Analyzing the Monetary Aggregates

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• In recent years, the Bank has put renewed emphasis on analyzing monetary variables and developing models that incorporate money as an active part of the transmission mechanism. This partly reflects an awareness of the uncertainties faced by policy-makers and of the need to base advice on a wide variety of data sources and models.

• The most important money-based model currently used in policy analysis at the Bank is the M1-VECM, an empirical model in which deviations in the money supply from the long-term demand for money cause changes in inflation.

• Other models are used to assess risks to the forecasts of the M1-VECM, including simple linear indicator models based on narrow money aggregates, non-linear neural networks, and an empirical model using the broad aggregate M2++.

When Canada abandoned money-growth targets in the early 1980s, the monetary aggregates became less central to ongoing analysis and advice regarding monetary policy. Even when monetary targets were in place, the main models incorporated explanations for inflation that were strongly based on the output gap. Any role played by money in these models was purely passive—money moved in reaction to other variables but was not an active part of the transmission mechanism. More recently, however, the Bank has placed renewed emphasis on developing money-based models of the transmission mechanism and on including analysis of monetary and financial variables in the policy-decision process as a complement to other models.

The Bank has placed renewed emphasis on developing money-based models of the transmission mechanism.

In part, the renewed interest in monetary variables comes from a greater awareness of the uncertainties faced by policy-makers. These include uncertainties about the kinds of shocks that are occurring and will occur in the future, about how those shocks are transmitted through the economy and into inflation, and about the speed and extent to which changes in interest rates affect activity. Given these uncertainties, no single model is likely to fully capture all aspects of the transmission mechanism or to be correct under all
circumstances. Monetary policy advice should, therefore, not be based on only one view of the world, but rather it should draw on a wide variety of data sources and use models that summarize different views, or "paradigms," of the transmission mechanism.\(^1\) The money-based paradigm focuses on money and financial behaviour as active determinants of inflation.

There are several perspectives from which to interpret money: pure time-series indicator models; structural vector autoregressive models (VARs); and choice-theoretic dynamic general-equilibrium models. This article provides an overview of how the monetary aggregates are used in the formulation of monetary policy analysis at the Bank. It describes the key components of the "money paradigm," followed by descriptions of the main tools and models used.

### The Money Paradigm

A key input into the Bank’s policy process is a projection based on the Quarterly Projection Model (QPM). The QPM is based on the expectations-augmented Phillips curve paradigm. According to this paradigm, the dynamics of inflation depend critically on the output gap—a measure of excess supply in the real economy. Interest rates and the exchange rate affect real output which, in turn, affects the output gap and inflation. In these models, the supply of money adjusts passively to demand, and money has no causal role.

An alternative view is provided by the "active-money" paradigm.\(^2\) In active-money models, changes in the quantity of money in the economy cause short-term changes in output and long-run changes in prices. While real variables may still be included and play an important role, money and credit are active parts of the transmission mechanism. Recent empirical work at the Bank of Canada supports an active role for money.\(^3\) The results are by no means conclusive, however, and there is considerable debate over the relative merits of the two paradigms in explaining the transmission mechanism. Currently, it is generally agreed that neither one provides a complete description of all aspects of the transmission mechanism.

A good description of the active-money view is given by Laidler (1999a and b). According to this view, the primary role of money is as a means of exchange, and money can be in disequilibrium; i.e., the supply of money may not always be equal to the demand for it. Money demand can be thought of as the target value of an inventory (or buffer stock) from which agents can make purchases of goods and services. The actual value of this inventory (or the actual money supply), however, will vary because of both exogenous shocks and the voluntary transactions of agents.

To illustrate how the money market can be in disequilibrium, consider the case in which the Bank of Canada cuts interest rates. When interest rates fall, demand for credit by both households and businesses increases, since the cost of credit has declined. Agents are more inclined to take out some form of loan, and those who do take out a loan receive a new deposit in their bank accounts—they increase their holdings of money. But agents do not want to hold this money for its own sake. Rather, they typically want to buy goods and services with it. Until they make these purchases, therefore, they are holding more money than is required for their long-term demand.

When holdings of real money balances are greater than demand, agents use their excess money to purchase goods and services. As they pay for these purchases, other agents experience unexpected increases in money holdings, which they in turn use to make purchases or to reduce loans. Thus, a series of transactions is set in motion. Over time, the increased demand for goods and services will cause firms to increase output and/or to increase their prices. As output increases, money demand may also increase, and as prices rise, real money balances fall. The reduction in loans also reduces the excess supply of money. These effects gradually act to bring money back into equilibrium. (Similarly, excess demand for money will cause agents to defer purchases and will be associated with downward pressure on output and prices.) Monetary disequilibria can also be caused by factors other than a policy-induced change in interest rates such as changes in money demand associated with persistent shocks to productivity.

Of course, not all financial assets are used as a means of exchange. In addition to transactions-related balances, some financial assets are used for savings. This is true, for example, of less-liquid assets such as fixed-term deposits, Canada Savings Bonds, and mutual funds.

In the example where the transactions-related money supply is greater than money demand, to the extent that individuals decide to use the excess supply to increase their holdings of savings balances, this takes

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2. There are, of course, many other interpretations of why money can predict income and prices, many of which give money a passive role.
3. For example, see Hendry (1995), Armour et al. (1996), and Engert and Hendry (1998).
money out of those balances intended for immediate 
transactions, thereby reducing the immediate pressure 
on output and inflation. Previous work at the Bank 
(McPhail 2000) suggests that deposits associated with 
savings play more of a passive role in the economy, 
with the amount being saved in part reflecting savers’ 
inflation expectations.

Since transactions balances are associated with the 
active-money paradigm, and savings balances appear 
to be more passive, it is important to be able to differ-
entiate between transactions and savings balances. 
For this reason, the monetary aggregates are often 
divided into the “narrow aggregates,” which encom-
pass those forms of money that are more liquid and 
thought to better represent money held to make pur-
chases of goods and services, and the “broad aggre-
gates,” which also include less-liquid deposits and 
which are thought to be associated with savings 
behaviour.

Measuring Narrow and Broad Money

Since the active-money paradigm is based on transac-
tions balances, much of our analysis uses this nar-
rower definition of money. In practice, however, it is 
often difficult to accurately measure transactions 
money. For some instruments, it is fairly evident—a 
5-year term deposit is likely to be held for savings 
rather than to complete immediate transactions. But it 
is often not this clear, since the amount will depend 
offered by banks have both transactions and savings 
characteristics. Current accounts, for example, are 
liquid and can therefore be used for transactions 
balances. But, if interest can be earned on these 
accounts, they may also attract savings. Moreover, 
financial innovations can change the nature of an 
account over time. For example, over the last decade, 
the spread of debit cards, ATMs, and telephone or 
personal-computer banking has increased the ease 
with which people can access a range of deposit 
accounts, thus making some savings-type accounts 
more relevant for transactions.

The aggregate M1 has traditionally been used as the 
main measure of transactions money. M1 is the sum of 
currency, personal chequing accounts, and current 
accounts. These are all very liquid forms of money 
from which it is easy to access funds and make trans-
actions. For this reason, they are thought to be domi-
nated by transactions balances. Personal chequing and 
current accounts are generally known as “demand” 
balances. Historically, demand accounts were clearly differenti-
ated from notice accounts, partly because the two 
kinds of accounts were subject to different reserve 
requirements. Over the 1992–94 period, however, 
reserve requirements on bank accounts were phased 
out. This reduced the distinction between notice and 
demand accounts and bank classification of deposit 
account became increasingly arbitrary (Aubry and 
Nott 2000). Notice accounts, which are not included in 
M1, thus became a close substitute for those accounts 
that are included. Two broader measures of transac-
tions money were therefore developed to try to take 
account of these changes: M1+, which includes cheq-
uable notice deposits, and M1++, which also includes 
non-chequable notice deposits.4

M1+ and M1++ do not represent transactions money 
perfectly because they also include funds held for 
savings purposes. The narrow aggregates also include 
balances held for making financial transactions rather 
than purchases of goods and services (for example, 
deposits held at investment dealers) and deposits held 
at banks by financial institutions. Unfortunately, data 
are not available at a fine enough level of disaggrega-
tion for us to be able to identify and remove all such 
deposits. As noted earlier, over time, financial innova-
tions can also change the nature of different accounts 
and the extent to which they are used for transactions 
and savings.

Given such problems, these aggregates can be affected 
by special factors unrelated to changes in transactions 
balances. These create instabilities in the relationships 
between narrow money and spending and in the 
demand function for transactions money. Dummy 
variables are one means of taking account of these 
instabilities when estimating regression equations for 
money demand. In their simplest form, these variables 
take a value of zero when there are no distortions 
affecting the data and a value of one when distortions 
are thought to be important. They are particularly 
useful where it is possible to identify specific periods 
when special factors were important. While such 
periods can be identified over history, it is often 
difficult to distinguish movements in transactions 
balances from the impact of special factors when new 
data are received. Thus, dummy variables may not be 
helpful in assessing current developments.

4. M1+ is defined as the sum of currency held by the public and all chequable 
(demand and notice) deposits at chartered banks, credit unions and caisses 
populaires (CUCPs), and trust and mortgage loan companies (TMLs). M1++ is 
the sum of M1+ and all non-chequable notice deposits at chartered banks, 
CUCPs, and TMLs.
There has been a considerable amount of research at the Bank of Canada and elsewhere trying to develop better measures of transactions balances. One measure used in the past few years at the Bank is a model-based definition of transactions money, which is described in the section dealing with the M1-VECM. Modelling transactions money should, however, be viewed as a work in progress. Currently, research is underway to determine whether statistical techniques based on the identification of common trends in the data can be used to differentiate between the transactions and savings processes within the money aggregates.

As already mentioned, broad money aggregates include less-liquid deposits and are dominated by savings balances. The broad money aggregate used most frequently at the Bank is $M2^{++}$. It includes all demand and notice deposits at banks and near-bank institutions, as well as personal savings deposits, Canada Savings Bonds, and mutual funds. $M2^{++}$ captures money held as a store of value and provides leading information about trends in inflation.

Money-Based Models

Various models are used at the Bank to help in the analysis of the monetary aggregates. Currently, the greatest emphasis is placed on an empirical model based on the active-money paradigm. Other models are also used to help provide indicators of near-term inflation and output and to help assess risks to the main forecast.

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5. For example, for early work on divisia indices see Cockerline and Murray (1981). For examples of aggregates developed outside the Bank of Canada, see Boessenkool, Laidler, and Robson (1997) and Robson and Aba (1999).

6. $M2$ includes net $M1$ plus personal savings and non-personal notice deposits at chartered banks; $M2^{++}$ adds to $M2$ the $M2$-like deposits at near-bank institutions, life insurance company annuities, and both money-market and other mutual funds.

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The M1-VECM: An empirical model restricted by theory

The main money-based model currently used at the Bank is the M1-VECM. The VECM (which stands for vector-error-correction model) is a system of four key equations in which changes in money, output, prices, and interest rates are functions of lagged changes in each of these variables, a number of exogenous variables, and the money gap.\(^7\)

The money gap in the model is defined as the difference between the current actual level of $M1$ and an estimate of long-run money demand. The main determinants of money demand are prices, real income, and interest rates:\(^8\):

$$money\ demand_t = constant + CPI_t + 0.6* income_t - 0.05* interest\ rate_t.$$

Money demand is assumed to increase one-for-one with increases in the price level (as measured by the consumer price index CPI), since people need to hold more money to cover the higher cost of the goods and services they wish to purchase. In the money-demand equation, therefore, the level of the CPI has a coefficient of one.\(^9\) The estimated coefficient on real income is 0.6. This suggests that an increase in real income increases money demand but by considerably less than proportionately; i.e., agents want to hold a lower proportion of additional income in highly liquid forms. The interest rate in the money-demand equation reflects the return that could be earned if money was held in less-liquid forms and is thus negatively related to money demand. Again, this is consistent with the sign of the coefficient on the interest rate, which is estimated at -0.05 (or -5, depending on the convention used to express interest rates).\(^10\)

The model is based on the active-money paradigm—that money demand and money supply are not

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7. A more detailed description of the VECM, as well as coefficient values, is provided in the appendix.

8. Money (gross $M1$), prices (the total CPI), and income (real GDP) are in log levels. The interest rate is calculated as the level of the overnight interest rate minus estimated structural policy shocks, as measured by the residuals from the interest rate forecasting equation. This gives a measure of the “policy-free” overnight rate. For the exact equation and further definitions of variables, see the appendix.

9. This hypothesis of long-run unit price elasticity was tested for and could not be rejected. See Hendry (1995).

10. In principle, an own-rate of interest could also be included to capture interest earned on transactions deposits. Currently, however, it is difficult to obtain a consistent series capturing interest earned on these accounts.
always in equilibrium and that periods of disequilibrium (or money gaps) cause changes in inflation. When the money supply is greater than the demand for money, agents will use their excess money balances to purchase goods and services. In the VECM, this results in increased prices and, over the short term, increased real output. The money gap persists until the money supply shock is reversed, or prices change to help restore monetary equilibrium.

When the money supply is greater than the demand for money, agents will use their excess money balances to purchase goods and services.

While the money gap is a key source of inflationary pressures in the model, a number of other variables are also important in determining the short-run dynamics of inflation. Within the forecasting model, the inflation equation is in terms of core inflation and can be characterized as follows:

\[
\text{Core inflation} = \Gamma \cdot \text{Lags of} \begin{bmatrix}
\text{Money growth} \\
\text{Inflation} \\
\text{Real output growth} \\
\text{Change in overnight interest rate}
\end{bmatrix}
\]

\[= + D_1 \text{output gap}_{t-1} \]
\[+ D_2 \Delta \text{exchange rate}_{t-1} \]
\[+ D_3 \Delta \text{US federal funds rate}_{t} \]
\[+ D_4 \text{money gap}_{t-1}. \]

The money gap and lagged changes in money growth have significant positive effects on inflation. The equation also includes past changes in interest rates and inflation. Changes in the level of output do not significantly affect inflation in this model, but the output gap has an important influence. (The output gap is the difference between actual output and an estimate of the long-run production potential of the economy.) An excess demand for goods (when output is above the long-run production potential of the economy) is associated with upward pressure on inflation.

Both the exchange rate and U.S. interest rates have a significant impact on Canadian inflation. A depreciation of the Canadian dollar is associated with price-level increases, consistent with pass-through of the higher costs of imported goods. An increase in U.S. interest rates has a small positive effect on inflation. This suggests that, in this model, U.S. interest rates are an indicator of activity in the rest of the world. Thus, a higher U.S. interest rate is associated with greater demand for Canadian goods and therefore a stronger domestic economy and upward pressure on prices.

In the active-money paradigm, a positive money gap leads to some combination of increases in real output and prices. The results from the VECM suggest, however, that any increases in real output occur only in the very short term. The money gap was found to be insignificant in the equation for output growth. However, lagged values of money growth do have strong positive short-run real effects on output.

The output equation can be characterized as follows:

\[
\text{Real output growth}_t = \Gamma \cdot \text{Lags of} \begin{bmatrix}
\text{Real money growth} \\
\text{Real output growth} \\
\text{Interest rate spread}
\end{bmatrix}
\]

\[= + F_1 \text{output gap}_{t-1} \]
\[+ F_2 \Delta \text{US federal funds rate}_t. \]

An increase in real money growth (where money is deflated by the core CPI) leads to an increase in output. Rather than the change in the overnight interest rate, the output equation includes a measure of the interest rate spread. (It is defined as the overnight rate minus the 10-year-and-over bond rate.) Other significant variables are the output gap and changes in the U.S. federal funds rate. The change in output is negatively related to the output gap. When the economy is in excess demand, and output is above the long-term growth potential of the economy, output growth can be expected to slow. As in the inflation equation, the

11. The VECM is used to forecast core inflation, which in this case was defined as the growth rate of the consumer price index excluding food, energy, and the effect of changes in indirect taxes.
U.S. federal funds rate is an indicator of the strength of external demand: an increase in U.S. rates is associated with increased demand outside Canada, which boosts Canadian output.

Originally, the VECM used gross M1 as the monetary aggregate, and dummy variables were used to offset instabilities associated with financial innovations. As mentioned above, however, it is difficult to assess changes in financial innovations as they are occurring. In order to get an ongoing measure of the distortions associated with such innovations, therefore, a model-based definition of transactions money was developed—adjusted M1.

Adjusted M1 was developed by Adam and Hendry (2000), and it attempts to correct for instabilities in M1 and to measure the size of distortions occurring. It is “model-based” in the sense that the VECM is used to forecast “distortion-free” M1 growth. Distortion-free M1 is calculated as the value for money predicted by the model over history, when all other variables are set at their actual historical values. This series is then regressed on the components of M1++. This generates weights for each of the components, which are then applied to the actual data. Adjusted M1 is thought of as the money series that would have occurred if financial innovations had not changed the relationship between money, output, prices, and interest rates from what it was in the early 1990s and if the model accurately represents reality. Thus, the difference between gross M1 and adjusted M1 is interpreted as a measure of the distortion in gross M1 since that time.

As mentioned above, while adjusted M1 has some advantages over other methods of accounting for distortions such as dummy variables, it is not as yet a fully satisfactory method of capturing transactions balances, and work to develop other measures is continuing.

A set of equilibrium conditions is imposed to better anchor the long-run forecasts of interest rates, the exchange rate, and the output gap. In the very long run, potential output is assumed to grow at around 2.3 per cent a year, inflation is 2 per cent, and money growth is 3.2 per cent, as implied by the long-run money-demand parameters and the assumptions for output and price growth. The overnight rate is assumed to be 4.8 per cent in the long run.

The VECM is an estimated model and should be judged, in part, by its ability to forecast. Armour et al. (1996) and Engert and Hendry (1998) both find that forecasts of the eight-quarter inflation rate from the VECM outperform those from a simple autoregressive model and a Phillips curve model. Chart 1 compares forecasts from the VECM with actual core inflation. It can be seen that the model forecasts capture the decline in inflation in the early 1990s as well as the upward trend in core inflation over the last two years.

The VECM can provide policy advice in two forms: (i) what would be the extent of inflation pressures if interest rates remained at current levels, and (ii) what path of interest rates would be required to ensure that inflation reaches the midpoint of the target range eight quarters into the future. (The eight-quarter time horizon is based on the horizon over which it is believed that the monetary authority can best influence inflation.)

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12. Personal chequing accounts are excluded to remove the effects of deposits at investment dealers (free-credit balances).

13. A more detailed description of adjusted M1, including the regression coefficients, is included in the appendix.

14. In a VECM model, if no long-run restrictions are imposed, variables will return to their sample mean rates of growth. Inflation, for example, would return to 4 per cent. When the model is estimated, the steady-state conditions are introduced via dummy variables. The conditions are not imposed over the whole sample. The steady-state condition of 2 per cent inflation, for example, is imposed only over the inflation-targeting period.

15. The long-run overnight rate is calculated as the U.S. real commercial paper rate plus the steady-state inflation rate of 2 per cent year-over-year and an estimated risk premium.

16. The VECM forecasts are one-year-ahead inflation forecasts obtained by estimating the model each quarter and calculating the out-of-sample forecast.
A measure of monetary policy stance can be derived, based on the difference between the rate of growth of M1 that would occur if policy remained unchanged (the fixed-interest-rate scenario) and the rate of growth of M1 needed to bring inflation back to 2 per cent (i.e., the second simulation). If, for example, forecast money growth (assumed policy is unchanged) is below that needed to bring inflation to 2 per cent in two years’ time, the monetary stance measure would be negative, suggesting that policy may be too tight.

**Solely empirical models**

While a more structural model is necessary for longer-term forecasts and for providing policy advice, a number of very simple empirical models are also used as indicators of output growth over a fairly short-term horizon. Simple correlations between growth in narrow money aggregates and growth in output suggest that money provides the most information about output two to three quarters ahead (see Cockerline and Murray 1981; Hostland, Poloz, and Storer 1988; and Muller 1992). The narrow aggregates provide useful leading information about output growth, particularly when lags in the release of data are taken into account. (The monetary aggregates are published a few weeks after month-end, while the National Accounts are received with a two-month lag.) Chart 2 illustrates the correlation between real gross M1 and real output growth. Real M1 predicted, for example, the increase in output growth over the period 1991 to 1994, as well as the decline in output growth in 1994. Based on past correlations, however, current output growth is surprisingly weak compared with real gross M1.

Simple linear models are used to exploit these correlations. In these models, the quarterly change in real output depends on the growth in real money balances in previous quarters. Increases in real money balances are followed by increased expenditures and increased output. Shift dummies are included to take account of structural breaks caused mainly by financial innovations. This model can be written:

\[
\text{real output}_t = \text{constant} + \beta_1 \text{real money}_{t-1} + \beta_2 \text{real money}_{t-2} + \ldots + \beta_3 \text{dumv}_t + \epsilon_t,
\]

where \(t\) stands for period \(t\); \(t-1, t-2\) represent lagged variables; output and real money are in terms of quarterly growth rates; \(dumv\) is a dummy variable; and \(\epsilon\) is the error term.\(^{17}\)

Recent work at the Bank suggests that an indicator model based on the narrow aggregate M1 provides the best money-based forecasts of output two to three quarters ahead. The M1 indicator model is, however, also the most dependent on shift dummies. As mentioned above, it can be difficult to know how to treat dummy variables in real time, since it is difficult to identify (on a month-by-month basis) which movements in the data are due to special factors or data errors that may be revised, as opposed to changes in the underlying trend. The model based on M1++, which is less dependent on shift variables, is therefore also used. Table 2 in the appendix shows the summarized coefficient values for the M1 and M1++ models. There are two models for each aggregate, generating one- and two-quarter-ahead forecasts of output growth.

Over a longer time horizon, models that also include the term spread have provided more reliable forecasts of output than those based purely on lagged money. (The term spread is calculated as the difference between the 10-year-and-over Government of Canada bond rate and the 90-day commercial paper rate.) A neural net model that includes the term spread is used, therefore, to forecast output growth one year ahead.

Neural networks are general models that can capture underlying non-linear relationships between a number of explanatory variables and an endogenous variable, in this case, real output growth. They are black-box models in that there is no economic

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\(^{17}\) Real output is national accounts GDP, and real money is defined as the nominal monetary aggregate deflated by the total consumer price index.
structure imposed on the equations, and the exact effects of any individual variable are difficult to identify. The money-based neural net used at the Bank contains four variables: the growth of real GDP, the term spread, the real 90-day rate (the 90-day commercial paper rate minus the four-quarter growth rate of the consumer price index), and the growth of real M1 (Tkacz 2001). Compared with the simple linear models, the neural net has a considerably more complex structure; in particular, it is able to capture non-linearities in relationships that the simpler models cannot. It also has a greater number of variables and takes into account the effect of interest rates and the term spread on output.

**A Broad-Money Model**

The models described above are all associated with the narrow money aggregates and the active-money story. Another model currently used is based on the broader money aggregate M2++. McPhail (2000) found little evidence that monetary disequilibria in M2++ are important in explaining inflation or output, but she found that growth in M2++ does help to forecast inflation even after output and interest rates are taken into account. (Chart 3 shows the general correlation between year-over-year growth in M2++ and both core and total CPI inflation.) She concludes that M2++ has a more passive role in the transmission mechanism than the narrow aggregates, evolving in response to other economic variables. In particular, McPhail suggests that growth in M2++ reflects agents’ expectations of future inflation.

**Chart 3**

**M2++ and Inflation**

Year-over-year percentage change

The broad-money model is not, therefore, developed in terms of deviations from a long-run demand for money but is one in which inflation, interest rates, output, and money are modelled as functions of lagged values of each other. The M2++ VAR contains four variables: core inflation, real output growth, growth in M2++, and the spread between 90-day commercial paper and 3-to-5-year government bonds. The latter was interpreted originally by McPhail as a measure of the opportunity cost of holding money but can also be thought of as a measure of expectations of future interest rates. (The general model and summarized coefficient values are included in the appendix.)

In practice, the model has been found to be less reliable than the M1-VECM in predicting inflation. In particular, it is very sensitive to starting-point shocks. Nevertheless, it is useful for assessing risks around the VECM forecast, especially during periods when broad money is showing a somewhat different trend than narrow money.

**Theoretical models calibrated to fit the characteristics of the data**

A third type of model being developed at the Bank is the theoretical dynamic general-equilibrium (DGE) model. The principle behind DGE models is that modelling economic activity, even for the aggregate economy, should begin with the economic problems faced by individual agents. It is the aggregation of all these decisions that forms the macroeconomic reality. These models are based on individual decision rules where agents are assumed to be maximizing utility. Moran (2000–2001) provides an overview of the ways in which DGE models are being used in monetary policy research. At the Bank, they are currently being used to better understand different aspects of the monetary transmission mechanism. It is hoped that such models will eventually be sufficiently well-developed to be used for forecasting.

**Applying the Money-Based Models**

The information from the models is combined, along with judgment, to forecast output and inflation based on the monetary indicators and to assess the risks associated with the forecast. These models may also be used to support specific risk analyses that consider “what if” scenarios. For example, if there is uncertainty about special factors affecting money growth, different assumptions can be made about the growth in transactions balances and, on this basis, the risks to inflation can be assessed.
In practice, our analysis has led us to focus on certain characteristics of money as particularly important for providing information on future trends in inflation and output. A simple, but nevertheless significant, element is to focus on the trends in money growth and to ignore month-to-month volatility. For this reason, we often place more weight on longer averages such as the three-month, six-month, and year-over-year growth rates, rather than on monthly rates of growth. Chart 4, for example, shows year-over-year, three-month, and monthly growth of M1. Clearly, the monthly numbers are volatile and can show sharp drops or increases that are not necessarily indicative of the trend. Year-over-year growth gives a better sense in this regard. The three-month growth rate is far less stable than year-over-year growth, but because it reacts more quickly to changes, it is sometimes a better indicator of turning points.

Persistent deviations of money from long-run money demand are associated with significant changes in inflation.

A second important aspect of the data is the extent to which money is estimated to be in disequilibrium. In particular, persistent deviations of money from long-run money demand are associated with significant changes in inflation. Chart 5 shows estimates of the money gap from the M1-VECM and the year-over-year increase in the core CPI. While increasing through the 1990s, the money gap is estimated to have been negative over this period. In other words, money supply has been below money demand. This suggests that, for much of this period, the money gap has had a moderating impact on inflation.

Conclusion

In conclusion, the analysis of money at the Bank draws on a variety of different monetary aggregates and a number of models. The models range from simple linear empirical models to those based more on economic theory. These models are used to provide forecasts that reflect not only an outlook that assumes money is an indicator of future activity, but also an outlook based on theory, where money plays an active role in the transmission mechanism. This analysis is then presented as one of the elements of the policy advice provided to the Governing Council prior to the fixed announcement dates for the target overnight rate (Longworth and Freedman 2000). By comparing this information with the QPM-based forecast and other indicators, such as those coming from the regional offices of the Bank and various measures of capacity utilization, policy-makers are able to ensure that their actions are based on a wide range of data and that they take into account different possible models of the economy.
Literature Cited


APPENDIX

Adjusted M1
The VECM is used to forecast “distortion-free” M1 growth from 1992Q1 to the current period. Distortion-free M1 is calculated as the value for money predicted by the model over history, when all other variables are set at their actual historical values. This series is regressed on currency, non-personal demand and notice deposits, and personal notice deposits. The regression is divided into two periods: 1992Q1 to 1994Q3 and 1994Q4 to 1999Q1. This reflects the fact that the parameter estimates after 1994Q3 are substantially different from those prior to that period. Based on data up to 2000Q3, the following equations are obtained:

1992Q1 to 1994Q3:
\[
\text{adjusted } M1 = 1.58(\text{currency}) + 0.29(\text{non-personal})
\]

1994Q4 to present:
\[
\text{adjusted } M1 = 1.60(\text{currency}) + 0.09(\text{non-personal}) + 0.11(\text{personal}).
\]

VECM
The full equations for the VECM are based on Adam and Hendry (2000).

The VECM is based on a system of four key equations. The model is an error-correction model because the variables are assumed to react to the deviation of money demanded from money supplied (the money gap). The equations have the general form:

\[
\Delta X_t = \Gamma(L)\Delta X_t + DZ_t + \alpha\beta'[X_{t-1}, D80a_{t-1}].
\]

where:
\[
X_t = [M1_t, CPIXFET_t, Y_t, RONf_t].
\]

\[
RONf_t = \text{level of “policy-free” interest rate = overnight rate minus estimated structural policy shocks, as measured by the residuals from the interest rate forecasting equation.}
\]

\[
M1_t = \log \text{level of adjusted M1}
\]
\[
Y_t = \log \text{level of real output}
\]
\[
CPIXFET_t = \log \text{level of the core consumer price index}
\]
\[
Z_t = [\text{constant, 3 seasonal dummies, output gap}_{t-1}, \Delta \log (exchange rate) \text{ from } t \text{ to } t-3, \Delta USFF_t, (D80b)^*NPN_t, D80a_t, RDIFF_{t-1}]
\]

\[
output \ gap_{t-1} = Y_t - \text{Bank of Canada’s estimate of potential output from QPM}
\]
\[
USFF_t = \text{U.S. federal funds rate}
\]
\[
D80b = 0 \text{ for 1979Q4 and before, and 1 thereafter}
\]
\[
NPN_t = \text{non-personal notice deposits}
\]
\[
D80a_t = 0 \text{ for 1979Q4 and before, and 1 for 1983Q1 and after. Increases linearly from 0 to 1 from 1980Q1 to 1982Q4.}
\]
\[
RDIFF_t = \text{difference between interest rates in Canada and interest rates in the United States}
\]
\[
\Gamma(L) = \text{matrix of parameters for a fourth-order lag process}
\]

The model is estimated in two steps. For technical reasons related to the cointegration estimation procedure that was used, it was best to estimate the \( \beta \) parameters using non-seasonally adjusted data in step 1. However, we are primarily interested in the movements of seasonally adjusted data, so the model was re-estimated in step 2 using that data.

In the first step, the coefficients of long-run demand are estimated (i.e., the values of \( \beta \)) by estimating the model with non-seasonally adjusted data over the period 1956Q1 to 2000Q2. Long-run money demand is modelled as a unique long-run cointegrating relationship between money, prices, output, and interest rates. The money gap is calculated as

\[
MGAP_t = c + M1_t - CPIXFET_t - \beta_{yt}Y_t + \beta_{rt}RONf_t + \beta_{d80t}D80a_t,
\]

where \( MGAP_t = M1 - M^D \)

\[
c = \text{long-run constant to ensure that the gap converges to 0 in steady state}
\]

\[
\beta_{yt}, \beta_{rt}, \beta_{d80t} = \text{estimates of the long-run parameters}
\]

In step 2, the forecasting model is estimated with seasonally adjusted data. The model has the same general format as the model in step 1, with some specification differences. Most important is the inclusion of the
overnight rate as an endogenous variable, as opposed to the “policy-free” overnight rate used in the first step. The forecasting model uses the values of the $\beta$ coefficients obtained in the first step. Furthermore, the inflation equation is used to forecast core rather than total CPI inflation.

Additional variables in the forecasting equations (Table 1):

- $ON_t = \text{level of the overnight rate}$
- $RPPP_t = \Delta \log(\text{exchange rate})_t - \Delta CPI_t + \Delta US CPI_t$
- $\text{spread}_t = \text{overnight rate} - 10\text{-year-and-over government bond rate}$
- $DPOLICY_t = \text{zero for 1992Q4 and before, increases linearly to one by 1999Q4, one thereafter}$
- $\text{MONPOL}_{t-1} = \text{zero for 1987Q4 and before, the 4-quarter inflation rate, less target inflation thereafter (where target inflation is 3 per cent from 1988Q1 to 1992Q4, 2 per cent from 1995Q4 on, and decreases linearly from 1993Q1 to 1995Q3)}$
- $D89_t = \text{zero for 1988Q4 and before, one from 1989Q1 to 1996Q2, and zero thereafter}$
- $D91_t = \text{zero for 1990Q4 and before, and one thereafter}$
- $UIIP_{t-1} = \text{deviation from uncovered interest rate parity}$
- $D60(Q1)_t = \text{one-period dummy with a value of one in 1960Q1}$
- $LPCOM_t = \log\text{level of commodity prices from the Quarterly Staff Projection}$
- $D73_t = \text{permanent shift dummy with a value of one from 1973Q1 and zero before}$

Over the forecast period, values are needed for the following variables:

- The exchange rate—this is obtained from an equation based on relative purchasing-power parity.
- The U.S. inflation rate and U.S. real federal funds rate—for forecasting purposes, the profiles are based on values in the staff projection.

**M2++ VAR**

The general form of the model is:

$$\Delta X_t = \Gamma(L)\Delta X_t + \varepsilon_t,$$

where $X$ is a vector of M2++, CPI excluding food and energy, real GDP, and the spread between the 90-day commercial paper rate and 3-to-5-year government bonds. With the exception of the interest rate, all variables are expressed in logarithms. The lag length of the model is three quarters. To allow for homogeneity of prices with respect to money in the long run, the lagged coefficients of money and prices in the money and price equations are restricted to sum to one. To preserve the neutrality of money, the lagged coefficients of money and prices in the output and interest rate equations are restricted to sum to zero.
### Table 1

**Summarized Coefficients of the Adjusted M1 VECM**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Δ M1</th>
<th>Δ CPIXFET</th>
<th>Δ Y</th>
<th>Δ ON</th>
<th>(RPPP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΣΔ M1</td>
<td>0.2111</td>
<td>0.1205</td>
<td>0.2382</td>
<td>13.990</td>
<td>-0.0929</td>
</tr>
<tr>
<td></td>
<td>(1.29)</td>
<td>(2.54)</td>
<td>(2.90)</td>
<td>(1.72)</td>
<td>(-0.70)</td>
</tr>
<tr>
<td>ΣΔ CPIXFET</td>
<td>1.4466</td>
<td>0.1961</td>
<td>-0.2382</td>
<td>1.4466</td>
<td>-0.2382</td>
</tr>
<tr>
<td></td>
<td>(2.70)</td>
<td>(1.29)</td>
<td>(2.90)</td>
<td>(2.70)</td>
<td>(2.90)</td>
</tr>
<tr>
<td>ΣΔ Y</td>
<td>-0.2236</td>
<td>-0.0042</td>
<td>-0.3509</td>
<td>-0.2236</td>
<td>-0.3509</td>
</tr>
<tr>
<td></td>
<td>(-1.17)</td>
<td>(-1.20)</td>
<td>(-1.63)</td>
<td>(-1.17)</td>
<td>(-1.63)</td>
</tr>
<tr>
<td>ΣΔ ON</td>
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<td>0.0196</td>
<td>-0.0494</td>
<td>0.0005</td>
<td>-0.0494</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(2.84)</td>
<td>(2.90)</td>
<td>(0.33)</td>
<td>(2.90)</td>
</tr>
<tr>
<td>RPPP</td>
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<td>-0.0008</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td>-0.0008</td>
</tr>
<tr>
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<td>-0.0008</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td>-0.0008</td>
</tr>
<tr>
<td>Constant</td>
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<td>0.0091</td>
<td>0.0094</td>
<td>0.0091</td>
<td>0.0094</td>
</tr>
<tr>
<td></td>
<td>(-0.65)</td>
<td>(4.03)</td>
<td>(5.85)</td>
<td>(4.03)</td>
<td>(5.85)</td>
</tr>
<tr>
<td>output gap, t-1</td>
<td>0.1676</td>
<td>0.0970</td>
<td>-0.1654</td>
<td>0.1676</td>
<td>-0.1654</td>
</tr>
<tr>
<td></td>
<td>(1.46)</td>
<td>(3.23)</td>
<td>(3.75)</td>
<td>(1.46)</td>
<td>(3.23)</td>
</tr>
<tr>
<td>ΣΔ log (exchange rate)* from t to t-3</td>
<td>0.0449</td>
<td>0.1098</td>
<td>-33.0944</td>
<td>0.0449</td>
<td>-33.0944</td>
</tr>
<tr>
<td>USFFt</td>
<td>0.0005</td>
<td>0.0010</td>
<td>0.0021</td>
<td>0.0005</td>
<td>0.0021</td>
</tr>
<tr>
<td></td>
<td>(0.37)</td>
<td>(2.51)</td>
<td>(3.20)</td>
<td>(0.37)</td>
<td>(2.51)</td>
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<tr>
<td>MGAPt</td>
<td>-0.0649</td>
<td>0.0278</td>
<td>0.0112</td>
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<td>0.0112</td>
</tr>
<tr>
<td></td>
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<td>(4.36)</td>
<td>(2.22)</td>
<td>(-2.81)</td>
<td>(4.36)</td>
</tr>
<tr>
<td>D80b * Δ NPNt</td>
<td>-0.0138</td>
<td>0.0056</td>
<td>-0.0247</td>
<td>-0.0138</td>
<td>-0.0247</td>
</tr>
<tr>
<td></td>
<td>(-0.84)</td>
<td>(1.10)</td>
<td>(3.03)</td>
<td>(-0.84)</td>
<td>(1.10)</td>
</tr>
<tr>
<td>D80q4</td>
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<td>-0.0040</td>
<td>-0.0040</td>
<td>0.0060</td>
<td>-0.0040</td>
</tr>
<tr>
<td></td>
<td>(0.57)</td>
<td>(-3.43)</td>
<td>(-3.43)</td>
<td>(0.57)</td>
<td>(-3.43)</td>
</tr>
<tr>
<td>DPOLICYt</td>
<td>0.0005</td>
<td>-0.0040</td>
<td>-0.0040</td>
<td>0.0005</td>
<td>-0.0040</td>
</tr>
<tr>
<td>MODEL1t</td>
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<td>-0.0067</td>
<td>-0.0038</td>
<td>-0.0038</td>
<td>-0.0067</td>
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<tr>
<td></td>
<td>(1.31)</td>
<td>(1.42)</td>
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<td>(1.42)</td>
</tr>
<tr>
<td>D91t</td>
<td>-0.0008</td>
<td>0.0196</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td>0.0196</td>
</tr>
<tr>
<td></td>
<td>(-3.81)</td>
<td>(2.84)</td>
<td>(-3.81)</td>
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<td>(2.84)</td>
</tr>
<tr>
<td>RDIFft</td>
<td>-0.0008</td>
<td>0.0196</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td>0.0196</td>
</tr>
<tr>
<td></td>
<td>(-3.81)</td>
<td>(2.84)</td>
<td>(-3.81)</td>
<td>(-3.81)</td>
<td>(2.84)</td>
</tr>
<tr>
<td>UIPt</td>
<td>-0.0008</td>
<td>0.0196</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td>0.0196</td>
</tr>
<tr>
<td></td>
<td>(-3.81)</td>
<td>(2.84)</td>
<td>(-3.81)</td>
<td>(-3.81)</td>
<td>(2.84)</td>
</tr>
<tr>
<td>D60Q1t</td>
<td>-0.0008</td>
<td>0.0196</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td>0.0196</td>
</tr>
<tr>
<td></td>
<td>(-3.81)</td>
<td>(2.84)</td>
<td>(-3.81)</td>
<td>(-3.81)</td>
<td>(2.84)</td>
</tr>
<tr>
<td>Δ LFCOMt</td>
<td>-0.0008</td>
<td>0.0196</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td>0.0196</td>
</tr>
<tr>
<td></td>
<td>(-3.81)</td>
<td>(2.84)</td>
<td>(-3.81)</td>
<td>(-3.81)</td>
<td>(2.84)</td>
</tr>
<tr>
<td>D73*RPPPt</td>
<td>-0.0008</td>
<td>0.0196</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td>0.0196</td>
</tr>
<tr>
<td></td>
<td>(-3.81)</td>
<td>(2.84)</td>
<td>(-3.81)</td>
<td>(-3.81)</td>
<td>(2.84)</td>
</tr>
</tbody>
</table>

### Table 2

**Summarized Coefficients of Single-Equation Real GDP Indicator Models**

<table>
<thead>
<tr>
<th>Model</th>
<th>Real gross M1</th>
<th>Real M1++</th>
<th>Linear term spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 quarter</td>
<td>2 quarters</td>
<td>1 quarter</td>
</tr>
<tr>
<td>Constant</td>
<td>3.29</td>
<td>3.29</td>
<td>3.29</td>
</tr>
<tr>
<td></td>
<td>(10.19)</td>
<td>(11.27)</td>
<td>(13.08)</td>
</tr>
<tr>
<td>Money growth</td>
<td>0.32</td>
<td>0.29</td>
<td>0.35</td>
</tr>
<tr>
<td>Lags 1–4</td>
<td>(6.97)</td>
<td>(6.93)</td>
<td>(5.87)</td>
</tr>
<tr>
<td>Lags 2–5</td>
<td>(6.88)</td>
<td>(5.00)</td>
<td>(4.92)</td>
</tr>
<tr>
<td>Lags 4–8</td>
<td>(6.88)</td>
<td>(5.00)</td>
<td>(4.92)</td>
</tr>
<tr>
<td>Dummy variable</td>
<td>-3.06</td>
<td>-2.94</td>
<td>-2.85</td>
</tr>
<tr>
<td>Real CP90t</td>
<td>-6.07</td>
<td>-6.88</td>
<td>-5.00</td>
</tr>
<tr>
<td>spread*</td>
<td>0.0449</td>
<td>0.0196</td>
<td>-0.0008</td>
</tr>
<tr>
<td></td>
<td>(1.14)</td>
<td>(1.20)</td>
<td>(-0.70)</td>
</tr>
</tbody>
</table>

* Calculated as the 10-year-and-over bond rate minus the 90-day commercial paper rate.

### Table 3

**Summarized Coefficients of the M2++ VAR (1968Q1–2000Q1)**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Δ M2++</th>
<th>Δ CPIXFET</th>
<th>Δ Y</th>
<th>Δ spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΣΔ M2++</td>
<td>-0.005</td>
<td>-0.003</td>
<td>-0.0001</td>
<td>0.356</td>
</tr>
<tr>
<td></td>
<td>(2.13)</td>
<td>(-3.77)</td>
<td>(-0.07)</td>
<td>(2.35)</td>
</tr>
<tr>
<td>Δ M2++</td>
<td>0.777</td>
<td>0.230</td>
<td>0.290</td>
<td>-18.156</td>
</tr>
<tr>
<td></td>
<td>(8.92)</td>
<td>(3.36)</td>
<td>(2.08)</td>
<td>(-1.21)</td>
</tr>
<tr>
<td>Δ CPIXFET</td>
<td>0.223</td>
<td>0.710</td>
<td>-0.290</td>
<td>18.156</td>
</tr>
<tr>
<td></td>
<td>(2.44)</td>
<td>(8.97)</td>
<td>(-2.08)</td>
<td>(1.21)</td>
</tr>
<tr>
<td>Δ Y</td>
<td>0.134</td>
<td>0.001</td>
<td>0.349</td>
<td>-20.569</td>
</tr>
<tr>
<td></td>
<td>(1.50)</td>
<td>(0.07)</td>
<td>(3.25)</td>
<td>(-1.40)</td>
</tr>
<tr>
<td>Δ spread</td>
<td>-0.005</td>
<td>-0.003</td>
<td>0.004</td>
<td>-0.337</td>
</tr>
<tr>
<td></td>
<td>(-0.80)</td>
<td>(-0.30)</td>
<td>(2.10)</td>
<td>(-1.82)</td>
</tr>
</tbody>
</table>