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by

**Carlos de Resende,[1] Ali Dib,[1] and Maral Kichian[2]**

[1]International Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
cderesende@bankofcanada.ca
adib@bankofcanada.ca

[2]Canadian Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
mkichian@bankofcanada.ca



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# Abstract

Inflation-targeting central banks around the world often state their inflation objectives with regard to the consumer price index (CPI). Yet the literature on optimal monetary policy based on models with nominal rigidities and more than one sector suggests that CPI inflation is not always the best choice from a social welfare perspective. We revisit this issue in the context of an estimated multi-sector New-Keynesian small open economy model where sectors are heterogeneous along multiple dimensions. With key parameters of the model estimated using data from an inflation targeting economy, namely Canada, we particularly focus on (i) the role of sector-specific real rigidities, specially in the form of factor mobility costs, and (ii) welfare implications of targeting alternative price indices. Our estimations reveal considerable heterogeneity across sectors, and in several dimensions. Moreover, in contrast to existing studies, our welfare analysis comparing simple optimized policy rules based on alternative sectoral inflation rates provides support for CPI-based targeting policies by central banks. Capital mobility costs matter importantly in this regard.



*JEL classification: E4, E52, F3, F4*
*Bank classification: Inflation: costs and benefits; Inflation and prices; Inflation targets; Monetary policy framework; Monetary policy implementation*



# Résumé

Les banques centrales de par le monde qui poursuivent des cibles d'inflation fixent fréquemment leurs objectifs par référence à l'indice des prix à la consommation (IPC). Or, selon la littérature qui fait appel, pour l'analyse de la politique monétaire optimale, à des modèles multisectoriels englobant des rigidités nominales, l'IPC ne serait pas toujours le meilleur étalon sous l'aspect du bien-être social. Les auteurs examinent de nouveau la question en se servant d'un modèle de type nouveau keynésien représentant une petite économie ouverte composée de plusieurs secteurs hétérogènes à différents égards. Ils estiment les principaux paramètres du modèle au moyen de données tirées d'une économie à régime de cibles d'inflation, en l'occurrence le Canada. Ils étudient notamment le rôle des rigidités réelles sectorielles, tout particulièrement sous forme de coûts de mobilité des facteurs, et les conséquences pour le bien-être du choix d'indices de prix autres que l'IPC. Leurs estimations révèlent une hétérogénéité intersectorielle appréciable, et ce, à plusieurs titres. Qui plus est, leur analyse du bien-être, fondée sur une comparaison de règles de politique monétaire simples et optimales sur la base de différents taux d'inflation sectoriels, accrédite les politiques de ciblage de l'inflation d'après l'IPC pratiquées par les banques centrales, à l'opposé des études antérieures. Les coûts de mobilité du capital sont déterminants à ce chapitre.



*Classification JEL : E4, E52, F3, F4*
*Classification de la Banque : Inflation : coûts et avantages; Inflation et prix; Cibles en matière d'inflation; Cadre de la politique monétaire; Mise en œuvre de la politique monétaire*

# 1 Introduction

The nineties witnessed a substantial increase in the number of countries adopting inflation-targeting as their main monetary policy framework. These countries with explicitly-announced inflation targets often stated those objectives based on the consumer price index (CPI).[1] Yet, the literature on optimal monetary policy based on dynamic models with multiple sources of nominal rigidities (Erceg *et al.* 2000, Mankiw and Reis 2002; Woodford 2003; Benigno 2004; Huang and Liu 2005) suggests that the CPI inflation is not always the best choice from a social welfare perspective. This is the case because, unlike models with a single source of nominal rigidity (Aoki 2001), multi-sector models with sectoral heterogeneity in price and/or wage rigidities display a trade-off between the stabilization of deviations of output from its steady-state value (hereafter refered to as the output gap) and multiple sectoral relative prices, which are not allocative-neutral. This trade-off implies that stabilization of CPI inflation does not necessarily mean stabilization of the output gap.

In this paper, we revisit the choice of CPI inflation as an optimal guide to monetary policy in the context of an estimated multi-sector New-Keynesian small open economy model with four sectors (manufacturing, non-tradable, commodity, and imported goods) that are heterogeneous with respect to: (i) the degree of price and wage nominal rigidity, (ii) labour inputs, (ii) the adjustment cost of capital, and (iv) the stochastic process underlying the technology shocks. In particular, we use the welfare implications of the model to quantitatively assess the relative merits of simple, implementable policy reaction functions that include targeting the inflation rate of an aggregate price index (e.g., CPI) versus a sectoral price index (e.g., tradables, nontradables, imports).

Our contribution to the debate regarding which price is the most desirable to target is two-fold. First, in contrast with the existing literature which focuses on nominal rigidities, we highlight the role of real rigidities in the form of factor mobility costs. As sectors differ with respect to their degree of nominal rigidities and the economy is hit by sector-specific shocks, monetary policy will have asymmetric effects in different sectors, with potential allocative implications. To the extent that sectoral (re)allocation of resources becomes costly due to the introduction of imperfect labour mobility and sector-specific capital, we are able to explore another source of welfare loss in the economy, which will add to the losses induced by nominal rigidities and by monopolistic competition (i. e., price dispersion and suboptimal output) extensively discussed in the literature.

Second, the existing literature (see, for example, Beningo 2004) emphasizes that the optimal price index to target will depend on the interplay between the shares of the different sectors in the economy and the differences in the sectoral degrees of nominal rigidities. Our conjecture is that the extent of sector-specific capital adjustment costs should also matter in this regard. How much it matters is a quantitative question and, for this reason, it becomes important to use parameter

[1] These include, for example, Canada, New Zealand, United Kingdom, Finland, Sweden, Spain, France, Germany, and Switzerland; a few of these targeting CPI without the effects of indirect taxes and mortgage interest payments.

values that reflect the reality of a well-established inflation-targeting economy in order to study the welfare implications of targeting alternative price indices within policy reaction functions. We follow the numerical approach used in Kollmann (2002), Ambler *et al.* (2004), Bergin *et al.* (2007), and Schmitt-Grohe and Uribe (2007) to calculate optimized policy functions in which the monetary authority optimally chooses the Taylor rule coefficients to stabilize inflation and output.[2] In this paper, we focus on the case of Canada to estimate our key parameters.[3]

Furthermore, although the main motivation for including real rigidities into the factor structure is the need to capture some arguably realistic assumptions regarding the limited transferability of labour skills and capital across sectors, Christiano, Eichenbaum, and Evans (2005) and Eichenbaum and Fisher (2007) suggest a further reason for using imperfectly-mobile capital with capital adjustment costs in estimated medium-scale dynamic stochastic general equilibrium models. They argue that these features help generate average frequencies of price adjustments that are much more in line with values obtained from micro-based studies.

Since we use Canada as our benckmark inflation-targeting economy, a number of additional features have been introduced in the model to better replicate important aspects of the Canadian economy. For instance, we consider a monopolistically competitive importing sector that distributes differentiated goods imported from Canada's main trading partner − the United States − and the rest of the world. This distinction among trade partners is important because of the pre-eminent role played by the Canada-U.S. bilateral exchange rate in the dynamics of the various components of Canadian inflation. In addition, as the price in the importing sector (expressed in domestic currency) also displays stickiness, our set-up takes into account the evidence of incomplete exchange rate pass-through in Canada (see, for example, Gagnon and Ihrig 2004).

In the same spirit, we introduce a commodity sector that uses labour, capital, and natural resource endowment ("land"). As Canada is a net exporter of this type of good, commodity prices exogenously determined in the world economy have important effects on the real exchange rate and, thus, on inflation. Because commodity goods are used as inputs in the production of other sectors, they contribute to the (incomplete) pass-through of shocks to commodity prices into the Canadian economy.[4]

We also consider aspects related to the current globalized world economy that are likely to affect our small open economy directly. Thus, while we do not explain globalization itself, we account for

[2] As pointed out by De Paoli (2009), such a numerical approach is useful to evaluate optimal policy in complex and fairly realistic open economy settings. An alternative would have been to examine the role of real rigidities within a simpler framework and considering an analytical representation of the monetary authorithy's problem as in Benigno (2004).

[3] While our model is calibrated and estimated using Canadian data, its structure and the results would equally apply to other commodity-exporting small open economies such as Australia, New Zealand, and Norway.

[4] See Dib (2008) for the importance of having such a commodities sector in a small open-economy model for Canada. See also Bouakez, Cardia, and Ruge-Murcia (2009) who use input-output tables to describe in even finer detail the various levels of production.

the possibility that relatively-persistent shocks from foreign sources can directly and markedly alter the import and tradable sectors of our economy. Examples of such shocks can be, for instance, the removal or addition of trade quotas and tariffs, changes in home bias and tastes, improvements in the quality of imported goods, and declines in import costs due to increased internationalization and the opening-up of new markets, that can potentially alter trade flows importantly.

We estimate key structural parameters of the model using Bayesian methods as in Smets and Wouters (2003, 2007). We focus on parameters with a sector-specific dimension, such as Calvo-price and -wage rigidities, capital adjustment costs, and the stochastic processes for exogenous technology shocks, as well as the processes for the shocks to foreign variables. Our estimations find support for significant heterogeneity across sectors and along many dimensions. In particular, the sector that produces nontradable goods stands out as the one with the most rigid prices and wages, with the most costly stock of capital to adjust, and with the most persistent and volatile technology shock. As that sector's production is also the one with the highest share in the Canadian economy, its underlying characteristics turn out to be very important for our results.

Using the estimated parameters, we next solve the model's equilibrium conditions to a second-order approximation around the deterministic steady-state and compare the welfare implications of five alternative monetary policy rules. We focus on policy reaction functions with a "smoothing term," whereby the central bank changes the nominal interest rate in reaction to last period's interest rate, current inflation and the output gap. The alternative options differ with respect to the price index or inflation rate targeted by the monetary authority.[5] Ortega and Rebei (2006), using a similar approach in a two-sector model without sector-specific capital adjustment costs find that the best option is to target the inflation rate of nontradables, which they also identify as the sector with the stickiest prices. In contrast, our main finding is that the highest welfare level is achieved by targeting CPI inflation rather than any of the inflation rates of the remaining sectors, thus confirming the soundness of CPI-based targeting policy by the Bank of Canada. The ranking of policies is maintained when price-level targeting is considered. Interestingly, when we shut down the capital adjustment costs, targeting the inflation rate of the nontradable sector becomes the best option, which highlights the importance of real rigidities in this regard.

The paper is organized as follows. Section 2 presents the main features of the model. Section 3 describes the data, the calibration of selected parameters, as well as the estimation results. Section 4 reports and discusses the simulation results of the model. Section 5 offers some conclusions.

[5] In all our welfare calculations policy reaction functions include deviations of inflation (price index) from its steady-state value which is calibrated to reflect the target level of inflation (price index).

# 2 The Model

We consider a variant of the small open economy model proposed by Dib (2008) for Canada, which builds upon earlier work by Mendoza (1991), Erceg, Henderson and Levin (2000), and Kollmann (2001).[6] The economy consists of households, a government, a monetary authority (or central bank), and a multi-tiered production sector. The latter is structured as follows: a domestic commodity sector exports some of its production and supplies the rest domestically to the manufacturing and the non-tradables sectors of the economy. The manufacturing sector produces goods that are exported or used domestically, and the non-tradables sector produces goods that are destined only for the local market. Foreign goods are imported from different external sources and combined with manufactured goods to yield an aggregate tradable good. The final stage of the production process consists in aggregating tradable and non-tradable goods in order to produce final consumption goods.[7]

Households set monopolistically competitive wages. Monopolistic competition is also assumed in all the intermediate stages of production, while perfect competition is considered in the commodities and final goods' markets. In addition, various nominal rigidities are allowed in the economy. Wages, as well as domestic, imported, intermediate, and final goods' prices are considered to be sticky à la Calvo-Yun.

## 2.1 Households

Households in the economy are represented by a continuum indexed by $h \in [0, 1]$. Each household $h$ has preferences defined over consumption, $C_{ht}$, and labour hours, $H_{ht}$, described by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_{ht}, H_{ht}\right),$$

where $E_0$ denotes the mathematical expectations operator conditional on information available at period 0, $\beta$ is the subjective discount factor, and $U(\cdot)$ is a utility function that is strictly concave, strictly increasing in $C_{ht}$, and strictly decreasing in $H_{ht}$.

The instantaneous utility function is given by

$$U(\cdot) = \frac{C_t^{1-\tau}}{1-\tau} - \frac{H_t^{1+\chi}}{1+\chi}. \tag{1}$$

with $H_{ht} = \left[ H_{M,ht}^{\frac{1+\varsigma}{\varsigma}} + H_{N,ht}^{\frac{1+\varsigma}{\varsigma}} + H_{X,ht}^{\frac{1+\varsigma}{\varsigma}} \right]^{\frac{\varsigma}{1+\varsigma}}$, where $H_{M,ht}$, $H_{N,ht}$, and $H_{X,ht}$, represent hours worked by the household $h$ in manufacturing, non-tradable, and commodity sectors, which are indexed by $M, N$, and $X$, respectively. In addition, $\tau$ is the inverse of the elasticity of intertemporal

[6] A related paper that estimates a DSGE model using Canadian data is Justiniano and Preston (2009).

[7] Bouakez, Cardia, and Ruge-Murcia (2009) show that estimated models using a more detailed production structure can better approximate sectoral nominal rigidities.

substitution of consumption, $\varsigma$ denotes the labour elasticity of substitution across sectors, and $\chi$ is the inverse of the Frisch wage elasticity of labour supply. All three parameters are assumed to be strictly positive. Note that, except in the limit case where $\varsigma \to \infty$, the household $h$'s labour supply, $H_{ht}$, is different from $\sum_{i=M,N,X} H_{i,ht}$, which implies that households do not perfectly substitute their hours worked in the different sectors.

Household $h$ is a monopolistic supplier of differentiated labour services, selling these to a representative competitive firm (a union) that transforms these into aggregate labour inputs for each sector $i$ $(= M, N, X)$ according to the technology:

$$H_{i,t} = \left( \int_0^1 H_{i,ht}^{\frac{\vartheta-1}{\vartheta}} dh \right)^{\frac{\vartheta}{\vartheta-1}}, \; i = M, N, X, \tag{2}$$

In the above, $H_{M,t}$, $H_{N,t}$, and $H_{X,t}$ denote aggregate labour supplied to manufacturing, non-tradable, and commodity sectors, respectively, and the elasticity of substitution among the types of labour is given by $\vartheta > 1$. From the labour union's optimization problem, labour demand in sector $i$ is described by the equation

$$H_{i,ht} = \left( \frac{W_{i,ht}}{W_{i,t}} \right)^{-\vartheta} H_{i,t}, \tag{3}$$

with $W_{i,ht}$ as the nominal wage of household $h$ when working for sector $i$. The nominal wage index in sector $i$, $W_{i,t}$, is thus given by:

$$W_{i,t} = \left( \int_0^1 (W_{i,ht})^{1-\vartheta} dh \right)^{\frac{1}{1-\vartheta}}. \tag{4}$$

Households can buy or sell bonds denominated in foreign currency in incomplete international financial markets. We assume that all international bonds are denominated in U.S. dollars (USD) and denote $e_t^{us}$ as the nominal exchange rate between Canadian dollars (CAD) and USD. Household $h$ enters period $t$ with $K_{i,ht}$ units of capital in the sector $i$, $B_{ht-1}$ units of domestic treasury bonds, and $P_t^{us} B_{ht-1}^*$ units of foreign bonds denominated in USD, where $P_t^{us}$ is the U.S. price level and $B_t^*$ is the (real) foreign debt of the economy. During period $t$, the household supplies labour and capital to firms in all production sectors, receives total factor payment $\sum_{i=M,N,X}(Q_{i,t}K_{i,ht} + W_{i,ht}H_{i,ht})$, where $Q_{i,t}$ is the nominal rental rate of capital in the sector $i$, and receives factor payment of natural resources, $\varpi_h P_{L,t} L_t$, where $P_{L,t}$ is the nominal price of the natural resource input, $L_t$, and $\varpi_h$ is the share of the household $h$ in natural resource payments.[8] Furthermore, household $h$ pays a lump-sum tax $\Upsilon_{ht}$ to the government and receives dividend payments from intermediate goods producing firms, $D_{ht}$. The household uses some of its funds to purchase the final good at the nominal price $P_t$, which it then divides between consumption and investment in each production sector.

[8] Note that, $\int_0^1 \varpi_h dh = 1$.

Accordingly, the budget constraint of household $h$ is given by:

$$P_t(C_{ht}+I_{ht})+\frac{B_{ht}}{R_t}+\frac{e_t^{us}P_t^{us}B_{ht}^*}{\kappa_t R_t^*}\leq\sum_{i=M,N,X}\left(Q_{i,t}K_{i,ht}+W_{i,ht}H_{i,ht}\right)$$
$$+B_{ht-1}+e_t^{us}P_{t-1}^{us}B_{ht-1}^*+\varpi_h P_{L,t}L_t+D_{ht}-\Upsilon_{ht}, \tag{5}$$

where $I_t=I_{M,t}+I_{N,t}+I_{X,t}$ is total investment in the manufacturing, non-tradable, and commodity sectors, respectively; and $D_{ht}=D_{M,ht}+D_{N,ht}+D_{F,ht}$ is the total profit from the manufacturing, non-tradable and import sectors.

The foreign bond return rate, $\kappa_t R_t^*$, depends on the foreign interest rate $R_t^*$ and a country-specific risk premium $\kappa_t$. The foreign interest rate evolves exogenously according to the following AR(1) process:

$$\log(R_t^*)=(1-\rho_{R^*})\log(R^*)+\rho_{R^*}\log(R_{t-1}^*)+\varepsilon_{R^*,t}, \tag{6}$$

where $R^*>1$ is the steady-state value of $R_t^*$, $\rho_{R_*}\in(-1,1)$ is an autoregressive coefficient, and $\varepsilon_{R^*,t}$ is uncorrelated and normally distributed innovation with zero mean and standard deviations $\sigma_{R^*}$.

The country-specific risk premium is increasing in the foreign-debt-to-GDP ratio. It is given by

$$\kappa_t=\exp\left(-\varkappa\frac{e_t^{us}P_t^{us}B_t^*}{P_t Y_t}\right), \tag{7}$$

where $\varkappa>0$ is a parameter that determines the ratio of foreign debt to GDP, and $Y_t$ is total real GDP. The introduction of this risk premium ensures that the model has a unique steady state. It is assumed that the U.S. inflation rate, $\pi_t^*=P_t^{us}/P_{t-1}^{us}$, evolves according to:

$$\log(\pi_t^*)=(1-\rho_{\pi^*})\log(\pi^*)+\rho_{\pi^*}\log(\pi_{t-1}^*)+\varepsilon_{\pi^*,t}, \tag{8}$$

where $\pi^*>1$ is the steady-state value of $\pi_t^*$, $\rho_{\pi^*}\in(-1,1)$ is an autoregressive coefficient, and $\varepsilon_{\pi^*,t}$ is an uncorrelated and normally distributed innovation with zero mean and standard deviations $\sigma_{\pi^*}$.

The stock of capital in the sector $i$ evolves according to:

$$K_{i,ht+1}=(1-\delta)K_{i,ht}+I_{i,ht}-\Psi(K_{i,ht+1},K_{i,ht}), \tag{9}$$

where $\delta\in(0,1)$ is the capital depreciation rate common to all sectors and $\Psi(\cdot)=\frac{\psi_i}{2}\left(\frac{K_{i,ht+1}}{K_{i,ht}}-1\right)^2K_{i,ht}$ is sector $i$'s capital-adjustment cost function that satisfies $\Psi(0)=0$, $\Psi'(\cdot)>0$ and $\Psi''(\cdot)<0$. Since capital is sector-specific and costly to adjust, moving capital between two sectors requires adjustment costs to be paid both in the sector that is investing and in the sector where the capital stock is being reduced.

Household $h$ chooses $C_{ht}, K_{i,ht+1}, B_{ht}$, and $B^*_{ht}$ to maximize its lifetime utility, subject to Eqs. (5) and (9). The first-order conditions, expressed in real terms, are:

$$C_{ht}^{-\tau} = \lambda_t; \tag{10}$$

$$\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( q_{i,t+1} + 1 - \delta + \psi_i \left( \frac{K_{i,ht+2}}{K_{i,ht+1}} - 1 \right) \frac{K_{i,ht+2}}{K_{i,ht+1}} - \frac{\psi_i}{2} \left( \frac{K_{i,ht+2}}{K_{i,ht+1}} - 1 \right)^2 \right) \right] = \psi_i \left( \frac{K_{i,ht+1}}{K_{i,ht}} - 1 \right) + 1, \quad i = M, N, X; \tag{11}$$

$$\frac{\lambda_t}{R_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} \right]; \tag{12}$$

$$\frac{S_t^{us} \lambda_t}{\kappa_t R_t^*} = \beta E_t \left[ \frac{\lambda_{t+1} S_{t+1}^{us}}{\pi_{t+1}^*} \right], \tag{13}$$

in addition to the budget constraint, Eq. (5), to which the Lagrangian multiplier, $\lambda_t$, is associated; $q_{i,t} = Q_{i,t}/P_t$, $\pi_t = P_t/P_{t-1}$, and $S_t^{us} = e_t^{us} P_t^{us}/P_t$ denote real capital return in the sector $i$, the domestic CPI inflation rate, and the bilateral CAD/USD real exchange rate, respectively. Equations (12) and (13) together imply the uncovered interest rate parity (UIP) condition:

$$\frac{R_t}{\kappa_t R_t^*} = \frac{E_t e_{t+1}^{us}}{e_t^{us}}. \tag{14}$$

Furthermore, there are three first-order conditions for setting nominal wages in each sector $i$, $\widetilde{W}_{i,ht}$, when household $h$ is allowed to revise its nominal wages. As in Calvo (1983), this happens with probability $(1 - \varphi_i)$ in the sector $i$, at the beginning of each period $t$. If household $h$ is not allowed to change its nominal wage, it fully indexes its wage to the steady-state inflation rate, $\pi$, as in Yun (1996). Following Schmitt-Grohé and Uribe (2007), household $h$ sets his optimized nominal wage in the sector $i$, $\widetilde{W}_{i,ht}$, to maximize the flow of its expected utility, so that

$$\max_{\widetilde{W}_{i,ht}} E_0 \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \left\{ U\left(C_{ht+l}, H_{i,ht+l}\right) + \lambda_{t+l} \pi^l \widetilde{W}_{i,ht} H_{i,ht+l}/P_{t+l} \right\} \right],$$

subject to $H_{i,ht+l} = \left( \frac{\pi^l \widetilde{W}_{i,ht}}{W_{i,t+l}} \right)^{-\vartheta} H_{i,t+l}$, where $i = M, N, X$. The first-order condition derived for $\widetilde{W}_{i,ht}$ is

$$E_0 \left[ \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} \left( \frac{\pi^l \widetilde{W}_{i,ht}}{W_{i,t+l}} \right)^{-\vartheta} H_{i,t+l} \left\{ \zeta_{i,t+l} - \frac{\vartheta - 1}{\vartheta} \frac{\widetilde{W}_{i,ht}}{P_t} \frac{\pi^l P_t}{P_{t+l}} \right\} \right] = 0, \tag{15}$$

where $\zeta_{i,t} = -\frac{\partial U / \partial H_{i,ht}}{\partial U / \partial C_{ht}}$ is the marginal rate of substitution between consumption and labour type $i$. Dividing Eq. (15) by $P_t$ and rearranging yields:

$$\widetilde{w}_{i,ht} = \frac{\vartheta}{\vartheta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} \zeta_{i,t+l} w_{i,t+l}^{\vartheta} H_{i,t+l} \prod_{k=1}^{l} \pi^{-\vartheta l} \pi_{t+k}^{\vartheta}}{E_t \sum_{l=0}^{\infty} (\beta\varphi_i)^l \lambda_{t+l} w_{i,t+l}^{\vartheta} H_{i,t+l} \prod_{k=1}^{l} \pi^{l(1-\vartheta)} \pi_{t+k}^{\vartheta - 1}}, \tag{16}$$

where $\widetilde{w}_{i,ht} = \widetilde{W}_{i,ht}/P_t$ and $w_{i,t} = W_{i,t}/P_t$ are the household $h$'s real optimized wage and real wage index in the sector $i$, respectively.

The nominal wage index in the sector $i$ evolves over time according to the following recursive equation:

$$(W_{i,t})^{1-\vartheta} = \varphi_i(\pi W_{i,t-1})^{1-\vartheta} + (1-\varphi_i)(\widetilde{W}_{i,t})^{1-\vartheta}, \tag{17}$$

where $\widetilde{W}_{i,t}$ is the average wage of those workers who are allowed to revise their wage at period $t$ in the sector $i$. Dividing (17) by $P_t$ yields:

$$(w_{i,t})^{1-\vartheta} = \varphi\left(\frac{\pi w_{i,t-1}}{\pi_t}\right)^{1-\vartheta} + (1-\varphi_i)(\widetilde{w}_{i,t})^{1-\vartheta}, \tag{18}$$

In a symmetric equilibrium, $\widetilde{w}_{i,t} = \widetilde{w}_{i,ht}$ and $H_{i,t} = H_{i,ht}$ for all $t$. Therefore, we can rewrite equation. (16) in a non-linear recursive form following Schmitt-Grohé and Uribe (2007):

$$\widetilde{w}_{i,t} = \frac{\vartheta}{\vartheta-1}\frac{f^1_{i,t}}{f^2_{i,t}}; \tag{19}$$

where

$$\begin{aligned} f^1_{i,t} &= E_t\left[\sum_{l=0}^{\infty}(\beta\varphi_i)^l \lambda_{t+l}\zeta_{i,t+l}H_{i,t+l}w^{\vartheta}_{i,t+l}\prod_{k=1}^{l}\pi^{-\vartheta l}\pi^{\vartheta}_{t+k}\right]; \\ &= \lambda_t\, H_{i,t}\zeta_{i,t}w^{\vartheta}_{i,t} + \beta\varphi_i E_t\left[(\pi_{t+1}/\pi)^{\vartheta} f^1_{i,t+1}\right]; \end{aligned} \tag{20}$$

and

$$\begin{aligned} f^2_{i,t} &= E_t\left[\sum_{l=0}^{\infty}(\beta\varphi_i)^l \lambda_{t+l}H_{i,t+l}w^{\vartheta}_{i,t+l}\prod_{k=1}^{l}\pi^{l(1-\vartheta))}\pi^{\vartheta-1}_{t+k}\right]; \\ &= \lambda_t\, H_{i,t}w^{\vartheta}_{i,t} + \beta\varphi_i E_t\left[(\pi_{t+1}/\pi)^{\vartheta-1} f^2_{i,t+1}\right]. \end{aligned} \tag{21}$$

In addition, Eqs. (16) and (18) permit us to derive the standard New Keynesian Phillips curve, $\hat{\pi}^{w_i}_t = \beta\hat{\pi}^{w_i}_{t+1} + \frac{(1-\beta\varphi_i)(1-\varphi_i)}{\varphi_i}[\hat{\zeta}_{i,t} - \hat{w}_{i,t}]$, where $\pi^{w_i}_t = W_{i,t}/W_{i,t-1}$ is wage inflation in the sector $i$ and hats over the variables denote deviations from steady-state values.

## 2.2 Final good

To facilitate the exposition of the production structure, we start from the final stage of production (explained in this section) and move through the intermediate phase to the initial stages of production. Diagram 1 displays the production structure of the model.

At the last stage of production, a perfectly-competitive representative firm combines a composite tradable good, $Y_{T,t}$, and non-tradable goods, $Y_{N,t}$, with corresponding prices $P_{T,t}$ and $P_{N,t}$, respectively, using the following CES function to produce a homogeneous final good, $Z_t$:

$$Z_t = \left[(1-\omega_N)^{\frac{1}{\nu}}Y_{T,t}^{\frac{\nu-1}{\nu}} + \omega_N^{\frac{1}{\nu}}Y_{N,t}^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{22}$$

where $\omega_N$ is the share of non-tradables in the final good and $\nu > 0$ is the elasticity of substitution between non-tradable and tradable goods.

Taking the price of the final good, $P_t$, as given, the final good producer chooses $Y_{T,t}$ and $Y_{N,t}$ to maximize its profit. Formally:

$$\max_{\{Y_{T,t},Y_{N,t}\}} P_t Z_t - P_{T,t}Y_{T,t} - P_{N,t}Y_{N,t},$$

subject to (22). Profit maximization implies the following demand functions for the aggregate tradable good and the non-tradable good:

$$Y_{T,t} = (1-\omega_N)\left(\frac{P_{T,t}}{P_t}\right)^{-\nu} Z_t, \quad \text{and} \quad Y_{N,t} = \omega_N \left(\frac{P_{N,t}}{P_t}\right)^{-\nu} Z_t. \tag{23}$$

The zero-profit condition implies that the final-good price level, which we interpret as the consumer-price index (CPI), is linked to tradable and non-tradable goods' prices via the equation:

$$P_t = \left[(1-\omega_N)P_{T,t}^{1-\nu} + \omega_N P_{N,t}^{1-\nu}\right]^{1/(1-\nu)}. \tag{24}$$

### 2.3 The non-tradable sector

**Wholesale producers.** The non-tradable good input used to produce $Z_t$ in (22) is generated by a competitive wholesale firm that aggregates differentiated goods produced by a continuum of intermediate good producers which are indexed by $j \in [0,1]$. The wholesale firm uses the following CES technology:

$$Y_{N,t} = \left(\int_0^1 (Y_{N,jt})^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \tag{25}$$

where $Y_{N,jt}$ is the demand for intermediate input $j$ and $\theta > 1$ is the constant elasticity of substitution between the differentiated goods. The maximization problem of the wholesaler is given by:

$$\max_{\{Y_{N,jt}\}} P_{N,t}Y_{N,t} - \int_0^1 P_{N,jt}Y_{N,jt}dj,$$

subject to (25). The solution to this problem implies a demand function and a corresponding price, $P_{N,jt}$, of the differentiated good, given, respectively, by:

$$Y_{N,jt} = \left(\frac{P_{N,jt}}{P_{N,t}}\right)^{-\theta} Y_{N,t}, \tag{26}$$

and

$$P_{N,t} = \left(\int_0^1 (P_{N,jt})^{1-\theta} dj\right)^{\frac{1}{1-\theta}}. \tag{27}$$

**Intermediate good producers.** Each intermediate good firm $j$ in sector $N$ produces output, $Y_{N,jt}$, using capital, $K_{N,jt}(=\int_0^1 K_{N,jht}dh)$, labour, $H_{N,jt}(=\int_0^1 H_{N,jht}dh)$, and a commodity input, $Y^N_{X,jt}$. The production function is represented by the following Cobb-Douglas technology:

$$Y_{N,jt} \leq A_{N,t}\left(K_{N,jt}\right)^{\alpha_N}\left(H_{N,jt}\right)^{\gamma_N}\left(Y^N_{X,jt}\right)^{\eta_N}, \quad \alpha_N, \gamma_N, \eta_N \in (0,1), \tag{28}$$

where $\alpha_N + \gamma_N + \eta_N = 1$; $\alpha_N$, $\gamma_N$, and $\eta_N$ are shares of capital, labour, and commodity inputs in the production of non-tradable goods, respectively. $A_{N,t}$ is a technology shock specific to the non-tradable sector. It is assumed that this shock evolves exogenously according to:

$$\log(A_{N,t}) = (1-\rho_{AN})\log(A_N) + \rho_{AN}\log(A_{N,t-1}) + \varepsilon^N_{A,t}, \tag{29}$$

where $A_N > 0$ is the steady-state value of $A_{N,t}$, $\rho_{AN} \in (-1,1)$ is an autoregressive coefficient, and $\varepsilon^N_{A,t}$ is uncorrelated and normally distributed innovation with zero mean and standard deviations $\sigma_{AN}$.

As in the wage-setting decisions of households, Calvo-Yun price rigidity applies: firm $j$ is allowed to revise its optimally chosen price, $\widetilde{P}_{N,jt}$, with probability $(1-\phi_N)$ for the period $t$, and it fully indexes its price to the steady-state CPI inflation rate, $\pi$, otherwise. In that case, it will use the non-optimal rule $P_{N,jt} = \pi P_{N,jt-1}$. The firm chooses $\widetilde{P}_{N,jt}$, $K_{N,jt}$, $H_{N,jt}$, and $Y^N_{X,jt}$, to maximize the expected discounted flow of its profits according to:

$$\max_{\{K_{N,jt},H_{N,jt},Y^N_{X,jt},\widetilde{P}_{N,jt}\}} E_0\left[\sum_{l=0}^{\infty}(\beta\phi_N)^l\lambda_{t+l}D_{N,jt+l}/P_{t+l}\right],$$

subject to (28), the demand function[9]

$$Y_{N,jt+l} = \left(\frac{\pi^l\widetilde{P}_{N,jt}}{P_{N,t+l}}\right)^{-\theta} Y_{N,t+l},$$

and the (nominal) profit function

$$D_{N,jt+l} = \pi^l\widetilde{P}_{N,jt}Y_{N,t+l} - Q_{N,t+l}K_{N,t+l} - W_{N,t+l}Y_{N,t+l} - e^{us}_{t+l}P^*_{X,t+l}Y^N_{X,jt+l}, \tag{30}$$

where $Q_{N,t+l}$ is the nominal rental rate of capital in period $t+l$, $W_{N,t+l}$ is the nominal wage rate in sector $N$; the nominal commodity price, $P^*_{X,t}$, is determined exogenously in world markets and denominated in U.S. dollars. The latter is multiplied by the nominal CAD/USD exchange rate, $e^{us}_t$, to yield the cost of commodity inputs in terms of domestic currency. The firm takes commodity prices and the nominal exchange rate as given. Finally, $\beta^l\lambda_{t+l}$ is the producer's discount factor, where $\lambda_{t+l}$ denotes the marginal utility of consumption in period $t+l$.

[9] Which is obtained from equation (26) combined with the non-optimal pricing rule.

The first-order conditions (in real terms) with respect to $K_{N,jt}$, $H_{N,jt}$, and $Y^N_{X,jt}$ are:

$$\begin{aligned} q_{N,t} &= \alpha_N Y_{N,jt}\xi_{N,t}/K_{N,jt}; && (31)\\ w_{N,t} &= \gamma_N Y_{N,jt}\xi_{N,t}/H_{N,jt}; && (32)\\ S^{us}_t p^*_{X,t} &= \eta_N Y_{N,jt}\xi_{N,t}/Y^N_{X,jt}, && (33) \end{aligned}$$

where $\xi_{N,t}$ is the real marginal cost in the non-tradable sector that is common to all intermediate firms, $q_{N,jt} = Q_{N,jt}/P_t$ and $w_{N,jt} = W_{N,jt}/P_t$ are real capital return and real wages in the non-tradable sector. In addition, $p^*_{X,t} = P^*_{X,t}/P^{us}_t$ is the real commodity price in units of U.S. goods, and $S^{us}_t = e^{us}_t P^{us}_t/P_t$ represents the real CAD/USD exchange rate, where $P^{us}_t$ is the U.S. price level. The condition (33) indicates that the real marginal cost in the non-tradable sector is also directly affected by real exchange rate and commodity price movements, since commodities enter the production of non-tradable goods as inputs.

The firm that is allowed to revise its price, which happens with probability $(1-\phi_N)$, chooses $\widetilde{P}_{N,jt}$ so that

$$\widetilde{p}_{N,jt} = \frac{\theta}{\theta-1}\frac{E_t\sum_{l=0}^{\infty}(\beta\phi_N)^l\lambda_{t+l}\xi_{N,t+l}p^{\theta}_{N,t+l}Y_{N,t+l}\prod_{k=1}^{l}\pi^{-\theta l}\pi^{\theta}_{t+k}}{E_t\sum_{l=0}^{\infty}(\beta\phi_N)^l\lambda_{t+l}p^{\theta}_{N,t+l}Y_{N,t+l}\prod_{k=1}^{l}\pi^{l(1-\theta)}\pi^{\theta-1}_{t+k}}, \qquad (34)$$

where $\widetilde{p}_{N,jt} = \widetilde{P}_{N,jt}/P_t$ is the real optimized price in the non-tradable sector and $p_{N,t} = P_{N,t}/P_t$ is the relative price of non-tradable goods.

The real non-tradable price index evolves as follows:

$$(p_{N,t})^{1-\theta} = \phi_N\left(\frac{\pi p_{N,t-1}}{\pi_t}\right)^{1-\theta} + (1-\phi_N)\left(\widetilde{p}_{N,t}\right)^{1-\theta}. \qquad (35)$$

## 2.4 Composite Tradable goods

A representative firm acting in a perfectly competitive market combines a domestically-produced manufactured good, $Y^d_{M,t}$ with imports, $Y_{F,t}$ to produce an aggregate tradable good, $Y_{T,t}$. Henceforth, we refer to the latter simply as the tradable good. The aggregation technology is given by a CES function as follows:

$$Y_{T,t} = \left[(1-\omega_{F,t})^{\frac{1}{\mu}}\left(Y^d_{M,t}\right)^{\frac{\mu-1}{\mu}} + \omega_{F,t}^{\frac{1}{\mu}}Y_{F,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}, \qquad (36)$$

where $\mu > 0$ is the elasticity of substitution between domestically-used manufactured and imported goods in the production function and $\omega_{F,t}$ denotes the shares of imported goods in the production of the tradable good. The share $\omega_{F,t}$ is time-varying and its evolution follows an exogenous AR(1) process given by:

$$\log(\omega_{F,t}) = (1-\rho_F)\log(\omega_{F,t}) + \rho_F\log(\omega_{F,t-1}) + \varepsilon_{F,t}, \qquad (37)$$

where $\omega_F > 0$ is the steady-state value of $\omega_{F,t}$, $\rho_F \in (-1, 1)$ is an autoregressive coefficient, and $\varepsilon_{F,t}$ is uncorrelated and normally distributed innovation with zero mean and standard deviations $\sigma_F$. The time-varying share of foreign goods in the tradable goods aggregator, $\omega_{F,t}$, is intended to capture some exogenous factors affecting the overall degree of "home bias" in the Canadian economy. For instance, it captures factors related to globalization often associated with a decline in various trade barriers, an increased variety of foreign goods available, or improvements in the quality of these goods.

Given $P_{F,t}$, $P_{M,t}$, and the price of tradables, $P_{T,t}$, the firm chooses $Y_{F,t}$ and $Y^d_{M,t}$ to maximize its profits. The corresponding maximization problem is:

$$\max_{\{Y_{F,t}, Y^d_{M,t}\}} P_{T,t}Y_{T,t} - P_{F,t}Y_{F,t} - P_{M,t}Y^d_{M,t}, \tag{38}$$

subject to (36). The following demand functions for domestically used manufacturing goods and imported goods are obtained:

$$Y_{F,t} = \omega_{F,t}\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} Y_{T,t}, \quad Y^d_{M,t} = (1-\omega_{F,t})\left(\frac{P_{M,t}}{P_t}\right)^{-\mu} Y_{T,t}.$$

Finally, the tradable good's price is given by:

$$P_{T,t} = \left[(\omega_{F,t}P_{F,t}^{1-\mu} + (1-\omega_{F,t})P_{M,t}^{1-\mu}\right]^{1/(1-\mu)}. \tag{39}$$

The production of inputs $Y_{F,t}$ and $Y^d_{M,t}$ used in the above equation (36) are explained in some detail in next sections.

## 2.5 The manufacturing sector

As in the non-tradable sector, there are wholesale and intermediate-good producers in the manufacturing sector. The competitive wholesale firm aggregates differentiated manufactured goods produced by a continuum of manufactured intermediate-good producers which are indexed by $j \in [0, 1]$. It uses the following CES technology:

$$Y_{M,t} = \left(\int_0^1 (Y_{M,jt})^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \tag{40}$$

where $\theta$, as defined before, is the constant elasticity of substitution between the intermediate goods. The maximization problem of the wholesaler is given by:

$$\max_{\{Y_{M,jt}\}} P_{M,t}Y_{M,t} - \int_0^1 P_{M,jt}Y_{M,jt}dj,$$

subject to (40), and implies the following demand functions for the intermediate manufactured goods:

$$Y_{M,jt} = \left(\frac{P_{M,jt}}{P_{M,t}}\right)^{-\theta} Y_{M,t} \tag{41}$$

where the price of the manufactured good is:

$$P_{M,t} = \left(\int_0^1 (P_{M,jt})^{1-\theta}\, dj\right)^{\frac{1}{1-\theta}}. \tag{42}$$

The $j$th intermediate-good-producing firm produces its output, $Y_{M,jt}$, using capital $K_{M,jt}(= \int_0^1 K_{M,jht}dh)$, labour, $H_{M,jt}(= \int_0^1 H_{M,jht}dh)$, and the commodity input, $Y^M_{X,jt}$. Its production function is given by

$$Y_{M,jt} \leq A_{M,t}\,(K_{M,jt})^{\alpha_M}\,(H_{M,jt})^{\gamma_M}\,\left(Y^M_{X,jt}\right)^{\eta_M}, \quad \alpha_M, \gamma_M, \eta_M \in (0,1), \tag{43}$$

where $\alpha_M + \gamma_M + \eta_M = 1$; $\alpha_M$, $\gamma_M$ and $\eta_M$ are the shares of capital, labour, and commodity inputs in the production of manufactured goods, respectively. $A_{M,t}$ is a technology shock specific to the manufacturing sector, which evolves exogenously according to the AR(1) process:

$$\log(A_{M,t}) = (1-\rho_{AM})\log(A_M) + \rho_{AM}\log(A_{M,t-1}) + \varepsilon^M_{A,t}, \tag{44}$$

where $A_M > 0$ is the steady-state value of $A_{M,t}$, $\rho_{AM} \in (-1,1)$ is an autoregressive coefficient, and $\varepsilon^M_{A,t}$ is uncorrelated and normally distributed innovation with zero mean and standard deviations $\sigma_{AM}$.

The nominal profit of firm $j$ in period $t+l$, $D_{T,jt+l}$, is:

$$D_{M,jt+l} = \pi^l \widetilde{P}_{M,jt} Y_{M,jt+l} - Q_{M,t+l}K_{M,jt+l} - W_{M,t+l}H_{M,jt+l} - e^{us}_{t+l}P^*_{X,t+l}Y^M_{X,jt+l}. \tag{45}$$

Given the rental rate for capital in this sector, $Q_{M,t}$, the wage rate, $W_{M,t}$, and $P_t$, the intermediate-goods producer $j$ thus chooses $K_{M,jt}$, $H_{M,jt}$, and $Y^M_{X,jt}$ to maximize its profits. As in the non-tradables sector, the Calvo-Yun probability of price revision is given by $(1-\phi_M)$ with the non-optimized prices fully indexed to the steady-state CPI inflation rate. The price $\widetilde{P}_{M,jt}$ thus maximizes the expected discounted flows of profits based on the optimization problem given by:

$$\max_{\{K_{M,jt}, H_{M,jt}, Y^M_{X,jt}, \widetilde{P}_{M,jt}\}} E_0\left[\sum_{l=0}^{\infty}(\beta\phi_M)^l \lambda_{t+l} D_{M,jt+l}/P_{t+l}\right],$$

subject to (43), (45), and the demand function:

$$Y_{M,jt+l} = \left(\frac{\pi^l \widetilde{P}_{M,jt}}{P_{M,t+l}}\right)^{-\theta} Y_{M,t+l}.$$

The first-order conditions with respect to $K_{M,jt}$, $H_{M,jt}$, and $Y^M_{X,jt}$ (in real terms) are:

$$\begin{aligned} q_{M,t} &= \alpha_M Y_{M,jt}\xi_{M,t}/K_{M,jt}; &(46)\\ w_{M,t} &= \gamma_M Y_{M,jt}\xi_{M,t}/H_{M,jt}; &(47)\\ S^{us}_t p^*_{X,t} &= \eta_M Y_{M,jt}\xi_{M,t}/Y^M_{X,jt}, &(48)\end{aligned}$$

where $\xi_{M,t}$ is the common real marginal cost in the manufacturing sector, and $q_{M,t} = Q_{M,t}/P_t$ and $w_{M,t} = W_{M,t}/P_t$ are defined as in the non-tradable sector. In addition, the condition (48) has a similar interpretation to its counterpart (33) for the non-tradables sector.

The first-order condition with respect to $\widetilde{P}_{M,jt}$ is:

$$\widetilde{p}_{M,jt} = \frac{\theta}{\theta-1}\frac{E_t\sum_{l=0}^{\infty}(\beta\phi_M)^l\lambda_{t+l}\xi_{M,t+l}p^{\theta}_{M,t+l}Y_{M,t+l}\prod_{k=1}^{l}\pi^{-\theta l}\pi^{\theta}_{t+k}}{E_t\sum_{l=0}^{\infty}(\beta\phi_M)^l\lambda_{t+l}p^{\theta}_{M,t+l}Y_{M,t+l}\prod_{k=1}^{l}\pi^{l(1-\theta)}\pi^{\theta-1}_{t+k}}; \tag{49}$$

where $\widetilde{p}_{M,jt} = \widetilde{P}_{M,jt}/P_t$, is the real optimized price for domestic manufactured goods and $p_{M,t} = P_{M,t}/P_t$ is the sectoral relative price.

The real manufacturing price index evolves according to:

$$(p_{M,t})^{1-\theta} = \phi_M\left(\frac{\pi p_{M,t-1}}{\pi_t}\right)^{1-\theta} + (1-\phi_M)\left(\widetilde{p}_{M,t}\right)^{1-\theta}. \tag{50}$$

The total domestic production of manufactured goods is divided into two parts: $Y^d_{M,t}$, for domestic use in the production of tradable goods (see equation 36) and $Y^{ex}_{M,t}$, which is exported, so that $Y_{M,t} = Y^d_{M,t} + Y^{ex}_{M,t}$. Following Obstfeld and Rogoff (1995), we assume producer currency pricing (PCP) behavior in the manufacturing sector. Under this assumption, all firms set their price, $\widetilde{P}_{M,jt}$, for both home and foreign markets. Thus, the law of one price (LOP) holds and movements of the exchange rate are completely passed through into export prices. The aggregate foreign demand function for domestically manufactured-goods that are exported, under the assumption of PCP, is given by

$$Y^{ex}_{M,t} = \omega_{ex}\left(\frac{P_{M,t}}{e^{us}_t P^{us}_t}\right)^{-\nu} Y^*_t, \tag{51}$$

where $Y^*_t$ is foreign output. The price-elasticity of demand for domestic manufactured-goods by foreigners is $-\nu$, and $\omega_{ex} > 0$ is determines the sensitivity of domestic manufactured-goods exports to foreign output. Since it is a small economy, domestic exports form an insignificant fraction of foreign expenditures and have a negligible weight in the foreign price index.

It is assumed that foreign output is exogenous and evolves according to:

$$\log(Y^*_t) = (1-\rho_{Y^*})\log(Y^*) + \rho_{Y^*}\log(Y^*_{t-1}) + \varepsilon_{Y^*,t}, \tag{52}$$

where $Y^*$ is the steady-state value, $\rho_{Y^*} \in (-1,1)$ is an autoregressive coefficient, and $\varepsilon_{Y^*,t}$ is an uncorrelated and normally distributed innovation with zero mean and standard deviation $\sigma_{Y^*}$.

## 2.6 Import sector

There are both wholesale and intermediate-good firms in this sector. A continuum of importing firms indexed by $j \in [0,1]$ buy homogenous goods from two sources abroad, the U.S. and the rest-of-the-world, *costlessly* combine these into $j$ differentiated goods, $Y_{F,jt}$, and sell them at price $P_{F,jt}$ to a competitive wholesale firm. The latter then aggregates these differentiated intermediate import goods into a composite good according to the following CES technology

$$Y_{F,t} = \left( \int_0^1 (Y_{F,jt})^{\frac{\theta-1}{\theta}} \, dj \right)^{\frac{\theta}{\theta-1}}, \tag{53}$$

where $\theta$, as defined before, is the constant elasticity of substitution between the differentiated goods. The maximization problem of the wholesaler is therefore given by:

$$\max_{\{Y_{F,t}\}} P_{F,t} Y_{F,t} - \int_0^1 P_{F,jt} Y_{F,jt} dj,$$

subject to (53).The demand functions for the differentiated imported goods are given by:

$$Y_{F,jt} = \left( \frac{P_{F,jt}}{P_{F,t}} \right)^{-\theta} Y_{F,t} \tag{54}$$

with the sectoral import price that is charged for local sale given by:

$$P_{F,t} = \left( \int_0^1 (P_{F,jt})^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}}. \tag{55}$$

To sell $Y_{F,jt}$ on the market, each intermediate-good importing firm $j$ buys a fraction $\alpha$ of this imported good in USD at the price $P_t^{us}$, and the remaining fraction, $(1-\alpha)$, at the price $P_t^{rw}$ (representing a basket of currencies). As in previous sectors, the Calvo-Yun probability of changing price is $(1-\phi_F)$ with the non-optimized prices, fully indexed to the steady-state CPI inflation rate. We define $e_t^{rw}$ as the bilateral nominal exchange rate between the rest of the world and the United States, which is not affected by the Canadian economy and treated as exogenous.[10] Given the nominal exchange rates $e_t^{us}$ and $e_t^{rw}$ and the foreign price levels $P_t^{us}$ and $P_t^{rw}$, the price $\widetilde{P}_{F,jt}$ maximizes the expected discounted flows of profits based on the optimization problem given by:

$$\max_{\{\widetilde{P}_{F,jt}\}} E_0 \left[ \sum_{l=0}^{\infty} (\beta \phi_F)^l \lambda_{t+l} D_{F,jt+l} / P_{t+l} \right],$$

subject to

$$Y_{F,jt+l} = \left( \frac{\pi^l \widetilde{P}_{F,jt}}{P_{F,t+l}} \right)^{-\theta} Y_{F,t+l},$$

where the nominal profit function is:

$$D_{F,jt+l} = \left\{ \pi^l \widetilde{P}_{F,jt} - e_{t+l}^{us} \left[ \alpha P_{t+l}^{us} + (1-\alpha) e_{t+l}^{rw} P_{t+l}^{rw} \right] \right\} Y_{F,jt+l}. \tag{56}$$

[10] Note that the bilateral exchange rate between Canada and the rest of the world is given by $e_t^{us} e_t^{rw}$.

The presence of price rigidity means that the response of the imported goods price to exogenous shocks is gradual, implying an incomplete pass-through of exchange rate changes to the levels of prices in the economy. Gagnon and Ihrig (2004) find evidence of incomplete exchange rate pass-through in industrialized economies including Canada.

In period $t+l$, the importer's nominal marginal cost is $e^{us}_{t+l}\left[\alpha P^{us}_{t+l} + (1-\alpha)\, e^{rw}_{t+l} P^{rw}_{t+l}\right]$, so its real marginal cost is equal to the weighted average real exchange rate, $S_{t+l} = S^{us}_{t+l}\left[\alpha + (1-\alpha)\, S^{rw}_{t+l}\right]$, between the CAD/USD rate, $S^{us}_{t+l}$, and the real exchange rate USD/rest of the world, $S^{rw}_{t+l} = e^{rw}_{t+l} P^{rw}_{t+l} / P^{us}_{t+l}$.

The distinction among trade partners is potentially important for two reasons. First, there is the pre-eminent role played by the Canada-U.S.bilateral exchange rate in the dynamics of the various components of Canadian inflation, given the amount of trade between these two economies.[11] Second, the distinction may help us capture certain relative price effects due to the increasing participation of emerging economies (e.g, China) in the world trade generally described within the context of globalization. We assume that these effects, unlike those pertaining to emerging economies, impacted Canada and the U.S. − both members of the North-American Free-Trade Agreement − roughly similarly so that real relative price of goods manufactured in the U.S. vis-à-vis Canada would not have been affected by this particular channel. We capture the latter changes in real effective prices by considering the exogenous stochastic process for $S^{rw}_{t+l}$:[12]

$$\log(S^{rw}_t) = (1-\rho_{rw})\log(S^{rw}) + \rho_{rw}\log(S^{rw}_{t-1}) + \varepsilon_{rw,t}, \tag{57}$$

where $S^{rw} > 0$ is the steady-state value of $S^{rw}_t$, $\rho_{rw} \in (-1,1)$ is an autoregressive coefficient, and $\varepsilon_{rw,t}$ is uncorrelated and normally distributed innovation with zero mean and standard deviations $\sigma_{rw}$.

The first-order condition of the optimization problem implies:

$$\widetilde{p}_{F,jt} = \frac{\theta}{\theta-1}\frac{E_t \sum_{l=0}^{\infty}(\beta\phi_F)^l \lambda_{t+l} S_{t+l} p^{\theta}_{F,t+l} Y_{F,t+l} \prod_{k=1}^{l} \pi^{-\theta l}\pi^{\theta}_{t+k}}{E_t \sum_{l=0}^{\infty}(\beta\phi_F)^l \lambda_{t+l} p^{\theta}_{F,t+l} Y_{F,t+l} \prod_{k=1}^{l} \pi^{l(1-\theta)}\pi^{\theta-1}_{t+k}}, \tag{58}$$

where $\widetilde{p}_{F,jt} = \widetilde{P}_{F,jt}/P_t$ is the real optimized price in import sector, and $p_{F,t} = P_{F,t}/P_t$ is the relative price of imports. The relative (real) import price index evolves as:

$$(p_{F,t})^{1-\theta} = \phi_F\left(\frac{\pi p_{F,t-1}}{\pi_t}\right)^{1-\theta} + (1-\phi_F)\,(\widetilde{p}_{F,t})^{1-\theta}. \tag{59}$$

[11] Currently Canada imports close to 80 per cent of all its foreign goods from the United States.

[12] The (exogenous) effects of globalization that are not related to the relative prices of the rest-of-the-world vis-à-vis both Canada and the U.S. are likely to be reflected in the time-varying share of imported goods in the production of the tradeable good, $\omega_{F,t}$, described in (37).

## 2.7 The commodity sector

The commodity sector is indexed by the subscript $X$. Production in this sector is modelled to capture the importance of natural resources in the Canadian economy. In this sector, there is a perfectly competitive firm that produces commodity output, $Y_{X,t}$, using capital, $K_{X,t}(=\int_0^1 K_{X,ht}dh)$, labour, $H_{X,t}(=\int_0^1 H_{X,ht}dh)$, and a natural-resource factor, $L_t$. The presence of the natural resource factor in the production of commodities prevents the small open economy from specializing in the production of a single tradable good. In equilibrium, the commodity and manufactured goods sectors will coexist. The production function is:

$$Y_{X,t} \leq (K_{X,t})^{\alpha_X} (H_{X,t})^{\gamma_X} (L_t)^{\eta_X}, \quad \alpha_X, \gamma_X, \eta_X \in (0,1), \tag{60}$$

with $\alpha_X + \gamma_X + \eta_X = 1$, where $\alpha_X$, $\gamma_X$, and $\eta_X$ are shares of capital, labour, and natural resources in the production of commodities, respectively. The supply of $L_t$ evolves exogenously according to the following AR(1) process:

$$\log(L_t) = (1-\rho_L)\log(L) + \rho_L \log(L_{t-1}) + \varepsilon_{L,t}, \tag{61}$$

where $L$ is a steady-state value of $L_t$, $\rho_L \in (-1,1)$ is an autoregressive coefficient, and $\varepsilon_{L,t}$ is uncorrelated and normally distributed innovation with zero mean and standard deviations $\sigma_L$. A positive shock may be interpreted as an exogenous increase in the supply of the natural resource factor due to, for example, favourable weather or a new mining discovery.

The commodity output is divided between exports and domestic use as direct inputs in the manufacturing and non-tradable sectors, so that $Y_{X,t} = Y_{X,t}^{ex} + Y_{X,t}^{M} + Y_{X,t}^{N}$; where $Y_{X,t}^{ex}$ is the quantity of commodity goods exported, while $Y_{X,t}^{M}$ and $Y_{X,t}^{N}$ denote the quantities of commodity goods used as material inputs in the manufacturing and non-tradable sectors, respectively.

The commodity-producing firm takes the nominal commodity price, $P_{X,t}^*$, and the nominal CAD/USD exchange rate, $e_t^{us}$, as given. Thus, given $e_t^{us}$, $P_{X,t}^*$, $Q_{X,t}$, $W_{X,t}$, and the price of the natural-resource factor, $P_{L,t}$, the commodity-producing firm chooses $K_{X,t}$, $H_{X,t}$, and $L_t$ to maximize its real profit flows. Its maximization problem is:

$$\max_{\{K_{X,t},H_{X,t},L_t\}} \left[e_t^{us} P_{X,t}^* Y_{X,t} - Q_{X,t}K_{X,t} - W_{X,t}H_{X,t} - P_{L,t}L_t\right]/P_t,$$

subject to the production technology, Eq. (60).

The first-order conditions, with respect to $K_{X,t}$, $H_{X,t}$, and $L_t$, in real terms are:

$$q_{X,t} = \alpha_X S_t^{us} p_{X,t}^* Y_{X,t}/K_{X,t}; \tag{62}$$

$$w_{X,t} = \gamma_X S_t^{us} p_{X,t}^* Y_{X,t}/H_{X,t}; \tag{63}$$

$$p_{L,t} = \eta_X S_t^{us} p_{X,t}^* Y_{X,t}/L_t, \tag{64}$$

where $q_{X,t} = Q_{X,t}/P_t$, $w_{X,t} = W_{X,t}/P_t$, and $p_{L,t} = P_{L,t}/P_t$ are real capital returns, real wages, and real natural resource prices in the commodity sector, respectively.

The demand for $K_{X,t}$, $H_{X,t}$, and $L_t$ are given by equations. (62)– (64), respectively. These equations stipulate that the marginal cost of each input must be equal to its marginal productivity. Because the economy is small, the demand for commodity exports and their prices are completely determined in the world markets. It is assumed that real commodity price, $p^*_{X,t}$, evolves exogenously according to the following AR(1) process:

$$\log(p^*_{X,t}) = (1 - \rho_{p_X}) \log(p^*_X) + \rho_{p_X} \log(p^*_{X,t-1}) + \varepsilon_{p_X,t}, \tag{65}$$

where $p^*_X > 0$ is the steady-state value of the real commodity price, $\rho_{p_X} \in (-1, 1)$ is an autoregressive coefficient, and $\varepsilon_{p_X,t}$ is uncorrelated and normally distributed innovation with zero mean and standard deviations $\sigma_{p_X}$. As long as the small economy is a net commodity exporter, a positive shock to $p^*_{X,t}$ is a favourable shock to its terms of trade.

## 2.8 Government

It is assumed that government's revenues include lump-sum taxes, $\Upsilon_t$, and newly issued debt, $B_t/R_t$. The government uses its revenues to finance its spending, $P_tG_t$ and repay its debt, $B_{t-1}$. The government's budget is given by

$$P_tG_t + B_{t-1} = \Upsilon_t + B_t/R_t. \tag{66}$$

Government spending evolves exogenously according to the following process

$$\log(G_t) = (1 - \rho_G) \log(G) + \rho_G \log(G_{t-1}) + \varepsilon_{Gt}, \tag{67}$$

where $G$ is the steady-state value $G_t$, $\rho_G \in (-1, 1)$ is an autoregressive coefficient, and $\varepsilon_{Gt}$ is an uncorrelated and normally distributed innovation with zero mean and standard deviations $\sigma_G$.

## 2.9 Current account

Combining the household's budget constraint, government budget, and single-period profit functions of commodity producing firm, manufactured and non-tradable goods producing firms, and foreign goods importers yields the following current account equation in real terms, under the PCP assumption:

$$\frac{B^*_t}{\kappa_t R^*_t} = \frac{B^*_{t-1}}{\pi^*_t} + p^*_{X,t}\left(Y_{X,t} - Y^M_{X,t} - Y^N_{X,t}\right) + \frac{p_{M,t}}{S^{us}_t} Y^{ex}_{M,t} - [\alpha + (1 - \alpha)S^{rw}_t]Y_{F,t}. \tag{68}$$

### 2.10 Monetary authority

We assume that the monetary authority uses the short-term nominal interest rate, $R_t$, as its instrument. The corresponding interest-type monetary policy reaction function is given by:

$$\log\left(\frac{R_t}{R}\right) = \varrho_R \log\left(\frac{R_{t-1}}{R}\right) + \varrho_\pi \log\left(\frac{\pi_t}{\pi}\right) + \varrho_Y \log\left(\frac{Y_t}{Y}\right) + \varepsilon_{Rt}. \tag{69}$$

where $R$, $\pi$, and $Y$ are the steady-state values of $R_t$, $\pi_t$, and $Y_t$. Parameter $\varrho_R$ captures interest-rate smoothing while $\varrho_\pi$ and $\varrho_Y$ are the policy coefficients measuring the central bank's responses to deviations of inflation, $\pi_t$, and output, $Y_t$, from their steady-state values, respectively. It is further assumed that the error term $\varepsilon_{Rt}$ is a serially uncorrelated and normally distributed monetary policy shock with zero mean and standard deviations $\sigma_R$. Note that while the above policy reaction function is written in terms of CPI inflation, $\pi$, one of the objectives of this paper is precisely to determine whether this particular inflation rate is indeed the right rate to use. We shall modify this reaction function appropriately (in section 4) given our objective of determining the best inflation rate to target.

### 2.11 Symmetric equilibrium

In equilibrium, the final good is divided between consumption, $C_t$, private investment in the three production sectors, $I_t$, and government spending, $G_t$, so that $Z_t = C_t + I_t + G_t$, with $I_t = I_{M,t} + I_{N,t} + I_{X,t}$. We consider a symmetric equilibrium, in which all households, intermediate goods-producing firms, and importers make identical decisions. Therefore, $C_{ht} = C_t$, $H_{ht} = H_t$, $B_{ht} = B_t$, $B^*_{ht} = B^*_t$, $I_{i,ht} = I_{i,t}$, $\widetilde{w}_{i,ht} = \widetilde{w}_{i,t}$, $H_{i,ht} = H_{i,jt} = H_{i,t}$, $K_{i,ht} = K_{i,jt} = K_{i,t}$, $Y^X_{M,jt} = Y^X_{M,t}$, $Y^X_{N,jt} = Y^X_{N,t}$, $Y_{\iota,jt} = Y_{\iota,t}$, and $\widetilde{p}_{\iota,jt} = \widetilde{p}_{\iota,t}$, for all $h \in [0,1]$, $j \in [0,1]$, $i = N, M, F$. Furthermore, the market-clearing conditions $P_t G_t = \Upsilon_t$ and $B_t = 0$ must hold for all $t \geq 0$. In addition, since $\widetilde{p}_{ij,t} = \widetilde{p}_{i,t}$ and $Y_{ij,t} = Y_{i,t}$ for $i = N, M, F$, we can rewrite equations (34), (49), and (58) in a non-linear recursive form as:

$$\widetilde{p}_{i,t} = \frac{\theta}{\theta - 1}\frac{x^1_{i,t}}{x^2_{i,t}}, \tag{70}$$

where

$$x^1_{i,t} = \lambda_t Y_{i,t}\xi_{i,t} p^\theta_{i,t} + \beta\phi_i E_t\left[(\pi_{t+1}/\pi)^\theta x^1_{i,t+1}\right], \tag{71}$$

$$x^2_{i,t} = \lambda_t\, Y_{i,t} p^\theta_{i,t} + \beta\phi_i E_t\left[(\pi_{t+1}/\pi)^{\theta-1} x^2_{i,t+1}\right]. \tag{72}$$

with $\xi_{F,t} = S_t$.[13]

Finally, note that the manufacturing and non-tradable sectors use commodity goods as material inputs in production of $Y_{M,t}$ and $Y_{N,t}$, which are defined as gross output. The value-added output

[13] When log-linearizing the three sets of two equations formed by (34) and (35), (49) and (50), and (58) and (59) around the steady-state, standard New Keynesian Phillips curves are obtained as: $\hat{\pi}_{i,t} = \beta\hat{\pi}_{i,t+1} + \frac{(1-\beta\phi_i)(1-\phi_i)}{\phi_M}\hat{\xi}_{i,t}$, for $i = N, M.F$, where hats over the variables denote deviations from their steady-state values.

in each sector, $Y_{M,t}^{va}$ and $Y_{N,t}^{va}$, can be constructed by subtracting commodity inputs as follow: $Y_{i,t}^{va} = Y_{i,t} - S_t^{us} p_{X,t}^* Y_{X,t}^i / p_{i,t}$ for $i = M, N$. Hence, aggregate GDP is defined as:

$$Y_t = p_{M,t} Y_{M,t}^{va} + p_{N,t} Y_{N,t}^{va} + S_t^{us} p_{X,t}^* Y_{X,t}. \tag{73}$$

# 3 Calibration and estimation

We use the Dynare program and Canadian data, at a quarterly frequency, extending from 1981Q1 to 2007Q2, to estimate the non-calibrated structural parameters. Output in the commodity sector is measured by the total real production in primary industries (agriculture, fishing, forestry, and mining) and resource processing, which includes pulp and paper, wood products, primary metals, and petroleum and coal refining. The output of non-tradables includes construction, transportation and storage, communications, insurance, finance, real estate, community and personal services, and utilities. The manufactured goods are measured by the total real production in different manufacturing sectors in the Canadian economy. Real commodity prices are measured by deflating the nominal commodity prices (including energy and non-energy commodities) by the U.S. GDP deflator. The nominal interest rate is measured by the rate on Canadian three-month treasury bills. Government spending is measured by total real government purchases of goods and services. The real exchange rate is measured by multiplying the nominal USD/CAN exchange rate by the ratio of U.S. to Canadian prices. Foreign inflation is measured by changes in the U.S. GDP implicit price deflator. Foreign output is measured by U.S. real GDP per capita. The series of commodities, manufactured goods, non-tradables, and government spending are expressed in real terms and per capita using the Canadian population aged 15 and over. Finally, we obtain import and exchange rate data from the IFS from which we construct the US effective exchange rate.

We focus on the estimation of key sector-specific and monetary policy-related parameters, and calibrate the rest to capture salient features of the Canadian economy, given that some of the parameters are poorly-identifiable. Many of these values are taken from Dib (2008) and are fairly standard in the existing literature.[14] Table 1 displays steady-state values of selected variables used in the calibration and Table 2 reports the values of the calibrated parameters.

The discount factor, $\beta$, is set at 0.991, which implies an annual steady-state real interest rate of 4% that matches the average observed in the estimation sample. The curvature parameter in the utility function, $\tau$, is given a value of 2, implying an elasticity of intertemporal substitution of 0.5. Following Bouakez *et al.* (2009), we set both $\varsigma$ and $\chi$, the labour elasticity of substitution across sectors and the inverse of the elasticity of intertemporal substitution of labour, at unity. The capital depreciation rate, $\delta$, is assigned a value of 0.025; this value is commonly used in the literature and assumed to apply to the three production sectors.

[14] See also Dib (2006).

The shares of capital, labour, and natural resources in the production of commodities, $\alpha_X$, $\gamma_X$, and $\eta_X$ are assigned values of 0.41, 0.39, and 0.2, respectively. The shares of capital, labour, and commodity inputs in production of manufactured (non-tradable) goods, $\alpha_M$ ($\alpha_N$), $\gamma_M$ ($\gamma_N$), and $\eta_M$ ($\eta_N$) are set equal to 0.26 (0.28), 0.63 (0.66), and 0.11 (0.06), respectively. All these shares are taken from Macklem *et al.* (2000) who calculated these from Canadian 1996 input-output tables.[15]

The parameter $\theta$, which measures the degree of monopoly power in intermediate-goods markets, is set equal to 6, implying a steady-state price markup of 20%. Parameter, $\vartheta$, which measures the degree of monopoly power in labour markets, is set equal to 10. Parameter $\nu$, which captures the price-elasticity of demand for imports and domestic goods (and it is also the elasticity of substitution between imports, manufactured and non-tradable goods in the final good), is set equal to 0.6. The parameter $\omega_{ex}$ is a normalization that ensures the ratio of manufactured exports to GDP is equal to the one observed in the data, and is, therefore, set to 0.21. Parameters $\omega_F$ and $\omega_N$, which are associated with the shares of imports and non-tradable goods in the final good, are calibrated to match the average ratios observed in the data for the estimation period. We set these equal to 0.50, and 0.60, respectively.

In addition, households are assumed to allocate, on average, one third of their available time to market activities. Therefore, the steady-state hours worked, $H_M$, $H_N$, and $H_X$, are set equal to 0.07, 0.21, and 0.05, respectively. The steady-state stock of natural resources, $L$, and of technology levels in manufacturing and non-tradable sectors, $A_M$ and $A_N$, are assigned values to match the ratios of commodity, manufactured, and non-tradable goods in Canadian GDP. Finally, the steady-state level of the exogenous variables $p_X^*$ and $Y^*$ are set equal to unity.

The remaining parameters are estimated using Bayesian procedures as in Smets and Wouters (2007). Prior distributions are conjectured for the various parameters and that provide starting points for the optimization algorithm. Tables 3, 4, and 5 report the estimation outcomes, as well as the prior distributions assumed for the estimated parameters.

The estimation shows that, consistent with what we expected to find, prices and wages are stickiest in the non-tradable sector. Furthermore, capital adjustment costs in this sector are also the highest. In contrast, price stickiness is lower in manufacturing, and both capital adjustment costs and wage stickiness in the commodity sector are also estimated to be relatively lower.

We also find that technology shocks, particularly in the non-tradable sector, are quite persistent, and that both U.S./rest of the world real exchange rate and government spending shocks exhibit a fair bit of sluggishness as well. Foreign shocks, in contrast, are estimated to have smaller autoregressive coefficient values, and the estimated parameters of the monetary policy reaction function have values that put a bigger weight on inflation than on output, reflecting similar findings in

[15] Macklem et al. (2000) have obtained these shares from 1996 current-dollar input-output tables at the medium level of aggregation, which disaggregates input-output tables into 50 industries and 50 goods.

other studies for Canada. As for the standard deviations, we find the supply of natural resource and the commodity price to have the two highest estimated values, followed by manufacturing and government spending. In contrast, the volatilities of non-tradable goods and of the various foreign variables are estimated to be considerably lower. Interestingly, the standard deviation of the exogenous shock to the import share is estimated to be relatively important, with a value of 0.024.

### 3.1 Variance decompositions

In order to gauge the extent to which the different shocks in our model affect variables of interest, we proceed with a variance decomposition for these shocks and report the outcomes in Table 6. From this table, it is quite apparent that technology shocks, and, in particular, the technology shock impacting the non-tradable sector, $A_N$, are the principal drivers of the movements in most of the considered variables. In particular, $A_M$ explains 66 per cent and 81 per cent of movements in the inflation rate and output of the manufacturing sector, respectively, as well as 14 per cent and 30 per cent of the corresponding variables in the tradables sector. On the other hand, apart from explaining big shares of the changes in variables in the non-tradable sector, $A_N$ explains 68 per cent of consumption, 70 per cent of GDP, and 48 per cent of CPI inflation.

The other interesting fact that can be observed from the table is that the shock to import share $\omega_F$ also explains a substantial portion of movements in key variables. In particular, in addition to accounting for around 40 per cent of fluctuations in the output of imports $(Y_F)$ and in the import content of manufacturing $\left(Y_M^d\right)$, and 30 per cent of the output of tradables, it also explains 18 per cent of interest rate movements, and 34 per cent of CPI inflation. Interestingly, we found that shutting down these two shocks resulted in technology shocks taking on their explanatory role. In contrast, the shock to competitivity, as characterized by $S^{rw}$, turns out to have very little role in explaining fluctuations on our variables of interest. One possible reason for why this shock is not instrumental is that we fix $\alpha$, the parameter in equation (68). Allowing this parameter to fluctuate endogenously is likely to make this shock more effective.

## 4 Welfare analysis

We solve the model to a second-order approximation around its deterministic steady-state to analyze and compare the welfare implications of alternative optimized monetary policy reaction functions.[16] The alternative "rules" differ with respect to the sectoral price index or inflation rate targeted by the monetary authority. For $j = CPI, N, T, M, F$, we consider the following reaction functions

[16] We use the Dynare program, which relies on Sims' (2002) algorithm, to first obtain the model's solution to a second-order approximation around its deterministic steady state and, then, to calculate the theoretical first and second moments of the endogenous variables, including period utility. See Juillard (2002).

that feature inflation or price level-targeting:

- Inflation-targeting (IT) reaction function: $\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\varrho_R} \left(\frac{\pi_{j,t}}{\pi}\right)^{\varrho_\pi} \left(\frac{Y_t}{Y}\right)^{\varrho_y} \exp\left(\varepsilon_t^R\right)$,
- Price Level-Targeting (PLT) reaction function: $\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\varrho_R} \left(\frac{P_{j,t}}{\widetilde{P}_{j,t}}\right)^{\varrho_\pi} \left(\frac{Y_t}{Y}\right)^{\varrho_y} \exp\left(\varepsilon_t^R\right)$,

where $\widetilde{P}_{j,t} = \pi \widetilde{P}_{j,t-1}$ and policy alternatives indexed by $j = CPI$ refer to the agregate price level, $P_t$.

The procedure to conduct the welfare analysis is similar to the one used in Schmitt-Grohé and Uribe (2004) and Ambler *et al.* (2004). First, we compute a reference welfare measure as the unconditional expectation of lifetime utility in the deterministic steady state, in which all shocks are set to zero and there is no uncertainty. This measure is convenient for welfare comparisons because the deterministic steady-state is invariant across all policy regimes considered. Then, using the deterministic steady-state as the initial state, we run stochastic simulations of the model and compute the conditional expectation of lifetime utility under the different alternative policy reaction functions. Finally, we rank the alternative policies according to their welfare losses relative to the reference welfare measure.

To compute the welfare losses associated with a particular policy rule, we use the compensating variation in consumption. This measures the percentage change in consumption at the deterministic steady state that would give households the same conditional expected utility in the stochastic economy. Because the model is solved using a second-order approximation, the variance of the shocks affects the means and the variances of the endogenous variables in the stochastic equilibrium. The latter implies a permanent shift in the stochastic steady state level of consumption.

We can decompose the total welfare effect into level and variance effects, as follows. We first calculate a second-order Taylor expansion of the single-period utility function (1) around the deterministic steady-state values $C$ and $H$:

$$U\left(C_t, H_t\right) \approx U\left(C, H\right) + \Lambda_m\left(\widehat{C}_t, \widehat{H}_t\right) - \Lambda_v\left(\widehat{C}_t, \widehat{H}_t\right),$$

where $U\left(C,H\right) = \frac{C^{1-\tau}}{1-\tau} - \frac{H^{1+\chi}}{1+\chi}$ is the utility at the steady-state, and $\widehat{C}_t$ and $\widehat{H}_t$ are the log-deviations of $C_t$ and $H_t$ from their deterministic steady state values. Functions $\Lambda_m\left(\widehat{C}_t, \widehat{H}_t\right) \equiv C^{1-\tau}(\widehat{C}_t) - H^{1+\chi}(\widehat{H}_t)$ and $\Lambda_v\left(\widehat{C}_t, \widehat{H}_t\right) \equiv \frac{\tau}{2}C^{1-\tau}(\widehat{C}_t^2) + \frac{\chi}{2}H^{1+\chi}(\widehat{H}_t^2)$ are convenient to compute the effects of the variance of the shocks on the mean and the variance of the endogenous variables, respectively.

Second, let $U\left(\left(1+\lambda_m\right)C, H\right)$ be the utilty derived by the household when a permanent shift in steady state of consumption is originated from the level (mean) effect. The level effect, $\lambda_m$, is the percent increase in steady-state consumption, $C$, that makes the household as well-off under an alternative policy rule as under the reference measure given by the new steady-state. From $U\left(\left(1+\lambda_m\right)C,H\right)=U\left(C,H\right)+\Lambda_m\left(\widehat{C}_t,\widehat{H}_t\right)$, $\lambda_m$ is solved as:

$$\lambda_m=\left[1+(1-\tau)E(\widehat{C}_t)-(1-\tau)\frac{H^{1+\chi}}{C^{1-\tau}}E(\widehat{H}_t)\right]^{\frac{1}{1-\tau}}-1. \tag{74}$$

Similarly, let $\lambda_v$ denote the variance effect, which is the solution for $U\left(\left(1+\lambda_v\right)C,H\right)=U\left(C,H\right)-\Lambda_v\left(\widehat{C}_t,\widehat{H}_t\right)$:

$$\lambda_v=\left[1-\frac{\tau(1-\tau)}{2}E(\widehat{C}_t^2)-\frac{\chi(1-\tau)}{2}\frac{H^{1+\chi}}{C^{1-\tau}}E(\widehat{H}_t^2)\right]^{\frac{1}{1-\tau}}-1. \tag{75}$$

Third, for each alternative optimized policy regime, we use the parameter values from the calibration and estimation procedures, with the exception of the parameters in the monetary policy reaction function, for which we conduct a grid-search and select the combination of parameters that imply the lowest welfare loss relative to the reference (deterministic steady-state) welfare measure. For the smoothing coefficient, $\varrho_R$, we consider three cases: zero inertia ($\varrho_R=0$), historical inertia (based on the estimated parameter value, $\varrho_R=0.6754$), and high inertia ($\varrho_R\approx 1$). For the coefficients related to the policy reaction to both inflation ($\varrho_\pi$) and output $\left(\varrho_y\right)$, we consider an equally-spaced grid in the $[0,6]$ interval with incremental step equal to 0.05. This numeric approach follows Kollmann (2002), Ambler *et al.* (2004), Bergin *et al.* (2007), and Schmidt-Grohe-Uribe (2007).

Table 7 displays the results of the welfare comparisons. We consider the following alternative policy reaction functions (hereafter refered to as rules for simplicity): (1) the historical rule, which is the one using the estimated/calibrated coefficients discussed in Tables 1-5, (2) the five inflation-targeting rules, in which the monetary policy reacts to the CPI inflation or to sectoral inflation rates (non-tradable, tradable, manufacturing, and imported goods), and (3) three price level-target rules (CPI, non-tradables, and tradables). Columns 1-3 show the combination of parameters $\left(\varrho_R,\varrho_y,\varrho_\pi\right)$ used in the (optimized) monetary policy rule for each alternative; Columns 4-7 display the average levels of consumption, hours-worked, GDP, and the lifetime utility of households obtained from a stochastic simulation of the model; Columns 8-10 show the welfare gains measured as percentage of the deterministic steady-state consumption, decomposed into the gains due to the change in the average levels and variances of consumption and hours-worked. For convenience, we include (second row) the results for the case of flexible prices and wages under a strict CPI inflation targeting rule, as well as the deterministic steady state levels of key variables, including welfare (bottom row).

Note that the two dominant options among optimized rules are the targeting of the CPI level or inflation.[17] Targeting the price level or the inflation rate in the non-tradables sector is the best

[17] A complementary study to ours is that of Shukayev and Ueberfeldt (2010). Rather than focusing on sectoral inflation rates, they examine the welfare implications of targeting alternatively weighted CPI baskets for Canada

option among the alternatives involving only sectoral indices. Thus, abstracting from the economy-wide price index (i.e., the CPI), the results in Table 7 are consistent with results by Aoki (2001) and Benigno (2004) whereby a higher weight should be attached to the inflation rate in sectors with high degrees of nominal rigidities. In those studies the welfare losses are associated with the suboptimal output produced by monopolistically competitive firms facing nominal rigidities. If we consider an economy hit by shocks that have asymmetric effects across sectors, the monetary policy can influence the cross-sector allocation of resources by targeting a sectoral price index. In the absence of factor mobility costs, welfare losses due to nominal rigidities prevail. However, when it is costly to move resources across sectors, this second source of welfare losses must also be added to the effect of suboptimal output and of price dispersion.

Although not the main focus of this paper, Table 7 also provides some implications for a comparison between reacting to the alternatives based on the inflation rate versus those based on corresponding price levels. In particular, it shows that the optimized rules based on the reaction of the monetary authority to the CPI inflation rate and to the CPI level are virtually equivalent from a welfare perspective. This result can partly be explained by the fact that the optimized value for the smoothing parameter in the CPI-inflation-based rule is equal to the historical value (which is relatively high at approximately 0.68), whereas the optimized rule based on the CPI level shows no inertial behaviour. Because of the important inertia in the inflation-based rule, actions in the past matter for the current period and bygones are therefore not full bygones. As a result, the coefficient on the CPI inflation rate in the optimized rule is higher (at 2.45) than the coefficient on the CPI inflation level in the corresponding rule (at 1.10). Finally, we note that the welfare-maximizing ordering of the rules with various inflation rates is maintained for the corresponding level-based rules, thus emphasizing again the roles of the real and nominal rigidities discussed above.

Table 8, which displays the simulated second moments of selected variables under the alternative optimized rules, helps us understand the main results. Observe that targeting the inflation or the price level in the non-tradable sector reduces inflation volatilty in that sector but induces more volatility in the inflation rates of other sectors. That means additional volatility in relative prices. Compared to the rules that target the CPI, the volatilities of consumption, exchange rate, output, CPI inflation, and even non-tradable output are higher under rules that target the non-tradable inflation or price level. Since the economy seems to be less volatile in the case of rules that target an economy-wide index, rather that a sectoral index, more resources are lost in the transfer of production factors across sectors in the latter case.

within a different framework. They conclude that using the current CPI weights in the monetary policy reaction fucntion is nearly optimal, which is consistent with our results.

## 4.1 Impulse response analysis

Next, we examine the dynamic impact of some of the model's shocks under alternative monetary policy rules that differ with respect to the targeted inflation rate, including the estimated historical rule. Figures 1 to 5 display the impulse responses (in percent deviation from steady state) of selected variables. Figure 1 shows the quarterly responses to a one percent increase in the manufacturing-related technology shock, $A_M$. With a few exceptions, responses are qualitatively similar whichever inflation rate is targeted in the policy rule, though quantitatively, there are important differences for some variables.

Regardless of the monetary policy rule, a shock to $A_M$ produces an immediate increase in manufacturing output and a concurrent decline in prices in that sector. In all cases, the monetary authority will cut interest rates in response to the fall in the relevant inflation being targeted. The effects on the remaining sectors' output will depend on the overall wealth effect generated by the increase in manufacturing output vis-a-vis the substitution effect induced by the cheaper traded goods. With the exception of total output in the importing sector, $Y_{F,t}$, under the cases of $\pi_M$-targeting and $\pi_F$-targeting rules, the demand-driven output in all sectors increases, which explains the increase in inflation in the other sectors than manufacturing. The only exception is $\pi_N$ in the case of the estimated historical rule. In all cases, total output increases. Since part of the manufacturing output is exported, the increase in exports will require a depreciation of the real CAD/USD exchange rate to accomodate the higher level of exports and increase in the current account. The decline in interest rates, a reaction to the fall in inflation, also contributes to the depreciation of the real CAD/USD exchange rate.

Note that under a monetary policy rule targeting manufacturing-price inflation, the policy rate responds more sharply to accommodate the shock that occurs in that very sector. As the monetary authority does not care about other sectors' inflation rates, their increase dominates the fall in $\pi_M$ and the overall CPI inflation increases.

If, on the other hand, policy were targeting inflation in the non-tradables' sector, the positive technology shock to the manufacturing sector would result in an increase in manufacturing output, but would not lead to important changes in demand for non-tradables or its prices, with a very subdued policy reaction. Manufacturing goods being cheaper compared to imports, and the income effect not being strong enough, import demand would somewhat decline, so that there would be practically no change in the exchange rate. Accordingly, we would witness a very modest increase in consumption and output.

Finally, we see that with CPI inflation-targeting, responses would quantatively lie between their corresponding levels for the above two policy scenarios. That is, total output and consumption would increase a little, the exchange rate would somewhat depreciate, the policy rate would be reduced but by less than half its increase under the manufacturing-price inflation-targeting case, and headline inflation would decline just a small amount and for a short time.

Figure 2 shows the responses of variables to a one percent technology shock in the non-tradables sector. Given the high price stickiness in this sector, targeting the non-tadable goods' inflation rate produces subtantially bigger effects than would have been produced under any other inflation targeting rule. In particular, output of non-tradables jumps quite high immediately and remains robust for a long while. The substitution effect between tradables and non-tradables is weaker than the generated wealth effect, and manufacturing output also rises in the short run. Consumption and total output subsequently increase, remaining at fairly high levels for a considerable amount of time. Inflation rates in all but the non-tradable sector also increase in the short run. However, the decline in $\pi_N$ dominates and CPI inflation falls, triggering an interest rate cut, except in the case of CPI-inflation targeting, when it increases in reponse to a (slight) increase in total inflation. The wealth effect also causes imports to rise, and the increase in the price of tradables relative to that of non-tradables amounts to a depreciation in the exchange rate.

We now turn to the effect of an unexpected increase in the share of imports in tradables. Figure 3 shows the responses of various macroeconomic variables to a positive shock in this share (for example, following a change in quota). We see that the increase in imports crowds out manufacturing demand and output in the latter sector declines, except under the historical rule which increases, but only on impact. Inflation in that sector, also declines (except for the case where monetary policy targets $\pi_T$), while import price inflation rises accompanied by a depreciation of the exchange rate. The net effect on the tradable goods sector is an increase in output and a decline in prices. Given the complementary nature of tradable and non-tradable goods, output is somewhat also driven up in the latter sector. As a result, overall output rises while overall inflation falls, leading in a temporary increase in consumption in the domestic economy.

Interestingly, a shock to import price results in a somewhat different dynamics for the variables of interest. Figure 4 shows the effect of a negative one percent price shock on these variables. We first note that imports immediately increase while the inflation rate in that sector falls. The resulting income effect is usually strong enough that it dominates the sustitution effect between domestically-manufactured and imported goods, except for the $\pi_N$-targeting and $\pi_M$-targeting cases. Thus, manufacturing output also increases, helping to also raise output in non-tradables. Regardless of the targeted inflation rate in the policy rule, overall output increases, followed by a similar increase in consumption. The decline in import prices is enough to generate even a small decline in overall inflation in the domestic economy, except when the targeted inflation in the policy rule is that of import prices that is deliberately countered by a substantial decline in interest rates.

Finally, Figure 5 displays how the asymmetric effects of monetary policy change depending on the different policy rules considered.

### 4.2 The role of factor mobility costs

To test our conjecture that costly resource realocation produces welfare losses that ultimately determine the choice of CPI inflation as the best option to target, we shut down the factor mobility costs in the model and conduct the same welfare analysis exercise as above. First, we consider a case with no labour mobillity costs ($\varsigma \to \infty$) and equal capital adjustment costs across sectors $\left(\psi_M = \psi_N = \psi_X = \frac{\psi_M+\psi_N+\psi_X}{3}\right)$. Second, we also shut down all sectoral capital adjustment costs ($\psi_M = \psi_N = \psi_X \approx 0$). The results shown in Table 9 indicate that the optimal CPI inflation rule still gives a higher welfare than the optimal non-tradable inflation rule when only labour is perfect mobile across sectors (case 1), but the reverse result prevails when all costs, including capital adjustment costs, are set to zero (case 2). As the non-tradable sector has the highest share in Canadian output and reveals the highest degree of nominal and real rigidities, for the targeting of the non-tradable inflation to be the best choice it suffices to set the adjustment cost of capital in that sector to zero (case 3).

From the above panels, we can conclude that capital mobility costs play an important role in the choice of the best inflation rate to include in the reaction function. To explore this issue in more detail, we conduct a set of additional experiments. Figures 6 and 7 show the comparisons between impulse responses of key variables to the two technology shocks for the case of CPI-inflation targeting with and without capital adjustment costs in the non-tradable sector. Notice that, for inovations in both types of technology shocks, setting $\psi_N = 0$ generally implies a stronger response of output and inflation and a weaker response of consumption, relative to the baseline case. Since non-tradable output represents a large share (60%) of the final good aggregator, when capital adjustment costs are shut down in this sector, aggregate investment becomes more volatile inducing higher volatility in the return on capital. The increased volatility of capital revenue earned by the households translates into more uncertainty in their total income. In turn, this induces precautionary savings and, thus, less consumption. From Table 9, also observe that the dampening effect of the higher income volatility on the level of consumption explains most of the positive difference in welfare between targeting inflation in the baseline case relative to case 3, for which the best option becomes reacting to the inflation of non-tradable goods. This shows that factor mobility costs, specially in the form of sector-specific capital adjustment costs, play a paramount role in the choice of the welfare maximizing monetary policy rule.

## 5 Conclusion

This paper proposed and estimated a structural multi-sector small open economy model for Canada to study whether targeting the CPI inflation rate, as many central banks around the world do, is the best option compared to targeting other inflation rates. We focused in particular on (i) the role of sector-specific real rigidities, specially in the form of factor mobility costs, and (ii) welfare implications of targeting alternative price indices having estimated key parameter values with data from Canada, a well-established inflation-targeting economy. The model features multiple sectors that allow for heterogeneity along three dimentions: price- and wage-rigidities, capital adjustment costs, and idiosyncratic technology shocks. Our estimations showed considerable heterogeneity across sectors. Comparing welfare implications of alternative monetary policy rules, where these differ with respect to the price index or the inflation rate targeted by the monetary authority, we found that maximal welfare is achieved by targeting CPI level or inflation. Capital mobility costs matter importantly in this regard, since shutting down these costs generates a different optimal targeting choice for the monetary authority.

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# Diagram 1: The Production Structure

Final Goods: $$Z_t = \left[ (\omega_N)^{\frac{1}{\nu}} (Y_{N,t})^{\frac{\nu-1}{\nu}} + (1-\omega_N)^{\frac{1}{\nu}} (Y_{T,t})^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}$$

$\mu > \nu$

Non-tradable goods

$$Y_{N,t} = \left[ \int_0^1 (Y_{Nj,t})^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

Tradable goods

$$Y_{T,t} = \left[ (\omega_{F,t})^{\frac{1}{\mu}} (Y_{F,t})^{\frac{\mu-1}{\mu}} + (1-\omega_{F,t})^{\frac{1}{\mu}} (Y^d_{M,t})^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}$$

Imported goods

$$Y_{F,t} = \left[ \int_0^1 (Y_{Fj,t})^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

Manufactured goods (used domestically)

$$Y^d_{M,t} = \left[ \int_0^1 (Y_{Mj,t})^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - Y^{ex}_{M,t}$$

$H_i, K_i, Y^i_X$

$H_X, K_X, L$

**Table 1: Steady-State Values**

| Variable | Definition | Values |
|---|---|---|
| $R$ | gross domestic nominal rate | 1.0185 |
| $\pi$ | gross domestic inflation rate | 1.0085 |
| $R^*$ | gross foreign nominal interest rate | 1.0158 |
| $\pi^*$ | gross foreign inflation rate | 1.0070 |
| $H_X$ | labour supply in commodity sector | 0.05 |
| $H_M$ | labour supply in manufacturing sector | 0.07 |
| $H_N$ | labour supply in nontradable sector | 0.21 |
| $L$ | supply of natural resources | 0.1 |
| $Y^*$ | world output | 1.0 |
| $A_M$ | manufacturing sector total factor productivity | 1.0 |
| $A_N$ | non-tradable sector total factor productivity | 1.0 |

**Table 2: Calibration**

| Parameters | Definition | Values |
|---|---|---|
| $\beta$ | discount factor | 0.991 |
| $\tau$ | inverse intertemporal elasticity of consumption | 2 |
| $\varsigma$ | inverse labour elasticity of substitution across sectors | 1 |
| $\chi$ | intertemporal elasticity of labour | 1 |
| $\delta$ | capital depreciation rate | 0.025 |
| $\alpha_X$ | share of capital in commodity output | 0.41 |
| $\gamma_X$ | share of labour in commodity output | 0.39 |
| $\eta_X$ | share of natural resources in commodity output | 0.20 |
| $\alpha_M$ | share of capital in manufactured goods | 0.26 |
| $\gamma_M$ | share of labour in manufactured goods | 0.63 |
| $\eta_M$ | share of commodity inputs in manufactured goods | 0.11 |
| $\alpha_N$ | share of capital in non-tradable goods | 0.28 |
| $\gamma_N$ | share of labour in non-tradable goods | 0.66 |
| $\eta_N$ | share of commodity inputs in non-tradable goods | 0.06 |
| $\omega_F$ | share of imports in the tradable goods at the steady state | 0.50 |
| $\omega_N$ | share of non-tradables in the final goods | 0.60 |
| $\alpha$ | share of imports from the US in total imports | 0.62 |
| $\theta$ | intermediate-goods elasticity of substitution | 6 |
| $\vartheta$ | elasticity of substitution within each sector's labour aggregator | 10 |
| $\nu$ | elasticity of substitution between tradable and non-tradable goods | 0.6 |
| $\mu$ | elasticity of substitution between domestically-manufactured and foreign goods | 1.2 |
| $\kappa$ | constant associated with risk premium | 0.0115 |
| $\omega_{ex}$ | constant associated with the share of exports in home GDP | 0.21 |

Table 3: **Results from Posterior Maximization - Structural Parameters**

| | Parameter | Prior Distribution | | | Posterior Distribution | |
|---|---|---|---|---|---|---|
| | | Density | Mean | Std Dev | Mode | 95% Conf. Interval |
| $\varrho_R$ | mon. policy, smoothing | beta | 0.60 | 0.06 | 0.68 | [0.59, 0.72] |
| $\varrho_\pi$ | mon. policy, inflation | gamma | 0.50 | 0.05 | 0.67 | [0.62, 0.80] |
| $\varrho_Y$ | mon. policy, output | normal | 0.10 | 0.01 | 0.06 | [0.04, 0.08] |
| $\phi_M$ | Calvo price, manufacturing | beta | 0.67 | 0.05 | 0.45 | [0.40, 0.51] |
| $\phi_N$ | Calvo price, nontradables | beta | 0.67 | 0.05 | 0.85 | [0.79, 0.92] |
| $\phi_F$ | Calvo price, imports | beta | 0.67 | 0.05 | 0.59 | [0.53, 0.65] |
| $\varphi_M$ | Calvo wage, manufacturing | beta | 0.67 | 0.05 | 0.59 | [0.49, 0.68] |
| $\varphi_N$ | Calvo wage, nontradables | beta | 0.67 | 0.05 | 0.68 | [0.62, 0.73] |
| $\varphi_X$ | Calvo wage, commodity | beta | 0.67 | 0.05 | 0.49 | [0.41, 0.55] |
| $\psi_M$ | $K$ adj. cost, manufacturing | gamma | 15.0 | 5.00 | 17.8 | [12.2, 22.7] |
| $\psi_N$ | $K$ adj. cost, nontradables | gamma | 15.0 | 5.00 | 24.5 | [18.1, 29.6] |
| $\psi_X$ | $K$ adj. cost, commodity | gamma | 15.0 | 5.00 | 16.6 | [11.5, 21.4] |

Table 4: **Results from Posterior Maximization - AR(1) Coefficients of shocks**

| | Parameter | Prior Distribution | | Posterior Distribution | |
|---|---|---|---|---|---|
| | | Density | Mean | Mode | 95% Conf. Interval |
| $\rho_M$ | technology, manufacturing | beta | 0.80 | 0.72 | [0.65, 0.79] |
| $\rho_N$ | technology, non-tradables | beta | 0.80 | 0.96 | [0.95, 0.97] |
| $\rho_G$ | government spending | beta | 0.80 | 0.84 | [0.81, 0.87] |
| $\rho_{R^*}$ | foreign interest rate | beta | 0.60 | 0.61 | [0.58, 0.64] |
| $\rho_{Y^*}$ | foreign output | beta | 0.60 | 0.63 | [0.58, 0.70] |
| $\rho_{\pi^*}$ | foreign inflation | beta | 0.60 | 0.63 | [0.61, 0.66] |
| $\rho_{p_x}$ | commodity price | beta | 0.60 | 0.76 | [0.69 0.82] |
| $\rho_L$ | natural resources | beta | 0.60 | 0.64 | [0.52, 0.74] |
| $\rho_{rw}$ | rest of the world REER | beta | 0.80 | 0.84 | [0.76, 0.92] |
| $\rho_F$ | import share | beta | 0.80 | 0.80 | [0.75, 0.85] |

Table 5: **Results from Posterior Maximization - Standard Deviations of Shocks**

| | Parameter | Prior Distribution | | Posterior Distribution | |
|---|---|---|---|---|---|
| | | Density | Mean | Mode | 95% Conf. Interval |
| $\sigma_M$ | tech., manufacturing | inv. gamma | 0.01 | 0.022 | [0.008, 0.026] |
| $\sigma_N$ | tech., nontradables | inv. gamma | 0.01 | 0.029 | [0.027, 0.031] |
| $\sigma_G$ | gov't spending | inv. gamma | 0.01 | 0.030 | [0.025, 0.032] |
| $\sigma_R$ | mon. policy | inv. gamma | 0.01 | 0.003 | [0.001, 0.005] |
| $\sigma_{R^*}$ | foreign interest rate | inv. gamma | 0.01 | 0.065 | [0.008, 0.011] |
| $\sigma_{Y^*}$ | foreign output | inv. gamma | 0.01 | 0.062 | [0.006, 0.013] |
| $\sigma_{\pi^*}$ | foreign inflation | inv. gamma | 0.01 | 0.003 | [0.002, 0.004] |
| $\sigma_{p_x}$ | commodity price | inv. gamma | 0.01 | 0.043 | [0.039, 0.049] |
| $\sigma_L$ | natural resources | inv. gamma | 0.01 | 0.076 | [0.068, 0.087] |
| $\sigma_{rw}$ | rest of the world REER | inv. gamma | 0.01 | 0.012 | [0.011, 0.014] |
| $\sigma_F$ | import share | inv. gamma | 0.01 | 0.024 | [0.019, 0.027] |

**Table 6: Variance Decompositions (%) - Historical Rule**

| | $A_M$ | $A_N$ | $G$ | $p_X$ | $L$ | $\varepsilon_R$ | $R^*$ | $\pi^*$ | $Y^*$ | $\omega_F$ | $S^{rw}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ | 5.6 | 68.3 | 17.7 | 1.5 | 0.1 | 1.4 | 1.5 | 0.1 | 0.0 | 3.7 | 0.1 |
| $H$ | 0.9 | 81.9 | 12.6 | 1.2 | 0.0 | 2.4 | 0.2 | 0.0 | 0.0 | 0.8 | 0.0 |
| $S^{us}$ | 2.1 | 50.1 | 2.0 | 11.8 | 0.6 | 2.4 | 20.9 | 1.4 | 0.2 | 8.7 | 0.1 |
| $Y$ | 8.7 | 70.1 | 9.1 | 5.7 | 0.9 | 2.7 | 0.7 | 0.1 | 0.0 | 1.8 | 0.1 |
| $R$ | 8.6 | 58.3 | 0.2 | 0.2 | 0.0 | 8.1 | 5.8 | 0.4 | 0.0 | 18.1 | 0.2 |
| $\pi$ | 10.6 | 47.5 | 1.1 | 0.2 | 0.1 | 1.9 | 4.2 | 0.3 | 0.0 | 34.0 | 0.2 |
| $\pi_N$ | 3.8 | 91.0 | 1.1 | 0.1 | 0.0 | 0.7 | 0.8 | 0.0 | 0.0 | 2.5 | 0.1 |
| $\pi_T$ | 13.7 | 16.3 | 0.5 | 0.5 | 0.1 | 1.8 | 6.5 | 0.4 | 0.0 | 60.0 | 0.3 |
| $\pi_M$ | 66.0 | 19.8 | 0.5 | 0.8 | 0.1 | 2.3 | 3.3 | 0.2 | 0.1 | 6.7 | 0.0 |
| $\pi_F$ | 4.7 | 44.1 | 1.5 | 5.9 | 0.4 | 4.6 | 30.9 | 2.0 | 0.1 | 3.1 | 2.8 |
| $Z$ | 5.4 | 67.6 | 15.7 | 0.8 | 0.1 | 2.4 | 2.2 | 0.1 | 0.0 | 5.7 | 0.1 |
| $Y_N$ | 0.5 | 87.1 | 8.7 | 0.3 | 0.0 | 2.0 | 0.5 | 0.0 | 0.0 | 0.9 | 0.0 |
| $Y_T$ | 30.1 | 8.1 | 19.2 | 5.0 | 0.4 | 1.3 | 7.4 | 0.4 | 0.1 | 27.4 | 0.6 |
| $Y_M$ | 80.7 | 0.7 | 3.0 | 4.1 | 0.1 | 0.7 | 1.5 | 0.1 | 0.2 | 9.0 | 0.0 |
| $Y_M^d$ | 52.9 | 1.1 | 5.0 | 0.7 | 0.0 | 0.3 | 0.5 | 0.0 | 0.1 | 39.4 | 0.0 |
| $Y_M^{ex}$ | 64.0 | 1.2 | 1.2 | 9.5 | 0.3 | 0.8 | 11.8 | 0.8 | 1.7 | 8.7 | 0.0 |
| $Y_F$ | 5.6 | 8.4 | 13.5 | 14.2 | 0.8 | 1.1 | 10.0 | 0.6 | 0.3 | 43.6 | 2.0 |

**Table 7: Welfare Implications of Alternative Monetary Policy Rules**

| Rule | $Y_t$ | $\pi_t$ | Average levels $C_t$ | $H_t$ | $Y_t$ | $u_t$ | Welfare Gain (% of SS $C_t$) Level | Variance | Total |
|---|---|---|---|---|---|---|---|---|---|
| **Historical rule** | 0.061 | 0.672 | 0.4377 | 0.2293 | 0.7083 | -2.3128 | -0.63 | -0.09 | -0.72 |
| **Flex-Prices** | 0.000 | 6.000 | 0.4406 | 0.2274 | 0.7111 | -2.2989 | 0.02 | -0.14 | -0.11 |
| **CPI Inflation Targeting** | | | | | | | | | |
| No inertia: | 0.000 | 6.000 | 0.4402 | 0.2287 | 0.7115 | -2.3007 | -0.07 | -0.13 | -0.19 |
| Hist. inertia: | 0.000 | 2.450 | 0.4404 | 0.2286 | 0.7116 | -2.2998 | -0.03 | -0.12 | -0.15 |
| High inertia: | 0.000 | 1.950 | 0.4403 | 0.2287 | 0.7117 | -2.2999 | -0.04 | -0.12 | -0.16 |
| **Nontradable Inflation Targeting** | | | | | | | | | |
| No inertia: | 0.000 | 4.200 | 0.4398 | 0.2288 | 0.7115 | -2.3037 | -0.17 | -0.16 | -0.33 |
| Hist. inertia: | 0.000 | 1.250 | 0.4400 | 0.2286 | 0.7115 | -2.3024 | -0.12 | -0.15 | -0.27 |
| High inertia: | 0.050 | 1.175 | 0.4400 | 0.2284 | 0.7115 | -2.3019 | -0.10 | -0.14 | -0.25 |
| **Imported Goods Inflation Targeting** | | | | | | | | | |
| No inertia: | 0.025 | 6.000 | 0.4384 | 0.2294 | 0.7088 | -2.3096 | -0.49 | -0.09 | -0.58 |
| Hist. inertia: | 0.000 | 2.350 | 0.4384 | 0.2293 | 0.7089 | -2.3092 | -0.48 | -0.08 | -0.56 |
| High inertia: | 0.000 | 1.525 | 0.4384 | 0.2293 | 0.7089 | -2.3090 | -0.47 | -0.08 | -0.56 |
| **Tradable Inflation Targeting** | | | | | | | | | |
| No inertia: | 0.100 | 5.725 | 0.4380 | 0.2304 | 0.7092 | -2.3125 | -0.58 | -0.13 | -0.71 |
| Hist. inertia: | 0.000 | 1.475 | 0.4384 | 0.2300 | 0.7093 | -2.3100 | -0.50 | -0.10 | -0.60 |
| High inertia: | 0.025 | 0.900 | 0.4384 | 0.2300 | 0.7093 | -2.3097 | -0.49 | -0.10 | -0.59 |
| **Manufacturing Inflation Targeting** | | | | | | | | | |
| No inertia: | 0.075 | 4.075 | 0.4375 | 0.2305 | 0.7083 | -2.3151 | -0.71 | -0.11 | -0.82 |
| Hist. inertia: | 0.000 | 0.950 | 0.4376 | 0.2303 | 0.7083 | -2.3139 | -0.67 | -0.10 | -0.77 |
| High inertia: | 0.025 | 0.625 | 0.4376 | 0.2303 | 0.7083 | -2.3141 | -0.68 | -0.10 | -0.78 |
| **Consumer Price Level Targeting** | | | | | | | | | |
| No inertia: | 0.000 | 1.100 | 0.4403 | 0.2285 | 0.7115 | -2.2998 | -0.04 | -0.11 | -0.15 |
| Hist. inertia: | 0.000 | 0.575 | 0.4403 | 0.2286 | 0.7115 | -2.3000 | -0.05 | -0.11 | -0.16 |
| High inertia: | 0.000 | 0.625 | 0.4402 | 0.2289 | 0.7115 | -2.3006 | -0.07 | -0.12 | -0.19 |
| **Nontradable Price Level Targeting** | | | | | | | | | |
| No inertia: | 0.075 | 1.100 | 0.4402 | 0.2284 | 0.7116 | -2.3013 | -0.08 | -0.14 | -0.22 |
| Hist. inertia: | 0.100 | 0.450 | 0.4402 | 0.2282 | 0.7114 | -2.3008 | -0.07 | -0.13 | -0.20 |
| High inertia: | 0.175 | 0.700 | 0.4401 | 0.2282 | 0.7112 | -2.3014 | -0.10 | -0.13 | -0.22 |
| **Tradable Price Level Targeting** | | | | | | | | | |
| No inertia: | 0.000 | 1.025 | 0.4385 | 0.2300 | 0.7094 | -2.3096 | -0.48 | -0.10 | -0.58 |
| Hist. inertia: | 0.000 | 0.350 | 0.4384 | 0.2300 | 0.7093 | -2.3098 | -0.49 | -0.10 | -0.59 |
| High inertia: | 0.000 | 0.150 | 0.4382 | 0.2303 | 0.7091 | -2.3109 | -0.53 | -0.10 | -0.64 |
| **Deterministic Steady-state:** | | | 0.4404 | 0.2269 | 0.7089 | -2.2962 | | | |

**Coefficients on $R_{t-1}$: No inertia = 0; Historical inertia = 0.6754; High inertia = 1**

Table 8: Simulation Results ($2^{nd}$ Order Approximation) - Standard Deviations

| | Hist. | CPI | $\pi_N$ | $\pi_F$ | $\pi_T$ | $\pi_M$ | $PLT^{CPI}$ | $PLT^{N}$ | $PLT^{T}$ |
|---|---|---|---|---|---|---|---|---|---|
| $C$ | 0.0131 | **0.0152** | **0.0169** | 0.0128 | 0.0143 | 0.0141 | **0.0149** | **0.0157** | 0.0140 |
| $H$ | 0.0115 | 0.0101 | 0.0065 | 0.0136 | 0.0148 | 0.0153 | 0.0101 | 0.0072 | 0.0147 |
| $S^{us}$ | 0.0200 | **0.0230** | **0.0276** | 0.0152 | 0.0212 | 0.0216 | **0.0228** | **0.0250** | 0.0209 |
| $Y$ | 0.0215 | **0.0264** | **0.0307** | 0.0213 | 0.0252 | 0.0253 | **0.0258** | **0.0268** | 0.0244 |
| $R$ | 0.0100 | 0.0121 | 0.0101 | 0.0078 | 0.0159 | 0.0103 | 0.0068 | 0.0061 | 0.0079 |
| $\pi$ | 0.0090 | **0.0040** | **0.0081** | 0.0075 | 0.0057 | 0.0074 | **0.0042** | **0.0073** | 0.0058 |
| $\pi_N$ | 0.0086 | 0.0041 | 0.0031 | 0.0070 | 0.0075 | 0.0081 | 0.0043 | 0.0024 | 0.0075 |
| $\pi_T$ | 0.0176 | **0.0134** | **0.0227** | 0.0162 | 0.0092 | 0.0133 | **0.0137** | **0.0210** | 0.0100 |
| $\pi_M$ | 0.0169 | **0.0137** | **0.0228** | 0.0153 | 0.0101 | 0.0063 | **0.0135** | **0.0208** | 0.0099 |
| $\pi_F$ | 0.0102 | **0.0095** | **0.0155** | 0.0021 | 0.0086 | 0.0107 | **0.0094** | **0.0130** | 0.0089 |
| $Z$ | 0.0210 | **0.0266** | **0.0295** | 0.0218 | 0.0257 | 0.0252 | **0.0260** | **0.0265** | 0.0251 |
| $Y_N$ | 0.0153 | **0.0185** | **0.0218** | 0.0151 | 0.0164 | 0.0164 | **0.0181** | **0.0201** | 0.0161 |
| $Y_T$ | 0.0094 | 0.0110 | 0.0103 | 0.0098 | 0.0118 | 0.0112 | 0.0107 | 0.0094 | 0.0114 |
| $Y_M$ | 0.0075 | **0.0075** | **0.0079** | 0.0075 | 0.0077 | 0.0084 | **0.0075** | **0.0076** | 0.0076 |
| $Y_M^d$ | 0.0052 | **0.0052** | **0.0054** | 0.0054 | 0.0051 | 0.0055 | **0.0052** | **0.0053** | 0.0052 |
| $Y_M^{ex}$ | 0.0036 | 0.0038 | 0.0038 | 0.0033 | 0.0040 | 0.0041 | 0.0038 | 0.0036 | 0.0039 |
| $Y_F$ | 0.0103 | 0.0117 | 0.0111 | 0.0107 | 0.0124 | 0.0110 | 0.0115 | 0.0104 | 0.0121 |

Table 9: The Role of Capital Adjustment Costs

| Rule | | | | Average levels | | | | Welfare Gain (% of SS $C_t$) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $C_t$ | $H_t$ | $Y_t$ | $u_t$ | Level | Variance | Total |
| | $R_{t-1}$ | $Y_t$ | $\pi_t$ | | | | | | | |
| **Historical rule** | 0.675 | 0.061 | 0.672 | 0.4377 | 0.2293 | 0.7083 | -2.3128 | -0.63 | -0.09 | -0.72 |
| **Flex-Prices** | | | | | | | | | | |
| **$\pi$ - targeting** | 0.000 | 0.000 | 6.000 | 0.4406 | 0.2274 | 0.7111 | -2.2989 | 0.02 | -0.14 | -0.11 |
| **$\pi_N$ - targeting** | 0.000 | 0.000 | 6.000 | 0.4398 | 0.2289 | 0.7116 | -2.3041 | -0.17 | -0.17 | -0.34 |
| **Baseline: Imperfect labour mobility ($\varsigma$ = 1) and sector-specific $K$-adj. costs** | | | | | | | | | | |
| **$\pi$ - targeting** | 0.675 | 0.000 | 6.000 | 0.4402 | 0.2287 | 0.7115 | -2.3007 | -0.07 | -0.13 | -0.19 |
| **$\pi_N$ - targeting** | 1.000 | 0.050 | 1.175 | 0.4400 | 0.2284 | 0.7115 | -2.3019 | -0.10 | -0.14 | -0.25 |
| **Case 1: Perfect labour mobility ($\varsigma$ = 10000) and common sectoral K-adj. costs ($\psi_N = \psi_M = \psi_X$ = 19.6)** | | | | | | | | | | |
| **$\pi$ - targeting** | 0.675 | 0.000 | 2.775 | 0.4405 | 0.3318 | 0.7117 | -2.3280 | -0.01 | -0.12 | -0.13 |
| **$\pi_N$ - targeting** | 1.000 | 0.050 | 1.250 | 0.4401 | 0.3324 | 0.7116 | -2.3310 | -0.11 | -0.15 | -0.26 |
| **Case 2: Perfect labour mobility ($\varsigma$ = 10000) and low sectoral K-adj. costs ($\psi_N = \psi_M = \psi_X$ = 0.02)** | | | | | | | | | | |
| **$\pi$ - targeting** | 0.675 | 0.400 | 6.000 | 0.4397 | 0.3322 | 0.7109 | -2.3323 | -0.20 | -0.11 | -0.31 |
| **$\pi_N$ - targeting** | 1.000 | 0.250 | 6.000 | 0.4409 | 0.3316 | 0.7127 | -2.3263 | 0.09 | -0.14 | -0.05 |
| **Case 3: Imperfect labour mobility ($\varsigma$ = 1) with low K-adj. cost only in non-tradables ($\psi_N$ = 0.02)** | | | | | | | | | | |
| **$\pi$ - targeting** | 0.675 | 0.500 | 6.000 | 0.4389 | 0.2291 | 0.7101 | -2.3071 | -0.36 | -0.10 | -0.46 |
| **$\pi_N$ - targeting** | 1.000 | 0.350 | 6.000 | 0.4403 | 0.2285 | 0.7120 | -2.3010 | -0.05 | -0.15 | -0.20 |
| **Deterministic Steady-state:** | | | | 0.4404 | 0.2269 | 0.7089 | | | | |

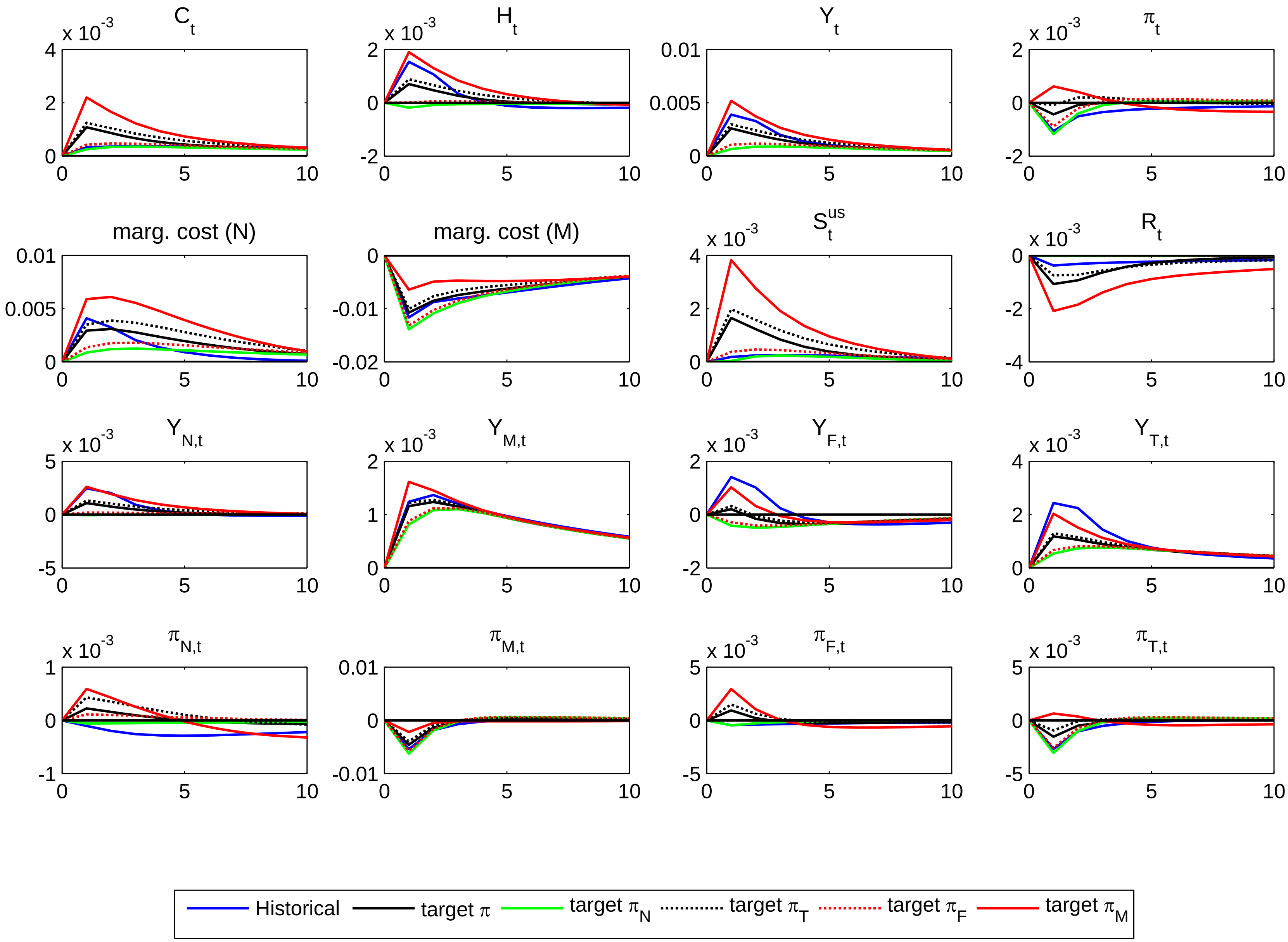


Figure 1: Responses to a One Percent Technology Shock in Manufacturing

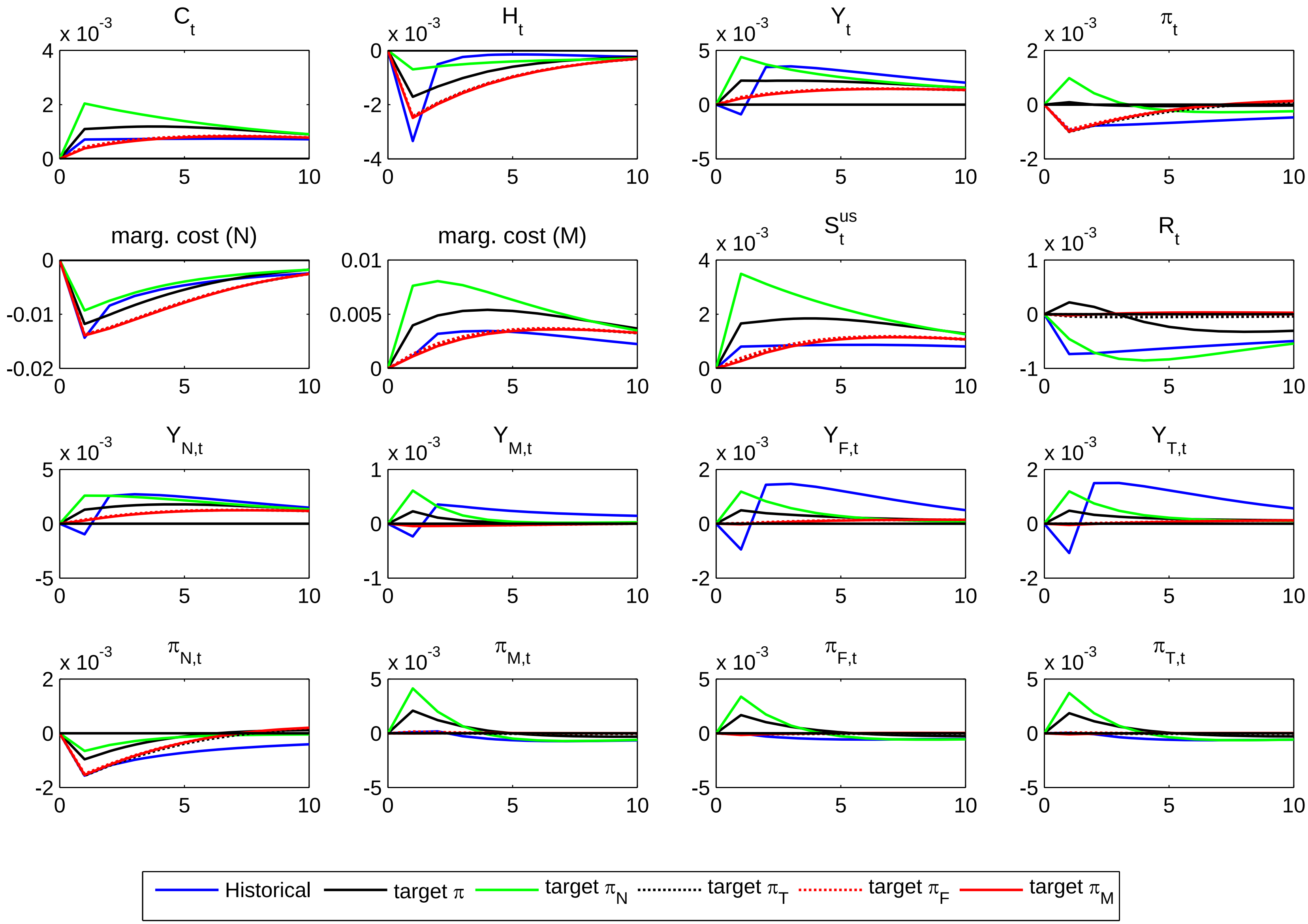


Figure 2: Responses to a One Percent Technology Shock in Non-tradables

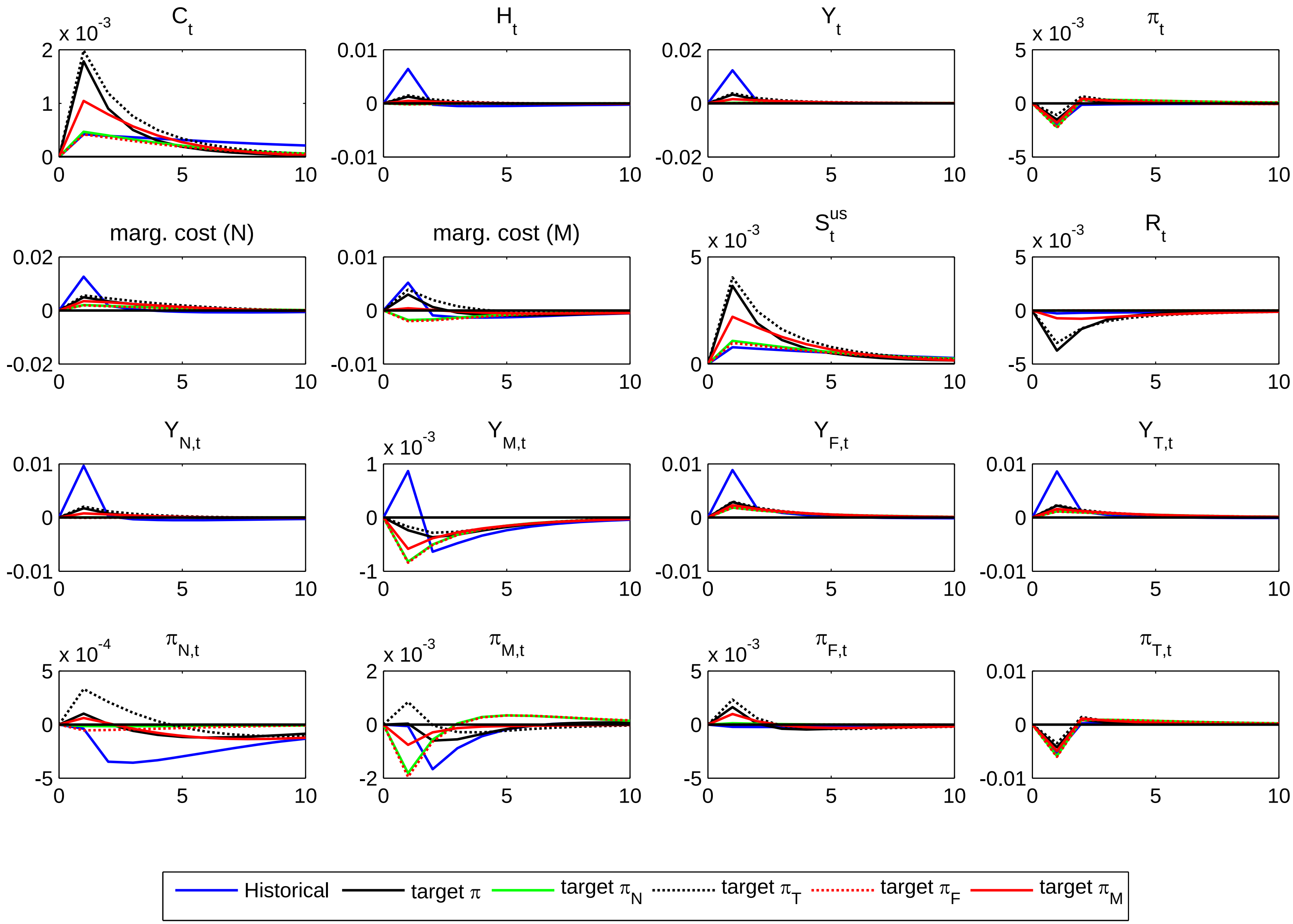


Figure 3: Responses to a One Percent Point Increase in the Share of Imports in Tradables

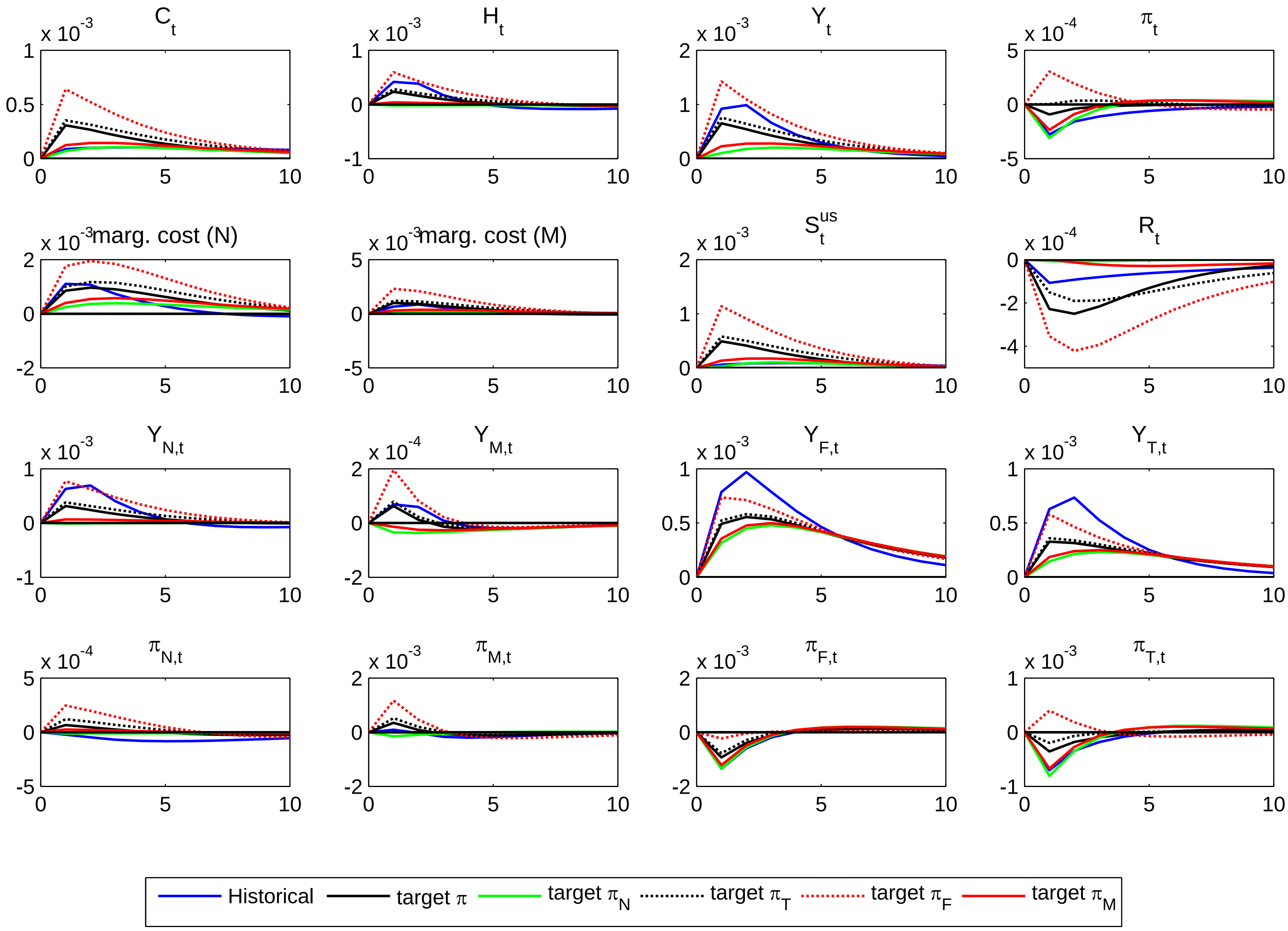


Figure 4: Responses to a Negative One Percent Shock in the USD/RW REER

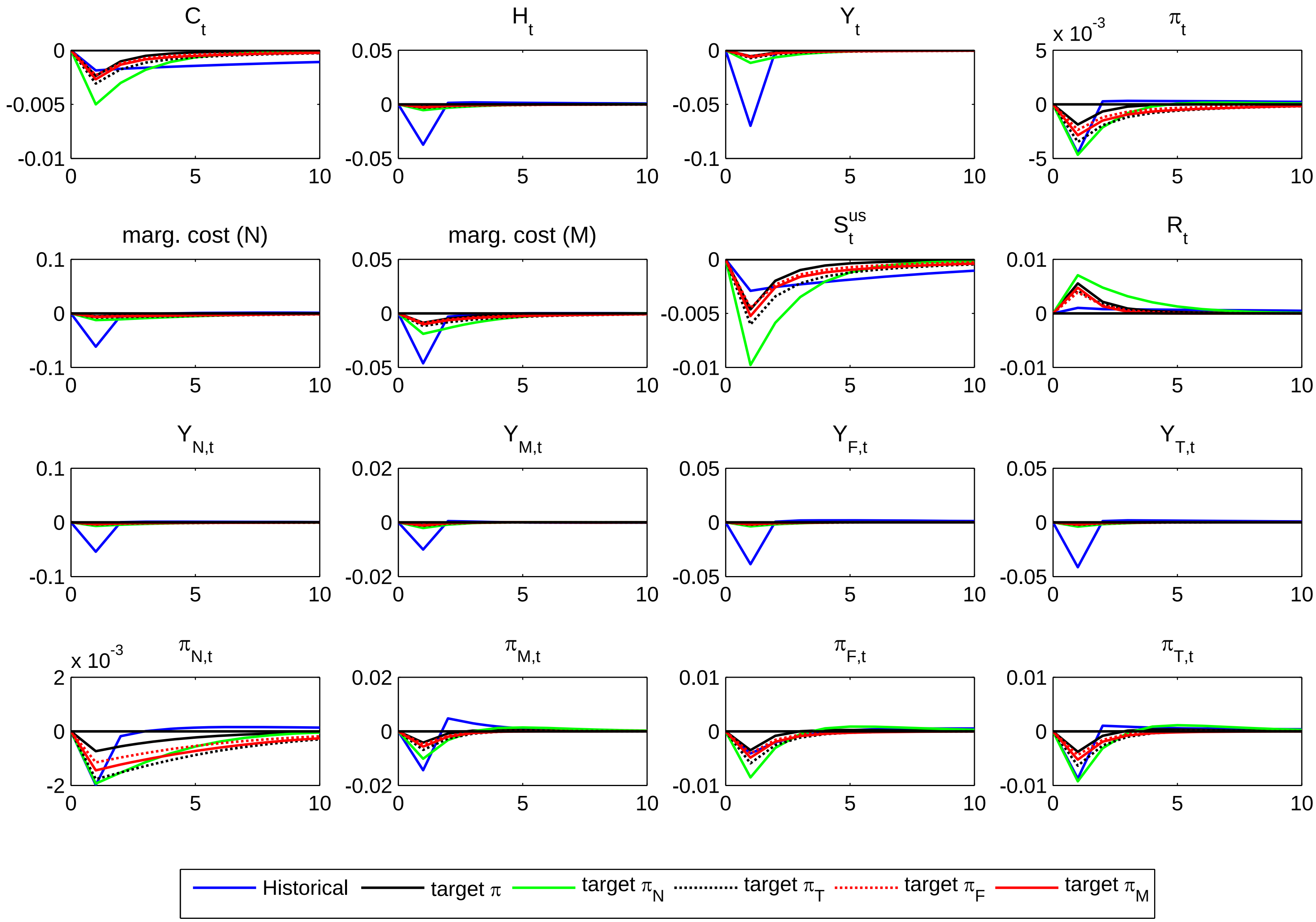


Figure 5: Responses to a One Percent Monetary Policy Shock

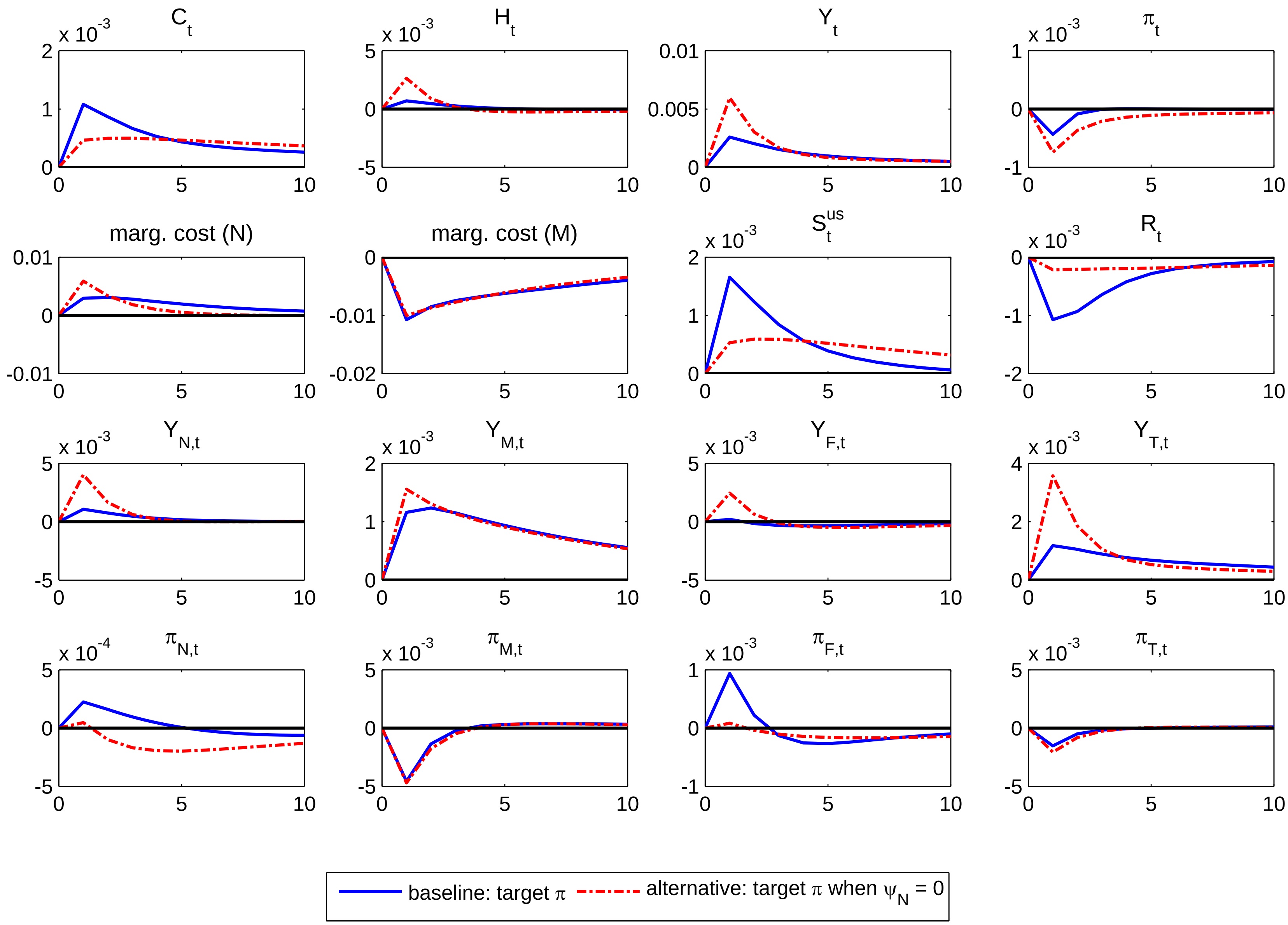


Figure 6: The Role of K-Adjustment Costs for Optimal CPI-Inflation Targeting Rules
IRF's to a 1% Technology Shock in the Manufacturing Sector

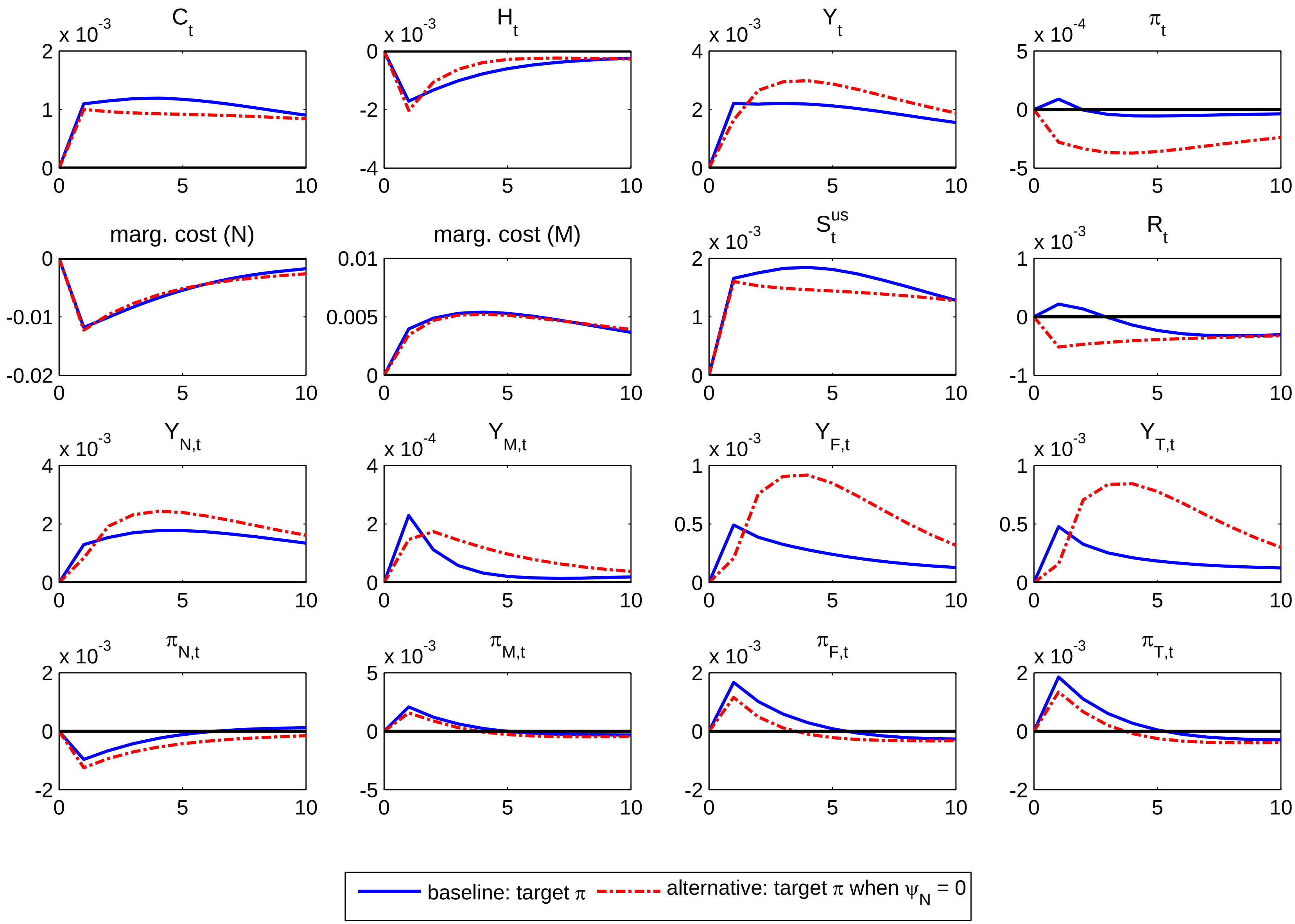


FIgure 7: The Role of K-Adjustment Costs for Optimal CPI-Inflation Targeting Rules
IRF's to a 1% Technology Shock in the Non-tradables Sector