The New Keynesian Phillips Curve When Inflation Is Non-Stationary: The Case for Canada

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Introduction

In its purest form, the New Keynesian Phillips curve (NKPC) relates inflation in period t to expected inflation in period $t+1$ and a cyclical indicator; NKPC can be derived by assuming optimizing behaviour on the side of firms that set their prices following a time-dependent rule, as in Calvo (1983) (see Sbordone 2005). Traditionally, the NKPC has been estimated under the assumption that inflation is stationary (see Galí and Gertler 1999 for estimates on US data and Galí, Gertler, and López-Salido 2001 for estimates on euro-area data). Existing estimates of the NKPC on Canadian data are also based on this assumption (Gagnon and Khan 2005; Guay, Luger, and Zhu 2003; Khan 2004).

The assumption that inflation is stationary, however, is questionable for Canadian data. Figure A2.1 (in Appendix 2), plotting annualized quarterly changes in the GDP deflator for Canada since 1973Q1, suggests that Canadian inflation over the past 30 years may in fact be integrated of order 1—a phenomenon that could stem from a variety of factors, including quasi-

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rational inflation expectations, inflation indexing, autocorrelation in the cyclical indicator, a shift in the mean of inflation over time (due, for example, to a change in the anti-inflation preferences of the monetary authorities), private sector learning about shifts in the policy target of inflation, or a combination of any of the above. If true, this is problematic, because non-stationarity invalidates many estimation techniques often used in the literature to estimate NKPCs for Canada, including the generalized method of moments (GMM) (and more refined versions of it, such as the continuous updated estimator by Newey and Smith 2000) and the full-information maximum-likelihood (FIML) procedure.

In this paper, we employ a method developed by Johansen and Swensen (1999) to examine whether non-stationarity in inflation is a property of the system underlying Canadian inflation dynamics. This method suggests that inflation in Canada is indeed non-stationary, even when shorter samples are considered to account for shifts in monetary regimes, such as the move to inflation targeting in 1991. The method provides a framework for detecting the cointegration rank of the system and for testing the cointegrating restrictions implied by rational expectations, starting from an unrestricted system. Following Batini, Jackson, and Nickell (2005), we specify the NKPC assuming more general technologies than Cobb-Douglas, and we account for the fact that Canada is an open economy.

This method offers three main advantages. First, it advances from other methods (used in the literature on data for other countries) that model inflation as $I(1)$, but assumes the cointegration specification a priori (e.g., Sbordone 2002; Kozicki and Tinsley 2002).

Second, like the methods used by Rudd and Whelan (2001), Sbordone (2002, 2003), McAdam and Willman (2003), and Banerjee and Batini (2003), this method ensures that model-consistent expectations are tested and subsequently imposed in estimation, and is thus to be preferred to previous methods, such as the GMM, that assume—without imposing and testing for—the rational-expectations assumption implicit in the model.

Finally, this method eliminates the identification problem raised by Ma (2002) and Mavroeidis (2002), who have stated that empirical methods commonly used to estimate the NKPC either cannot identify, or can only weakly identify, the parameters in the estimated regression. We show that when the NKPC is estimated within the Johansen and Swensen (1999) framework, we do not need to make any ex ante assumption on the process of the forcing variable in order for the estimation method to identify the parameters, and so we are spared the trouble of using exact analytical methods.

We find the following:

(i) The NKPC offers a good representation of inflation dynamics in Canada for some—but not all—measures of marginal costs.

(ii) The degree of forward-looking behaviour in price setting varies wildly over time. Split-sample regressions indicate that Canadian price-setters have become slightly more forward looking, especially after the move to inflation targeting in the early 1990s—a finding also reached by Batini (2005). Recently, Canadian firms have exhibited a degree of forward-looking behaviour that is comparable to that observed in the United States or the euro area.

(iii) The empirical method used here is capable of identifying the forward- and backward-looking terms in the NKPC, and our results using this method indicate that both terms are important in determining Canadian inflation.

(iv) Real marginal cost is a significant determinant of Canadian inflation, especially when it is adjusted for the cost of imported intermediates—a finding that mirrors observations for other open economies (in line with Banerjee and Batini 2003, but that is contrary to results by Gagnon and Khan 2005). Its importance varies greatly, however, depending on the method of measurement.

(v) The estimated weights on lagged and expected inflation generally sum to one, which suggests that Canadian inflation does not depend on real factors in the long run—the superneutrality result.

Section 1 of this paper reviews the recent related literature on Canadian data and discusses estimation methods. Section 2 specifies a simple dynamic system that can characterize inflation dynamics under the NKPC paradigm. Section 3 describes the Johansen and Swensen (1999) method and formulates the restrictions implied by the NKPC on such a statistical model. Section 4 estimates the NKPC on Canadian data using this method, and the final section offers concluding remarks and suggestions for future research.

1 Recent Related Literature

A number of empirical papers estimate open economy NKPCs for Canada using GMM. Most of these papers use a version of the NKPC, as in Galí and Gertler (1999), that allows for the fact that a fraction of firms set their prices in a myopic hybrid NKPC.

For example, Gagnon and Khan (2005) follow Sbordone (2002) and fit various NKPC specifications to Canadian data using alternative measures of marginal costs derived from assuming different kinds of production technologies. They find that an NKPC based on constant elasticity of substitution (CES) technology fits the Canadian data well over the period 1970–2000, and more so than an NKPC derived from assuming a Cobb-Douglas technology. They also find that, for this sample, the backward-looking component in the Canadian NKPC is stronger than for the United States and the euro area.

Khan (2004) and Leith and Malley (2002) also estimate hybrid NKPCs for Canada. In particular, Khan estimates a rolling NKPC regression and shows that the NKPC in Canada may have flattened over time. This is consistent with increasing competition among firms over part of the period, under the assumption that price contracts in Canada are set as in Calvo (1983). Estimating both CES and Cobb-Douglas-based NKPCs over the period 1960Q1–1999Q4, Leith and Malley find that they fit Canadian data, and show that, generally, Canada enjoys less price inertia than other G-7 countries but similar inertia to the United States and the United Kingdom.

Kozicki and Tinsley (2002) estimate several different NKPC specifications on Canadian data, over the period 1990Q2–2001Q4 using two-stage least squares, once with inflation and once with inflation gaps defined as the difference between inflation and an estimated "perceived" target. They find that hybrid NKPCs fit better than forward-looking ones, especially when based on Taylor contracts.

Banerjee and Batini (2003) estimate NKPCs for Canada and other open economies over the period 1970Q1–2002Q1 using the maximum-likelihood estimator and assuming various contracting specifications. They find that an NKPC based on time-dependent contracts, as in Dotsey, King, and Wolman (1999), fits the Canadian data well and better than when NKPCs are based on Calvo or Taylor contracts. Banerjee and Batini also find that Canadian firms are predominantly backward looking when setting prices.

Finally, Guay, Luger, and Zhu (2003) estimate NKPCs on Canadian data using a bias-corrected estimator (continuous updating estimator, CUE) as proposed by Newey and Smith (2001) in conjunction with an automatic lag-selection procedure proposed by Newey and West (1994) to calculate estimates of the variance-covariance matrix of the moment conditions. This empirical approach attenuates the potential bias of GMM estimates when there are many instruments and the low power of specification tests based on over-identifying restrictions (see Guay, Luger, and Zhu 2003; Galí, Gertler, and López-Salido 2005). They find that contrary to estimates of the NKPC on Canadian data obtained using standard GMM, the CUE-based estimates do not fit the data well and the NKPC is statistically rejected.

In addition to standard modelling and data-measurement issues, three issues are particularly important when estimating NKPCs. The empirical work on

Canadian data discussed above has dealt with some of the issues but never all at once and within a unified framework, such as the one suggested here. These issues are:

- Non-validity of estimates when inflation is non-stationary

Existing estimates of the NKPC for Canada assume that inflation is a stationary variable. If Canadian inflation is non-stationary, this approach is not ideal. Modelling variables as stationary when they contain a unit root invalidates the estimation result, as in the case of GMM (or CUE) and FIML. If inflation is non-stationary, the asymptotic distributions of the GMM and FIML estimators are not necessarily Gaussian normal, which implies that the estimated NKPC's coefficients and standard errors are invalid; therefore, any inference about the parameter values is incorrect, and more efficient estimators can be obtained by taking into account the unit root. Pre-transforming the data to make them stationary on a priori assumptions about the source of non-stationarity of the data may lead to results that are not robust to alternative hypotheses about the nature of the common trend.

- Testing for model-consistent expectations

A number of empirical studies using GMM (notably, Leith and Malley 2002; Gagnon and Khan 2005; Khan 2004) characterize the NKPC model based only on a very weak property of rational expectations, namely, that the expectational error $(\pi_{t+1} - E_t\pi_{t+1})$ should be unforecastable by variables dated at time t or earlier. In the context of GMM, this boils down to choosing instruments for inflation expectations that are correlated with the portion of π_{t+1} that is orthogonal to π_{t-1} and the cyclical indicator at time t. Estimation is carried out on the assumption that the chosen instruments accomplish this requirement—in other words, rational expectations are simply assumed on the presumption that instrument orthogonality is indeed met. However, rational expectations should be model-consistent: expectations for the next period's inflation rate should be consistent with the process for inflation described by the model. As shown by Fuhrer (1997), Sbordone (2002, 2003), Lindé (2002), Rudd and Whelan (2001), McAdam and Willman (2002), Kozicki and Tinsley (2002), and Banerjee and Batini (2003), this additional prediction yields specific, testable implications for how inflation expectations in the NKPC are modelled. These studies address the issue by following the present-value approach of Campbell and Shiller (1987) and Fuhrer (1997) and cast the NKPC in a system of equations. This procedure computes the expected present value of the driving variable (the cyclical factor in the NKPC case) under the assumption that this follows a specific process, and then determines what fraction of inflation is accounted for by this present-value term. If the present value is well characterized as a function of lags of the driving variable, this method will be equivalent to that originally suggested by Hansen and Sargent (1980).

- Parameter identification

Several recent papers have drawn attention to the problem of identifying the parameters in the NKPC, and more specifically, to whether existing estimation methods can correctly distinguish between backward- and forward-looking solutions. Mavroeidis (2002) demonstrates how identification of the parameters depends on the uniqueness of the solution to the system containing the NKPC and the equations governing the exogenous variables. In the GMM framework, this involves making assumptions on the process of the forcing variable, which is largely ignored in the NKPC literature using Canadian data.[1] Ma (2002) points out that GMM estimation relies on a quadratic, concentrated objective function, because GMM solves a locally quadratic minimization problem. For the hybrid NKPC, the objective function is non-quadratic with respect to the share of firms that set prices in a backward-looking manner, and, consequently, the coefficients on the expected and lagged inflation are only weakly identified.

Nason and Smith (2005) examine this for a number of countries, including Canada. They use Anderson-Rubin (1949) exact analytic methods to examine the identification problem in several statistical environments: under strict exogeneity, in a vector autoregression (VAR), and in the context of a small closed economy “aggregate supply-IS” model augmented with a monetary policy rule. Nason and Smith find that, when these methods are used, the NKPC model is rejected on Canadian data for a different set of instruments over the sample 1963Q1–2000Q4.

In the following sections, we re-estimate the NKPC on Canadian data, addressing these key issues simultaneously within the Johansen and Swensen (1999) unified framework.

2 The Theoretical Model

Consider the hybrid version of the NKPC in Galí, Gertler, and López-Salido (2001):[2]

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t(\pi_{t+1}) + \lambda z_t, \tag{1}$$

1. See also Bårdsen, Eitrheim, Jansen, and Nymoen (2005).
2. This specification assumes that both steady-state inflation and the equilibrium markup are constant. See Ascari (2003) and Batini, Jackson, and Nickell (2005) for a discussion of how this specification changes when these assumptions are relaxed.

where π_t is inflation at time t, z_t is the cyclical indicator—typically the output gap or real marginal cost, and E_t is the expectation operator indicating expectations formed at time t. The complete dynamic system also contains an equation for the real marginal cost; we assume that it is an autoregressive process given by

$$z_t = \delta_1 z_{t-1} + \delta_2 \pi_{t-1} + \eta_t, \tag{2}$$

where η_t is a white-noise residual.

The parameters γ_b, γ_f, and λ depend on “deep” or “structural” parameters, including the probability that firms reset prices at any given time; the discount factor; the fraction of rule-of-thumb firms that set their prices in a backward-looking, myopic way (as in Galí and Gertler 1999); or the degree of indexation to past prices of the firms that are not allowed to reoptimize at time t (as in Christiano, Eichenbaum, and Evans 2005). Direct estimation of γ_b and γ_f, as opposed to the structural equations expressed in terms of deep parameters, facilitates the interpretation of the NKPC under a broad class of sticky-price models and modelling assumptions. γ_b and γ_f must satisfy the following restrictions:

$$\gamma_b, \gamma_f \geq 0$$

$$\gamma_b + \gamma_f \leq 1. \tag{3}$$

Written in the closed-form solution, the interpretation of equation (4) is that fundamental inflation equals the discounted stream of expected future real marginal costs, taking into account the backward-looking behaviour:

$$\pi_t = \delta_1 \pi_{t-1} + \frac{\lambda}{\delta_2 \gamma_f} \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2}\right)^k E_t(z_{t+k}), \tag{4}$$

where δ_1 and δ_2 are, respectively, the stable and unstable roots of the dynamic system. Equations (1) and (2) are examples of an exact rational-expectations hypothesis, in the sense that the econometric test of this hypothesis involves determining whether expectational error is the only error present.

In the NKPC literature (see Lindé 2002), as well as in the inflation-dynamics model by Fuhrer and Moore (1995), it is commonly assumed that

$$\gamma_b + \gamma_f = 1, \tag{5}$$

a restriction often referred to as "dynamic price homogeneity" or "superneutrality." In practice, it is not easy to distinguish between restrictions on the NKPC that guarantee superneutrality and those that do not contradict it, because superneutrality refers to the invariance of flexible-price values of real variables to the steady-state inflation rate. Therefore, the NKPC might violate superneutrality if γ_b and γ_f do not sum to one. But even when they do sum to one, superneutrality need not hold if some other aspect of the model (e.g., the tax system and thus labour market conditions) relevant for the determination of the flexible-price value of z_t made real variables like z_t sensitive to the inflation rate in the long run. And even with $\gamma_b + \gamma_f < 1$, the NKPC might not violate superneutrality whenever there is a specific constant term restriction that involves the mean of π_t (perhaps because of some static indexation).[3] If equation (5) holds, inflation does not depend on real factors in the long run—legitimizing the use of monetary policy for the exclusive pursuit of price stability in the long run.

3 Estimation Method

This paper uses the FIML (full-information maximum-likelihood) method proposed by Johansen and Swensen (1999). It is based on the idea that the mathematical expectation conditional on a theoretical model and the observed data can be used to substitute for the forward-looking term in an estimated model.

The method comprises three steps.

Step 1. The first step requires specifying and estimating an unrestricted VAR containing the relevant variables, under the assumption that at least one variable has a unit root. Diagnostic tests are run to ensure that residuals are white noise and the number of stationary relations in the system are determined. This is done using the trace statistic of Johansen and Juselius (1990)—a test comparing the likelihood of the unrestricted VAR with the likelihood of a cointegrated VAR. If the test indicates that there are fewer stationary relations than variables, the assumption that at least one variable has a unit root is confirmed.

Step 2. The second step requires estimating the parameters of the structural system via maximum likelihood.

Step 3. This step consists of testing the restrictions implied by the rational-expectations model (here the NKPC). This implies parameterizing the cointegrated VAR model to account for any forward-looking term. Appendix 1 explains our parameterization.

3. We thank Edward Nelson for pointing this out.

Below, we offer further details on key aspects of the estimation procedure. In particular, Section 3.1 sketches the numerical optimization methods that we use in step 2 to derive the maximum-likelihood estimators of the Canadian NKPC when the parameters of the rational-expectations model are unknown, as in our case. Section 3.2 formulates the testable hypothesis in terms of the NKPC, where restrictions on the expectations also entail restrictions on the cointegration relationships—as required by step 3. The test of the rational-expectations model compares the likelihood of the cointegrated VAR with the likelihood of the cointegrated VAR with the restrictions imposed. It then describes the associated maximum-likelihood estimator and likelihood-ratio tests. Section 3.3 describes the maximum-likelihood ratio tests that we also use in step 3.

3.1 Numerical maximization methods

The Johansen and Swensen (1999) method provides a test of rational expectations for models in which coefficients are known. If coefficients are unknown, as in our case, with this method—unlike the method used by Campbell and Shiller (1987)—it is still possible to derive maximum-likelihood estimators and likelihood-ratio tests by evaluating the likelihood at every fixed value of the coefficients.

To find the maximum value of the likelihood function, we use the Broyden-Fletcher-Goldfarb-Shanno (BFGS) numerical optimization method for non-linear functions.[4] This method is invariant to the scaling of parameters but not to sample sizes. The BFGS optimization starts from initial values and maximizes the function using a quasi-Newton method based on numerical derivatives. The convergence decision, in turn, is based on the likelihood elasticities and the one-step-ahead relative change in parameter values.[5] We also carry out a grid search for the parameters. This is a simple approach, where we calculate the value of the log likelihood function for each possible combination of values, within given intervals, for the parameters $\{\gamma_b, \gamma_f, \lambda\}$. However, because of the dimension of the parameter matrix, using a grid search we cannot include a constant in the estimated NKPC equation, and we therefore rely mainly on the BFGS numerical optimization results for estimation.

4. We maximize the log likelihood controlling for the number of observations; $-\frac{T}{2}\{\log|S_{11}| + \log|\tilde{\Sigma}_{22}| - \log|a'a| - \log|b'b|\}$.

5. See Fletcher (1987) for details.

3.2 The testable rational-expectations restrictions for the NKPC

Consider the p-dimensional autoregressive process $X_t = (\pi_t, z_t)$ defined for $t = 1, \ldots, T$ by the equations:

$$X_t = \Pi_1 X_{t-1} + \ldots + \Pi_k X_{t-k} + \mu_0 + \varepsilon_t \tag{6}$$

for fixed values of $X_{-k+1}, \ldots, X_0$ and $\varepsilon_t \sim N_p(0, \Omega)$. The parameter space is given by the unrestricted parameters $(\Pi_1, \ldots, \Pi_k, \Omega)$. μ_0 is the coefficient of the constant term. Equation (6) can equivalently be written in a vector-error-correction form as:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \phi D_t + \varepsilon_t, \tag{7}$$

where

$$\Pi = \sum_{i=1}^{k} \Pi_i - I \text{ and } \Gamma_i = -\sum_{j=i+1}^{k} \Pi_j .$$

The first step of the procedure consists in testing the rank of the matrix Π. If there is at least one unit root in the system, and if the NKPC as specified in equation (1) holds, there must be one cointegration vector.[6] Hence, the rank of the matrix must be 1.

Given the reduced rank of the matrix Π, the restrictions implied by the NKPC in equation (1) can be tested as a restriction on the parameters in the matrix Π. However, to find the maximum-likelihood estimators with respect to freely varying parameters, rather than under constraints, we need to reformulate the restrictions according to those in Johansen and Swensen (1999). This is a simple reparameterization of the statistical model and implies that the model's parameters are uniquely identified. The details of the reparameterization, as well as of the complete formulation of the restrictions implied by rational expectations, can be found in Appendix 1. For $X_t = (\pi_t, z_t)$, the restrictions take the form

$$E_t(c'_1 X_{t+1} | \varphi_t) + c'_0 X_t + c'_{-1} X_{t-1} + c = 0 \tag{8}$$

$$E_t(c'_1 \Delta X_{t+1} | \varphi_t) - d'_1 X_t + d'_{-1} \Delta X_{t+c} = 0 . \tag{9}$$

6. Where the number of cointegration vectors equals the number of rational-expectations hypotheses to be tested is a special case and simplifies the estimation procedure. Only this case is described below; see Johansen and Swensen (1999) for details.

In terms of the NKPC, this is expressed as

$$\gamma_f E_t(\pi_{t+1}) - \pi_t + \lambda z_t + \gamma_b \pi_{t-1} = 0 \tag{10}$$

$$\gamma_f E_t(\Delta\pi_{t+1}) - (1 - \gamma_f - \gamma_b)\pi_t + \lambda z_t - \gamma_b \Delta\pi_t = 0 . \tag{11}$$

Hence, to test the NKPC within the Johansen and Swensen framework, we can rewrite the coefficients of the statistical model in terms of the parameters in the NKPC model

$$\begin{aligned} c_1 &= (\gamma_f, 0) \equiv b \\ c_0 &= (-1, \lambda) \\ c_{-1} &= (\lambda_b, 0) \end{aligned} \tag{12}$$

with

$$\begin{aligned} d_1 &= ((1 - \gamma_f - \gamma_b), -\lambda) \equiv d \\ d'_{-1} &= (-\gamma_b, 0) . \end{aligned}$$

d is the cointegration vector. The second step of the method consists in testing the validity of the restrictions implied by the NKPC using a maximum-likelihood ratio test.

3.3 The maximum-likelihood estimators and the maximum-likelihood ratio test

The maximum-likelihood ratio test compares the likelihood of the unrestricted cointegrated VAR with the likelihood of the cointegrated VAR under rational-expectations restrictions. Under the reparameterization mentioned above, the likelihood of the cointegrated VAR under the rational-expectations restrictions is the product of the conditional model for the cyclical factor and the marginal model for inflation.

To estimate the conditional model, one should regress $a'\Delta X_t$ on $b'\Delta X_t + d'_{-1}\Delta X_{t-1}$, $d'X_{t-1}$, ΔX_{t-1}, and the constant term. In terms of the present model, we regress Δz_t on $\gamma_f \Delta\pi_t - \gamma_b \Delta\pi_{t-1}$, $(1 - \gamma_f - \gamma_b)\pi_{t-1} - \lambda z_{t-1}$, ΔX_{t-1}, and the constant term. Denote by R_{1t} the residuals in the regression and define S_{11} as the sum of squared residuals:

$$S_{11} \equiv \frac{1}{T} \sum_{t=1}^{T} R_{1t} R'_{1t} . \tag{13}$$

The part of the maximized likelihood function from the conditional model is then

$$L_{1.2\,\max}^{-2} = \frac{|S_{11}|}{|a'a|} . \tag{14}$$

The marginal model is given by

$$b'\Delta X_t = d'X_{t-1} - d'_{-1}\Delta X_{t-1} , \tag{15}$$

which, for the specific model here, is equivalent to

$$\gamma_f \Delta\pi_t = (1 - \gamma_f - \gamma_b)\pi_{t-1} - \lambda z_{t-1} + \gamma_b \Delta\pi_{t-1} . \tag{16}$$

Define $\tilde{\Sigma}_{22}$ as the sum of squared residuals in the marginal model.

$$\tilde{\Sigma}_{22} \equiv \frac{1}{T} \sum_{t=1}^{T} (\gamma_f \Delta\pi_t - (1 - \gamma_f - \gamma_b)\pi_{t-1} + \lambda z_{t-1} - \gamma_b \Delta\pi_{t-1})$$
$$\times (\gamma_f \Delta\pi_t - (1 - \gamma_f - \gamma_b)\pi_{t-1} + \lambda z_{t-1} - \gamma_b \Delta\pi_{t-1})' .$$

The part of the maximized likelihood function from the marginal model is then

$$L_{2\,\max}^{-2/T} = \frac{|\tilde{\Sigma}_{22}|}{|b'b|} . \tag{17}$$

Consequently, the maximum value of the likelihood function under the rational-expectations hypothesis is given by

$$L_{H\,\max}^{-2/T} = \frac{|\hat{S}_{11}|}{|a'a|} \frac{|\tilde{\Sigma}_{22}|}{|b'b|} . \tag{18}$$

The maximum-likelihood ratio test compares the log likelihood under the rational-expectations hypothesis to the log likelihood of the unrestricted cointegrated VAR, where the asymptotic distribution of the test statistic is χ^2 with degrees of freedom equal to the difference in the number of the parameters in the unrestricted case and under the hypothesis, corrected for the number of estimated parameters.

4 The Data

To estimate the NKPC for Canada, we use inflation, marginal cost, and real import price data for Canada from 1973Q1 to 2003Q4. The inflation rate is measured as the log difference of the (officially seasonally adjusted) implicit price deflator of GDP at market prices.[7] The real marginal cost is measured by the deviation of the labour share from its sample mean. We look at two measures of the share. First, we use an unadjusted measure as in Galí and Gertler (1999) for the United States and Gagnon and Khan (2005) for Canada. Then, in line with the adjustment proposed by Batini, Jackson, and Nickell (2005) and used on Canadian data by Guay, Luger, and Zhu (2003), we also use a measure of the share that is net of indirect taxes. It includes partial remuneration of the self-employed that constitutes a return to labour rather than to capital, and is adjusted to remove public sector inputs and outputs from the expression for the share.[8] Finally, to allow for the openness of the Canadian economy, we follow Batini, Jackson, and Nickell (2005) and modify marginal cost to account for the role of imported material input prices under more general technologies than Cobb-Douglas. This implies adding to (the log of) marginal cost the log of the ratio of import prices to the GDP deflator, weighted by a time-varying indicator for the openness for the economy (export plus import volumes divided by GDP).[9] The time series of the data used are plotted in Appendix 2.

5 The Results

We present the results in five subsections. Subsection 5.1 presents estimates of the unrestricted VARs. Section 5.2 describes stationarity results obtained using multivariate tests.[10] Subsection 5.3 presents parameter estimates of the NKPC system (equation 1) and the likelihood-ratio test for the NKPC obtained using Canadian data. There we also discuss what occurs if steady-state inflation is time varying and, accordingly, allow for a non-zero constant term in the NKPC. Subsection 5.3 also considers the possibility that shifts in

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7. We thank Nicolas Raymond at the Bank of Canada for help with these data.

8. These specific adjusted labour share data for Canada are those used by Guay, Luger, and Zhu (2003). We thank Zhenhua Zhu for providing us with these data. Following Batini, Jackson, and Nickell (2005), Gagnon and Khan (2005) first modified Canadian marginal cost in this way for estimates on Canadian data.

9. Following Batini, Jackson, and Nickell (2005), Gagnon and Khan (2005) first modified Canadian marginal cost in this way for estimates on Canadian data.

10. We also conducted univariate tests. The results are plausible and largely consistent with the results from the multivariate tests, but they tend to be less reliable, as explained in Johansen (1995). Because of space limitations, however, we report only the results from the multivariate tests.

the mean of inflation have occurred, arising, for example, through shifts in Bank of Canada anti-inflation preferences, and thus presents estimates on two different samples—one pre- and one post-inflation targeting. Subsection 5.4 discusses results from likelihood-ratio tests for variable exclusion. Finally, subsection 5.5 repeats the analysis in subsections 5.1 through 5.4, using a measure of the labour share adjusted for net indirect taxes, self-employment, and the public sector.

5.1 Step 1

This first step requires estimating an unrestricted VAR on the variables of the system. Since Canadian inflation seems to exhibit a break around 1990—possibly in conjunction with the shift to inflation targeting in 1991—one question is whether we should conduct the analysis on the full sample or on split samples.

To ascertain this, we check for a possible break at the time of the regime shift using the Chow break-point test. In line with findings in Ravenna (2000), who documents a large post-1990 drop in inflation persistence, the break-point Chow test confirms a structural break in 1991Q3, the year Canada shifted to inflation targeting. (The sequences of the Chow-test statistics normalized by their critical values are plotted in Figures A2.2 and A2.3 in Appendix 2.) Levin and Piger (2002) also find evidence of structural breaks for various measures of Canadian inflation around this time—but not GDP price inflation—using a variety of tests for structural breaks.

Testing for structural breaks is complicated by a number of factors, and results can differ sensibly according to the test method and underlying assumptions (see Stock 2004). Consequently, we do not take a stand here on whether a break has occurred, and we proceed by looking at results on the full sample and on the split sample. In line with our test finding and with the literature, we choose 1993Q3 as the time of the break for our split sample.[11]

We estimate an unrestricted three-lag VAR in Canadian GDP price inflation and the unadjusted labour share modified for open economy considerations, as well as a constant term over the full sample and two similar VARs on the split samples. The lag length was determined using standard lag length

11. Each step of the parameter estimation involves testing for the cointegrating rank, estimating the reduced rank VAR, and then estimating the parameters in a reduced-rank restricted VAR. To date, no automated software exists to perform this task. Due to the complexity of the estimation method and the fact that the estimates do not change much when we consider one split sample, we decided to leave recursive estimation for future research.

information criteria (see Table A2.2 in Appendix 2). Tests suggest that the residuals are not white noise for the full sample. However, similar tests indicate that residuals are no longer misspecified when the sample is split (see Table A2.3), implying that it is sensible to proceed with the Johansen and Swensen method, at least on split samples.

5.2 Stationarity tests

Figure A2.1 in Appendix 2 plots Canadian GDP price inflation and our (unadjusted) measure of open economy marginal cost. They look highly correlated contemporaneously, with inflation moving from high and more volatile levels in the 1970s and 1980s to a lower and more stable level after 1990; they therefore lend visual support to the hypothesis that inflation in Canada has a unit root. We test for the existence of a unit root in inflation both over the entire sample and on the split samples.

For the full sample, multivariate unit-root tests using the trace test for cointegration give strong evidence of one stationary relationship and one common trend in the system (see Table 1).[12] This is in line with inflation being integrated of order 1 over the full sample. For the full sample, we can thus proceed to the analysis of the cointegrated VAR model with one cointegrating relationship.

Results on the split sample are more mixed and also depend on whether the constant is restricted to the cointegration space, the lag length of the VAR, and the start and end date of the sample. The Bartlett-corrected trace statistic does not find strong evidence of cointegration in the first sample, but there is evidence of cointegration in the second sample. Bootstrapping the *p*-values does not seem to considerably change the results. It is clear that there is a trade-off between robust cointegration rank results, which are obtained on the full sample, and well-behaved residuals, which are obtained on the split samples. Appendix 1 discusses additional cointegration evidence.

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12. In Table 1, λ denotes the eigenvalues of the Π-matrix, and the test is for the number of non-zero eigenvalues. Due to the small sample, the tests are Bartlett-corrected. The trace-test statistics, the asymptotic critical values (simulated with 2,000 replications), and the bootstrapped critical values (simulated with 1,999 replications) were obtained with the software program Structural VAR, created by Anders Warne. Taking into account the three lags, the estimation sample starts in 1974Q3.

Table 1
Cointegration rank test statistics

1973–2003				
λ	$H_0: r = p$	Trace statistic	p-value (asymptotic)	p-value (bootstrap)
	$p = 0$	23.11	0.02	0.02
0.149	$p \leq 1$	4.06	0.41	0.43
0.039				
1973–90				
λ	$H_0: r = p$	Trace statistic	p-value (asymptotic)	p-value (bootstrap)
	$p = 0$	14.85	0.23	0.29
0.164	$p \leq 1$	5.49	0.25	0.25
0.060				
1991–2003				
λ	$H_0: r = p$	Trace statistic	p-value (asymptotic)	p-value (bootstrap)
	$p = 0$	20.99	0.04	0.06
0.253	$p \leq 1$	6.72	0.15	0.16
0.128				

5.3 Parameter estimates and restriction tests

Results in the previous section indicate that there is a unit root in the NKPC system (both in the full and in the split samples), and that variables in the system are linked through one stationary relationship. Given this, we are interested in testing whether such a stationary relationship satisfies the restrictions implied by the NKPC (restrictions (equation 3)) *for reasonable parameter values* of $\{\gamma_b, \gamma_f, \lambda\}$.[13] Under the assumption that inflation has a unit root, three possible cases exist. First, the stationary relationship is not the NKPC. This would be the case where the maximum-likelihood estimates of the parameters in the stationary relationship do not correspond to plausible parameter estimates for the NKPC. Second, the stationary relationship is the NKPC, and there is a cointegration relationship between the inflation rate and the real marginal cost. This implies that both the inflation rate and the real marginal cost are integrated of order 1. Third, it is possible that the stationary relationship is the NKPC, but it can be represented by a relationship between the change in inflation and the real marginal cost. In this case, there is no cointegration, since the inflation rate is integrated of order 1, but the real marginal cost is stationary. In the following, we use the numerical optimization methods described in section 3.1 to obtain the maximum-likelihood estimates, the maximized

13. In the literature, the parameters are commonly restricted by $0 < \gamma_b, \gamma_f, \lambda < 1$.

value of the likelihood function, and the likelihood-ratio test statistic for the NKPC.

Tables 2 and 3 show parameter estimates obtained maximizing the likelihood via BFGS numerical methods on full and split samples. Likelihood-ratio tests indicate that on the split sample, the NKPC fits the Canadian data well, when the share of labour accounts for openness considerations but is not adjusted for net indirect taxes or other data considerations. The fit, however, is not as good for the full sample, since the likelihood-ratio test restrictions implied by the NKPC can be rejected at the 5 per cent significance level.

Both on full and split samples, the estimated weight on the lag of inflation, γ_b, is much lower than the estimated weight on the lead of inflation, γ_f, suggesting that price-setters in Canada are predominantly forward looking. The estimated value of γ_b is, in general, smaller than in Gagnon and Khan (2005) and Guay, Luger, and Zhu (2003), and the estimated value of γ_f is also generally larger. However, Nason and Smith (2005) find very similar estimates when a large set of instruments is included, but in their estimations, the estimated value of the coefficient on the real marginal cost is much smaller than in our case. For both the full sample and the later sample, our estimates of the coefficient on real marginal cost are generally on the higher side of the range found in the literature for comparable measures of the share (see, notably, Gagnon and Khan 2005). The analysis on split samples points to parameter instability of the estimates over time: in the later sample, price-setters seem to have become more forward looking—in line with estimates of NKPC for the United States and the euro area on samples starting in the 1970s. Likewise, inflation seems to have become almost three times more sensitive to the real marginal cost than in the earlier sample.

When the model is estimated with the restriction

$$\gamma_b + \gamma_f = 1,$$

the likelihood-ratio test statistic is 3.96 for the first sample and 3.32 for the second sample. Given that the critical value for the likelihood-ratio test with one degree of freedom at the 5 per cent significance level is 3.84, the restriction cannot be rejected for the second sample. This probably indicates that the Phillips curve in Canada is vertical in the long run, with obvious implications for monetary policy.

Table 2
Parameter estimates with BFGS, 1973–2003

γ_b	γ_f	λ	$LR(5)$	***p*-value**
0.271	0.729	0.376	23.36	< 0.05

Note:
BFGS: Broyden-Fletcher-Goldfarb-Shanno.

Table 3
Parameter estimates with BFGS, 1973–90 and 1973–2003

Period	γ_b	γ_f	λ	$LR(5)$	***p*-value**
1973–90	0.326	0.714	0.165	6.30	> 0.25
1973–2003	0.269	0.721	0.415	5.52	> 0.25

Note:
BFGS: Broyden-Fletcher-Goldfarb-Shanno.

5.4 Likelihood-ratio test for variable exclusion

The null hypothesis that one of the variables can be excluded from the NKPC can be checked by doing a test on the ratio of the likelihood of the NKPC with the variable and the likelihood of the NKPC without the variable, under the assumption that the restriction implied by the NKPC is valid. This test is distributed as $\chi^2(1)$, given the difference in the numbers of the estimated parameters. The likelihood-ratio test statistics are shown in Table 4, together with approximate *p*-value. They imply that all of the coefficients, except for the one on the real marginal cost during 1973–91, are highly significant.

5.5 What happens when we adjust the labour share for data considerations?

As explained in Batini, Jackson, and Nickell (2005), the labour share appears easy to compute: take the total compensation of employees in the economy and divide it by the national income. In practice, however, there are three issues to bear in mind when computing the labour share.

First, the share must be derived relative to a measure of value added that is net of indirect taxes. Conceptually, firms and workers can only lay claim on revenue (in terms of output per head) that accrues to the firm. By definition, this will be net of taxes on value added, because the latter go to the government and are not received by the firm.

Table 4
Likelihood-ratio tests for variable exclusion
(*p*-values in brackets)

Period	γ_b	γ_f	λ
1973–2003	7.42	4.68	4.00
	[0.01]	[0.05]	[0.05]
1973–90	16.04	10.78	0.42
	[0.01]	[0.01]	[0.50]
1991–2003	12.42	13.78	3.62
	[0.01]	[0.01]	[0.10]

Second, as Bentolila and Saint-Paul (1999) emphasize, because the share represents the remuneration of employees in value added, it ignores the portion of remuneration of the self-employed that constitutes a return to labour rather than to capital. Two ways can be used to adjust for this. We can either augment the numerator of the ratio defining the labour share to include the fraction of total compensation of the self-employed that relates to labour; or we can subtract the amount of value added generated by the self-employed from the denominator of that ratio.

A final consideration when deriving a measure of labour share is related to the contribution of the public sector. It might be argued that the concept of labour and capital shares only really makes sense with regard to the market sector of the economy. In this spirit, we may amend the labour share to remove the public sector's inputs from the numerator and the denominator of its expression. We do so by subtracting from the numerator of the self-employed adjusted share, the compensation of employees by the general government, and by removing from the denominator of that share the general government total resources, essentially a measure of general government gross value added. Figure A2.4 in Appendix 2 plots the labour share adjusted for open economy considerations alone vis-à-vis the labour share adjusted also for net indirect taxes, self-employment, and public sector considerations.

In this section, we thus repeat the analysis of the previous three subsections using this adjusted measure of the share. Misspecification tests and cointegration results are reported in Tables A2.4 and A2.5 in Appendix 2. The cointegration tests indicate one stationary relationship in the full sample. There is no evidence, however, of a stationary relationship in the first sample, and no evidence of a stationary relationship in the second sample. When this adjusted measure of the share is used for 1973–90, the parameters that maximize the maximum-likelihood function are reasonable, as shown in Table 5. However, when we attempt to estimate the NKPC

Table 5
Parameter estimates with BFGS, 1973–90 and 1991–2003, labour share adjusted for taxes, public sector, and self-employment

Period	γ_b	γ_f	λ	$LR(5)$	***p*-value**
1973–90	0.681	0.318	0.743	10.26	> 0.05
1991–2003	–	–	–	–	–

Note:
BFGS: Broyden-Fletcher-Goldfarb-Shanno.

parameters from 1991 to 2003, the BFGS algorithm does not converge for any reasonable parameters.

Conclusion and Future Research

We have used a new method for estimating linear rational-expectation models containing $I(1)$ variables to estimate the NKPC on Canadian data (1973–2003). Our results strongly indicate the presence of a unit root in the Canadian GDP price inflation rate over the full sample and give evidence of a unit root in inflation over the earlier period of the sample when we split the data into a pre- and a post-inflation-targeting period. We find that the NKPC offers a good representation of inflation dynamics in Canada for some—but not all—measures of marginal cost. Accounting for open economy considerations seems particularly important for the fit. Contrary to much previous literature, estimates of the NKPC based on this method and this assumption also support the superneutrality result. In addition, estimation of the NKPC in the Johansen and Swensen (1999) framework overcomes the problem of identification associated with GMM estimation. Hence, it is possible to discern empirically between forward-looking and backward-looking NKPC specifications. We find that both terms are important in determining inflation.

One interesting avenue of research could include estimating on Canadian data the NKPC jointly with a wage equation, as in Sbordone (2005) and Sbordone and Cogley (2004), under the hypothesis of a unit root in price inflation or on price and wage inflation.

Appendix 1
The Statistical Model and the Rational-Expectations Hypothesis

Assume that the p-dimensional vectors of observation are generated according to the VAR model[1]

$$X_t = A_1 X_{t-1} + \ldots + A_k X_{t-k} + \mu + \phi D_t + \varepsilon_t$$
$$\text{for } t = 1, \ldots, T, \tag{A1.1}$$

where $X_{-k+1}, \ldots, X_0$ are assumed to be fixed, and $\varepsilon_1, \ldots, \varepsilon_T$ are independent identically distributed shocks with mean zero and covariance matrix Σ. The matrices $D_t, t = 1, \ldots, T$, consist of deterministic series orthogonal to the constant term, μ. The VAR in equation (A1.1) can be reparameterized as

$$\Delta X_t = \Pi X_{t-1} + \Pi_2 \Delta X_{t-1} + \ldots + \Pi_k \Delta X_{t-k} + \mu + \phi D_{t+1} + \varepsilon_t$$
$$\text{for } t = 1, \ldots, T, \tag{A1.2}$$

where

$$\Pi = A_1 + \ldots + A_k - I$$

and

$$\Pi_i = -(A_1 + \ldots + A_k) \quad \text{for } i = 2, \ldots, k.$$

For the process X_t to be $I(1)$, we assume that the matrix Π has reduced rank $0 < r < p$ and hence may be written as

$$\Pi = \alpha\beta',$$

where α and β are $p \times r$ matrices of full column rank.

The formulation of the rational-expectations hypothesis takes the form

$$E_t(c'_1 X_{t+1} | \varphi_t) + c'_0 X_t + c'_{-1} X_{t-1} + \ldots$$
$$+ c'_{-k+1} X_{t-k+1} + c = 0. \tag{A1.3}$$

---

1. This part follows Johansen and Swensen (1999) closely.

$E_t(c'_1 X_{t+1} | \varphi_t)$ denotes the conditional expectation given variables $X_1, \ldots, X_t$. The $p \times q$ matrices c_i, $i = -k+1, \ldots, 1$ are known matrices, possibly equal to zero. Assume that the two matrices c_1 and $c_{-k+1}, \ldots, c_0, c_1$ are of full column rank. Defining

$$d_{-i+1} = \sum_{j=i-1}^{k-1} c_{-j} \quad \text{for } i = 0, \ldots, k, \tag{A1.4}$$

the restriction in equation (A1.3) may be reformulated as

$$E_t(c'_1 X_{t+1} | \varphi_t) - d_1 X_t + d'_{-1} \Delta X_t + \ldots$$

$$+ d'_{-k+1} \Delta X_{t-k+2} + c = 0. \tag{A1.5}$$

Reformulate restrictions (A1.3) and (A1.5) as restrictions on the coefficients of the statistical model in equation (A1.2). Taking the conditional expectations of ΔX_{t+1} given $X_t, \ldots, X_0$, and multiplying equation (A1.2) by c'_1, we obtain

$$c'_1 E_t(\Delta X_{t+1} | \varphi_t) = c'_1 \Pi X_{t-1} + c'_1 \Pi_2 \Delta X_{t-1} + \ldots$$

$$+ c'_1 \Pi_k \Delta X_{t-k} + c'_1 \mu + c'_1 \phi D_{t+1}. \tag{A1.6}$$

Inserting this expression into equation (A1.5) implies that the following conditions must be satisfied:

$$c'_1 \Pi = d'_1$$

$$c'_1 \Pi_i = -d'_{-i+1} \quad \text{for } i = 2, \ldots, k$$

$$c'_1 \mu = -c$$

$$c'_1 \phi = 0.$$

Expressed in terms of the statistical model in equation (A1.2),

$$\beta \alpha' c_1 = -\sum_{j=-1}^{k-1} c'_{-j} = d_1$$

$$c_1\Pi_i = \sum_{j=i-1}^{k-1} c'_{-j} = -d'_{-i+1} \quad \text{for } i = 2, \ldots, k$$

$$c'_1\mu = -H\omega$$

$$c'_1\phi = 0 .$$

Note that the first part of the restriction implies that the vector d_1 must belong to the space spanned by the columns of β, i.e., d_1 is a cointegration vector. Also, multiplying both sides by the matrix $(\beta'\beta)^{-1}\beta'$, one obtains the following restrictions on the adjustment parameters in α: $\alpha'c_1 = (\beta'\beta)^{-1}\beta'd_1$. Hence, the restrictions implied by the rational-expectations hypothesis are simultaneous restrictions on all parameters.

Appendix 2
The Data

Table A2.1
Univariate unit-root test, 1973–2003

Series	Period	Level	Difference
π_t	1973Q1–2003Q4	–2.48	–12.48 (**)
π_t	1973Q1–1990Q4	–1.60	–9.70 (**)
π_t	1991Q1–2003Q4	–3.56 (*)	–8.16 (**)
z_t	1973Q1–2003Q4	–1.30	–4.67 (**)
z_t	1973Q1–1990Q4	–2.37	–9.68 (**)
z_t	1991Q1–2003Q4	–2.45	–7.18 (**)

Table A2.2
Information criteria for determination of lag length, 1973–2003

Model	Schwartz	Hannan-Quinn	Akaike
VAR(1)	**–6.69**	–6.78	–6.84
VAR(2)	–6.59	–6.73	–6.82
VAR(3)	–6.60	**–6.79**	**–6.93**
VAR(4)	–6.46	–6.71	–6.88

Table A2.3
Misspecification tests

	1973–2003		1973–90		1991–2003	
AR 1–5	$F(20, 198)$	0.98 [0.48]	$F(20, 94)$	0.65 [0.87]	$F(16, 72)$	1.70 [0.07]
Normality	$\chi^2(4)$	69.48 [0.00]	$\chi^2(4)$	31.36 [0.00]	$\chi^2(4)$	8.00 [0.09]

Table A2.4
Misspecification tests, labour share adjusted for taxes, public sector, and self-employment

	1973–2003		1973–90		1991–2003	
AR 1–5	$F(20, 200)$	2.05 [0.01]	$F(20, 96)$	2.30 [0.00]	$F(16, 74)$	2.10 [0.027]
Normality	$\chi^2(4)$	37.68 [0.00]	$\chi^2(4)$	8.15 [0.06]	$\chi^2(4)$	7.44 [0.11]

Table A2.5
Cointegration rank test statistics,
labour share adjusted for taxes, public sector, and self-employment

1973–2003		
λ	$H_0: r = p$	Trace statistic
	$p = 0$	28.599 (**)
0.143	$p \leq 1$	10.404
0.084		
1973–90		
λ	$H_0: r = p$	Trace statistic
	$p = 0$	15.032 (**)
0.199	$p \leq 1$	3.296
0.060		
1991–2003		
λ	$H_0: r = p$	Trace statistic
	$p = 0$	15.478 (*)
0.147	$p \leq 1$	5.025 (*)
0.073		

Figure A2.1
Inflation and labour share adjusted only for open economy considerations, 1973–2003

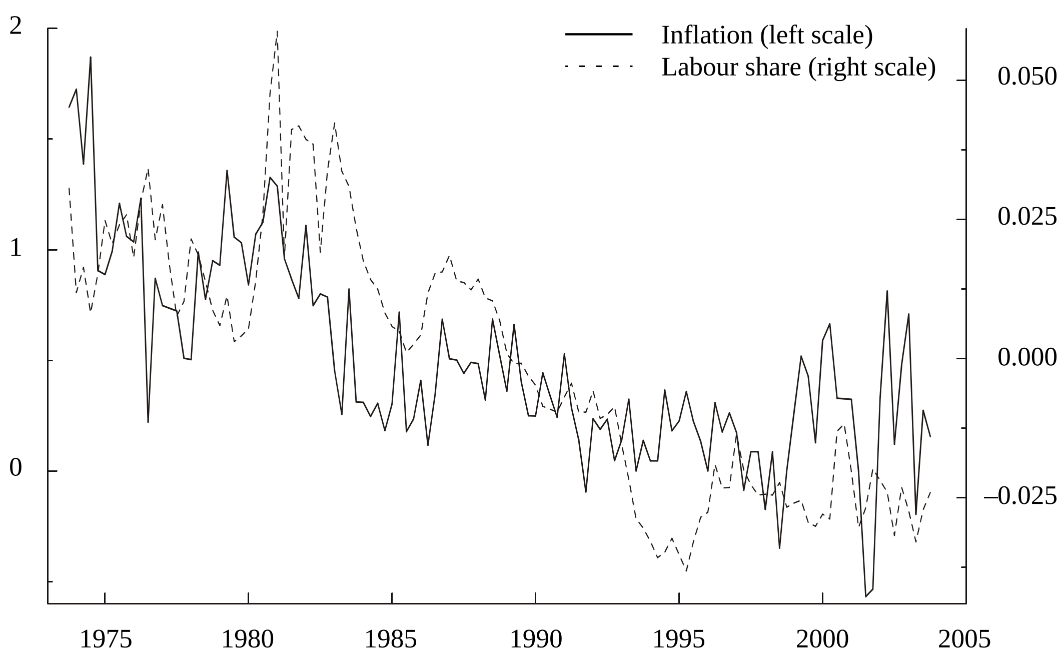


Figure A2.2
Sequence of the Chow test statistics for the AR inflation process normalized by critical values

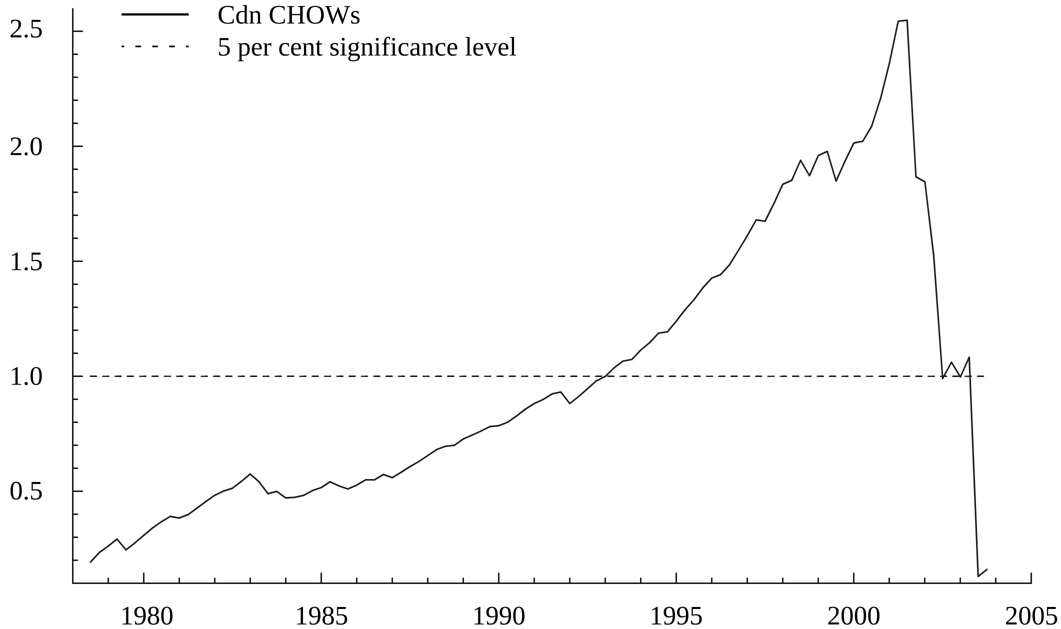


Figure A2.3
Sequence of Chow test statistics for the AR labour share process normalized by critical values

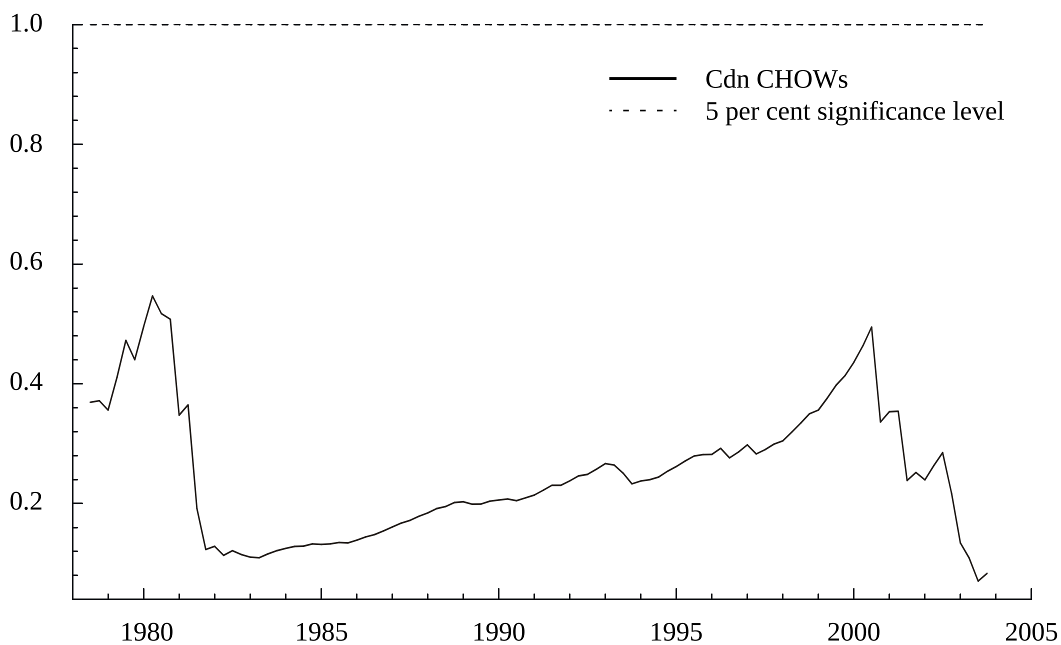
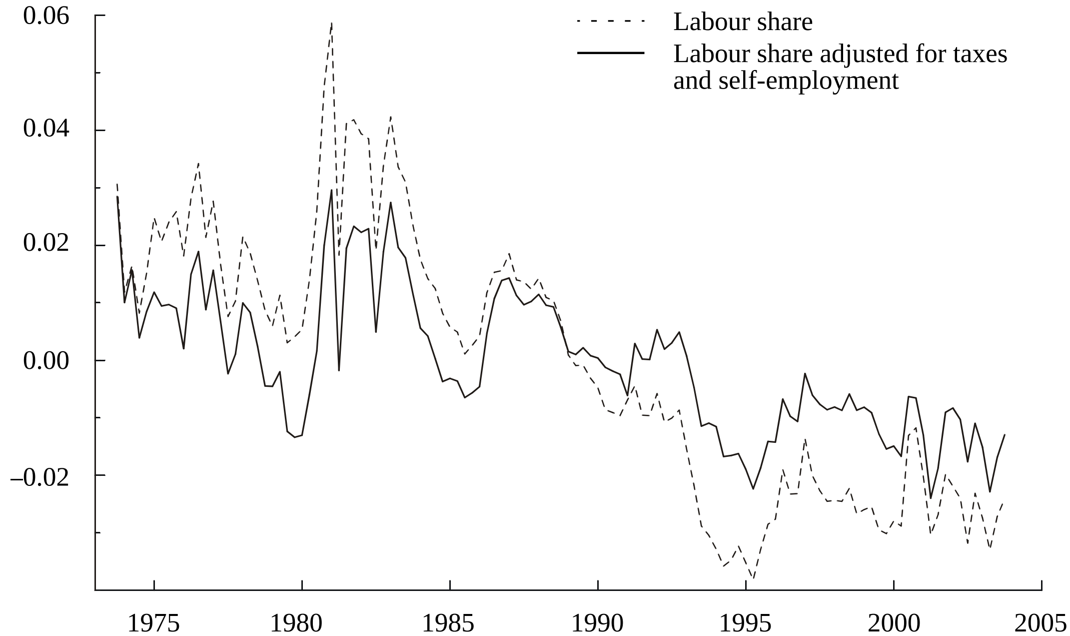


Figure A2.4
Labour share, adjusted and unadjusted for net indirect taxes, self-employment, and public sector considerations



Appendix 3
Empirical Evidence of Cointegration

The trace-test statistic finds strong evidence of cointegration between the inflation rate and the real marginal cost in the full sample, and this is supported by the graphical representation, since the time series seem to have the same stochastic trend. When splitting the sample, the cointegration evidence becomes less clear-cut. While the trace test supports cointegration from 1991 to 2003 (see Figure A3.2), it cannot accept cointegration from 1973 to 1990 (see Figure A3.1). In the following paragraphs, we investigate further the evidence for cointegration in the split samples.

Roots of the companion matrix

The roots of the companion matrix indicate the number of unit roots in the system, whereas the cointegration rank test indicates the number of unit roots in the cointegration matrix. If the unit roots in the companion matrix disappear by imposing the correct number r of cointegrating relationships, they belong to the cointegration matrix, implying that there must be cointegration. While one cointegration vector is consistent with the evidence in the second sample, the second large root in the first sample only becomes marginally smaller when one cointegration vector is imposed.

Plots of the cointegration relationships

The cointegration relationships should represent stationary relationships of the variables. While the trace-test statistics and the roots of the companion matrix suggest that there is more evidence of cointegration in the second sample, the cointegration relationship for the first sample looks more stationary.

Figure A3.1
Estimated cointegration relationship, 1973–1990

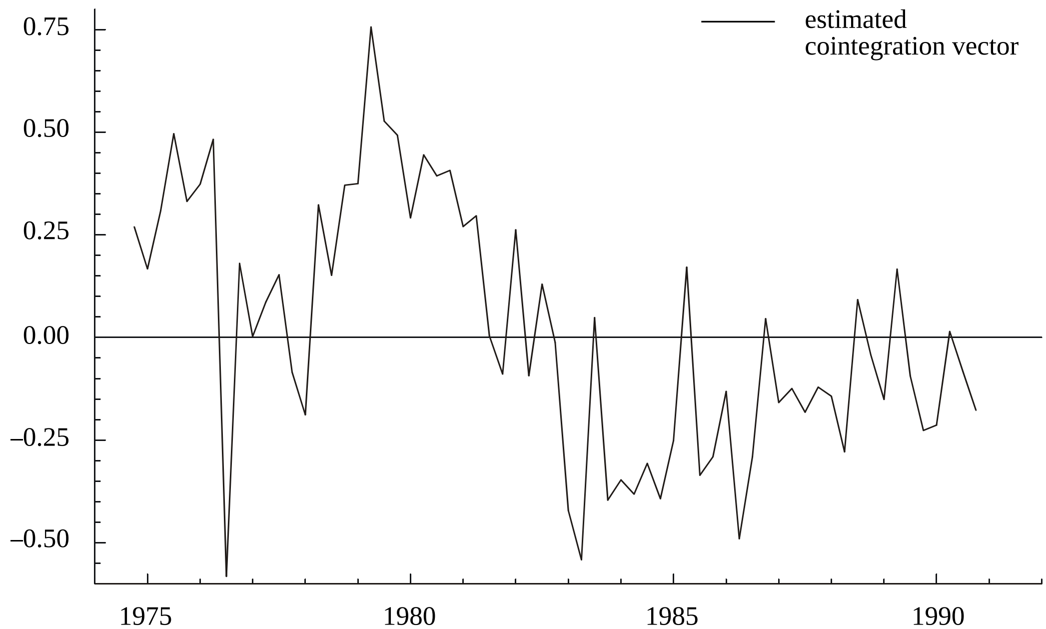
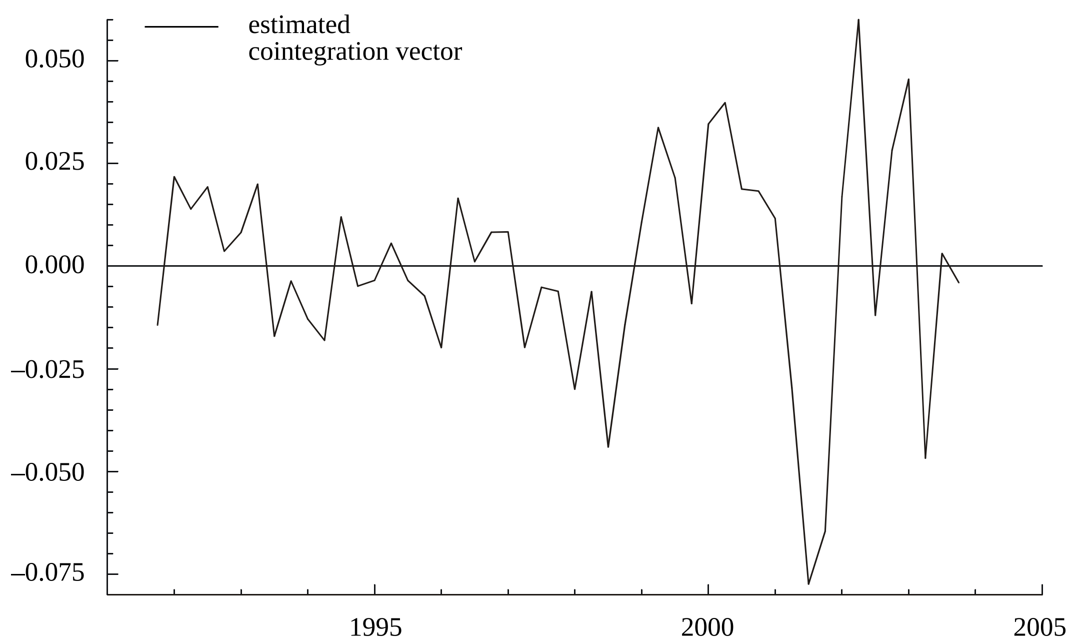


Figure A3.2
Estimated cointegration relationship, 1991–2003



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