

# Discussion

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It is helpful to think of this paper as an empirical application of the Gray (1978) model. This is not an entirely accurate view, however, since the authors also discuss some of the more recent refinements of Gray's analysis, but the Gray model is the canonical statement of the main proposition the authors are interested in: that contract length is a decreasing function of real and monetary variability. My discussion will therefore focus first on the appropriateness of the empirical analysis in light of the Gray model, then on the limitations of the model as a way of understanding the data.

The essential point of Gray's theory is that both real and nominal uncertainty reduce optimal contract length. The model consists of the following equations for output,  $Y_t$ , money supply,  $M_t^s$ , and money demand,  $M_t^D$ , given labour,  $L_t$ , and price level,  $P_t$ , and shocks  $\alpha$  and  $\beta$  to productivity and money supply, respectively:

$$\ln Y_t = \delta \ln L_t + \alpha_t \quad \text{var}(\alpha) = V_\alpha$$

$$\ln M_t^s = \bar{M} + \beta_t \quad \text{var}(\beta) = V_\beta$$

$$\ln M_t^D = \ln \kappa + \ln P_t + \ln Y_t.$$

Gray assumes that  $\alpha$ ,  $\beta$  follow independent Wiener processes.

Labour is supplied according to contracts that set wages,  $w$ , and indexation  $\gamma$  for  $l$  periods in the future; employment is determined by labour demand after the current shock is realized:

$$\ln L_t^D = -\eta(\ln w_t - \alpha_t) + \eta \ln \delta$$

$$\ln L_t^s = \varepsilon \ln w_t + \eta \ln \delta.$$

The price level adjusts each period to clear the money market. Note that in an equilibrium without wage rigidity, output would be given by  $Y_t^* = \delta \ln L_t^* + \alpha$ , where the labour quantity,  $L_t^*$ , represents full employment. Contract characteristics are chosen by a planner to minimize the loss function, which is assumed to be proportional to deviations from full-employment output:

$$\min_{l, \gamma} \left\{ \frac{\lambda}{l} \int_0^l E[\ln Y_t - \delta \ln L_t^*]^2 dt + C \right\}.$$

The solution of the model is an optimal indexation and an optimal contract length,  $(\gamma^*, l^*)$ , such that the following conditions hold:

$$\gamma^* = \theta + (1 - \theta) \frac{\varepsilon}{1 - \varepsilon}$$

$$l^* = \frac{\varepsilon + \eta}{\delta \eta^2} \phi,$$

where

$$\phi \equiv \left[ \frac{2C/\lambda(V_\alpha \eta^2 + V_\beta)}{V_\alpha V_\beta} \right]$$

$$\theta \equiv \frac{V_\beta}{\frac{\eta^2(1 + \varepsilon)}{\varepsilon + \eta} V_\alpha + V_\beta}.$$

This solution implies that the optimal contract length is decreasing in the variance of the two types of shocks, and that the optimal indexation is a weighted sum of the optimal response to a monetary shock and the optimal response to a real shock. These are the properties at the heart of the Fay-Lavoie paper.

The paper's main strategy is to regress average contract length at a particular point in time on measures of the real and nominal uncertainty. In most cases, uncertainty is equated with variability, so the measure of uncertainty is a moving average of first differences. In addition, the authors present some results in which the dependent variable is the proportion of contracts with some positive level of indexing.

There are several reasons why this procedure is not the ideal way to implement the model, as the authors themselves recognize, but the choice of variables is driven by data constraints. The essential result, that real uncertainty does not seem to affect contract length, but nominal uncertainty does, confirms results by other researchers in the United States and Canada, and consequently one might hope that this indicates a robustness of the underlying relationship that is reflected indirectly through the proxy variables used in these and previous studies.

It seems to me that the mapping from the proxy variables to real and nominal uncertainty is somewhat hypothetical; how seriously should we believe that average variability reflects subjective uncertainty at a point in time? Is the duration of non-government labour contracts really representative of Canadian labour contracts, both explicit and implicit? Let us take the quality of the proxies for granted, however, and ask instead how to interpret the regression in light of the model.

A fairly direct empirical form of the model can be found by taking logs of the equation for optimal contract length:

$$\begin{aligned}\log J^* &= \log\left(\frac{\varepsilon + \eta}{\delta\eta^2}\right) + \log\phi \\ &= b_0 - \log V_\beta - \log V_\alpha + \log(V_\alpha\eta^2 + V_\beta).\end{aligned}$$

This corresponds fairly closely to the regression we see in the paper, except that we learn that the effect of uncertainty on contract length depends directly on another parameter that is not discussed: the wage elasticity of labour demand. The model assumes that this is constant over time, but it is clear that in real life, or in a more modern model, this is going to be determined by the profit-maximization problem of the firm. A deeper issue is that this equation assumes explicitly that the contract will have the optimal degree of wage indexation. Gray shows that if we allow the degree of indexation to deviate from the optimum,  $\gamma^*$ , the optimal contract length is decreasing in the size of the deviation. Since  $\gamma^*$  depends on both  $V_\alpha$  and  $V_\beta$ , then the model tells us that variations in contract length may be due to failure of the contract process to arrive at the optimal degree of indexation.

In other words, the apparent failure of contract length to respond to reductions in real variability may be the result of constraints on indexation. Indeed, Fay and Lavoie's analysis of the data seems to suggest that contracts are trending towards zero indexation, which is not optimal as long as there is a positive probability of real shocks. Whether this trend to suboptimal indexation is related to the general weakening of labour unions in Canada is perhaps beyond the purview of the paper, but the trend to long-term

contracts without indexation seems to be a clear shifting of the burden of uncertainty from employers to workers, rather than the optimal response of the labour market under the Gray model to reduced monetary variability.

So far I have considered the model at face value. It is clear, however, that the model takes a very simplified view of the labour market, and it would be helpful to know whether labour contracts are in fact as rigid as Gray and the subsequent literature have construed them. In particular, the assumption of a fixed wage seems counterfactual; suppose instead that the wage depended on how many hours the worker was asked to supply. This seems to be the essence of overtime pay, and gives labour markets a way of responding to real shocks that improves upon the rigid wage contract.

Overall, the current empirical study is a valuable first step towards understanding the effect of changes in Canada's monetary policy. This paper demonstrates that regardless of the choice of proxy variables for real and nominal uncertainty, contract duration in Canada is negatively related to nominal uncertainty, and unrelated to real uncertainty. However, to establish that a less volatile monetary policy has indeed increased contract duration by reducing monetary uncertainty requires at the very least an explicit accounting for the prevalence of zero-indexing contracts. If, in addition, we believe that the wage elasticity of labour demand varies over time, possibly in response to the variability of these shocks, then a natural second step would be to estimate a dynamic model of equilibrium in the labour market, in which other changes experienced by the Canadian labour market are fully accounted for.

## Reference

- Gray, J.A. 1978. "On Indexation and Contract Length." *Journal of Political Economy* 86 (1): 1–18.