Asset Pricing in Consumption Models: A Survey of the Literature

Benoît Carmichael

Introduction

This paper attempts to provide a survey of asset-pricing models based on the principle of maximization of expected utility. I will begin my analysis by setting out a simplified, discrete-time version of the model that was developed independently by Lucas (1978) and Breeden (1979). Since those studies appeared, intertemporal general-equilibrium models have come to occupy an increasingly important place in the economic literature on asset pricing. A common characteristic of those models is that prices and yields of financial assets are linked, in a general-equilibrium context, to investors' decisions about consumption and savings. The yield structure predicted by these models is therefore intimately tied to the nature of investors' preferences and, in particular, to the parameters of risk aversion and intertemporal substitutions. Moreover, in contrast to the capital-asset-pricing (CAPM) model of Sharpe (1964) and Lintner (1965), intertemporal general-equilibrium models identify clearly the underlying economic forces that influence the risk-free real interest rate and the compensation that investors earn by accepting risk.[1]

My analysis begins in Section 1 by developing a fundamental asset-pricing equation derived from the Lucas model. This equation links the excess return expected from a risky asset to the covariance of its yield with the intertemporal marginal rate of substitution of consumption. I will then

1. The CAPM model deals with the question of how asset prices and yields are determined, under the hypothesis that the risk-free interest rate and market return are variables determined outside the model.

discuss in detail the extent to which this restriction is compatible with observed empirical phenomena. We shall see, in particular, that preferences that fail to dissociate the concepts of risk aversion and intertemporal substitution cannot explain simultaneously the level of real interest rates and the level of the equity premium. I will conclude the second section with a brief discussion of two possible modifications to the structure of preferences that bring the model more in line with reality, especially regarding the real interest rate level.

Section 2 looks at the pricing of zero-coupon discount bonds or “strip” bonds. I focus here on the way prices and yields are set on the spot and forward markets. First, I will show that the price of a forward contract is, in general, a combination of the expected future spot price plus a risk premium. I will then examine to what extent the level and variability of the risk premiums predicted by the model are compatible with empirical observations. Once again, there are some major tensions between the model and the data. These are especially apparent when investors’ preferences fail to dissociate the concepts of risk aversion and intertemporal substitution. The section concludes with a brief discussion of options pricing.

The impact of inflation and of monetary growth on asset pricing is discussed in Section 3. Money is introduced into the model by means of a Clower cash-in-advance constraint. I will show that the uncertainty surrounding the purchasing power of money modifies the systematic risk of financial assets and, in general, gives rise to an inflation-risk premium. In the case of bonds, this premium reflects solely the covariance of the marginal rate of intertemporal substitution of consumption with the rate of appreciation of the purchasing power of money. In the case of equities, on the other hand, it also reflects the impact of the inflation tax path on the uncertainty surrounding future returns in the form of capital gains.

1 Prices and Returns in Consumption Models

1.1 The Lucas model

In this section of the literature survey I develop the basic elements of the consumption model as it relates to asset pricing. The primary objectives here are: to understand the factors that determine the systematic risk of financial assets in this type of model; and to isolate the factors underlying the determination of the real interest rate. I will address these questions by means of a discrete-time model proposed by Lucas (1978). Once I have developed the model’s structure, I will examine in detail the extent to which the model’s predictions are consistent with reality. In particular, I will conclude that this type of model requires a high coefficient of risk aversion in order to explain the observed level of risk premiums. We shall also see that the model's predictions are compatible with a relatively low level of real interest rates, as long as preferences dissociate the concepts of risk aversion and intertemporal substitution. A synthesis of the Lucas model is given below.

Lucas analyzes the portfolio and consumption choices of a representative agent who maximizes expected intertemporal utility over an infinite planning horizon. For each period, this agent has the choice of investing in two different kinds of financial assets. The agent can acquire equities, which promise an uncertain return, or invest in bonds, which have a fixed yield that is known in advance. In this first section, I assume that there are J equities in circulation, and that the only bonds available are strip bonds with a certain term to maturity. The portfolio choice facing the agent, who at the beginning of period t has a portfolio containing b_τ bonds and z_τ^j shares of J stocks, gives rise to the following dilemma of intertemporal maximization:

$$\underset{z_\tau^j, b_\tau, C_\tau}{MAX} \; E_t\left[\sum_{\tau = t}^{\infty} \beta^{\tau - t} U(C_\tau)\right] \qquad 0 < \beta < \infty, \tag{1}$$

under the constraint that

$$C_\tau + \sum_{j=1}^{J} q_\tau^{z_j} \bullet z_{\tau+1}^j + q_\tau^f \bullet b_{\tau+1} \leq \sum_{j=1}^{J} \left[D_\tau^j + q_\tau^{z_j}\right] \bullet z_\tau^j + b_\tau$$

$$\tau = t, \ldots, \infty, \tag{2}$$

where: $U(\bullet)$ is the investor's momentary utility function;[2] C_τ is consumption in period t; $q_\tau^{z_j}$ is the price of the equity j at period t after the distribution of dividends; D_τ^j and q_τ^f are the price at period t of a bond guaranteeing return of a unit of consumption at period $t+1$; and $E_t[\bullet]$ is the conditional expectations operator for all the information that the investor possesses at period t.

Lucas (1978) shows that the agent's optimum portfolio must satisfy, for each period, the following two Euler conditions:

$$U'(\bullet_t) \bullet q_t^{z_j} = \beta E_t\left[U'(\bullet_{t+1}) \bullet \left(D_{t+1}^j + q_{t+1}^{z_t}\right)\right] \qquad j = 1, \ldots, J \tag{3}$$

2. The instantaneous utility function has the usual characteristics: the marginal utility of consumption is positive but decreasing, and Inada conditions are respected.

$$U'(\bullet_t) \bullet q_t^f \;=\; \beta E_t\left[U'(\bullet_{t+1})\right]. \tag{4}$$

Conditions (3) and (4) have the following intuitive interpretations. In the first of these two conditions, an investor who acquires at period t an additional share of equity j must sacrifice q_t^{zj} units of consumption, which at the margin generates a utility loss equal to

$$U'(\bullet_t) \bullet q_t^{zj}$$

units. This investment, however, will bring, in period $t+1$, capital and interest equal to

$$(D_{t+1}^j + q_{t+1}^{zj})$$

units, the consumption of which will enhance the investor's welfare by

$$U'(\bullet_{t+1}) \bullet (D_{t+1}^j + q_{t+1}^{zj})$$

units. Given the uncertainty of this return, and the fact that the agent discounts future utility by a factor β, the marginal benefit expected from this investment is equal to

$$\beta E_t\left[U'(\bullet_{t+1}) \bullet (D_{t+1}^j + q_{t+1}^{zj})\right].$$

Condition (3) therefore simply expresses the fact that the agent optimizes portfolio management by equalizing the marginal cost and marginal benefit of investment in equity *j*. Similarly, an agent who buys an additional unit of the safe asset in period t must reduce current consumption by q_t^f units. This produces an immediate utility loss of

$$U'(\bullet_t) \bullet q_t^f$$

units. That loss, however, is offset by the gain realized on this investment at period $t+1$. This gain is equal to

$$\beta E_t\left[U'(\bullet_{t+1})\right]$$

units of expected utility. Once again, condition (4) shows that efficient portfolio management requires the agent to invest in safe (i.e., riskless) assets up to the break-even point between the marginal benefit and marginal cost of the investment.

The consumption model's predictions about the pricing of bonds and equities flow from a general-equilibrium estimation of conditions (3) and (4). To this point, we have discussed conditions (3) and (4) solely from the perspective of individual choices, where the market values (i.e., the current prices) of assets are given. Under general equilibrium, prices must constantly adjust to maintain the balance between supply and demand on all markets simultaneously. In the specific case of a representative-agent model, market equilibrium is reached when:

$$C_t = \sum_{j=1}^{J} D_t^j \tag{5}$$

$$z_t^j = 1 \qquad j = 1, \ldots, J \tag{6}$$

$$b_t = 0; \tag{7}$$

that is to say, when the agent: consumes all economic endowment; is willing to hold all equities in circulation; and carries no debt.

In this literature, the momentary utility function for a representative agent often takes the following isoelastic form:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \tag{8}$$

where the parameter γ is the Arrow–Pratt risk-aversion coefficient.

The utility function (8) has several interesting properties that deserve examination. First, (8) is compatible with risk neutrality (i.e., $\gamma = 0$), and it also includes, when γ tends towards unity, the case where preferences are logarithmic. Second, with this functional form, the risk premiums predicted by the model are resistant to changes in wealth levels and in the size of the economy. Third, to the extent that economic agents share the same utility function, we can aggregate individual choices, even if agents have different levels of wealth. This property offers some theoretical support for using aggregate consumption, rather than individual consumption, in econometric studies on the determination of returns. Finally, with the isoelastic utility function, the parameter γ determines simultaneously the relative risk-aversion coefficient and the elasticity of intertemporal substitution, ρ.

In fact, with this functional form, the elasticity of intertemporal substitution is the reciprocal of the relative risk-aversion coefficient (i.e., $\gamma = 1/\rho$). Hall (1988) points out that this property of the isoelastic utility function is not necessarily desirable. In theory, there should be no such rigid link between these two distinct preference aspects. Risk aversion influences the rate at which the agent is prepared to exchange units of consumption between different states of nature, whereas the elasticity of intertemporal substitution reflects the agent's willingness to exchange units of consumption between periods. Risk aversion is a notion that can exist only in the presence of uncertainty, and it need not have a temporal dimension. On the other hand, the notion of intertemporal substitution arises in a situation of full certainty, even if it makes no real sense in an atemporal setting. At the end of this section we shall look at two alternative and more-general formulations of preferences, attributable to Campbell and Cochrane (1995), Epstein and Zin (1989 and 1991), and Weil (1989), which allow us to dissociate these two important aspects of preferences.

The consumption model's predictions about the prices and returns of bonds and equities flow from a general-equilibrium estimation of conditions (3) and (4). Ignoring speculative bubbles, the equilibrium prices for risky and safe assets are given by the following two equations:

$$q_t^{zj} = E_t\left[\sum_{i=1}^{\infty} \beta^i \frac{U'(C_{t+i})}{U'(C_t)} \bullet D_{t+i}^j\right] \tag{9}$$

$$q_t^f = E_t\left[\beta^i \frac{U'C_{t+1}}{U'C_t}\right]. \tag{10}$$

Equation (10), which follows directly from condition (4), suggests that the equilibrium price of bonds reflects the expected marginal rate of intertemporal substitution of consumption. Note that this equation links the price of bonds to the predicted growth of consumption when preferences are isoelastic, i.e.:

$$\beta\frac{U'(C_{t+1})}{U'(C_t)} = \beta \bullet \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

The equilibrium value of q_t^{zj} is obtained by recursive substitutions of equation (3). In this model, q_t^{zj} is equal to the present value of expected future dividend flows, where the discount factor for dividends in period $t+i$ is the expected marginal rate of intertemporal substitution of consumption between periods t and $t+i$.

Alternatively, the first-order conditions (3) and (4) may be expressed in terms of asset yields. Let us define

$$1 + r_{t+1}^{zj} = \left(D_{t+1}^j + q_{t+1}^{zj}\right)/q_t^{zj}$$

as the gross return on equity j between the periods t and $t+1$, and

$$1 + r_{t+1}^f = 1/q_t^{zj}$$

as the gross return on bonds over the same time span. After manipulation, conditions (3) and (4) become:

$$1 = E_t\left[S_{t,\,t+1} \bullet \left(1 + r_{t+1}^{z_j}\right)\right] \qquad j = 1, \ldots, J \tag{11}$$

$$1 = E_t\left[S_{t,\,t+1}\right] \bullet \left(1 + r_{t+1}^{f}\right), \tag{12}$$

where the variable $S_{t,\,t+1}$ represents, for brevity's sake, the marginal rate of intertemporal substitution of consumption

$$\beta\frac{U'(C_{t+1})}{U'(C_t)}.$$

Equation (11) is often identified as the "canonical asset-pricing equation;" see for example Ferson (1995) and Campbell, Lo, and MacKinlay (1997). In this equation, the variable $S_{t,\,t+1}$ plays the role of a stochastic discount factor.[3] In a consumption model, the stochastic discount factor is in fact assimilated into the consumer's marginal rate of intertemporal substitution. Note as well that the riskless return, r_{t+1}^{f}, appears in equation (12) outside the mathematical expectations operator, because it is known from the beginning of period *t*.

Conditions (11) and (12) impose several restrictions on the behaviour of expected real returns on bonds and equities. We shall now discuss in turn the role played by each of these restrictions, beginning with those that are imposed on the real interest rate. Equation (12) shows that the real interest rate—the return on riskless assets—is determined by the marginal rate of intertemporal substitution of consumption.

$$r_{t+1}^{f} = \left\{E_t\left[S_{t,\,t+1}\right]\right\}^{-1} - 1. \tag{13}$$

We can delve further into the restrictions imposed by this equation if we assume that preferences are of the isoelastic kind, and that consumption follows a conditional lognormal distribution. Under these two assumptions, equation (13) becomes[4]

3. This variable is also sometimes known as the "asset-pricing kernel."

4. A variable χ that follows a conditional lognormal law has the property that

$$\ln E_t\left[\chi\right] = E_t\left[\ln\chi\right] + \frac{1}{2} \bullet \mathrm{Var}_t(\ln\chi).$$

$$r_{t+1}^{f} = \delta + \gamma \bullet E_t\left[\Delta c_{t+1}\right] - \frac{\gamma^2}{2} \bullet \mathrm{var}_t(\Delta c_{t+1}). \tag{14}$$

From (14) it can be seen that the real interest rate is determined by three separate factors. First, the real interest rate tends to be high if δ, the agent's time preference, is great. Second, the real interest rate is high if the forecast growth rate of consumption $E_t\left[\Delta c_{t+1}\right]$ is high, since in this case the agent will be inclined to borrow on the credit market in order to smooth out the consumption profile. The importance of this second effect is inversely proportionate to the elasticity of intertemporal substitution. Finally, for reasons of precautionary savings, the real interest rate tends to be low when $\mathrm{Var}_t(\Delta c_{t+1})$, the conditional variance of the consumption growth rate, is high. The strength of this effect depends on the square of the relative risk-aversion coefficient.

We shall now turn our attention to the restrictions imposed by equation (11) on the expected return on equities. In particular, we will isolate the condition under which the expected return on equities differs from the real interest rate. Using the definition of a covariance, we can express the right-hand side of equation (11) as a product of expectations plus a covariance term. Thus:

$$1 = E_t\left[S_{t,\,t+1}\right] \bullet E_t\left[1 + r_{t+1}^{z_j}\right] + \mathrm{cov}_t\left(S_{t,\,t+1}, 1 + r_{t+1}^{z_j}\right). \tag{15}$$

Next, equations (13) and (15) let us isolate predictions concerning the spread between risky and riskless returns:

$$E_t\left[r_{t+1}^{z_j}\right] - r_{t+1}^{f} = -\left(1 + r_{t+1}^{f}\right) \bullet \mathrm{cov}_t\left(S_{t,\,t+1}, r_{t+1}^{z_j} - r_{t+1}^{f}\right). \tag{16}$$

Equation (16) is an alternative form of the canonical asset-pricing equation. This highlights the general measure of the systematic risk of a risky asset *j* in a consumption model. An asset is considered to be risky if its excess return has a negative covariance with the marginal rate of intertemporal substitution of consumption. A negative covariance means that the asset tends to offer a higher (lower) excess return than expected when the marginal utility of consumption is weaker (stronger) than expected, i.e., when consumption is stronger (weaker) than expected. The risk is systematic in the sense that it is linked to the rate of growth of the marginal utility of aggregate consumption. In the specific case where the agent's preferences are isoelastic, we can take this relationship further to show that

$$E_t\left[r_{t+1}^{z_j}\right] - r_{t+1}^{f} = \gamma \bullet \mathrm{corr}_t\left(\Delta c, r^{z_j}\right) \bullet \sigma_t\left(\Delta c\right) \bullet \sigma_t\left(r^{z_j}\right). \quad (17)$$

Here, $\sigma_t(\Delta c)$ is the conditional standard deviation of the consumption growth rate, $\sigma_t(r^{z_j})$ is the conditional standard deviation of the return on risky asset *j*, and $\mathrm{corr}_t(\Delta c, r^{z_j})$ is the conditional correlation coefficient between Δc and r^{z_j}. Equation (17) shows that the expected excess return offered by the risky asset *j* depends on three different elements. Risk premiums depend first on the quantity of risk being assumed. This flows from $\sigma_t(\Delta c)$ and $\sigma_t(r^{z_j})$. Risk premiums depend secondly on agents' risk sensitivity which, in turn, is determined by the relative risk-aversion coefficient γ. Finally, the presence of risk, and sensitivity to it, does not necessarily mean that a risky asset will yield a higher return than a safe one. In order for that to be the case, the return on the asset in question must be positively correlated with the non-diversifiable risk factor Δc. A risky asset may even offer a negative premium and yield less than a safe asset, if its return is negatively correlated with the consumption growth rate. Intuitively, it is advantageous for agents to hold such an asset, since it protects them by offering a relatively higher return during periods of falling consumption.

1.2 Extensions of the Lucas model

The restrictions imposed by equation (16) on expected excess returns conflict with several empirical observations. The best known of these, or course, is the "equity-premium puzzle." Mehra and Prescott (1985) estimate that the average annual excess return on all equities in the United States over the 1889–1978 period was 6.18 per cent. Calibration exercises conducted by Mehra and Prescott, and by others, show that it is very difficult to generate a significant premium (more than 2 per cent) with isoelastic preferences when the risk-aversion coefficient is kept below 10. Mehra and Prescott's point can be readily demonstrated using equation (17). Given that, over the 1889–1978 period, the correlation between Δc and r^m is about 0.4, the standard deviation of the consumption growth rate is 0.036, and the standard deviation of excess market returns is 0.167, an average overall risk premium of 6.18 per cent is only possible, according to (17), if $\gamma = 25$.[5] Mehra and Prescott consider such a value to be beyond "reasonable" bounds for this

5. Mankiw and Zeldes (1991) show that the required aversion coefficient must approach 100 when the sample is limited to the post-war period.

parameter, in light of the microeconomic literature on the subject.[6] The fact that excess returns are positive is not in itself a problem. Positive premiums flow naturally enough from equation (15). The puzzle is that the predicted premiums are too small for “reasonable” values of the relative risk-aversion coefficient.[7]

Weil (1989) points to another puzzle, associated with riskless returns, that is illustrated by equation (14). Over the period studied by Mehra and Prescott, the average annual growth rate of consumption was 0.018, with a variance of 0.0013. Yet, unless the relative risk-aversion coefficient is very weak and the elasticity of intertemporal substitution is consequently very high, the annual average real interest rate predicted by equation (14) is several times higher than 0.80 per cent, the level observed by Mehra and Prescott between 1889 and 1978. For example, even if the value of γ is as low as 2, equation (14) predicts a real interest rate that is higher than the time preference rate of 3.34 per cent. Hence, unless γ is negative, equation (14) is incapable of predicting the observed real interest rate.

Several recent studies using U.S. data have shown that a small but significant portion of the fluctuations in excess returns can be predicted on the basis of information at hand at the beginning of the period. The empirical work of Campbell (1987), Campbell and Shiller (1988), and Fama and French (1988) concluded that a change in short-term or long-term interest rates, the dividend/price ratio, and the spread between long-term and short-term interest rates can all be used to predict future movements in U.S. excess returns. Carmichael and Samson (1996) found that the same is true for Canadian excess returns over the 1969:M1–1992:M12 period. Can all this be explained by the consumption model? Equation (17) suggests that predictable movements in excess returns should be associated with predictable movements in $\sigma_t(\Delta c)$ and/or in $\mathrm{corr}_t(\Delta c, r^j)$. For the moment, however, empirical evidence does not support either of these two possibilities. Moreover, calibration exercises generally produce simulated premiums that vary little compared with those observed.

In the face of these empirical problems, some authors have suggested modifications to the consumption model to make it more general. In particular, they have tried to determine whether the mixed empirical results might be attributable to the auxiliary assumptions used for deducing

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6. Accepting a γ value of 26 as reasonable does not, however, solve all the problems. In fact, Obstfeld and Rogoff (1996) show that it is especially difficult in this case to explain why investors who are so risk-sensitive seek so little international diversification in their portfolios.

7. Kocherlakota (1996) offers an excellent survey of the literature on the equity-premium puzzle.

equations (14) and (17). Essentially, consumers in the consumption-based capital-asset-pricing model (C-CAPM) use financial assets above all as a means of smoothing out the marginal utility of consumption over time. In principle, there is no constraint that would require marginal utility for the period t to be dependent solely on consumption for period t. It is reasonable to assume that $U_c(\bullet_t)$ may also be influenced by other variables, such as the consumption of leisure or the level of consumption attained in the recent past. In this case, the covariance of excess returns with these other variables will also influence risk premiums, and may even help alleviate some of the empirical difficulties noted earlier. Research in this area has taken two different directions.

Epstein and Zin (1989 and 1991) and Weil (1989) introduced the notion of “non-expected utility” preferences into the model. Adopting such preferences allows some loosening of the independence hypothesis about the marginal utility of consumption between different states of nature. With such preferences, the marginal utility of consumption in good times is not independent of the level of consumption in bad times. Another important property of Epstein and Zin’s preferences is that risk aversion and the elasticity of intertemporal substitution are determined by different parameters. Thus, in contrast to isoelastic preferences, a risk-averse consumer may still be highly willing to substitute consumption intertemporally with non-expected utility preferences. Epstein and Zin (1991) maintain that separating the concepts of risk aversion and intertemporal substitution can help to resolve some of the anomalies that arise when preferences are isoelastic. Weil (1989) studied this question in detail, and concludes that, with such preferences, risk premiums are determined by the risk-aversion coefficient, and that the real interest rate is influenced by the elasticity of intertemporal substitution. Consequently, the non-expected utility hypothesis does nothing to solve the equity-premium puzzle posed by Mehra and Prescott. Yet these preferences do offer a solution to the riskless-returns puzzle. In fact, Weil manages to reproduce exactly the observed levels of the equity premium and the real interest rate, by setting the risk-aversion and elasticity of intertemporal substitution coefficients at 45 and 0.10, respectively. A risk-aversion coefficient at this value should produce, in the case of isoelastic preferences, an intertemporal substitution elasticity parameter of $0.022 = 1/45$. From this observation, we may conclude that Epstein and Zin’s preferences solve the riskless-return puzzle by allowing for, simultaneously, high levels of risk aversion and intertemporal substitution.

In the other direction are found the works of Constantinides (1990), Ferson and Constantinides (1991), and Campbell and Cochrane (1995), who have tried to resolve the above anomalies by introducing non-separability of preferences over time into the model. This is done by means of a mechanism for consumption habit-forming. These authors make use of the simple but intuitive idea that the utility of current consumption is not independent of consumption levels attained in past periods. In technical terms, Campbell and Cochrane replace the utility function (8) with the following function:

$$U(C - X) = (C - X)^{1-\eta}, \tag{18}$$

where X represents the accustomed level of consumption and η influences the preferences curve. The accustomed level of consumption is modelled as a variable that adjusts gradually to variations in consumption. With these preferences, the marginal utility of consumption rises as consumption approaches its accustomed level. For this reason, the risk-aversion coefficient varies with the business cycle, according to this relationship:

$$\gamma_t = \eta \frac{C_t}{C_t - X_t}. \tag{19}$$

Equation (19) shows that a consumer's risk aversion rises as his level of consumption approaches the accustomed level. Consequently, the preferences curve parameter is no longer the only determinant of risk aversion. Introducing this consumption habits mechanism also allows a loosening of the close link between the concepts of risk aversion and intertemporal substitution that is imposed by isoelastic preferences. In particular, the risk-aversion coefficient can be very high, even if the value of η is low, since it is the η parameter that governs the elasticity of intertemporal substitution. For this reason, a model with consumer habits can reproduce the observed level of the equity premium by allowing the risk-aversion premium to be high even if the η parameter is weak. However, this is not really a new solution to the equity premium puzzle, since other models also reproduce the level of the premium when risk aversion is significant.

This model does, however, offer a new explanation of the riskless-returns puzzle. With isoelastic preferences, an increase in the risk-aversion coefficient, which is needed to reproduce the equity premium, leads to an increase in the level and the variability of the real interest rate. A model with consumption habits gets around this problem by adding a precautionary or "rainy day" savings component that will counter the upward pressure on the real interest rate. Intuitively, consumers who are aware of their consumption habits will become more averse to risk when their current consumption drops relative to the accustomed level. This will induce them to save more, in order to protect themselves against any further drop. Thanks to this precautionary savings mechanism, the model can reproduce a level for the real interest rate that is stable and low, and can thereby offer a solution to the riskless-returns puzzle.

Consumption habit-forming models also offer an explanation for the variability of estimated risk premiums. This explanation lies in the countercyclical behaviour predicted by equation (19) for the risk-aversion coefficient. At a time of recession, falling consumption makes agents more sensitive to risk and thus induces higher risk premiums; this would seem consistent with observed behaviour.

To sum up, consumption models define the systematic asset risk as the covariance between asset returns and the marginal rate of intertemporal substitution of consumption. Given the low variability of aggregate consumption, the model has difficulty explaining the relatively high levels of risk premiums, unless very high risk aversion is assumed. This becomes especially clear when consumer–investor preferences are isoelastic. Modifying preferences by incorporating consumption habits or the notion of non-expected utility does not change this result. Both models reproduce the observed level of the equity premium only when risk aversion is high. After more than a dozen years of intensive research, Mehra and Prescott's puzzle has still not been solved, at least not within reasonable levels of risk aversion.

The development of consumption models has also helped us better understand the factors underlying the determination of the real interest rate. Today we can conclude that the enigma of the risk-free rate, which was identified initially by Weil (1989), disappears when we assume preferences that allow for the separation of the concepts of risk aversion and intertemporal substitution.

2 Prices of Other Kinds of Financial Assets

The principles developed in Section 1 are equally applicable to the pricing of other kinds of market-traded assets. In this section, I will expand the discussion to predictions of the model for pricing securities with a term to maturity of more than one period. I will also address the question of options pricing.

Looking first at bonds, I will pay particular attention to price-setting on the spot and forward markets. I will also examine the restrictions imposed on rates of return according to maturity.

Let us imagine that, in addition to the kinds of securities discussed in the previous section, financial markets also offer a series of risk-free strip bonds with different maturities. Each bond gives the right to one unit of consumption on maturity. The existence of such securities modifies an agent's budget constraint. Let us define $b_{j,t}$ as the quantity of bonds, maturing in *j* periods, held by an agent at the beginning of period *t* and $q_{j,t}$ as the spot price of those bonds. The budget constraint in period *t* becomes

$$C_t + \sum_{j=1}^{N} b_{j,t+1} \bullet q_{j,t} \le y_t + \sum_{j=0}^{N} b_{j,t} \bullet q_{j,t}, \tag{20}$$

where N is the longest maturity offered on the bond market. For simplicity's sake, budget constraint (20) ignores the elements attributed to the stock market in our earlier analysis. Variable y_t includes all of the agent's other income. Also, by definition, $q_{0,t} \equiv 1$. For each of the available maturities, efficient portfolio management must satisfy the following Euler condition:

$$U'(\bullet_t) \bullet q_{j,t} = \beta E_t\left[U'(\bullet_{t+1}) \bullet q_{j-1,t+1}\right] \qquad j = 1, \ldots, N. \tag{21}$$

This condition has an interpretation similar to that developed in the previous section. The purchase in period *t* of a bond with maturity *j* entails at the margin a utility loss equal to $U'(\bullet_t) \bullet q_{j,t}$ units. However, this investment allows consumption in period $t+1$ to rise by $q_{j-1,t+1}$ since in $t+1$ this bond can be sold at the price of those with maturity $j-1$. Looked at from period *t*, this future benefit has an expected value of

$$\beta E_t\left[U'(\bullet_{t+1}) \bullet q_{j-1,t+1}\right]$$

units of utility. Once again, condition (21) shows that our agent's bond portfolio is optimized when the marginal cost and benefit of investment are equal for every available maturity.

Under general equilibrium, the net offer of bonds by maturity is equal to 0, $b_{j,t+1} = 0$. Hence, the equilibrium spot price of a bond with maturity *j* is obtained by recursive substitutions of equation (21):

$$q_{j,t} = \beta^j E_t\left[S_{t,t+j}\right]. \tag{22}$$

The equilibrium price at the end of period *t* for a bond with maturity *j* reflects fully the marginal rate of intertemporal substitution of consumption between periods *t* and $t+j$. When preferences are isoelastic, equation (22) links the price of the bonds to the expected growth rate of consumption. *Ceteris paribus*, moreover, the price of a bond with maturity *j* is relatively high (low) when the market expects a low (high) consumption growth rate between periods *t* and $t+j$, because investors will seek to make massive bond purchases in order to reorient their consumption profile to the future. The analytical emphasis here is on prices. I will also develop predictions of the yield to term on bonds. The yield to term of a bond, which is denoted as $r_{j,t}$, is entirely determined by its purchase price. Thus:

$$q_{j,\,t} \;=\; \left(\frac{1}{1+r_{j,\,t}}\right)^{j}. \tag{23}$$

In effect, the yield $r_{j,\,t}$ of a strip bond is simply the constant rate at which the bond's price must rise in order for its value to be equal to one unit of consumption at maturity $t+j$. Predictions about the term structure of interest rates flow directly from equations (22) and (23).

I will now focus on predictions about the prices of forward contracts for bonds of varying maturities. A forward contract constitutes a commitment to consummate a transaction at a specified date in the future, under pre-established conditions. We shall define $f^{k}_{n,t}$ as the price set at period t for a contract for delivery in period $t+n$ of a bond maturing in period $t+k$, where $n<k$. The benefit realized on this contract at the time of delivery $t+n$ by an investor taking a long position is entirely determined by the spread between $q_{k-n,\,t+1}$, the spot price at period $t+n$ of a bond maturing at period $t+k$, and the delivery price $f^{k}_{n,t}$. Portfolio managers have an interest in trading on the forward market until all opportunities for profit have been exhausted. This situation is reached when:

$$0 \;=\; E_t\left[S_{t,\,t+n}\bullet(q_{k-n,\,t+1}-f^{k}_{n,t})\right], \tag{24}$$

the present value of the benefit expected from a long position, is nil.[8] Since $f^{k}_{n,t}$ is known as of period t, equation (24) constrains the delivery price of a forward contract to respect the following condition:

$$f^{k}_{n,t} \;=\; \left\{E_t\left[S_{t,\,t+1}\right]\right\}^{-1}\bullet E_t\left[S_{t,\,t+1}\bullet q_{k-n,\,t+n}\right]. \tag{25}$$

Developing the covariance term appearing on the right-hand side of equation (25), we can show that the forward contract price is a combination of the expected future spot price plus a risk premium:

$$f^{k}_{n,t} \;=\; E_t\left[q_{k-n,\,t+n}\right]+\frac{1}{q_{n,\,t}}\bullet \mathrm{cov}_t\left(S_{t,\,t+n},q_{k-n,\,t+n}\right). \tag{26}$$

The forward contract price is higher (lower) than the expected future spot price when the conditional covariance between $S_{t,\,t+1}$ and $q_{k-n,\,t+n}$ is positive (negative). Intuitively, when the conditional covariance is positive, an investor taking a long position on the forward market will enter into a

8. Investors are in a long (short) position if they are committed to purchase (sell) a bond at a delivery price of $f^{k}_{n,t}$.

contract that will deliver him an asset with a spot market value that is higher than expected when the marginal utility of consumption is also higher than expected. In this case, the forward contract is an excellent instrument for smoothing out consumption over time, and investors are prepared to pay a premium for this desirable characteristic. The risk premium $fp_{n,t}^{k}$ on forward contracts is conventionally defined as

$$fp_{n,t}^{k} = E_t\left[q_{k-n,\, t+n}\right] - f_{n,t}^{k},$$

the spread between the expected future spot price and the forward price. Equation (26) shows that the sign of the premium is determined by the sign of the conditional covariance between $q_{k-n,\, t+n}$ and $f_{n,t}^{k}$. In particular, a positive premium is possible only if the conditional covariance is negative. Backus, Gregory, and Zin (1989) show that the sign of the covariance between $S_{t,\, t+n}$ and $q_{k-n,\, t+n}$ results directly from the sign of the covariance between $S_{t,\, t+n}$ and $S_{t+n,\, t+k}$ in light of equation (22). In particular, a negative covariance is only possible if the autocorrelation coefficient of the marginal rate of intertemporal substitution is also negative. As we shall see below, this restriction allows some versions of the model to be rejected. Finally, it should be noted that the risk premium $fp_{n,t}^{k}$ is not necessarily constant, since there is nothing to prevent the conditional covariance that appears in equation (26) from varying systematically with the state of nature.

The factors influencing expected bond returns by maturity can also be analyzed with the help of equation (21). We define h_{t+1}^{k} as the actual return between periods t and $t+1$ on a bond maturing in period $t+k$. In this case, $h_{t+1}^{k} = q_{k-n,\, t+n}/q_{k,\, t}$ is determined entirely by the capital gain realized between periods t and $t+1$. The first-order condition (21) constrains to respect the condition

$$1 = E_t\left[S_{t,\, t+1} \bullet h_{t+1}^{k}\right] \qquad 1 \le k \le N. \tag{27}$$

Because the return on a bond maturing in period $t+1$ is known in advance, i.e.,

$$h_{t+1}^{1} = 1/q_{1,t} \text{ and } q_{1,t} = E\left[S_{t,\, t+1}\right],$$

condition (27) implies that

$$E_t\left[h_{t+1}^{k}\right] - h_{t+1}^{1} = -h_{t+1}^{1} \bullet \operatorname{cov}_t\left(S_{t,\, t+1}, h_{t+1}^{k}\right). \tag{28}$$

The expected return on a bond with maturity *k* is different from the riskless return—i.e., the return on a one-period bond—if the conditional covariance between $S_{t,\,t+1}$ and h^k_{t+1} is other than 0.

According to the theory of expectations, the slope of the term structure of interest rates is determined by market expectations about future interest rates. The slope will be positive (negative) and rates will rise (fall) with the length of maturity, when the market anticipates that interest rates will increase (decline). In this case, the delivery price of forward contracts will reflect entirely and solely the market's expectations about future spot prices. The C-CAPM model is compatible with this prediction if the conditional covariance terms appearing in equations (26) and (28) are nil. Backus et al. (1989) studied this question, and concluded that covariances are nil when: (i) agents are risk-neutral, or (ii) marginal rates of substitution $S_{t,\,t+1}$ are independent. In the first case, the marginal rate of intertemporal substitution is constant and equal to β under all states of nature. For this reason, equations (22), (25), and (27) imply: (i) constant values for

$$q_{j,\,t},\ f^{\,k}_{n,t},\ \text{and}\ h^k_{t+1},$$

and (ii) nil risk premiums for

$$fp^k_{n,t}\ ,\ \text{and}\ E_t\left[h^k_{t+1}\right] - h^1_{t+1},$$

under all possible conditions. Alternatively, if marginal rates of intertemporal substitution are independent, equation (22) becomes simply

$$q_{j,\,t} = E_t\left[\prod_{i=1}^{j} S_{t+i-1,\,t+i}\right] = \prod_{i=1}^{j} E_t\left[S_{t+i-1,\,t+i}\right]$$

$$= \prod_{i=1}^{j} E_t\left[q_{1,\,t+i-1}\right], \tag{29}$$

and the price of a bond maturing in *j* periods is the product of expected future prices. Consequently, the delivery price for a forward contract reflects solely the expected future spot price of the underlying security, and so the premium $fp^k_{n,t}$ is nil. For the same reason, expected returns on bonds between periods *t* and $t + 1$ are independent of maturity, i.e.,

$$E_t\left[h^k_{t+1}\right] = (1/E_t)\left[S_{t,\,t+1}\right],$$

and maturity premiums are nil.

The term structure of interest rates is one of the most thoroughly studied relationships in economics and finance. The voluminous work on this question has revealed two characteristic features of the term structure. On one hand, studies show that the mean risk premium on forward contracts, fp_n^k, is small but positive. On the other hand, evidence shows that these premiums are highly variable and partially predictable. Both these phenomena are very robust, and do not seem to depend on any particular period or sample. Roll (1970) and Fama (1976 and 1984) provide several statistics that confirm this. In their review of the literature, Shiller, McCulloch, and Huston (1987) conclude that these two phenomena indicate outright rejection of the theory of expectations.

Backus et al. (1989) attempted to discover whether fluctuations in bond prices and risk premiums are compatible with the predictions of a general-equilibrium model of the kind developed by Breeden (1979) and Lucas (1978). Equations (26) and (28) show that, in principle, this model is compatible with risk premiums that are positive and variable.

The question, instead, is whether the model can explain risk-premium behaviour in quantitative terms. To examine this question, Backus et al. calibrated a two-state model with isoelastic preferences. Their simulation exercises reveal two important discrepancies between the model's predictions and empirical observations. First, with isoelastic preferences, the model appears unable to reproduce the mean risk premium observed on the forward market when the risk-aversion coefficient has a value of less than 7 or 8. This result brings to mind Mehra and Prescott's equity-premium puzzle. Second, as noted earlier, the forward-market risk premium predicted by the artificial economy is positive only when the autocorrelation coefficient of the marginal rate of intertemporal substitution is negative. With isoelastic preferences, this means that the autocorrelation coefficient of the consumption growth rate must also be negative. Yet empirical evidence provides virtually no support for a negative autocorrelation coefficient. For example, during the period studied by Backus et al., the autocorrelation coefficient of the consumption growth rate in the United States was weakly positive.

Backus et al. (1989) also apply econometric tests to simulated data to see whether risk premiums generated by the artificial economy fluctuate enough that the prediction of the theory of expectations can be rejected. Here again, the results lend little support to their version of the model. The simulated data reject the theory of expectations only when the risk-aversion and consumption autocorrelation parameters are given extreme values. Invariably, the simulated data accept the restrictions imposed by the theory of expectations when the parameters are set at reasonable levels.

The results obtained by Backus et al. derive in part from the structure of preferences they used in calibrating their model. As we noted in Section 1, the behaviour of the marginal rate of intertemporal substitution is intimately linked to that of the consumption growth rate when preferences are isoelastic. The smooth and unpredictable behaviour of the consumption growth rate is, under these circumstances, difficult to reconcile with the estimated behaviour of risk premiums.

Gregory and Voss (1991) test the robustness of the results obtained by Backus et al. by adopting more general preferences. In particular, they examine whether the preferences suggested by Constantinides (1990), incorporating consumption habits and the non-expected utility preference proposed by Epstein and Zin (1989), allow a better match of the observed behaviour of premiums. The simulation results show that adopting more general preferences is a step in the right direction. In fact, with the preferences proposed both by Constantinides (1990) and by Epstein and Zin, we can reproduce both the level and the variability of forward-market risk premiums. Moreover, the simulated premiums are sufficiently variable to indicate rejection of the theory of expectations when the simulated data are subjected to the battery of econometric tests that led to empirical rejection of the theory.

Yet, while these results are encouraging from a theoretical viewpoint, the simulations reveal significant tensions between the model and the data, particularly with respect to the variability of bond prices. In fact, the standard deviation of the simulated bond price is at best 30 times greater than that observed in the data. This simply reflects the fact that the bond price must be variable if the artificial economy is to reproduce the level of observed risk premiums. From this viewpoint, the term structure of interest rates remains an enigma for general-equilibrium theory.

I will conclude this section with a brief discussion of options pricing. One of the major strengths of the consumption model is that it offers a unified approach to the pricing of financial assets. More specifically, an asset's value can always be determined once we know the time structure of the payments it produces. Equation (9) is an illustration of this general principle. We shall now see how this principle applies to options. To do so, we must first define clearly the structure of payments that options offer. We shall then see how equation (9) allows us to estimate options prices.

An option gives its holder the right to buy or to sell an asset within a given period of time, at a predetermined price. This price is known as the "exercise" or "strike" price. An American-style call (put) option gives its holder the right to exercise the option and to buy (sell) the underlying assets at the strike price at any time up to and including the expiry date of the option. European-style options are more restrictive; the holder can exercise the option and acquire the underlying assets only at the option's expiry date. In contrast to the forward contracts discussed above, then, option holders are not obligated to exercise their call or put privileges.

I will look first at the determination of the premium for a call option on an equity j expiring in one period.[9] Note that American and European options are equivalent in the period up to expiry. Suppose that the strike price of an option is equal to K. To find the premium for this option at period t, we must determine how much an investor is willing to pay for the right to buy the equity at period $t+1$ at the option's strike price. If, at period $t+1$, the equity's value is greater than the option's strike price (i.e., $q_{t+1}^{z_j} > K$), the investor will have an interest in exercising the option and taking advantage of the gap $q_{t+1}^{z_j} - K$. If, on the other hand, $q_{t+1}^{z_j}$ is lower than K, the investor will have nothing to gain, and will let the option expire. Thus, the equilibrium premium at period t for an American call option in $t+1$ is:[10]

$$PA_j^a(t, t+1) = E_t\left[S_{t,\,t+1} \bullet \max\left(0, q_{t+1}^{z_j} - K\right)\right], \tag{30}$$

where the marginal rate of intertemporal substitution $S_{t,\,t+1}$ is used to discount future benefits. The equilibrium premium for a longer-term European option can readily be deduced using similar reasoning. For example, the premium for a European call option expiring at period $t+n$ must be equal to:

$$PE_j^a(t, t+n) = E_t\left[S_{t,\,t+n} \bullet \max\left(0, q_{t+n}^{z_j} - K\right)\right], \tag{31}$$

where the discount factor is now the marginal rate of intertemporal substitution between periods t and $t+n$.

The equilibrium premium for a comparable American option is more difficult to obtain because its holder can exercise the option at any time during its life. Nevertheless, as we have just seen in equation (30), the premium can be readily evaluated for the period preceding the option's expiry. This means that the premium at period $t+n-1$ for an American option expiring at period $t+n$ is

9. The market jargon term for the price of an options contract is the "option premium."

10. By analogy, the equilibrium premium at period t of a put option in $t+1$ is:

$$PA_j^v(t, t+1) = E_t\left[S_{t,\,t+1} \bullet \max\left(0, K - q_{t+1}^{z_j}\right)\right].$$

$$PA_j^a(t+n-1, t+n) = E_{t+n-1}\left[S_{t+n-1,\, t+n} \bullet \max\left(0, q_{t+n}^{z_j} - K\right)\right].$$

A portfolio manager who acquires this call option at period $t+n-2$ will have the choice, at period $t+n-1$, of keeping the option, which at that time will have a value of $PA_j^a(t+n-1, t+n)$, or of exercising the option and making a profit of $q_{t+n}^{z_j} - K$. The equilibrium premium must then be:

$$PA_j^a(t+n-2, t+n)$$

$$= E_{t+n\text{-}2}\left[S_{t+n-2,\, t+n-1} \bullet \max\left(PA_j^a(t+n-1, t+n), q_{t+n}^{z_j} - K\right)\right]$$

at period $t+n-2$. Through a recursive process, we can deduce that the equilibrium premium for an American call option expiring in period $t+n$ is:

$$PA_j^a(t, t+n)$$

$$= E_t\left[S_{t,\, t+1} \bullet \max\left(PA_j^a(t+1, t+n), q_{t+n}^{z_j} - K\right)\right]. \qquad (32)$$

Generally speaking, we may conclude from equation (32) that the value of options in a consumption model is not independent of the preference parameters, in particular risk aversion and intertemporal substitution. The empirical results obtained by Garcia and Renault (1998) tend to confirm this conclusion.

3 Inflation and Financial Markets

Up to this point, I have been examining how yields are determined, without considering monetary factors. In many situations, this omission is justified and unimportant. However, it cannot be ignored when analyzing all the factors underlying the yield structure. In this section, I will introduce money into the pricing model developed earlier, in order to examine whether the risk surrounding the purchasing power of money is one of the risk factors that financial markets take into account. In particular, I will try to discover the conditions under which the real return expected from nominal bonds incorporates an inflation-risk premium. I will also attempt to identify the mechanisms by which inflation affects equity prices and real returns.

Discussions of monetary factors always run up against the problem of scale. Macroeconomic and financial models are always short on sound macroeconomic fundamentals relating to money demand. This gap, however, has not prevented theoretical research from making at least some progress on monetary questions. There are now several more or less ad hoc ways of introducing money into macroeconomic models. Models with cash-in-advance constraints currently seem to be the most popular.[11] The works of Lucas (1982, 1984), Svensson (1985a, 1985b), Lucas and Stockey (1987), Labadie (1989), and Giovannini and Labadie (1991) show that introducing money into a macroeconomic model in this way is useful as a means of isolating the financial market impact of monetary factors.

In a model with cash-in-advance constraints, transactions on the goods and services market must be paid for in money. In other words, goods are exchanged for money and money is exchanged for goods, but goods are not exchanged directly for other goods. There are two major variants of this model that differ, depending on whether financial markets operate at the beginning or the end of the period. Lucas (1982) adopts the convention that: (i) agents observe existing economic conditions, as determined by the availability of goods and the rate of growth of the money supply at the beginning of the period, before taking decisions; and (ii) financial markets come into play at the beginning of the period, when the goods and services market is inactive. Svensson (1985b) uses the opposite scenario, activating the goods and services market at the beginning of the period and the financial market at the end. Svensson also assumes that economic agents become aware, as soon as the goods market opens, of the value of monetary transfers that they will receive from the monetary authorities at the end of the period, when the financial transactions are conducted. Whatever the scenario, the key factor is that, in a model with cash-in-advance constraints, financial markets are inaccessible at the time agents are making their transactions on the goods and services market.

For purposes of analysis, I adopt the scenario proposed by Lucas. As well, I will limit the discussion to the equities market and the short-term securities market, as I did in Section 1. However, in contrast with the preceding sections, payments are now made in money. In particular, repayments of securities at maturity and dividend payments to shareholders are made in money. Financial assets are securities that give the right to monetary payments.

Given the sequential opening of markets, agents are subject to two distinct budget constraints, one for financial markets and the other for the goods and services market. Financial markets come into play first. At this point, agents choose their monetary transactions balances, M_t^d, for the current period, and compose their portfolios using equities z_t and strip

11. The cash-in-advance constraint first appeared in the literature with the publication of Clower (1967).

bonds B_t. Financial market choices must respect the following budget constraint:

$$\frac{Q_t^z \bullet z_t}{P_t} + \frac{Q_t \bullet B_t}{P_t} + \frac{M_t^d}{P_t} = \frac{H_t}{P_t} + \frac{Q_t^z \bullet z_{t-1}}{P_t}, \qquad (33)$$

where the variables P_t and H_t measure, respectively, the general price level and agents' cash holdings at the beginning of period *t*. The bond price Q_t and the equity price Q_t^z are in nominal terms. Transactions balances at the beginning of period *t* are determined by dividend receipts from the previous period $P_{t-1} \bullet D_{t-1} \bullet z_{t-1}$ by the redemption of bonds maturing in B_{t-1}, by the amount of transactions balances $M_t^d - P_{t-1} \bullet C_{t-1}$ that have not been spent on the goods market in period $t-1$, and by a lump-sum transfer received from the monetary authorities, Γ_t. Thus:

$$H_t \equiv P_{t-1} \bullet D_{t-1} \bullet z_{t-1} + B_{t-1} + (M_{t-1}^d - P_{t-1} \bullet C_{t-1}) + \Gamma_t.$$

The goods and services market operates only after financial markets have closed. Consumer expenditures in this market must be financed entirely from cash balances. This restriction gives rise to the cash-in-advance constraint

$$C_t \le \frac{M_t^d}{P_t}. \qquad (34)$$

Agents choose among M_t^d, B_t, z_t, and C_t so as to maximize their expected intertemporal utility

$$\max_{M_t^d, B_t, z_t, C_t} E_t\left[\sum_{\tau = t}^{\infty} \beta^{\tau - t} U(C_\tau)\right] \qquad 0 < \beta < 1, \qquad (35)$$

under constraints (33) and (34). The optimizing choices of z_t and B_t must respect the following two first-order conditions:

$$U'(C_t) \bullet q_t^z = \beta E_t\left[U'(C_{t+1}) \bullet (D_t \bullet \pi_{t+1}^{-1} + q_{t+1}^z)\right] \qquad (36)$$

$$U'(C_t) \bullet Q_t = \beta E_t\left[U'(C_{t+1}) \bullet \pi_{t+i}^{-1}\right], \qquad (37)$$

where $q_t^z = Q_t^z / P_t$ is the relative price of equities and $\pi_{t+1} = P_{t+1} / P_t$ is inflation between periods *t* and *t*+1. The interpretation of conditions (36) and (37) is almost identical to that for conditions (3) and (4). Conditions

(36) and (37) take into account the fact that financial investment earnings fluctuate with the general price level in a monetary economy—hence, the presence of π_{t+1} in these equations.

Markets are in equilibrium when: money and equities in circulation are held willingly, $M_t^d = M_t$ and $z_t = 1$; the net supply of bonds is nil, $B_t = 0$; and the goods market is in equilibrium, $C_t = D_t$. For simplicity's sake, I will restrict the discussion to the case where the cash-in-advance constraint is always binding at equilibrium.[12] Under this hypothesis, the circulation velocity of money is always constant, and the general price level obeys a strict version of the quantity theory of money:

$$P_t = \frac{M_t}{P_t}. \tag{38}$$

This hypothesis also means that inflation $\pi_{t+1} = P_{t+1}/P_t$ between periods t and $t+1$ is equal to the quotient $(\pi_{t+1} = \omega_{t+1}/\lambda_{t+1})$ of the money supply growth factor $\omega_{t+1} = M_{t+1}/M_t$ divided by the goods endowment growth factor $\lambda_{t+1} = C_{t+1}/C_t$.

The model's predictions as to the effect of inflation on asset prices and yields flow from the first-order conditions (36) and (37) estimated at general equilibrium. Using these equations, we will explore how the variability of inflation affects the determination of prices and yields on financial markets. My analysis relies in particular on the work of Labadie (1989) and Giovannini and Labadie (1991), and to a lesser extent on the studies of Fama and Farber (1979), Leroy (1984), and Svensson (1985b). I will first look at the impact on bond yields. For analysis purposes, I define the inflation-risk premium ϕ_{t+1}^{π} as the spread between the expected real

12. The Kuhn–Tucker multiplier μ_t associated with the cash-in-advance constraint (34) is interpreted as the liquidity service of money. Using (37) and the first-order condition for the choice of M_t^d (an unspecified condition here) it can be shown that:

$$\frac{\mu_t}{P_t} = \beta E_t\left[U'(\bullet_{t+1}) \bullet \frac{1}{P_{t+1}}\right] \bullet i_t.$$

This equation demonstrates that in a cash-in-advance constraint model the nominal interest rate serves to compensate investors for the liquidity shortcomings of bonds. For this reason, the cash-in-advance constraint is relaxed (i.e., $\mu_t = 0$) only when the nominal interest rate is nil. The hypothesis that the cash-in-advance constraint is always satisfied at equality is thus equivalent to limiting our analysis to situations where the nominal interest rate is always positive. In theory, it should be possible, using the alternative scenario of Svensson (1985b), to achieve an equilibrium where the liquidity constraint is relaxed under certain conditions, even if the nominal interest rate is positive. Simulation results from Hodrick, Kocherlakota, and Lucas (1991) show, however, that this is mainly a theoretical possibility, since in practice the cash-in-advance constraint always seems to be binding, at least when the parameters are set at “reasonable” values.

return on a nominal bond and the risk-free real interest rate, r^f_{t+1}. The inflation-risk premium, by this definition, is equal to:

$$\phi^{\pi}_{t+1} = E_t\left[r_{t+1}\right] - r^f_{t+1}. \tag{39}$$

The risk-free real interest rate corresponds to the real return on an indexed bond. In the Lucas (1982) model, r^f_{t+1} is determined, as in equation (13), by the reciprocal of the marginal rate of intertemporal substitution of consumption.[13] The expected real return of a nominal bond is equal to:

$$1 + E_t\left[r_{t+1}\right] = \frac{E_t\left[\pi^{-1}_{t+1}\right]}{Q_t}. \tag{40}$$

According to Fisher's theorem,

$$E_t\left[r_{t+1}\right]$$

is equal to a risk-free real interest rate. By substituting in (40) the equilibrium value of Q_t obtained from equation (37) and decomposing the covariance term that appears, an expression is derived that shows the condition under which the Fisher theorem holds true:

$$1 + E_t\left[r_{t+1}\right] = \left(1 + r^f_{t+1}\right) \bullet \left[\frac{E_t\left[\pi^{-1}_{t+1}\right]}{E_t\left[\pi^{-1}_{t+1}\right] + \dfrac{\text{cov}_t(S_{t,\,t+1}, \pi^{-1}_{t+1})}{E_t\left[S_{t,\,t+1}\right]}}\right]. \tag{41}$$

Equation (41) shows that the Fisher theorem's validity, and hence the existence of an inflation-risk premium, relies on the value of the conditional covariance between the real return on money, π^{-1}_{t+1}, and the marginal rate of intertemporal substitution of consumption, $S_{t,\,t+1}$. The Fisher theorem is valid if covariance is nil. In all other cases, the model predicts the existence of an inflation-risk premium, the sign of which is determined by the sign of the covariance, $\text{cov}_t(S_{t,\,t+1}, \pi^{-1}_{t+1})$. Intuitively, a positive (negative) $\text{cov}_t(S_{t,\,t+1}, \pi^{-1}_{t+1})$ means that the real return on money, π^{-1}_{t+1}, is generally greater (less) than predicted when the marginal rate of intertemporal substitution of consumption is greater (less) than

13. For this reason, r^f is solely a function of real shocks. On the other hand, in Svensson's model, the real interest rate is determined by the marginal rate of intertemporal substitution of illiquid wealth, which is generally influenced by both real and monetary shocks.

predicted. In this case, nominal bonds are a better (worse) investment than indexed bonds for smoothing out consumption over time, because they offer a high return when consumption is low.[14] This analysis can be carried even further if one assumes that preferences are isoelastic. In this case,

$$\mathrm{cov}_t(S_{t,\,t+1}, \pi_{t+1}^{-1}) = \beta \bullet \mathrm{cov}_t(\lambda_{t+1}^{-\gamma}, \lambda_{t+1} \bullet \omega_{t+1}^{-1}), \tag{42}$$

and the sign of the inflation premium depends on the contemporary covariance between λ and ω. For example, if the inflation-risk premium is to be negative, the contemporary covariance between these two variables must be positive. In other words, inflation and monetary transfers must be procyclical.

Inflation also affects the stock market in the guise of an inflation tax that is levied implicitly on dividend income. An equity yields its owner a nominal dividend of $P_t \bullet D_t$ at the end of period t. This income can be used to pay for consumption, at the earliest, in period $t + 1$, at a time when its real value will be $D_t \bullet \pi_{t+1}^{-1}$. If $\pi_{t+1} \neq 1$, the change in the purchasing power of money will represent either a tax of $(\pi_{t+1} > 1)$ or a subsidy of $(\pi_{t+1} < 1)$ to shareholders. Carmichael (1985) showed that, in an environment without uncertainty, this inflation tax has a negative effect on equity prices in steady state. In a broader framework, where the volume and growth of the money supply are random, the uncertainty surrounding the value of the inflation tax is an additional element that affects the systematic risk of equity investments in two ways. I will define r_{t+1}^z as the real return on an equity investment between periods t and $t + 1$. Taking the structure of the Lucas model, the value of r_{t+1}^z is equal to

$$r_{t+1}^z = \frac{C_t \bullet \pi_{t+1}^{-1} + q_{t+1}^z}{q_t^z} - 1. \tag{43}$$

Under conditions of general equilibrium, (36) means that the expected value of r_{t+1}^z must satisfy the following condition:

$$1 + E_t\left[r_{t+1}^z\right] = \frac{1 - \mathrm{cov}_t(S_{t,\,t+1}, r_{t+1}^z)}{E_t\left[S_{t,\,t+1}\right]}. \tag{44}$$

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14. With the alternative scenario of Svensson (1985b), the inflation-risk premium depends instead on the conditional covariance of π_{t+1}^{-1} with the marginal rate of intertemporal substitution of illiquid wealth, because financial markets come into play at the end of the period, after the goods and services market has closed.

As with equation (15), the systematic risk of equity investments is determined by the conditional covariance between $S_{t,\,t+1}$ and r^z_{t+1}. To see how inflation affects this covariance, it is useful to reformulate it, using (43), as a sum of the two elements representing the risk due to returns in the form of dividends and returns in the form of capital gains:

$$\text{cov}_t\left(S_{t,\,t+1}, r^z_{t+1}\right) = \left(q^z_t\right)^{-1}$$

$$\bullet \left[C_t \bullet \text{cov}_t\left(S_{t,\,t+1}, \pi^{-1}_{t+1}\right) + \text{cov}_t\left(S_{t,\,t+1}, q^z_{t+1}\right)\right]. \qquad (45)$$

It is easy enough to explain the first covariance. Equities yield income that is held as money for a time before it is spent or invested. It is thus natural that the risk surrounding the purchasing power of money during this waiting period (which is determined by the covariance between $S_{t,\,t+1}$ and π^{-1}_{t+1}) should be one of the elements affecting the systematic risk of equity investments. To appreciate the impact of inflation on the second covariance, we must explain the link between the equilibrium price of equities and the future path of the inflation tax. The equilibrium value of q^z_t is obtained by recursive substitutions of equation (36):

$$q^z_t = E_t\left[\sum_{j=1}^{\infty} S_{t,\,t+1} \bullet C_{t+j-1} \bullet \pi^{-1}_{t+j}\right]. \qquad (46)$$

Solution (46), in contrast with that obtained in Section 1, is a function of all the future expected values of the inflation tax. For this reason, the systematic risk of equity investments is also influenced, through returns in the form of capital gains, by the uncertainty surrounding the future path of the inflation tax.

To sum up, in the Lucas model, the inflation risk affects the systematic risk of equities in two ways. First, because the covariance between the real return on money and the marginal rate of intertemporal substitution of consumption is not nil; second, because the future path of the inflation tax influences the distribution of future capital gains and thus modifies the covariance between $S_{t,\,t+1}$ and q^z_{t+1}.

In principle, the level and the variability of inflation premiums, as discussed, here can significantly affect the stochastic process of asset returns. Could the inclusion of monetary factors and inflation-risk premiums help us to understand some of the anomalies set out in Section 1? Simulation results from Labadie (1989) and Giovannini and Labadie (1991) show that, in qualitative terms, the inclusion of monetary factors generally steers the model's predictions in the right direction. Yet these effects are quantitatively small when preferences are isoelastic. For example, Labadie finds a maximum inflation premium at an absolute value of 0.3 per cent for nominal bonds. Giovannini and Labadie simulate market premiums of the order of 0.42 per cent to 1.91 per cent. These values, while much greater than those found by Mehra and Prescott (1985), are still far from the value of 6.18 per cent observed in the data. Moreover, the simulated premiums are not very variable. Giovannini and Labadie arrive at the conclusion that fluctuations in expected returns are essentially due to fluctuations in the marginal rate of intertemporal substitution of consumption.

Conclusions

This paper has attempted to review the asset-pricing predictions of intertemporal models under conditions of general equilibrium. I have also discussed the extent to which the predictions of these models conform to reality.

My analysis has focused primarily on a fundamental asset-pricing equation that links expected returns on assets to the covariance of their yields with the marginal rate of intertemporal substitution of consumption. The intensive research conducted over the past 20 years has shown significant discrepancies between the model and the data, particularly when preferences take an isoelastic form that does not dissociate the concept of risk aversion from that of intertemporal substitution. While it may not have resolved all the problems, the development of more generalized preference structures that can dissociate the concepts of risk aversion and intertemporal substitution has nonetheless helped clarify how these two concepts influence yields. I have shown that in these models risk aversion affects primarily the risk premium, while the elasticity of intertemporal substitution determines the real interest rate. I have also shown that models based on unexpected utility and on consumption habit formation are better able to explain reality because, each in its own way, they allow simultaneously for high levels of risk aversion and of intertemporal substitution.

The introduction of monetary factors, by means of a Clower-type cash-in-advance constraint, shows that uncertainties as to the purchasing power of money can modify the systematic risk of financial assets. Yet the simulation results obtained to date reveal inflation premiums that are low and not very variable.

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