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# A Non-Paradoxical Interpretation of the Gibson Paradox

**by**

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# Contents

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## Abstract

In this study, we show how, to yield the real cost of borrowing, the price level can be combined with the nominal interest rate in a monetary regime where the level of prices is trend stationary. We show that the price level then conveys intertemporal information in a way similar to nominal interest rates. We estimate real interest rate series for the gold-standard period in the United Kingdom under the assumption the agents expect the price level to come back to its long-run equilibrium value. The positive correlation between the price level and the nominal interest rate—known as the Gibson paradox and far from being paradoxical—helps explain why the nominal interest rate was so stable in a period characterized by numerous wars and important gold discoveries. The new real interest rate series provides the opportunity to re-examine Barro's (1987) finding on the effect of temporary military spending on interest rates. It also relaxes the assumption that the nominal long-term interest rate is also the expected real rate.

## Résumé

L'auteur de l'étude montre qu'il est possible de calculer le taux d'intérêt réel en combinant le niveau des prix avec le taux d'intérêt nominal dans le cadre d'un régime monétaire où le niveau des prix est stationnaire par rapport à sa tendance. Il constate en effet que, sous un tel régime, le niveau des prix véhicule de l'information intertemporelle tout comme les taux d'intérêt nominaux. L'auteur estime une série chronologique de taux d'intérêt réels pour la totalité de la période de l'étalon-or au Royaume-Uni, en postulant que les agents s'attendent à ce que le niveau des prix retourne à sa valeur d'équilibre à long terme. Loin d'être paradoxale, la corrélation positive observée entre le niveau des prix et le taux d'intérêt nominal (ou paradoxe de Gibson) aide à expliquer pourquoi le taux d'intérêt nominal est resté si stable durant une période marquée par les guerres et d'importantes découvertes d'or. La nouvelle série de taux d'intérêt réels obtenue permet à l'auteur de réexaminer le résultat de Barro (1987) concernant l'incidence des variations temporaires des dépenses militaires sur les taux d'intérêt et de lever l'hypothèse que le taux d'intérêt nominal à long terme est également le taux réel attendu.

## 1. Introduction

The positive correlation between the nominal interest rate and the price level observed during the gold-standard period was called the Gibson Paradox by Keynes (1930). Figure 1 provides a striking illustration of this phenomenon, which has been observed by many economists and which has been the topic of a host of studies.[1] Keynes (1930, 198) described the Gibson paradox as: "one of the most completely established facts in the whole field of quantitative economics." More recently, Benjamin and Kochin (1984, 587) noted that the Gibson paradox was still "one of the best-known and least understood of all economic regularities."

What is paradoxical in the correlation between the nominal interest rate and the price level is that in equilibrium, the price level—which has the dimension of money—depends on the quantity of money in circulation. However, the rate of interest—which has the dimension of a pure number—would not. Doubling the stock of money would double the price level but leave unchanged the equilibrium rate of interest. As Friedman and Schwartz (1982, 527) put it:

> On theoretical grounds, there is no reason to expect any direct relation between the nominal rate of interest and the level of prices. The rate of interest is a pure number . . . the level of prices is not a pure number; it has the dimension of dollars.

The nominal rate of interest depends on the expected rate of growth of the money stock, hence the rate of inflation, as specified in the Fisher equation. If annual series of recent U.K. data are examined, one can easily see the adjustment of the nominal interest rate to the inflation rate. Between 1960 and 1996, the correlation coefficient between the first difference of the log of the price-level series and the yearly average yield on long-maturity government bonds is 0.85. But this correlation is only 0.02 between 1717 and 1913, a period during which the correlation coefficient between the long-run yield and the log of the price level is 0.63.

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[1] In particular, see Wicksell (1907), Fisher (1907; 1930), Keynes (1930, Chapter 30), Cagan (1965), Sargent (1973), Shiller and Siegel (1977), Friedman and Schwartz (1982, Chapter 10), Lee and Petruzi (1986; 1987a; 1987b), Barsky and Summers (1988), Corbae and Ouliaris (1989), Chen and Lee (1990), Mills (1990), and Sumner (1993). Since Lee and Petruzi (1987a) and Barsky and Summers (1988), the Gibson paradox has been interpreted as a phenomenon associated with the gold standard.

In this study, we will show that in a monetary regime where the level of prices is stationary, the price level conveys intertemporal information, which complements that conveyed by nominal interest rates. By comparing the actual price level with its long-run equilibrium level, a rational agent extracts information from the price system pertinent to intertemporal choices. In the modern world of fiat currency where the price level is integrated of order one, all the information pertinent to the intertemporal allocation of resources is conveyed by nominal interest rates.

The analysis is based on a simple model of individuals' choices that is presented in the following section. In Section 3, we use the relationship between the level of prices and the nominal interest rate that is suggested by the theoretical model when the price level is stationary to estimate real interest rate series for the gold-standard period in the United Kingdom. The analysis of Section 3 leads to a new interpretation rather than a new explanation of the Gibson paradox, since we do not explain why the price level was positively correlated with the nominal interest rate during the gold standard. Rather, we highlight the consequences for agents' intertemporal choices of the positive correlation between the nominal interest rate and the price level when the latter is stationary. In Section 4, following Barro (1987), we analyze the empirical relationship between the new real interest rate series estimated in Section 3 and temporary military spending during the gold-standard period in the United Kingdom. We conclude by drawing some lessons for the choice of a monetary regime.

## 2. The Fisher equation when the price level is stationary

In this section, we consider the intertemporal optimization problem of a consumer in two different monetary regimes. We focus attention on the relationship between price expectations and the Euler equation. From a stylized macro framework, we derive the time path of the price level under two alternative monetary regimes. Individual choices are then analyzed under the assumption that consumers know which regime they are in.

### 2.1 Two monetary regimes

Consider a stylized macro framework where $y_t$, the natural logarithm of aggregate output, follows a random walk:

$$y_t = y_{t-1} + \eta_t, \qquad (1)$$

and $\eta_t$ is white noise. Suppose that movements in the price level are explained by the quantity theory of money:

$$m_t = p_t + y_t + \delta_t \ . \tag{2}$$

The policy variable $m_t$ is the natural log of the quantity of money at time $t$; $p_t$ is the natural log of the price level; and $\delta_t$, the inverse of velocity, is a random walk (i.e., $\delta_t - \delta_{t-1} = \nu_t$ is white noise). This stochastic element, integrated of order one, captures a permanent shock to the velocity of money circulation. In the first monetary regime, R1, the central bank follows a constant growth rate rule:

$$m_t - m_{t-1} = \pi \ , \tag{3}$$

where $\pi$ is the target inflation rate. Given this reaction function, equations (1) and (2) imply that the price level is I(1) and the inflation rate is stationary:

$$p_t = p_{t-1} + \pi - \epsilon_t \ , \tag{4}$$

where $\epsilon_t = \nu_t + \eta_t$ is white noise.

In the second regime, R2, the central bank aims to keep the log of the price level stationary[2] around a deterministic trend $\mu_t = p_0 + \pi t$. The target level of $p_t$ increases at rate $\pi$ over time. The price stability regime that will be considered later is a special case of R2 when the target trend growth rate ($\pi$) is zero. The reaction function of the central bank is:

$$m_t = m_{t-1} + \pi + \alpha(\mu_{t-1} - p_{t-1}) \ , \tag{5}$$

where $\alpha$ falls between zero and one. The central bank therefore adjusts the money supply by following a partial error-correction process.[3] It is easily seen in this case that, given (1) and (2), the deviation between the price-level logarithm and its trend follows a stationary AR(1) process:

$$p_t - \mu_t = (1-\alpha)(p_{t-1} - \mu_{t-1}) - \epsilon_t \ . \tag{6}$$

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[2] A rule of this type—where the trend growth rate of prices is zero—was proposed by Simons (1936). See also Barro (1986) and Yeager (1992). A number of studies have focused on price-level stationarity issues. In particular, see Fillion and Tetlow (1994); Gavin and Stockman (1998; 1991); Lebow, Roberts, and Stockton (1992); McCallum (1990a; 1990b); and McCulloch (1991).

[3] McCulloch (1991) proposes a reaction function of this type to maintain price stability.

## 2.2 Individual choices

To examine the implications of these two different monetary regimes for price expectations, we consider a very simple model of an individual who is planning consumption $c$ for two periods under the certainty equivalent assumption.[4] Utility is of the form

$$U = U(c_t,\ c_{t-1}) \tag{7}$$

where $c$ is a non-storable composite good and $U$ is increasing in $c$ and concave. At time $t$, the individual receives an endowment of $w_t$, in money, the numeraire, whose price is normalized to one. The budget constraint for time $t$ is:

$$P_t c_t + s_t = w_t\ , \tag{8}$$

where $P_t$ is the monetary price of the consumer good at time $t$. The savings at time $t$, $s_t$, motivates a demand for an interest-bearing asset between the two periods. The individual consumes the savings at time $t$+1, plus accrued interest of ($r_t$):

$$P_{t+1} c_{t+1} = (1+r_t) s_t. \tag{9}$$

The individual makes plans for both periods at time $t$, knowing $P_t$ and $r_t$. The asset that conveys the savings over time is a fixed-term contract in money terms. At time $t$, however, $P_{t+1}$ is unknown. The individual's problem, therefore, involves selecting a bundle of goods ($c_t$, $c_{t+1}$) that maximizes the following Lagrangian function:

$$L = U(c_t, c_{t+1}) - \lambda(P_t c_t + \frac{P_{a,t+1} c_{t+1}}{1+r_t} - w_t), \tag{10}$$

where $P_{a,\,t+1}$ is the anticipated monetary price for $t$+1 at time $t$. From the first-order conditions of the maximization problem, it can be shown that the utility-maximizing individual will choose consumption over time so as to equalize the marginal rate of intertemporal substitution to a price ratio. This maximization rule is expressed in logarithmic form as:

$$MRIS \equiv \ln\ U_t - \ln\ U_{t+1} = \ln\ (1+r_t) + p_t - p_{a,t+1}\ , \tag{11}$$

[4] The optimizing problem could be very easily generalized to allow for dynamic stochastic optimization in an infinite time horizon. The substance of the results will be exactly the same.

where $p$ and *MRIS* are the natural logs of, respectively, the monetary price and the marginal rate of intertemporal substitution.

The monetary regime affects the maximization problem through the expected monetary price $p_{a,\,t+1}$. Assume that $c$ is a basket of goods and its monetary price is the price level. Let us assume that the individual forms expectations for $p_{t+1}$ that are consistent with the rule chosen by the monetary authority. What happens when individuals form expectations for $p_{t+1}$ that are consistent with R1? In this case, the anticipated price level for $t$+1 at time $t$ is:

$$p_{a,t+1} = p_t + \pi$$

and the individual maximizing his utility adjusts the *MRIS* to:

$$MRIS = \ln\,(1 + r_t) - \pi \;\cong\; r_t - \pi\ . \tag{12}$$

Consumption is adjusted over time to make the *MRIS* equal to the real expected interest rate, which is subjectively estimated by the individual as the difference between the nominal interest rate and the constant expected trend rate of inflation. The individual does not care about the *level* of prices. The nominal interest rate (adjusted for trend inflation) is the only market price that conveys information pertinent for intertemporal choices. A change in the trend inflation rate does not affect individuals' choices if the nominal inflation rate adjusts according to the usual Fisher effect, leaving the *MRIS* unchanged.

Under the monetary regime R2, if individuals form expectations consistent with the monetary rule, the *MRIS* of an individual seeking to optimize his return takes the following form:

$$MRIS \cong r_t + \alpha(p_t - \mu_t) - \pi. \tag{13}$$

The current price level now contributes to the individual's subjective evaluation of the real interest rate. If the price level at time $t$ exceeds its trend value ($\mu_t$), agents expect a price-level decrease in the proportion $\alpha(p_t - \mu_t) - \pi$ for the following period. In this case, the real ex ante interest rate is higher than the difference between the nominal interest rate and the expected trend inflation rate ($\pi$). Similarly, if the price level is below its trend value, the expectation of its reversion towards long-term equilibrium decreases the subjective assessment of the real interest rate. Note, too, that the trend inflation rate always exerts the same effect on the subjective assessment of the real interest rate through a Fisher effect. This effect should not, however, be confused with the relation between the *level* of prices and the nominal interest rate, which contributes to determining the subjective assessment of the real interest rate.

The information pertinent for intertemporal choices emerges from a comparison of the price level $p_t$ against a standard of value $\mu_t$. What happens if the central bank chooses a standard of value that is constant over time? Under these circumstances ($\pi$=0), which we call price stability, equation (13) can be expressed as:

$$MRIS \cong r_t + \alpha(p_t - p_0) \ , \qquad (14)$$

where $p_0$ is the long-term equilibrium level of prices, or the standard of value. The individual evaluates the real interest rate on the basis of the information jointly conveyed by the current price level and the nominal interest rate. Because the standard of value is constant, the price level can be considered an intertemporal relative price.[5]

### 2.3 Nominal interest rate floor and the real interest rate

Intertemporal substitution plays a key role in macroeconomic models. The microeconomic analysis of Section 2.2 shows that implications of price-level movements for intertemporal choices change substantially according to whether the price level is I(1) or I(0). We conclude this section by discussing the consequences for the macroeconomic adjustment mechanism of modelling price expectations in a manner that is consistent with a monetary regime where price-level stability is observed.

Macroeconomists have long recognized the risk that a price-level decrease could have a destabilizing effect on the macroeconomic adjustment mechanism.[6] The traditional analysis suggests that, under conditions of excess supply of goods and services, a price-level decrease could prove destabilizing if it triggers deflationary expectations. These can give rise to an increase in the real interest rate, which could offset the drop in the nominal interest rate. The signals sent by the price system under these circumstances could work against the requirements for a restoration of

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[5] To that end, the standard of value might be defined in terms of a basket of goods whose intrinsic qualities do not change over time. On this topic, see the discussion in Yeager (1992).

[6] See Tobin's (1980, Chapter 1) analysis of Fisher's (1933) debt-deflation theory and the analysis of Blanchard and Fischer (1989, 548) on the Keynes-Mundell-Tobin effect.

macroeconomic equilibrium. However, under a stationary price-level regime, the impact of price expectations on the real interest rate is reversed and necessarily stabilizing, because a drop (rise) in the level of prices is equivalent, to some degree, to a drop (rise) in the nominal interest rate.

Taking the stabilizing effect of expectations under price-level stability into account sheds new light on Summers' (1991) analysis in which he advocates a monetary regime with a positive inflation rate on average. His argument is based on the assumption that the real interest rate cannot go negative in a regime with a rate of trend inflation of zero, because the nominal interest rate cannot be lower than zero. In his view, this could pose a binding constraint on monetary policy, making it at least more difficult to maintain full employment under certain circumstances. In the context of the simple model developed above, Summers' concern is justified if the price level is integrated of order one, since the subjective real interest rate given by (12) cannot be negative if the trend inflation rate is zero. The trend inflation rate must be positive if the subjective real interest rate is to become negative when needed. Summers' (1991) analysis, however, does not apply under a monetary regime where the level of prices is trend stationary even if the trend inflation rate is zero. As equation (13) demonstrates, even if the nominal interest rate $r$ cannot drop below a certain threshold (for example, $r_{min}$), the perceived (or expected) real interest rate will indeed be negative if the price level drops far enough below its long-term equilibrium level:

$$p_t < p_0 - \frac{r_{min}}{\alpha} .$$

This analysis shows that the nominal interest rate might not be able to convey the full range of information pertinent to intertemporal choices in a zero inflation regime where the price level is I(1). Under a regime where the price level evolves around a standard of value, fluctuations in prices can reinforce the effect of interest rate movements if nominal interest rates move procyclically with the price cycle.[7] In the next section, we will show that this is precisely what took place during the gold-standard period.

[7] There is less need for the nominal interest rate to adjust in this case. This prediction is borne out by the results of the stochastic simulations of a macroeconomic model of the Canadian economy carried out by Black, Macklem, and Rose (1998). The introduction of a function for an anticipated partial price reversion like equation (14) in the context of a monetary rule aimed at maintaining price stability produces a significant decrease in the variability of nominal interest rates.

## 3. The Gibson paradox reconsidered

From 1717–1914, the pound sterling was convertible into gold at a fixed rate. This type of monetary framework can ensure a stationary price level if the relative price of gold is itself stationary.[8] In this section, we estimate the subjective real interest rate during the gold standard in the United Kingdom, based on equation (14).

For the purpose of the statistical analysis, the gold-standard period is defined as starting in 1717 when, *de facto*, the United Kingdom was on a gold standard following the undervaluation of silver by Sir Isaac Newton, the Master of the Mint. The gold-standard period ends in 1914 with the suspension of convertibility and the outbreak of World War I. We exclude the suspension period 1793–1815 associated with the Napoleonic wars. *De jure*, the United Kingdom adopted the gold standard in 1816 and fixed the return to convertibility to 1819 at the same exchange rate established in 1717 by Newton. The model underlying the formation of expectations in our analysis cannot be used for the 1793–1815 period, since the model assumes a continuous regime in which individuals expect the price level to reverse toward a standard of value.

As in previous studies of the Gibson paradox, the choice of the time series for the price level and the interest rate is dictated by the availability of historical data. The nominal interest rate for British government perpetual annuities (consols) over the period is that calculated by Homer (1963). The commodity price index developed by Mitchell and Deane (1962) and Mitchell and Jones (1971) is used as a measure of the price level.

### 3.1 A stationary price level

Economic historians have traditionally described the evolution of the price level in the United Kingdom during the classical gold-standard period as a stationary process characterized by long

[8] For a theoretical analysis of the gold standard, see Barro (1979); for a detailed chronology, refer to Hawtrey (1947). The relative price of gold could be shown to be stationary in comparison with other commodities by using a classical theory of value (e.g., Smith 1776, Chapter 7) where the natural price of a good is based on its production costs. If the relative price of gold rises above its long-term equilibrium price, resources from other sectors of the economy shift to gold exploration and mining. This phenomenon represents an implicit error-correction mechanism, akin to equation (5) in Section 2.

swings.[9] However, to our knowledge, macroeconomists have not yet attempted to interpret the evolution of the price level and interest rates during this period under the assumption that the former is I(0). This is not surprising, because a stationary time series characterized by long swings can be represented by a stochastic, autoregressive process with a quasi-unit root. It is, therefore, difficult to determine in this case whether the series is stationary or integrated of order one.[10]

Formal tests of the null hypothesis that the log of the price level is integrated of order one yield ambiguous results over samples that include periods in which convertibility is suspended. For example, for the log of the price level over the 1717–1931 sample, the null hypothesis of a unit root is rejected at the 5 per cent critical level based on the Augmented Dickey-Fuller (ADF) test with a *t*-statistic of -2.95, which is slightly below the critical value of -2.88.[11] However, the null hypothesis can only be marginally rejected in the neighbourhood of the 10 per cent level over the same sample using the Phillips-Perron (PP) test. Note that the 1717–1931 period includes the suspension following World War I. Between 1914 and 1920, the average annual inflation rate was 18 per cent and post-war deflations brought the price level back to its 1914 level by 1932. If one restricts the sample to the 1717–1913 period, the results are not ambiguous since the null hypothesis that the price level follows a unit root cannot be rejected at the 10 per cent level for both the ADF and the PP tests.

One could conclude from this analysis that the price level was not stationary during the gold standard. However, as mentioned before, this sample includes the suspension period of 1793–1815. Between 1793 and 1801, the annual inflation rate was 6.3 per cent on average. The convertibility

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[9] Mills (1990) comes to this conclusion. The discovery of the New World, however, caused a sustained price rise throughout the 17th century. Beginning in the 18th century, the price level fluctuated around a relatively constant value over time, once the inflationary period due to the massive influx of New World gold had ended. For a thorough study on the subject, see Jastram (1977).

[10] On the basis of Dickey-Fuller tests, Mills (1990) shows that the unit root hypothesis can be rejected for the price level in the United Kingdom for the period 1729–1931 at the 5 per cent significance level. Kuchciak (1997) comes to a similar conclusion on the basis of Phillips-Perron (PP) and augmented Dickey-Fuller (ADF) tests for the period 1717–1931. Barro (1987) concludes in his analysis of the price level in the United Kingdom that one cannot reject the unit root hypothesis over the joint-sample 1705–1796, 1822–1913 at the 5 per cent level. This result is based on the estimation of $\log(p_t)$ as a function of $\log(p_{t-1})$ and the current temporary military spending as an instrumental variable. See Section 4 below for the use of temporary military spending.

[11] In the statistical discussion that follows, we assume that the log of the price series contains a constant term but no time trend. For the ADF tests, we used the procedure proposed by Perron (1992) to select the lag number $k$ for the parametric correction. For the Phillips-Perron tests, interpretations of the results are based on the *t*-statistics for truncation lags of 2 to 6.

period was characterized by sharp increases and decreases in the price level. But such a steady increase for an 8-year period is unprecedented in the 1717–1913 sample.[12]

However, the stationarity tests for the log of the level of prices over the joint-sample period 1717–1792 and 1815–1913 reveal a quite different story. For both the ADF and the PP tests, the null hypothesis of a unit root is rejected at the 1 per cent critical level. The ADF *t*-statistic (with two lagged differences) is -3.65 and, for the PP test, the *t*-statistic varies between -3.72 and -4.24 for lag truncations between 2 and 6. For both tests, the 1 per cent critical value is -3.47. Price-level stationarity during the gold standard represents a reasonable hypothesis in this case.

### 3.2 Estimating price expectations

According to Bordo and Kydland (1995), the gold standard in the United Kingdom could be considered a policy rule that was well understood and anticipated by the public. In this section, we assume that individuals form price-level expectations that are consistent with estimated representations of the evolution of the price level. In order to illustrate the robustness of the analysis, two representations of the AR processes for the log of the price level are estimated. First, the sample is split into pre- and post-Napoleonic periods and AR processes are estimated for the two subperiods 1717–1792 and 1815–1913. Second, an AR process is estimated for the joint-sample 1717–1792, 1815–1913.

The Box and Jenkins (1976) method was used to identify the best ARMA process describing the evolution of the price level $p_t$ for each of the two subperiods 1717–1792, 1815–1913 and for the joint sample. An AR(1) process was selected for the pre-Napoleonic war period, and AR(2) processes were selected for both the post-Napoleonic period and the joint period.

[12] The null hypothesis that the AR process estimated below for the 1717–1792 could be used to forecast the log of the price level during the 1793–1815 period is rejected by a Chow Forecast Test at well below the 1 per cent critical level. Similarly, the null hypothesis that the coefficients of the AR representation of the log of the price level over the 1717–1913 period are constant over the subsamples 1717–1792, 1792–1815, 1815–1913 is rejected by a Chow Breakpoint Test at well below the 1 per cent critical level.

For the pre-Napoleonic period, the estimated process is:

$$p_t = \underset{(0.006)}{0.43} + \underset{(0.05)}{0.90}\, p_{t-1}$$

$$R^2 = 0.79,\ s.e.e. = 0.052,\ MEAN(p) = 4.31$$

For the post-Napoleonic sample, the estimated process is:

$$p_t = \underset{(0.006)}{0.39} + \underset{(0.10)}{1.12}\, p_{t-1} - \underset{(0.09)}{0.21}\, p_{t-2}$$

$$R^2 = 0.89, \quad s.e.e. = 0.061 \quad MEAN(p) = 4.47$$

Finally, for the joint sample, the estimated process is:

$$p_t = \underset{(0.005)}{0.41} + \underset{(0.08)}{1.07}\, p_{t-1} - \underset{(0.07)}{0.166}\, p_{t-2}$$

$$R^2 = 0.89, \quad s.e.e. = 0.057, \quad MEAN(p) = 4.40$$

Standard errors are shown in parentheses. For the second period, the AR(2) process has two real characteristic roots of 0.89 and 0.23; for the joint-sample estimate, the two real roots are 0.89 and 0.19. Since the dominant root is close to one, those processes are marked by long swings very similar to those predicted by the AR(1) process for period 1, for which the root is 0.90.

### 3.3 Estimating the short-term real interest rate from the consol rate

The estimated real interest rate is calculated from the annual series on consols and is the expected real rate of return discounted on a consol held for a 1-year period. It is a subjective evaluation of the 1-year-ahead real interest rate based on the presumption that the price level will move back partially toward its normal value in the following year. It is implicitly assumed that the expected capital gain for the holding of the perpetuities is zero.[13] Rates of return observations are annual averages. Ideally, we should have used the data on the nominal return of a 1-year debt security at the start of the year to estimate the real interest rate for the current year. Only data on the rates of return on perpetual annuities, however, are available to reconstruct a reasonably reliable chronological series to track interest rate developments in the 18th and 19th centuries.

For the pre-Napoleonic period, equation (14) can be used directly to estimate the real interest rate. For the second subperiod, the log of the price level follows the AR(2) process:

$$p_t - p_0 = (1-\alpha_1)(p_{t-1}-p_0) + \alpha_2(p_{t-2}-p_0) + \epsilon_t ,$$

where $\epsilon_t$ is white noise. In this case, it can be easily shown that equation (14) becomes:[14]

$$MRSI \cong r_t + (\alpha_1 - \alpha_2)(p_t - p_0) + \alpha_2(p_t - p_{t-1}) . \tag{15}$$

This equation was used to estimate the real interest rate for the post-Napoleonic period and for the joint-sample estimate.

The interpretation of the AR(1) process in the light of (14) is straightforward. The real subjective interest rate would be equal to the nominal interest rate plus 10 per cent of the gap between the current price level and its long-run equilibrium value. We estimate this long-run value as the mean of the log of the price level over the sample under study. Interpreting the AR(2) representation from equation (15) is slightly different. Given the estimates for $\alpha_1$ and $\alpha_2$ for the post-Napoleonic period (1.12 for 1-$\alpha_1$ and - 0.21 for $\alpha_2$), we can see that, interestingly, the real subjective interest rate is equal to the nominal interest rate plus 9.0 per cent of the gap between the current price

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[13] Clearly, expected capital gains are zero if the consol rate follows a random walk. There is some indication, however, of stationarity of the consol rate during the gold-standard period. We have estimated the expected capital gain during the 1717–1920 period under the assumption that agents form expectations of the nominal rate that are compatible with the AR representation. As anticipated by Barro (1987, 231), the capital gains are relatively large. The standard deviation of the expected capital gain series for holding a consol during one year is 1.67 per cent and the series varies between a minimum of -4.79 per cent and a maximum of 3.62 per cent. In this study, we follow the conventional approach to the Gibson paradox and abstract from capital gains.

[14] For example, in the framework of the stylized macroeconomic model of Section 2.1, an equation like (15) could be easily derived if equation (1) is replaced by $y_t = y_{t-1} + \rho(y_{t-1} - y_{t-2}) + \epsilon_t$. In this case, the theoretical MRIS equals $r$-$\rho(p_t - p_{t-1}) + \alpha(1-\rho)(p_t - p_0)$.

level and its standard of value, minus 21 per cent of the inflation rate from *t*-1 to *t*. For the joint-sample AR(2) processes, the real interest rate is estimated by the nominal interest rate plus 9.6 per cent of the gap between the current price level and its normal value minus 16.6 per cent of the current inflation rate. For the AR(2) representations, to determine the real interest rate, a weight is assigned to the gap observed between the price level and the standard of value, as well as a weight to the current inflation rate. The results of the exercise are presented in Figures 2a, 2b, and 3, and in Table 1.

For the pre-Napoleonic period (Table 1a), the estimated real interest rate ranges from 1.33 per cent to 7.59 per cent, with a standard deviation of 1.57 per cent and a mean of 3.57 per cent. By comparison, the consol rate for this period has a standard deviation of 0.63 per cent and ranges from 2.83 per cent to 5.41 per cent. The correlation coefficient between the contribution of prices expectations to the real interest rate (hereafter referred to as the price contribution) and the consol rate is 0.57. Fifty-one per cent of the real rate's variance is due to the price-contribution variance, 16 per cent to consol rate variance, and 33 per cent to the covariance.

The real interest rate estimated from the split sample is somewhat more variable during the second period (Table 1b): the standard deviation is 2.28 per cent, the mean is 3.28 per cent, and maximum/minimum values are 9.56 per cent and -1.10 per cent respectively. Interestingly, the standard deviation of the consol rate during this period is only 0.45 per cent. The correlation coefficient between the consol rate and the price contribution is 0.78. Therefore, for the two subperiods, the Gibson correlation implies that price-level expectations amplify real interest rate variations. For the post-Napoleonic period, most real interest rate movements (71 per cent) are explained by the direct contribution of price expectations, since the consol rate varies very little.

The real interest rate series generated from the AR(2) representation of the joint sample (Table 1c) has a correlation coefficient with the split sample estimates of 0.88 (Table 1d). The descriptive statistics are very close to the ones from the split-sample estimates. The time series on the short-term expected real interest rate (from the consol rate) from the joint sample and the split sample are used in Section 4 to analyze the incidence of temporary military spending on the short-term real interest rate.

### 3.4 Some historical considerations

In general, the estimated real interest rate peaks at times of war. It reaches a historical high at the end of the Napoleonic wars and is generally high during the tumultuous second half of the 18th century, which saw the the United Kingdom involved in the Seven Years' War, the American War of Independence, and the French revolutionary wars. The empirical relationship between military spending and the estimated real interest rate is studied in Section 4 below.

The only two periods in which negative real interest rates are estimated coincide with major gold discoveries and rapid increases in the world gold supply. The year 1853 follows on the heels of the Australia and California gold rushes and, according to Hawtrey (1947), the increase in the world's gold supply that year was the most rapid of the century until the 1890s. At the end of the 19th century, gold production was boosted by the Alaskan, Yukon, and South African gold rushes, and by the development of the cyanide extraction process (see Barsky and Summers, 1988).

When the short-term real interest rate is abnormally low, so too is the expected marginal product of physical capital. The short-term return to investment for the production of goods and services is low and economic agents switch to alternative economic activities. The production of money, if possible, then becomes a profitable alternative. In a gold-standard regime, a negative short-term real interest rate calls for a switch of economic activities from the production sector to the exploration and the production of gold.

### 3.5 Comparisons with post-1950 estimates of the real rate

How does the real interest rate series estimated during the gold standard compare with real rate estimates in the inflationary post-WW II era? Thanks to the numerous switches of monetary regimes since the 1950s, the modelling of inflationary expectations is a fragile exercise and the choice of modelling technique certainly affects the statistical properties of the real interest rate series. For comparison purposes, we chose to make use of the expected inflation rate from a Markov model utilized by the Bank of Canada Research Department to estimate future inflation in models of the Canadian economy.[15] Characteristics of the resulting expected real interest rate in Canada for the

[15] On this topic, see Laxton, Ricketts, and Rose (1994) and Ricketts (1996).

inflationary period 1952–1994 are shown in Table 2. The nominal rate of return is the average return on long-term Government of Canada bonds, since this is the closest to the return on the perpetual annuities used during the gold-standard period.

The real interest rate ranges from -2.32 per cent to 9.46 per cent and its standard deviation is 2.71 per cent. The real interest rate since 1952 thus shows greater variability than that estimated for the gold-standard period on the basis of the expected price-level return. The nominal interest rate is much more variable than during the gold-standard period, with a standard deviation of 3.63 per cent. Most of the nominal interest rate variability occurs in the 1970–1982 period, which saw very large interest rate fluctuations. Even the periods before 1970 and after 1982, however, show greater nominal interest rate variability than during the gold-standard period. The standard deviation of the nominal rate is 1.13 per cent from 1956–1969 and 1.38 per cent during the post-1982 period.

This comparison shows just how far the nominal interest rate has had to adjust in an inflationary world. Under a regime of price-level stability, the nominal interest needs to adjust less, for two reasons. First, the expected trend rate of inflation is constant at zero. Second, part of the intertemporal information is conveyed by the price level. If the price level is positively correlated with the nominal interest rate, the nominal interest rate will be less variable. Essentially, this is our non-paradoxical interpretation of the Gibson Paradox.

### 3.6 Estimating the short-term real interest rate from the short-run yield

To verify the robustness of the empirical analysis presented in this section, we re-estimated the real ex ante interest rate using a series of short-term nominal interest rates, again for the case of the United Kingdom. This series consists of the monthly averages of 3-month rates since 1824.[16] The same AR(2) process estimated for the post-Napoleonic period is used for calculating the price contribution to the real interest rate. The results are shown in Figures 4 and 5 and in Table 3.

The short-term nominal interest rate series is much more variable than the consol rate, with a standard deviation almost three times larger. The correlation coefficient between the two series is 0.48. The correlation between the price contribution (from the split-sample estimate) and the short-term nominal interest rate remains positive (0.34), but it is much weaker than that observed with

[16] The data are taken from Mitchell and Deane (1962).

consol rates (0.74 over the 1824–1913 sample).[17] The real interest rate estimated from short-term rates is slightly more variable than the one estimated from consol rates (the standard deviation is 2.36 compared with 1.94 for the 1824–1913 sample). The correlation between the two estimates (0.90) is considerably stronger than between the two nominal rates. Figure 5 indicates to what extent the estimates of the real interest rate produced by the two methods are comparable. The series generated by consol rates follows closely the series produced by short-term rates, with a short lag. From a theoretical point of view, the two series should be fairly comparable, since they both measure the real ex ante interest rate associated with holding debt instruments that are, at least to some extent, mutual substitutes.[18]

### 3.7 Estimating the long-term expected real rate on consols

When the price level is stationary, at any date, the long-run expected annual inflation rate approaches zero asymptotically as the time horizon goes to infinity. This result holds even if, for example, at time *t*, the price level is far from its long-run equilibrium value. However, that does not mean that the long-term expected real rate of return on a consol is equal to its nominal return. When the price level is below its long-run value, agents expect the price level to come back gradually to its normal value in the near future and the expected rate of changes in prices is greater than zero over the transitory adjustment period. To evaluate the expected real return on a consol, agents discount the present value, in real terms, of the constant (in nominal terms) yield on the bond. As a result, more weight is given to shorter time horizons. Thus, the expected real return in this case would be higher than the nominal return. In the appendix, we show how the estimates presented in Figure 6 and Table 4 of the long-term expected real interest rate were calculated from the present value formula of the real return, given the AR(1) and AR(2) process estimated for the log of prices in the split sample.

The long-term real expected return has a standard deviation of 0.87 per cent, 52 per cent larger than the standard deviation of the nominal return, and it varies between a maximum of

[17] Shiller and Siegel (1977, 892-3) call the positive correlation between the short-term nominal interest rate and the price level the Kitchin phenomenon, referring to Joseph Kitchin, who noted this correlation in 1923.

[18] The variance of the price contribution, a component common to both estimates of the real interest rate, accounts for 69 per cent of the covariance between the series. The remainder is associated with the positive covariances between the price contribution and the long-term nominal interest rate (10 per cent), between the price contribution and the short-term nominal interest rate (17 per cent), and between the two nominal interest rate series (4 per cent).

6.14 per cent and a minimum of 1.55 per cent. Not surprisingly, the long-term real expected return is much closer to the nominal return than our estimate of the 1-year expected real interest rate (Figure 6).[19] The correlation coefficient between the nominal return and the long-term expected real return is 0.94. This time series is used in Section 4 to analyze the incidence of temporary military spending on real interest rates.

### 3.8 Correlation between the price level and the real interest rate

According to the interpretation of the Gibson paradox offered in this section, if the price level is positively correlated with the nominal interest rate, it should also be correlated with the real interest rate. This prediction is compatible with the conclusions of Sargent (1973) and Barsky and Summers (1988), who calculated the real interest rate on the basis of equity returns. Sargent concludes his exhaustive study of the Gibson paradox as follows (1973, 446–47):

> The Gibson paradox appears to have characterized nominal and real interest rates alike. It follows that it is desirable to have an explanation of the Gibson paradox that focuses on the relationship between movements in real rates of return and the price level. Our empirical results imply that to explain the Gibson paradox it is not adequate to hypothesize a one-way influence directed from inflation to the interest rate (or for that matter, from interest to inflation). Instead, within the context of bivariate models, interest and inflation appear mutually to influence one another.

A unidirectional causal relationship between the expected rate of inflation and the nominal interest rate is the result of the Fisher effect, when the nominal rate adjusts to changes in the expected inflation rate.

### 3.9 Switching from price stability to an inflationary world

Our reinterpretation of the Gibson paradox also sheds some light on the early post-WWII period. As the price level switched from a stationary variable to at least an I(1) variable with drift, and the economy ostensibly moved from a stable price-level regime to an inflation regime after the price level failed to come back to its long-run value after World War II, we gradually leave Gibson's world and enter Fisher's. This is consistent with Friedman and Schwartz's (1982, 535) observation about

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[19] From the analysis in the appendix, the effect of price expectations on the long-term real rate is, on average, approximately 1/4 ( or $r/(r+\alpha)$ following the notation of the appendix) of the effect on the short-term real rate.

the 1960s that, "when interest rates start to parallel price *changes*, they start departing from parallelism with price *levels*." The transition from the gold standard to the new post-WWII monetary regime has been analyzed in depth by Klein (1975). On page 477, he writes that the change in expectations explains why the St. Louis macroeconomic model included a dummy variable in its interest rate equation for the post-1960 period. Klein (1975, 472) also notes, like Gordon (1973, 462), that on the basis of J. A. Livingston's surveys of economists' price expectations, economists consistently expected a price decrease from 1946 to 1952, except for the two years of the Korean War (1951–1952). Friedman and Schwartz (1963, 584) also observed this phenomenon from 1946 to 1948 on the basis of a comparison of equity and bond rates of return. During the transition period from the stationary-price monetary regime to the post-war monetary regime, individuals for a long time expected price declines that never materialized.[20]

## 4. Revisiting Barro (1987) on military spending and interest rates

### 4.1 Background

Benjamin and Kochin (1984) use annual British data on military spending going back to the 18th century to propose a "martial solution" to the Gibson paradox. They argue that the positive effect of temporary military spending on the price level and the long-term nominal interest rate can account for the positive correlation between the price level and the nominal interest rate. Barsky and Summers (1988) point out, however, that the martial solution is unsatisfactory because the Gibson's correlation was observed during the gold standard for non-war periods as well. If the war periods are excluded from the 1717–1913 sample (see the shaded areas in Figure 7), the correlation coefficient between the log of the price level and the nominal interest rate remains positive but falls slightly from 0.63 to 0.51. However, for the overall 1717–1792, 1815–1913 joint-sample period where the United Kingdom was on a gold standard, this correlation is even lower at 0.40.

Following Benjamin and Kochin (1984), Barro (1987) uses the historical British data to analyze the effects of government spending on interest rates. Many economic models predict that an increase in government spending should increase the real interest rate in a closed economy. In the 18th and 19th centuries in the United Kingdom, temporary changes in military spending account for

[20] It is not surprising, therefore, that nominal interest rates remained very low during this period, since the expectation of a reversion of prices towards their standard of value increased agents' subjective evaluation of the real interest rate.

most of the changes in government purchases. Owing to the availability of long series of annual data and the numerous wars involving the United Kingdom, both Benjamin and Kochin (1984) and Barro (1987) turn to the British experience as a promising data set for isolating the effects of government spending on interest rates. Barro (1987) concludes that, as predicted by the theory, temporary increase in military spending had a positive effect on interest rates. This empirical analysis has subsequently been presented as a textbook case study that supports the theoretical prediction.[21] In this section, Barro's empirical analysis is revisited in light of the real interest rate series estimated in Section 3.

Consider the dynamic effect of government purchases on interest rates in the framework of an infinite horizon neoclassical growth model of a closed economy with consumer optimization, as analyzed by Romer (1996, Section 2.7). Suppose that government spending increases unexpectedly at time $t_0$ and households know that it will come back to its normal level at time $t_1$. The increase in government spending decreases consumption at time $t_0$ but households expect their consumption to increase after $t_1$. The short-term real interest rate has to increase between $t_0$ and $t_1$ to match the increase in the marginal rate of intertemporal substitution. The real interest rate peaks at $t_1$ and comes back gradually thereafter to its initial level. If the increase in government purchases is short-lived, the intertemporal substitution of consumption is small and the effect on the short-term real interest rate is minimal. A relatively long-lasting increase in government purchases will increase the short-term real interest rate more for a more extended period to allow for consumption smoothing.

### 4.2 Replicating Barro (1987)

The data on military expenditures and nominal interest rates used in the actual empirical analysis come from the same data sources as in Barro (1987).[22] The temporary military spending variable, however, is different. Barro deflates the military spending series by the price level and then divides real military expenditure by the trend real GNP. To detrend real GDP, he simply uses a trend line in the log series with a change in the slope in 1771. We prefer to stay closer to the historical data

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[21] Mankiw (1994, Case Study 3-2) and Romer (1996, Section 2.7) used Barro's (1987) study to illustrate the effect of government purchases on interest rates.

[22] The data on military expenditure are taken from Mitchell and Deane (1962). We followed the same procedure as Benjamin and Kochin (1984, f. 6) and Barro (1987, f. 4) to build the military spending series. The data on GNP are from Feinstein (1972), Deane (1968), and Deane and Cole (1967). See Barro (1987, f. 5).

by simply dividing the nominal military spending by the nominal GDP. Barro (1987) proposes two alternative temporary military spending variables but finds that the effects on interest rates do not differ greatly when one or the other is used as the explanatory variable. Following one suggestion of Barro, we measure temporary military spending as the difference between the actual military spending ratio and its mean (0.041).

The temporary military spending variable is displayed in Figure 7. To a large degree, it has a pattern similar to the two time series proposed by Barro (1987, Fig. 5), with one notable exception. Our variable is much higher during the Napoleonic wars than Barro’s measures. This period, however, is excluded from our analysis that it is restricted to the convertibility period of 1729–1792, 1817–1913.

Barro (1987) reports regression results only with the long-term nominal consol rate as the dependent variable, since short-term interest rates are not available for the period prior to 1824. He points out that “much of the action in the government spending variable would be lost” (1987, 228) if the sample started in 1824. Barro regresses the nominal interest rate *R* on its temporary military spending variable *(GB)* and includes a first-order serial correlation correction because the residuals were serially correlated. Over the 1729–1913 sample, Barro (1987) reports the following results:

$$R_t = \underset{(0.20)}{3.54} + \underset{(1.3)}{6.1}\; GB_t, \qquad \lambda = \underset{(0.03)}{0.91}$$

$$R^2 = 0.89, \qquad s.e.e. = 0.243, \qquad D.W. = 2.2$$

Where λ is the coefficient of first-order serial correlation and numbers in parentheses are standard errors. With our temporary military spending variable *(GM)*, over the convertibility period 1729–1792, 1817–1913, the results for the regression of the same long-term nominal interest rate are:

$$R_t = \underset{(0.15)}{3.23} + \underset{(1.2)}{5.7}\; GM_t, \qquad \lambda = \underset{(0.03)}{0.90}$$

$$R^2 = 0.87, \qquad s.e.e. = 0.196, \qquad D.W. = 2.2$$

The exclusion of the Napoleonic-wars period accounts for the smaller constant term and standard error of the regression. The estimated coefficients of the military spending variables are both significant at the 5 per cent critical level. A typical war of the 18th century generates a 10 per cent increase in the ratio of temporary military spending to GNP, resulting in a rise of only between 0.57 and 0.61 percentage point in the long-term nominal interest rate.

### 4.3 Regressions of real interest rates

According to Romer's (1996) textbook presentation of Barro's empirical analysis, "the most significant complication he faces is that, instead of having data on short-term real interest rates, he has data only on long-term nominal interest rates" (page 62). However, since the long-term interest rate is a function of the weighted average of expected short-term rates, it too should be affected although to a lesser extent. The key issue is really the use of the nominal interest rate instead of the real interest rate. Regarding this problem, Barro (1987) concedes that "up to now, I have been unsuccessful in generating reliable quantitative measures of long-term inflationary expectations" (page 228). Since the long-term inflation rate during the gold standard is close to zero, Barro assumes that the long-term nominal interest rate is also the real expected rate. However, he does not rule out the possibility that movements in nominal rates might represent changes in inflationary expectations rather than changes in the real expected rate.

The real interest rate series presented in Section 3 provide the opportunity to re-examine Barro's finding while relaxing the assumption that the nominal long-term interest rate is also the expected real rate. The use of the short-term expected real rate from the consol rate is particularly interesting since it can be used over the full sample analyzed by Barro. Also, as illustrated in Figure 5, it is very closely associated with the short-term real rate from the short-run yield, which is available only since 1824. After having corrected for serial correlation, regression of the short-term expected real rate from the split-sample exercise *(RRSS)* over the 1729–1792, 1917–1913 convertibility period yields the following results:

$$RRSS_t = \underset{(0.47)}{3.20} + \underset{(6.9)}{16.4}\, GM_t, \qquad \lambda = \underset{(0.05)}{0.81}$$

$$R^2 = 0.70, \qquad s.e.e. = 1.10, \qquad D.W. = 2.1$$

The results for the regression of the short-term real interest rates from the joint-sample exercise *(RRJS)* over the same sample are:

$$RRJS_t = \underset{(0.52)}{3.23} + \underset{(6.4)}{13.2}\ GM_t, \qquad \lambda = \underset{(0.04)}{0.84}$$

$$R^2 = 0.75, \qquad s.e.e. = 1.03, \qquad D.W. = 2.2$$

The estimated coefficients of the temporary military spending variable are significant at the 5 per cent critical level for both regressions. The $R^2$ is slightly smaller in both case than for the regressions using the nominal interest rate, but remains remarkably high given that short-rates are considerably more volatile than the consol rate. As predicted by the theory, the incidence of temporary military spending on short-term real rates is much larger. A 10 per cent increase in temporary military spending raises short-term real rates between 1.3 and 1.6 percentage points.

The regression of the long-term real interest estimated in Section 3.7 over the 1729–1792, 1817–1913 sample yields the following results:

$$RRLT_t = \underset{(0.30)}{3.15} + \underset{(1.5)}{4.2}\ GM_t, \qquad \lambda = \underset{(0.02)}{0.93}$$

$$R^2 = 0.91, \qquad s.e.e. = 0.250, \qquad D.W. = 1.96$$

The estimated coefficient on *GM* is a bit smaller than in the regression of the consol rate à la Barro. However, the null hypothesis that the two estimated coefficients on *GM* are equal cannot be rejected by a Wald test at the 10 per cent level. The incidence of temporary military spending on short-term real rates is then three to four times higher than on long-term real rates.

## 5. Concluding remarks

In this study, I have shown how the price level can be combined with the nominal interest rate in a monetary regime with a stationary price level to calculate the real interest rate. In such a monetary regime, the price level conveys part of the information necessary for intertemporal choices, just as nominal interest rates do. Viewed in this light, the positive correlation between the price level and the nominal interest rate during the gold-standard period is not so paradoxical after all. The theory also explains why the nominal interest rate did not adjust much in a period that was characterized by numerous long-lasting wars and important gold discoveries. The nominal interest rate did not need to adjust as much because it evolved procyclically with the price cycle. Part of the adjustment in the marginal rate of intertemporal substitution follows from the impact on the real interest rate of expectations that the price level would return toward its standard of value. The resulting estimates of real interest rates during the gold standard were then used to revisit Barro's (1987) study of the incidence of temporary military spending on interest rates. As predicted by the theory, a temporary increase in military spending increases the short-term real interest rate. The estimated effect on short-term real rates is three to four times higher than that on long-term real rates.

To date, post-WWII monetary policies have proven unable (or unwilling) to preserve the value of fiat currencies. The sustained rise in prices, at first barely noticeable, became rampant, steepened, and then skyrocketed in all major industrial countries—in some more than in others—culminating with the second oil-price shock of the 1970s. Subsequently, central banks declared all-out war on inflation. Since the early 1990s, inflation has receded further in most countries to the point that a number of central banks are now concerned with defining a non-inflationary monetary regime.[23] Given this context, this study contributes to our understanding of agents' behaviour under alternative monetary regimes. We conclude this study by drawing some lessons from our analytical framework for the formulation of monetary policy objectives.

Under a monetary regime where the price level is integrated of order one, the nominal interest rate is the only price that conveys intertemporal information. The nominal interest rate must, therefore, adjust freely to movements in the marginal rate of intertemporal substitution. As pointed out by Summers (1991), the existence of a nominal interest rate floor precludes the possibility of a negative real interest rate in a zero-inflation world. In this case, a low but positive inflation target might be preferable to zero inflation because it gives more room for stabilization policy to work through intertemporal substitution. However, Summers' analysis does not apply to a regime of price stability. As shown in Section 2, it is theoretically possible for the short-term real interest rate to become negative under a monetary regime where the trend inflation rate is zero. This actually occurred twice in the United Kingdom—in the 1850s and again at the end of the 1890s.

[23] On this, refer to Bank of Canada (1994; 1998).

From an operational point of view, central banks control the evolution of the price level through their influence on short-term nominal interest rates. If central banks choose a price-level target, they will have to increase nominal interest rates when the price level is above target and to decrease nominal interest rates when the price level is below. This study shows that it would be essential then for monetary authorities to recognize that the price level is an intertemporal price. If central banks choose to restore price-level stability, agents will gradually forget to focus on the first difference of the price-level series and will gradually learn to look at the difference between the actual price level and its long-run equilibrium value. We might then finally leave Fisher's world to return to the world of Gibson.

## Appendix: The long-run real return on consols under price stability[24]

The relationship between a constant bond's yield ($C$) as a percentage of the face value, its market value ($V$), and its nominal rate of return ($r$) is:

$$V = \int_{t=0}^{\infty} C e^{-rt} dt = C / r.$$

The real expected rate of return ($\rho$) on the consol is:

$$V = \int_{t=0}^{\infty} C \frac{P_0}{P_t} e^{-\rho t} dt$$

where $P_0$ is the actual price level and $P_t$ is the price level at time $t$. Let us suppose that the price level is stationary and its evolution can be described by the following differential equation:

$$\frac{P_t}{P_0} = \frac{Q}{P_0}(1 - e^{-\alpha t}) + e^{-\alpha t}$$

where $Q$ is the long-run equilibrium value of the price level. For the purpose of the estimation of the real rate, $\alpha$ is set to 0.1 for the 1717–1792 period and to 0.11 for the 1815–1913 period, given the dominant roots of the AR processes estimated in Section 3 for the split-sample exercise.[25] $Q$ was estimated as the mean of the price level in each subsample. For the given path of $P_t$, the solution of the integral yields:

$$\rho = r + \alpha \left( \frac{r}{\alpha + \rho} \right) \left( \frac{P_0 - Q}{Q} \right).$$

[24] The derivation of the formula for the real expected interest rate on consols was provided by Pierre Duguay from the Bank of Canada.

[25] 0.9 is the root of the AR(1) process for the 1717–1992 period. The roots of the AR(2) process for the 1815–1913 period is 0.89 and 0.23.

The two solutions of this quadratic equation in ρ, are:

$$2\rho = r - \alpha \pm (r + \alpha)\sqrt{1 + \left(\frac{P_0 - Q}{Q}\right)\left(\frac{4\alpha r}{(r + \alpha)^2}\right)}$$

For $(P_0 - Q)/Q$ = zero, one can see that ρ= $r$ for the following solution:

$$2\rho = r - \alpha + (r + \alpha)\sqrt{1 + \left(\frac{P_0 - Q}{Q}\right)\left(\frac{4\alpha r}{(r + \alpha)^2}\right)}$$

Whereas ρ=α in the other. The last equation is used to estimate the real expected return in Section 3.7.

**Table I**

**Estimation of the short-term real interest rate from the consol rate in the United Kingdom**

| **1A: 1717-1792 sample (in %)** | | | | |
|---|---|---|---|---|
| | Mean | Maximum | Minimum | Std. Dev. |
| Consol rate | 3.57 | 5.41 | 2.83 | 0.63 |
| Price contribution | 0 | 2.42 | -1.87 | 1.12 |
| Short-term real rate | 3.57 | 7.59 | 1.33 | 1.57 |

| **1B: 1815-1913 sample (in %)** | | | | |
|---|---|---|---|---|
| | Mean | Maximum | Minimum | Std. Dev. |
| Consol rate | 3.20 | 5.02 | 2.25 | 0.45 |
| Price contribution | 0.09 | 4.99 | -3.45 | 1.92 |
| Short-term real rate | 3.28 | 9.56 | -1.10 | 2.28 |

| **IC: Joint sample 1717-1792, 1815-1913 (in %)** | | | | |
|---|---|---|---|---|
| | Mean | Maximum | Minimum | Std. Dev. |
| Consol rate | 3.36 | 5.41 | 2.25 | 0.57 |
| Price contribution | 0 | 5.53 | -3.18 | 1.80 |
| Short-term real rate | 3.35 | 10.02 | -0.58 | 2.08 |

| **1D: Split sample 1717-1792, 1815-1913 (in %)** | | | | |
|---|---|---|---|---|
| | Mean | Maximum | Minimum | Std. Dev. |
| Short-term real rate | 3.40 | 9.56 | -1.10 | 2.00 |

**Table 2**
**Descriptive statistics on the estimated real interest rates (in %)**
**in Canada, 1954-1994**

| | Mean | Maximum | Minimum | Std. Dev. |
|---|---|---|---|---|
| Nominal rate of return | 8.25 | 15.22 | 3.63 | 2.87 |
| Expected inflation | 4.25 | 11.51 | 1.17 | 2.95 |
| Real rate of return | 3.99 | 9.46 | -2.32 | 2.71 |

Data were provided by Nicholas Ricketts, Research Department, Bank of Canada.
The expected inflation is an annual average estimated from a Markov model (see Laxton, Ricketts and Rose (1994) for a similar model).
The nominal rate of return is the average return on Government of Canada bonds with terms of 10 years and over.

**Table 3**
**Estimation of the short-term interest rate from the short-term yield in the United Kingdom, 1824-1913 sample (in %)**

| | Mean | Maximum | Minimum | Std. Dev. |
|---|---|---|---|---|
| Short-term nominal rate | 3.35 | 7.00 | 0.96 | 1.15 |
| Price contribution | -0.17 | 3.71 | -3.45 | 1.69 |
| Short-term real rate | 3.18 | 8.21 | -1.71 | 2.37 |

**Table 4**
**Estimation of the long-term real interest rate**
**on the consol rate in the United Kingdom, 1717-1992, 1815-1913 (in %)**

| | Mean | Maximum | Minimum | Std. Dev. |
|---|---|---|---|---|
| Consol rate | 3.36 | 5.41 | 2.25 | 0.57 |
| Long-term real rate | 3.38 | 6.14 | 1.55 | 0.87 |

Figure 1: The Gibson Paradox in U.K., 1717-1931

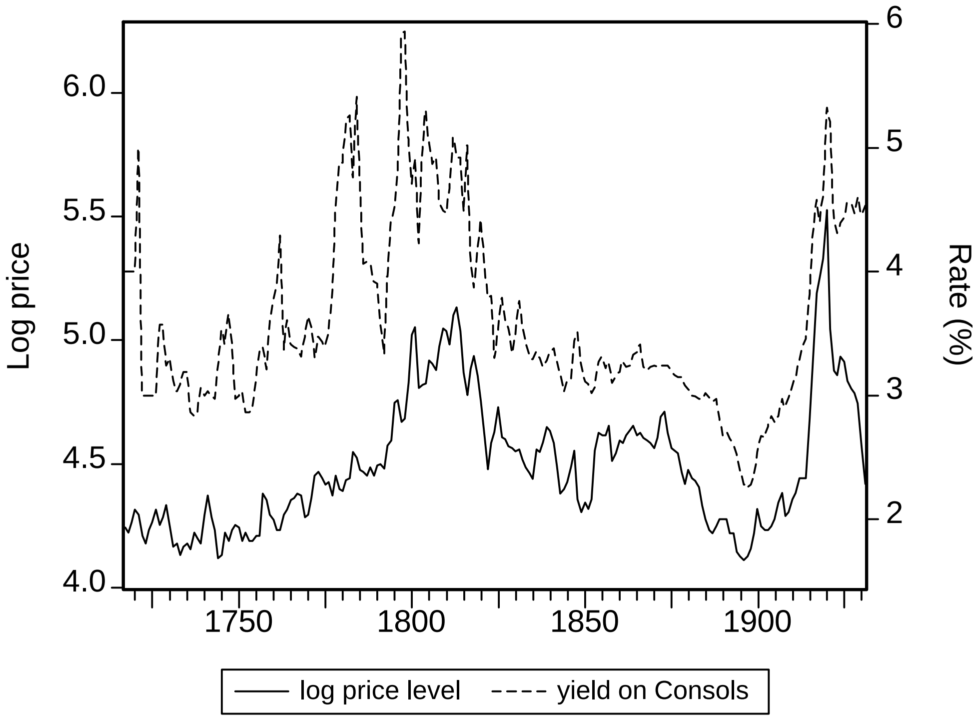


Figure 2a: The Real Interest Rate in U.K., 1717-1792

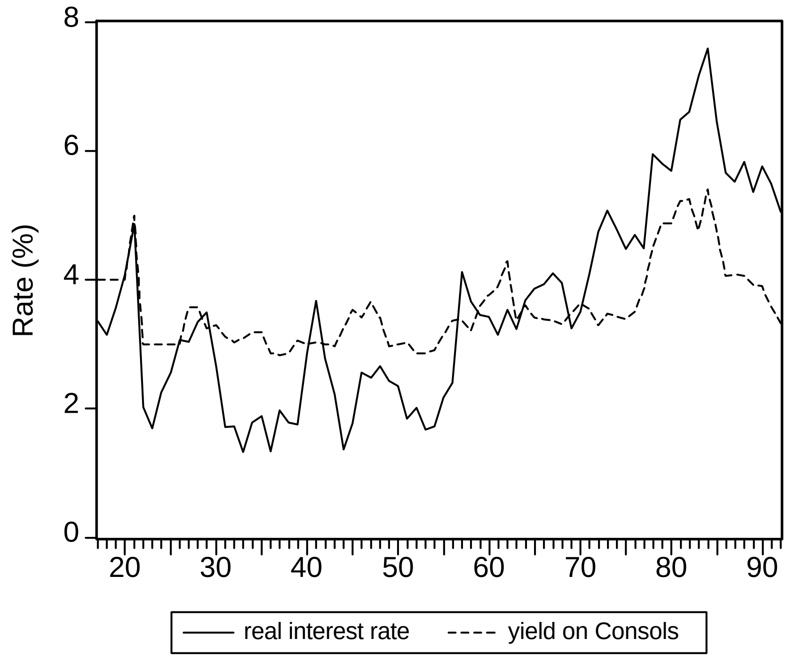



Figure 2b: The Real Interest Rate in U.K., 1817-1913

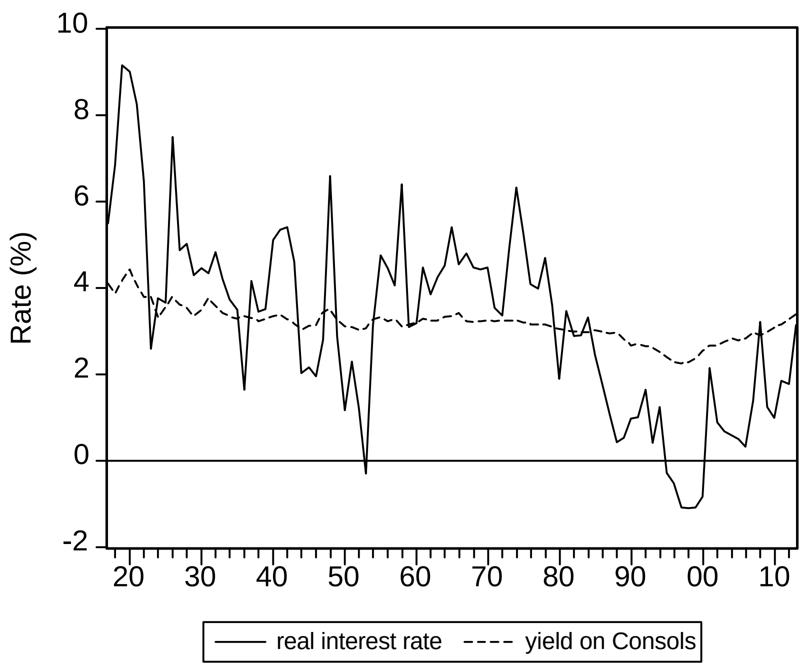


Figure 3: The Real Interest Rate in U.K.
from the Joint Sample Estimates, 1717-1913

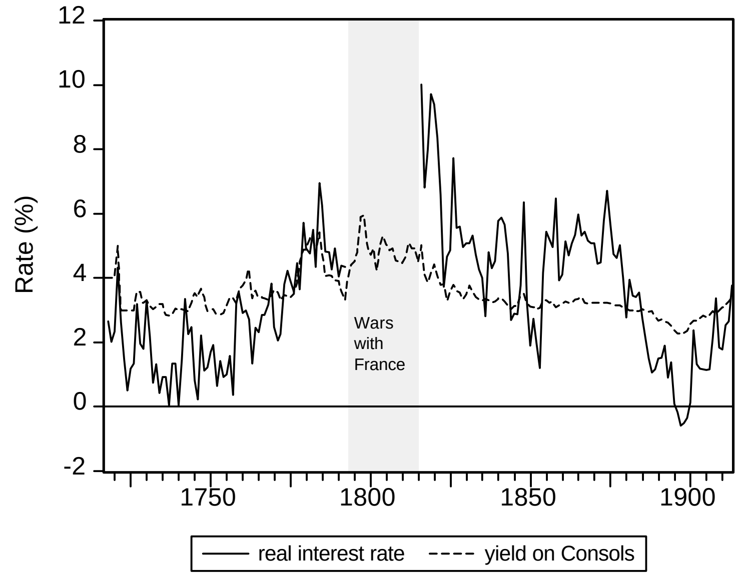



Figure 4: The Nominal and the Real Interest Rate
in U.K. from the Short-run Yield, 1825-1913

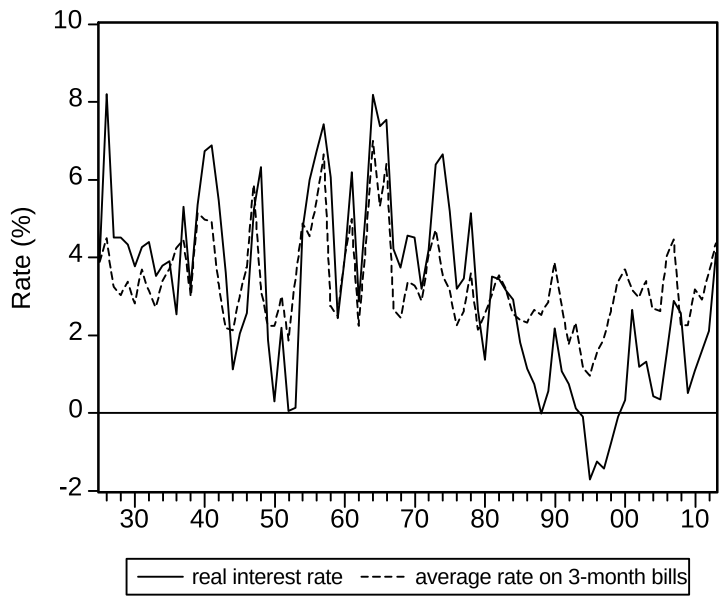


Figure 5: The Real Interest Rate in U.K.
from the Short- and the Long-run Yield, 1825-1913

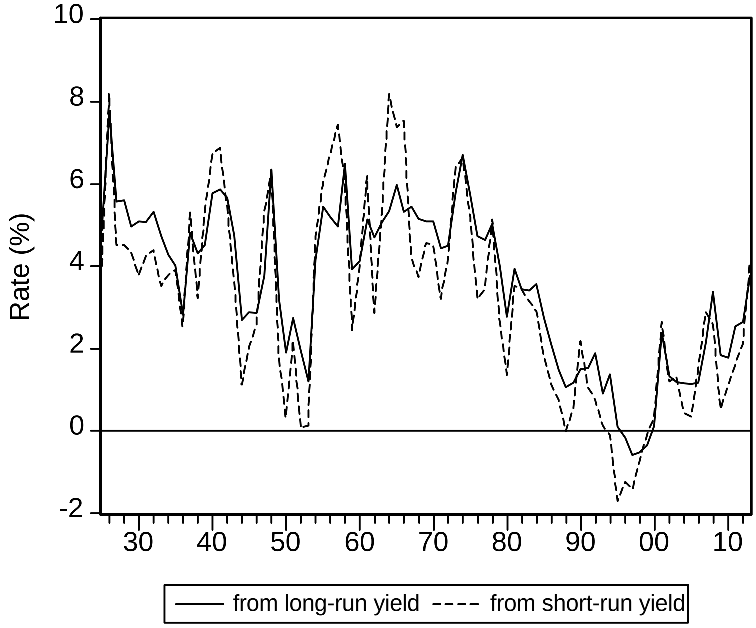



Figure 6: The Long-term Expected Rate of Retun
on Consols in U.K., 1717-1913

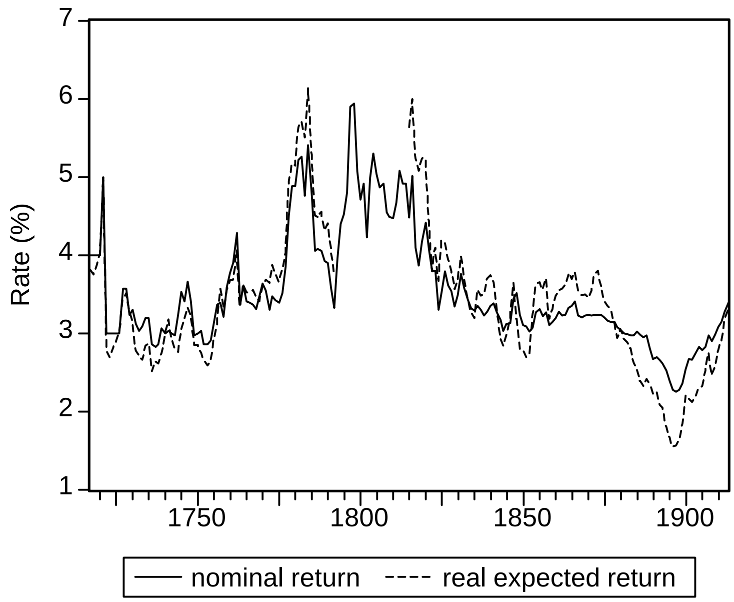
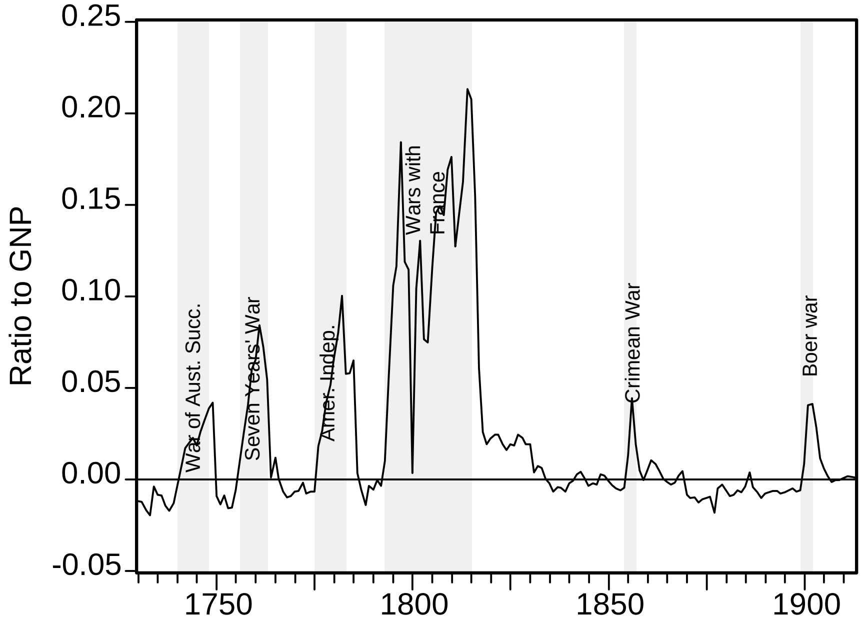


Figure 7: Temporary Military Spending in the U.K., 1730-1913



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