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PROVINCIAL CREDIT RATINGS IN CANADA: An Ordered Probit Analysis
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This paper is intended to make the results of Bank research available in preliminary form to other economists to encourage discussion and suggestions for revision. The views expressed are those of the author. No responsibility for them should be attributed to the Bank of Canada.

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Abstract

The author estimates the relationship between the provincial credit ratings, as assessed by Standard & Poor's, and a number of economic variables, using the *ordered probit* methodology. All the variables in her estimation prove to be significant. In particular, she finds that downgrades take place at almost the same speed at different levels of the debt-to-GDP ratio, based on a pooled sample of nine provinces. Based on smaller pools of provinces with similar economic charateristics, she finds that downgrades take place at different speeds at different levels of the debt-to-GDP ratio.

To assess the likelihood of further provincial government rating downgrades, the author generates probabilistic projections of the provincial credit ratings over the years 1996 to 2000 under various assumptions about government revenue and spending growth. The projections suggest that no further downgrades are likely for provinces whose policies result in only moderate increases or decreases in the debt-to-GDP ratio. Some provinces may be upgraded if they benefit from income growth or follow fiscal policies that result in a declining debt-to-GDP ratio or an improvement in other explanatory variables.

Résumé

L'auteure utilise la méthode des *probits ordonnés* pour estimer les relations entre les cotes de solvabilité octroyées aux provinces par Standard & Poor's et un certain nombre de variables économiques. Les résultats s'avèrent significatifs dans le cas de toutes les variables estimées. En particulier, l'auteure constate, sur la foi d'un échantillon regroupant les données de neuf provinces, que les déclassements se produisent à peu près au même rythme à différents niveaux du ratio de la dette au PIB. Lorsque l'estimation porte sur de plus petits échantillons de provinces ayant des profils économiques semblables, les déclassements surviennent à des rythmes qui varient avec le niveau du ratio de la dette au PIB.

Afin d'évaluer les chances que survienne un nouvel abaissement des cotes de solvabilité provinciales, l'auteure effectue des projections probabilistes de ces dernières sur la période 1996-2000, en s'appuyant sur diverses hypothèses relatives à la croissance des recettes et des dépenses publiques. Les résultats semblent indiquer qu'un nouveau déclassement est peu probable dans le cas des provinces dont le ratio de la dette au PIB n'augmente ou ne diminue que légèrement sur la période de projection. Certaines provinces pourraient voir leur cote monter si la croissance économique ou leurs politiques budgétaires donnaient lieu à une réduction du ratio de la dette au PIB ou à une amélioration d'autres variables explicatives.

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1. Introduction

Credit ratings represent rank orderings of the likelihood of default on loan payments of interest and principal. Credit ratings are thought to influence borrowing conditions through two channels: the cost of credit and the availability of credit. For instance, a bond rating downgrade would increase the risk premium demanded by the market and therefore the cost of borrowing; a rating downgrade could also result in a reduction of the potential market for the downgraded debt, as some investors are either unable (due to institutional constraints) or unwilling to hold debt below a certain rating.

Against a background of rapid debt accumulation, most provinces were downgraded during the 1980s and/or the early 1990s. However, Saskatchewan, where debt accumulation was particularly rapid, stands out as the province that experienced the largest number of downgrades as well as the largest cumulative reduction in its rating. Based on Standard & Poor's ratings, the long-term debt of Saskatchewan was downgraded in five steps between 1986 and 1992 from AA+ to BBB+. The number of prospective institutional buyers of Saskatchewan bonds is estimated to have fallen from 125-140 when the province's rating was AA+, to about 25-30 at the current rating of BBB+. Boothe (1993a) summarizes this experience as an inverse non-linear relationship between per capita debt and bond rating, under which successively smaller increases in per capita debt led to rating downgrades.

Based on the Saskatchewan experience, Boothe (1993a) conjectures that other provinces that were downgraded in the early 1990s, such as Alberta and Ontario, would experience further downgrades over the remainder of the current decade if they were to continue the spending and revenue trends of the early 1990s.

However, as Boothe (1993b) acknowledges, the relationship between per capita debt and bond rating identified for Saskatchewan is only an imperfect guide to the future bond rating experience of other provinces. Ontario, for instance, has a more diversified economic structure and stronger economic growth prospects than resource-based Saskatchewan, which is more vulnerable to market and weather conditions. Moreover, Boothe's (1993b) finding was based on an estimation that ignored the qualitative ordinal nature of the dependent variable (i.e., the credit rating index).

The present paper attempts to estimate a model for the provincial bond ratings based on the pooled experience of Canadian provinces over the fiscal years 1969-70 to 1994-95. An *ordered probit* method is used to estimate the probabilities for the different bond ratings assigned to the provinces for given values of the explanatory variables. The empirical search for explanatory variables is guided by the significance and consistency

of the variables in affecting the credit ratings. In addition to the debt indicator, I find that provincial bond rating is correlated with the employment ratio (the number of persons employed as a fraction of the population aged 15 and above), provincial GDP as a share of total Canadian GDP, federal transfers as a proportion of total provincial revenues, and the proportion of provincial personal income represented by unemployment insurance (UI) benefits.

In contrast to Boothe's (1993b) finding, my results show that downgrades take place at similar speeds at different levels of the debt-to-GDP ratio, based on the average experience of the nine provinces rated by Standard & Poor's. Moreover, downgrades take place at lower speeds at higher levels of the debt-to-GDP ratio for the average experience of the three Atlantic provinces, although downgrades take place at higher speeds at higher levels of the debt-to-GDP ratio for the average experience of British Columbia, Alberta, Ontario, and Quebec and for the average experience of Saskatchewan and Manitoba. Probabilistic projections of the provincial credit ratings over the years 1996 to 2000 are then generated under various assumptions about government revenue and spending growth in order to assess the likelihood of further provincial government downgrades. The projections suggest that no further downgrades are likely for provinces whose policies result in only moderate increases or decreases in the debt-to-GDP ratio. Some provinces may be upgraded if they benefit from income growth or follow fiscal policies that result in a declining debt-to-GDP ratio and/or an improvement in other explanatory variables.

The paper is organized as follows. Section 2 reviews Standard & Poor's credit rating scale (also used by Boothe) and the history of ratings for Canadian provincial governments. Section 3 describes the ordered probit model. Section 4 discusses the specification and reports estimation results. Section 5 generates projections for provincial credit ratings over the remainder of the 1990s under different economic scenarios. Section 6 offers concluding remarks.

2. Standard & Poor's provincial credit ratings

The ratings assigned by Standard & Poor's are expressed in letter form, ranging from AAA, the highest, to CCC, the lowest. These ratings represent an attempt to divide a continuum of risk into discrete risk classes based on an assessment of the capacity of the debt issuer to pay interest and repay the principal in accordance with the terms of the issue. The assessment of the credit risk of provincial governments takes into account actual and projected developments in a wide number of areas, including demographic,

^{1.} Prince Edward Island is not rated by Standard & Poor's.

economic, financial and debt indicators. Informed judgment by the rating agency analysts also plays an important role. The highest rating, AAA, indicates an extremely strong capacity to pay interest and repay principal, while the lowest rating, CCC, indicates a serious vulnerability to default on payment of interest and/or repayment of principal. Debt rated from AAA to BBB is considered "investment" grade, while debt rated at BB and below is considered "speculative" grade. Table 1 describes in more detail the current meaning of Standard & Poor's debt ratings (the meaning of the ratings may have varied over time).² Since 1974, plus and minus symbols have been attached to the ratings from AA to CCC in order to show relative standing within the major rating categories.³ Standard & Poor's also introduced rating outlooks in the summer of 1989, which indicate the likely direction of the next rating change if current economic and financial conditions continue.

Standard & Poor's began to rate Canadian provinces in July 1966. Table 2 and Chart 1 summarize the history of Standard & Poor's ratings for the long-term debt of the Canadian provinces. 4 At the moment, no province has a triple-A rating, although British Columbia, Alberta and Ontario had triple-A ratings at times during the last two decades. Ontario was the last province to have a triple-A rating, in the late 1980s and early 1990s. The current ratings range from AA+ for British Columbia to BBB+ for Saskatchewan and Newfoundland. From July 1966 to October 1967, Newfoundland had the lowest rating ever assigned to a Canadian province by Standard & Poor's - BBB - a rating Newfoundland and Saskatchewan are currently only a notch above. Ontario, British Columbia and Alberta usually receive higher ratings than the other provinces.

Most provinces were upgraded in the 1970s and/or in 1980 and 1981; not a single downgrade took place between 1966 and 1981. However, between 1982 and 1994, 21 downgrades, affecting eight provinces, took place in a context of rising provincial debtto-GDP ratios (see Charts 2 to 10), leaving many provinces in 1994 with their lowestever ratings. Although the general increase in provincial debt-to-GDP ratios during the recessions of the early 1980s and 1990s had an important cyclical component, the continuation of an upward trend in those ratios between the two recessions indicates structural imbalances between spending programs (including debt service) and the revenue-raising capacity of the provinces.

Tables and charts are grouped at the end of the paper.
 The rating of Moody's, the other major U.S. rating agency, has historically been viewed by market participants as roughly equivalent to Standard & Poor's. This view seems validated by formal statistical analysis, as in Cantor and Packer (1994).
 Usually the same rating is assigned to both the long-term direct debt and long-term guaranteed

obligations of provincial governments.

New Brunswick stands out as the only province to have a higher rating now than at the start of the 1980s. The province was upgraded in 1991 to AA- from A+, a rating it had had since 1976. This upgrade occurred after eight years of sustained reduction in the province's deficit-to-GDP ratio, efforts to diversify the province's economic structure, and a public commitment to responsible fiscal planning. With this upgrade, New Brunswick also became the only Atlantic province to have ever received a double-A rating from Standard & Poor's.

3. The ordered probit model

Credit ratings can be viewed as resulting from a continuous, unobserved creditworthiness index. Each credit rating corresponds to a specific range of the creditworthiness index, with higher ratings corresponding to a higher range of creditworthiness values. Since the credit rating representation of creditworthiness is a qualitative ordinal variable, the estimation of a model for such a dependent variable necessitates the use of a special technique.

Consider the simple case of a qualitative unordered dichotomous dependent variable, i.e., a variable that can take only two values (such as *yes* or *no*, *on* or *off*). Assume that this variable, represented as a 0-1 binary variable, is modelled as a linear function of a set of explanatory variables and of an error term. The predicted values from the estimation of this model should fall mainly within the 0-1 interval, suggesting that they could be interpreted as probabilities that the dependent variable takes the value 0 (or 1), given the values of the explanatory variables. However, such estimated probabilities can fall outside the 0-1 range. Various distribution functions are available to constrain the estimated probabilities to lie in the range (0,1), the most frequently used being the cumulative standard normal probability function and the logistic function. The probit model makes use of the former, while the logit model makes use of the latter. If the qualitative dependent variable can be classified into more than two categories (i.e., if it is a polychotomous variable), estimation can be undertaken by means of the multinomial probit or the multinomial logit models, which are generalizations of the binary probit and logit models.

However, the credit rating representation of creditworthiness is not only a polychotomous qualitative variable; it is also an ordinal variable, i.e., a variable with an inherent order (unlike a polychotomous variable representing, say, choices of colours or travel destinations). An ordinal polychotomous dependent variable would usually be coded as 0, 1, 2, 3, and so on. This representation reflects only a ranking; it is not known to what extent going from 0 to 1 is different from (or equivalent to) going from 2 to 3. For

such an ordinal dependent variable, using multinomial probit or logit would not be efficient, because these models would misspecify the data-generating process in assuming that there is no order in the different categories that the dependent variable can take. OLS regression estimation would also be inappropriate, since OLS would consider the difference in the dependent variable between a 1 and a 2 as equivalent to the difference between a 2 and a 3. For example, the difference from AAA to AA+ will be treated the same as from AA+ to AA in OLS. The *ordered multinomial probit* (OMP) is used for estimation in the context of an ordinal polychotomous dependent variable. While taking into account the existence of a ranking, the OMP also assumes that the size of the difference between any two adjacent ratings is not known but does not matter to the carrying out of the analysis, unlike, for example, the usual regression techniques, where the size of the difference between adjacent elements is known and matters to the carrying out of the analysis.

Assume that the unobserved continuous measure, creditworthiness (\tilde{y}) , is a linear function of a set of explanatory variables x, with parameter vector β , and an error term ε:

$$\tilde{y} = \beta' x + \varepsilon$$

As usual, \tilde{y} is unobserved. What is observed are the credit ratings assigned to the provinces, which range from AAA to BBB+ over the years 1970-1995.⁵

$$y = AAA$$
 if $\tilde{y} \le \mu_1$,
 $= AA+$ if $\mu_1 < \tilde{y} \le \mu_2$,
...
$$= BBB+$$
 if $\mu_7 \le \tilde{y}$.

This is a form of censoring. The µ's are unknown partition boundaries (or cut points) that define the ranges of the creditworthiness index (i.e., AAA, AA+, ... BBB+); these parameters must be estimated in conjunction with the β vector. Estimation proceeds by maximum likelihood (because of the asymptotic properties of ML estimators: consistency, normal distribution and efficiency).

^{5.} The range of y is constrained by the range of observed ratings assigned during the years that are included in the sample. This means that the probability distribution of a rating outside the range of observed ratings (i.e., BBB) cannot be estimated .

6. If an intercept is included in the equation for \tilde{y} , it is customary to normalize by setting μ_1 equal to

zero.

It is assumed that ϵ is normally distributed across observations, and the mean and variance of ε are normalized to zero and one. With the normal distribution, the following probabilities result (for simplicity, AAA, AA+, ..., BBB+ is recoded as 1, 2, ... 8.), where Φ is the cumulative function of a normal distribution:

Prob
$$(y = AAA \text{ or } 1) = \Phi(\mu_1 - \beta' x)$$
,
Prob $(y = AA+ \text{ or } 2) = \Phi(\mu_2 - \beta' x) - \Phi(\mu_1 - \beta' x)$,
...

Prob
$$(y = BBB + or 8) = 1 - \Phi(\mu_7 - \beta'x)$$
.

A likelihood function can be formed as follows:⁷

$$L(y/x) = \sum_{k=1}^{n} \left\{ r_{1k} \bullet \log \Phi(\mu_1 - x_{k'}\beta) + \sum_{i=2}^{7} r_{ik} \bullet \log \left[\Phi(\mu_i - x_{k'}\beta) - \Phi(\mu_{i-1} - x_{k'}\beta) \right] + r_{8k} \bullet \log \left[1 - \Phi(\mu_7 - x_{k'}\beta) \right] \right\}$$

where Y_{ik} is an indicator variable that takes on the value one if the realization of the kth observation y_k is the *i*th rating, and zero otherwise. Once the likelihood function is formed, the estimation of the unknown parameters μ 's and β 's can be undertaken. The estimated cutoff points, μ's, along with the estimated β's, maximize the log-likelihood function stated above.⁸

Note that the impact of a change in an explanatory variable on the estimated probabilities of the highest and lowest of the ordered classifications is unequivocal. For instance, if β_i is positive, an increase in the value of x_i (an increase in the conditional mean $\beta'x$) definitely decreases the probability of having the top rating (AAA) and definitely increases the probability of having the bottom rating (BBB+). However, the impact on the estimated probabilities of intermediate classifications (AA+ to A-) can be in either direction.

Note also that an estimated β value in a probit (or a logit) model does not estimate the change in the probability of a given outcome due to a unit change in the relevant explanatory variable. This probability change is given by the partial derivative of the expression for prob (y = i) with respect to x_j , which is a function of β_j and of normal density functions at the value of x_i , at which the partial derivative is calculated. For example, the probability of getting the AA rating is

For the model of unordered multiple choices, see Greene (1993, 664-72). As mentioned earlier, this model can also be estimated with the ordered logit. This is a trivial modification of the formulation and appears to make virtually no difference in practice.

prob
$$(y = AA) = \Phi(\mu_3 - \beta' x) - \Phi(\mu_2 - \beta' x)$$

The marginal effects of changes in x_i on the probability of having an AA rating is

$$\frac{\partial Prob(y=AA)}{\partial x_{j}} = (\phi(\mu_{3} - \beta'x) - \phi(\mu_{2} - \beta'x))\beta_{j}$$

where $\boldsymbol{\phi}$ is the density function of a standard normal distribution. 9

4. Estimation

4.1 Identifying and modelling explanatory variables

We begin our search for the explanatory variables by using one debt indicator, as in Boothe (1993b). While Boothe used per capita debt, I prefer to use the gross direct debt-to-GDP ratio (simplified as debt-to-GDP ratio), which is a more appropriate indicator of the relative burden of government debt on the economy. The net direct debt is not used here because it is negative in some periods of time for some provinces, which prevents the use of the logarithmic transformation of the variable; however, it should be noted that the ratios of net and gross direct debt to GDP show similar trends over time (see Charts 2 - 10).

Gross direct debt series for the provincial governments are from the Financial Management System (FMS). As debt is measured at the end (31 March) of the fiscal years 1969/70 to 1994/95, there is a total of 26 observations for each province. The debt-to-GDP ratios are calculated by using nominal GDP at market prices for the calendar year ending during the fiscal year.

A sample size of 26 observations for each province is not large enough for the *ordered probit* model, estimated with the *maximum-likelihood* method, to produce good results. To overcome this problem, a cross-sectional data set is constructed by pooling the data of the nine provinces rated by Standard & Poor's. For estimation purposes, the credit rating scale is translated into numbers as follows: AAA = 1, AA+ = 2, AA = 3, AA- = 4, A+ = 5, A= 6, A- = 7, and BBB+ = 8.

Using other "debt" indicators, such as per capita gross direct debt, per capita net direct debt, direct debt as a proportion of government revenue, or total direct and guaranteed debt as a proportion of government revenue yields inferior results in terms of log likelihood. I then searched for additional explanatory variables, based on the list of demographic, economic, and financial indicators used by Standard & Poor's when

^{9.} For more information on ordered probit models, see Greene (1993, 672-76).

assessing credit ratings (see Table 3)¹⁰ and on my own assessment of variables that could play a significant role in explaining ratings changes, such as the diversification of a province's economy and the presence of federal automatic stabilizers that mitigate the deterioration of a province's fiscal situation during economic slowdowns. A stepwise procedure is used to determine which variable(s) would provide the best fit. 11 After trying a number of combinations with the stepwise procedure, I found that five variables consistently produce the best results, using the pooled sample of nine provinces: gross direct debt-to-GDP ratio (DDGDP), the employment ratio (ER, the product of labour force participation rate and employment rate), 12 federal transfers as a proportion of total provincial revenues (TRREV), provincial GDP as a share of the Canadian total (SIZE), and the proportion of provincial personal income represented by UI benefits (*UIBPI*). ¹³ The *SIZE* variable can be seen as a proxy for economic diversification since the larger provinces are usually more diversified than the smaller provinces. More diversified economies are less vulnerable to adverse sectorial shocks.

Therefore, the best empirical specification obtained for the nine provinces pooled together is:

$$RATE_{it} = \beta_0 + \beta_1 \bullet DDGDP_{it}^{(\lambda)} + \beta_2 \bullet ER_{i(t-1)} + \beta_3 \bullet SIZE_{i(t-1)} + \beta_4 \bullet TRREV_{it} + \beta_5 \bullet UIBPI_{i(t-1)} + \epsilon_{it}$$

where *i* is an index for the provinces; *t* is a time index, ranging from 1970 to 1995; $RATE_{it}$ is the credit rating of province i on 31 December of year t; $DDGDP_{it}^{(\Lambda)}$ is the Box-Cox transformation of the debt-to-GDP ratio (the debt is as of 31 March in year *t*, and GDP is nominal GDP of province *i* in year *t-1*), equalling $(DDGDP^{\lambda} - 1)/\lambda$ (where $\lambda \ge 0$; ¹⁴ $ER_{i(t-1)}$ represents the employment ratio of province i in year t-1; $SIZE_{i(t-1)}$ is province is GDP as a share of the Canadian total in year t-1, $TRREV_{it}$ is federal transfers as proportion of total provincial revenues in year t, $\mathit{UIBPI}_{\mathit{I(t-1)}}$ is the proportion

^{10.} Using more than one variable from the same category is avoided to prevent possible multicollinearity problems.

^{11.} Stepwise regression (Efroymson 1960) combines the two procedures of forward selection and backward elimination, following backward elimination by forward selection until both fail to change the model. In forward selection, the best unselected covariate satisfying the selection criterion is added at each stage until no further candidates remain. Backward elimination begins with the full set and eliminates the worst covariates one by one until all remaining covariates are

^{12.} Using the product of the two variables rather than one of them is more suitable, especially for

^{12.} Using the product of the two variables rather than one of them is more suitable, especially for provinces with many discouraged workers.
13. The transfers and revenues series are from FMS and calculated for the fiscal year ended on 31 March, while the employment ratio, UIB, and provincial personal income series are calculated for the calendar year ending during the fiscal year. Some of the explanatory variables are chosen by Bayoumi et al. (1993) when estimating an equation for the yield spread for state government bonds in the United States.
14. By l'Hôpital's Rule, lnx is the limit of (x^λ - 1)/λ as λ → 0.

of provincial personal income in province *i* represented by UI benefits in year *t-1*, and ϵ_{it} is the error term. 15

The Box-Cox transformation of the debt-to-GDP ratio allows this explanatory variable to enter the conditional mean ($\beta'x$) linearly [$\lambda = 1$] or non-linearly [$0 \le \lambda < 1$ or $\lambda > 1$]. The estimated curvature of the Box-Cox transformation will play an important role in the measurement of rating changes. Moreover, the explanatory variables are used in a lagged fashion in the estimation to allow some time for the credit agency to respond to new information and update their ratings accordingly when necessary.

4.2 Estimation results

The maximum-likelihood estimation is carried out by a packaged ordered probit command in STATA. However, a joint estimation of the Box-Cox parameter λ and other ordered probit parameters via maximum likelihood is not available in STATA or any other package. A separate program is required to do the joint estimation. The alternative to programmed joint estimation used here is a grid-search method, where regressions are run for a predetermined range of values for the Box-Cox parameter λ (going from 0 to 3.0 in 0.1 increments) and the λ is chosen that maximizes the log-likelihood function (using smaller increments would lead to slightly different likelihood-maximizing λ values).

The estimation results of Equation (1) for the nine provinces pooled together are reported under Estimation (1) in Table 4. A pseudo R^2 of 0.31 is quite good for the amount of information in the estimation. The employment ratio, provincial GDP as a share of the Canadian total, and the proportion of provincial personal income represented by UI benefits are significant and have a favourable impact on the credit ratings. For example, a higher employment ratio decreases the conditional mean $\beta'x$, and given the partition boundaries, a lower conditional mean implies a higher probability of observing a more extreme high rating. In addition, *UIBPI* is acting as a balancing factor for ER, for a province with low employment. For example, UI benefit payments will differ for the same case across provinces, since they are adjusted for the unemployment rate in different regions. Federal transfers as a proportion of total provincial revenues have an unfavourable impact on credit ratings, since higher dependency on transfers rather than own-source revenues signals the incapability and

^{15.} In this paper, the error term is assumed to be independently, identically, and normally distributed (i.i.d.). The STATA program, which I use in this paper to do estimations, does not allow non-identical errors for ordered probit. It will be worth redoing the estimations when this feature becomes available in the future.

^{16.} An example of a programmed joint estimation of a Box-Cox parameter and ordered probit parameters is found in Hausman, Lo and MacKinlay (1992).

vulnerability of a province's economy. The results also suggest a linear relationship between the debt-to-GDP ratio and the conditional mean of the provincial credit rating distribution (Chart 11, Curve 1¹⁷). In other words, the debt-to-GDP ratio increases the conditional mean at the same speed at all levels, *ceteris paribus*. However, this does not necessarily suggest the same linear relationship between the debt-to-GDP ratio and the change of credit ratings.

The change in the probability of getting a particular rating also depends on the partition boundaries, the μ 's or cut points as described at the end of Section 3. In order to infer the real relationship between the debt-to-GDP ratio and the change of credit ratings, I calculate and then graph the probability distributions of all the credit ratings in the sample on various levels of the debt-to-GDP ratio (Charts 12a & 12b), holding other explanatory variables constant at their sample means. In Chart 12b (and Charts 13b–to 15b), the x axis denotes the debt-to-GDP ratio, the y axis denotes the level of probability of having one credit rating, and the curves, each of which is associated with a rating, show how the probability of each particular rating changes when the debt-to-GDP ratio changes. Therefore, the upper envelope of the different curves represents the credit rating with the highest probability at each level of the debt-to-GDP ratio.

If it is the case that "successively smaller increases in debt load trigger downgrades" as stated by Boothe (1993b, 5), the horizontal distances between two crossover points of the upper envelope of the different curves should become smaller at higher debt-to-GDP ratios. However, Chart 12b suggests that the horizontal distances between two crossover points are very similar at different levels of the debt-to-GDP ratio.

Chart 12b also shows how fast the next lower rating becomes the one with the highest probability when the debt-to-GDP ratio increases. To find out the probability of a downgrade when the debt-to-GDP ratio increases, one can compare the rise in the probabilities of relevant portions of the curves that represent different ratings. The relevant portion of each curve is the portion just before the curve becomes part of the upper envelope. As seen in Chart 12b, the rise in the probabilities of downgrades, which are indicated by thick lines, are very similar at different levels of the debt-to-GDP ratio, suggesting that downgrades would take place at similar speeds at different levels of the debt-to-GDP ratio, in contrast to Boothe's (1993b) finding. Two things are worth mentioning again here: 1) Boothe's (1993b) finding was based on an estimation for a

^{17.} Setting other explanatory variables at their sample means, I graph the conditional mean versus the level of DDGDP ranging from 0.05 to 1. Curve 1 in Chart 11 uses results from Estimation (1) reported in Table 4 for the nine provinces together.

single province (Saskatchewan) and assumed that each credit step was of equal importance; 2) our results are based on the average experience of nine provinces.

In order to assess how sensitive the results are to smaller groups of provinces, Equation (1) is re-estimated for three subgroups: the four largest and economically most diversified provinces, i.e., Alberta, British Columbia, Ontario, and Quebec; the Prairie provinces, i.e., Manitoba and Saskatchewan; and the three Atlantic provinces rated by Standard & Poor's, i.e., New Brunswick, Newfoundland, and Nova Scotia. Estimation results for these groups are reported under Estimations (2) - (8) in Table 4.

For each group, the log likelihood is significantly higher than for the nine provinces pooled together, although *UIBPI* is no longer significant in any of the subgroups, while *SIZE* is not significant in the four largest provinces' group. In addition, the signs of the coefficients for *SIZE* and *TRREV* change for the Atlantic provinces' group. Further investigation – running separate regressions for each of the variables *ER*, *SIZE*, and *TRREV* with the debt-to-GDP ratio – shows that none of these variables are significant. Thus, the significance of the three variables in Estimations (6) and (7) seems to be produced by correlation among them. Given that and the fact that the *SIZE* variable has the wrong size in Estimations (6) and (7), I decide to keep the debt-to-GDP ratio only as in Estimation (8) to identify the relationship between this variable and the credit rating for the Atlantic provinces.

The estimated values for β_1 indicate that an increase in the debt-to-GDP ratio would lower the conditional mean of the credit rating non-linearly (Chart 11, Curves 2, 3, and 4), but to a different extent in each subgroup, owing to different values for λ (3.2 for the four largest provinces, 2.2 for the Prairie provinces, and 0 for the Atlantic provinces). However, it is difficult to make meaningful comparisons of λ and β_1 across groups, since the conditional mean is scaled by the standard deviation of the error term. Therefore, to obtain a measure of the changes in credit ratings that could be compared across subgroups, the impact on the conditional mean must be translated into an impact on probabilities (as was done for the pooled sample).

Charts 13b to 15b show the results for the three subgroups (Chart 12b shows the results for the nine provinces together). For the four largest provinces and the Prairie provinces, the rise in the probability of getting a lower credit rating when debt-to-GDP ratio increases is faster at a higher level of debt-to-GDP ratio. However, for the Atlantic provinces, the results suggest that the rise in the probability of getting a lower credit rating decreases as the debt-to-GDP ratio increases.

^{18.} Using Estimations (3), (5) and (8) in Table 4.

To summarize, a negative linear relationship between the debt-to-GDP ratio and the credit rating is found for the pooled sample of nine provinces at all levels of *DDGDP*. A non-linear relationship is identified for the four largest provinces' group, the Prairie provinces' group, and the Atlantic provinces' group separately, with successively smaller increases in *DDGDP* triggering downgrades in the first two groups of provinces, the opposite tendency being suggested for the third group. The different results among the three groups of provinces reflect their different debt situations as well as the market expectations. It may be that higher ratings are more fragile than the lower ratings. When the debt level is low or the credit rating is high, the ratings change downward faster, but when the debt level is high or the rating is low, the downgrades happen more slowly. Therefore, the better the economic conditions of the province, the higher the nonlinearity in *DDGDP*. This is shown by the higher λ value and the steeper slopes exhibited for the four largest provinces. With different λ values and different mean values for the other explanatory variables, the estimated credit ratings with the highest probabilities are different for the same level of DDGDP across different groups of provinces (Charts 13a and 13b, 14a and 14b, and 15a and 15b).

4.3 Fitted ratings with the highest probabilities

In Table 5, the actual and the fitted ratings are compared with the highest probabilities for the four largest provinces, the Prairie provinces, and the Atlantic provinces, based on the estimated results by their group samples. ¹⁹ The discrepancies between the actual and fitted ratings can be classified into three categories: leads in predicting rating changes, lags in predicting rating changes, and persistent deviations.

The problem of the model's leads and lags in predicting rating changes might be related to missing variables. The actual ratings would be downgraded more quickly if a variable that is considered by the rating agent but which is not in our model signals an obvious deterioration of the province's credit quality. And in like fashion, the actual ratings would be upgraded more quickly if a variable that is considered by the rating agent but which is not in our model signals an obvious improvement of the province's credit.

Among the nine provinces, the estimated model produces the most accurate fitted ratings for Quebec, which had experienced relatively fewer rating changes over the history. There are only three variations between the actual and fitted ratings with the

^{19.} Results from Estimation (3) in Table 4 are used for the four largest provinces; results from Estimation (5) are used for the Prairie provinces; however, results from Estimation (1) with the nine provinces are used for the Atlantic provinces, since they include other explanatory variables with which assumptions have been made in the projection of two other groups.

highest probabilities (in 1972, 1982, and 1990, when there is only one grade difference); of these variations, one comes from the model's lagging in predicting a downgrade (in 1982) and two from one-period deviations produced by the model's mechanical responses to transitory changes of the values of explanatory variables (in 1972 and 1990). The model's lagging in predicting downgrades happened to other provinces as well, such as for Alberta from 1986 to 1988, for Saskatchewan from 1991 to 1992, for Manitoba from 1983 to 1988, and for Ontario from 1991 to 1995. At some other times, the model also lags in predicting upgrades, for example, for Nova Scotia from 1976 to 1978.

On the other hand, the model sometimes anticipates rating changes by Standard & Poor's. For instance, the model leads in predicting downgrades for British Columbia from 1983 to 1984, and for Saskatchewan from 1983 to 1985 and from 1987 to 1989. It also leads in giving upgrades for Alberta from 1975 to 1978, for Saskatchewan in 1975, and for New Brunswick from 1974 to 1975.

However, the model sometimes does present some persistent deviations from actual ratings, like most of the deviations for Ontario and the Atlantic provinces. The model has consistently predicted lower ratings than what Standard & Poor's actually gave for Ontario from 1977 to 1986 and for New Brunswick from 1991 to 1995. In 1991, New Brunswick was upgraded from A+ to AA- by Standard & Poor's, while our model rather predicts a downgrade to A one year later. The fact is that New Brunswick had achieved a sustained deficit reduction during the 1983-1984 to 1990-91 period but was not upgraded until July 1991. However, in 1991-92, New Brunswick's deficit increased sharply (to a level close to the historical peak in 1982-83). Perhaps Standard & Poor's would not have upgraded New Brunswick if it had waited longer. In addition, possible missing variables might have caused the deviations as well (i.e., a variable which represents the political stability of a province). However, as mentioned above, the model can capture only the variables whose influence on rating changes is relatively constant over time. If the weight of a variable used by a rating agency in determining rating changes differs from time to time, most likely it won't be found to be significant, and it will be excluded from our specification. Moreover, the persistent deviations might also be caused by the model's mechanical responses to the changes of the values of explanatory variables. Standard & Poor's might make ratings decisions not only on actual data, but also on the basis of expectations of the consequences of current policy in future economic variables, like debt-to-GDP ratio. A deterioration perceived to be permanent will lead to a downgrade. A deterioration perceived to be transitory (perhaps because the Minister

^{20.} The predicted probabilities of the actual ratings are very close to those of the fitted ratings in 1972 and 1990, where the errors take place.

of Finance is taking steps to correct it) will be more likely to be overlooked by Standard & Poor's. The model cannot make such distinctions.

5. **Projections of provincial credit ratings**

In this section, I present projections of the provincial credit ratings from 1996 to 2000 under different assumptions for growths in government revenue, expenditure, and provincial nominal GDP, and therefore, for the debt-to-GDP ratio, based on various assumptions for the other relevant explanatory variables. In all the projection scenarios, the following assumptions are maintained: (1) The employment ratio is set at the 1995²¹ level for the rest of the 1990s; (2) provincial personal income grows at the same rate as the provincial nominal GDP; (3) the shares of the provincial nominal GDP are unchanged from 1995 to 2000 at their 1994 levels; (4) the growth rate of federal transfers to each province is calculated according to the 1995 Federal Budget assumptions up to 1997-98; for 1998-99 and 1999-00, equalization payments grow at the same rate as provincial nominal GDP assumed in the relevant scenario, while other federal transfers remain flat; (5) the growth in UI benefit payments is projected as in the 1995 Federal Budget for 1995-96 and 1996-97 and is set to zero afterwards; (6) the projection for gross direct debt is obtained by accumulating the projected annual deficits (or surpluses), under the assumption that the stock of financial assets remains unchanged from 1994-95 on.

The projected ratings shown here are the ones with the highest probabilities.²² and they are produced in the same way as was done for the fitted ratings (see footnote 20). The projection is therefore constrained by the ratings history of the group of provinces included in the sample. For example, the projections of credit ratings for Quebec for the remainder of the 1990s are bounded within the AAA to A+ range because the regression results are based on the actual ratings assigned to the four largest provinces, which range from AAA to A+. This constraint will prove to be a problem only if very high probabilities are observed for the two extreme rating categories of a group of provinces.²³

Table 6 reports projection results under the assumption that expenditure, revenue, and nominal GDP increase at 5 per cent per year in each province over the

^{21.} As averaged over the first three quarters of 1995, the employment ratio ranges from 42.6 per cent

in Newfoundland to 65.6 per cent in Alberta.

22. In some cases, other ratings have only slightly lower probabilities but are not reported here.

23. AAA and A+ for the four largest provinces; AA+ and BBB+ for the Prairie provinces; AAA and BBB+ for the nine provinces together.

period 1995 to 2000.²⁴ As a result, the debt-to-GDP ratios are projected to decrease slowly for the rest of the decade, except for Ontario and Quebec, whose debt-to-GDP ratios would continue to increase. Under this scenario, British Columbia, Ontario, Quebec, and Nova Scotia would keep their predicted ratings for 1995 until the end of the decade. These ratings are lower for British Columbia and higher for Ontario and Nova Scotia than the actual ratings.²⁵ Quebec could be downgraded to a rating lower than any in its history (as seen from the table, the calculated debt-to-GDP ratio under the above assumption is increasing), but this is not shown in Table 6 because of the aforementioned constraint. The table shows that there is an extremely high probability that Quebec will keep its lowest rating of A+. Alberta would be upgraded to AAA in 1996 from AA. Saskatchewan would be upgraded to A+ in 1998 from its predicted and higherthan-actual rating of A in 1995. Moreover, Manitoba would be upgraded to A+ in 1996 from its predicted but lower-than-actual rating of A in 1995. New Brunswick would be upgraded to A+ in 1998 from its predicted and lower-than-actual rating A in 1995. Newfoundland would be upgraded to A- in 1996 and further to A in 1998, following a downgrade to BBB+ in 1994.

Tables 7 and 8 assume the same 5 per cent growth in nominal GDP and revenue, but lower growth of 2 per cent and 0 per cent, respectively, in expenditure. Under these two more stringent expenditure growth paths, the debt-to-GDP ratios decrease for all provinces and more extensively under the 0 per cent growth path. 26 The shaded areas in Tables 7 to 11 indicate changes in ratings from the base case scenario presented in Table 6 (5 per cent growth in revenue, expenditure, and nominal GDP). Under the 2 per cent expenditure growth scenario, Nova Scotia, British Columbia, and Manitoba and Quebec would receive upgrades in 1998, 1999, and 2000 respectively, and Saskatchewan and Newfoundland would have their upgrades one year earlier than under 5 per cent expenditure growth assumptions. Under the 0 per cent expenditure growth scenario, all provinces except for Alberta (which has been predicted to have the highest rating, AAA, in 1996 in the base scenario) would receive further upgrades and/or earlier upgrades by the end of the decade. Therefore, more spending restraint would result in more and faster upgrades, for given revenue and GDP growth.

24. The 5 per cent growth assumption is consistent with potential output growth of 3 per cent and

26. Under the 2 per cent expenditure growth scenario, Ontario's debt-to-GDP ratio would increase temporarily.

² per cent inflation.
25. The actual ratings for 1995 are: British Columbia (AA+), Saskatchewan (BBB+), Manitoba (A+), Ontario (AA-), New Brunswick (AA-), and Nova Scotia (A-). However, our model predicts lower ratings for British Columbia (AA-), Manitoba (A), and New Brunswick (A) and higher ratings for Saskatchewan (A), Ontario (AA), and Nova Scotia (A).
26. Yellow and CAP and Nova Scotia (A).

To test the sensitivity of the results to the assumption about revenue and nominal GDP growth, it is assumed that the growth rates of both nominal GDP and government revenue are 2.5 per cent, half of what they were in the base scenario presented in Table 6, while government expenditure growth is maintained at 5 per cent. As shown in Table 9, under these assumptions, the debt-to-GDP ratios for all provinces except for Alberta would increase, rather than decrease. In light of the 5 per cent growth assumptions for revenue, expenditure, and nominal GDP, there would not be any upgrades for Saskatchewan, Manitoba, and New Brunswick; an upgrade would take place one year later for Alberta; downgrades would happen for British Columbia and Ontario. And although Newfoundland would be upgraded to A- in 1997 (a year later than in the base scenario), it would be downgraded back to BBB+ in 1999.

In order to assess how sensitive the results are to the assumption about the employment ratio, the employment ratio for each individual province is increased by 1.5 percentage points. Note that the 1.5 percentage point difference represents the average gap between the 1989 (prerecession) and 1995 levels of the employment ratio in the nine provinces. The 5 per cent revenue, expenditure, and nominal GDP growths assumed in Table 6 are kept. The results with the higher employment ratio are reported in Table 10. In comparing the results in these two tables, I find that the projected upgrades of credit rating would take place one year earlier for Newfoundland and two years earlier for Saskatchewan. Nova Scotia and Manitoba would receive new upgrades in 1998 and 2000 respectively. Although there are no rating changes for the other five provinces under this scenario, the probability associated with the next better rating generally increases.

In order to see if the assumption about the ratio of federal transfers to total provincial revenues affects the results, I decrease this ratio by 5 percentage points for each individual province and keep the 5 per cent revenue, expenditure, and nominal GDP growths assumed in Table 6. The results are reported in Table 11. Since dependence on transfers is a sign of weakness that has an unfavourable impact on credit ratings, a decrease in the proportion of revenues coming from federal transfers, provided with the total revenues remaining the same, would generally improve the projected ratings and/or the probability associated the better rating next to the projected rating. Under this scenario, Saskatchewan and Newfoundland would be upgraded one year earlier in 1997; Nova Scotia and British Columbia would receive upgrades in 1998 and 1999 respectively. It should, of course, be recognized that the relationship between the ratio of federal transfers to provincial revenues and the credit rating is based on the fact

^{27.} Five per cent GDP growth is assumed from 1994 to 1995, as the provincial GDP for 1995 was not available on CANSIM when this paper was prepared.

that the ratio is in fact an indicator of the economic and fiscal health of the province. The lower the ratio the more the province is able to rely on its own resources. A change to the structure of federal-provincial transfers that raised or lowered the ratio for all provinces would not normally have the same impact on their credit ratings.

This last point, as well as the limitation of the range of possible outcomes to the range of observed ratings, suggests that projection results produced by the ordered probit model should be used with caution, especially when the initial conditions are at the limit of the allowable range of categories for the dependent variable. However, the ordered probit model remains more powerful than other models for investigating the historical relationship between a discrete qualitative dependent variable and continuous explanatory variables.

6. Conclusions

The ordered probit framework makes it possible to capture and summarize the historical relationship between discrete-valued credit ratings and continuous-valued regressors – this without taking the risk of assuming the magnitude of the discreteness, which is not known and which one would have to assume to be known in a linear regression. However, the model has some shortcomings for projection purposes.

Using the 1969-70 to 1994-95 provincial data from the Financial Management System, I find that the debt-to-GDP ratio, the employment ratio, provincial GDP as a share of total Canadian GDP, federal transfers as a proportion of total provincial revenues, and the proportion of provincial personal income represented by UI benefits do affect the conditional distribution for provincial credit ratings, with the first variable affecting the credit rating non-linearly at different degrees for the nine provinces. The results do not support Boothe's (1993b, 5) statement that "successively smaller increases in debt load trigger downgrades." Using an ordered probit model, I find that downgrades take place at almost the same speed at different levels of debt-to-GDP ratio, based on a pooled sample of nine provinces. Based on smaller pools, I find that for New Brunswick, Newfoundland, and Nova Scotia, downgrades seem to take place at lower speeds at higher levels of debt-to-GDP ratio; for Alberta, British Columbia, Quebec, and Ontario as well as for Manitoba and Saskatchewan, the evidence supports Boothe's finding that downgrades take place at higher speeds at higher levels of debt-to-GDP ratio. However, this finding seems to be largely attributable to the vulnerability of high credit ratings to increases in the debt-to-GDP ratio. While Boothe (1993b) found that only one variable (per capita debt) was significant in relation to bond rating, I found that a number of other variables were significant (such as the employment ratio, the relative size of the

provincial economy, the government's dependence on federal transfers, and the relative importance of UI benefits). As well, different experiences in rating changes for the same explanatory variables are exploited for different provinces.

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Table 1: What Standard & Poor's debt ratings mean^a

AAA	Capacity to pay interest and repay principal extremely strong.
AA	Very strong capacity to pay interest and repay principal; differs from the AAA-rated issues only in small degree.
A	Strong capacity to pay interest and repay principal, but somewhat more susceptible to adverse changes in economic and financial conditions than debt in higher-rated categories.
BBB	Adequate capacity to pay interest and repay principal, but adverse economic and financial conditions more likely to weaken capacity to pay interest and repay principal than in higher-rated categories. Lowest investment-grade rating.
BB	Any debt rated BB or below (B, CCC, CC and C) is regarded as having predominantly speculative characteristics with respect to capacity to pay interest and repay principal. Debt rated BB has less near-term vulnerability to default than other speculative issues, but it faces major uncertainties or exposure to adverse financial and economic conditions that could lead to inadequate capacity to meet timely interest and principal payments.
В	Greater vulnerability to default but still has the capacity to meet interest payments and principal repayments. Adverse financial and economic conditions would likely impair capacity or willingness to pay interest and repay principal.
CCC	Debt rated CCC has a currently identifiable vulnerability to default and is dependent upon favourable economic and financial conditions to meet timely payments of interest and principal. In the event of adverse conditions, it is not likely to have the capacity to pay interest and repay principal. ^b

a. Source: Standard & Poor's Canadian Focus, November 1994.

b. Standard & Poor's also uses a CC rating for debt subordinated to senior debt that is assigned a CCC rating, a C rating for debt subordinated to senior debt that is assigned a CCC- rating, and a D rating for debt that is in payment default.

Table 2: History of Standard & Poor's ratings for Canadian provincial governments

Province	Rating	Effective:	Province	Rating	Effective:
British Columbia	AA+ AA AA+ AAA AA	June 1989 August 1985 August 1983 June 1980 December 1966	Quebec	A+ AA- AA	June 1993 July 1982 December 1966
Alberta	AA AA+ AAA AA	July 1990 October 1986 February 1979 April 1967	New Brunswick	AA- A+ A	July 1991 October 1976 November 1966
Saskatchewan	BBB+ A- A AA- AA- AA+ AA	June 1992 November 1991 August 1989 March 1987 January 1986 June 1981 November 1976 July 1966	Nova Scotia	A- A A+ A	August 1985 July 1982 February 1976 October 1967
Manitoba	A+ AA- AA	July 1986 May 1983 March 1969	Newfoundland	BBB+ A- A BBB	May 1994 August 1985 October 1967 July 1966
Ontario	AA- AA AA+ AAA AA+ AAA AA	November 1993 May 1992 June 1991 July 1988 November 1985 December 1977 July 1966	Prince Edward Island		Not Rated

Table 3: Selected indicators used by Standard & Poor's in assessing credit quality of provincial governments^a

Demographic Indicators:	Population growth Population distribution by broad age category
Economic Indicators:	 Per capita income Gross domestic product growth Investment patterns Employment growth Labour force participation rate Unemployment rate
Financial Indicators:	 Operating budget balance as per cent of budgetary revenues Overall budget balance as per cent of budgetary revenues Budgetary revenue growth Intergovernmental transfers as per cent of budgetary revenues Operating expenditure growth Capital spending as per cent of total spending Expenditure as per cent of GDP Per cent of capital spending financed by non-debt sources
Debt Indicators:	 Direct debt as per cent of GDP Direct debt as per cent of budgetary revenues Total local government sector debt as per cent of GDP Tax-supported debt as per cent of budgetary revenues Net debt service costs as per cent of budgetary revenues

a. *Sourc*e: Standard & Poor's *CreditReview*, 13 December 1993.
b. The operating budget balance is the difference between budgetary revenues and operating expenditures, where operating expenditures correspond to total budgetary expenditures less capital outlays.

Table 4: Ordered probit estimates

	Nine		argest	Pra			Atlantic	
	provinces	prov	inces	prov	inces]	provinces	3
				Estima	tions			
RATEit	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$DDGDP_{it}^{\hat{\lambda}}$	5.11	29.94	38.62	5.04	4.65	1.30	1.42	1.64
(z)	(8.38)	(5.50)	(5.75)	(2.61)	(2.47)	(1.80)	(2.03)	(3.17)
(P> z)	(0.00) (0.00)		(0.00)	(0.01)	(0.01)	(0.07)	(0.04)	(0.02)
λ	1.0	2.8	3.2	1.9	2.2	0.0	0.0	0.0
$ER_{i(t-1)}$	-10.22	-12.72	-10.78	-34.77	-35.79	-25.10	-25.29	
(z)	(-3.85)	(-2.77)	(-2.63)	(-3.34)	(-3.61)	(-3.14)	(-3.18)	
(P> z)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	
$SIZE_{i(t-1)}$	-1.76	-0.98		-409.35	-394.61	173.56	146.05	
(z)	(-2.27)	(-0.97)		(-4.34)	(-4.35)	(2.57)	(2.65)	
P> z	(0.02)	(0.33)		(0.00)	(0.00)	(0.01)	(0.01)	
TRREV _{it}	3.41	12.66	12.43	6.15	5.39	-5.90	-7.58	
(z)	(2.48)	(2.93)	(2.94)	(1.85)	(1.67)	(-1.16)	(-1.69)	
(P> z)	(0.01)	(0.00)	(0.00)	(0.07)	(0.10)	(0.25)	(0.09)	
$UIBPI_{i(t-1)}$	-16.96	-17.90		-50.11		7.14		
(z)	(-2.81)	(-0.89)		(-0.93)		(0.71)		
(P> z)	(0.01)	(0.37)		(0.35)		(0.48)		
Cut points:								
CUT1	-6.19	-6.60	-5.01	-36.27	-36.02	-13.69	-15.51	-2.72
(Std. error)	(1.86)	(3.39)	(2.99)	(7.41)	(7.34)	(5.41)	(4.76)	(0.44)
CUT2	-5.46	-5.85	-4.29	-33.96	-33.69	-12.49	-14.31	-1.63
(Std. error)	(1.84)	(3.37)	(2.98)	(7.34)	(7.27)	(5.39)	(4.74)	(0.41)
CUT3	-3.84	-3.72	-2.16	-33.24	-32.97	-11.27	-13.09	-0.50
(Std. error)	(1.84)	(3.37)	(2.98)	(7.29)	(7.23)	(5.37)	(4.72)	(0.37)
CUT4	-3.08	-1.33	0.34	-31.98	-31.75	-9.56	-11.40	1.03
(Std. error)	(1.85)	(3.40)	(3.04)	(7.14)	(7.09)	(5.32)	(4.64)	(0.43)
CUT5	-2.13			-30.77	-30.58			
(Std. error)	(1.84)			(7.04)	(6.99)			
CUT6	-1.07			-30.55	-30.37			
(Std. error)	(1.84)			(7.05)	(6.99)			
CUT7	-0.15							
(Std. error)	(1.85)							
No. of obs.	234	104 104 52		52	78	78	78	
chi2(2)	281.07	91.67	90.46	67.68	66.87	22.34	21.84	10.27
Prob > chi ²	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Pseudo R ²	0.31	0.33	0.32	0.38	0.38	0.11 0.10		0.05
Log Like.	-308.49	-94.14	-94.75	-54.69	-55.09	-93.33	-93.58	-99.37

Table 5: Actual ratings and fitted ratings with highest probabilities (The probabilities are reported under the fitted ratings.)

	B.C. Alberta		erta	Saskat	chewan	Man	itoba	Ont	ario	Qu	ebec	N	.В.	N	.S.	Newfor	ındland	
Year	Act.	Fitt.	Act.	Fitt.	Act.	Fitt.	Act.	Fitt.	Act.	Fitt.	Act.	Fitt.	Act.	Fitt.	Act.	Fitt.	Act.	Fitt.
1970	AA	AA	AA	AA	A	<i>A</i> +	AA	AA	AA	AA	AA	AA	Α	A	A	A	A	BBB+
1071		.561	١	.538	١.	.441		.720		.507		.698		.387		.403	١.	.499
1971	AA	AA .686	AA	AA .603	A	<i>BBB+</i> .452	AA	AA .754	AA	AA .615	AA	AA .503	A	A .400	A	A .402	A	<i>BBB+</i> .600
1972	AA	AA	AA	AA	Α	A	AA	AA	AA	AA	AA	AA -	A	A	Α	A	A	BBB+
		.690		.680		.437		.703		.673		.509		.404		.388		.7 0 5
1973	AA	AA .637	AA	AA .591	A	<i>BBB+</i> .579	AA	AA .665	AA	AA .645	AA	AA .612	A	A .339	A	A .394	A	<i>BBB+</i> .690
1974	AA	AA	AA	AA	A	A	AA	AA	AA	AA	AA	AA	Α	A+	A	A	A	BBB+
		.519		.536		.422		.742		.493		.696		.355		.367		.512
1975	AA	AA .583	AA	AAA .370	A	AA .693	AA	AA .726	AA	AA .488	AA	AA .676	Α	A+ .362	A	A .341	A	<i>BBB+</i> .538
1976	AA	AA	AA	AAA	AA	AA	AA	AA	AA	AA	AA	AA	A+	A+	A+	\boldsymbol{A}	A	BBB +
		.647		.549		.750		.735		.675		.626		.361		.354		.448
1977	AA	AA .648	AA	AAA .601	AA	AA .694	AA	AA .752	AAA	AA .703	AA	AA .643	A+	A+ .362	A+	A .346	A	<i>BBB+</i> .471
1978	AA	AA	AA	AAA	AA	AA	AA	AA	AAA	AA	AA	AA	A+	A+	A+	A	A	<i>BBB+</i>
1070		.579		.744	١	.755		.426		.579		.562		.340	١.	.344	١.	.413
1979	AA	AA .502	AAA	AAA .801	AA	AA .750	AA	AA .547	AAA	AA .546	AA	AA .594	A+	A+ .353	A+	A+ .353	A	<i>BBB+</i> .440
1980	AAA	AA	AAA	AAA	AA	AA	AA	AA	AAA	AA	AA	AA	A+	A+	A+	A+	A	A -
1001		.447		.881		.701		.568		.479		.587		.361		.344		.353
1981	AAA	AA .396	AAA	AAA .910	AA+	AA+ .638	AA	AA .532	AAA	AA .497	AA	AA .633	A+	A+ .356	A+	A .350	A	A .347
1982	AAA	AAA	AAA	AAA	AA+	AA+	AA	AA	AAA	AA	AA-	AA	A+	A+	Α	Α	Α	Α
		.447		.884		.802		.737		.415		.613		.358		.361		.352
1983	AA+	AA .448	AAA	AAA .820	AA+	AA .565	AA-	AA .538	AAA	AA .459	AA-	AA- .629	A+	A .354	A	A .352	A	A .361
1984	AA+	AA	AAA	AAA	AA+	AA	AA-	AA	AAA	AA	AA-	AA-	A+	A+	A	.332 A +	A	.501 A
		.674		.723		.626		.35 8		.571		.756		.339		.345		.357
1985	AA	AA .686	AAA	AAA .647	AA+	AA .754	AA-	AA .613	AA+	AA .511	AA-	AA- .726	A+	A .343	A-	A+ .352	A-	A- .348
1986	AA	AA	AA+	AAA	AA	AA	A+	AA	AA+	AA	AA-	AA-	A+	A+	A-	A+	A-	A-
		.673		. 648		.717		.572		.412		.746		.343		.340		.355
1987	AA	AA .680	AA+	AAA .355	AA-	AA .337	A+	AA	AA+	AAA .417	AA-	AA- .682	A+	A+ .359	A-	A+ .364	A-	A- .347
1988	AA	AA	AA+	AAA	AA-	.337 A+	A+	.611 AA	AAA	AAA	AA-	.062 AA-	A+	.339 A+	A-	.304 A+	A-	.347 A
1000		.707	1227	.371		.440		.389		.511		.592		.358		.360	''	.356
1989	AA+	AA	AA+	AA	AA-	A	A+	A+	AAA	AAA	AA-	AA-	A+	A+	A-	A+	A-	A
1990	AA+	.633 AA	AA	.445 AA	A	. 410 A	A+	.396 A+	AAA	.649 AAA	AA-	.449 AA	A+	.361 A+	A-	.361 A+	A-	.403 A
1990	AA+	.488	AA	.449	A	.411	A+	.400	AAA	.694	AA-	.538	A+	.362	A-	.362	A-	.403
1991	AA+	AA	AA	AA	A-	A	A+	A+	AA+	AAA	AA-	AA-	AA-	A+	A-	A+	A-	A
1992	AA+	.509 AA	AA	.484 AA	BBB+	.421 A	A+	.333 A	AA	.622 AAA	AA-	.546 AA-	AA-	.355 A	A-	.362 A+	A-	.399 A
1332	AA+	.676	AA	.561	+000	.426	A+	.410	AA	.357	AA-	.770	AA-	.342	A-	.355	A-	.403
1993	AA+	AA	AA	AA	BBB+	BBB+	A+	A	AA-	AA	A+	A+	AA-	A	A-	A	A-	<i>A</i>
1004	ΛΛ.	.638		.708	DDD.	.404	Λ.	.440		.645	۸.	.575	ΛΛ	.403	Α	.360	DDD.	.366
1994	AA+	AA .517	AA	AA .694	BBB+	BBB+ .413	A+	A .441	AA-	AA .679	A+	A+ .941	AA-	A .402	A-	A .392	BBB+	A- .357
1995	AA+	AA	AA	AA	BBB+	\boldsymbol{A}	A +	\boldsymbol{A}	AA-	AA	A+	A+	AA-	\boldsymbol{A}	A-	\boldsymbol{A}	BBB+	BBB+
		.680		.544		.419		.440		.703		.978		.402		.398		.380

Table 6: Projected ratings with highest probabilities 5% annual growth in expenditure, revenues, and nominal GDP

Year	B.C.		B.C. Alberta		lberta Saskat		Man	nitoba On		ario	Quebec		N.B.		N.S.		Newfoundland	
	Debt-to- GDP	Rating (Prob.)																
1996	.406	AA (.708)	.299	AAA (.375)	.648	A (.440)	.724	A+ (.388)	.363	AA (.710)	.567	A+ (.987)	.658	A (.401)	.592	A (.398)	.713	A- (.357)
1997	.396	AA (.686)	.266	AAA (.635)	.614	A (.400)	.701	A+ (.448)	.380	AA (.694)	.577	A+ (.987)	.626	A (.374)	.589	A (.383)	.689	A- (.346)
1998	.387	AA (.607)	.235	AAA (.796)	.582	A+ (.426)	.680	A+ (.457)	.397	AA (.691)	.588	A+ (.991)	.595	A+ (.344)	.585	A (.363)	.665	A (.361)
1999	.378	AA (.554)	.205	AAA (.840)	.551	A+ (.448)	.660	A+ (.448)	.414	AA (.712)	.597	A+ (.996)	.566	A+ (.356)	.582	A (.363)	.643	A (.374)
2000	.370	AA (.503)	.176	AAA (.873)	.522	A+ (.457)	.661	A+ (.431)	.429	AA (.706)	.607	A+ (.999)	.538	A+ (.362)	.579	A (.360)	.622	A (.385)

Table 7: Projected ratings with highest probabilities 2% annual growth in expenditure and 5% in revenues and nominal GDP

Year	B.C.		B.C.		Alberta		Saskatchewan		Manitoba		Ontario		Quebec		N.B.		N.S.		Newfoundland	
	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	ı	Rating (Prob.)	ı	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)		Rating (Prob.)		Rating (Prob.)	Debt-to- GDP	Rating (Prob.)		
1996	.399	AA (.713)	.294	AAA (.392)	.640	A (.438)	.716	A+ (.399)	.357	AA (.704)	.559	A+ (.975)	.658	A (.398)	.592	A (.395)	.702	A- (.356)		
1997	.377	AA (.624)	.251	AAA (.670)	.591	A+ (.390)	.678	A+ (.457)	.364	AA (.653)	.554	A+ (.921)	.626	A (.354)	.589	A (.366)	.657	A (.349)		
1998	.350	AA (.436)	.204	AAA (.832)	.535	A+ (.453)	.635	A+ (.433)	.365	AA (.580)	.552	A+ (.774)	.595	A+ (.361)	.585	A+ (.348)	.603	A (.398)		
1999	.318	AAA (.421)	.155	AAA (.873)	.476	A+ (.455)	.587	A+ (.380)	.360	AA (.546)	.523	A+ (.551)	.566	A+ (.355)	.582	A+ (.360)	.543	A (.401)		
2000	.282	AAA (.561)	.104	AAA (.896)	.411	A+ (.428)	.534	AA (.346)	.351	AA (.489)	.478	AA- (.701)	.538	A+ (.319)	.579	A+ (.362)	.475	A (.370)		

Table 8: Projected ratings with highest probabilities 0% annual growth in expenditure and 5% in revenues and nominal GDP

Year	B.C.		Alberta		Saskatchewan		Manitoba		Ontario		Quebec		N.B.		N.S.		Newfoundland	
	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	1	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)		Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)
1996	.395	AA (.713)	.291	AAA (.401)	.635	A (.437)	.710	A+ (.407)	.353	AA (.699)	.553	A+ (.960)	.644	A (.396)	.579	A (.392)	.695	A- (.355)
1997	.365	AA (.576)	.240	AAA (.692)	.575	A+ (.407)	.663	A+ (.458)	.353	AA (.618)	.539	A+ (.821)	.584	A+ (.339)	.549	A (.353)	.636	A (.367)
1998	.326	AAA (.375)	.185	AAA (.848)	.505	A+ (.458)	.606	A+ (.406)	.343	AA (.486)	.512	AA- (.552)	.514	A+ (.361)	.509	A+ (.360)	.564	A (.404)
1999	.280	AAA (.552)	.124	AAA (.883)	.428	A+ (.439)	.540	AA (.326)	.327	AA (.408)	.476	AA- (.786)	.435	A+ (.327)	.459	A+ (.360)	.479	A (.368)
2000	.227	AAA (.688)	.059	AAA (.900)	.342	A+ (.388)	.467	AA (.457)	.302	AAA (.412)	.430	AA- (.617)	.347	AA (.344)	.500	A+ (.335)	.383	A+ (.361)

Table 9: Projected ratings with highest probabilities 5% annual growth in expenditure and 2.5% in revenues and nominal GDP

Year	B.C.		Alberta		Saskatchewan		Manitoba		Ontario		Quebec		N.B.		N.S.		Newfoundland	
	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)		Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	ı	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)		Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)
1996	.421	AA (.670)	.312	AA (.392)	.671	A (.438)	.748	A (.414)	.375	AA (.711)	.587	A+ (.999)	.682	A (.404)	.613	A (.403)	.740	BBB+ (.364)
1997	.432	AA (.683)	.295	AAA (.526)	.666	A (.438)	.755	A (.388)	.411	AA (.695)	.624	A+ (1.000)	.679	A (.401)	.637	A (.402)	.750	A- (.357)
1998	.448	AA (.662)	.282	AAA (.675)	.668	A (.431)	.770	A (.380)	.452	AA (.573)	.667	A+ (1.000)	.685	A (.397)	.667	A (.403)	.769	A- (.357)
1999	.470	AA (.530)	.276	AAA (.700)	.677	A (.434)	.791	A (.406)	.496	AA- (.737)	.717	A+ (1.000)	.698	A (.401)	.704	A (.399)	.798	BBB+ (.394)
2000	.497	AA- (.678)	.274	AAA (.713)	.694	A (.439)	.819	A (.431)	.545	AA- (.612)	.773	A+ (1.000)	.719	A (.404)	.748	A (.376)	.837	BBB+ (.473)

Table 10: Projected ratings with highest probabilities 5% annual growth in expenditure and 5% in revenues and nominal GDP 1.5 percentage point increase for employment ratio

Year	B.C.		Alberta		Saskatchewan		Manitoba		Ontario		Quebec		N.B.		N.S.		Newfoundland	
	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)		Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)		Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)
1996	.406	AA (.713)	.299	AAA (.438)	.648	A+ (.391)	.724	A+ (.457)	.363	AA (.695)	.567	A+ (.981)	.658	A (.389)	.592	A (.384)	.713	A- (.352)
1997	.396	AA (.656)	.266	AAA (.694)	.614	A+ (.451)	.701	A+ (.439)	.380	AA (.667)	.577	A+ (.980)	.626	A (.349)	.589	A (.361)	.689	A (.347)
1998	.387	AA (.557)	.235	AAA (.838)	.582	A+ (.455)	.680	A+ (.387)	.397	AA (.662)	.588	A+ (.986)	.595	A+ (.357)	.585	A+ (.340)	.665	A (.382)
1999	.378	AA (.499)	.205	AAA (.877)	.551	A+ (.439)	.660	A+ (.354)	.414	AA (.702)	.597	A+ (.994)	.566	A+ (.362)	.582	A+ (.341)	.643	A (.392)
2000	.370	AA (.445)	.176	AAA (.903)	.522	A+ (.415)	.661	AA (.331)	.429	AA (.713)	.607	A+ (.998)	.538	A+ (.361)	.579	A+ (.343)	.622	A (.399)

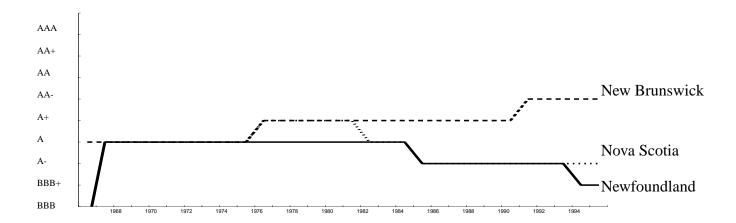
Table 11: Projected ratings with highest probabilities 5% annual growth in expenditure and 5% in revenues and nominal GDP 5 percentage point decrease for federal transfers as proportion of total provincial revenues

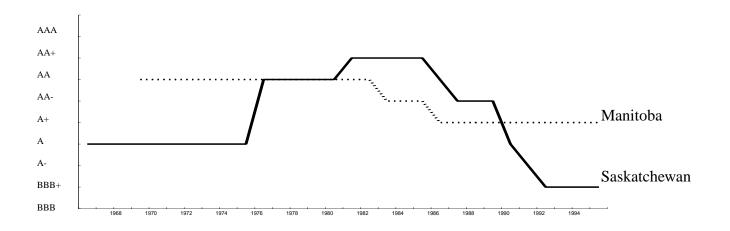
Year	B.C.		Alberta		Saskatchewan		Manitoba		Ontario		Quebec		N.B.		N.S.		Newfoundland	
	Debt-to- GDP	Rating (Prob.)		Rating (Prob.)	Debt-to- GDP	Rating (Prob.)	Debt-to- GDP	Rating (Prob.)										
1996	.406	AA (.659)	.299	AAA (.619)	.648	A (.420)	.724	A+ (.435)	.363	AA (.594)	.567	A+ (.946)	.658	A (.387)	.592	A (.382)	.713	A- (.351)
1997	.396	AA (.521)	.266	AAA (.833)	.614	A+ (.417)	.701	A+ (.458)	.380	AA (.539)	.577	A+ (.945)	.626	A (.346)	.589	A (.359)	.689	A (.350)
1998	.387	AA (.395)	.235	AAA (.926)	.582	A+ (.455)	.680	A+ (.434)	.397	AA (.531)	.588	A+ (.960)	.595	A+ (.358)	.585	A+ (.343)	.665	A (.384)
1999	.378	AAA (.380)	.205	AAA (.947)	.551	A+ (.458)	.660	A+ (.411)	.414	AA (.610)	.597	A+ (.981)	.566	A+ (.362)	.582	A+ (.343)	.643	A (.394)
2000	.370	AAA (.438)	.176	AAA (.961)	.522	A+ (.449)	.661	A+ (.382)	.429	AA (.666)	.607	A+ (.993)	.538	A+ (.361)	.579	A+ (.345)	.622	A (.400)

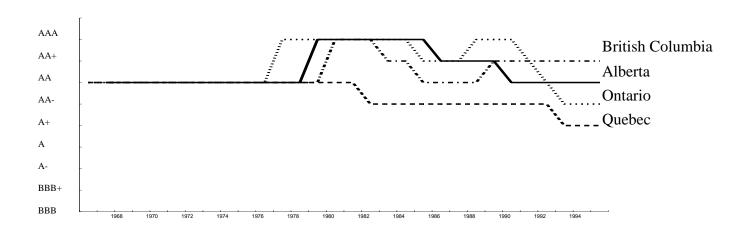
Table 12: Debt-to-GDP ratios needed for downgrades to BBB+ in 2000 $\,$

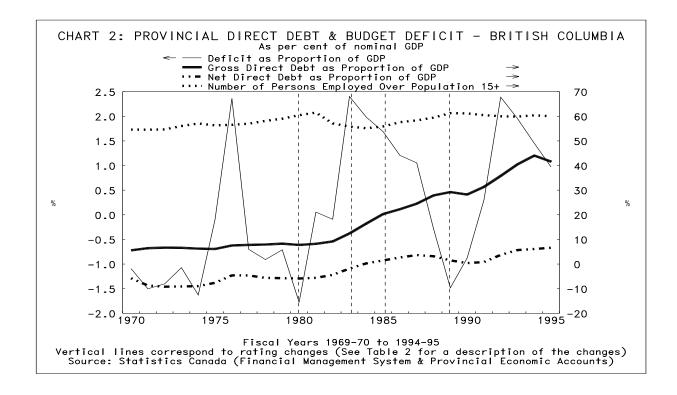
	Year	1996	1997	1998	1999	2000	
	Debt-to- GDP	.602	.685	.807	.974	1.188	
Quebec	Rating (Prob.)	A+ (.356)	A+ (.359)	A (.391)	A (.343)	BBB+ (.596)	
A31	Debt-to- GDP	.360	.465	.670	1.007	1.513	
Alberta	Rating (Prob.)	AA (.562)	AA (.577)	AA (.348)	A (.396)	BBB+ (.824)	

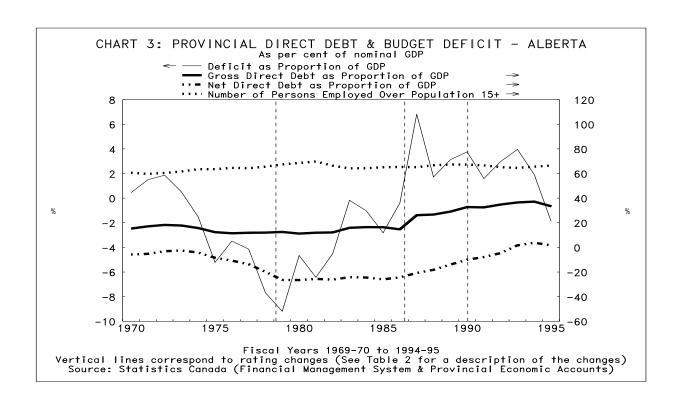
CHART 1: HISTORY OF PROVINCIAL CREDIT RATINGS

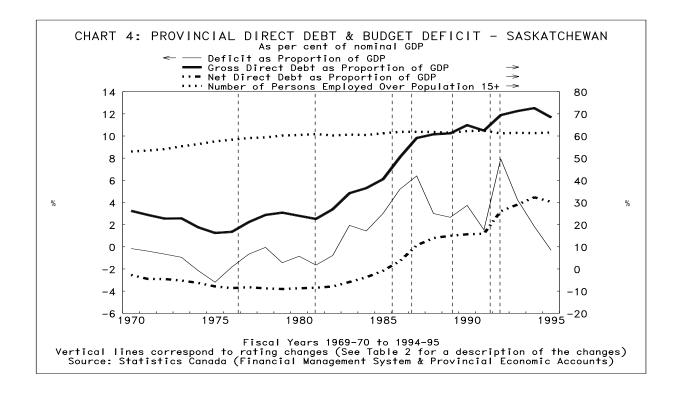


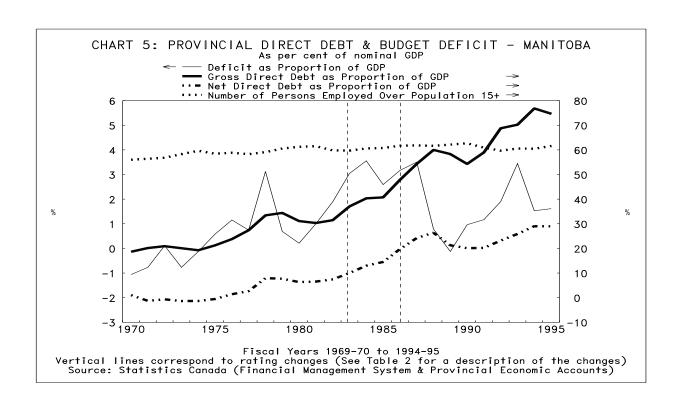


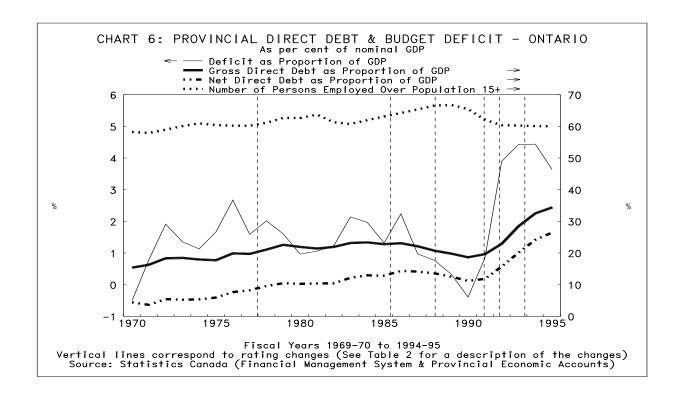


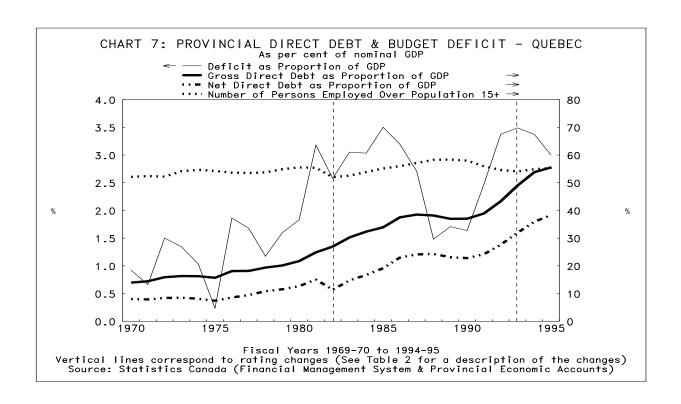


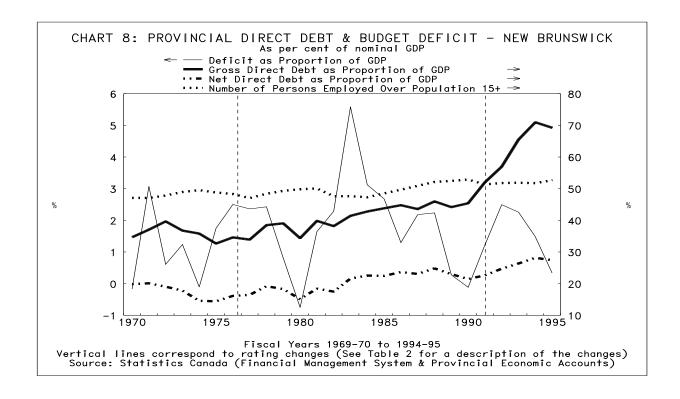


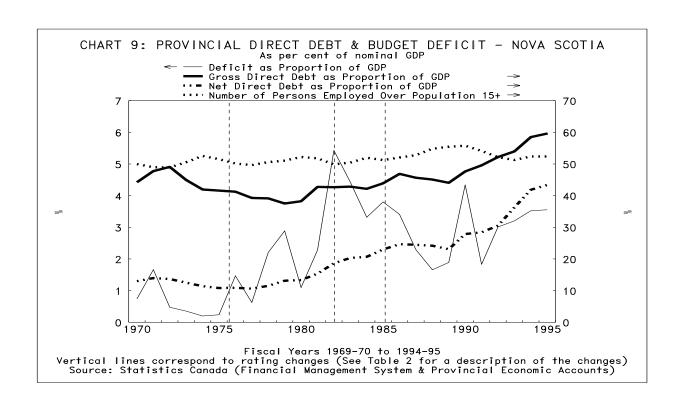


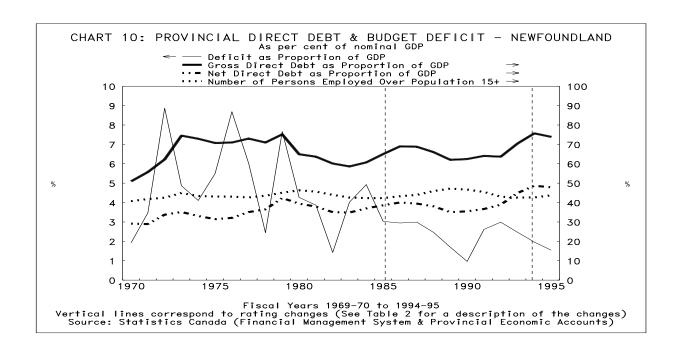












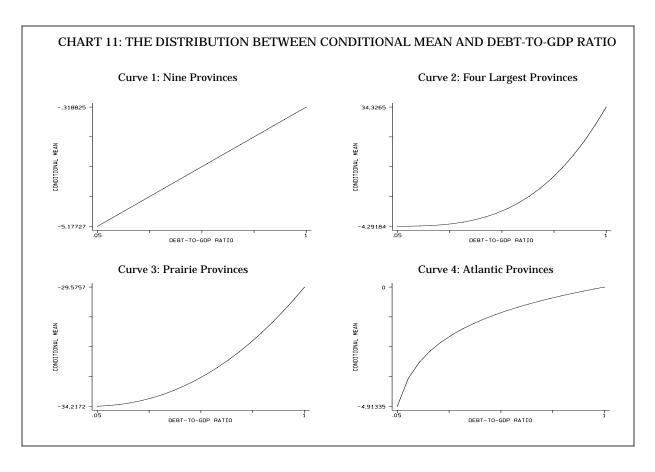
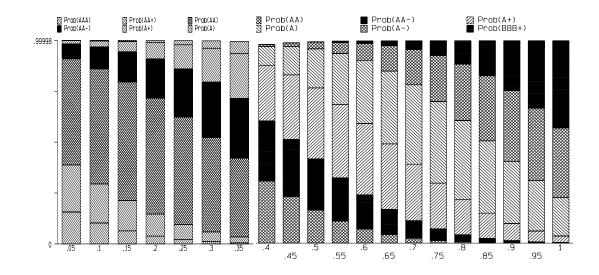


CHART 12: PROBABILITY DISTRIBUTIONS FOR CREDIT RATINGS NINE PROVINCES



a

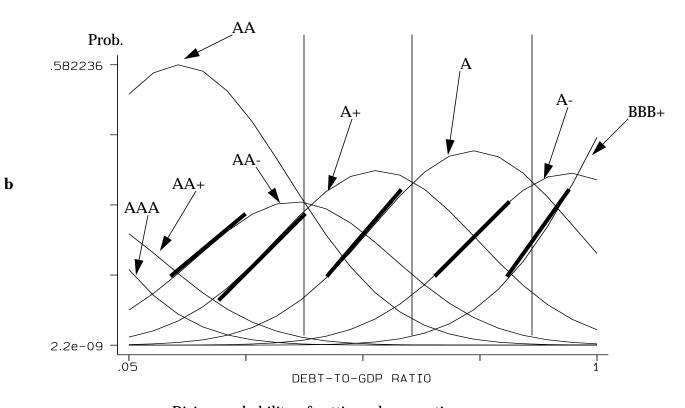
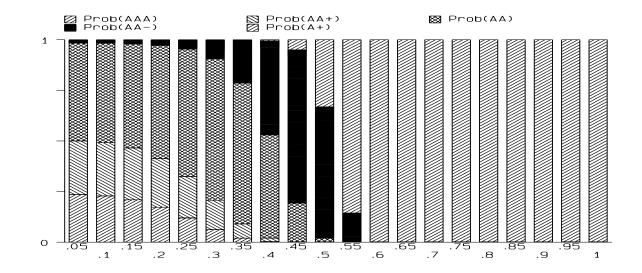
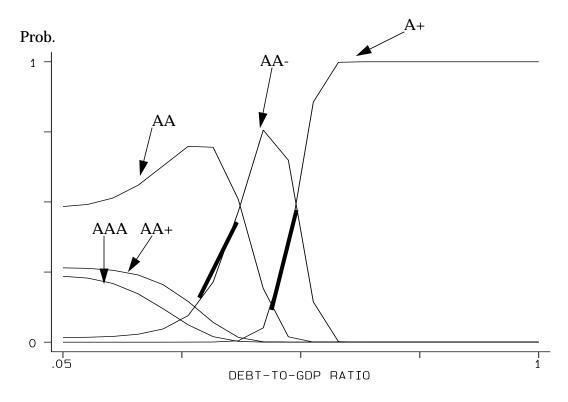


CHART 13: PROBABILITY DISTRIBUTIONS FOR CREDIT RATINGS FOUR LARGEST PROVINCES



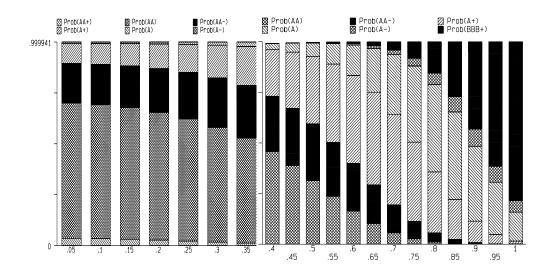


Rising probability of getting a lower rating

a

b

CHART 14: PROBABILITY DISTRIBUTIONS FOR CREDIT RATINGS PRAIRIE PROVINCES



a

b

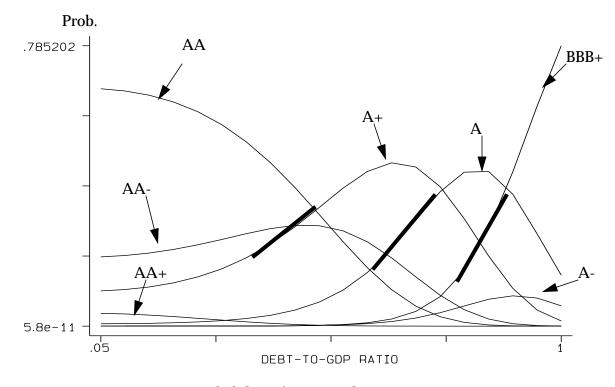
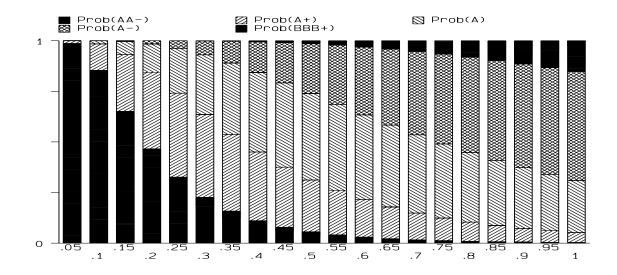
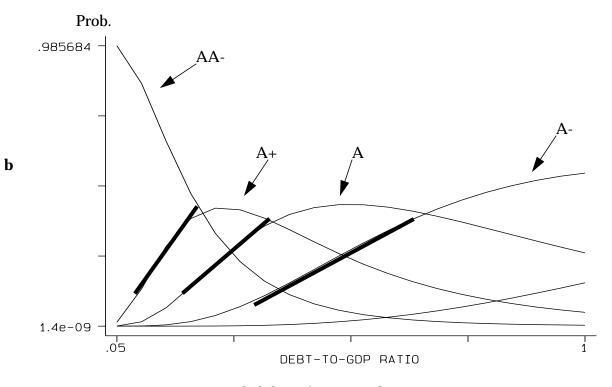


CHART 15: PROBABILITY DISTRIBUTIONS FOR CREDIT RATINGS ATLANTIC PROVINCES





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