Canada has a very open economy which is broadly integrated into the global economy. Accordingly, the Bank of Canada has developed several models to analyze and forecast economic developments in the rest of the world. Given the importance of Canada’s economic relationship with the United States, the Bank of Canada places considerable importance on generating internal forecasts of U.S. economic activity as an input to the Canadian projection. Over the past couple of years, Bank of Canada staff have been using a new macroeconometric model, MUSE, to analyze and forecast developments in the U.S. economy. The model is a system of equations that describe the interactions among the principal U.S. macroeconomic variables, including gross domestic product, inflation, interest rates, and the exchange rate. At the same time, a smaller forecasting model for Europe, NEUQ (New European Union Quarterly Model) has been introduced.

Although the main goal of MUSE and NEUQ is to provide forecasts of foreign economic variables as inputs to the Canadian projection, these models can also be used independently to address other issues, such as understanding the responses of foreign economies to different shocks. It is hoped that these models will result in a better-informed perspective on current and future external economic developments.

The analysis and forecasting of economic developments in the rest of the world plays a critical role in the formulation of Canadian monetary policy. In particular, the Bank of Canada places considerable importance on generating internal forecasts of U.S. economic activity as an input to the Canadian projection (Macklem 2002). This focus stems from the close real and financial linkages between the Canadian and U.S. economies. Although different generations of Canadian economic models used at the Bank have undergone significant changes in terms of theoretical underpinnings or macroeconomic structure, they have always relied on other models or sources of information for estimates of external economic activity. Over the past couple of years, Bank staff have been using a new macroeconometric model, MUSE (Model of the U.S. Economy), to analyze and forecast developments in the U.S. economy.¹ The model is a system of equations that describe the interactions among the principal U.S. macroeconomic variables, such as gross domestic product (GDP), inflation, interest rates, and the exchange rate. MUSE contains more than 30 behavioural equations, most of which are estimated. At the same time, a smaller forecasting model for Europe, NEUQ (New European Union Quarterly Model) has been introduced. In addition, to improve our understanding of global economic issues and to complement other Bank models, the staff are currently developing BoC-GEM (Bank of Canada Global Economy Model), an adaptation of GEM, the multi-country dynamic stochastic general-equilibrium model built at the International Monetary Fund. While this article mainly focuses on MUSE, brief descriptions

¹. MUSE has been developed at the Bank. A detailed discussion of the specification of the model can be found in Gosselin and Lalonde (2005).
of the specifications and purpose of NEUQ (Box 1) and BoC-GEM (Box 2) are also included.

**Importance of International Projections**

There is a conceptual difference between the Bank staff’s projection for Canada and those for the rest of the world. The Canadian projection is the staff’s assessment of the most likely path for the economy. It includes a recommendation to the Bank’s Governing Council on the optimal profile for the overnight interest rate that will return inflation to the 2 per cent midpoint of the inflation-control target range. In contrast, the primary role of the international projection is not to provide specific recommendations for monetary policy, but to forecast external economic activity and inflation, together with the probable interest rate response by foreign monetary authorities. The models involved in this process are therefore not as oriented towards policy analysis as is the Canadian model.

Although the main goal of MUSE and NEUQ is to provide forecasts of foreign economic variables as inputs to the Canadian projection, these models can also be used independently to address other issues of interest, including understanding the responses of foreign economies to different shocks.

Although the main goal of MUSE and NEUQ is to provide forecasts of foreign economic variables as inputs to the Canadian projection, these models can also be used independently to address other issues of interest, including understanding the responses of foreign economies to different shocks (e.g., oil-price shocks, productivity shocks, and fiscal policy shocks). Such analysis can also be useful for comparison with the Canadian economy.

**Basic Structure of MUSE**

Long-term planning and the presence of costs in adjusting economic activities both play a key role in MUSE. Thus, the specification of most behavioural equations relies on the polynomial adjustment-cost (PAC) approach, which is widely used in the Federal Reserve Board’s FRB/US model (Brayton et al. 1997). In PAC models, households and firms make informed decisions based on expectations about future circumstances, and these decisions rest on forecasts of desired goals that would be selected in the absence of adjustment costs. While firms and households modelled under the PAC approach behave optimally, planning lags, contractual requirements, and other frictions prevent them from moving immediately towards the desired level. Decisions subject to higher adjustment costs require longer planning horizons.

**PAC models stand halfway between general-equilibrium models, in which the dynamics are entirely explained by theory, and reduced-form models, which are based solely on data.**

The PAC specification is akin to an error-correction model. It contains two equations: one for the desired level (often based on a cointegrating relationship) and one for the dynamic path of the variable under consideration (in growth rates). The dynamic behaviour of the variable of interest is determined by the lagged gap between the actual and desired values of the variable, the lagged values of the dependent variable, and expected future changes in the desired level of the variable. By using a very general description of adjustment costs, these models are able to closely match the persistence in the historical time-series data. As a result, they stand halfway between general-equilibrium models, in which the dynamics are entirely explained by theory, and reduced-form models, which are based solely on data.

The stock-flow equilibrium is fully described in MUSE. In steady state, the model defines specific equilibrium values for all stocks. For instance, the equilibrium ratio of business capital to output is determined by firms’ financing costs and asset-specific depreciation rates and relative prices. The tax rate adjusts to meet a target

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2. The number of lags of the dependent variable depends on a very general description of the order of adjustment costs. In PAC models, it can be costly to adjust the variable of interest in terms of level, change, rate of acceleration, and so on (Tinsley 1993).
level of government debt in the steady state. The model also converges to a constant ratio of net foreign assets to GDP. Finally, households’ human wealth in equilibrium is influenced by personal income, taxes, and government transfers.

Nominal dynamics in MUSE are determined by a short-run Phillips curve that describes the positive relation between inflation and the output gap. Thus, the difference between actual and potential output is a key driver of inflation in the model. In addition, inflation is determined in the context of forward-looking rational expectations, and the persistence in the inflation process is explained by adjustment costs. Monetary policy is modelled according to a simple forward-looking rule. The central bank reacts with the objective of closing both the current output gap and the expected gap between actual inflation and an implicit inflation target. The other key adjustment mechanism in the model is the real exchange rate, which reacts to interest rate differentials and the current account balance so as to restore the target position for net foreign assets.

Box 1: The NEUQ Model*

The euro area and the United Kingdom (U.K.) combined account for approximately 5 per cent of Canadian exports and 11 per cent of Canadian imports and represent Canada’s second most important trading partner after the United States. Although total Canadian trade and financial linkages with the two economies are smaller than those with the United States (U.S.), they are magnified by the indirect effect that the euro area and the United Kingdom have on the U.S. economy and on world commodity prices. To account for these direct and indirect effects on the Canadian economy, projections of key macroeconomic variables for the euro area and the United Kingdom are used as inputs into MUSE, ToTEM, and the Bank of Canada internal projection for commodity prices.

In March 2005, the Bank of Canada incorporated NEUQ, its New European Union Quarterly projection model, into the projection process (see Piretti and St-Arnaud 2006 for details). NEUQ is a small estimated reduced-form model built using the same “conventional”—or Phillips curve—paradigm as MUSE, but at a higher level of aggregation. NEUQ is primarily designed as a tool to project the future path of real output, inflation, and the policy rate for the euro area and U.K. economies.

The NEUQ model consists of two country blocs (one for the euro area and one for the United Kingdom), which are endogenous to each other via the foreign demand channel. Each country bloc features three behavioural equations. Aggregate demand, the first equation, relates real output to the interest rate, the real effective exchange rate, and foreign activity (U.K., U.S. and Asian demand for the euro area, and euro area, U.S. and Asian demand for the United Kingdom). Aggregate supply, the second equation, is modelled using a forward-looking Phillips curve where inflation is determined by the output gap, the real effective exchange rate, and the real price of oil. As in MUSE, inflation dynamics are modelled using the PAC (polynomial adjustment-cost) approach to account for expected inflation and some persistence in the adjustment process of inflation.2 The model is closed by a third endogenous equation, an estimated forward-looking interest rate rule for monetary policy. It relates the nominal short-term interest rate to the deviation of forecast inflation from the monetary authority’s inflation target, and to the deviation of real output from potential output. As in MUSE, each dynamic equation has a steady state to which the model converges in the long run.

The NEUQ model delivers reasonably accurate projections of key U.K. and euro area macro variables at a variety of horizons and also provides a useful tool for policy analysis. An interesting feature of the model is that the simulation results suggest that output and inflation exhibit a greater degree of persistence to shocks in the euro area than in the United Kingdom.

* This box was written by Denise Côté.

1. The euro area and the United Kingdom account for around 20 per cent of U.S. exports and 17 per cent of U.S. imports, and represent the second-largest U.S. trading partner after Canada (International Monetary Fund 2006).

2. As part of the aggregate supply side, potential outputs for the U.K. and euro area economies are estimated by means of a Hodrick-Prescott filter, conditioned by an equilibrium path generated by a structural vector autoregression (St-Arnaud 2004).
Real GDP
In MUSE, U.S. real GDP is decomposed into household spending, business investment, government spending, and international trade. Hence, MUSE can be used to analyze the impact of a wide variety of shocks on the U.S. economy.

In MUSE, U.S. real GDP is decomposed into household spending, business investment, government spending, and international trade.

Household spending
Household spending is modelled in MUSE as the sum of total consumption and residential investment. According to the permanent-income hypothesis, a household’s consumption in any given period depends on its permanent income, defined as the current value of household wealth (human and non-human). In this context, consumption changes when agents modify their expectations of future income, or when agents make expectational errors. Most economists, however, reject the pure permanent-income hypothesis, arguing that the existence of credit-constrained households limits its applicability and that households could also choose to save for precautionary reasons. In both cases, consumption may be more closely related to current than to permanent income.

On this basis, the desired level of household purchases in MUSE is consistent with the permanent-income hypothesis, but deviations from the hypothesis are allowed along the dynamic path (Gosselin and Lalonde 2003). The desired level of household spending is a function of real interest rates, expected future disposable labour income (human wealth), and stocks of real estate and financial wealth.3 In addition to the standard determinants from the dynamic PAC specification, short-run movements in household spending are influenced by current disposable income. The percentage of households that are credit constrained is 27 per cent; 73 per cent are forward looking, and their behaviour is represented by the permanent-income hypothesis. Consequently, the spending patterns of households adjust relatively sluggishly to differences between actual expenditures and their desired level. To account for credit-constrained consumers, a negative effect of higher oil prices is also included to proxy the impact of this variable on disposable income.

Business investment
In MUSE, firms can invest in three types of capital goods: non-residential structures, high-tech equipment, and other equipment excluding high-tech. Modelling business investment using these different categories of expenditure allows for substitution and complementarity effects among the three types of capital. The user cost of capital and the level of output are the key determinants of long-run movements in desired capital stocks.4 Desired investment flows are derived from the desired capital stocks. As can be imagined, reaching the desired investment levels entails significant adjustment costs, which can derive from such diverse sources as information gathering, plant or product design, testing, and regulatory approval. The dynamic path of investment is specified in terms of the PAC structure. In some cases, output growth is also included in order to capture cash-flow effects for some subset of financially constrained firms. In all cases, investment exhibits substantial inertia to movements in output or user costs.

The trend of labour input and total factor productivity, which are based on exogenous assumptions, combined with the forecasted capital stock, feed into a Cobb-Douglas production function to generate a projection for potential output.5

Government spending
Several channels through which the government sector affects the economy are modelled in MUSE. Aside from government consumption and investment, which feed directly into GDP, taxes and transfers partly determine personal disposable income. Government debt influences consumption through its effect on

3. Real estate wealth is a function of the stock of residential capital and of house prices. The main determinant of residential investment flows and house prices is the real mortgage rate. Financial wealth is anchored on businesses’ capital stock, government debt, and net foreign assets.

4. The user cost of capital depends on the relative prices of investment goods, interest rates, and depreciation rates.

5. Over the historical period, potential output is measured using the eclectic approach, which consists of Hodrick-Prescott filters to which an equilibrium path generated by a structural vector autoregression is added as information conditioning the filters. We use this approach to estimate the two components of potential GDP: the full-employment labour input and trend labour productivity. The full-employment labour input is a function of the population, the equilibrium participation rate, the equilibrium unemployment rate, and the equilibrium hours worked (Gosselin and Lalonde 2006).
household financial wealth. It also influences the risk premium on Treasury bonds, which affects the cost of capital for businesses and mortgage rates. Total government expenditures (including transfers) are a function of the output gap, which reflects the operation of automatic stabilizers. The greater the recession or degree of excess supply, the higher are government expenditures in the form of transfers to the household sector.

There is a fiscal policy rule in MUSE: the government adjusts revenues to achieve an exogenous debt target in the long run. Political constraints, budget deliberations, and implementation lags prevent the government from adjusting the tax rate to its desired level in the near term. Thus, the aggregate tax rate adjusts slowly to its target level.

International trade

MUSE is a one-good model: it does not differentiate between traded and non-traded goods. Desired export and import volumes are modelled similarly: both react to relative prices, and they react to foreign and domestic income, respectively. Adjustment costs can be important in the tradable goods sector. These costs reflect costs of changing suppliers or markets, such as costs owing to a lack of familiarity with the commercial practices of foreign markets, commercial policies (e.g., taxes and tariffs), or other border effects. Given such costs, profit-maximizing firms must be forward looking in their behaviour, anticipating domestic and foreign growth in order to reduce the costs of sudden shifts in demand (Gagnon 1989). Such frictions justify using the PAC approach to model dynamic movements in trade volumes and relative prices.

In MUSE, the desired level of real imports is a function of private domestic demand, openness to global trade, and the relative price of imports. The desired level of real exports is also determined by the standard paradigm of income and relative prices, augmented by a proxy for globalization. In the dynamic PAC specification of imports, the change in the output gap is added to account for the fact that the short-run income elasticity of imports is much higher than its long-run value (Hooper, Johnson, and Marquez 2000). For the same reason, the foreign output gap is included in the

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6. Openness to trade is proxied by the volume of trade between the countries that belong to the Organisation for Economic Co-operation and Development. For more details, see Gosselin and Lalonde (2004).

Box 2: BoC-GEM

Economists at the Bank of Canada are currently adapting GEM (the Global Economy Model), developed by the International Monetary Fund, to the Bank’s needs.

In this version of GEM, the BoC-GEM, the global economy is divided into five countries or country groups: Canada, the United States, emerging Asian countries that import raw materials (primarily China and India), commodity-exporting countries (including members of the Organization of Petroleum-Exporting Countries), and the rest of the world (notably Europe and Japan). Owing to the size of the natural resource sector in Canada, this version of GEM incorporates, in addition to the tradable and non-tradable goods sectors, the oil and gas sector, as well as other commodities.

BoC-GEM is a dynamic general-equilibrium (DGE) model. Its theoretical and microeconomic foundations are therefore highly developed and are modelled on the principles of supply and demand. The model’s parameters were calibrated using data and microeconomic studies or by drawing on other DGE models. Overall, the properties of BoC-GEM are compatible with those of estimated or partially estimated models. Because of its complexity and specific characteristics, BoC-GEM is complementary to the MUSE, NEUQ, and ToTEM models. It will be used primarily for studying issues that require a global perspective, such as global imbalances, causes and effects of the rise in oil prices observed in recent years, and the impact of open markets on competitiveness and price levels. The model will also be used in the context of the international projection to guide the staff’s judgment on monetary policy issues that require an integrated global perspective. Finally, BoC-GEM can help Bank staff to ensure that economic forecasts generated using MUSE, NEUQ, and ToTEM form a consistent whole.
dynamic exports equation. The desired levels of the relative prices of imports and of exports are a function of the real exchange rate, the relative price of oil, and a downward deterministic trend that captures the higher productivity in the traded-goods sector relative to the non-traded-goods sector. Import prices play an important role in MUSE, since they help to determine import volumes and feed directly into the inflation process.

U.S.-Dollar Real Effective Exchange Rate

As mentioned earlier, in the steady state, MUSE reaches a target ratio of net foreign assets to GDP. This convergence is facilitated by the adjustment of the real effective exchange rate. Given the steady-state version of the model, there is a unique value of the exchange rate such that the ratio of net foreign assets to GDP converges to its target level. Therefore, the real exchange rate generates the movements in the trade balance that are required to attain the target ratio for net foreign assets.

A partly calibrated error-correction equation governs exchange rate fluctuations. The key short-run determinants are the gap between the actual and equilibrium exchange rates and real interest rate differentials between the United States and its major trading partners. There is a dichotomy between the short-run and long-run behaviour of the exchange rate. For instance, following an increase in domestic demand, the exchange rate appreciates in the short run because of positive interest rate differentials, but then depreciates in order to generate a trade surplus consistent with a restoration of the target for net foreign assets.

Inflation

Inflation persistence due to sticky prices can be modelled in many ways, from menu costs to price-setting behaviour in the fashion of Calvo (1983) and Taylor (1980). More recent research focuses on New Keynesian Phillips curves or their variants. Hybrid specifications, such as that of Galí and Gertler (1999), can identify significant inflation persistence with the use of lagged values of inflation.

Instead of choosing one of these approaches, we do not take a rigid stance on the theory of inflation determination. Like Kozicki and Tinsley (2002), we use a more general PAC approach and let the data determine the persistence of inflation, rather than impose it by specification. This approach assumes rational economic agents that balance the cost of price adjustments against the costs of diverging from the desired price level. The costs associated with changing prices lead firms to smooth the inflation profile, generating persistence in the inflation process. In addition to the leads and lags of inflation that capture inflation expectations and adjustment costs, the inflation process in MUSE is driven by the current output gap and past movements in the relative price of imports.

Monetary Policy

A number of interest rates are modelled in MUSE and, in turn, influence various elements of the model. They are all anchored, in one way or another, to the U.S. federal funds rate. The nominal federal funds rate in MUSE follows a Taylor (1993) rule. This type of monetary rule is a good description of the Federal Reserve’s actions and is consistent with the monetary authority’s dual mandate of maintaining low and stable inflation while supporting maximum sustainable employment. Based on the work of English, Nelson, and Sack (2002), the rule is specified in terms of the neutral rate, the future gap between inflation and the implicit inflation target, the current output gap, and a smoothing coefficient. The neutral rate is fixed at its steady-state value. This value is endogenous in MUSE: it is equal to the unique value of the real interest rate that makes aggregate demand equal to aggregate supply in the steady state.

Shock Analysis

Several relevant shocks can be used to illustrate the dynamic behaviour of MUSE. We simulate the impact...
of three temporary shocks: a shock to demand, a shock to the federal funds rate, and an inflation shock. We also look at a permanent shock to total factor productivity.\(^8\)

**A temporary shock to private domestic demand**

In this scenario, an increase in private domestic demand generates a positive output gap that lasts about two years. The opening of the output gap yields a small but persistent increase in inflation. In reaction to these two developments, the monetary authority raises the federal funds rate and engineers a small degree of excess supply to bring inflation back to the target. This rate increase feeds through the term structure and raises longer-term rates, thereby pushing household spending and investment back to the control scenario.\(^9\) Investment is the slowest to return to equilibrium, since adjustment costs are higher for this component. Since fiscal policy is countercyclical, government transfers decrease following the shock. Lower transfers reduce personal disposable income flows and human wealth, which depresses household spending.

Real imports rise in the short run because part of the increase in demand is for imported goods and services. Since this scenario assumes no response in foreign variables, real exports fall in response to a short-term appreciation in the real exchange rate that results from higher domestic interest rates. The deterioration in the trade balance leads to a temporary worsening of the net foreign assets position, thereby requiring a depreciation of the real exchange rate in the medium run in order to return net foreign assets to the target position (Chart 1, page 27).

**A temporary shock to the federal funds rate**

This shock illustrates the various channels of the U.S. monetary policy transmission mechanism that are modelled in MUSE. In this simulation, the Federal Reserve raises the nominal federal funds rate by 100 basis points in the first period and maintains it above control for about six quarters, which reflects a preference for interest rate smoothing. Through the term structure of interest rates, the change in short-term interest rates affects all other interest rates in the model. Higher interest rates reduce both consumption and investment in the early years of the simulation. Again, investment is slower to return to control. The impact on consumption would be greater absent the fiscal response, which generates an increase in government transfers, lending support to disposable income.

Positive interest rate differentials generate an appreciation of the dollar, leading to lower exports in the short run. Imports fall as well because the short-run effect from the reduction in private income dominates the relative price effect. Higher interest rates raise the interest costs associated with net foreign assets. To compensate, MUSE must generate a depreciation of the dollar in the longer run to improve the trade balance and return net foreign assets to its target. The excess supply combined with the higher U.S. dollar has a negative impact on inflation (Chart 2, page 28).

**An inflation shock**

In this simulation, we look at the effects of higher inflation on the U.S. economy. The shock is relatively persistent, since inflation remains above the implicit target for about three years. This reflects the extent of the adjustment costs inherent in the inflation process. The Federal Reserve reacts quickly, but tightens policy by a relatively small amount. Two reasons explain this behaviour. The first is that more than half of the inflation shock dissipates after one quarter. The second explanation pertains to the Fed’s dual mandate: the interest rate tightening is limited by the fact that it generates a negative output gap, which by itself would necessitate a monetary easing. Yet, the Fed creates the excess supply that is required to bring inflation back to the implicit target (Chart 3, page 29).

**A permanent shock to total factor productivity**

In this scenario, we simulate the effects of an unexpected permanent increase in the level of total factor productivity. This increase raises potential output immediately. The adjustment costs related to the components of demand are such that, initially, demand does not react as fast as supply. Therefore, the shock creates a short-lived but nevertheless significant excess supply, which leads to a temporarily lower inflation rate. The Fed responds by reducing interest rates, which speeds up the adjustment of demand. Household spending is positively affected by the permanent increase in human wealth, while investment flows rise in order to reach firms’ equilibrium ratio of capital to output.

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8. In all simulations, we assume that foreign output and foreign interest rates do not respond to shocks in the United States.

9. The control scenario refers to the variables’ profile in the absence of shocks.
Owing to the negative output gap, government transfers increase significantly in the short run. Government expenditures rise by the same amount as output in the long run, which restores the steady-state size of the government sector. Imports rise permanently, in line with the permanent increase in private domestic demand. The increase in imports in the longer run deteriorates net foreign assets such that, in order to bring net foreign assets back to its target, a permanent depreciation of the exchange rate is needed. This depreciation raises real exports in the steady state and dampens the increase in real imports (Chart 4, page 30). MUSE generates a completely different response in the context of an expected productivity shock. For instance, an increase in total factor productivity that is expected to occur two years from now is inflationary, not deflationary, since it initially creates excess demand: agents anticipate the shock to future income and increase demand immediately. In this case, the Fed raises rates and creates excess supply, which eventually brings inflation back to target.

**Conclusions**

Through an extensive application of PAC models, we have developed in MUSE what we believe to be a good balance between theoretical structure and forecasting accuracy. Importantly, MUSE can also be used for policy simulations. It can, for example, be used to examine issues ranging from how the U.S. economy might react to inflationary pressures to the consequences of sustained productivity gains. As a result, it is hoped that this model will result in a more enlightened perspective on current and future economic developments in the United States.

The NEUQ model is a useful complement to MUSE in the context of the international economic projection. Furthermore, given the high and rising real and financial linkages within the global economy, the BoC-GEM model will be key to examining economic issues from an international perspective. Taken together, these three models are valuable tools in the formulation of Canadian monetary policy.
Chart 1
Results of a Demand Shock
The second variable is indicated by the dashed line.

Household expenditures and business investment

Output gap

Inflation rate
Quarter-over-quarter annual rates, percentage points

Real and nominal federal funds rate
100 basis points = 1

Exports and imports

Real effective exchange rate
Chart 2

Results of a 100-Basis-Point Shock to the Federal Funds Rate

The second variable is indicated by the dashed line.

Real and nominal federal funds rate

Real effective exchange rate

Household expenditures and business investment

Exports and imports

Output gap

Inflation rate

Quarter-over-quarter at annual rates, percentage points
Chart 3

Results of an Inflation Shock
The second variable is indicated by the dashed line.

Inflation rate
Quarter-over-quarter at annual rates, percentage points

Real and nominal federal funds rate
100 basis points = 1

Household expenditures and business investment

Output gap

Real effective exchange rate

Exports and imports
Chart 4
Results of a Permanent Shock to Total Factor Productivity
The second variable is indicated by the dashed line.

Output and potential output

Output gap

Inflation rate
Quarter-over-quarter at annual rates, percentage points

Real and nominal federal funds rate
100 basis points = 1

Household expenditures and business investment

Real effective exchange rate
Literature Cited


