of the fact that American short-term interest rates are difficult to predict because of the behaviour of the Federal Reserve,[6] systematic forecast errors, or time-varying term premiums. More specifically, systematic forecast errors imply that ω_{t+1} does not follow a white noise process and may also be correlated with the interest rate

5. For this to be the case, markets must be efficient, investors must be risk-neutral, and they must form their expectations rationally.

6. In the conduct of monetary policy, the Federal Reserve targets the Fed funds rate and intervenes to mitigate its volatility. According to Rudebusch (1995), this imposes a random walk behavior on the Fed funds rate, which is then propagated to Treasury bill maturities.

variables. Similarly, a time-varying term premium implies that x_t varies systematically.[7] On the other hand, Brenner and Kroner (1995) maintain that the ϕ coefficient estimated from equation (4) will be biased, even if the term premium is constant and expectations are rational. This occurs when there is a cointegrating relation between the forward and spot rates,[8] which the single-equation regression method does not take into account.

Nevertheless, studies that test the validity of the EHTS using the single-equation regression approach obtain more favorable results with non-U.S. interest rates. Hardouvelis (1988) and Gerlach and Smets (1995) find that, outside the United States, the behaviour of short-term rates conforms to the EHTS predictions in all countries studied. Studies performed on Canadian data, including Paquette and Stréliski's (1998), analyze the validity of the EHTS and find that it is not rejected for most maturities across the money market forward term structure. Stréliski finds, however, that the ϕ coefficients vary sharply depending on the subsample used. Still, this study is, to our knowledge, the first to test the validity of the EHTS in a framework of cointegrating system of equations with Canadian money market data.

1.2 The VECM

Assuming that the interest rate series are non-stationary and cointegrated, they can be expressed as a VECM with the following specification:

$$\Delta f_t^k = \lambda_f(f_{t-1}^k - \beta r_{t-1} - \mu) + \sum_{i=1}^{q} b_i^{ff} \Delta f_{t-i}^k + \sum_{i=1}^{q} b_i^{fr} \Delta r_{t-i} + \varepsilon_{f,t}$$

$$\Delta r_t = \lambda_r(f_{t-1}^k - \beta r_{t-1} - \mu) + \sum_{i=1}^{q} b_i^{rf} \Delta f_{t-i}^k + \sum_{i=1}^{q} b_i^{rr} \Delta r_{t-i} + \varepsilon_{r,t}, \quad (5)$$

7. The ϕ coefficient depends on: variation in the term premium; variation in expected spot rate changes; and the correlation between changes in the term premium and expected changes in the spot rate. Even if there is no correlation between the variation in the term premium and the variation in expected spot rate changes, the estimated ϕ coefficient is biased downward if the term premium is in fact variable over time. For more details, see Gerlach and Smets (1995) and Mankiw and Miron (1986).

8. The estimated measure of ϕ will be biased for two reasons. First, under the hypothesis that forward and spot rates follow $I(1)$ processes, $(f_t^k - r_t)$ will be stationary only if the cointegrating vector is [1, –1]. Second, even if the cointegrating vector is [1, –1], the single-equation error-correction approach implicitly assumes that the forward rate is weakly exogenous (that the spot rate determines the forward rate). If the forward rate is not [$\lambda_r \neq 0$ in equation (5)], then the estimation of ϕ within a single-equation approach will necessarily be biased.

where the residuals ($\varepsilon_{1,t}$ and $\varepsilon_{2,t}$) are assumed to follow a white noise process. This VECM formulation relates changes in each of the forward and spot rates (Δf_t^k and Δr_t) to the error-correction term ($f_{t-1}^k - \beta r_{t-1} - \mu$), and to lagged changes in the rates. The error-correction term represents the long-run equilibrium properties of the spot–forward relation. The lagged changes in the rates are interpreted as influencing the short-run properties of each rate. The coefficients in the error-correction term denote the cointegrating vector, which in this case is equal to $[1, -\beta]$. The loading factors or error-correcting adjustment coefficients (λ_f and λ_r) measure the single-period adjustments of the forward and spot rate to the preceding period's departure from equilibrium.

The procedure we use to test the necessary but non-sufficient conditions imposed by the EHTS consists of two steps. In the first step, we determine the presence of a cointegrating relation between forward rates of various maturities and the spot rate. In the second step, we test the null hypothesis that this cointegrating vector is the one implied by the EHTS, i.e., equal to [1, –1] in the case of a two-rate system, and more generally that the sum of the cointegrating vector coefficients is 0 in the case of systems of more than two interest rates. If these two necessary conditions hold, one may then test the short-run conditions imposed by the EHTS.[9]

Recent empirical studies have used this multiple-equation cointegration model to test the validity of the EHTS. These studies estimate the cointegrating relations in a system of equations that allows better use to be made of all available information on long- and short-term variations in each of the variables. EHTS validity tests using this kind of model have a greater tendency to support the hypothesis than those using a single-equation regression model. Cuthbertson (1996) and Rossi (1996) provide two examples. They find results that generally lend support to the EHTS for the United Kingdom at the short end of the term structure maturity spectrum. Engsted and Tanggaard (1994), Shea (1992), and Hall, Anderson, and Granger (1992) are examples of studies using this cointegration technique to test the validity of the EHTS for the United States. Generally, these studies found little conclusive empirical support for the EHTS. No study to date has used this technique to test the validity of the EHTS in Canada at the short end of the term structure.[10]

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9. This short-run condition requires that ex post forward forecast errors have a zero mean, be independent, and be identically normally distributed. We do not test this condition as we find in Section 3 that long-run EHTS conditions do not hold.

10. Côté and Fillion (1997) use this technique to test the EHTS in Canada in two-interest rates systems that combine rates of short and long maturities. Their results, as ours, demonstrate the EHTS's frailness in Canada.

Once the VECM has been estimated, the variable component of the term premium can be measured by the error-correction term $(f_t^k - \hat{\beta} r_t - \hat{\mu})$, which will be equal to $\hat{x}_t$, the zero-mean measure of the variable component of the expected term premium.[11] The new measure of expected 3-month Canadian interest rates that we propose in this study is calculated as follows:

$$E_t[r_{t+k} - r_t] = f_t^k - r_t - \hat{\alpha} - \hat{x}_t, \tag{6}$$

where $E_t[r_{t+k} - r_t]$ is the expected change in 3-month rates, and $\hat{\alpha}$ is an estimate of the constant part of the term premium.[12]

Results that indicate rejection of the EHTS are generally assumed to be caused by a time-varying term premium, by systematic expectation errors, or by a combination of both of these factors. Expectation errors are defined as occurring in periods when investors incorrectly anticipate the behaviour of the spot rate, so that non-zero or systematic expectation errors, when viewed ex post, appear to be irrational.

Using our methodology to estimate the time-varying term premium, we are implicitly assuming that: (i) expectations are rational; (ii) both forward and spot rates are driven by a common permanent component; and (iii) daily changes in the forward–spot rate spread are proportional to variations in the term premium.[13] Specifically, any variation in the forward–spot rate spread that departs from the long-term equilibrium relation, represented by the estimated error-correction term $(f_t^k - \hat{\beta} r_t - \hat{\mu})$, is interpreted as a change in the term premium. Consequently, the term premium estimated within the VECM framework is a proper measure of the true term premium only when variations in the premium are the major cause of variations in the forward-spot spread. Appendix 2 proposes two possible explanations of a cointegrating vector β coefficient that differs from 1. For instance, β will be different from 1 when investors hold what appear ex post to be irrational expectations.

2 The Data

Our empirical analysis is based on daily closing yields of 3-month Canadian bankers' acceptances (referred to in the tables as BA90) and forward contracts derived from them, i.e., those that have settlement dates in

11. For more insight on the assumptions that lead the variable component of the term premium to equal the cointegrating vector, see Appendix 2.

12. The measure of the constant-term premium in Canada comes from the estimation of α in equation (4) by imposing the constraint $\phi = 1$. It is important to note that $\hat{\alpha}$ is not equal to $\hat{\mu}$, as $\hat{\mu}$ is an estimated constant that forces the error-correction term to have a zero mean.

13. See Appendix 2.

one, two, three, and up to nine months in the future (FRA 1x4, 2x5, 3x6, 6x9, and 9x12).[14]

The EHTS is tested on different systems of interest rates during the period of 9 August 1988 to 16 January 1998. As Figure 1 shows, during the period studied, Canadian 3-month forward (in three months) interest rates generally declined, other than in 1989–90, 1994–95, and 1997–98. To accurately measure the long-run relationship between spot and forward rates, it would be preferable to use a longer time span covering several interest-rate (or economic) cycles and inflection points. That would allow the estimated parameters to fully reflect the time-series properties of the term structure. However, data on Canadian forward rates are not available prior to 1988. In Section 4, we use a longer data set (1982 to 1998) of several treasury bill maturities to assess the robustness of our results.[15]

In modelling the term premium by means of a cointegration technique, we are implicitly assuming that interest rates are themselves integrated or non-stationary. To verify the validity of this assumption, we conducted unit root tests on the interest rates series used in this study. Table 1 shows the results of augmented Dickey–Fuller (ADF) tests applied to 90-day BAs and on FRA rates (1x4, 3x6, 6x9, and 9x12). The number of lags used in the ADF tests was chosen using the method proposed in Campbell and Perron (1991). This method consists of selecting a large number of lags of the dependent variable, and then testing down until the last lag is found to be significant.

The unit root test results do not reject the hypothesis that the series for spot and forward interest rates used in this study are $I(1)$ variables.[16] We also performed ADF tests on the differenced series to determine whether the interest rates could be considered $I(2)$ variables. The results show the rejection of the unit root hypothesis. None of these series, then, can be considered $I(2)$ variables. These results indicate that spot and forward rates satisfy the necessary conditions for use in a cointegration framework.[17]

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14. The reasons for using FRAs rather than futures contract rates is to formulate the interest rate expectations model on the basis of money market rates that are quoted daily, with fixed terms to maturity (30, 60... 270 days).

15. Non-interrupted daily 1-year treasury bill rates are available only from 1982, while treasury bill rates for shorter maturities are available from 1979.

16. It is possible, however, that changes in the level of the Canadian inflation rate that occurred between 1990 and 1992 have biased the unit root tests towards not rejecting the unit root hypothesis.

17. The same conclusions are drawn for the FRA series not shown in Table 1.

Table 1

Unit Root Tests on BAs and FRAs, ADF Tests

Series	Level	Lags (L)	ADF test *t*-statistics
BA	Level	21	0.028
	First difference	21	–11.603**
FRA1x4	Level	21	0.013
	First difference	21	–11.938**
FRA3x6	Level	21	0.018
	First difference	21	–11.404**
FRA6x9	Level	21	0.021
	First difference	21	–11.145**
FRA9x12	Level	21	0.037
	First difference	21	–11.935**

FRA and BA rates are daily quotes covering the period of 8 August 1988 to 16 January 1998. The number of lags is selected using the Campbell and Perron (1991) procedure; a maximum of 21 lags are allowed, corresponding to one month of daily observations.

** indicates rejection of the unit root null hypothesis, at significance level of 1 per cent.

3 Empirical Results

To test the validity of the long-run conditions imposed by the EHTS, we analyze the cointegrating relation between the interest rates that make up the forward yield curve for 3-month BAs. In the first part of this section, we present the results of our cointegration tests and test the validity of the conditions imposed by the EHTS, under the assumptions that the spot and forward interest rates are non-stationary. In the second part, we estimate the cointegrating vector for each combination of spot and forward rates, so as to estimate the variable component of the term premium for each maturity on the yield curve. This allows us to compare the measure of market interest rate expectations in Canada derived from the VECM with the one based on the EHTS, using a constant-term premium. In the third part, we examine whether the estimated time-varying component of the term premium can be interpreted as a risk premium that is dependent on interest rate volatility.

3.1 The validity of the EHTS

We use the maximum likelihood estimation technique of a VECM introduced by Johansen and Juselius (1990) to test the conditions imposed by the EHTS.[18] This approach allows one to simultaneously analyze

18. For more information about the estimation technique of Johansen and Juselius (1990), as well as the detailed form of the VECM, see Hansen and Juselius (1994) and Paquet (1994).

systems of two or more non-stationary interest rates. The EHTS implies that interest rates in such systems should be driven by a common stochastic trend. In a system with p interest rates of different maturities, the conditions imposed by the EHTS therefore imply that we should identify $(p-1)$ cointegrating vectors. Furthermore, the EHTS implies that the sum of the cointegrating vector coefficients related to the p interest rates must be 0.[19] To test the EHTS cointegration conditions, we used the following statistical procedure. First, we determine the number of cointegrating vectors in systems of p interest rates. Then, we test the restriction whereby the sum of the coefficients of each cointegrating vector must be 0. Rejection of either one of these two long-run conditions would indicate that the EHTS does not hold.

Tables 2 and 3 show the results of our cointegration tests. They present the test results for the number of cointegrating vectors and for the restriction on the sum of the coefficients of each cointegrating vector. We examine systems of two, three, four, and five interest rates. We determine the number of lags to be included in the VECM using the Sims (1980) technique, taking a maximum number of 21 lags representing one month of observations. The number of lag lengths to remove the residual autocorrelation ranged between 14 and 19. The residuals, however, displayed autoregressive conditional heteroscedasticity, or ARCH-type heteroscedasticity and non-normality. We shall discuss these problems in detail later in this section.

The two-rate systems combining the 90-day BA rate and one of the FRA rates (Table 2) have the appropriate number of EHTS-imposed cointegrating vectors (one, in this case). They therefore satisfy the necessary condition for the use of a cointegration technique to measure variable-term premiums. However, in certain three-rate systems (see the top of Table 3), we cannot reject the hypothesis that the number of cointegrating vectors is less than two. That means that there is only one cointegrating vector in some of the three-interest-rate systems. Furthermore, the hypothesis of two or fewer cointegrating vectors is not rejected in the four-rate systems, and the hypothesis of three or fewer vectors is not rejected in the five-rate systems (see the bottom of Table 3). The first EHTS necessary condition is thus rejected in systems of more than two interest rates.

In the sixth column of Tables 2 and 3, we see that the null hypothesis that the sum of the coefficients of each cointegrating vector is equal to 0 is rejected at a significance level of 1 per cent in all the interest rate systems. These results show that the second long-run condition implied by the EHTS

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19. For more details on the conditions imposed by the EHTS in a system of more than two interest rates, see Engsted and Tanggaard (1994).

Table 2

Cointegration Tests on Two-Interest-Rate Systems, 8 August 1988 to 16 January 1998

Pair	Number of lags[2]	Cointegration tests (r)[1]							Cointegrating vector restriction test [5]
		H_0	Max V[3]	Trace	PGp[4]	Number of cointegrating vectors	λ_f (t-statistic)	λ_r (t-statistic)	
BA-FRA1x4	19	$j = 0$	33.82*	35.92*	35.43*	1	−0.033	0.021	
		$j \leq 1$	2.10	2.10	2.08		(–2.918)	(3.259	0.000++
BA-FRA3x6	19	$j = 0$	29.56*	31.69*	31.28*	1	−0.013	0.017	
		$j \leq 1$	2.13	2.13	2.10		(–3.007)	(3.210)	0.000++
BA-FRA6x9	19	$j = 0$	23.22*	25.23*	24.92*	1	−0.006	0.010	
		$j \leq 1$	2.01	2.01	1.99		(–2.259)	(3.161)	0.000++
BA-FRA9x12	19	$j = 0$	19.75*	21.67*	21.41*	1	−0.004	0.008	
		$j \leq 1$	1.92	1.92	1.90		(–2.008)	(3.185)	0.000++

1. The optimal number of VECM lags was determined using the Sims (1980) procedure with a maximum number of 21 lags.
2. Tests for the number of cointegrating vectors use the critical values from Table 1 in Osterwald-Lenum (1992), in which the author assumes that the data generation process is designed with only one constant in the error-correction vector.
3. These "maximal eigenvalue" statistics, and the "trace" statistics in the next column to the right, are cointegrating test statistics suggested by Johansen and Juselius (1990).
4. "Corrected Pitavakis–Gonzalo" statistics; see Paquet (1994) for details.
5. The statistic shown is the p value of the null hypothesis test that the cointegrating vector β coefficient equals 1.

* Indicates rejection of the null hypothesis at a critical level of 10 per cent.

++ indicates rejection of the null hypothesis at significance levels of 1 per cent.

Table 3

Cointegration Tests on Three-, Four-, and Five-Interest-Rate Systems, 8 August 1988 to 16 January 1998

Pair	Number of lags[1]	Cointegration tests (r)[2]				Number of vectors	Cointegrating vector restriction test [5]
		H_0	Max V[3]	Trace	PGp[4]		
BA-FRA1x4–FRA3x6	19	$j \le 1$	28.25*	30.32*	29.69*	2	
		$j \le 2$	2.08	2.08	2.04		0.000^{++}
BA-FRA1x4–FRA6x9	19	$j \le 1$	16.25*	18.07*	17.71	2	
		$j \le 2$	1.82	1.82	1.79		0.002^{++}
BA-FRA1x4–FRA9x12	19	$j = 0$	41.49*	56.73*	55.50*	1	
		$j \le 1$	13.48	15.24	14.95		na
BA-FRA3x6–FRA6x9	17	$j = 0$	36.46*	51.12*	50.14*	1	
		$j \le 1$	12.82	14.66	14.41		na
BA-FRA3x6–FRA9x12	17	$j = 0$	34.51*	47.72*	46.80*	1	
		$j \le 1$	11.33	13.21	12.98		na
BA-FRA6x9–FRA9x12	17	$j = 0$	33.98*	47.74*	46.83*	1	
		$j \le 1$	11.80	13.75	13.52		na
BA-FRA1x4–FRA3x6-FRA6x9	21	$j \le 1$	39.08*	54.06*	52.32*	2	
		$j \le 2$	13.15	14.98	14.53		na
BA-FRA1x4–FRA3x6-FRA9x12	19	$j \le 1$	40.61*	54.21*	52.60*	2	
		$j \le 2$	11.71	13.59	13.22		na

(continued)

Table 3 (cont'd)

Cointegration Tests on Three-, Four-, and Five-Interest-Rate Systems, 8 August 1988 to 16 January 1998

Pair	Number of lags[1]	Cointegration tests (r)[2]					Cointegrating vector restriction test [5]
		H_0	Max V[3]	Trace	PGp[4]	Number of vectors	
BA-FRA1x4–FRA6x9-FRA9x12	19	$j \le 1$	40.73*	54.79*	53.16*	2	na
		$j \le 2$	12.22	14.05	13.67		
BA-FRA3x6–FRA6x9-FRA9x12	17	$j \le 1$	35.33*	49.06*	47.77*	2	na
		$j \le 2$	11.90	13.74	13.40		
BA-FRA1x4-FRA3x6–FRA6x9-FRA9x12	18	$j \le 2$	35.33*	38.33*	37.33*	3	na
		$j \le 3$	12.78	14.59	14.10		

1. The optimal number of VECM lags was determined using the Sims (1980) procedure with a maximum number of 21 lags.
2. Tests for the number of cointegrating vectors use the critical values from Table 1 in Osterwald-Lenum (1992), where the authors assume that the data generation process is designed with only one constant in the error-correction vector.
3. These "maximal eigenvalue" statistics, and the "trace" statistics in the next column to the right, are cointegrating test statistics suggested by Johansen and Juselius (1990).
4. "Corrected Pitavakis–Gonzalo" statistics; see Paquet (1994) for details.
5. The statistic shown is the p value of the null hypothesis test that the sum of the coefficients of each cointegrating vector is 0.

* Indicates rejection of the null hypothesis at a critical level of 10 per cent.

++ indicates rejection of the null hypothesis at significance levels of 1 per cent.

na means when we do not find the EHTS-imposed number of cointegrating vectors, the null hypothesis test regarding the zero-sum cointegrating vector restriction is not applicable.

does not hold. One explanation for this result (ignoring the rejection of the first necessary condition for the moment), under the assumption that the process driving interest rates is inflation, is that β may not equal 1 in Canada over the sample used, because investors are slow to adjust to the changing trend in the Canadian inflation rate (announced during speeches made by the Governor of the Bank of Canada throughout 1988 and formalized in February 1991).[20]

On another matter, tests performed on the adjustment coefficients of the error-correction terms λ_f and λ_r presented in Table 2 show that we can reject the null hypothesis of a zero value, thus implying the rejection of weak exogeneity of either variable. Term-structure estimation methods using a single-equation cointegrating technique would thus be invalid, for they assume that the forward rate is weakly exogenous.

We then perform diagnostic tests on the residuals of each two-rate cointegration equation;[21] the results are shown in Table 4. For each equation, the Lagrange Multiplier tests show that the null hypothesis of no autocorrelation is not rejected. The ARCH and Bera–Jarque tests show, however, that the residuals of each equation suffer from ARCH effects and are non-normal. Studies by Lee and Tse (1996) and by Cheung and Lai (1993) show that Johansen and Juselius cointegration tests are reasonably robust both to kurtosis and to ARCH effects present in the residuals. We are not aware, however, of any study analyzing the influence that non-normal residuals and ARCH effects on tests of the $\beta = 1$ hypothesis. These residuals' characteristics could bias the results of the hypothesis tests, and could therefore influence the test of the EHTS $\beta = 1$ imposed condition.

To analyze the influence that ARCH effects might have on the $\beta = 1$ hypothesis tests, we re-estimate the two-interest-rate system VECM parameters by modelling the residuals in a multivariate generalized autoregressive heteroscedasticity (GARCH) process. To do so, we relax the assumption that the residuals' variance–covariance matrix elements are constant, and we model them by a process that depends on past matrix elements (variances and covariances) as well as past residuals. This process, a multivariate GARCH estimation technique called BEKK, was developed by Engle and Kroner (1995). Furthermore, within the VECM maximum likelihood estimation, we assume that residuals follow a *t* distribution to

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20. See Appendix 2 for more details.

21. Results of diagnostic tests applied to the residuals of three-, four- and five-interest-rate systems are not presented in Table 4, as they lead to the same conclusions as those of two-rate systems.

Table 4

Diagnostic Tests of the Two-Interest-Rate System VECM Residuals

Equations	Number of lags (L)	Autocorrelation LM Test [1]	ARCH χ^2 (L) Statistic[2]	Bera–Jarque Normality Test [3]
FRA1x4 BA90	19	7.931	369.61** 42.40**	5,951.79** 5,898.48**
FRA3x6 BA90	19	5.032	282.99** 49.76**	4,610.76** 6,167.41**
FRA6x9 BA90	19	2.771	323.86** 58.79**	1,996.34** 6,615.05**
FRA9x12 BA90	19	1.798	255.31** 63.60**	2,349.24** 7,327.14**

1. Lagrange Multiplier (LM) multivariate test of no autocorrelation.
2. Chi-squared test for absence of ARCH processes (L degrees of freedom).
3. Bera–Jarque chi-squared normality test (two degrees of freedom).

** indicates rejection of the tests at significance thresholds of 5 per cent and 1 per cent, respectively.

properly take into account their non-normality.[22] We therefore re-estimate the two-rate VECM parameters, while allowing residuals to take the heteroscedastic GARCH functional form. We find that the resulting estimated β parameter to be slightly smaller for each maturity. Furthermore, *p* values of the null hypothesis test that $\beta = 1$ are also smaller than those obtained from the original VECM estimation. These results therefore simply reinforce the rejection of the EHTS illustrated in Table 2.[23]

3.2 Estimating the new measure of interest rate expectations

In this subsection, we estimate the cointegration vector for each combination of spot and forward rates (BA–FRA 1x4, BA–FRA 2x5, and so on up to BA–FRA 9x12) in order to estimate the variable component of the term premium across all short-term maturities. Each cointegration vector comes from the estimation of the VECM described in the previous section. The variable measure of the term premium is then used to calculate the new (VECM) measure of interest rate expectations. This measure of interest rate expectations in Canada is then compared with that based on the EHTS, using constant-term premiums.

22. The empirical literature finds that a *t* distribution seems to properly assess the non-normality present in high-frequency data.

23. Detailed results of the cointegrating vector parameters and of the $\beta = 1$ test statistics (from the re-estimated VECM in which the residuals were modelled as a multivariate GARCH process using a *t* distribution) are available from the authors.

As we saw in Table 2, none of the combinations of spot rates and forward rates conform to the EHTS-imposed cointegration vector, i.e., the vector (1, –1). Since this is a necessary (long-term) condition for the EHTS to hold, its rejection implies that daily forecasts of the behaviour of 3-month interest rates in Canada using these forward rates, and based on the EHTS, are biased in the short term. As well, the second column in Table 5 shows that the longer the maturity for the forward rate, the further the cointegration vector deviates from its –1 value dictated by the EHTS.[24]

The time-series properties of the variable component of the term premium are shown in Table 5. We find that the variance of the time-varying component of the term premium increases continuously along the maturities. During periods of interest rate variability, the variable compensation required by investors for investing in longer-term assets seems to increase with the maturity.[25] Ljung–Box autocorrelation tests show that the estimated variable component of each term premium is strongly autocorrelated. Moreover, we tested each term premium for skewness and kurtosis. Results indicate that the skewness and the kurtosis of term premiums differ statistically from that of a normally distributed time series. We also find a positive skewness for all the term premiums, except for that with a maturity of 1 month.

Although we have shown that a measure of market expectations based on the EHTS hypothesis is biased, it is possible to obtain a more accurate measure of interest rate expectations. To do so, we use the equilibrium relation depicted in equation (2). Specifically, to obtain a measure of expectations about the behaviour of 3-month Canadian interest rates, we use equation (7) below.

In this section, we compare our VECM measure of market expectations, equation (7), to that using constant-term premiums. The latter, based on the EHTS, is defined as follows:

$$E_t[r_{t+k}] = f_t^k - c,$$

where c is the constant-term premium. This constant-term premium is obtained by the estimation of α in equation (4) under the constraint that the β coefficient is equal to 1. The results of the estimation of this constant-term premium show that it increases with the maturity of the forward rate, going from 6 to 20, and from 58 to 100 basis points when the maturity increases

24. This is consistent with the first half of what is known in literature as the “predictability smile.” For more details on this concept, see Roberds and Whiteman (1997).

25. Notice that the joint increase in the term premium and the maturity we are dealing with here relates only to the time-varying component of the term premium, not to the fixed component of the term premium.

Table 5

Time-Series Properties of the Variable Component of the Term Premium

Combinations	Cointegration vector:[1] $f_t - \hat{\beta} r_t - \hat{\mu} = \hat{x}_t$	Variance	Skewness	Kurtosis	Autocorrelation (*p* value of the Ljung–Box test)[2]
BA–FRA1x4	$f_t - 0.962 r_t - 0.324$	0.035	−0.134 +	3.847 +	0.000**
BA–FRA2x5	$f_t - 0.917 r_t - 0.672$	0.102	0.536 +	1.223 +	0.000**
BA–FRA3x6	$f_t - 0.881 r_t - 0.969$	0.172	0.674 +	0.833 +	0.000**
BA–FRA4x7	$f_t - 0.837 r_t - 1.311$	0.242	0.785 +	0.613 +	0.000**
BA–FRA5x8	$f_t - 0.801 r_t - 1.594$	0.302	0.815 +	0.700 +	0.000**
BA–FRA6x9	$f_t - 0.772 r_t - 1.841$	0.342	0.888 +	0.573 +	0.000**
AB–FRA7x10	$f_t - 0.739 r_t - 2.118$	0.393	0.913 +	0.532 +	0.000**
AB–FRA8x11	$f_t - 0.717 r_t - 2.320$	0.429	0.934 +	0.548 +	0.000**
AB–FRA9x12	$f_t - 0.698 r_t - 2.505$	0.450	0.877 +	0.338 +	0.000**

1. Notice here that $\hat{\mu}$ is not equal to $\hat{\alpha}$. See footnote 12.
2. *p* values for the Ljung–Box test of the no-autocorrelation hypothesis (with 21 lags).

** indicates rejection of the null hypothesis at significance level of 1 per cent.

+ indicates rejection of null hypotheses that skewness and kurtosis are equal to 0 and 3, respectively (at the 1 per cent significance level).

from 1 to 3, and from 6 to 9 months, respectively.[26] This measure of interest rate expectations is compared with the VECM measure, which includes the variable component of the term premium, and is calculated as follows:

$$E_t[r_{t+k}] = f_t^k - c - \hat{x}_t. \tag{7}$$

By way of example, to obtain the measure of the expected rate on 90-day BAs in 3 months with a constant-term premium measure, we subtract 20 basis points (the constant component of the term premium found in the second column of Table 5) from the FRA 3x6 rate at a certain date. Our VECM measure of expectations subtracts from that used above the estimated variable component of the term premium $\hat{x}_t$ [i.e., as indicated in Table 5 ($FRA3x6_t$ – 0.881*$BA90_t$ – 0.969)].

In Figures 1 and 2, we compare the measures of 3-month interest rate expectations for 3 and 9 months hence.[27] The solid line is the measure of expectations, adjusted by the variable premium (VECM measure), and the dotted line is the measure using a constant-term premium. The variations of the variable component of the term premiums are presented at the bottom of Figures 1 and 2. The variable component of the term premium is negative during certain periods. The divergence between the two expectation measures increases during periods of greater volatility or when there is a changing trend in 3-month interest rates. In fact, Figure 1 shows that during periods of high volatility the variable component of the term premium becomes sharply positive or negative.[28] For example, during the Mexican crisis in 1994, 3-month Canadian interest rates went up suddenly and remained volatile until 1995. The variable component of the term premium for the 3-month maturity (Figure 1) reacted by increasing sharply during summer of 1994 to reach a value of more than 100 basis points. It remained high during much of 1994, reducing considerably the VECM measure of expectations in comparison with the EHTS measure based on a fixed-term premium. Then, following several positive events such as the improved Canadian government budgetary status and some other economic changes,

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26. As a comparison, the same constant-term premium is at 47 basis points for a 9-month maturity for the United States. Constant-term premiums increase less rapidly with maturities than their Canadian equivalent.

27. For brevity's sake, we show only the comparison of the two expectations measures for the spot rate 3 and 9 months hence. The divergence of these measures (i.e., the behaviour of the variable component of the term premium) behaves in essentially the same way for other maturities. Note, however, that the scale of the variable component of the term premium increases with the maturity of the forward rate.

28. When interest rates drop suddenly, the estimated variable component of the term premium tends to becomes positive, whereas it becomes negative when the 3-month spot rate increases sharply relative to the FRA rate.

Figure 1

Market Expectations Comparisons: Constant Premium vs. Vector Error-Correction Model

(Samples: 8 August 1988 to 16 January 1998, FRA 3x6)

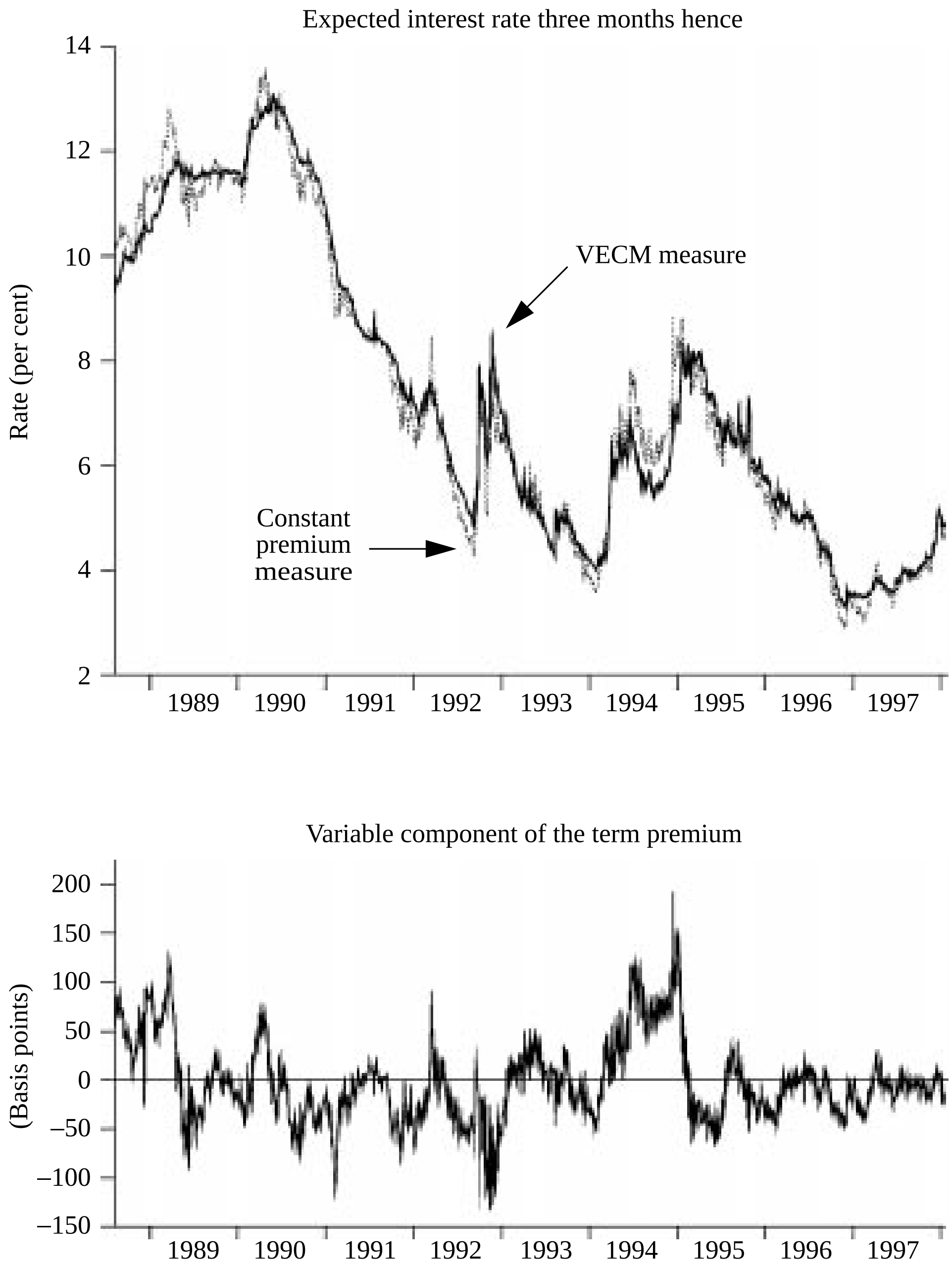
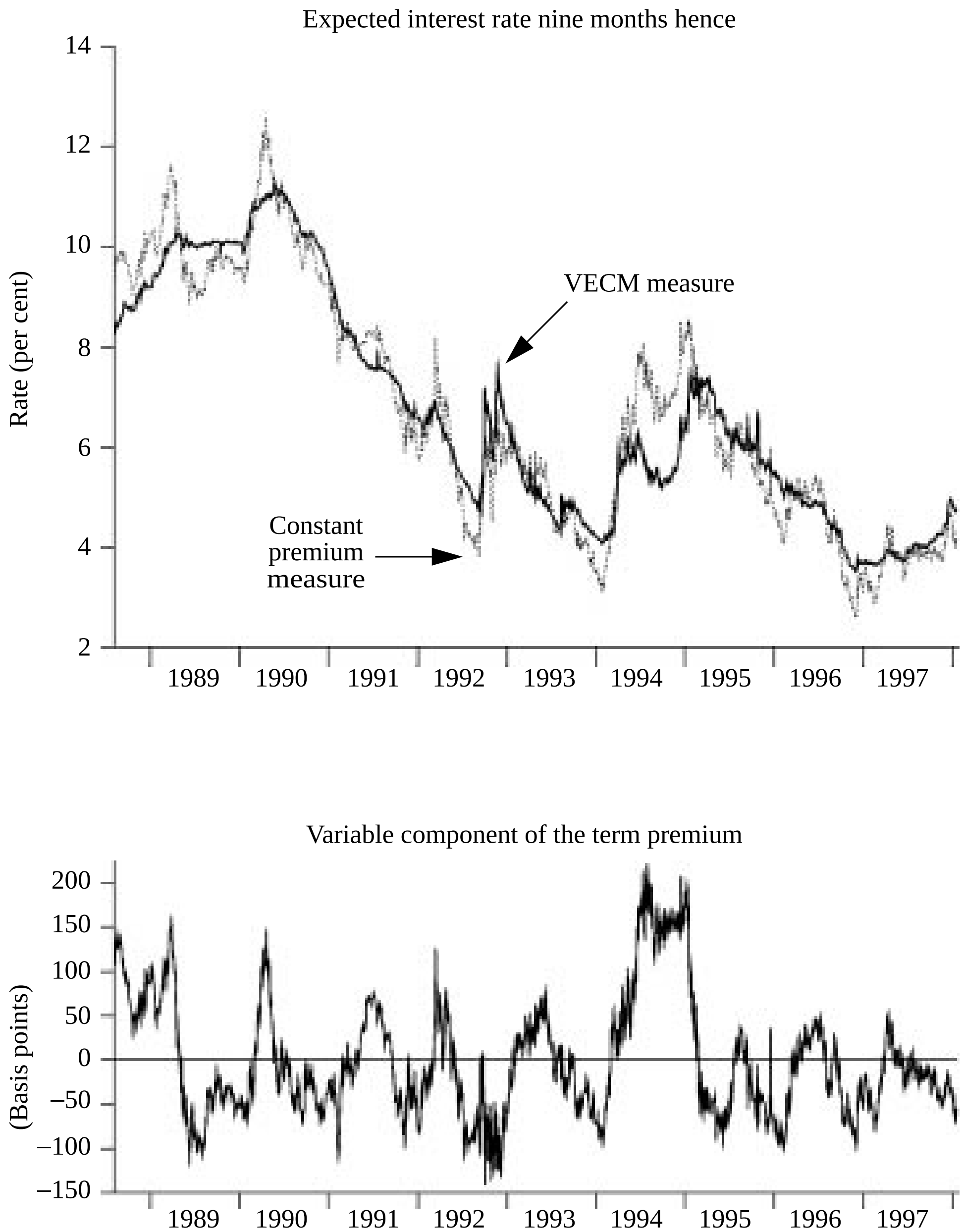


Figure 2

Market Expectations Comparisons: Constant Premium vs. Vector Error-Correction Model

(Samples: 8 August 1988 to 16 January 1998, FRA 9x12)



the volatility of interest rates settled down, and, eventually, the time-varying component of the term premium for 3-month-hence expected interest rates came back to its long-term value of 0. In fact, between mid-1995 and 1998, the variable part of the term premium has remained stable around the same low values, i.e., between plus and minus 50 basis points for the 3-month maturity (Figure 1) and between plus and minus 85 basis points for the 9-month maturity (Figure 2). Consequently, since the beginning of 1995, the VECM measure of interest rate expectations has been almost identical to the expectations measure based on a constant-term premium. During periods of stability of interest rates, the VECM measure reverts to the EHTS-based measure of expectations.[29]

In Figures 3 and 4, we compare for several dates the measures of interest rate expectations based on the constant-term premiums with the measure extracted from the VECM. Figure 3 shows, for 14 December 1994 and 15 March 1995, FRA rates for time horizons between 1 and 9 months, and both measures of the expected movements of BAs extracted from these rates. Two events strongly influenced the evolution of Canadian interest rates between these two dates: the Mexican Peso crisis, and the credit-rating downgrade of the Canadian government's debt by national and international credit-rating agencies. On 14 December 1994, while both events were already perturbing financial markets, the two measures of expectations differ greatly. The variable component of the term premium widens greatly and compensates for a very steep and positive term structure of FRA rates. The term structure of expected interest rates calculated within the VECM keeps a negative slope, but the expected term structure based on constant-term premiums takes a very positive slope. On 15 March 1995, when both economic shocks had already been absorbed by financial markets, the two measures of expectations are very similar.

In Figure 4, we compare the two term structures of expected 3-month treasury bill rates on 14 October 1997 and on 9 February 1998. These dates were chosen as they correspond, on the one hand, to the beginning and the end of the Asian "crisis," and on the other, to precise dates on which expectations of 3-month treasury bill rates in 4 and 13 months were gathered from Canadian financial institutions. On 14 October 1997, both measures of

29. The main goal of this study is to obtain a precise measure of market expectations of future movements of 3-month Canadian interest rates. It would therefore be useful to compare the VECM measure of expectations to a measure obtained from a survey of market participants' expectations. Unfortunately, survey data on the average expected movements in 3-month interest rates are not available on a regular basis in Canada. However, as of January 1996, Canadian financial institutions' expectations of 3-month treasury bill rates in 4 and 13 months are available from the *Consensus Forecast* monthly publication. These consensus survey expectations are illustrated in Figure 4.

Figure 3

Expected 3-Month Bankers' Acceptance Rates, All Expectations Horizons, 1 to 9 months

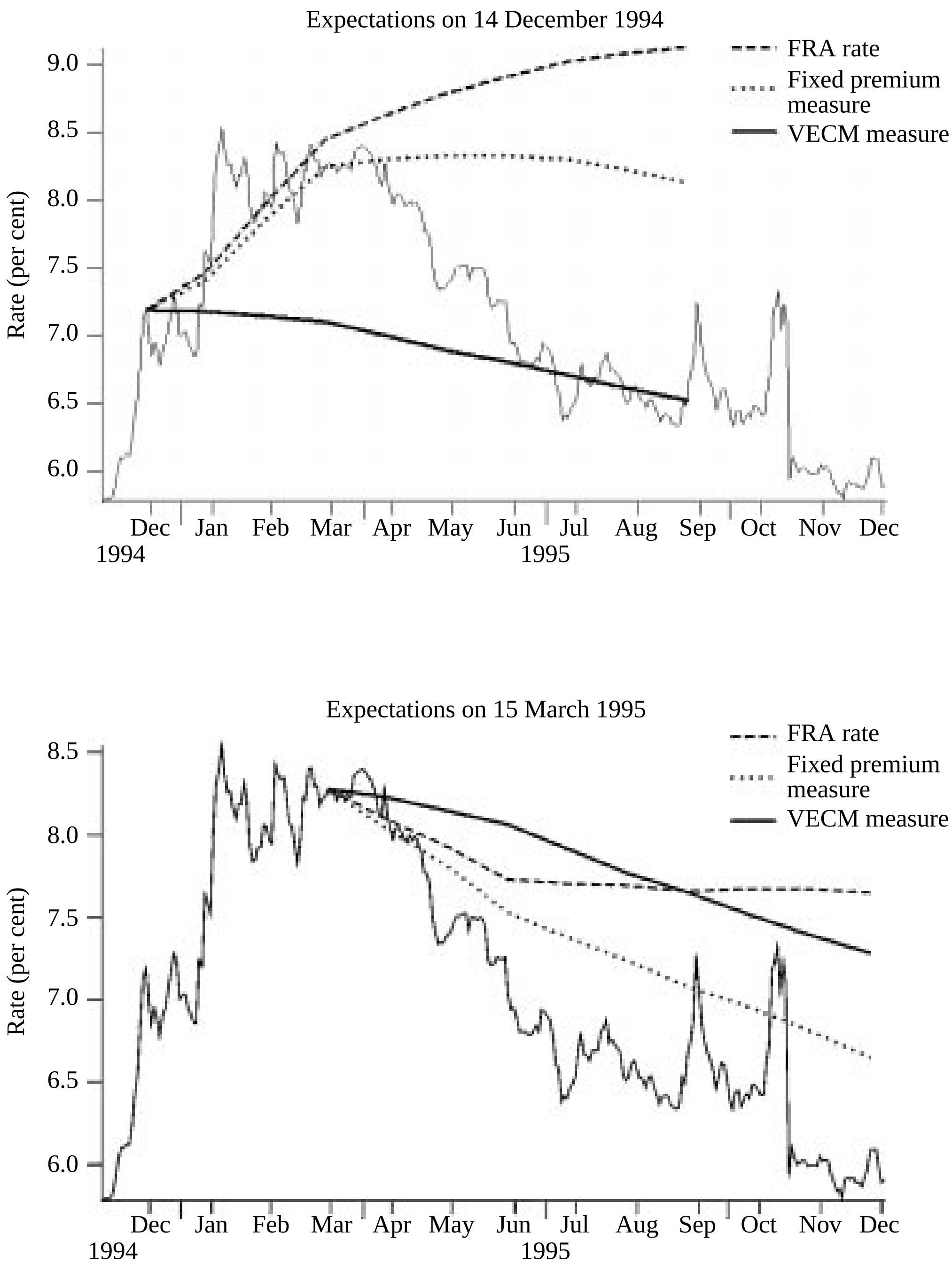
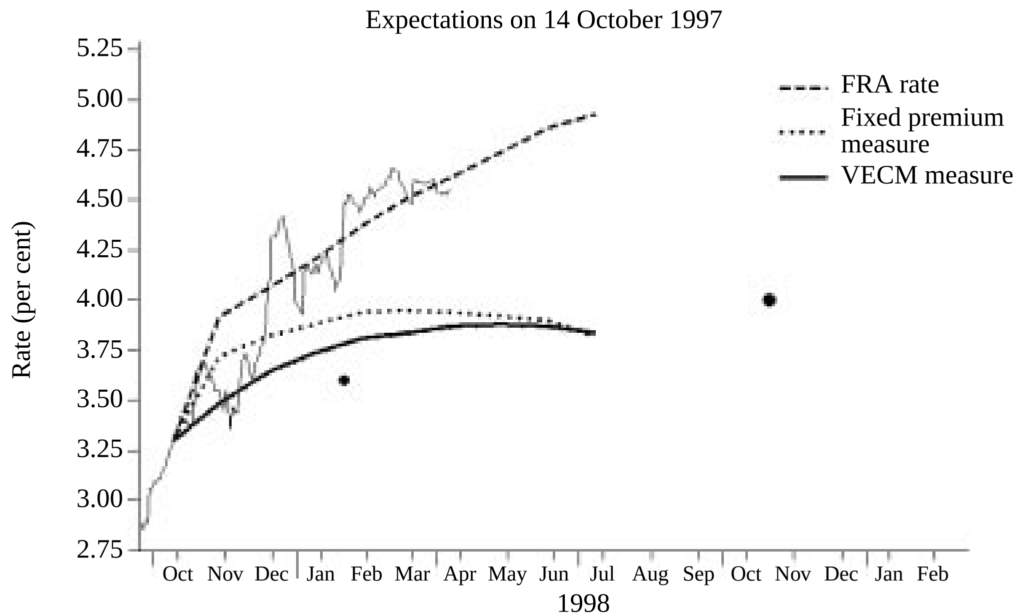
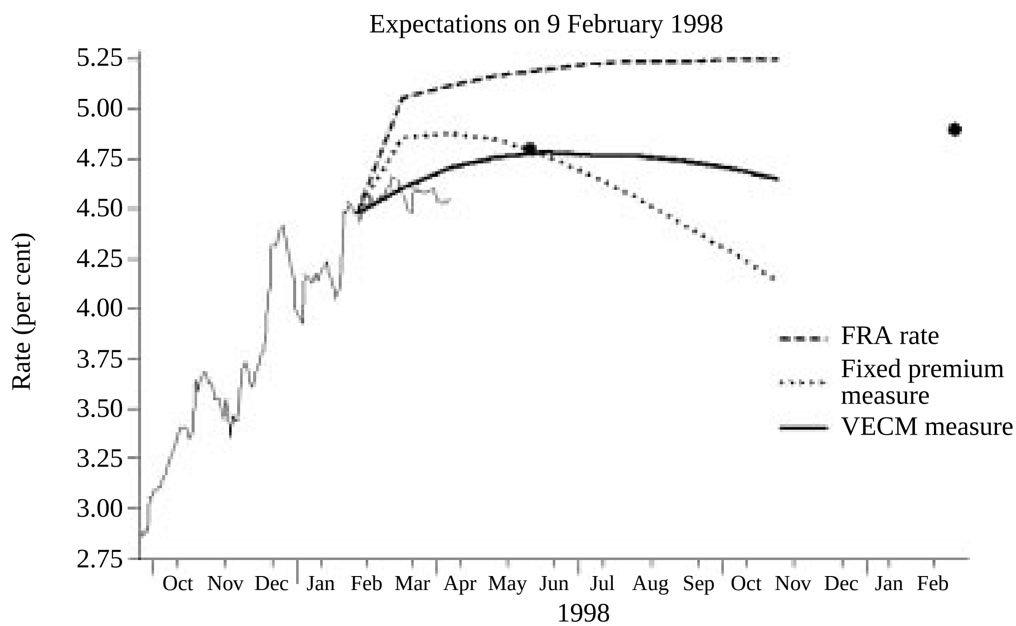


Figure 4

Expected 3-Month Treasury Bill Rates, All Expectations Horizons, 1 to 9 months





- Consensus survey forecast of 3-month treasury bill rates in 4 and 13 months.

expectations produce similar structures of expected interest rates. However, the VECM measure of expectations seems to better follow consensus expectations gathered from surveys (represented by bullets). On 15 January 1998, during a period of increased volatility in Canadian interest rates mainly attributable to the effects of the Asian "crisis," the two estimated term structures of expected interest rates differ greatly. The VECM measure of expectations indicates an moderate increase in interest rates over the next four months, followed by stable rates over the following five. However, the measure of expectations based on a constant-term premium indicates a pronounced increase in interest rates until April, followed by a decline over the next six months. Diverging views between these two measures of interest rates expectations therefore seem to be linked with the volatility of interest rates. On 9 February 1998, the VECM measure of expectations seems to be more in line with financial market survey expectations than with the constant-term premium measure. Both the VECM measure and consensus survey data indicate that financial markets were expecting a slight increase in interest rates followed by stable rates.

3.3 Is the term premium a measure of risk?

There is an extensive literature that assumes that the term premium is variable over time, and that it represents a measure of risk.[30] This type of research, pioneered by Engle, Lilien, and Robins (1987), models the term premium on a GARCH-M model, where the level of the term premium is a linear function of its own time-varying conditional variance. This variance is taken as a measure of the underlying risk associated with the asset.[31] These studies regularly find a positive statistical relationship between the term premium (excess returns) and its conditional volatility. If our measure of the term premium corresponds in part to a risk premium, then the variable component estimated for the term premium, $\hat{x}_t$, should have a significant positive dependence on its own conditional volatility.

Results of the GARCH-M (1,1) models of the term premium are presented in Table 6. Except for the 1-month horizon, we find that there is a positive and significant relation, ($\gamma > 0$), between each estimated variable-

30. More accurately, the literature models the time series characteristics of excess returns, which are equivalent to the expected term premium under standard no-arbitrage conditions—see Ilmanen (1996). The conditional variance of excess returns is a good proxy for the volatility of the underlying asset, since excess returns are in fact a linear function of the underlying yield of the asset in question.

31. These GARCH-M models assume that variance is autoregressive and conditionally heteroskedastic. Engle, Ng, and Rothschild (1990), Engle and Ng (1993), and Lee (1995) are other studies that have modelled the variable-term premium using a GARCH-M model.

Table 6

Estimation of the GARCH-M Model (1,1)

$$\textbf{Model: } \hat{x}_t = \alpha + \gamma h_t^{1/2} + \varepsilon_t$$
$$h_t = a + bh_{t-1} + c\varepsilon_{t-1}^2$$

	a	γ	a	b	c
TP1x4*	−0.0277	−0.0147	0.0003	0.5450	0.5334
	(–11.654)	(–0.484)	(4.479)	(22.946)	(12.290)
TP3x6	−0.1742	0.1146	0.0019	0.3847	0.6424
	(–17.530)	(3.6616)	(5.974)	(11.979)	(14.727)
TP6x9	−0.3770	0.3304	0.0048	0.2207	0.7781
	(–26.072)	(10.809)	(8.564)	(6.315)	(16.564)
TP9x12	−0.4933	0.4849	0.0039	0.3584	0.6377
	(–46.963)	(18.298)	(5.699)	(8.790)	(13.739)

Notes: $\hat{x}_t$ is the estimated zero-centred variable component of the term premium.
The student test statistics are shown in parentheses.
* indicates that a *t* distribution forms the basis of the log-likelihood in the maximization used to estimate parameters.

term premium and its conditional volatility, h_t.[32] Moreover, the conditional volatility itself depends not only on past squared events ($c > 0$), but also on past conditional volatility (b is strongly significant). These results suggest that the time-series properties of the estimated variable-term premium are consistent with those estimated in the literature, where the term premium is assumed to be a measure of risk. In addition, high volatility today indicates that the future term premium will also be high, since the time variation of conditional volatility is persistent. These results support the observed behaviour of the term premiums estimated with the VECM, in subsection 3.2, which tend to rise during periods of high volatility.

4 The Robustness of the VECM Results

As noted earlier, our data set may appear relatively small, considering that we are estimating cointegration relationships, which usually requires data spanning a longer period. This data set does not cover a full economic cycle, and does not include all of the inflection points in the trend of the interest rate with which it is associated. To gauge the robustness of our results, we evaluate the validity of the necessary conditions of EHTS over several different sample lengths using the same cointegration technique applied in Section 3.

32. In GARCH-M models, the level of the estimated variable, $\hat{x}_t$, depends on its own variance, which is itself modelled by a GARCH process.

First, we tested the EHTS with three-, four-, and five-rate systems of treasury bill rates, to verify that the rejection of the EHTS conditions presented in section 3.1 is not due to the characteristics of the FRAs and the BAs in the sample used. We present the results of the estimations, based on a sample beginning 7 January 1982 and ending 16 January 1998 and, for comparison purposes, data covering 1988 to 1998, in Table A1.1 in Appendix 1. We find that all the systems of interest rates have the number of cointegration vectors required by the EHTS, even the sample period 1988 to 1998. The rejection of this condition in Section 3.1 on some of the three-, four-, and five-interest-rate (BA and FRA) systems may therefore be specifically attributable to the relation between FRA and the spot rate. We also observe that the null hypothesis that the sum of the cointegration vector coefficients is 0 is rejected to a significance level of 1 per cent in all systems, except for one using the 1988–1998 sample, and in 11 of the 16 systems using the 1982–1998 sample. Thus, the second long-term EHTS-imposed condition is in general rejected.

The results in Table A1.1 indicate that the EHTS does not hold in Canada when it is evaluated over an extended period (1982 to 1998) on systems of three, four or five interest rates. In order to test the validity of EHTS in two-interest-rate systems, we estimate the cointegration vector between 90-day treasury bill rates and an implicit forward rate (IFR 3x6). This IFR is extracted from 90- and 180-day treasury bill rates on the sample periods 1979 to 1998, 1988 to 1998, and 1991 to 1998.[33] The results are presented in Table A1.2 in Appendix 1. We find that there is a cointegration vector in all the samples, thus satisfying the first necessary EHTS condition. However, we reject the null hypothesis that the sum of coefficients of the cointegration vector is equal to 0 for all the samples, except for the most recent one, 1991 to 1998. The EHTS does not hold when it is analyzed over an extended period spanning several economic and interest rate cycles.

The EHTS stipulates that the different interest rates are driven by a common stochastic component. In the literature, it is sometimes assumed that this common component is linked to inflation.[34] On 26 February 1991, the Bank of Canada and the Department of Finance jointly announced that price

33. With the spot rates for 90- and 180-day treasury bills (available since January 1979) we can calculate an IFR 3x6 (91x181) using the following no-arbitrage relation:

$$\left[(1+TB_{91\text{ days}})\left(\frac{91}{365}\right)\right][(1+IFR_{3\text{x}6})] = \left[(1+TB_{181\text{ days}})\left(\frac{181}{365}\right)\right].$$

The implied forward rate 3x6 is, however, the only implicit rate that we can calculate from available maturities of treasury bills that is comparable to an FRA.

34. See, for example, Engsted and Tanggaard (1994). These authors use the Fisher hypothesis postulating that the inflation rate is the permanent trend that leads the nominal term structure.

stability was to be defined as a core inflation rate of less than 2 per cent. To reach their goal, they defined successive bounds to the inflation rate, progressively directing it toward the 2-per-cent target. In addition, there was a change of direction in interest rates in May 1990. After that date, 3-month Canadian interest rates tended to decline across the entire 1988–1998 sample.

It is reasonable to believe that the behaviour of interest rates is difficult to predict when the monetary policy stance is shifting. Stréliski (1998) found, using the single-equation regression technique, that if he excluded the data for the first year (August 1988 to July 1989) of the 1988–1997 sample, the test results in fact supported the EHTS. For these reasons, we re-estimated the BA–FRA rate combinations for the period beginning 2 January 1991 and ending on 16 January 1998. The results are shown in Table A1.3 in Appendix 1. We cannot reject the hypothesis that there is a cointegration vector for all combinations. Table A1.3 shows, however, a marked difference from the results obtained with the full sample (1988–1998). In no case can we reject the hypothesis that the sum of the cointegration vector coefficients is equal to 0. It would seem, then, that the long-term conditions of the EHTS hold for a period where interest rates do not change trend, and during which inflationary expectations are homogeneous, such as 1991–1998. This would seem to indicate that regime changes tend to have a negative impact on the validity of the EHTS over the long run.

Conclusions

The primary objective of this study was to obtain an accurate measure of market expectations about the future behaviour of 3-month interest rates in Canada. According to the EHTS, each long-term rate represents the average of market expectations about short-term rates over the life of a long-term security, plus a constant-term premium. The EHTS thus offers a practical analytical framework for estimating average market expectations about the behaviour of short-term rates on the basis of the term structure of interest rates.

To accurately test the validity of the necessary conditions imposed by the expectations hypothesis in Canada, we use daily rates for 90-day BAs and for Canadian FRAs for maturities of 1 to 9 months. The analysis is conducted using a cointegration technique. The use of a VECM lets us test the long-term conditions of the EHTS in systems with two, three, four, and five interest rates. We find that in an environment in which interest rates are assumed (and found) to be non-stationary, the EHTS is invalid for Canada. The sensitivity analysis tests performed with daily data on treasury bill rates over longer periods (1979 to 1998 and 1982 to 1998) confirm this conclusion. A measure of expectations of short-term interest rates based on constant-term premiums is therefore incorrect. Nevertheless, we find that the EHTS is valid when evaluated over a period when Canadian 3-month interest rates did not change their trend and when expectations about inflation were homogeneous.

The rejections of the EHTS are perhaps in part due to the variability of the term premium. In principle, a daily measure of the time-varying component of the term premium should help us to statistically quantify market expectations. We therefore use an arbitrage relation that assumes that there is a time-varying term premium, and we apply it to the FRA and 90-day BA rates. This arbitrage relation is more general than the EHTS, which is in fact a particular case of that relation. In order to accurately measure the variable component of the term premium for different forward rates, we estimate a VECM. This technique lets us calculate a time-variable, but stationary, measure of the term premium. The measure of 3-month interest rate expectations that we propose adjusts the measure of expectations based on a constant-term premium by adding to it a variable component.

We find that the discrepancy between the VECM measure of interest rate expectations and that based on a fixed-term premium increases during periods of heightened interest rate volatility. When interest rates are stable, the VECM measure seems to come back to the EHTS-based expectations measure, which is a particular case of the equilibrium relationship used. Using a GARCH-M approach, we show that there is a positive relation between the variable-term premium and the volatility of interest rates. This finding supports the view that the variable-term premium can be viewed as a risk premium that rises in periods of heightened interest rate volatility.

It would be interesting to pursue this analysis of the variable component of the term premium. Other economic variables, such as the volatility of exchange rates and the spread of output with respect to its potential, could be added as explanatory variables in a multivariate version of the GARCH-M model presented in Section 3.

We assumed in this paper that any variation in the forward–spot rate spread that departs from the estimated long-term equilibrium relation is directly proportional to a change in the term premium. We concede that this assumption is debatable. So, it would be interesting to investigate the alternative of assigning a proportion Θ of forward-premium variation to a variable-term premium and $(1-\Theta)$ to expected interest rate changes. To this end, we believe that using the Kalman filtering technique to estimate a time-varying term premium should be studied. This technique provides the added advantage of estimating an equilibrium relation that allows for systematic forecast errors and the possibility of measuring the proportional parameter, Θ. It would also capture the cointegrating relationship between spot and forward interest rates dictated by the EHTS.

Appendix 1

Table A1.1

Treasury Bill Cointegration Tests in 3-, 4-, and 5-Equation Systems

		1 July 1982 to 16 January 1998					8 September 1988 to 16 January 1998				
Rates in the system	H_0	**Lags**[ϕ]	**Max V**[1]	**Trace**	**PGp**[2]	***p* value for** $H_0 \sum_{i=1}^{p} \beta_i = 1$	**Lags**[ϕ]	**Max V**[1]	**Trace**	**PGp**[2]	***p* value for** $H_0 \sum_{i=1}^{p} \beta_i = 1$
TB30-60-90	$r \leq 1$	21	73.37*	78.25*	76.99*	0.0858	21	40.51*	42.59*	41.56*	0.0979
	$r \leq 2$		4.88	4.88	4.82			2.08	2.08	2.04	
TB30-60-180	$r \leq 1$	21	49.43*	54.92*	54.12*	0.0663	21	34.34*	36.34*	35.49*	0.0005
	$r \leq 2$		5.49	5.49	5.43			2.00	2.00	1.96	
TB30-60-360	$r \leq 1$	21	33.14*	38.80*	38.27*	0.0232	21	21.83*	23.64*	23.11*	0.0018
	$r \leq 2$		5.66	5.66	5.59			1.80	1.80	1.77	
TB30-90-180	$r \leq 1$	21	38.34*	43.95*	43.34*	0.0614	21	30.57*	32.57*	31.82*	0.0000
	$r \leq 2$		5.62	5.62	5.55			2.00	2.00	1.96	
TB30-90-360	$r \leq 1$	21	30.28*	35.75*	35.27*	0.0170	21	18.73*	20.59*	20.14*	0.0007
	$r \leq 2$		5.47	5.47	5.40			1.86	1.86	1.82	
TB60-90-360	$r \leq 1$	21	30.95*	36.40*	35.90*	0.0120	21	19.86*	21.79*	21.31*	0.0000
	$r \leq 2$		5.44	5.44	5.38			1.94	1.94	1.90	
TB60-180-360	$r \leq 1$	21	33.31*	38.49*	37.97*	0.0006	20	17.67*	19.48*	19.07*	0.0044
	$r \leq 2$		5.18	5.18	5.12			1.81	1.81	1.78	
TB90-180-360	$r \leq 1$	20	33.56*	38.91*	38.40*	0.0003	19	16.17*	18.02*	17.66	0.0086
	$r \leq 2$		5.35	5.35	5.29			1.85	1.85	1.82	
TB30-60-90-180	$r \leq 2$	21	38.22*	43.71*	42.88*	0.0608	21	31.41*	33.45*	32.38*	0.0000
	$r \leq 3$		5.49	5.49	5.40			2.04	2.04	1.98	

(continued)

Table A1.1 (cont'd)

Treasury Bill Cointegration Tests in 3-, 4-, and 5-Equation Systems

		1 July 1982 to 16 January 1998					8 September 1988 to 16 January 1998				
Rates in the system	H_0	**Lags**[ϕ]	**Max V**[1]	**Trace**	**PGp**[2]	***p* value for** $H_0 \sum_{i=1}^{p} \beta_i = 1$	**Lags**[ϕ]	**Max V**[1]	**Trace**	**PGp**[2]	***p* value for** $H_0 \sum_{i=1}^{p} \beta_i = 1$
TB30-60-90-360	$r \le 2$	21	30.24*	35.65*	34.98*	0.0103	21	19.05*	20.95*	20.31*	0.0002
	$r \le 3$		5.40	5.40	5.31			1.90	1.90	1.85	
TB30-60-180-360	$r \le 2$	21	32.14*	37.32*	36.63*	0.0001	20	17.91*	19.70*	19.12*	0.0099
	$r \le 3$		5.19	5.19	5.09			1.79	1.79	1.74	
TB30-90-180-360	$r \le 2$	21	31.07*	36.29*	35.61*	0.0001	21	17.33*	19.24*	18.65*	0.0009
	$r \le 3$		5.22	5.22	5.13			1.91	1.91*	1.85	
TB60-90-180-360	$r \le 2$	21	31.59*	36.86*	36.17*	0.0003	21	17.06*	19.01*	18.42*	0.0000
	$r \le 3$		5.27	5.27	5.18			1.95	1.95	1.89	
TB30-60-90-180-360	$r \le 3$	18	38.22*	43.71*	42.88*	0.0003	18	31.41*	33.45*	32.38*	0.0000
	$r \le 4$		5.14	5.14	5.04			1.94	1.94	1.87	

1. These "maximal eigenvalue" statistics, and the "trace" statistics in the next column to the right, are cointegrating test statistics suggested by Johansen and Juselius (1990).
2. "Corrected Pitavakis–Gonzalo" statistics; see Paquet (1994) for details.

* Denotes rejection of the null at the 10 per cent critical level. Critical values are taken from Osterwald-Lenum (1992), Table 1, derived under the assumption that the data-generating process includes a constant in the cointegrating vector only.

ϕ The lag lengths of the VECMs were selected using the Sims (1980) modified likelihood ratio statistic.

Table A1.2

Cointegration Tests Results for Implied Forward-rate Term Structure Relation, 8 January 1979 to 16 January 1998

Pair	Lags[ϕ]	H_0	Max V[1]	Trace	PGp[2]	p values for H_0: $\hat{\beta} = 1$	Cointegrating vector estimates: $f_t - \hat{\beta} r_t - \hat{\mu} = \hat{x}_t$
8 January 1979 to 16 January 1998							
Treasury bill	18	$r = 0$	65.87*	68.18*	67.63*	0.0002++	$f_t - 0.922 r_t$
IF3x6		$r \leq 1$	2.31	2.31	2.30		$- 1.039$
8 September 1988 to 16 January 1998							
Treasury bill	20	$r = 0$	32.41*	34.38*	33.90*	0.0002++	$f_t - 0.893 r_t$
IF3x6		$r \leq 1$	1.97	1.97	1.95		$- 1.153$
2 January 1991 to 16 January 1998							
Treasury bill	20	$r = 0$	20.46*	26.91*	26.46*	0.3365	$f_t - 0.953 r_t$
IF3x6		$r \leq 1$	6.45	6.45	6.36		$- 0.938$

1. These “maximal eigenvalue” statistics, and the “trace” statistics in the next column to the right, are cointegrating test statistics suggested by Johansen and Juselius (1990).
2. “Corrected Pitavakis–Gonzalo” statistics; see Paquet (1994) for details.

* Denotes rejection of the null at the 10 per cent critical level. (The critical values are 13.75 and 7.52 for $r = 0$ and $r = 1$ respectively). Critical values are taken from Osterwald-Lenum (1992), Table 1, derived under the assumption that the data generating process includes a constant in the cointegrating vector only.

ϕ The lag lengths of the VECMs were selected using the Sims (1980) modified likelihood ratio statistic.

++ denotes the rejection of H_0: $\beta = 1$ at the 1 per cent critical level.

Table A1.3

Cointegration Test Results for Shortened Sample, 2 January 1991 to 16 January 1998

Pair	Lags[ϕ]	H_0	Max V[1]	Trace	PGp[2]	*p* values for H_0: $\hat{\beta} = 1$	Cointegrating vector estimates: $f_t - \hat{\beta} r_t - \hat{\mu} = \hat{x}_t$
BA–FRA1x4	19	$r = 0$ $r \leq 1$	21.62* 6.79	28.41* 6.79	27.95* 6.69	0.2737	$f_t - 0.970 r_t - 0.279$
BA–FRA3x6	19	$r = 0$ $r \leq 1$	17.65* 6.49	24.14* 6.49	23.76* 6.39	0.4117	$f_t - 0.937 r_t - 0.667$
BA–FRA6x9	19	$r = 0$ $r \leq 1$	14.10* 6.18	20.28* 6.18	19.97* 6.09	0.6890	$f_t - 0.941 r_t - 0.963$
BA–FRA9x12	19	$r = 0$ $r \leq 1$	12.83 6.18	19.01* 6.18	18.72* 6.09	0.7789	$f_t - 0.945 r_t - 1.233$

1. These "maximal eigenvalue" statistics, and the "trace" statistics in the next column to the right, are cointegrating test statistics suggested by Johansen and Juselius (1990).
2. "Corrected Pitavakis–Gonzalo" statistics; see Paquet (1994) for details.

* Denotes rejection of the null at the 10-per-cent critical level. (The critical values are 13.75 and 7.52 for $r = 0$ and $r = 1$ respectively). Critical values are taken from Osterwald-Lenum (1992), Table 1, derived under the assumption that the data-generating process includes a constant in the cointegrating vector only.

ϕ The lag lengths of the VECMs were selected using the Sims (1980) modified likelihood ratio statistic.

β = 1 at the 1 per cent critical level.

Appendix 2

A Common-Trends Cointegrating Representation of the Term Structure

A common-trends representation for the term structure is simply an alternative interpretation of the cointegration relationship between various *I*(1) interest rates. Recasting the term structure relation between the forward and spot money market interest rates within a common-trends representation model is also a useful way to derive the time-varying term premium measure extracted from the cointegrating term structure relation. This method is also flexible enough to illustrate how the estimated cointegrating vector coefficient, β, can deviate from the value implied by the expectations hypothesis.

Throughout this appendix, we assume that the spot and forward interest rates are non-stationary [*I*(1)] variables. Second, we assume that both the forward and spot time-series variables can be decomposed into their *permanent* [non-stationary or *I*(1)] and *transitory* [stationary or *I*(0)] components. Thus, the forward and spot rate processes are written as:

$$\begin{aligned} r_t &= r_{1,t} + r_{0,t} \\ f_t &= f_{1,t} + f_{0,t}, \end{aligned} \qquad \text{(A2.1)}$$

where the variables with a 1 and 0 subscript represent the permanent and transitory components, respectively.

Given this decomposition, we can show how the cointegrating relation between the spot and forward rates can be interpreted as what is referred to in the literature as common-trends cointegration.[1] This common-trends representation of the forward–spot term structure relation allows one to more easily understand how these rates may move together in the long run. Specifically, the common-trends representation assumes that there is an unobserved unit root process common to both interest rate processes such that:

$$\begin{aligned} r_t &= r_{1,t} + r_{0,t} = \tau_t + r_{0,t} \\ f_t &= f_{1,t} + f_{0,t} = \gamma\tau_t + f_{0,t}, \end{aligned} \qquad \text{(A2.2)}$$

1. This interpretation of cointegration states that if X_t is cointegrated with rank ρ, then each element of the *n*x1 vector X_t is a linear function of the same set of $n - \rho$ stochastic trends. This implies that if the forward and spot rates are cointegrated with cointegrating rank $\rho = 1$, then both the forward and spot rates have one common stochastic trend, because $n - \rho = 1$ in the case of two-rate systems.

where $\tau_t = \tau_{t-1} + \varepsilon_t$ and ε_t is a white noise process. In this case, the forward and spot rates have one common permanent component, τ_t, and transitory components specific to each interest rate process.[2] Because both variables in the system have a common permanent component, and because this permanent component influences the long-run path of each variable, these variables will have a long-run equilibrium, or cointegrating relation. As well, the expectations hypothesis demands that the forward and spot rates must not only be cointegrated, but also imply the added restriction that $\gamma = 1$ in (A2.2).

The time-varying term premium measure calculated in this study is derived by identifying the transitory component of the forward rate, $f_{0,t}$, with the time-series behaviour of the term premium, and by assuming that the spot rate's transitory component is simply a white noise process.[3] In this case, the system of equations in (A2.2) can be written as

$$f_t - \gamma r_t = f_{0,t} - \gamma r_{0,t} = \theta_t + \varepsilon_t,$$

where θ_t is the time-varying term premium, ε_t is a white noise process and $(f_t - \gamma r_t)$ forms the cointegrating vector. Therefore, the mean-zero time-varying term premium measure presented in Section 3 is based on the estimated cointegrating vector, $(f_t - \hat{\beta} r_t - \hat{\mu})$, which includes a scaling constant.

In the above discussion, estimated cointegrating coefficients that deviated from the theoretic value implied by the expectations hypothesis are assumed to occur because $\gamma \neq 1$ in (A2.2). However, it is difficult to find plausible economic explanations as to why a long-run shock that enters via the common stochastic trend does not have a proportionally equal impact on the *permanent* components of both the forward and spot rates. We offer two explanations of how the estimated cointegrating coefficient $\hat{\beta}$ may still deviate from the theoretical value implied by the expectations hypothesis, even when γ in (A2.2) is assumed to equal 1.

The following discussion is made easier by decomposing the term premium and forecast error terms from the equilibrium relation [equation (2)] into their permanent and transitory components. Using the notation

2. The permanent component often has an intuitive economic interpretation. For term structure relations, the permanent component can be associated with current or expected inflation, expected movements in the central bank's target rate, or with trend growth in money. See Engsted and Tanggaard (1994) and Anderson, Hall, and Granger (1992) for more on this.

3. These conditions in turn imply that changes in the forward rate that are related to changes in the expected future spot rate are associated with movements in the stochastic trend.

introduced above, this implies that the term premium can be written as $\theta_t = \theta_{0t} + \theta_{1t}$ and the forecast error as $\omega_{t+1} = \omega_{0t+1} + \omega_{1t+1}$.[4]

The first explanation relaxes the assumption that the forecast errors are white noise. This formulation of the forecast errors draws upon Evans and Lewis (1994), which suggests that the forecast errors will be non-stationary when investors rationally expect shifts in the time-series process driving the spot rate. Under the added assumption that the permanent component of the forecast errors is driven by the same unit root process driving the forward and spot rates such that $\omega_{1t+1} = \phi\tau_t$, one can write the following expression for the forward premium:[5]

$$f_t - r_t = \theta_{0t} + \omega_{0t+1} + \varepsilon_{t+1} + \omega_{1t+1} = \phi\tau_t + \theta_{0t} + \text{WN terms}.$$

Rewriting this expression in terms of a stationary cointegrating vector, $f_t - \gamma r_t$, we obtain:

$$f_t - \gamma r_t = f_t - (1+\phi)r_t = \theta_{0t} + \text{WN terms}. \tag{A2.3}$$

Thus, the estimated coefficient for γ (from the VECM) will diverge from the value of 1 as ϕ diverges from 0.

The second explanation is similar to the first, and also stems from Evans and Lewis (1994). In this case, it is the term premium's permanent component that is assumed to be driven by the common stochastic trend, with $\theta_{1t} = \phi\tau_t$. The forecast error process is assumed to be white noise. Under these assumptions, the forward premium can be written as:

$$f_t - r_t = \theta_{0t} + \omega_{0t+1} + \varepsilon_{t+1} + \theta_{1t+1} = \phi\tau_t + \theta_{0t} + \text{WN terms}.$$

Rewriting this expression in terms of the stationary cointegrating vector $f_t - \gamma r_t$ again yields equation (A2.3). Thus, the estimated coefficient for γ (from the VECM) will diverge from the value of 1 as ϕ diverges from 0.

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4. Note that ω_{1t+1} is defined as:

$$\omega_{t+1} = E_t[(r_{t+1} - r_t)] - (r_{t+1} - r_t).$$

Under the usual assumption of rational expectations, $\omega_{0t+1} = 0$ is a white noise process and $\omega_{1t+1} = 0$.

5. Note that equation (A2.2) includes an expected change in the spot rate term. We have assumed that the change in the spot rate is a white noise process ε_{t+1}, which in turn implies that the expected change in the spot rate can be written as:

$$E_t[(r_{t+1} - r_t)] = (r_{t+1} - r_t)\omega_{t+1} = \varepsilon_{t+1} + \omega_{t+1}.$$

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