Non-Linearities in the Output-Inflation Relationship

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Introduction

Since the beginning of the 1990s, several countries have experienced low inflation; at the same time, several central banks have explicitly committed themselves to low inflation targets. This recent experience has raised questions concerning the output losses associated with disinflation and also the issue of the economic adjustment in an environment characterized by low inflation. The standard approach in the literature is to use a linear Phillips curve to assess the loss of output throughout the disinflation period. In this approach, the short-run trade-off between output and inflation is assumed to be constant over time, and the change in inflation relative to expected inflation to be simply proportional to the deviation of output from potential—the output gap. In particular, the size of the effect of the output gap on inflation relative to expectation is assumed not to vary with the initial level of inflation, the sign of the output gap, or other economic indicators. However, a strand of the theoretical literature allows for an output-inflation trade-off that depends on the initial state of the economy, and recently some studies have found empirical evidence for a variety of different possible non-linearities in the Phillips curve. From a policy perspective, the source of any non-linearity in the Phillips curve is important, since the different theoretical motivations for non-linearity have quite different policy implications.

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In this study we analyse the dynamic process of inflation, and examine whether the variation in the relationship between inflation and output over time (that is, the time variation) can be detected. We also attempt to identify the source of the time variation in the relationship. Our tests are designed to identify, within the short-run Phillips curve framework, those variables that affect the output-inflation trade-off. In contrast to most other studies, our methodology allows us to test different types of non-linearity at the same time. The study concentrates on Canada and the United States. In the first section of the paper, we survey the models that imply a time-varying sacrifice ratio, and we comment on the implications that these models have for monetary policy. In the second section, we perform a simple test of non-linearity in the output-inflation relationship. The third section provides a more detailed study of the time variation and links it to variables that can be associated with different models of non-linear behaviour. The final section provides concluding remarks.

1 A Literature Survey

The shape of the short-run Phillips curve is a long-standing issue in macroeconomics that has recently attracted renewed attention. A common assumption is that expectations can be modelled as a simple weighted average of past inflation rates, which gives rise to the accelerationist version of the Phillips curve.[1] It is now recognized that expectations formation may be sensitive to the monetary policy regime, among other things, so that constant parameter weights on past inflation may be inappropriate. This has led to the search for proxies for expectations, such as survey measures of inflation expectations, and to the separation of expectations dynamics from the intrinsic dynamics due to price inertia.

The functional form traditionally favoured by researchers is the linear Phillips curve. To some extent, the linear model has always been viewed as a simplification, made necessary because there are not enough data to estimate any non-linearities reliably. More recently, with the availability of larger data sets, the possibility of non-linearity in the Phillips curve has come under greater scrutiny. One strand of this empirical literature has focussed on the evidence for a convex Phillips curve, which gives rise to an asymmetry between the effects on inflation of excess demand and excess supply. Another set of studies relates the sacrifice ratio to the level of inflation or to its volatility.

1. For more details on an accelerationist Phillips curve, see Cozier and Wilkinson (1990), Dupasquier and Girouard (1992), and Duguay (1994).

1.1 Theoretical models of state dependence in the output-inflation trade-off

Several theoretical models of price-setting behaviour predict that the slope of the Phillips curve will itself be a function of macroeconomic conditions. For policymakers, this implies that it is not only the existence of non-linearity that is important, but also its source. The policy implications of a sacrifice ratio that is a function of the level of inflation are quite different from those of a ratio that is a function of the sign of the output gap. In this section, we briefly describe five different approaches that may give rise to an asymmetric relationship between output and inflation or to time variation in an otherwise linear relationship. (The linear relationship and the five different models are illustrated in graph form in Appendix 1.)

The first model, the *capacity constraint model*, supposes that some firms find it difficult to increase their capacity to produce in the short run. Thus, when an economy experiences strong aggregate demand, the impact on inflation will be greater when more firms are restricted in their ability to raise output in the short run. This model implies that inflation becomes increasingly sensitive to excess demand. In this particular framework, the short-run Phillips curve has a convex shape as shown in the second graph in Appendix 1. This is consistent with the early empirical work on the Phillips curve, including Phillips (1958), which assumed that the relationship was non-linear, and predicted that excess demand would increase inflation more than excess supply would reduce it. A simplified version of the model allows for a higher sacrifice ratio in periods of excess supply than in periods of excess demand.

In the capacity constraint model, the costs of a disinflation are independent of the initial level of inflation, as in the simple linear model. However, the capacity constraint model has important implications for the conduct of monetary policy aimed at controlling inflation.[2] In particular, a direct implication of a convex Phillips curve is that the more stable the output is, the higher the level of output will be in the economy, on average. Given the lags in the effects of monetary policy, this provides an incentive for pre-emptive monetary responses to inflationary pressure. This conclusion is generally offered on the basis of a comparison of policies for controlling inflation under linear and non-linear Phillips curves (see, for example, Laxton, Meredith, and Rose 1995.) The thinking behind this result is that a pre-emptive tightening in response to an inflationary pressure helps to prevent the economy from moving too far up the Phillips curve where

2. See Macklem (1997) for a full discussion of this model and its implications for monetary policy.

inflation begins to rise more rapidly, thereby avoiding the need for a larger negative output gap in the future to reverse this large rise in inflation.

The second model, the *misperception* or *signal extraction model*, was proposed by Lucas (1973). In this model, a relationship between output and inflation arises because firms are unable to distinguish precisely between aggregate and relative price shocks, since these shocks are not directly observable but must be inferred from the behaviour of individual prices. Output decisions are based on estimated relative price movements. What matters then, in determining the measured statistical relationship between output and inflation, is the amount of noise in the price signal. If aggregate prices are extremely volatile, then little can be inferred about relative price shocks, and most of the variation in individual prices is attributed to aggregate price shocks. Under these conditions, output will respond less to aggregate demand shocks when the volatility of prices is high than when the volatility is low. In this case, the short-run Phillips curve may be linear, but its slope will vary with the volatility of inflation. This type of relationship is shown in the third graph in Appendix 1.

A third model, the *costly adjustment model*, implies a relationship between output and inflation that varies with the level of inflation. For example, Ball, Mankiw, and Romer (1988) show that, in the presence of menu costs, not all firms will change their prices in response to a particular demand shock. However, the more firms that do decide to change their prices, the more responsive the aggregate price level will be to demand shocks. As the level of inflation rises, and as firms adjust the timing and size of their price changes, aggregate demand shocks will have less effect on output and more effect on the price level. Ball and Mankiw (1994) discuss another implication of menu costs. In the presence of trend inflation, prices should be more flexible upwards than downwards because some firms are able to obtain relative price declines from trend inflation without changing their own prices and incurring real costs. The model could thus imply a convex Phillips curve that becomes linear as inflation falls.

Another example relates to the duration of contracts. The process of negotiating wages and benefits between firms and workers is costly. It thus could be optimal, in an environment characterized by low inflation, to negotiate longer contracts on average in order to lower the costs faced by the firms. In this case, when a shock occurs, even though prices and wages are fully flexible in the long run, the existence of the contracts makes it difficult to adjust quickly. The implication from the costly adjustment model is that the Phillips curve is steeper—and possibly convex—at higher rates of inflation than at lower rates. This relationship can be represented as in the fourth graph of Appendix 1, where it is the average level of inflation that controls the slope.

In the costly adjustment model, the impact of the output gap on the deviation of actual from expected inflation is a function of the average level of inflation. In this case, monetary authorities may find it much more difficult to achieve lower inflation when current inflation is low than when it is relatively high. This means that the benefits of lower inflation have to be greater in order to justify a disinflation when inflation is already low. By the same token, it also implies that inflation control may be easier at low rates of inflation, since the adjustment to excess demand shocks is slower, giving the monetary authority more time to react. A monetary policy that reacts more slowly allows more information to be gathered about the state of excess demand.

Another model that can explain a potential asymmetric relationship between output and inflation is the *downward nominal wage rigidity model*. Stiglitz (1986) and Fisher (1989) give excellent overviews of the type of theoretical models that can generate wage rigidity. In this model, workers are more reluctant to accept a decrease in their nominal wages than a decrease in their real wages because of money illusion, or institutional or behavioural factors. Therefore, in an environment characterized by a low rate of inflation, relative wages could adjust at a slow pace, leading to allocative inefficiencies. Provided that full adjustment to individual demand shocks eventually occurs, this model has two implications for the shape of the short-run Phillips curve. First, it implies that the effects of nominal wage floors are more likely to be important at low rates of inflation, since the higher the average level of inflation, the less likely it is that a nominal wage cut will be required for a given decline in real wages. Second, if the rigidity applies only to downward wage adjustment, then at low rates of inflation excess supply might have less effect on inflation than excess demand, leading to an asymmetry with respect to the output gap. Recently Akerlof, Dickens, and Perry (1996) proposed a model in which downward nominal wage rigidity also leads to a long-run trade-off between inflation and output.[3] In our empirical section we limit our focus to models of short-run trade-offs. The fifth graph in Appendix 1 shows the output-inflation relationship for those models that specify a short-run trade-off.

A final model, the *monopolistically competitive model*, refers to the strategic pricing behaviour of firms in monopolistically competitive or oligopolistic markets (see Stiglitz 1984, for example). In such an environment, producers might be inclined to lower prices quickly to avoid being undercut by rivals. However, they might be reluctant to raise prices, even in the face of generally rising prices, hoping to keep out potential new

3. Fortin and Prud'homme (1984) also discuss the issue of nominal wage rigidity and the hypothesis of a non-linear Phillips curve.

competitors. This last type of model is consistent with a concave short-run Phillips curve as shown in graph 6 of Appendix 1.

1.2 Empirical results

The empirical evidence regarding the nature of the short-run trade-off between output and inflation has been pursued from a number of different directions. One strand of the literature looks for evidence that nominal demand shocks have different effects on output in different countries, and links the differences across countries to variables suggested by a particular model. Another branch of the literature looks for evidence of a non-linear Phillips curve using either single-country or multicountry data. However, most studies do not attempt to test for the type of non-linearity in a framework that considers more than one possibility at a time.

Laxton, Rose, and Tetlow (1993) and Clark, Laxton, and Rose (1996), among others, look at different functional forms to capture the effect of asymmetry in the capacity constraint model. A simplification that captures the essentials of the model results in the following estimation equation:

$$\pi_t = A(L)\pi_{t-1} + B(L)\pi^e_{t+1} + \beta_1 GAP_t + \beta_2 GAPPOS_t + \varepsilon_t\,, \qquad (1)$$

where π is the inflation rate, π^e is the expected inflation rate, *GAP* is the output gap, and ε is a random shock.

The variable *GAPPOS* takes the value of the output gap if the economy is in excess demand, and is equal to zero when the economy is in excess supply. Within this framework, Laxton, Rose, and Tetlow find significant asymmetry in the Canadian output-inflation trade-off over the period 1975 to 1991.[4] Clark, Laxton, and Rose find the same type of result for the United States over the period 1964Q4 to 1990Q4.[5]

Fillion and Léonard (1997) re-examine the shape of the Phillips curve in Canada over the period 1968Q4 to 1994Q4. In addition to the existence of an asymmetric relationship between the output gap and inflation, they argue that the process by which inflation expectations are formed has changed over time. Like Laxton, Rose, and Tetlow and Clark, Laxton, and Rose, they

4. Laxton, Rose, and Tetlow use annual data, while Clark, Laxton, and Rose use quarterly data.

5. Turner (1995) studies the process of inflation for the G-7 countries; his results support the conclusion that the output-inflation trade-off is asymmetric in the United States, Japan, and Canada, while not rejecting a linear model for the other four G-7 countries. Debelle and Laxton (1996) present further evidence for Canada.

find evidence of an asymmetry related to the capacity constraint model in Canadian data.[6]

Lucas (1973) used data on nominal and real gross domestic product (GDP) and on inflation variability for a number of countries to find evidence of a relationship between the volatility of prices and the effect of nominal demand shocks on real output. Ball, Mankiw, and Romer (1988) (hereafter BMR), also using international data, showed that the output-inflation trade-off, over the sample period 1948-86, is affected by the average rate of inflation. They also found that accounting for the inflation-level effect reduces the significance of the measures of volatility used by Lucas. In countries with low average inflation, the short-run Phillips curve is relatively flat; fluctuations in nominal aggregate demand have large effects on output. In countries with high average inflation, the Phillips curve is steep; fluctuations in demand are quickly reflected in the price level. Yates and Chapple (1996) examine the robustness of the BMR results using a larger time series of data from each country. They also examine the issue of whether the speed of disinflation matters for the size of the sacrifice ratio. They find that the BMR results are quite robust, even though numerically the effect they measure is smaller. Their own results, though weak, also suggest that fast disinflations are not more costly, in terms of output losses, than slow disinflations. However, Koelln, Rush, and Waldo (1996) dispute the methodology used by BMR to measure the effects of aggregate demand shocks. They argue that when one allows for different government spending and monetary policy multipliers, the evidence disappears for an inflation-level effect consistent with the costly adjustment model.

Kiley (1996) studies the movements of inflation and output in 43 countries over the BMR sample period and also a sample including recent years. His results reveal a relationship between high inflation and low output persistence that meshes nicely with the work of BMR and Lucas. Kiley also shows that the small real effects of nominal shocks, as well as their low persistence, when inflation is high, follow directly from lower price stickiness in high-inflation environments, a conclusion that reinforces the results presented by BMR.[7] Amano and Macklem (1997) apply the menu-cost model of price adjustment, and find some support for the predictions of this model with Canadian data.

Coulton (1993) and Bean (1993) look individually at models that specify the non-linearity of the Phillips curve in terms of the level of inflation and the output gap. Coulton finds that, in the United Kingdom, the

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6. Fillion and Léonard caution that their estimates of the size of the asymmetry may be imprecise.

7. Loungani, Razin, and Yuen (1997) reach a similar conclusion.

coefficient on the output gap is a function of the level of inflation. Bean finds further evidence for this relationship over a historical sample going back to the 1850s.

By contrast, Eisner (1997) presents evidence from U.S. data that the Phillips curve is concave—that is, flatter when the unemployment rate is below the conventional non-accelerating-inflation rate of unemployment (NAIRU) and steeper when the unemployment rate is above the conventional NAIRU. Eisner challenges the conventional model of the natural rate, arguing that during the recent episode, even though unemployment was below the NAIRU, there was no acceleration of inflation. Stiglitz (1997) also reports results carried out by the U.S. Council of Economic Advisers suggesting that the shape of the Phillips curve is concave.

Gordon (1996) provides new estimates of linear Phillips curves using a measure of a time-varying NAIRU, and also tests for some forms of non-linearity. His evidence conflicts with that of Eisner on concavity in the short-run Phillips curve. Gordon concludes his work by mentioning that "the short-run Phillips curve [for the United States] is resolutely linear, at least within the range of inflation and unemployment values observed over the 1955-96 period" (Gordon 1996, 28).[8]

One of the few papers to tackle the problem from an encompassing framework is Evans (1992). Using a structural vector autoregression (VAR) approach with time-varying parameters and GARCH residual covariances,[9] he studies the trade-off of output and inflation in the United States over the period 1953 to 1991. He specifies a model in which it is possible to link the non-linearity to the volatility of inflation and the level of inflation at the same time. He finds that, before 1974, the output-inflation trade-off varied independently of both average inflation and the variance of inflation, while after 1974 the effect on real output of nominal shocks was inversely related to the level of inflation.

One of the difficulties with the Phillips curve framework is that of identification of the different parameters. To test whether non-linearity is at play, ideally one ought to identify the effects of non-linearity separately with respect to the level of inflation, its volatility, and the state of excess demand, but this may not be possible. Some studies have found that the volatility and

8. Lown and Rich (1997) estimate a linear Phillips curve for the consumer price index (CPI) in the United States and conclude that the recent behaviour of inflation and its determinants has not been unusual. King and Watson (1994), however, find that there is clear evidence of a structural change in the behaviour of post-war U.S. inflation and unemployment that takes place at the beginning of the 1970s.

9. GARCH is generalized autoregressive conditional heteroscedasticity.

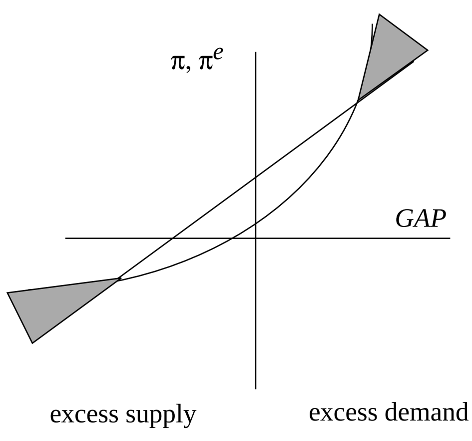
the level of inflation may be strongly positively correlated, making it difficult to distinguish between the misperception model and the costly adjustment model. One might also expect that it would be difficult to distinguish between the costly adjustment model and the capacity constraint model since high inflation is generally associated with periods of large excess demand. Clark, Laxton, and Rose (1995) point out a further difficulty in identifying asymmetry that is associated with the sign of the output gap. If excess demand causes larger movements in unanticipated inflation than does excess supply, then monetary policy aimed at stabilizing the level of inflation must generate a negative output gap on average. This means that traditional measures of the gap will be biased since they generally assume a gap that historically is zero on average. Clark, Laxton, and Rose also show that a failure to take this into account can lead to conclusions that are biased against this type of non-linearity.

2 A Simple Test of Asymmetry

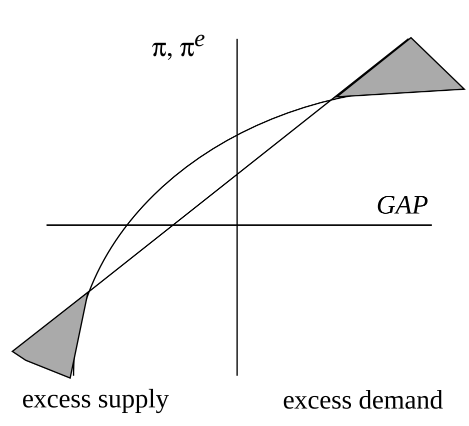
In this section of the empirical work, we perform simple experiments to detect the presence of asymmetries in the output-inflation trade-off. The idea behind this test is to explore the ability of a simple linear model to forecast inflation by testing for systematic forecast errors.

The two accompanying figures show a convex and a concave Phillips curve. By using a linear model to generate inflation forecasts when inflation is generated by a non-linear model, one should find systematic inflation forecast errors at extreme values of the output gap. If the extreme values of the output gap tend to correspond to underpredictions of inflation, it implies that the true model is convex. However, if the correlations between the extreme values of the output gap and the forecast errors of inflation are negative, the inflation rate will be systematically overpredicted, corresponding to the shaded areas of the concave model.

The convex model



The concave model



The test is performed within a three-variable structural VAR framework (SVAR), which defines the first difference of inflation, of output, and of the real interest rate as a stochastic process responding to three types of structural shocks.[10] SVARs have been used previously to identify potential output and output gaps based on long-run restrictions.[11] They have been used extensively in the identification of monetary policy innovations and have also been used to test for asymmetries in the transmission mechanism of monetary policy. We use the SVAR to identify the unanticipated components of changes in output and inflation. These will be composed of different combinations of the underlying structural shocks. The shocks that have a permanent effect on output are called supply shocks, while monetary and non-monetary demand shocks are related to the shocks with temporary effects on output. Within this framework, we estimate an output gap that is defined as the cumulative effect of temporary structural shocks on output. Appendix 2 provides a more detailed description of the SVAR approach to measuring the output gap. Appendix 3 describes the data in greater detail.

The reduced-form equation estimating the dynamics of prices is used to compute the forecast errors of inflation. The errors are calculated using the two-step-ahead forecasts.[12] In Table 1, we present a parametric and a non-parametric test to detect the existence of asymmetry. Since we are unable to identify the cutoff value of the output gap that bounds the shaded areas in the diagrams, we use the squared value of the output gap. This minimizes the influence of small gap values in the calculated correlations. The correlations and the rank correlations for Canada are generally positive, which is consistent with a convex Phillips curve, but are never statistically different from zero, so there is no compelling evidence of asymmetry in the relationship between output and inflation. In the case of the United States, the evidence of a convex Phillips curve is slightly stronger. The correlations are all positive, and now two of the rank correlations are significantly different from zero at the 95 per cent confidence level.

This first test was designed to distinguish between two types of asymmetry with respect to the output gap. The results provide somewhat

10. DeSerres and Guay (1995) show that it is important to include a sufficient number of lags in the estimated autoregression vectors. We use the likelihood-ratio test to establish at eight the optimal number of lags for the estimated autoregression vectors. Lagrange multiplier tests applied to the residuals of the estimated equations show that these are not autocorrelated.

11. Dupasquier, Guay, and St-Amant (1997) compare alternative methodologies and discuss the advantages of the SVAR approach.

12. We performed the same type of tests with one-step-ahead and four-step-ahead forecasts, and overall the results did not change.

Table 1

Correlations: Inflation Forecast Errors and the Squared Output Gap, Canada and the United States

	1963-95		1973-95	
Type of correlations	**Total CPI[a]**	**CPIXFE**	**Total CPI**	**CPIXFE**
Canada				
Correlation	0.06 (−0.13; 0.12)	0.06 (−0.13; 0.13)	0.04 (−0.19; 0.19)	0.15 (−0.17; 0.18)
Rank correlation	0.08 (−0.13; 0.13)	−0.02 (−0.13; 0.14)	0.02 (−0.18; 0.17)	0.11 (−0.17; 0.18)
United States				
Correlation	0.14 (−0.18; 0.16)	0.10 (−0.17; 0.15)	0.16 (−0.22; 0.22)	0.20 (−0.21; 0.21)
Rank correlation	0.25 (−0.14; 0.14)	0.12 (−0.15; 0.15)	0.18 (−0.18; 0.19)	0.23 (−0.18; 0.18)

Notes: The 95 per cent confidence intervals are shown in parentheses, and are generated using Monte Carlo simulations in RATS. Shading indicates correlations that are significantly different from zero at the 95 per cent confidence level. CPI refers to consumer price index, and CPIXFE is CPI less food and energy.

a. For Canada: total CPI excluding GST, QST, and tobacco tax.

weak evidence for a convexity in the relationship between inflation and output. The estimated structural VAR was specified to be consistent with the non-rejection of a unit root in inflation as reported in Appendix 3. However, this specification assumes that inflation expectations are formed in the same way in periods of high, unstable inflation and in periods of low, stable inflation. The results of Fillion and Léonard suggest that this may be an important factor. In the next section we present an empirical model that can distinguish between time variation in the output-inflation relationship linked to the capacity constraint model, the Lucas misperception model, and the costly adjustment model.

3 State-Space Framework

In this section we present evidence for different types of asymmetry from estimates of reduced-form Phillips curves. This framework has been used extensively by researchers to quantify the effects of asymmetry or non-linearity in terms that are useful for policymakers. Typically, this has involved allowing the parameter measuring the output-inflation trade-off to vary with the size or sign of the output gap or with the level of inflation. Our analysis is similar in this respect, but in our estimated models we also test jointly for different types of asymmetry or non-linearity. We treat the output-inflation trade-off as an unobserved state variable that can be forecast using different types of conditioning information. Because the state variable is unobserved, we also quantify the uncertainty surrounding estimates of the trade-off parameter and its variation over time.

Our estimation framework consists of three parts. The first part is the basic Phillips curve equation, which we treat as the observation equation of a state-space model.

$$\pi_t = a \bullet \pi_t^e + (1 - a) \bullet \pi_{t-1} + \beta_t \bullet GAP_t + \varepsilon_t . \quad (2)$$

The second part specifies the form of the transition equation for the trade-off parameter β_t. The transition equation specifies the dynamics of the state variable and the set of conditioning information that should be useful in predicting its value. The general form of the transition equation is

$$\beta_t = \alpha + \rho \bullet \beta_{t-1} + \gamma \bullet X_{t-1} + \mu_t , \quad (3)$$

where X_{t-1} represents the conditioning information set. The inclusion of the error term μ_t means that we allow parameter variation that cannot be explained by the elements of X_{t-1}. It may be that none of the theories we examine explain all the variation in β_t. It may also be that some of the estimated movements in β_t are the result of misspecification of the measurement equation.

The third part of the estimated model specifies the variables that enter the information set and their relationship to the state variable. The variables entering the information set depend on the model that generates the non-linearity or asymmetry. The capacity constraint model would imply that the sign or magnitude of the output gap should be positively related to the level of the trade-off parameter, while the costly adjustment model would imply that some measure of the average level of inflation should be a useful predictor. In the misperception model, a measure of the conditional volatility of inflation would enter the information set.

3.1 Data and information sets

The measurement equation requires that we specify proxies for inflation expectations and the output gap. Both variables are unobservable and hence our results will be dependent upon the effect of errors introduced by our choice of proxies. One way of attempting to control for these effects is to determine the robustness of the results to different proxies. For this purpose we use two measures of inflation expectations and the output gap for the United States. One measure of inflation expectations, which has been used elsewhere in this context, is derived from the Michigan Survey data.[13]

13. The Michigan Survey is conducted by the Survey Research Center at the University of Michigan and is designed to be representative of households in the United States.

This measure uses a weighted average of the quarterly forecasts from the survey data. Another measure is generated from a three-state Markov switching model (MSM) applied to the inflation data.[14] Each of the three states is described by a different long-run mean and autoregressive process, so that inflation expectations are generated differently in each state. We use two measures of the output gap, one of which uses a Hodrick-Prescott (HP) filter to separate output into a trend and a cyclical component, and another that estimates potential output from a structural VAR of output, inflation, and real interest rates. For Canada, we do not have a survey measure of inflation expectations on a quarterly basis, so we use just the measure generated from a three-state MSM. One of the gap measures for Canada is derived from an extended multivariate filter (EMVF)—the filter used to generate potential output in the Bank of Canada's Quarterly Projection Model (QPM) (see Butler 1996). The other comes from the same type of structural VAR used for the United States. The U.S. gap measures are quite similar except for the period between 1985 and 1987. However, the output gaps for Canada are quite different after 1980. There is greater volatility in the EMVF gap measure than in the SVAR measure, and there is more excess supply at the end of the sample in 1994. The gap measures are graphed in Figures 1 and 2, and the various inflation expectations in Figures 3 and 4.

The Markov switching model provides three useful outputs for our purposes. First, the one-period-ahead predictions of inflation can be used as proxies for expected inflation. These predictions are not based solely on the current behaviour of inflation. They also take into account the changing nature of the inflation process over time. In this sense they have something of a forward-looking element because they adjust for the possibility of future changes in process as well. This introduces an additional degree of uncertainty in inflation forecasts because the current inflation state is never known with certainty.

Another output of the MSM is a set of probabilities for each state over time. The probability assigned to a particular state indicates the extent to which the current behaviour of inflation fits the description of inflation in that state. The estimated model allows for the states to be identified by different long-run means, different autoregressive dynamics, and different volatility of the shocks directly affecting the level of inflation. However, in the estimated model, the dynamics in state 3 are restricted to impose a unit root. The three-state MSM generally divides the data according to the mean rate of inflation, although exceptions occur when short periods of extreme volatility interrupt the sample. Thus, the MSM endogenously picks out

14. Laxton, Ricketts, and Rose (1994); Ricketts and Rose (1995); and Ricketts (1996) have used the MSM structure to characterize inflation and proxy expected inflation.

Figure 1

U.S. Output Gaps

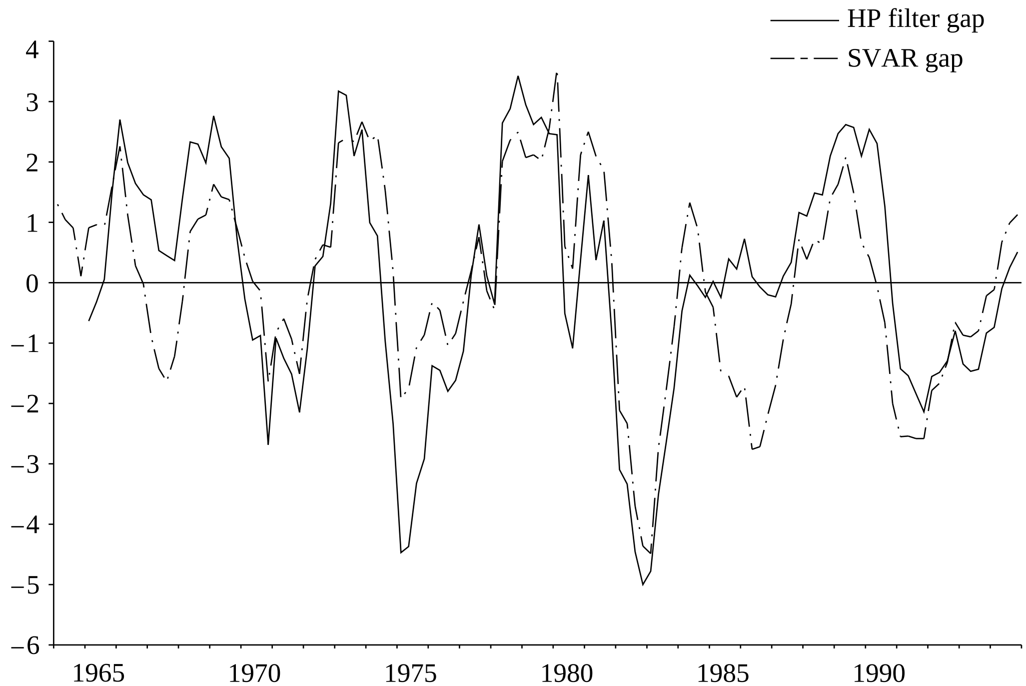


Figure 2

Canadian Output Gaps

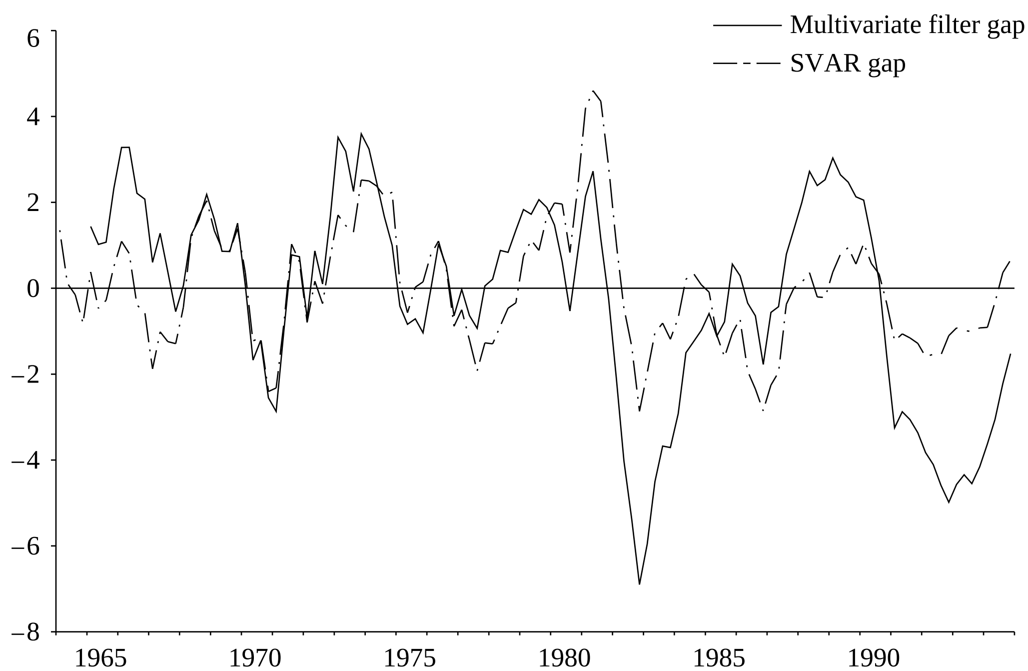


Figure 3

U.S. Inflation and Inflation Expectations

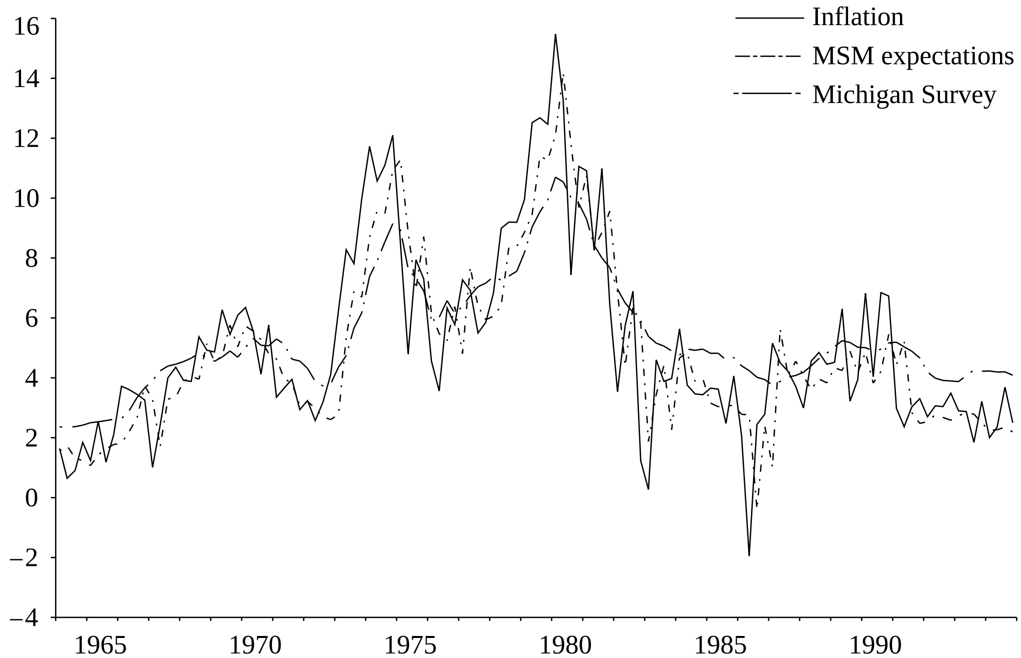
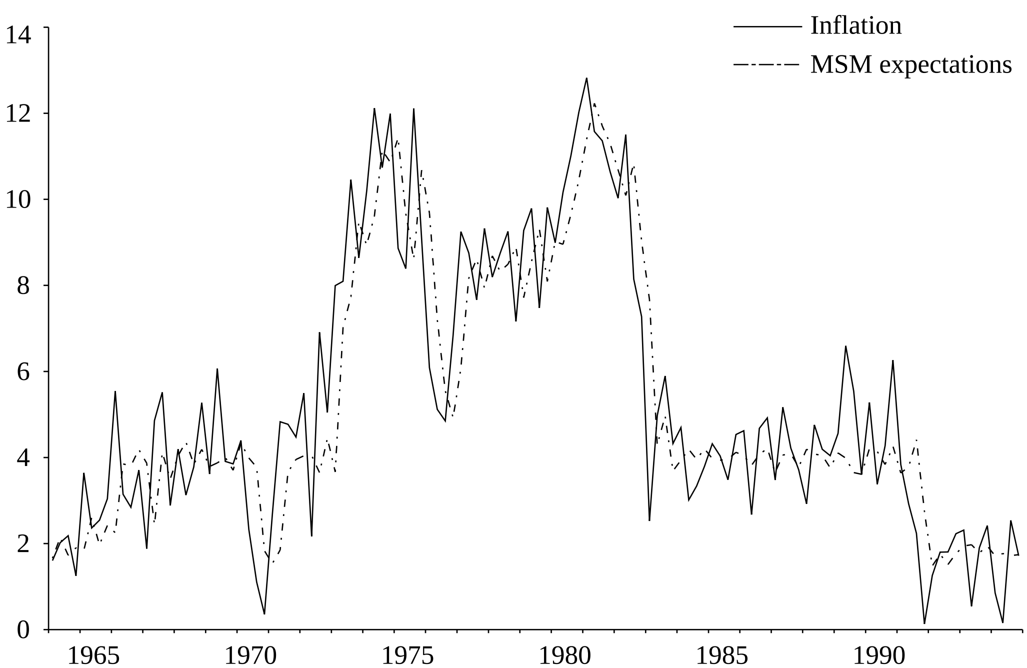


Figure 4

Canadian Inflation and Inflation Expectations



periods of low, medium, and high inflation, and the ex post probabilities serve as indicator variables for the three states. The third useful output from the MSM is a measure of the conditional volatility of inflation. Because the volatility of shocks to inflation is allowed to differ across states, and because there is always some uncertainty about the inflation state, the conditional volatility measure will vary within and across states. The state probabilities and conditional volatility measures are pictured in Figures 5 and 6. Whereas the state probabilities estimated for Canada seem to be closely associated with periods of low, moderate, and high inflation, this is not clearly the case for the U.S. state probabilities. There, the probability of state 3, indicated by the dashed line, seems associated more with rapidly rising and falling inflation than with any particular level of inflation.

We include four variables in the conditioning information set for the transition equation. These are a dummy variable that is equal to unity when the output gap is positive; the ex post probabilities for states 2 and 3—that is, $\Pr(S_{t-1} = 2)$ and $\Pr(S_{t-1} = 3)$ from the MSMs—interpreted as indicators of moderate-inflation and high-inflation periods, respectively; and the conditional volatilities, $V_{t-1}(\pi_t)$, calculated from the MSMs. In keeping with the Clark, Laxton, and Rose (1996) analysis, a negative average output gap is estimated when testing for asymmetry associated with the sign of the gap. To ensure its identification, this additional parameter adjusts the output gap in both the measurement equation and the transition equation. Since the costly adjustment model predicts that the trade-off parameter will be different according to the mean rate of inflation, the ex post state-probabilities can proxy for the periods when the long-run means appeared to be different. In this case we are not attempting to measure a continuous relationship between the trade-off parameter and the average level of inflation, but rather to separate the data into periods in which the trade-off parameter would be measurably different. The conditional volatilities from the MSMs reflect uncertainty about the size of shocks to inflation in each state as well as uncertainty about the state itself. If both these factors influence actual inflation uncertainty for individuals, then this measure can proxy the inflation uncertainty predicted by the misperception model to have an effect on the output-inflation trade-off.

3.2 Estimation results

The state-space model parameters are estimated using maximum likelihood (ML). The unobserved state variable is estimated using a Kalman filter that was initialized with values obtained from a linear ordinary least squares (OLS) regression of the observation equation. The data are quarterly from 1964 to 1994. Appendix 4 provides details of the estimation and filtering procedure.

Figure 5a

Ex Post State Probabilities for the United States

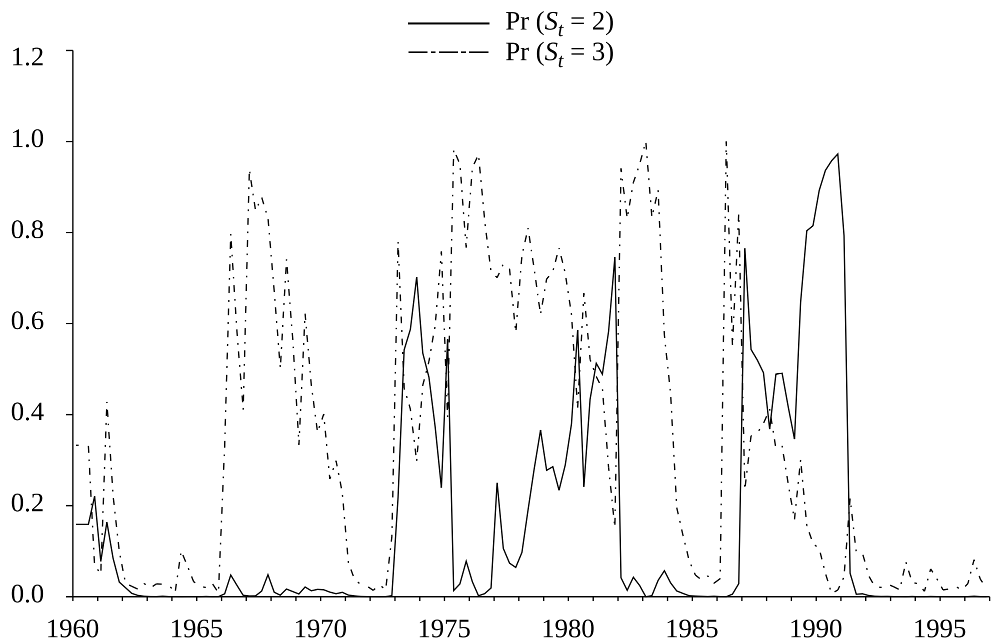


Figure 5b

Conditional Variance of U.S. Inflation

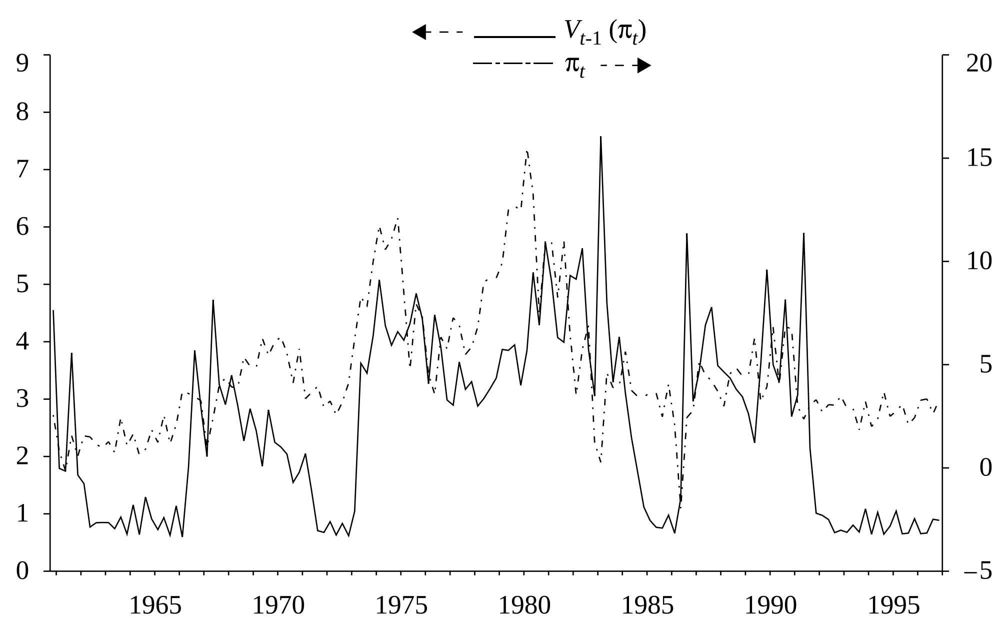


Figure 6a

Ex Post State Probabilities for Canada

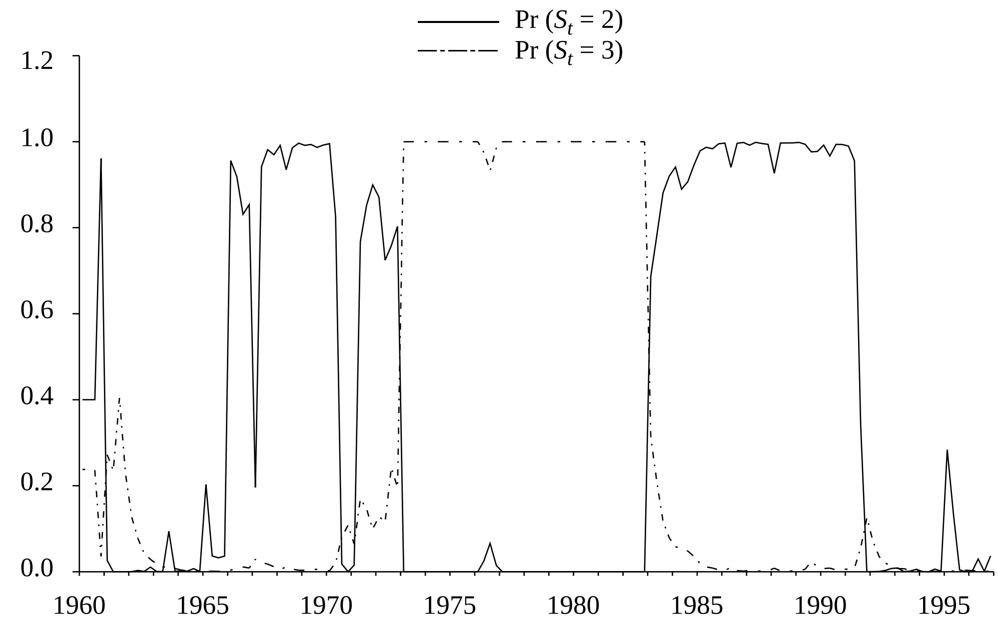
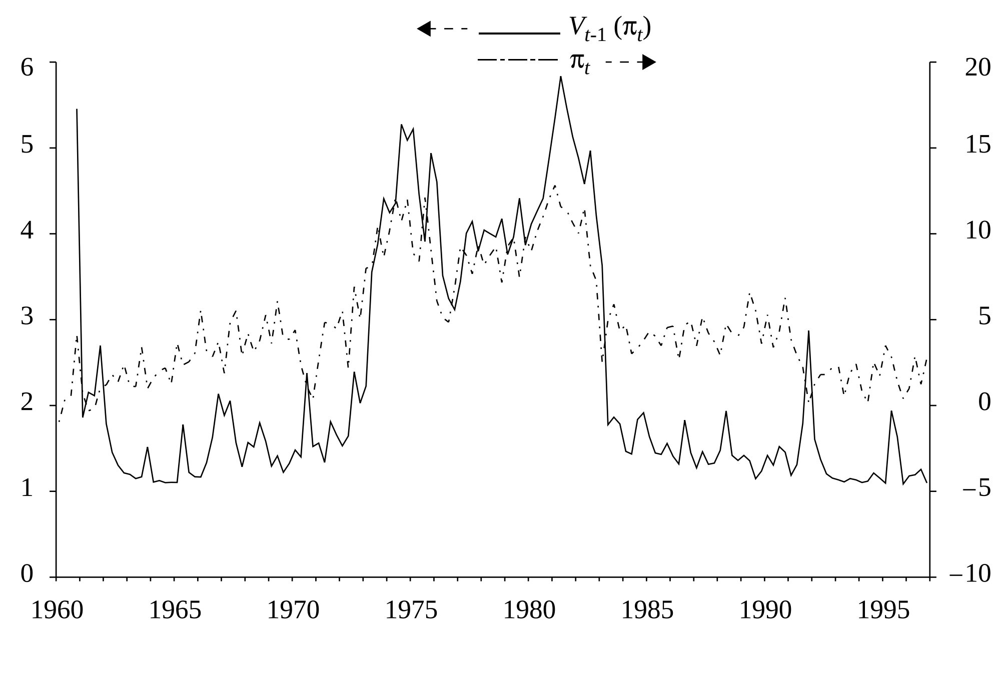


Figure 6b

Conditional Variance of Canadian Inflation



3.2.1 Results for the United States

The estimation results for the U.S. data are presented in Tables 2 to 5. For Tables 2 and 3 the output gap is derived using an HP filter. Tables 4 and 5 use an output gap measure generated from a three-variable SVAR. Tables 2 and 4 use expectations data from the Michigan Survey, while Tables 3 and 5 report results for expectations generated from the MSM applied to inflation data. The main effect of changing the expectations variable was the change in the estimated coefficient on expected inflation in the measurement equation. This coefficient was estimated to be very close to 1.0 when MSM expectations were used. This change may be due to the large backward-looking element in these expectations, although the weights on past inflation do change over time. Thus, the dynamics associated with slow price adjustment, which are meant to be captured by lagged inflation, may be captured in the expectations variable.

The variables associated with the three theoretical models were tested separately in the transition equations, reported in columns 1 to 3 of the tables, and then jointly, reported in column 4 of the tables. Only the measure of the conditional volatility of inflation, $V_{t\text{-}1}(\pi_t)$, was found to be insignificant when entered separately in the transition equation. The misperception model has little support in the U.S. data.

The results do appear sensitive to the output gap measure, however. When the gap is measured using an HP filter, the non-linearity is more closely associated with the probability assigned to state 2—the periods of moderate inflation. When the gap is generated from a three-variable SVAR, the positive gap measure is the most significant indicator of asymmetry. The fact that it is state 2 that appears to be significant in the transition equation, while state 3 is not, means that this evidence is not strictly in accord with the costly adjustment model. In fact, since state 3 is associated more with periods of high inflation, it is surprising not only that the estimated value of the coefficient associated with this state is for the most part insignificant, but also that its value is small and sometimes negative.

Another perspective on the variations in the output-inflation trade-off is presented in Figures 7 to 21. Here we show the predicted value of the trade-off parameter together with its 90 per cent confidence region based on the full sample information. The width of the confidence region varies over time, but the size of the variation is not large. This measure of the uncertainty about the value of the output-inflation trade-off does not include the uncertainty about the parameter estimates in the transition equation. These estimates are taken as given by the Kalman filter when calculating the conditional variance of the trade-off parameter. Taking into account the parameter uncertainty as well would increase the uncertainty about the value of the trade-off. The graphs also show that for some of the estimates the non-systematic variation at times includes negative values of the estimated trade-off parameter. These occur when there are large overpredictions of inflation associated with a positive gap, or large underpredictions associated with a negative gap, in the observation equation. This might be an indication of mismeasurement of the output gap or inflation expectations or of model misspecification, highlighting periods in which the forecasting power of the model is weakest.

Figures 7 to 9 show the output-inflation trade-off corresponding to columns 1, 2, and 3 of Table 2. Although there is substantial variation in the trade-off over time, in most cases high values of the trade-off are within the 90 per cent confidence region of those times when there is no systematic variation. As well, the confidence regions for the high values of the trade-off generally encompass the values when the trade-off is at its base level. This implies that the values of the trade-off during periods when it is systematically higher cannot be confidently distinguished as different from its base level. Therefore, even though in column 4 of Table 2, the parameter associated with state 2 is significant in predicting the value of the output-inflation trade-off, we cannot be confident that there is in fact systematic variation in the trade-off over time.

Systematic variation in the trade-off seems to be more evident in Figures 10 and 11, which correspond to columns 1 and 2 of Table 3. In this case, the difference between the high values of the trade-off and its base value is larger and in Figure 11 the uncertainty is smaller. As the last column in Table 3 shows, it is the probability of state 2 that is significant in predicting the variations in the trade-off. The changes in the trade-off parameter are also economically significant. From a base level of about 0.2 it rises to between 0.5 and 0.9, more than doubling the effect of the gap on inflation.

Figures 12 to 14 show the output-inflation trade-off corresponding to columns 1 to 3 of Table 4. Here the confidence regions allow us to rule out linearity only for the case of variation associated with the positive gap. This reinforces the significance of the parameter estimates in Table 4, which imply that the positive gap is the likely source of non-linearity in this case. This is the picture that also emerges from Figures 15 and 16, corresponding to columns 1 and 2 of Table 5. Again, the variation between the base level of the trade-off and its value during episodes of excess demand is substantial.

Table 2

Maximum-Likelihood Estimates of Measurement and Transition Equations: United States, Total CPI

Expectations from Michigan Survey; Potential Output – HP Filter Trend

Variables	Capacity constraint model	Costly adjustment model	Misperception model	Joint test
π_t^e	0.48 (0.10; 0.00)	0.47 (0.09; 0.00)	0.48 (0.10; 0.00)	0.43 (0.10; 0.00)
π_{t-1}	0.52	0.53	0.52	0.57
α (constant)	0.30 (0.13; 0.01)	0.23 (0.10; 0.01)	0.28 (0.14; 0.02)	0.33 (0.15; 0.01)
Pr $(S_{t-1} = 2)$	—	0.64 (0.37; 0.04)	—	0.89 (0.43; 0.02)
Pr $(S_{t-1} = 3)$	—	—	—	—
$V_{t-1}(\pi_t)$	—	—	0.05 (0.05; 0.16)	−0.04 (0.06; 0.27)
$(GAP_{t-1} + b)^+$	0.34 (0.26; 0.09)	—	—	0.25 (0.28; 0.18)
b	−0.78 (0.44; 0.04)	—	—	−0.86 (0.48; 0.04)
ρ (lagged state)	−0.05 (0.33; 0.45)	—	−0.19 (0.27; 0.23)	−0.22 (0.16; 0.09)
Standard error (μ_t)	0.32 (0.09; 0.00)	0.33 (0.11; 0.00)	0.33 (0.11; 0.00)	0.27 (0.10; 0.00)
Var(ε_t)	1.86 (0.40; 0.00)	1.86 (0.43; 0.00)	1.92 (0.45; 0.00)	1.84 (0.38; 0.00)
Mean likelihood	−1.81873	−1.81581	−1.83152	−1.79697
Mean(β_t)	0.40	0.35	0.36	0.37
Min(β_t)	−0.07	−0.10	−0.03	−0.13
max(β_t)	0.87	1.00	0.78	1.11

Notes: Heteroscedasticity-consistent standard errors and p-values in parentheses.

Measurement equation: $\pi_t = e \bullet \pi_t^e + (1 - e) \bullet \pi_{t-1} + \beta_t \bullet (GAP_t + b) + \varepsilon_t$.

Transition equation: $\beta_t = \alpha + \rho \bullet \beta_{t-1} + \gamma_1 \bullet \Pr(S_{t-1} = 2) + \gamma_2 \bullet \Pr(S_{t-1} = 3) + \gamma_3 \bullet V_{t-1}(\pi_t) + \gamma_4 \bullet 1_{(GAP_{t-1} + b > 0)} + \mu_t$.

Table 3

Maximum-Likelihood Estimates of Measurement and Transition Equations: United States, Total CPI

Expectations from Markov Switching Model; Potential Output – HP Filter Trend

Variables	Capacity constraint model	Costly adjustment model	Misperception model	Joint test
π_t^e	1.00 (0.19; 0.00)	1.04 (0.18; 0.00)	1.03 (0.17; 0.00)	1.00 (0.19; 0.00)
π_{t-1}	0.00	−0.04	−0.03	0.00
α (constant)	0.12 (0.10; 0.11)	0.22 (0.17; 0.10)	0.23 (0.18; 0.11)	0.07 (0.10; 0.22)
$\Pr(S_{t-1} = 2)$	—	1.05 (0.43; 0.01)	—	0.88 (0.30; 0.00)
$\Pr(S_{t-1} = 3)$	—	0.05 (0.05; 0.42)	—	0.00 (0.12; 0.49)
$V_{t-1}(\pi_t)$	—	—	0.05 (0.06; 0.17)	—
$(GAP_{t-1} + b)^+$	0.58 (0.22; 0.00)	—	—	0.25 (0.26; 0.16)
b	−0.80 (0.44; 0.04)	—	—	−0.45 (0.60; 0.22)
ρ (lagged state)	−0.01 (0.07; 0.47)	−0.26 (0.22; 0.12)	−0.16 (0.42; 0.35)	—
Standard error (μ_t)	0.31 (0.07; 0.00)	0.17 (0.19; 0.19)	0.26 (0.13; 0.02)	0.22 (0.12; 0.03)
$\mathrm{Var}(\varepsilon_t)$	1.58 (0.33; 0.00)	1.81 (0.41; 0.00)	1.83 (0.45; 0.00)	1.73 (0.37; 0.00)
Mean likelihood	−1.77838	−1.74438	−1.7819	−1.74083
$\mathrm{mean}(\beta_t)$	0.31	0.34	0.33	0.33
$\mathrm{Min}(\beta_t)$	−0.14	0.03	0.14	–0.07
$\mathrm{Max}(\beta_t)$	0.89	1.01	0.64	1.30

Notes: Heteroscedasticity-consistent standard errors and p-values in parentheses.
Measurement equation: $\pi_t = e \bullet \pi_t^e + (1-e) \bullet \pi_{t-1} + \beta_t \bullet (GAP_t + b) + \varepsilon_t$.
Transition equation: $\beta_t = \alpha + \rho \bullet \beta_{t-1} + \gamma_1 \bullet \Pr(S_{t-1} = 2) + \gamma_2 \bullet \Pr(S_{t-1} = 3) + \gamma_3 \bullet V_{t-1}(\pi_t) + \gamma_4 \bullet 1_{(GAP_{t-1} + b > 0)} + \mu_t$.

Table 4

Maximum-Likelihood Estimates of Measurement and Transition Equations: United States, Total CPI

Expectations from Michigan Survey; Potential Output – SVAR

Variables	Capacity constraint model	Costly adjustment model	Misperception model	Joint test
π_t^e	0.70 (0.10; 0.00)	0.62 (0.10; 0.00)	0.64 (0.10; 0.00)	0.68 (0.11; 0.00)
π_{t-1}	0.30	0.38	0.36	0.32
α (constant)	0.29 (0.14; 0.02)	0.44 (0.12; 0.00)	0.45 (0.17; 0.00)	0.41 (0.20; 0.00)
$\Pr(S_{t-1} = 2)$	—	0.47 (0.38; 0.10)	—	−0.04 (0.22; 0.42)
$\Pr(S_{t-1} = 3)$	—	—	—	−0.33 (0.45; 0.23)
$V_{t-1}(\pi_t)$	—	—	0.03 (0.05; 0.25)	0.01 (0.09; 0.44)
$(GAP_{t-1} + b)^+$	0.60 (0.25; 0.01)	—	—	0.69 (0.29; 0.01)
b	−0.78 (0.21; 0.00)	—	—	−0.83 (0.19; 0.00)
ρ (lagged state)	0.30 (0.21; 0.07)	—	—	0.24 (0.40; 0.27)
Standard error (μ_t)	0.34 (0.13; 0.00)	0.41 (0.12; 0.00)	0.44 (0.12; 0.00)	0.32 (0.11; 0.00)
$\mathrm{Var}(\varepsilon_t)$	1.56 (0.31; 0.00)	1.64 (0.35; 0.00)	1.63 (0.35; 0.00)	1.55 (0.29; 0.00)
Mean likelihood	−1.73809	−1.77278	−1.77952	−1.72925
$\mathrm{Mean}(\beta_t)$	0.70	0.53	0.53	0.71
$\mathrm{Min}(\beta_t)$	0.04	0.07	0.11	−0.03
$\mathrm{Max}(\beta_t)$	1.54	1.13	1.20	1.59

Notes: Heteroscedasticity-consistent standard errors and p-values in parentheses.

Measurement equation: $\pi_t = e \bullet \pi_t^e + (1-e) \bullet \pi_{t-1} + \beta_t \bullet (GAP_t + b) + \varepsilon_t$.

Transition equation: $\beta_t = \alpha + \rho \bullet \beta_{t-1} + \gamma_1 \bullet \Pr(S_{t-1} = 2) + \gamma_2 \bullet \Pr(S_{t-1} = 3) + \gamma_3 \bullet V_{t-1}(\pi_t) + \gamma_4 \bullet 1_{(GAP_{t-1} + b > 0)} + \mu_t$.

Table 5

Maximum-Likelihood Estimates of Measurement and Transition Equations: United States, Total CPI

Expectations from Markov Switching Model; Potential Output – SVAR

Variables	Capacity constraint model	Costly adjustment model	Misperception model	Joint test
π_t^e	1.18	1.03	1.13	1.16
	(0.16; 0.00)	(0.18; 0.00)	(0.43; 0.00)	(0.16; 0.00)
π_{t-1}	−0.18	−0.03	−0.13	−0.16
α (constant)	0.06	0.33	0.27	0.02
	(0.11; 0.29)	(0.19; 0.04)	(0.32; 0.20)	(0.12; 0.42)
$\Pr(S_{t-1} = 2)$	—	0.67	—	0.38
		(0.49; 0.08)		(0.38; 0.16)
$\Pr(S_{t-1} = 3)$	—	−0.20	—	—
		(0.32; 0.26)		
$V_{t-1}(\pi_t)$	—	—	−0.01	—
			(0.13; 0.46)	
$(GAP_{t-1} + b)^+$	0.66	—	—	0.64
	(0.23; 0.00)			(0.23; 0.00)
b	−0.69	—	—	−0.72
	(0.22; 0.00)			(0.24; 0.00)
ρ (lagged state)	0.32	−0.07	0.33	0.24
	(0.24; 0.09)	(0.34; 0.42)	(1.81; 0.43)	(0.28; 0.20)
Standard error (μ_t)	0.49	0.47	0.52	0.49
	(0.14; 0.00)	(0.14; 0.00)	(0.33; 0.06)	(0.13; 0.00)
Var(ε_t)	1.26	1.49	1.43	1.25
	(0.31; 0.00)	(0.41; 0.00)	(0.43; 0.00)	(0.30; 0.00)
Mean likelihood	−1.72995	−1.75879	−1.77583	−1.72329
Mean(β_t)	0.44	0.35	0.35	0.42
Min(β_t)	−0.93	−0.61	−0.76	−0.95
Max(β_t)	1.43	1.13	1.16	1.50

Notes: Heteroscedasticity-consistent standard errors and p-values in parentheses.

Measurement equation: $\pi_t = e \bullet \pi_t^e + (1-e) \bullet \pi_{t-1} + \beta_t \bullet (GAP_t + b) + \varepsilon_t$.

Transition equation: $\beta_t = \alpha + \rho \bullet \beta_{t-1} + \gamma_1 \bullet \Pr(S_{t-1} = 2) + \gamma_2 \bullet \Pr(S_{t-1} = 3) + \gamma_3 \bullet V_{t-1}(\pi_t) + \gamma_4 \bullet 1_{(GAP_{t-1} + b > 0)} + \mu_t$.

Figure 7

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to Positive Gap

Column 1, Table 2

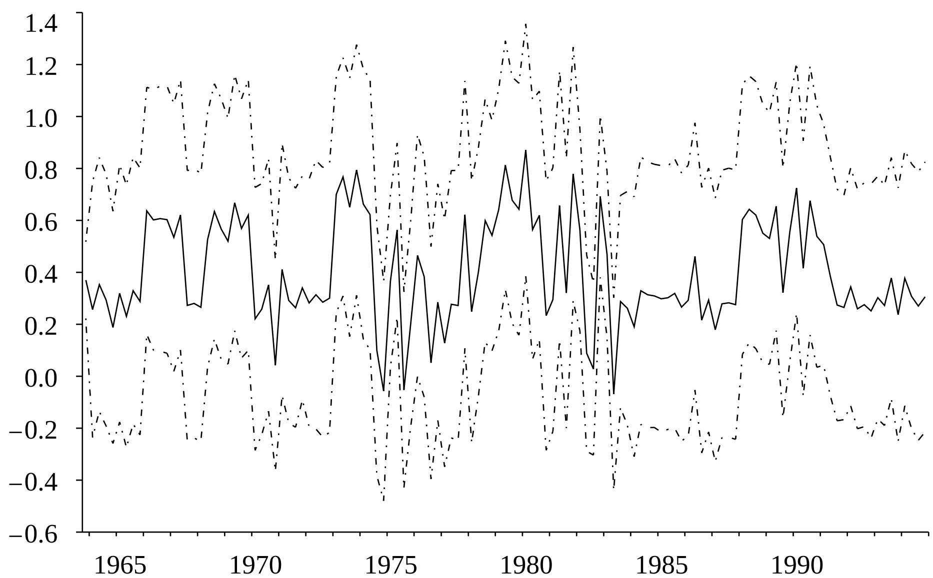


Figure 8

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to State 2 of Markov Switching Model

Column 2, Table 2

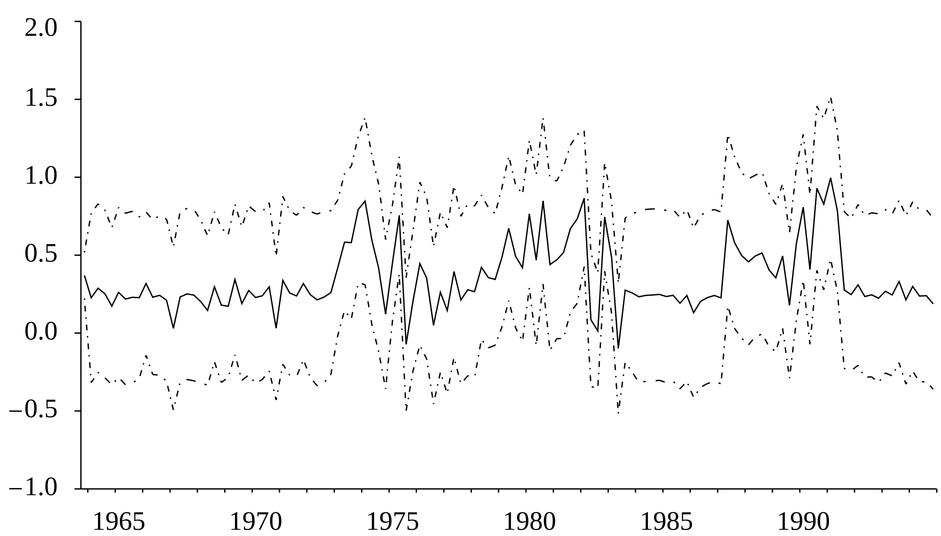


Figure 9

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to Conditional Volatility

Column 3, Table 2

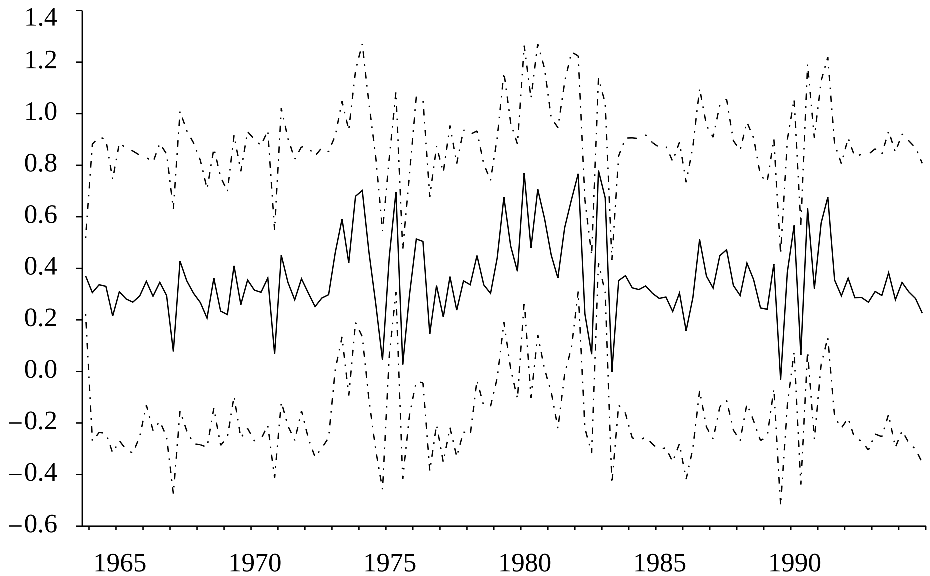


Figure 10

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to Positive Gap

Column 1, Table 3

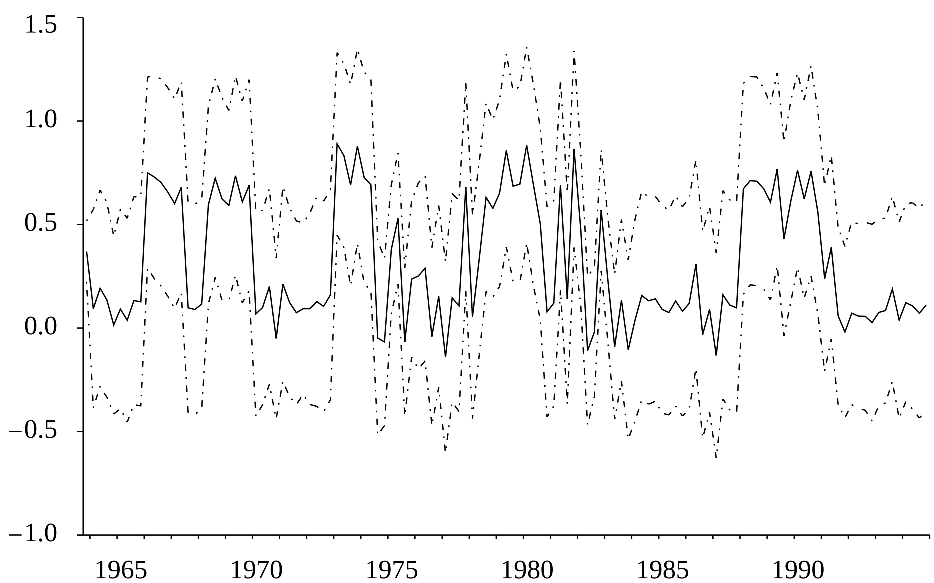


Figure 11

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to State 2 of Markov Switching Model

Column 2, Table 3

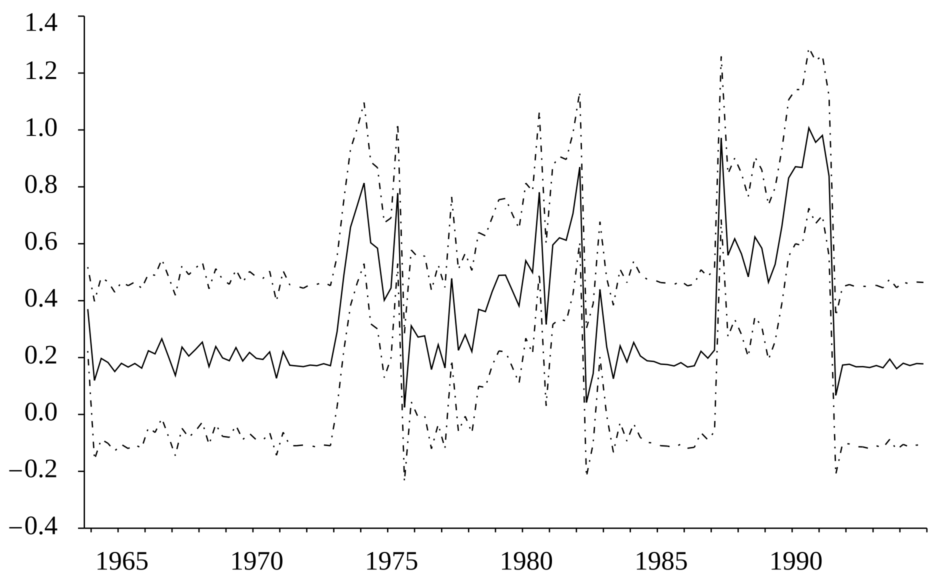


Figure 12

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to Positive Gap

Column 1, Table 4

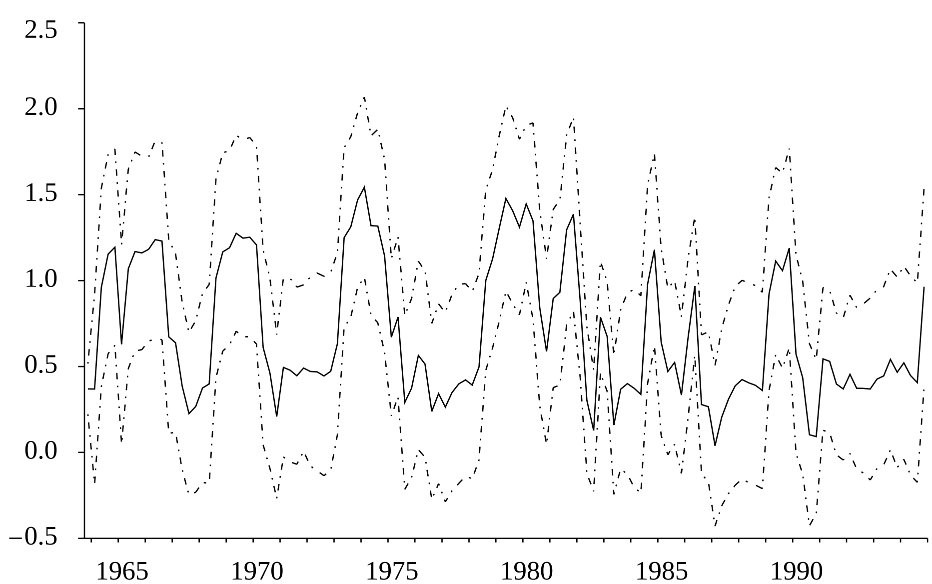


Figure 13

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to State 2 of Markov Switching Model

Column 2, Table 4

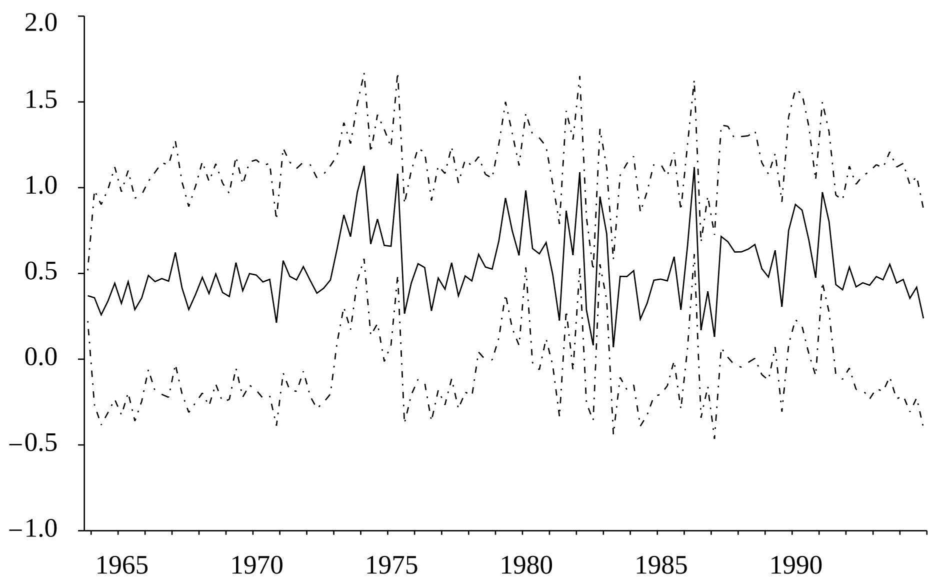


Figure 14

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to Conditional Volatility

Column 3, Table 4

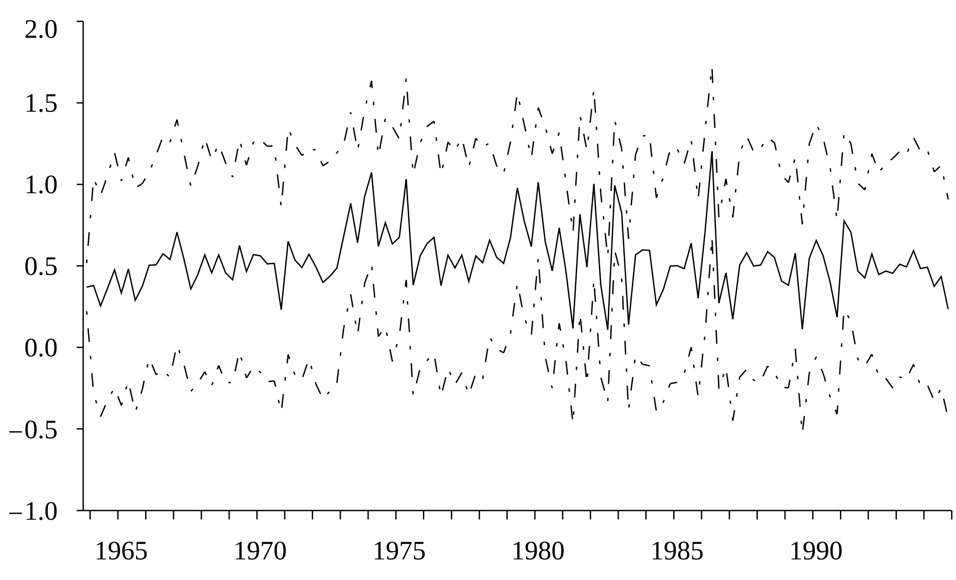


Figure 15

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to Positive Gap

Column 1, Table 5

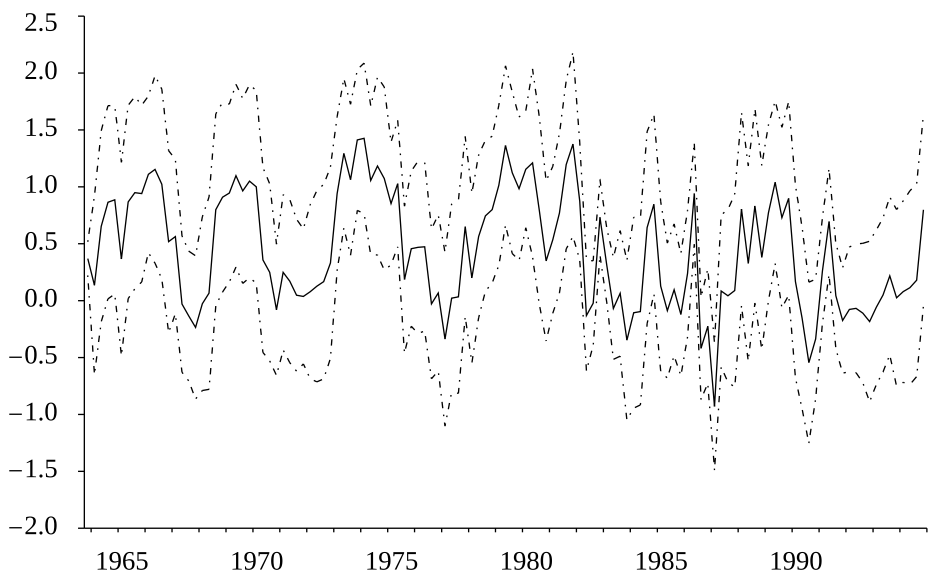
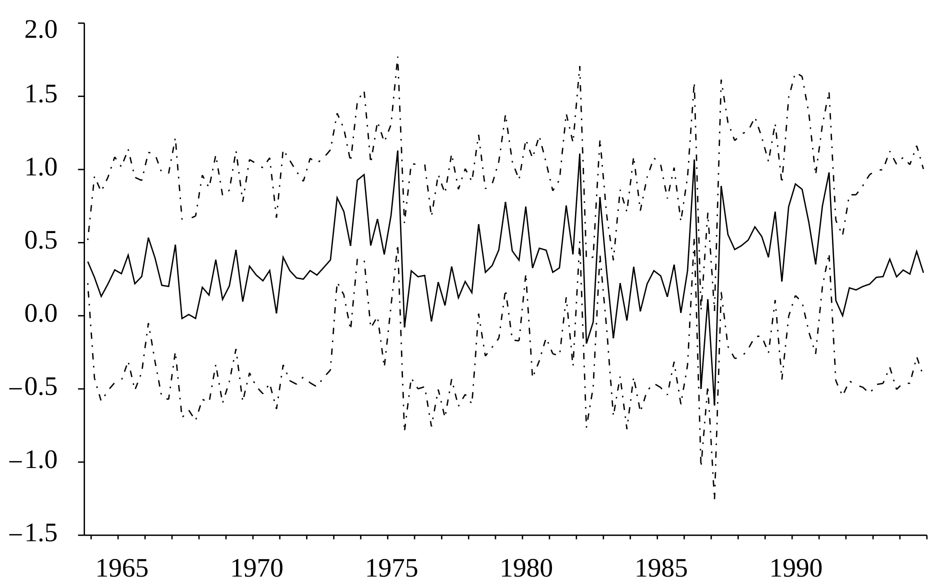


Figure 16

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to State 2 of Markov Switching Model

Column 2, Table 5



3.2.2 Results for Canada

The estimates for Canada are presented in Tables 6 and 7. Table 6 corresponds to the use of the extended multivariate filter to estimate potential output, while for Table 7 the output gap was generated using a three-variable SVAR. Inflation expectations are generated by a Markov switching model applied to the total CPI less GST, QST, and tobacco tax.

For the Canadian data, the estimates point to a systematic non-linearity in Table 6 but not in Table 7.[15] The fact that the measure of the output gap can influence our results for Canada means that we can be less certain about the nature of a non-linearity for Canada than for the United States. As indicated previously, however, the SVAR that we use to estimate the output gap used for Table 7 assumes a unit root in inflation. This is not consistent with the MSM expectations in the low- and moderate-inflation states.

The results in column 4 of Table 6 show that it is not possible to rule out either conditional volatility or the positive gap as the source of the non-linearity. It is also not possible to rule out the level of inflation as the source of systematic variation in the trade-off because, in this case, the high-inflation state also corresponds to a period of high conditional volatility of inflation, as can be seen from Figure 6. However, the graphs of the 90 per cent confidence region for the variation in the trade-off (Figures 17 to 19) show evidence of significant systematic variation associated more with the positive output gap than with the level of inflation or its volatility.

For those cases where a non-linearity is identified for Canada, the measured variation in the trade-off is again substantial and of economic significance. For example, if the variation is the result of capacity constraint, then the effect of excess demand on inflation is more than six times the effect of excess supply. Since our model looks only for a level change in the trade-off associated with the sign of the gap, we cannot be too precise in determining the value of the trade-off in periods of excess demand as opposed to periods of excess supply. There may in fact be a continuous non-convexity with respect to the gap. According to the point estimates, the trade-off parameter would be only 0.08 in periods of excess supply, compared with 0.51 during periods of excess demand. These values are less than the point estimates reported by Laxton, Rose, and Tetlow (1993) and Fillion and Léonard (1997), but the relative increase in the trade-off parameter is generally comparable. The unexplained variation—represented

15. The Phillips curve used in the extended multivariate filter to estimate potential output is non-linear. This may play a role in the results shown in Table 6.

Table 6

Maximum-Likelihood Estimates of Measurement and Transition Equations: Canada, Total CPI Less GST, QST, and Tobacco Tax

Expectations from Markov Switching Model; Potential Output – QPM Multivariate Filter

Variables	Capacity constraint model	Costly adjustment model	Misperception model	Joint test
π_t^e	0.93	0.87	0.87	0.91
	(0.13; 0.00)	(0.13; 0.00)	(0.13; 0.00)	(0.12; 0.00)
π_{t-1}	0.07	0.13	0.13	0.09
α (constant)	0.08	0.19	0.06	−0.07
	(0.06; 0.10)	(0.15; 0.10)	(0.13; 0.34)	(0.10; 0.24)
$\Pr(S_{t-1} = 2)$	—	−0.03	—	—
		(0.18; 0.43)		
$\Pr(S_{t-1} = 3)$	—	0.26	—	—
		(0.20; 0.09)		
$V_{t-1}(\pi_t)$	—	—	0.10	0.08
			(0.06; 0.05)	(0.05; 0.07)
$(GAP_{t-1} + b)^+$	0.43	—	—	0.39
	(0.27; 0.05)			(0.25; 0.06)
b	−0.87	—	—	−0.87
	(0.53; 0.05)			(0.50; 0.04)
ρ (lagged state)	−0.02	−0.49	−0.58	−0.15
	(0.13; 0.44)	(0.30; 0.05)	(0.27; 0.02)	(0.18; 0.21)
Standard error (μ_t)	0.17	0.13	0.11	0.10
	(0.10; 0.03)	(0.07; 0.03)	(0.08; 0.08)	(0.14; 0.23)
$\mathrm{Var}(\varepsilon_t)$	1.83	1.87	1.88	1.89
	(0.28; 0.00)	(0.29; 0.00)	(0.29; 0.00)	(0.29; 0.00)
Mean likelihood	−1.76562	−1.76202	−1.75807	−1.75352
$\mathrm{Mean}(\beta_t)$	0.24	0.18	0.19	0.24
$\mathrm{Min}(\beta_t)$	−0.03	0.00	−0.05	−0.03
$\mathrm{Max}(\beta_t)$	0.62	0.49	0.48	0.72

Notes: Heteroscedasticity-consistent standard errors and p-values in parentheses.

Measurement equation: $\pi_t = e \bullet \pi_t^e + (1 - e) \bullet \pi_{t-1} + \beta_t \bullet (GAP_t + b) + \varepsilon_t$.

Transition equation: $\beta_t = \alpha + \rho \bullet \beta_{t-1} + \gamma_1 \bullet \Pr(S_{t-1} = 2) + \gamma_2 \bullet \Pr(S_{t-1} = 3) + \gamma_3 \bullet V_{t-1}(\pi_t) + \gamma_4 \bullet 1_{(GAP_{t-1} + b > 0)} + \mu_t$.

Table 7

Maximum-Likelihood Estimates of Measurement and Transition Equations: Canada, Total CPI Less GST, QST, and Tobacco Tax

Expectations from Markov Switching Model; Potential Output – SVAR

Variables	Capacity constraint model	Costly adjustment model	Misperception model	Joint test
π_t^e	0.90	0.87	0.87	0.87
	(0.15; 0.00)	(0.13; 0.00)	(0.14; 0.00)	(0.13; 0.00)
π_{t-1}	0.10	0.13	0.13	0.13
α (constant)	0.03	0.06	0.13	0.11
	(0.17; 0.43)	(0.17; 0.37)	(0.08; 0.06)	(0.05; 0.02)
Pr $(S_{t-1} = 2)$	—	0.08	—	—
		(0.24; 0.37)		
Pr $(S_{t-1} = 3)$	—	0.05	—	—
		(0.22; 0.41)		
$V_{t-1}(\pi_t)$	—	—	−0.01	—
			(0.03; 0.36)	
$(GAP_{t-1} + b)^+$	0.07	—	—	—
	(0.31; 0.41)			
b	0.47	—	—	—
	(1.80; 0.40)			
ρ (lagged state)	0.66	0.50	0.56	0.52
	(0.11; 0.00)	(0.28; 0.04)	(0.24; 0.01)	(0.17; 0.00)
Standard error (μ_t)	0.14	0.15	0.12	0.15
	(0.36; 0.35)	(0.23; 0.25)	(0.39; 0.37)	(0.24; 0.27)
Var(ε_t)	2.05	2.08	2.09	2.09
	(0.44; 0.00)	(0.33; 0.00)	(0.36; 0.00)	(0.33; 0.00)
Mean likelihood	−1.79349	−1.79910	−1.79923	−1.79962
Mean(β_t)	0.22	0.22	0.23	0.23
Min(β_t)	0.06	0.10	0.11	0.12
Max(β_t)	0.39	0.36	0.32	0.35

Notes: Heteroscedasticity-consistent standard errors and p-values in parentheses.

Measurement equation: $\pi_t = e \bullet \pi_t^e + (1 - e) \bullet \pi_{t-1} + \beta_t \bullet (GAP_t + b) + \varepsilon_t$.

Transition equation: $\beta_t = \alpha + \rho \bullet \beta_{t-1} + \gamma_1 \bullet \Pr(S_{t-1} = 2) + \gamma_2 \bullet \Pr(S_{t-1} = 3) + \gamma_3 \bullet V_{t-1}(\pi_t) + \gamma_4 \bullet 1_{(GAP_{t-1} + b > 0)} + \mu_t$.

Figure 17

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to Positive Gap

Column 1, Table 6

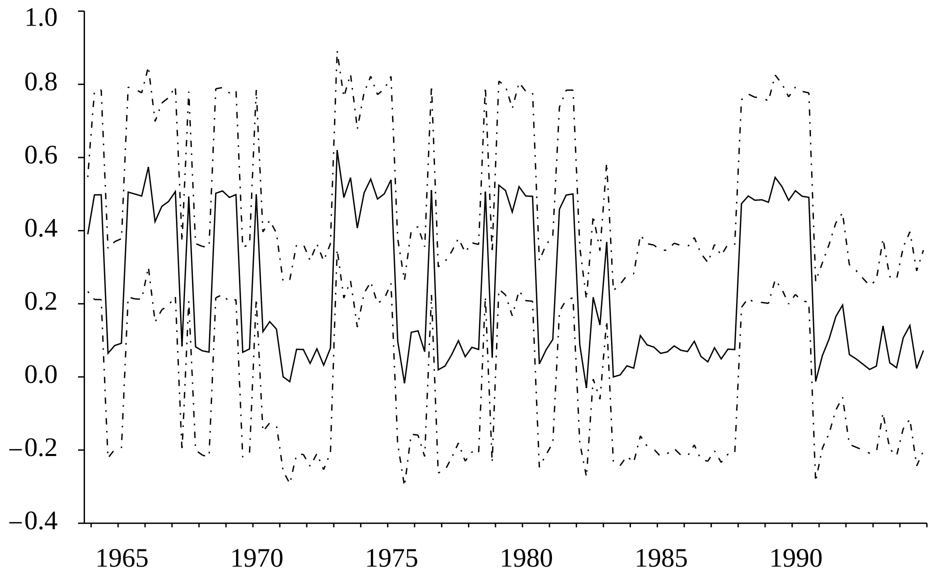


Figure 18

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to State 2 and State 3 of Markov Switching Model

Column 2, Table 6

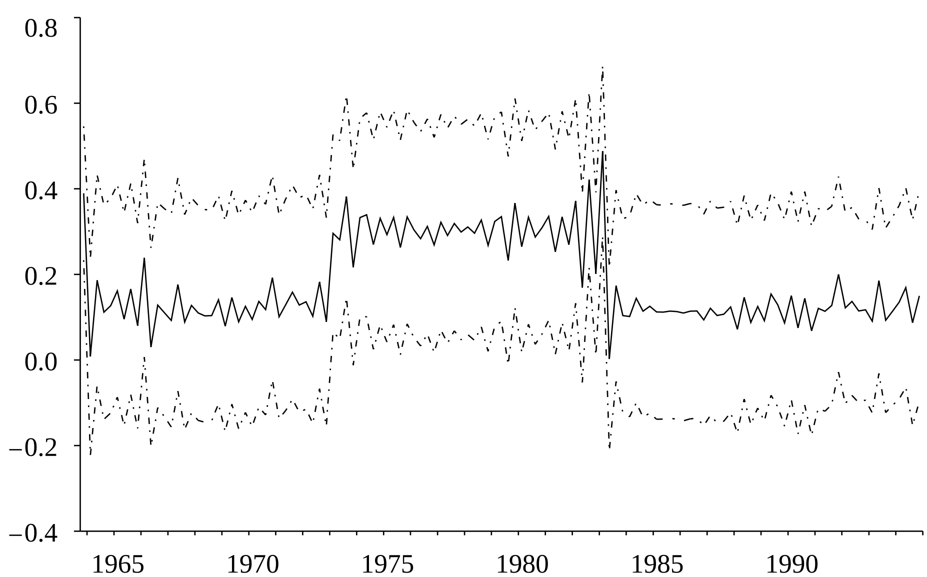


Figure 19

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to Conditional Volatility

Column 3, Table 6

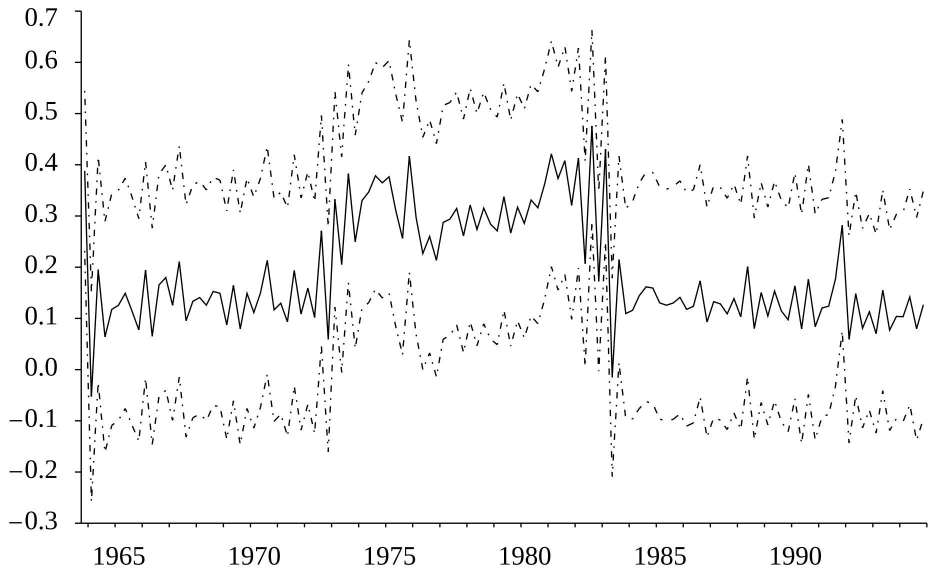


Figure 20

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Variation Due to Conditional Volatility and Positive Gap

Column 4, Table 6

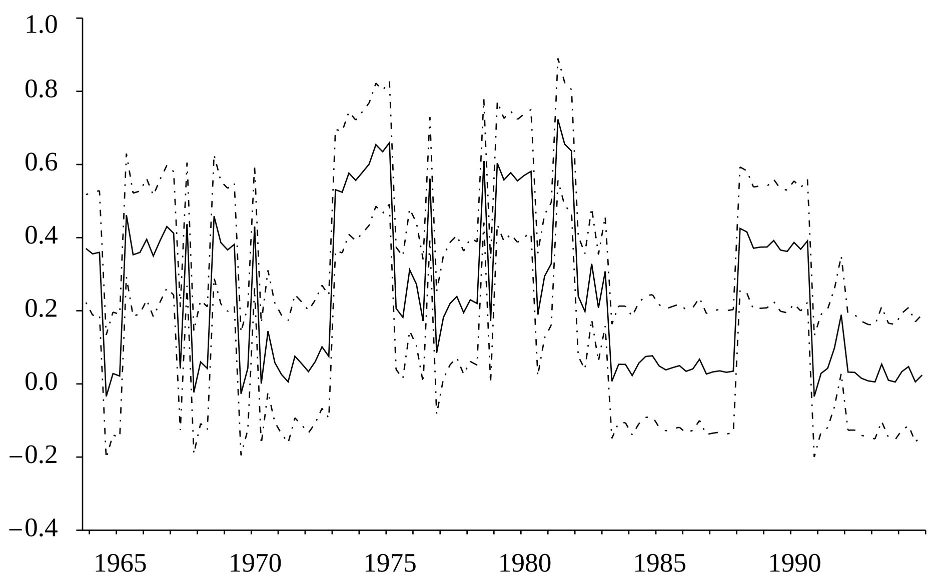
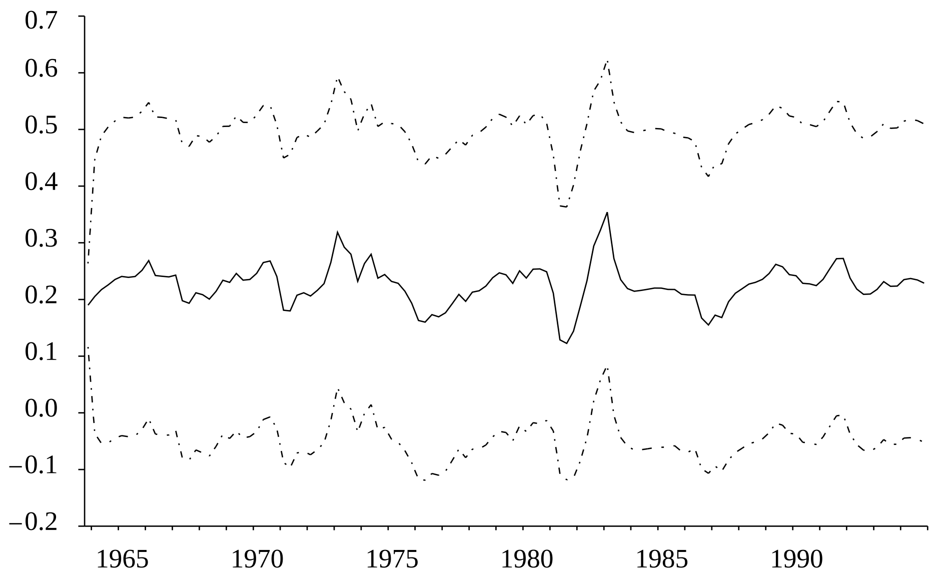


Figure 21

90 Per Cent Confidence Region of Output-Inflation Trade-Off: Non-Systematic Variation Only

Column 4, Table 7



by μ_t—is generally small but also seems to be more pronounced in the period after 1990.

If the variation in the trade-off parameter is due to changes in the average level of inflation, the magnitude of the variation is much smaller. This variation is shown in Figure 18, corresponding to the estimates in column 2 of Table 6. When inflation is at low or moderate levels, the point estimate is 0.13, but this rises to 0.31 when inflation is high. Thus the slope of the Phillips curve is a little more than twice as high when inflation is high as when inflation is low. The specification of the time variation in terms of three inflation regimes can yield only an approximation of the output-inflation trade-off if it is a continuous function of the average level of inflation. However, the finding that moderate-inflation periods appear not to have a significantly different trade-off from low-inflation periods suggests that this may not be the case.

Conclusions

In this paper we have presented some empirical evidence on the nature of the output-inflation trade-off. Our survey of the literature identifies five models of pricing behaviour that imply a non-linearity in the short-run adjustment of prices to aggregate demand shocks. It is important, though difficult, to be able to distinguish between these non-linearities because different types have different implications for monetary policy. Problems arise, especially in the Phillips curve framework, in the measurement of inflation expectations and the output gap. In this context, the tests that we have used lack power. At times they identified economically important shifts in the sacrifice ratio, but the statistical significance was small. Uncertainty about the unexplained variation in the output-inflation trade-off sometimes exceeded the size of the systematic variations.

In the framework of the traditional short-run Phillips curve, both the capacity constraint model and the monopolistically competitive model imply an asymmetry in the response of inflation to excess demand and excess supply. A simple test of this asymmetry produced some evidence for the United States of a convex relationship, as implied by the capacity constraint model. Of the other models that we covered, the costly adjustment model and the misperception model imply a Phillips curve slope that varies with the average level of inflation or with its conditional volatility, respectively. We used this information to construct an estimation framework in which evidence for these models could be tested separately and jointly.

Some of the evidence we have presented suggests that there is significant time variation in the output-inflation trade-off for Canada, but we are unable to distinguish definitively among the possible models generating the non-linearity—indeed, more than one may be at play. There is stronger evidence in favour of non-linearity for the United States; and overall, for both countries, the capacity constraint model seems to find more empirical support. In those cases where significant non-linearity is found, the changes in the trade-off are of substantial economic consequence as well.

Further work is required to identify proxies for the different theories of non-linearity that will enable us to obtain more definitive results regarding both the existence of non-linearity or asymmetry and its nature. In our current work we have not allowed for continuous variation in the trade-off parameter with respect to measures of the output gap and the average level of inflation.[16] These restrictions should be relaxed in future work. This will also involve methods of accounting for mismeasurement in the inflation expectations and output gap proxies. For example, perhaps we could implement instrumental variable techniques in the state-space framework. We might obtain more precise estimates of the parameter variation by pooling the U.S. and Canadian data. We must also take into account the possibly large influence of some of the important supply shocks. It has

16. Preliminary estimation indicates that lagged values of expected inflation are significant indicators of continuous variation in the trade-off parameter.

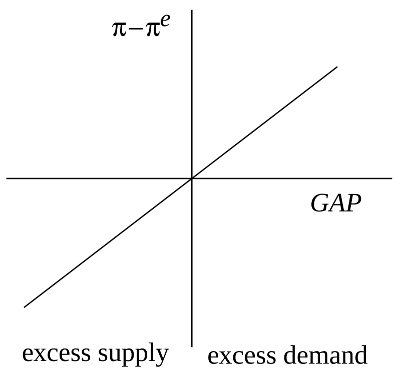
become standard within the Phillips curve framework to include measures that control separately for food, energy, and exchange rate shocks.

The implications of non-linearities in the Phillips curve are very important for the conduct of monetary policy. Further research along these lines is required to clarify our results.

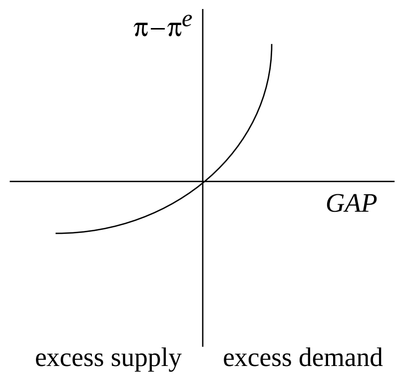
Appendix 1

Different Types of Output-Inflation Relationships

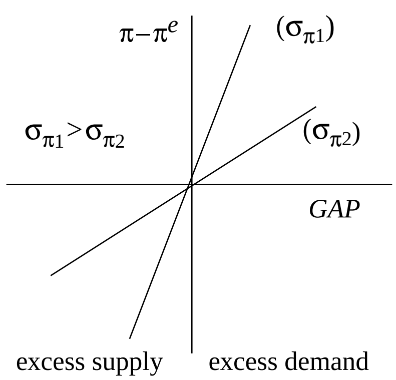
1. Linear model



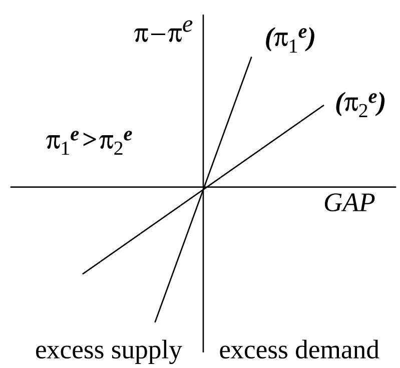
2. Capacity constraint model



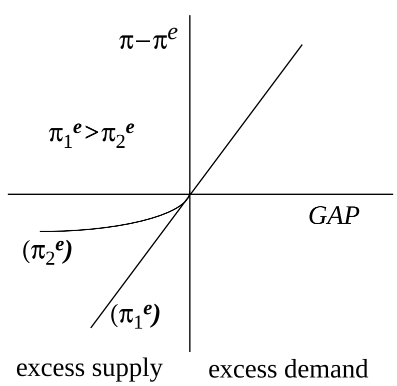
3. Misperception model



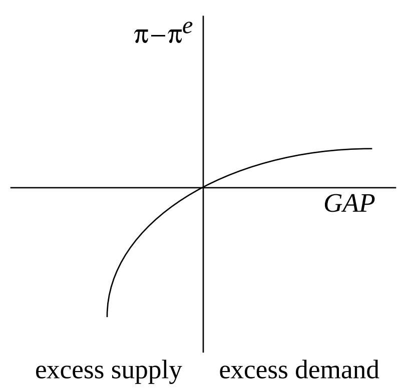
4. Costly adjustment model



5. Downward nominal wage rigidity model



6. Monopolistically competitive model



Appendix 2

Long-Run Restrictions Imposed on Output to Measure Potential Output

This appendix briefly presents the decomposition method based on long-run restrictions imposed on output (LRRO) to measure potential output.[1]

Let Z_t be an $n \times 1$ stationary vector including an n_1-vector of I(1) variables and an n_2-vector of I(0) variables such that $Z_t = (\Delta X_{1t}', X_{2t}')'$.[2] By the Wold decomposition theorem, Z_t can be expressed as the following reduced form:

$$Z_t = \delta(t) + C(L)\varepsilon_t, \tag{A2.1}$$

where $\delta(t)$ is deterministic; $C(L) = \Sigma_{i=0}^{\infty} C_i L^i$ is a matrix of polynomial lags; $C_0 = I_n$ is the identity matrix; the vector ε_t is the one-step-ahead forecast errors in Z_t, given information on lagged values of Z_t; $E(\varepsilon_t) = 0$; and $E(\varepsilon_t \varepsilon_t') = \Omega$ with Ω positive definite. We suppose that the polynomial det $|C(L)|$ has all its roots on or outside the unit circle, which rules out the non-fundamental representations emphasized by Lippi and Reichlin (1993).

Equation (A2.1) can be decomposed into a long-run component and a transitory component:

$$Z_t = \delta(t) + C(1)\varepsilon_t + C^*(L)\varepsilon_t, \tag{A2.2}$$

where $C(1) = \Sigma_{i=0}^{\infty} C_i$, and $C^*(L) = C(L) - C(1)$. We define $C_1(1)$ as the long-run multiplier of the vector X_{1t}. If the rank of $C_1(1)$ is less than n_1, there exists at least one linear combination of the elements in X_{1t} that is I(0).

The LRRO approach assumes that Z_t has the following structural representation:

$$Z_t = \delta(t) + \Gamma(L)\eta_t, \tag{A2.3}$$

where η_t is an *n*-vector of structural shocks, $E(\eta_t) = 0$, and $E(\eta_t \eta_t') = I_n$ (a simple normalization). From the estimated reduced form, we can retrieve

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1. For a more detailed presentation of the LRRO approach see Watson (1994); Dupasquier, Guay, and St-Amant (1997); or St-Amant and van Norden (1997).
2. I(d) denotes a variable that is integrated of order d.

the structural form (A2.3) using the following relationships: $\Gamma_0\Gamma_0' = \Omega$, $\varepsilon_t = \Gamma_0\eta_t$, and $C(L) = \Gamma(L)\Gamma_0^{-1}$.

The long-run covariance matrix of the reduced form is equal to $C(1)\Omega C(1)'$. From (A2.2) and (A2.3) we have:

$$C(1)\Omega C(1)' = \Gamma(1)\Gamma(1)'. \tag{A2.4}$$

This relationship suggests that we can identify matrix Γ_0 with an appropriate number of restrictions on the long-run covariance matrix of the structural form.

Let us assume that the log of output is the first variable in the vector Z_{1t}. It is then equal to:

$$\Delta y_t = \mu_y + \Gamma_1^p(L)\eta_t^p + \Gamma_1^c(L)\eta_t^c, \tag{A2.5}$$

where η_t^p is the vector of permanent shocks affecting output, and η_t^c is the vector containing shocks having only a transitory effect on output. Potential output based on the LRRO method is then:

$$\Delta y_t^p = \mu_y + \Gamma_1^p(L)\eta_t^p. \tag{A2.6}$$

Thus, "potential output" corresponds to the permanent component of output. The part of output due to transitory shocks is defined as the "output gap."

Appendix 3

Description of Data

We have used quarterly gross domestic product (GDP) as the measure of real output in Canada and the United States from 1964 to 1995. Canadian and U.S. inflation are measured by the total consumer price index (CPI) (excluding GST, QST, and tobacco tax, in the case of Canada) and CPIXFE, the CPI excluding food and energy. For the Canadian data, the seasonal adjustment is made at the Bank of Canada, while for the U.S. data it is done by Data Resources INC. Interest rates are defined as the overnight rate (RON) for Canada (for a description of RON, see Armour, Engert, and Fung 1996), and the federal funds rate for the United States.

We test for unit roots using augmented Dickey-Fuller statistics. On the basis of our tests, we cannot reject the hypothesis that production, inflation rates, and interest rates are first-order integrated.

Appendix 4

Maximum-Likelihood Estimation of the State-Space Model

The parameters of the state-space model are estimated using maximum likelihood (ML). A Kalman filter generates the prediction error decomposition form of the likelihood function as in Harvey (1993). Numerical maximization is implemented with GAUSS software.

The state-space model is defined by equations (2) and (3) in the text as follows:

$$\pi_t = a \cdot \pi_t^e + (1-a) \cdot \pi_{t-1} + \beta_t \cdot GAP_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2), \qquad \text{(A4.1)}$$

$$\beta_t = \alpha + \rho \cdot \beta_{t-1} + \gamma \cdot X_{t-1} + \mu_t \qquad \mu_t \sim N(0, \sigma_\mu^2). \qquad \text{(A4.2)}$$

The parameters to be estimated by ML are $\{a, \alpha, \rho, \gamma, \sigma_\varepsilon, \sigma_\mu\}$. These are called the hyper parameters of the model. The Kalman filter takes these parameters as given and produces time-series estimates of β_t and ε_t. Let $\beta_{t|s}$ denote the prediction of β_t given information up to period s, and let $P_{t|s}$ be the associated conditional variance. Then, given starting values for the elements of the distribution of β_0, denoted by $\beta_{0|0}$ and $P_{0|0}$, the Kalman filter proceeds iteratively for $t = 1$ to $t = T$ as follows:

$$\beta_{t|t-1} = \alpha + \rho \cdot \beta_{t-1|t-1} + \gamma \cdot X_{t-1} \qquad \text{(A4.3)}$$

$$P_{t|t-1} = \rho^2 \cdot P_{t-1|t-1} \qquad \text{(A4.4)}$$

$$\varepsilon_{t|t-1} = \pi_t - a \cdot \pi_t^e - (1-a) \cdot \pi_{t-1} - \beta_{t|t-1} \cdot GAP_t \qquad \text{(A4.5)}$$

$$H_t = P_{t|t} \cdot GAP_t^2 + \sigma_\varepsilon^2 \qquad \text{(A4.6)}$$

$$K_{t|t-1} = P_{t|t-1} \cdot GAP_t \cdot H_t^{-1} \qquad \text{(A4.7)}$$

$$\beta_{t|t} = \beta_{t|t-1} + K_{t|t-1} \cdot \varepsilon_{t|t-1} \qquad \text{(A4.8)}$$

$$P_{t|t} = (I - K_{t|t-1} \cdot GAP_t) \cdot P_{t|t-1}. \qquad \text{(A4.9)}$$

H_t in equation (A4.6) is the conditional variance of the prediction errors, $\varepsilon_{t|t-1}$. It incorporates parameter uncertainty about the slope of the Phillips curve in addition to uncertainty about the supply shocks. The prediction error decomposition form of the likelihood function for observation t is therefore:

$$\log(l_t) = -\frac{\log 2.\mathrm{pi}}{2} - \frac{\log H_t}{2} - \frac{\varepsilon^2_{t|t-1}}{2H_t}. \qquad \text{(A4.10)}$$

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