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Working Paper/Document de travail
2010-8

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Bank of Canada Working Paper 2010-8

February 2010

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ISSN 1701-9397

Acknowledgements

We thank Tom Carter, Dylan Thomas Hogg and Frances Donald for their research assistance.

Abstract

Various papers have suggested that Price-Level targeting is a welfare improving policy relative to Inflation targeting. From a practical standpoint, this raises an important yet unanswered question: What is the optimal price index to target? This paper derives the optimal price level targeting index defined over the eight main components of the Consumer Price Index. It finds that such an index places a heavier weight, relative to the expenditure weight, on sectors with slow price adjustments. However, using the expenditure weights instead of the optimal ones results in very small welfare cost.

JEL classification: E32, E52
Bank classification: Monetary policy framework

Résumé

Plusieurs études tendent à montrer que la poursuite d'une cible axée sur le niveau des prix apporte un gain en bien-être par rapport à un régime de ciblage de l'inflation. Sur le plan pratique, ce résultat soulève une question importante mais encore sans réponse : en fonction de quel indice des prix devrait-on définir cette cible? Les auteurs tentent d'établir la composition de l'indice cible optimal, à partir des huit grandes composantes de l'indice des prix à la consommation. Dans l'indice optimal obtenu, les secteurs dont les prix s'ajustent lentement sont affectés d'une pondération supérieure à celle des dépenses. Il reste qu'assigner aux différents secteurs un poids égal à celui des dépenses qui leur sont consacrées au lieu de pondérations optimales se traduit par un coût très faible en matière de bien-être.

Classification JEL : E32, E52
Classification de la Banque : Cadre de la politique monétaire

1 Introduction

Research on Inflation targeting and Price-level Targeting monetary policy regimes shows that a credible Price-level Targeting (PT) regime dominates an Inflation targeting regime.[1] The key factor behind this result is that under PT inflation expectations serve as an automatic stabilizer of economic activity which helps central banks attain their stabilization goals at lower costs than under IT. Most papers on this subject have one-sector models, where the targeted price level index, is simply the monetary price of the only available output. In such a setup, the price level can be interpreted as an expenditure weighted price-level index, such as the Personal Consumption Expenditure or GDP deflator. While those aggregate price indices are natural benchmarks, it is not clear whether targeting them is actually the optimal thing to do. Thus, from a practical standpoint two questions arise: What is the optimal price index that should be targeted in a PT regime? How costly is it to target some other price-level index, such as the CPI?[2] In this paper we are address these questions with the help of a multisector general-equilibrium model, calibrated to Canadian data. Our model has several consumption goods sectors which differ in their frequency of price adjustment, their average productivity growth (trend), and in the volatility of sectoral productivity shocks they experience. In this economy, we find the optimal price-level target weights on each sector and then compute the welfare loss associated with targeting an expenditure weighted price index. We find that the difference between the optimal weight on each sector and its expenditure weight is determined primarily by the frequency of price adjustments in that sector. Sectors with slow price adjustments get heavier weights. Other sources of heterogeneity in our model play a minor role. Our second main finding is that the welfare loss from targeting the expenditure weighted index, instead of the optimal index, is small. These results appear to be extremely robust in the

[1] See for example: Svensson (1999), Vestin (2001), Gaspar et al (2007), Meh et al (2008), Cateau (2008), Cateau et al (2008), Kryvtsov et al (2008), Dib et al (2008), Covas and Zhang (2008).

[2] The focus on the CPI seems quite natural, given that CPI based inflation is presently the reference measure used by many Inflation targeting central banks, such as those of Australia, Canada, and New-Zealand.

parameter space and to various modifications of the benchmark model.

The question of the optimal *inflation target* has been explored before. Aoki (2001) uses a two-sector economy with one sector having sticky prices and the other sector being completely flexible in its price setting. In such an environment he shows that it is optimal for the monetary authority to focus exclusively on stabilizing the sticky sector's inflation. The intuition for this result is straightforward: sticky prices create costly relative price distortions, which could be reduced or eliminated by stabilizing that sector's inflation. The flexible sector, on the other hand, accommodates all relative price changes without need for policy intervention. Aoki's result were generalized to other multisectoral models by Huang and Liu (2005), Kara (2009), Wolman (2009) and Eusepi, Hobjin and Tambalotti (2009), Benigno (2004). These papers focused on different sources of sectoral heterogeneity and relative price changes (aside from heterogeneity in the price-stickiness). Huang and Liu (2005) and Kara (2009) highlighted the distinction between final and intermediate goods. Kara (2009) shows that, given the existing microevidence on the frequency of price adjustments, the optimal inflation target should have most of its weight on the stickier final good prices. Wolman (2009) evaluated the optimal trend inflation in a model with relative prices, that trend due to differences of the sectoral productivity growth rates. Wolman finds that the optimal trend inflation is influenced primarily by the productivity growth rate in the stickiest sector.[3] Eusepi et al (2009) calibrated a multisector economy in which the consumption goods sectors differ in their labour shares of output. They find that the optimal weights in the inflation target are determined mostly by differences in the frequency of price adjustments, and very little by differences in labour shares. Finally, Benigno (2004) focuses on regional differences in price dynamics and also finds that the optimal inflation target puts a heavier weight (relative to size) on inflation in the regions with stickier prices. Thus, the common finding of the literature is that a stickier sector has a heavier

[3] Wolman (2009) abstracted from shocks and focused on the balanced growth path properties of the model.

weight (relative to its expenditure share) in the central bank's optimal inflation target. Our paper confirms that finding both for inflation and Price-level Targeting regimes: the optimal target weights are determined primarily by the relative degrees of price stickiness, and much less so by differences in the rates of productivity growth or the sectoral shock processes. Unlike the previous studies, however, we also evaluate the welfare losses of using the expenditure weights in the target. This welfare loss was found to be extremely small, which suggests that targeting the standard CPI index is close to optimal.[4] We also contribute to the literature by analytically characterizing the balanced growth path allocation in a J sector economy, J finite. The analytical solution for the balanced growth path is often crucial when solving large multisector models.

The rest of the paper is organized as follows: in Section 2 we discuss sectoral price dynamics in Canada. In Section 3 we present the benchmark model. Section 4 discusses the calibration of the model. Section 5 presents our benchmark results as well as the sensitivity checks performed. Section 6 concludes.

2 Properties of the CPI

The Consumer Price Index (CPI) is an indicator of the changes in consumer prices as experienced by a subset of the population[5]. It compares price changes over time by measuring the cost of a fixed basket of commodities. The CPI is measured at a monthly frequency, but some components change much less frequently and are not collected at a monthly frequency. To avoid any issues in that regard we use the CPI at the quarterly frequency. For any given month, the Canadian CPI contains about 70,000 price observations. Given our macroeconomic policy perspective, we make use of the main aggregation as provided by Statistics Canada, namely the split of the CPI into

[4] Preliminary findings by De Resende et al (2010) provide additional support for targeting the CPI in a small-open-economy model with capital, tradable/non-tradable goods, commodity exports and imports.

[5] The subset of the population is not necessarily representative, since certain groups are excluded from the sampling for practical reasons. For example, before 1995 only households in urban centres with a population larger than 30000 persons were included in the sample. Given that the *aim* of the CPI is to give an "informative, reliable and impartial" measure of the cost of living changes, we abstract from these sampling problems and assume that the goal is achieved.

its eight main components: Food, Shelter, Household operations and furnishings, Clothing and footwear, Transportation, Health and personal care, Recreation, education and reading, Alcoholic beverages and tobacco products. The basket composition is determined by the expenditure behavior of a subset of the population in a reference period. The basket is fixed over a certain interval of time, in the Canadian case this normally means over a 5 year period. Adjustments reflect changes in the spending behavior of Canadians. In principle the delayed adjustment of the basket is a problem since it ignores the substitution effect and instead focuses on the income effect of price changes. To consider this issue we look at data on the composition of the basket from 1986 to 2005. Most of the consumption changes in that period occurred between 1986 and 1996. Afterwards the largest systematic change was a 1.61%*pt* increase in expenditures for Recreation, Education and Reading.

Figure 1 shows the price indices for all eight components for the period 1980.I to 2008.IV. The series are seasonally adjusted using the X12 package of the U.S. Census Bureau. There are several facts that are visible, when analyzing these eight lines. First, all sectors experienced sizable growth over the 29 years. Second, there is a lot of heterogeneity present. Some components, like Alcoholic beverages and tobacco (AlcBev&Tob), have grown considerably faster than the rest, while others, like Clothing, have stagnated relative to the rest. Thus there are some commodity groups that drive up inflation, while others are keeping it low. Third, there are some trend breaks visible in the presented series. The most visible ones are displayed by AlcBev&Tob. The first main trend-break is due to a sales tax reform in 1991 and affects nearly all subcomponents of the CPI. The next break is due to a tax rollback regarding Tobacco. Finally, all price series show a sizable volatility despite the removal of seasonal effects. The main impression to take from the figure is that of large trend differences, but also large differences in volatility.

In Canada the stabilization of the CPI around 2% is the main stated objective of the current monetary policy. In this context the composition of the CPI is of secondary importance. The main reason to look at subcomponents in an Inflation Targeting (IT) regime is to understand whether recent price changes are temporary or persistent. Once that has been determined the appropriate policy response follows. One way the Bank of Canada (an inflation targeter) determines the persistent component of the CPI is by looking at Core inflation, which is CPI inflation less its eight most volatile components. The task is likely to be more complicated in a Price Level Targeting (PT) environment. Given that the central bank under PT commits to stabilizing a certain price level, different compositions of that targeted price index might have very different implications for the economy. In particular, some of the sectors with volatile prices (like Shelter and Transportation) have very large CPI expenditure shares, which means that the central bank may need to do more to stabilize a price index which gives a larger weight to those sectors. The long run trends in prices might be another concern for PT monetary policy. A very strong upward productivity trend in one sector might, under a PT regime, force the central bank to run a strongly contractionary policy in order to keep a price index that includes that sector close to the trend of the other included components.

So, under a PT regime there are three important aspects of the CPI to consider: 1) the CPI consists of different goods and services whose importance is determined by their relative share in consumer expenditures; 2) the subcomponents of the CPI have heterogenous patterns regarding their long term price growth; 3) the volatility across sectors is also very diverse. Now, we make some of these features more visible. We start by decomposing the CPI subcomponents into trend components and residuals. This is done with a Hodrick-Prescot (HP) filter with a parameter value of 1600. Table 1 shows the results of this decomposition. The first column of the table shows the names of the eight main CPI components. The second column shows the standard deviations of the

HP cyclical components. The third column shows the growth rates of HP trend components.[6] An alternative way of seeing the differences in price dynamics is to plot the results of the decomposition. This is done in Figure 2. Here the focus is on the relative movement, so all trends are divided by the HP-trend of the CPI. Four commodity groups are growing roughly in line with the CPI: Food, Shelter, Health and Recreation. Then there are two fast growing price components, Transportation and AlcBev&Tob, as well as two very slowly growing price components, Housing Operation and Clothing.

The division by volatility, see Figure 3, reveals that the fastest growing price indices are also the ones with the highest volatility: Transportation and AlcBev&Tob. The next most volotile sectors are Clothing and Shelter.

Looking at the fourth column of Table 1, it becomes visible that the expenditure shares are unevenly distributed over the eight components. AlcBev&Tob with a high growth rate and a high volatility has a very small expenditure share of 4%, while Transportation with similar characteristics has a large share of 18.7%. From a different angle, the three largest components cover 62.8% of all expenditures.

To investigate the importance of these heterogenous trends in growth, volatility and expenditure weights for monetary policy in the context of Price-level Targeting, we make use of our benchmark framework as presented in the next section.

3 Multiple sector model

In this part, we analyze a model with multiple consumption goods. Each good has its own degree of price stickiness, its own productivity growth rate, and is exposed to its own persistent productivity shock. The aim is to find the optimal weights of the different sector's inflation rates in the PT interest rate rule. We also wish to know how different the optimal weights are, from

[6] The data in the last two columns of Table 1 will be discussed in the calibration section.

the corresponding expenditure weights, i.e. CPI weights. Given that we calibrate the model to match certain characteristics of the Canadian economy, this will allow us to contemplate the current monetary policy as well as an optimized Price-level Targeting policy rule.

In the benchmark model we consider an economy with constant growth trends. In extensions, we also allow for stochastic trends in sector-specific productivity processes.

3.1 Household

Households have preferences over sequences of working time N_t and aggregate consumption C_t:

$$\begin{aligned} U\left(\{C_t, N_t\}_{t=0}^{\infty}\right) &= E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t\right), \\ \text{with } u\left(C_t, N_t\right) &= \frac{\left(C_t\right)^{1-\sigma}}{1-\sigma} - \alpha \frac{\left(N_t\right)^{1+\eta}}{1+\eta} \end{aligned}$$

where aggregate consumption is a CES composite of J sector specific consumption products $C_{j,t}$:

$$C_t = \left(\sum_{j=1}^{J} \mu_j \left(C_{j,t}\right)^{\rho}\right)^{1/\rho}.$$

Households decide on how much to consume, C_t, and how much to invest into nominal risk-free bonds, B_t. Working generates wage income, $u_t W_t N_t$. There are also transfers, T_t, that are coming from firms' profits. So the households solve the following problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t\right)^{1-\sigma}}{1-\sigma} - \alpha \frac{\left(N_t\right)^{1+\eta}}{1+\eta}\right]$$

s.t.

$$\sum_{j=1}^{J} P_{j,t} C_{j,t} + B_t \leq u_t W_t N_t + \left(1 + i_{t-1}\right) B_{t-1} + T_t \tag{1}$$

$$C_t = \left(\sum_{j=1}^{J} \mu_j \left(C_{j,t} \right)^{\rho} \right)^{1/\rho}. \tag{2}$$

Cost-minimization on the part of the households results in the demand functions:

$$C_{j,t} = \left(\frac{P_{j,t}}{\mu_j P_t} \right)^{1/(\rho-1)} C_t, \tag{3}$$

where P_t is the expenditure-weighted price index given by

$$P_t = \left(\sum_{j=1}^{J} \left(\mu_j \right)^{1/(1-\rho)} \left(P_{j,t} \right)^{\rho/(\rho-1)} \right)^{(\rho-1)/\rho}. \tag{4}$$

This price index P_t is the shadow price of real income for households and therefore is the welfare relevant price aggregate in our economy. In what follows, we allow the central bank to target other price aggregates, and compute the welfare gains of doing so relative to the case where the expenditure weighted aggregate P_t is the target.

The labour income in the budget constraint is disturbed by the cost-push shock u_t. This shock drives a time-varying wedge between the household's marginal rate of substitution (for labour and consumption) and the marginal product of labour. Variation in this wedge might be created by time varying taxes or by preference shocks.

3.2 Production

Production takes place in two steps. There are J consumption-good sectors, and J intermediate-goods sectors, which supply those final good sectors with inputs.

Final good sectors

A final good firm in the sector j uses a large variety of intermediate inputs $c_{j,t}(i)$ and produces $C_{j,t}$, that it sells in a perfectly competitive market to the households.[7] The problem of the firm can be summarized by the next problem:

$$\min \int_0^1 p_{j,t}(i)\, c_{j,t}(i)\, di$$

$$C_{j,t} \leq \left(\int_0^1 (c_{j,t}(i))^{(\theta-1)/\theta}\, di \right)^{\theta/(\theta-1)},$$

which results in two familiar optimality conditions:

$$c_{j,t}(i) = \left(\frac{p_{j,t}(i)}{P_{j,t}} \right)^{-\theta} C_{j,t} \tag{5}$$

for the demand functions, and

$$P_{j,t} = \left(\int_0^1 (p_{j,t}(i))^{1-\theta}\, di \right)^{1/(1-\theta)}$$

for the cost-minimizing, sector-specific price index.

Intermediate goods

There are intermediate goods producers who use labor to produce differentiated intermediate goods and sell them in a monopolistically competitive market to the final goods producer of each sector. We break their optimization problem into two parts. First, they solve their cost minimization problem. Then they solve their profit maximization problem.

[7] We assume that all sectors produce the respective composite good with the same elasticity of substitution. While this is likely too strong an abstraction it is not important for the main results of the paper, as is clear from the results presented in Eusepi et al (2009).

Assumption 1. Each sector is facing a productivity process $Z_{j,t}$ which has a growth component, growing at the rate γ_j, and a cyclical component determined by:

$$\log \tilde{Z}_{j,t} = \rho_j \log \tilde{Z}_{j,t-1} + \varepsilon_{j,t}.$$

So that $Z_{j,t} = Z_{j,0} \left(\gamma_j^t\right) \tilde{Z}_{j,t}$. Taking the logarithm we get

$$\log Z_{j,t} = \log Z_{j,0} + t \log \gamma_j + \rho_j \log \tilde{Z}_{j,t-1} + \varepsilon_{j,t}.$$

We allow for cross-sector correlations of the productivity innovations $\varepsilon_{j,t}$. We assume that

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{J,t} \end{bmatrix} \sim N\left(\mathbf{0}_{J\times 1}, \Omega_{J\times J}\right)$$

where the variance-covariance matrix

$$\Omega_{J\times J} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1J}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2J}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{J1}^2 & \sigma_{J2}^2 & \cdots & \sigma_{JJ}^2 \end{bmatrix}$$

allows for cross-sector correlations.

The cost minimization problem

$$\min \left(\frac{W_t}{P_t}\right) n_{j,t}(i) + \psi_{j,t}\left(c_{j,t}(i) - Z_{j,t} n_{j,t}(i)\right)$$

leads to the first-order optimality condition

$$Z_{j,t}\psi_{j,t} = \frac{W_t}{P_t}.$$

Notice that $\psi_{j,t}$ is a measure of the firm's marginal cost in period t. It is sector specific but not firm specific. The firm i's period t profit is given by:

$$\begin{aligned}
\Pi_t &= \left(\frac{p_{j,t}(i)}{P_t} - \psi_{j,t}\right) c_{j,t}(i) \\
&= \left(\left(\frac{p_{j,t}(i)}{P_t}\right)^{1-\theta} - \psi_{j,t}\left(\frac{p_{j,t}(i)}{P_t}\right)^{-\theta}\right)(\mu_j)^{\theta}\left(\frac{P_{j,t}}{\mu_j P_t}\right)^{1/(\rho-1)+\theta} C_t
\end{aligned}$$

where we used the demand functions (3) and (5) to substitute for $c_{j,t}(i)$.

We assume that firms are adjusting their prices in a Calvo fashion (see Calvo 1983). At each point in time, a random fraction $1-\omega_j$ gets a signal to adjust their prices. The other firms retain previously set prices. Let $\Delta_{t,t+\tau} = \beta^{\tau}(C_{t+\tau}/C_t)^{-\sigma}$ be the discount factor. The firm's profit maximization problem in a given period t thus becomes:

$$\max_{p_{j,t}(i)} E_t \sum_{\tau=0}^{\infty} \omega_j^{\tau} \Delta_{t,t+\tau}\left(\left(\frac{p_{j,t}(i)}{P_{t+\tau}}\right)^{1-\theta} - \psi_{j,t+\tau}\left(\frac{p_{j,t}(i)}{P_{t+\tau}}\right)^{-\theta}\right)(\mu_j)^{\theta}\left(\frac{P_{j,t+\tau}}{\mu_j P_{t+\tau}}\right)^{1/(\rho-1)+\theta} C_{t+\tau}$$

Notice that all firms in sector j that are allowed to adjust their prices in a given period are identical and will choose the same price $p_{j,t}(i) = p_{j,t}^{*}$. Then the first order condition can be reduced to:

$$\frac{p_{j,t}^{*}}{P_t} = \frac{\theta}{\theta-1}\frac{E_t \sum_{\tau=0}^{\infty}(\omega\beta)^{\tau}(C_{t+\tau})^{1-\sigma}\left(\psi_{j,t+\tau}\left(\frac{P_t}{P_{t+\tau}}\right)^{-\theta}\right)\left(\frac{P_{j,t+\tau}}{P_{t+\tau}}\right)^{1/(\rho-1)+\theta}}{E_t \sum_{\tau=0}^{\infty}(\omega\beta)^{\tau}(C_{t+\tau})^{1-\sigma}\left(\frac{P_t}{P_{t+\tau}}\right)^{1-\theta}\left(\frac{P_{j,t+\tau}}{P_{t+\tau}}\right)^{1/(\rho-1)+\theta}}$$

One key advantage of Calvo pricing is the simple evolution of the price level at the sectoral level

$$(P_{j,t})^{1-\theta} = (1-\omega_j)\left(p_{j,t}^{*}\right)^{1-\theta} + \omega_j (P_{j,t-1})^{1-\theta}.$$

3.3 Market clearing

The market clearing conditions are given by:

$$
\begin{aligned}
B_t &= 0 \\
C_t &= \left(\sum_{j=1}^{J} \mu_j \left(C_{j,t}\right)^{\rho} \right)^{1/\rho} \\
C_{j,t} &= Y_{j,t} \\
c_{j,t}(i) &= y_{j,t}(i) \\
\sum_{j=1}^{J} \int_0^1 n_{j,t}(i)\, di &= N_t
\end{aligned}
$$

Now we are left dealing with the determination of the interest rate. It is common in this regard to use a ...

3.4 ...Taylor rule

The rule we use in the case of a PT regime is given by:

$$
\log\left(\frac{1+i_t}{1+\bar{\imath}}\right) = \chi_R \log\left(\frac{1+i_{t-1}}{1+\bar{\imath}}\right) + (1-\chi_R)\left(\chi_P \log\left(\tilde{P}_t/\bar{P}_t\right) + \chi_c \log\left(\frac{C_t}{\bar{C}_t}\right)\right) + \varepsilon_t \tag{6}
$$

with $\bar{P}_t = P_0 \left(\tilde{\pi}\right)^t$ being the target trend and $\tilde{P}_t = \tilde{P}_{t-1}\tilde{\pi}_t$ being the targeted aggregate price index, with $\tilde{\pi}_t$ defined as in

$$
\log\left(\tilde{\pi}_t\right) = \sum_{j=1}^{J} \varphi_j \log\left(\pi_{j,t}\right). \tag{7}
$$

(7). Note that the weights φ_j are not restricted to be equal to the expenditure weights. In fact our objective will be to find the optimal weights φ_j^* and compare them to the expenditure weights.

$\bar{C}_t$ is the aggregate consumption trend consistent with the balanced growth path, which will be derived in the next subsection. There we also make clear why the aggregate target price index takes the particular form from equation 7.

Similarly, in the case of inflation targeting (IT) the Taylor rule is defined as

$$\log\left(\frac{1+i_t}{1+\bar{\imath}}\right) = \chi_R \log\left(\frac{1+i_{t-1}}{1+\bar{\imath}}\right) + (1-\chi_R)\left(\chi_\pi \log\left(\tilde{\pi}_t/\tilde{\pi}\right) + \chi_c \log\left(\frac{C_t}{\bar{C}_t}\right)\right) + \varepsilon_t. \tag{8}$$

3.5 Equilibrium conditions and balanced growth path

We characterize the equilibrium using the first order conditions of the above stated problems. Defining $x_{j,t} \equiv \frac{N_{j,t}Z_{j,t}}{C_{j,t}}$, $q_{j,t} \equiv \frac{p_{j,t}}{P_t}$, $u_{j,t} \equiv \frac{P_{j,t}}{P_t}$ and $\pi_{j,t} \equiv \frac{P_{j,t}}{P_{j,t-1}}$ we can state the equilibrium defining system of equations (for PT) as follows:

$$q_{j,t} = \frac{\theta}{(\theta-1)}\frac{\Lambda_{j,t}}{\Gamma_{j,t}} \tag{9}$$

$$\Lambda_{j,t} = (C_t)^{1-\sigma}\,\psi_{j,t}\,(u_{j,t})^{1/(\rho-1)+\theta} + \omega_j\beta E_t\left[\pi_{t+1}^{\theta}\Lambda_{j,t+1}\right]$$

$$\Gamma_{j,t} = (C_t)^{1-\sigma}\,(u_{j,t})^{1/(\rho-1)+\theta} + \omega_j\beta E_t\left[\pi_{t+1}^{\theta-1}\Gamma_{j,t+1}\right]$$

$$1 = (1-\omega_j)\left(\frac{q_{j,t}}{u_{j,t}}\right)^{1-\theta} + \omega_j\left(\pi_{j,t}\right)^{\theta-1}$$

$$x_{j,t} = (1-\omega_j)\left(\frac{q_{j,t}}{u_{j,t}}\right)^{-\theta} + \omega_j\left(\pi_{j,t}\right)^{\theta} x_{j,t-1}$$

$$\pi_t = \pi_{j,t}\frac{u_{j,t-1}}{u_{j,t}}$$

$$Z_{j,t}\psi_{j,t} = Z_{l,t}\psi_{l,t}$$

$$\alpha\left(N_t\right)^{\eta} = \psi_{1,t}Z_{1,t}\left(C_t\right)^{-\sigma}$$

$$1=\left(\sum_{j=1}^{J}\left(\mu_j\right)^{1/(1-\rho)}\left(u_{j,t}\right)^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$$

$$N_t=\left[\sum_{j=1}^{J}x_{j,t}\left(\frac{1}{\mu_j}u_{j,t}\right)^{1/(\rho-1)}\psi_{j,t}\right]\frac{C_t}{Z_{1,t}\psi_{1,t}}$$

$$1=\beta\left(1+i_t\right)E_t\left[\frac{1}{\pi_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right]$$

$$\log\left(\frac{1+i_t}{1+\bar{\imath}}\right)=\chi_R\log\left(\frac{1+i_{t-1}}{1+\bar{\imath}}\right)+\left(1-\chi_R\right)\left(\chi_P\log\left(\tilde{P}_t/\bar{P}_t\right)+\chi_c\log\left(\frac{C_t}{\bar{C}_t}\right)\right)+\varepsilon_t$$

$$\tilde{P}_t=\tilde{P}_{t-1}\tilde{\pi}_t$$

$$\log\left(\tilde{\pi}_t\right)=\sum_{j=1}^{J}\varphi_j\log\left(\pi_{j,t}\right). \tag{10}$$

A detailed derivation is provided in the appendix. The unknowns in this system are: $\{\Lambda_{j,t},\ \Gamma_{j,t},\ q_{j,t},\ u_{j,t},\ x_{j,t},\ \pi_{j,t},\ \psi_{j,t}\}_{j=1}^{J}$, π_t, $\tilde{\pi}_t$, i_t, C_t, N_t, P_t. Thus, per period, we have $7J+6$ equations, Equation 9 to 10, in $7J+6$ unkonwns.

The main step we have to take next is a transformation of the variables. This is necessary to get a stationary version of the above equations. We use the equations to find the balanced growth path (BGP) behavior of the economy for an arbitrary inflation target. A key result for finding the balanced growth path is the following:

Lemma 1. In the absence of shocks and given any inflation target $\tilde{\pi}$, a non-trivial balanced growth path exists, if $\sigma\to 1$ and $\rho\to 0$. The BGP growth rates of the endogenous variables in this case are:

$$\gamma_c=\prod\nolimits_{j=0}^{J}\left(\gamma_j\right)^{\mu_j}$$

for the aggregate consumption C_t, and

$$\begin{aligned}
\gamma_{u,j} &= \gamma_c/\gamma_j, \forall j \\
\gamma_{\psi,j} &= \gamma_{u,j} \\
\gamma_{q,j} &= \gamma_{u,j} \\
\gamma_{\Gamma,j} &= \left(\gamma_{u,j}\right)^{1/(\rho-1)+\theta} \\
\gamma_{\Lambda,j} &= \left(\gamma_{u,j}\right)^{\rho/(\rho-1)+\theta}
\end{aligned}$$

for the other variables.

Proof. The proof can be found in the appendix.

Note that the first restriction in Lemma 1 implies that the inflation rate of the expenditure-weighted price index P_t

$$\log(\pi_t) = \sum_{j=1}^{J} \mu_j \log(\pi_{j,t})$$

has the same functional form as the inflation of the targeted price index given in (7). Thus, the expenditure weighted price index is a special case of our general price-target aggregate index.

Once we remove trends from the equilibrium conditions we obtain the stationary version of the equilibrium conditions stated in terms of detrended variables

$$
\begin{aligned}
\tilde{u}_{j,t} &\equiv u_{j,t}/\gamma_{u,j}^{t} \\
\tilde{\psi}_{j,t} &\equiv \psi_{j,t}/\gamma_{\psi,j}^{t} \\
\tilde{q}_{j,t} &\equiv q_{j,t}/\gamma_{q,j}^{t} \\
\tilde{\Gamma}_{j,t} &\equiv \Gamma_{j,t}/\gamma_{\Gamma,j}^{t} \\
\tilde{\Lambda}_{j,t} &\equiv \Lambda_{j,t}/\gamma_{\Lambda,j}^{t} \\
\tilde{C}_{t} &\equiv C_{t}/\gamma_{c}^{t},
\end{aligned}
$$

as follows:

$$\tilde{q}_{j,t} = \frac{\theta}{(\theta-1)}\frac{\tilde{\Lambda}_{j,t}}{\tilde{\Gamma}_{j,t}}$$

$$\tilde{\Lambda}_{j,t} = \left(\tilde{C}_t\right)^{1-\sigma}\tilde{\psi}_{j,t}\left(\tilde{u}_{j,t}\right)^{1/(\rho-1)+\theta} + \omega_j\beta E_t\left[\pi_{t+1}^{\theta}\tilde{\Lambda}_{j,t+1}\gamma_{\Lambda,j}\right]$$

$$\tilde{\Gamma}_{j,t} = \left(\tilde{C}_t\right)^{1-\sigma}\left(\tilde{u}_{j,t}\right)^{1/(\rho-1)+\theta} + \omega_j\beta E_t\left[\pi_{t+1}^{\theta-1}\tilde{\Gamma}_{j,t+1}\gamma_{\Gamma,j}\right]$$

$$1 = (1-\omega_j)\left(\frac{\tilde{q}_{j,t}}{\tilde{u}_{j,t}}\right)^{1-\theta} + \omega_j\left(\pi_{j,t}\right)^{\theta-1}$$

$$x_{j,t} = (1-\omega_j)\left(\frac{\tilde{q}_{j,t}}{\tilde{u}_{j,t}}\right)^{-\theta} + \omega_j\left(\pi_{j,t}\right)^{\theta}x_{j,t-1}$$

$$\pi_t = \pi_{j,t}\frac{\tilde{u}_{j,t-1}}{\tilde{u}_{j,t}\gamma_{u,j}}$$

$$\tilde{Z}_{j,t}\tilde{\psi}_{j,t} = \tilde{Z}_{l,t}\tilde{\psi}_{l,t}$$

$$\alpha\left(N_t\right)^{\eta} = \tilde{\psi}_{1,t}\tilde{Z}_{1,t}\left(\tilde{C}_t\right)^{-\sigma}$$

$$1 = \left(\sum_{j=1}^{J}\left(\mu_j\right)^{1/(1-\rho)}\left(\tilde{u}_{j,t}\right)^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$$

$$N_t = \left[\sum_{j=1}^{J} x_{j,t} \left(\frac{1}{\mu_j}\tilde{u}_{j,t}\right)^{1/(\rho-1)} \tilde{\psi}_{j,t}\right] \frac{\tilde{C}_t}{\tilde{Z}_{1,t}\tilde{\psi}_{1,t}}$$

$$1 = \beta\left(1+i_t\right) E_t\left[\frac{1}{\pi_{t+1}}\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\gamma_c\right)^{-\sigma}\right]$$

$$\log\left(\frac{1+i_t}{1+\bar{\imath}}\right) = \chi_R \log\left(\frac{1+i_{t-1}}{1+\bar{\imath}}\right) + (1-\chi_R)\left(\chi_P \log\left(\frac{\tilde{P}_t}{\bar{P}_t}\right) + \chi_c \log\left(\frac{\tilde{C}_t}{\bar{C}}\right)\right) + \varepsilon_t$$

$$\tilde{P}_t = \tilde{P}_{t-1}\tilde{\pi}_t$$

$$\log\left(\tilde{\pi}_t\right) = \sum_{j=1}^{J} \varphi_j \log\left(\pi_{j,t}\right).$$

These detrended equations can be solved to derive the BGP values of the detrended variables. One of the contributions of our paper is that we derived the BGP in closed form solution for the model a finite number of sectors J. We think of this finding as very useful for building larger multisector models with growth trends.

The detrended welfare criterion that could be used to evaluate various policies can also be obtained by removing the growth trends:

$$\begin{aligned} U_0 &= E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(\left(\gamma_c\right)^t \tilde{C}_t\right) - \alpha\frac{\left(N_t\right)^{1+\eta}}{1+\eta}\right] - \log\left(\gamma_c\right)\sum_{t=0}^{\infty}\beta^t t \\ &= E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(\tilde{C}_t\right) - \alpha\frac{\left(N_t\right)^{1+\eta}}{1+\eta}\right] - \log\left(\gamma_c\right)\sum_{t=0}^{\infty}\beta^t t. \end{aligned}$$

Notice that for the standard case of $\beta \in [0,1)$ the term

$$\log\left(\gamma_c\right)\sum_{t=0}^{\infty}\beta^t t = \log\left(\gamma_c\right)\frac{\beta}{\left(1-\beta\right)^2}$$

does not affect the welfare comparisons and thus, can be safely ignored.

4 Calibration

The benchmark model has eight consumption goods sectors, with their respective expenditure shares, μ_j, being equal to the expenditure shares of the eight main components of the Consumer Price Index provided by Statistics Canada and shown in the fourth column of Table 1. The expenditure shares are the averages over the baskets from 1986 to 2005, as provided by Statistics Canada. The components, as already introduced in Section 2, are: Recreation, education and reading, Household operation and furnishings, Health and personal care, Clothing and footwear, Alcoholic beverages and tobacco, Food, Transportation and Shelter. Note that Table 1 presents these sectors arranged in the order of increasing frequency of price adjustments, shown in the last column of the table. We draw on the work done by Harchaoui, Michaud and Morceau at Statistics Canada (2007) who measured those frequencies, to calibrate the Calvo price adjustment parameters ω_j. The three authors make use of Canadian micro data for the period from 1995-2006. Among other things they report price changes at a monthly frequency. Using their results as reported in Table 1, we compute the quarterly values of ω_j by taking one minus their fractions, as a whole to the power three.

Our fundamental assumption regarding the productivity process is that it moves along a deterministic trend with disturbances following an VAR(1) process:

$$\begin{aligned}\log Z_{j,t} &= t \log \gamma_j + \log Z_{j,0} + \log \tilde{Z}_{j,t} \\ \log \tilde{Z}_{j,t} &= \rho_j \log(\tilde{Z}_{j,t-1}) + \varepsilon_{j,t}\end{aligned}$$

with sectoral innovations $\varepsilon_{j,t}$ drawn from a joint normal distribution

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{J,t} \end{bmatrix} \sim N\left(\mathbf{0}_{J\times 1}, \Omega_{J\times J}\right)$$

with the variance-covariance matrix

$$\Omega_{J\times J} = \begin{bmatrix} \sigma^2_{11} & \sigma^2_{12} & \cdots & \sigma^2_{1J} \\ \sigma^2_{21} & \sigma^2_{22} & \cdots & \sigma^2_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma^2_{J1} & \sigma^2_{J2} & \cdots & \sigma^2_{JJ} \end{bmatrix} \tag{11}$$

which allows for cross-sector correlations. To calibrate these processes we need estimates of γ_j, ρ_j, and $\Omega_{J\times J}$. The measure of productivity in the model is closest to that of real labor productivity in the data. The key issue to solve is to map the sectors in the North American Industry Classification System (NAICS) into the sectors of the CPI. The mapping we use is presented in Table 5. We realize that the mapping is imperfect and has limitations, but it is the best measure of productivity available for Canada at a disaggregate level. To give an example of the limitations of our measure: the labor productivity of the NAICS sector "Education" is not reported by Statistics Canada due to "confidentiality requirements of the Statistics Act". A notable other problem is that the consumption of typical Canadian households consists of imported and domestically produced goods. We only use the labor productivity of the domestic sectors and thus, most likely underestimate the labor productivity for all CPI sectors with a high import content. One way of thinking about this is that the labour productivities of the domestic and the foreign produced commodities of the same type need to move roughly proportionally to each other at least in the medium term, since otherwise the expenditures should decline for the most costly good driving either the domestic or the foreign producer from the market. The data we used for our parameter determination are labor productivity and total hours by NAICS sector as available through Statistics Canada. We use the total hours as weights to derive the aggregates we need for our analysis and then perform an econometric analysis of the derived aggregates. Our data have annual frequency for the period 1961 to 2004. We would have preferred to use quarterly frequency data, but those were only available for a short period of time, 1997 to 2003.

With basic econometric techniques we estimate γ_j, ρ_j, and $\Omega_{J\times J}$ from the constructed annual sectoral labour productivity series. Appendix A.4 outlines our procedure by which we infer the quarterly frequency stochastic processes from the annual data. Tables 2 and 3 present those estimated parameter values for the different sectors. Little is remarkable about the estimates, the one thing that is noteworthy is that for the "Health" sector the point estimate of the labour productivity growth rate was slightly negative, but insignificant. We set it to zero and re-estimated the parameters with that restriction.

The parameters of the model which are not sector specific are given in Table 6. Recall the following functional form for the utility function and the consumption aggregate.

$$\begin{aligned} u\left(C_t, N_t\right) &= \frac{\left(C_t\right)^{1-\sigma}}{1-\sigma} - \alpha \frac{\left(N_t\right)^{1+\eta}}{1+\eta} \\ C_t &= \prod\nolimits_{j=0}^{J} \left(C_{j,t}\right)^{\mu_j} \end{aligned}$$

The consumption aggregate is of the Cobb-Douglas type due to the balanced growth path restriction. For the cost-push shock we assume that its logarithm follows an AR(1) process:

$$\log\left(u_t\right) = \rho_u \log\left(u_{t-1}\right) + \varepsilon_{u,t} \ , \qquad \varepsilon_{u,t} \sim N\left(0, \sigma^2_{\varepsilon_u}\right) .$$

Some of the parameters are fairly standard: the discount rate $\beta = 0.99$, and the inverse of the Frisch elasticity of labour supply $\eta = 1$ were set in accordance with much of the recent business cycle literature. The weight on labour $\alpha = 13.9$ was calibrated to match the average labour supply to 0.25, roughly the average share of working hours in Canada, in the annual data per 5200 total available hours. Two parameters are determined through the balanced growth path restrictions stated above. These restrictions require the intertemporal elasticity of substitution, σ, to be one, and the elasticity of substitution parameter between the different sectors, ρ, to be zero. The next parameter we have to deal with, θ, determines the markup in the model. Here we make use of a 2008 study by Danny Leung. The study provides us with various markup estimates for different sectors of the Canadian economy from 1961 to 2004. We choose the average markup value across sectors as determined in the paper, 14.8%, to calibrate the markup in the model. This implies $\theta = 7.76$. We calibrate the stochastic process for the cost-push shocks by using the estimated labour wedges from the business cycle accounting study of Cociuba and Ueberfeldt (2008). Fitting an AR(1) process to the *annual* frequency labour wedge series gives an AR1 coefficient equal to 0.71 and a standard deviation of the innovations equal to 0.01. We take $\rho_u = 0.71^{0.25}$ and $\sigma_{\varepsilon_u} = \frac{0.01}{1+\rho_u^2+\rho_u^4+\rho_u^6}$, which are the implied *quarterly* estimates of the persistence and of the standard deviation of the cost-push shocks.[8]

Next, we outline the determination of the historic Taylor rule parameters which we use in some of our experiments

$$\log\left(\frac{1+i_t}{1+\bar{\imath}}\right) = \chi_R^{HIST}\log\left(\frac{1+i_{t-1}}{1+\bar{\imath}}\right) + \left(1-\chi_R^{HIST}\right)\left(\chi_\pi^{HIST}\log\left(\frac{\pi_t}{\tilde{\pi}^{HIST}}\right) + \chi_c^{HIST}\log\left(\frac{\tilde{C}_t}{\bar{C}}\right)\right) + \varepsilon_t. \tag{12}$$

To estimate this Taylor-type rule we use the following data: the interest rate is the quarterly average 'overnight money market financing rate' as provided by the Bank of Canada. The inflation rate is calculated using the Canadian Consumer Price Index. The inflation target, $\tilde{\pi}^{HIST}$, is set at 2%. Furthermore, we use real Canadian GDP at the quarterly rate relative to trend as our output gap measure. The trend output is determined by a polynomial of degree three. The period for the estimation is 1980.1 to 2008.4. For a discussion of 'policy reaction functions', we refer to Judd and Rudebusch (1998) and Rudebusch (2002). The estimation results of the Taylor rule for Canada

[8] Appendix D shows derivations for these transformations.

are reported in Table 4. The main implication from the Taylor rule analysis is that the monetary authority in Canada has placed a very strong emphasis on stabilizing inflation around the target rate.

Now, before we discuss our results, we would like to give some sense of how closely the model matches the aggregate inflation dynamics in Canada. Table 7 shows the standard deviations and the first-order autocorrelation coefficients for CPI inflation in the data and in the model. As we can see from the table, the model matches the persistence of CPI inflation in the data quite well, although it overpredicts its standard deviation.

5 Results

This section presents various results we derive from our model. First we find the optimal sectoral target weights for the benchmark calibration, and analyze the importance of the different sector specific aspects of the model for the optimal weights. Second, we consider how costly a deviation from the optimum might be. One particular case of interest here is that of the expenditure weights. Third, we conclude the results section with a list of extensions of our baseline model and their implications for our results.

5.1 The optimal weights and their determinants

Our first objective is to search over Taylor rule coefficients χ_P, χ_c and sectoral target weights $(\varphi_j)_{j=1}^{7}$ in order to maximize the unconditional expected welfare of the household in the economy.[9] From the analysis we wish to obtain two things: first, how do the optimal weights $(\varphi_j)_{j=1}^{8}$ compare to the expenditure weights $(\mu_j)_{j=1}^{8}$ used in the aggregation of the CPI, and second, what are the welfare consequences of using various suboptimal weights.

[9] For this part of the analysis, we set the Taylor coefficient on the lagged interest rate, χ_R, equal to zero. We found that doing otherwise did not change our results, because the welfare function was effectively flat in $\chi_R \in [0, 1)$ as long as we optimized over other parameters $\left(\chi_P, \chi_c, (\varphi_j)_{j=1}^{7}\right)$. Further, note that we make use of the restriction $\sum_{j=1}^{8} \varphi_j = 1$, by normalizing $\varphi_8 = 1 - \sum_{j=1}^{7} \varphi_j$.

We find the optimal weights, $\left(\varphi_j^*\right)_{j=1}^{8}$, jointly with the other optimized parameters χ_π^*, χ_c^* by searching for their (nonnegative) values, which maximize the average ex-post (de-trended) utility value $\frac{1}{(1-\beta)}\frac{1}{T}\sum_{t=1}^{T}\left[\log\left(\tilde{C}_t\right)-\alpha\frac{(N_t)^{1+\eta}}{1+\eta}\right]$, computed over T =10,000 quarters of simulation. In doing so we are relying on the Law of large numbers to obtain a good approximation of the unconditional expected utility of the household

$$\begin{aligned} U &= E\sum_{t=0}^{\infty}\beta^t\left[\log\left(\tilde{C}_t\right)-\alpha\frac{(N_t)^{1+\eta}}{1+\eta}\right] = E\left[\log\left(\tilde{C}\right)-\alpha\frac{(N)^{1+\eta}}{1+\eta}\right]\sum_{t=0}^{\infty}\beta^t \\ &= \frac{1}{1-\beta}E\left[\log\left(\tilde{C}\right)-\alpha\frac{(N)^{1+\eta}}{1+\eta}\right]. \end{aligned}$$

Column 3 of Table 8 shows the optimal weights and the optimal Taylor-rule coefficients for PT. Column 4 shows results for the optimized IT rule, and the last column of Table 8 shows the corresponding numbers for the estimated historical IT rule 12. Under the historical rule the target weights φ_j are equal to the CPI expenditure weights, μ_j. Compared to those weights, optimal weights under both PT and IT put much more emphasis on the sectors with a low price flexibility. For example Recreation, the least flexible (price) sector in the data, has its optimal weight nearly 3 times as big as its expenditure weight. At the other extreme, Shelter, the sector with the most flexible prices, has its optimal weight as less than a quarter of the sector's expenditure weight.[10] It is quite striking that most of the weight in the optimal target is falling on the three least flexible sectors. This is in stark contrast to the expenditure weights. While the combined expenditure weight of the three least flexible sectors is 27%, their combined optimal weight is 62%. Conversely, the three most flexible sectors have a combined expenditure weight of 63%, but a combined optimal weight of only 24%.

The second to last line of Table 8 shows the expenditure-weighted inflation rate $\bar{\pi}$ for each

[10] Note, that sectors are arranged in the order of increasing price flexibility.

of the regimes. Under the historical rule, it is set at 2 percent per annum, which has been the official target rate for Canada since 1993. Under both PT and IT, $\bar{\pi}$ is set at the value 0.19% per annum which maximizes the steady-state de-trended utility $\log\left(\tilde{C}\right) - \alpha\frac{(N)^{1+\eta}}{1+\eta}$.[11] Finally, the last row of Table 8 shows the welfare losses for each policy rule, relative to the welfare level attained in the counterfactual model specification with all the sectoral prices completely flexible. The welfare loss measure we use is the consumption equivalent (in percent of steady-state consumption) of the difference in expected utility between the model in question and the model with fully flexible prices.[12] Regarding the welfare consequences, we find that both optimized rules are quite close to the fully flexible benchmark with a welfare loss equivalent of only 0.0336 % of consumption. The welfare loss under the historical rule is bigger at 0.2067 percent.

The results above suggest that price stickiness is one of the main determinants of the optimal sectoral target weights. To get more insight into what determines the optimal weights, we conduct a sequence of counterfactual experiments. In each of these experiments we eliminate one aspect of sectoral heterogeneity and then report how the optimal PT target weights, φ_j^{PT}, are changing as a result of that modification.[13]

1. In the first counterfactual experiment, we assume that all sectors have the same long-run growth rate of productivity, which is set to be equal to the weighted average of sectoral growth rates in the data $\log\left(\bar{\gamma}\right) = \sum_{j=1}^{8}\mu_j \log\left(\gamma_j\right)$. The optimal sectoral weights found from this

[11] Note that the target inflation rate $\tilde{\pi}$ defined as $\log\left(\tilde{\pi}\right) = \sum_{j=1}^{J}\varphi_j \log\left(\bar{\pi}_j\right)$ depends on the weights φ_j, and thus, in general, is not equal to $\bar{\pi}$.

[12] That is if $U^{PT} = \frac{1}{(1-\beta)}\frac{1}{T}\sum_{t=1}^{T}\left[\log\left(\tilde{C}_t^{PT}\right) - \alpha\frac{\left(N_t^{PT}\right)^{1+\eta}}{1+\eta}\right]$ is the average level of utility attained under the optimized PT rule, and U^{FP} is the corresponding number in the flexible price model, we report $100 * \left(\frac{\exp((1-\beta)U^{FP})-\exp\left((1-\beta)U^{PT}\right)}{\tilde{C}}\right)$ as our measure of the welfare loss. Here and everywhere else $\tilde{C}$ is the steady-state (de-trended) consumption in the benchmark model with the *historical* IT rule.

[13] For each of the experiments we also found the optimal PT rule coefficients χ_P, χ_c (in addition to target weights, φ_j). Those coefficients were similar to the benchmark ones in Table 8, except when all the sectors had the same degree of price stickiness. In that case the optimal PT coefficients χ_P, χ_c were much bigger. Since our focus is on sectoral weights, we do not report those additional results.

experiment are shown in the third line (with results) of Table 9. For the ease of comparison, the previous two lines replicate the expenditure weights, and the benchmark optimal weights. Relative to the benchmark weights, we find some fairly small changes. The largest absolute changes were a 2 percentage point increase in the weights of Housing Operations and Alcohol & Tobacco. Overall, however, the pattern of the optimal weights did not change much relative to the benchmark case: stickier sectors get disproportionately larger weight in the optimal target price index.

2. In the second experiment we restore the sectoral productivity growth rate differences to what they were under the benchmark, and then remove the cross correlation of innovations to sectoral productivity processes. The idea is to make the sectors independent of each others' shocks. We do it by simply setting to zero all the off-the-main-diagonal elements of the variance-covariance matrix Ω in (11), while retaining the benchmark values on the main diagonal. Thus the variance covariance matrix becomes

$$\tilde{\Omega}_{J\times J} = \begin{bmatrix} \sigma_{11}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{22}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{JJ}^2 \end{bmatrix}.$$

This experiment lets us assess the importance of the cross-correlation of sectoral productivity processes for the optimal target weights. Plus, it lays the ground for our next counterfactual experiment in which we make all the sectors equally volatile. The fourth line of Table 9 shows the optimal sectoral weights found from this experiment. Comparing these numbers with the benchmark weights we can see that cross-correlations of productivity shocks have some effect on the optimal weights, but the overall pattern of optimal weights remains similar to the one from the benchmark case.

3. In our next experiment we further simplify the variance-covariance matrix $\tilde{\Omega}_{J\times J}$ by setting

all of its main diagonal elements equal to their weighted average:

$$
\begin{aligned}
\hat{\Omega}_{J\times J} &= \hat{\sigma}^2 \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \\
\hat{\sigma}^2 &= \mu_1\sigma_{11}^2 + \mu_2\sigma_{22}^2 + \cdots + \mu_8\sigma_{88}^2.
\end{aligned}
$$

In addition we make all of the persistence coefficients for the productivity processes equal to their weighted average:

$$
\begin{aligned}
\rho_1 &= \rho_2 = \cdots = \rho_8 = \hat{\rho} \\
\hat{\rho} &= \mu_1\rho_1 + \mu_2\rho_2 + \cdots + \mu_8\rho_8.
\end{aligned}
$$

These transformations make sure that all of our sectoral productivity processes are identical. The optimal sectoral weights found from this experiment are shown in the fifth line of Table 9. Once again we see some small changes in the weights, relative to the benchmark, but the general message remains intact: stickier sectors get disproportionately heavier weights. Notice that the main gains in terms of weights are going to sectors with very high volatility before the change, while the main decreases in weight go to sectors with a low volatility before the change.

4. From the previous three experiments, we can see that neither the productivity growth rate differences nor differences in the volatility of the productivity processes across sectors seem to be the key determinants of the optimal price-target weights. Thus, in this experiment we restore all the parameters to their benchmark values, and then let all of our sectors have the same degree of price flexibility. We do that by setting all the sectoral Calvo coefficients equal to the weighted average of those from the benchmark case: $\bar{\omega} = \frac{1}{8}\sum_{j=1}^{8} \mu_j\omega_j$. The effect of

this change on the optimal weights is quite remarkable, as is visible in the second to last line of Table 9. The new optimized index is very close to the original CPI (maximum absolute deviation is 1%pt). These results suggest that cross-sectoral differences in the degree of price flexibility are the key drivers of the optimal target weights. This finding is consistent with generalizations of Aoki's result to models with more than one sticky sector, as discussed in the literature review section. The monetary authority focuses its attention on the sectors that have the most costly price adjustment, namely the sticky sectors, while the flexible sectors are accommodating most of the relative price changes at a fairly low cost. We make this finding even more transparent in our last counterfactual experiment.

5. In Aoki's case, the last row of Table 9, we show results of an experiment in which all prices are fully flexible except for the Recreation sector, whose Calvo parameter is left unchanged. The outcome is that all the weight is now placed on the Recreation sector and no weight on the other 7 sectors. It is a confirmation of Aoki's theoretical result in our richer model.

To conclude this part of our analysis: our results show that for the baseline calibration the optimal weights are quite different from the expenditure weights used in the CPI. The main driver behind a sector's optimal weight is its degree of price stickiness relative to the other sectors, while sectoral volatility and growth differences matter little.

5.2 How costly are suboptimal weights?

Now after we have shown that the optimal weights can be quite different from the expenditure weights, a natural question to ask is: how costly is it to use suboptimal weights? This question is quite important given that: (a) it is a common practice for central banks to use the CPI as the targeted price aggregate; and (b) the CPI uses expenditure weights, which we have shown to be suboptimal.[14]

We next report results for three experiments which shed light on the welfare consequence of choosing suboptimal weights. The first experiment looks at the welfare cost of using expenditure weights. In this experiment we are optimizing over the Taylor rule coefficients, but keeping the sectoral weights fixed at their expenditure share values. The results of this experiment are summarized in the rightmost column of Table 10. The previous column restates the benchmark results. As we see from the last row of the table, the welfare loss (relative to the flexible prices model) is larger with the expenditure weights, but only by 0.005% of steady-state consumption. This suggests that at least for the optimized Taylor rules[15], it does not make much difference, welfare-wise, whether we optimize over sectoral target weights as well, or simply use the expenditure shares.

We try to generalize this statement somewhat in our next experiment, in which we vary the weights in a wider range, but keep the Taylor rule coefficients at their benchmark-optimal values. Table 11 shows the results for this case. The second column simply restates our benchmark optimized PT results. The next column shows results for the CPI weights and the last column shows results for the worst possible weights, which we found by minimizing welfare over the set of target weights. Notice that in all three cases the Taylor rule coefficients on the lagged interest rate, the price level and the output gap are held fixed at their benchmark values. The worst thing the central bank could do in these circumstances is to put all the weight on the price level of Alcoholic Beverages & Tobacco. This sector's measured labour productivity has the highest volatility over the sample period. As a result, if the central bank completely focuses on stabilizing the price level of that sector, it will force other sectors (half of which are stickier than Alcohol & Tobacco) to accommodate the relative price changes. The extra cost of those relative price

[14] There are very good reasons for using the CPI weights that are abstracted from in this paper. In the context of monetary policy the main once are the realtive ease with which the target can be communicated to the public and that the weights can't be manipulated by the central bank.

[15] Results for optimized IT rules are very similar.

distortions is approximately 0.08% of consumption as can be seen from the last row of Table 11. In relative terms the increase is fairly large: the welfare loss triples relative to the benchmark. In absolute terms the extra cost from using the worst target weights approaches 8% of Lucas' cost of business cycle (estimated at roughly 1 percent of consumption, see Lucas 1987). Table 12 gives a different perspective on the costs of suboptimal target weights (in the same experiment). It reports standard deviations of the CPI inflation, of the output gap, of the interest rate and of the change in the interest rate. These moments are often used in computing second-order loss functions. The volatility differences between various target weights are quite striking. Under the worst weights, the CPI inflation rate has a four times larger standard deviation of inflation than under the optimal weights. A seemingly surprising result is that CPI inflation is actually much less volatile under PT with CPI weights than under PT with optimal weights. The reason for that is quite simple. The optimal price index puts most of the weight on the small sticky sectors: the three stickiest sectors accounting for 27 percent of expenditures get 62 percent of the weight in the optimal target. As the price of the stickier sectors is being stabilized, the prices in the other, more flexible sectors accommodate most of the relative price changes. In particular, the three most flexible sectors: Food, Transportation and Shelter, accounting for 63 percent of consumer expenditures, get only 24 percent weight in the optimal PT index. Prices in these flexible sectors become quite volatile as a result of the optimal weighting. Because these three sectors constitute a big part of consumer expenditures, the whole CPI index becomes (optimally) more volatile. On the contrary, when the CPI index is being used as the price target, sectoral prices are being stabilized "in proportion" to their expenditure weights. This leads to smaller volatility of the CPI inflation shown in Table 12 in the last row of the "CPI weights" column. Our volatility results in Table 12 also suggest that the loss functions computed only from the volatilities of the aggregate variables, like CPI inflation, could give misleading welfare rankings of alternative policies in a multisector model.

Finally, Figures 4 and 5 show results for our last experiment, in which we vary the PT interest rate rule coefficients, χ_P or χ_c (one at a time), and compute the welfare loss differences between the WORST and the BEST sectoral weights. We find the best and the worst weights by first maximizing and then minimizing the (unconditional expected) utility of the household over the sectoral target weights, $\{\varphi_j\}$. In Figure 4 we plot the welfare loss difference (on the vertical axis, in percent of steady-state consumption) against various values of the Taylor coefficient on the price level (χ_P in the equation 6) holding all other parameters (including χ_c) constant at their benchmark values. In Figure 5 we plot the welfare loss difference against various values of the Taylor coefficient on the output gap (χ_c in the equation 6), now holding all other parameters (including χ_P) constant at their benchmark values. In effect, we wish to determine the robustness of our finding to variations in the Taylor rule coefficients. So, we look at the effect of varying different coefficients on the range of welfare differences due to variations in the sectoral weights. Figures 4 and 5 suggest that the range of welfare differences is essentially independent of the parameters χ_P and χ_c, at least for the fairly wide range of values shown. Our results for IT rules were very similar. So, our last experiment suggests that the magnitude of the welfare losses from using suboptimal target weights is very stable across various specifications of the monetary policy rule.

5.3 Extensions and their implications

To evaluate the robustness of our results, we analyzed the implications of a variety of extensions. None of these extensions changed the results presented above beyond minor quantitative differences.

The first extension is the introduction of stochastic growth trends to the sectoral productivity processes. We assume that the productivity shocks follow random walks with drifts:

$$
\begin{aligned}
\log Z_{j,t} &= \log\left(\gamma_j\right) + \log Z_{j,t-1} + \log \tilde{Z}_{j,t} \\
\log \tilde{Z}_{j,t} &= \rho_j \log \tilde{Z}_{j,t-1} + \varepsilon_{j,t}.
\end{aligned}
$$

The idea is that with stochastic growth trends, it might be costlier to maintain fixed sectoral weights under PT. This is due to the fact that with persistent stochastic trends it is possible for various price indices to have very different dynamic properties. However, after we re-calibrated the model economy with the stochastic productivity trends, we found that neither our results regarding the importance of the price stickiness for the optimal weights nor the findings regarding the small costs of sub-optimal (CPI) weights had changed.

The second extension considered is a modification of the intermediate input technology. Instead of constant returns we consider diminishing returns to labor in the intermediate input technology, i.e. $c_{j,t}\left(i\right) = Z_{j,t}\left[n_{j,t}\left(i\right)\right]^{\theta}$. This technology captures the idea that firms have some firm-specific capital which cannot be easily adjusted. This type of production function is often used in sticky price models in order to increase the real effects of monetary policy (or of monetary shocks) on the model economy. Despite that, we found that the resulting reduction of flexibility in moving resources across industries did very little to increase the welfare costs of suboptimal weights. The welfare differences between various target weights were only marginally larger than the ones from the benchmark model.

In our next extension, we assume that labour is completely immobile across-sectors. The total labour force is assumed to be permanently divided between sectors in proportion to their CPI expenditure shares. However, we still allow for within sector mobility across intermediate firms. As in the fixed capital case, the idea is to reduce the ability of the economy to respond to sector specific shocks. This in principle places more emphasis on the monetary authority to unwind the shocks for the less flexible sectors. While this extension clearly matters for welfare overall, its effect on the importance of the sectoral weights was quite small and the welfare consequences of suboptimal weights were also marginal.

6 Conclusion

We calibrate a multisector general equilibrium model to Statistics Canada's decomposition of the Consumer Price Index into eight main components, The sectors are heterogeneous in their degree of price flexibility, in their growth rates of productivity and in the volatility of their sector-specific productivity shocks. With that calibrated model we evaluate two questions:

1. What is the optimal price index to target?
2. How costly is it to target the expenditure weighted CPI index instead of the optimal index?

We find that the optimal price level targeting index puts a heavier weight, relative to the expenditure weight, on sectors with less flexible prices. The other sources of sectoral heterogeneity in the model were found to matter very little for the optimal weights. Regarding the second question, the paper finds that the welfare cost of targeting the CPI index, instead of the optimal index, is quite small. These results appear to be extremely robust in the parameter space and across various extensions. So, the main policy implication of the paper is that the payoffs to fine-tuning the target price-level index appear to be very small. Central banks do well by focusing on the CPI.

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Appendix

A1. Figures

Figure 1: The eight main subcomponents of the CPI, 1980.I to 2008.IV.

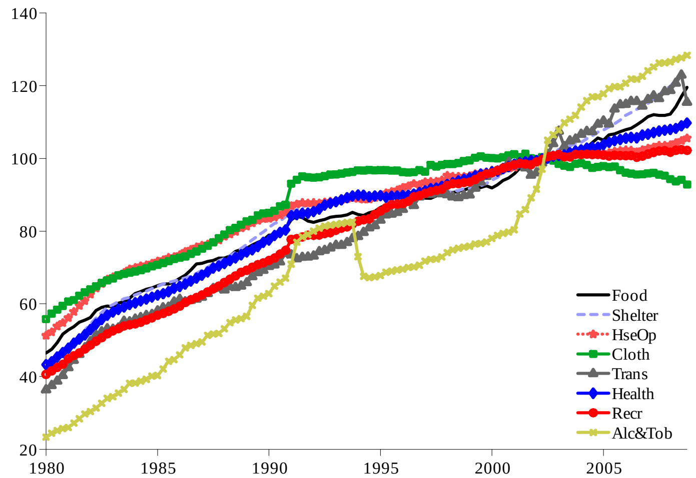


Figure 2: Analyzing growth trends of the eight main components.

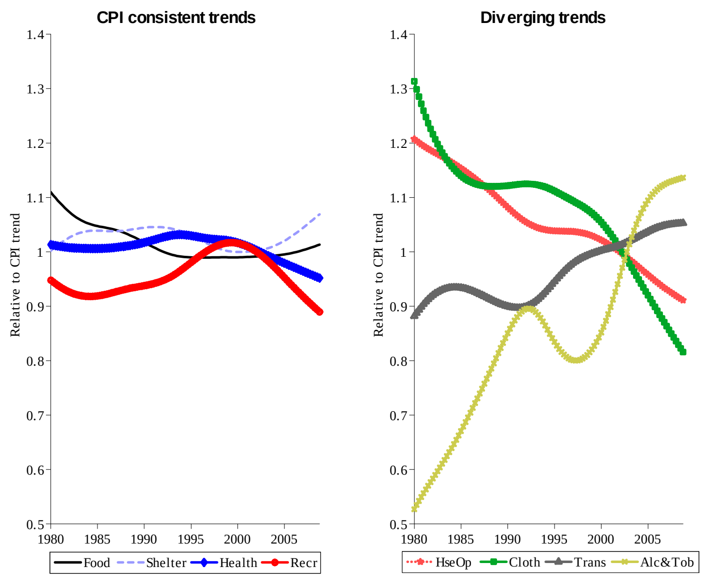


Figure 3: Analyzing the volatility of the eight main CPI components.

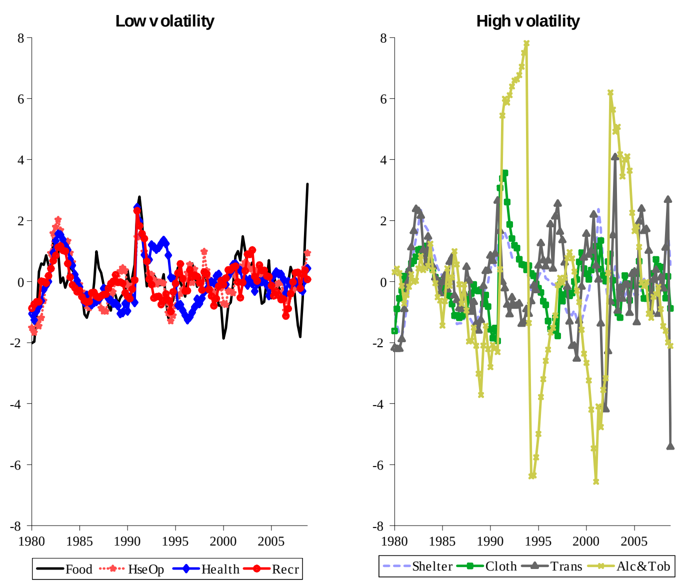


Figure 4: Range of welfare losses between the Best and the Worst sectoral weights

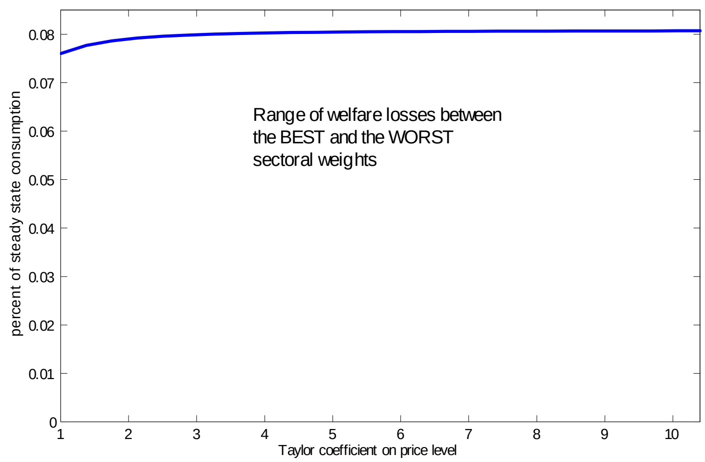
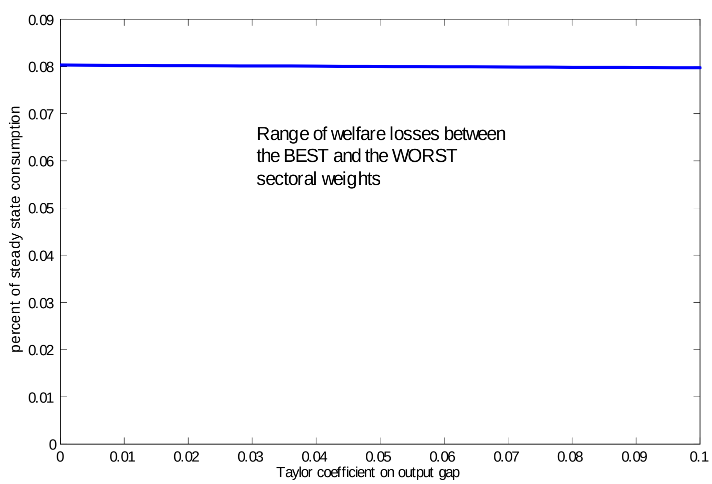


Figure 5: Range of welfare losses between the Best and the Worst sectoral weights



A2. Tables

Table 1: Price dynamics of the CPI components.

Components	Volatility *value*	Growth *annual rate*	Expenditure share† *in %*	Proportion of price changes‡ monthly, in %
Recreation	0.592	1.994	11.3	10.9
House Operation	0.717	1.801	10.8	11.0
Health	0.750	1.994	4.5	12.1
Clothing	0.985	1.632	6.5	14.7
Alcohol Bev. & Tobacco	3.162	2.727	4.0	17.8
Food	0.852	1.968	17.3	28.4
Transportation	1.471	2.203	18.7	35.9
Shelter	0.949	2.106	26.8	50.7

† Source: Statistics Canada provides the shares for various years on its web page. We average over the shares from 1986 to 2005.
‡ Source: Table 2 in Harchaoui et al. (2007).

Table 2: Sectoral labour productivity processes.

Components	Growth rate† *annual rate, in %* $100\left(\gamma_j - 1\right)^4$	Persistence† *annual estimate* ρ_j^4
Recreation	1.91	0.94
House Operation	1.35	0.75
Health	0.00	0.95
Clothing	1.73	0.91
Alcohol Bev. & Tobacco	1.78	0.77
Food	1.06	0.77
Transportation	0.88	0.92
Shelter	1.22	0.81

Source: Statistics Canada

† The parameter estimates are for annual data and have to be adjusted appropriately.

Table 3: Variance-covariance matrix of productivity innovations

Recreation	2.42	0.95	0.61	1.17	0.53	0.81	1.22	0.08
House Operation		1.9	0.41	1.28	1.01	0.91	0.68	0.14
Health			4.32	0.27	0.20	0.79	0.55	0.78
Clothing				4.33	0.69	0.82	0.18	-0.03
Alcohol Bev. & Tobacco					7.16	0.46	0.98	-0.98
Food						3.72	0.23	0.18
Transportation							3.46	0.18
Shelter								2.54

Source: Statistics Canada
Only the upper diagonal elements are shown
Variance-covariance parameter estimates multiplied by 10,000

Table 4: Historical Taylor rule estimates for Canada, 1980.1 to 2008.4.

χ_R	χ_π	χ_c	R^2
0.88 (23.88)	4.15 (2.79)	0.89 (2.72)	0.92

Note: The parameters values are followed by the t-statistic in brackets.

Table 5: Mapping from NACIS sectors into CPI sectors.

CPI sectors	NACIS-Sectors
Food	Crop and animal production Food manufacturing Accommodation and food services
Shelter	Finance insurance real estate and renting and leasing Construction
Household operation and furnishings	Electric power generation, transmission and distribution Natural gas distribution, water and others Personal and laundry services and private households Furniture and related product manufacturing Electrical equipment appliance and component manufacturing Waste management and remediation services
Clothes and footwear	Textile and textile product mills Clothing manufacturing Leather and allied product manufacturing
Transportation	Air, rail, water and scenic and sightseeing transportation Transit and ground passenger transportation
Health and personal care	Health care and social assistance (except hospitals)
Recreation, education and reading	Computer and electronic product manufacturing Paper manufacturing Motion picture and sound recording industries Broadcasting and telecommunications Publishing industries, information services and data processing services Printing and related support activities Arts, entertainment and recreation
Alcohol Bev. & Tobacco	Beverage and tobacco product manufacturing

Table 6: Calibration: Baseline / common parameter values.

Parameter	Content	Value
β	Time discounting – annual interest 4.1%	0.99
σ	Intertemporal Rate of Substitution	1
α	Preference for leisure	13.94
η	Inverse of the Frisch elasticity	1
ρ	Elasticity of substitution between sectors	0
θ	Elasticity of substitution between interm. goods	7.76
ρ_u	AR1 coef. of the labour wedge	0.92
σ_{ε_u}	St. dev. of residuals from AR(1) process for labour wedge	0.32 %

Table 7: Data and model moments of CPI inflation

Inflation moments, (1980q1-2008q4)	Data	Model
St. deviation of annualized inflation, percent	0.81	1.24
AR1 coefficient of inflation	0.82	0.83

Table 8: Benchmark results

Parameter	Description	Optimized PT rule	Optimized IT rule	Historical IT rule
	Target weight on:			
φ_1	Recreation	27	31	11
φ_2	House Operation	25	28	11
φ_3	Health	10	9	5
φ_4	Clothing	11	7	7
φ_5	Alcohol & Tobacco	4	6	4
φ_6	Food	10	10	17
φ_7	Transportation	8	7	19
φ_8	Shelter	6	4	27
	Taylor coefficient on:			
χ_R	lagged interest rate	0	0	0.88
χ_π	inflation	-	35.56	4.15
χ_P	price level	3.48	-	-
χ_c	output gap	0.03	0.13	0.89
$\bar{\pi}$	CPI inflation rate	0.19 %	0.19 %	2.00 %
LTCE	Welfare loss†	0.0336 %	0.0336 %	0.2067 %

† Relative to flexible price model in % of steady state consumption.

Table 9: Counterfactual experiments: optimal weights

	Recr	HseOp	Hlth	Clth	Alc&Tob	Food	Trans	Shelt
CPI	11	11	5	7	4	17	19	27
Benchmark PT	27	25	10	11	4	10	8	6
Counterfactual								
Average Growth	27	27	10	10	6	10	7	5
No cross-correlation	25	25	11	10	5	11	7	6
Identical productivity process	27	27	9	10	7	9	6	5
Average Stickiness	11	11	5	6	4	17	20	26
Aoki's Case	100	0	0	0	0	0	0	0

The numbers may not add up to 100 due to rounding

Table 10: Welfare comparison: Optimized PT rule and PT rule with CPI weights.

Parameter	Description	Optimized PT rule	PT with CPI weights
	Target weight on:		
φ_1	Recreation	27	11
φ_2	House Operation	25	11
φ_3	Health	10	5
φ_4	Clothing	11	7
φ_5	Alcohol & Tobacco	4	4
φ_6	Food	10	17
φ_7	Transportation	8	19
φ_8	Shelter	6	27
	Taylor coefficient on:		
χ_R	lagged interest rate	0	0
χ_P	price level	3.48	3.98
χ_c	output gap	0.03	0.22
LTCE	Welfare loss†	0.0336 %	0.0386 %

† Relative to flexible price model in % of steady state consumption.

Table 11: Welfare comparison: PT rule with BEST, CPI, and WORST weights.

Description	Optimized PT rule	CPI Weights	Worst Weights
Target weight on:			
Recr	27	11	0
HseOp	25	11	0
Health	10	5	0
Cloth	11	7	0
Alc&Tob	4	4	100
Food	10	17	0
Trans	8	19	0
Shelt	6	27	0
Taylor coefficient on:			
lagged interest rate	0	0	0
price level	3.48	3.48	3.48
output gap	0.03	0.03	0.03
Welfare loss†	0.0336 %	0.0386 %	0.1131 %

† Relative to flexible price model in % of steady state consumption

Table 12: Volatility comparison: PT rule with BEST, CPI, and WORST weights.

Standard deviation of:	Optimized PT with: BEST weights	CPI weights	WORST weights
CPI Inflation	1.44	0.25	6.69
Output gap	3.86	3.86	3.86
Interest rate	0.63	0.58	1.54
Change in interest rate	0.39	0.32	0.55

All numbers in percent at annualized rates

A3. Technical appendix

A1. First order conditions of the different decision makers

Household

The first order conditions of the household are given by:

$$C : \beta^t u_C(t) = \lambda_t$$

$$B_t : \lambda_t / P_t = (1 + i_t) E_t \frac{\lambda_{t+1}}{P_{t+1}}$$

$$N_t : -\beta^t u_N(t) = \frac{W_t}{P_t} \lambda_t$$

$$C_t + \frac{B_t}{P_t} \leq \frac{W_t}{P_t} N_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + T_t$$

$$C_t = \left(\sum_{j=1}^{J} \mu_j (C_{j,t})^{\rho} \right)^{1/\rho}$$

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t \mu_j} \right)^{1/(\rho-1)} C_t$$

$$P_t = \left(\sum_{j=1}^{J} (\mu_j)^{1/(1-\rho)} (P_{j,t})^{\rho/(\rho-1)} \right)^{(\rho-1)/\rho}$$

and the Transversality Condition is:

$$\lim_{T \to \infty} \beta^T u_C(C_T, N_T) B_T = 0.$$

The first four FOC's can be transformed into:

$$-u_N(t) = \frac{W_t}{P_t} u_C(t)$$

$$1 = E_t \left[(1 + i_t) \frac{P_t}{P_{t+1}} \frac{\beta u_C(t+1)}{u_C(t)} \right]$$

$$C_t + \frac{B_t}{P_t} \leq \frac{W_t}{P_t} N_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + T_t.$$

Consider the assumed functional form:

$$u(C_t, N_t) = \frac{(C_t)^{1-\sigma}}{1-\sigma} - \alpha \frac{(N_t)^{1+\eta}}{1+\eta},$$

then the FOC's reduce to:

$$\alpha (N_t)^{\eta} = \frac{W_t}{P_t} (C_t)^{-\sigma}$$

$$1 = \beta (1 + i_t) E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right]$$

$$C_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + T_t.$$

Production side

Final good production

The key FOC's are:

$$\begin{aligned} c_{j,t}(i) &= \left(\frac{p_{j,t}(i)}{P_{j,t}} \right)^{-\theta} C_{j,t} = \left(\frac{p_{j,t}(i)}{P_{j,t}} \right)^{-\theta} \left(\frac{P_{j,t}}{\mu_j P_t} \right)^{1/(\rho-1)} C_t \\ &= \left(\frac{p_{j,t}(i)}{P_t} \right)^{-\theta} \left(\frac{1}{\mu_j} \right)^{1/(\rho-1)} \left(\frac{P_{j,t}}{P_t} \right)^{1/(\rho-1)+\theta} C_t \end{aligned}$$

$$P_{j,t} = \left(\int_0^1 (p_{j,t}(i))^{1-\theta} \, di \right)^{1/(1-\theta)},$$

$$P_t = \left(\sum_{j=1}^{J} (\mu_j)^{1/(1-\rho)} (P_{j,t})^{\rho/(\rho-1)} \right)^{(\rho-1)/\rho}.$$

The intermediate good production

Using the definition of profits and the demand function:

$$
\begin{aligned}
\Pi_t &= \frac{p_{j,t}(i)}{P_t} c_{j,t}(i) - \left(\frac{W_t}{P_t}\right) n_{j,t}(i) \\
&= \left(\frac{p_{j,t}(i)}{P_t} - \psi_{j,t}\right) c_{j,t}(i) \\
&= \left(\frac{p_{j,t}(i)}{P_t} - \psi_{j,t}\right) \left(\frac{p_{j,t}(i)}{P_{j,t}}\right)^{-\theta} C_{j,t} \\
&= \left(\frac{p_{j,t}(i)}{P_t} - \psi_{j,t}\right) \left(\frac{p_{j,t}(i)}{P_t}\right)^{-\theta} (\mu_j)^{\theta} \left(\frac{P_{j,t}}{\mu_j P_t}\right)^{1/(\rho-1)+\theta} C_t \\
&= \left(\left(\frac{p_{j,t}(i)}{P_t}\right)^{1-\theta} - \psi_{j,t} \left(\frac{p_{j,t}(i)}{P_t}\right)^{-\theta}\right) (\mu_j)^{\theta} \left(\frac{P_{j,t}}{\mu_j P_t}\right)^{1/(\rho-1)+\theta} C_t
\end{aligned}
$$

The firm's profit maximization problem in a given period t then becomes:

$$
\max_{p_{j,t}(i)} E_t \sum_{\tau=0}^{\infty} \omega_j^{\tau} \Delta_{t,t+\tau} \left(\left(\frac{p_{j,t}(i)}{P_{t+\tau}}\right)^{1-\theta} - \psi_{j,t+\tau} \left(\frac{p_{j,t}(i)}{P_{t+\tau}}\right)^{-\theta}\right) (\mu_j)^{\theta} \left(\frac{P_{j,t+\tau}}{\mu_j P_{t+\tau}}\right)^{1/(\rho-1)+\theta} C_{t+\tau}
$$

Notice that all firms from the same sector j that are allowed to adjust their prices in a given period are identical and thus will choose the same optimal price $p_{j,t}^*$. Then the first order condition is:

$$
E_t \sum_{\tau=0}^{\infty} \omega_j^{\tau} \Delta_{t,t+\tau} \left((1-\theta)\left(\frac{1}{P_{t+\tau}}\right)^{1-\theta} + \theta \left(p_{j,t}^*\right)^{-1} \psi_{j,t+\tau} \left(\frac{1}{P_{t+\tau}}\right)^{-\theta}\right) (\mu_j)^{\theta} \left(\frac{P_{j,t+\tau}}{\mu_j P_{t+\tau}}\right)^{1/(\rho-1)+\theta} C_{t+\tau} = 0,
$$

or after some re-arrangements

$$
\frac{p_{j,t}^*}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{\tau=0}^{\infty} (\omega\beta)^{\tau} (C_{t+\tau})^{1-\sigma} \left(\psi_{j,t+\tau} \left(\frac{P_t}{P_{t+\tau}}\right)^{-\theta}\right) \left(\frac{P_{j,t+\tau}}{P_{t+\tau}}\right)^{1/(\rho-1)+\theta}}{E_t \sum_{\tau=0}^{\infty} (\omega\beta)^{\tau} (C_{t+\tau})^{1-\sigma} \left(\frac{P_t}{P_{t+\tau}}\right)^{1-\theta} \left(\frac{P_{j,t+\tau}}{P_{t+\tau}}\right)^{1/(\rho-1)+\theta}}.
$$

A2. Equilibrium conditions

We start by summarizing the PT equations that characterize the equilibrium for these aggregates. From the labor supply equation, we are able to obtain:

$$\sum_{j=1}^{J} \int_{0}^{1} n_{j,t}\left(i\right) di = N_t.$$

Define: $N_{j,t} = \int_0^1 n_{j,t}\left(i\right) di$. With the re-arrangements below:

$$\begin{aligned}
N_{j,t} &= \int_0^1 n_{j,t}\left(i\right) di \\
N_{j,t} &= \int_0^1 \frac{c_{j,t}\left(i\right)}{Z_{j,t}} di \\
N_{j,t} &= \int_0^1 \left(\frac{p_{j,t}\left(i\right)}{P_{j,t}}\right)^{-\theta} \frac{C_{j,t}}{Z_{j,t}} di
\end{aligned}$$

we obtain

$$\begin{aligned}
\frac{N_{j,t}Z_{j,t}}{C_{j,t}} &= \int_0^1 \left(\frac{p_{j,t}\left(i\right)}{P_{j,t}}\right)^{-\theta} di \\
\frac{N_{j,t}Z_{j,t}}{C_{j,t}} &= \left(1-\omega_j\right)\left(\frac{p^*_{j,t}}{P_{j,t}}\right)^{-\theta} + \omega_j \left(\frac{P_{j,t}}{P_{j,t-1}}\right)^{\theta} \frac{N_{j,t-1}Z_{j,t-1}}{C_{j,t-1}},
\end{aligned}$$

as well as:

$$\sum_{j=1}^{J} N_{j,t} = N_t.$$

Using the last equation together with

$$\frac{C_{j,t}}{C_t}\left(\frac{P_{j,t}}{\mu_j P_t}\right)^{-1/(\rho-1)} = 1$$

$$Z_{j,t}\psi_{j,t} = Z_{l,t}\psi_{l,t} = \frac{W_t}{P_t}; \forall l, j$$

we can get:

$$\begin{aligned}\sum_{j=1}^{J}\frac{N_{j,t}}{N_t}\frac{C_t}{C_{j,t}}\left(\frac{P_{j,t}}{\mu_j P_t}\right)^{1/(\rho-1)}\frac{Z_{j,t}\psi_{j,t}}{Z_{1,t}\psi_{1,t}} &= 1\\ \sum_{j=1}^{J}\frac{N_{j,t}Z_{j,t}}{C_{j,t}}\left(\frac{P_{j,t}}{\mu_j P_t}\right)^{1/(\rho-1)}\psi_{j,t} &= \frac{N_t Z_{1,t}\psi_{1,t}}{C_t}.\end{aligned}$$

Next, we consider the relationship between the aggregate and the sectoral price levels:

$$1=\left(\sum_{j=1}^{J}(\mu_j)^{1/(1-\rho)}\left(\frac{P_{j,t}}{P_t}\right)^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$$

and we also have:

$$\begin{aligned}C_{j,t} &= \left(\frac{P_{j,t}}{\mu_j P_t}\right)^{1/(\rho-1)}C_t\\ \frac{N_{j,t}Z_{j,t}}{x_{j,t}} &= \left(\frac{P_{j,t}}{\mu_j P_t}\right)^{1/(\rho-1)}C_t.\end{aligned}$$

Then, we make use of the optimal pricing equation to derive three relationships:

$$\frac{p^*_{j,t}}{P_t}=\frac{\theta}{\theta-1}\frac{E_t\sum_{\tau=0}^{\infty}(\omega_j\beta)^{\tau}(C_{t+\tau})^{1-\sigma}\left(\psi_{j,t+\tau}\left(\frac{P_t}{P_{t+\tau}}\right)^{-\theta}\right)\left(\frac{P_{j,t+\tau}}{P_{t+\tau}}\right)^{1/(\rho-1)+\theta}}{E_t\sum_{\tau=0}^{\infty}(\omega_j\beta)^{\tau}(C_{t+\tau})^{1-\sigma}\left(\frac{P_t}{P_{t+\tau}}\right)^{1-\theta}\left(\frac{P_{j,t+\tau}}{P_{t+\tau}}\right)^{1/(\rho-1)+\theta}},$$

or re-written:

$$\frac{p^*_{j,t}}{P_t}=\frac{\theta}{(\theta-1)}\frac{\Lambda_{j,t}}{\Gamma_{j,t}}$$

$$\Lambda_{j,t}=E_t\sum_{\tau=0}^{\infty}(\omega_j\beta)^{\tau}(C_{t+\tau})^{1-\sigma}\left(\psi_{j,t+\tau}\left(\frac{P_t}{P_{t+\tau}}\right)^{-\theta}\right)\left(\frac{P_{j,t+\tau}}{P_{t+\tau}}\right)^{1/(\rho-1)+\theta}$$

$$\Gamma_{j,t}=E_t\sum_{\tau=0}^{\infty}(\omega_j\beta)^{\tau}(C_{t+\tau})^{1-\sigma}\left(\frac{P_t}{P_{t+\tau}}\right)^{1-\theta}\left(\frac{P_{j,t+\tau}}{P_{t+\tau}}\right)^{1/(\rho-1)+\theta}.$$

We can write the last two equations in recursive form

$$\Lambda_{j,t} = (C_t)^{1-\sigma} \psi_{j,t} \left(\frac{P_{j,t}}{P_t} \right)^{1/(\rho-1)+\theta} + \omega_j \beta E_t \left[\pi_{t+1}^{\theta} \Lambda_{j,t+1} \right]$$

$$\Gamma_{j,t} = (C_t)^{1-\sigma} \left(\frac{P_{j,t}}{P_t} \right)^{1/(\rho-1)+\theta} + \omega_j \beta E_t \left[\pi_{t+1}^{\theta-1} \Gamma_{j,t+1} \right].$$

Furthermore, we use the aggregate price updating formula under Calvo to get:

$$\begin{aligned} (P_{j,t})^{1-\theta} &= (1-\omega_j) \left(p_{j,t}^{*} \right)^{1-\theta} + \omega_j \left(P_{j,t-1} \right)^{1-\theta} \\ 1 &= (1-\omega_j) \left(\frac{p_{j,t}^{*}}{P_{j,t}} \right)^{1-\theta} + \omega_j \left(\frac{P_{j,t-1}}{P_{j,t}} \right)^{1-\theta}. \end{aligned}$$

Making use of the Euler equation, we find:

$$1 = \beta \left(1 + i_t\right) E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right]$$

Then from the labor-consumption first order condition in the household's problem:

$$\alpha \left(N_t\right)^{\eta} = \psi_{j,t} Z_{j,t} \left(C_t\right)^{-\sigma}$$

where we substitute in $\psi_{j,t} Z_{j,t} = W_t / P_t$.

The model is closed by the Taylor-Rule, which in the case of IT is:

$$\begin{aligned} \log \left(\frac{1+i_t}{1+\bar{\imath}} \right) &= \chi_R \log \left(\frac{1+i_{t-1}}{1+\bar{\imath}} \right) + (1-\chi_R) \left(\chi_{\pi} \log \left(\frac{\tilde{\pi}_t}{\tilde{\pi}} \right) + \chi_c \log \left(\frac{C_t}{\bar{C}_t} \right) \right) + \varepsilon_t \\ \log \left(\tilde{\pi}_t \right) &= \sum_{j+1}^{J} \varphi_j \log \left(\pi_{j,t} \right) \end{aligned}$$

Now, we have all the equations that we need. The main adjustment we have to make is a change of variables. The following transformations are used: $x_{j,t} \equiv \frac{N_{j,t}Z_{j,t}}{C_{j,t}}$, $q_{j,t} \equiv \frac{p_{j,t}}{P_t}$, $u_{j,t} \equiv \frac{P_{j,t}}{P_t}$ and $\pi_{j,t} \equiv \frac{P_{j,t}}{P_{j,t-1}}$. The first variable is the endogenous state variable that measures the degree of distortion in the economy. The second variable shows how far the optimal reset price is away from the prevailing price level. The last variable is just the definition of inflation. From this we get, for all sectors j:

$$q_{j,t} = \frac{\theta}{(\theta-1)} \frac{\Lambda_{j,t}}{\Gamma_{j,t}}$$

$$\Lambda_{j,t} = (C_t)^{1-\sigma} \psi_{j,t} (u_{j,t})^{1/(\rho-1)+\theta} + \omega_j \beta E_t \left[\pi_{t+1}^{\theta} \Lambda_{j,t+1}\right]$$

$$\Gamma_{j,t} = (C_t)^{1-\sigma} (u_{j,t})^{1/(\rho-1)+\theta} + \omega_j \beta E_t \left[\pi_{t+1}^{\theta-1} \Gamma_{j,t+1}\right]$$

$$1 = (1-\omega_j) \left(\frac{q_{j,t}}{u_{j,t}}\right)^{1-\theta} + \omega_j (\pi_{j,t})^{\theta-1}$$

$$x_{j,t} = (1-\omega_j) \left(\frac{q_{j,t}}{u_{j,t}}\right)^{-\theta} + \omega_j (\pi_{j,t})^{\theta} x_{j,t-1}$$

$$\pi_t = \pi_{j,t} \frac{u_{j,t-1}}{u_{j,t}}$$

$$Z_{j,t}\psi_{j,t} = Z_{l,t}\psi_{l,t}$$

$$1 = \left(\sum_{j=1}^{J} (\mu_j)^{1/(1-\rho)} (u_{j,t})^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$$

$$\alpha (N_t)^{\eta} = \psi_{j,t} Z_{j,t} (C_t)^{-\sigma}$$

$$N_t = \left[\sum_{j=1}^{J} x_{j,t} \left(\frac{1}{\mu_j} u_{j,t}\right)^{1/(\rho-1)} \psi_{j,t}\right] \frac{C_t}{Z_{1,t}\psi_{1,t}}$$

$$1 = \beta (1+i_t) E_t \left[\frac{1}{\pi_{t+1}} \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right]$$

$$\log\left(\frac{1+i_t}{1+\bar{\imath}}\right) = \chi_R \log\left(\frac{1+i_{t-1}}{1+\bar{\imath}}\right) + (1-\chi_R)\left(\chi_\pi \log\left(\frac{\tilde{\pi}_t}{\tilde{\pi}}\right) + \chi_c \log\left(\frac{C_t}{\bar{C}_t}\right)\right) + \varepsilon_t$$

$$\log(\tilde{\pi}_t) = \sum_{j+1}^{J} \varphi_j \log(\pi_{j,t}).$$

The unknowns in this system are: $\left\{\Lambda_{j,t}, \Gamma_{j,t}, q_{j,t}, u_{j,t}, x_{j,t}, \pi_{j,t}, \psi_{j,t}\right\}, \pi_t, i_t, C_t, N_t, \tilde{\pi}_t$. Thus, per period, we have $7J+5$ equations in $7J+5$ unkonwns.

As the next step, we use the equations to analyze the balanced growth path behavior of the economy.

A3. Finding a balanced growth path

We assume that a balanced growth path exists and thus:

$$\begin{aligned}
\pi_j &= \bar{\pi}_j \\
Z_{j,t+1} &= \gamma_j Z_{j,t} \\
C_{t+1}/C_t &= \gamma_c \\
N_{t+1}/N_t &= 1 \\
i_t &= \bar{\imath}
\end{aligned}$$

and the other variables might grow at their own rates. Define

$$
\begin{aligned}
\tilde{u}_{j,t} &\equiv u_{j,t}/\gamma_{u,j}^{t} \\
\tilde{\psi}_{j,t} &\equiv \psi_{j,t}/\gamma_{\psi,j}^{t} \\
\tilde{q}_{j,t} &\equiv q_{j,t}/\gamma_{q,j}^{t} \\
\tilde{\Gamma}_{j,t} &\equiv \Gamma_{j,t}/\gamma_{\Gamma,j}^{t} \\
\tilde{\Lambda}_{j,t} &\equiv \Lambda_{j,t}/\gamma_{\Lambda,j}^{t} \\
\tilde{C}_{t} &= C_{t}/\gamma_{c}^{t}.
\end{aligned}
$$

With these detrended variables we can restate the equilibrium conditions as follows:

$$\gamma_{q,j}^{t}\tilde{q}_{j,t} = \frac{\theta}{(\theta-1)}\frac{\tilde{\Lambda}_{j,t}\gamma_{\Lambda,j}^{t}}{\tilde{\Gamma}_{j,t}\gamma_{\Gamma,j}^{t}}$$

$$\tilde{\Lambda}_{j,t}\gamma_{\Lambda,j}^{t} = \left(\tilde{C}_{t}\gamma_{c}^{t}\right)^{1-\sigma}\tilde{\psi}_{j,t}\gamma_{\psi,j}^{t}\left(\tilde{u}_{j,t}\gamma_{u,j}^{t}\right)^{1/(\rho-1)+\theta} + \omega_{j}\beta E_{t}\left[\pi_{t+1}^{\theta}\tilde{\Lambda}_{j,t+1}\gamma_{\Lambda,j}^{t+1}\right]$$

$$\tilde{\Gamma}_{j,t}\gamma_{\Gamma,j}^{t} = \left(\tilde{C}_{t}\gamma_{c}^{t}\right)^{1-\sigma}\left(\tilde{u}_{j,t}\gamma_{u,j}^{t}\right)^{1/(\rho-1)+\theta} + \omega_{j}\beta E_{t}\left[\pi_{t+1}^{\theta-1}\tilde{\Gamma}_{j,t+1}\gamma_{\Gamma,j}^{t+1}\right]$$

$$1 = (1-\omega_{j})\left(\frac{\tilde{q}_{j,t}\gamma_{q,j}^{t}}{\tilde{u}_{j,t}\gamma_{u,j}^{t}}\right)^{1-\theta} + \omega_{j}\left(\pi_{j,t}\right)^{\theta-1}$$

$$x_{j,t} = (1-\omega_{j})\left(\frac{\tilde{q}_{j,t}\gamma_{q,j}^{t}}{\tilde{u}_{j,t}\gamma_{u,j}^{t}}\right)^{-\theta} + \omega_{j}\left(\pi_{j,t}\right)^{\theta}x_{j,t-1}$$

$$\pi_{t} = \pi_{j,t}\frac{\tilde{u}_{j,t-1}\gamma_{u,j}^{t-1}}{\tilde{u}_{j,t}\gamma_{u,j}^{t}}$$

$$\tilde{Z}_{j,t}\gamma_{j}^{t}\tilde{\psi}_{j,t}\gamma_{\psi,j}^{t} = \tilde{Z}_{l,t}\gamma_{l}^{t}\tilde{\psi}_{l,t}\gamma_{\psi,l}^{t}$$

$$\alpha\left(N_{t}\right)^{\eta} = \tilde{\psi}_{1,t}\gamma_{\psi,1}^{t}\tilde{Z}_{1,t}\gamma_{1}^{t}\left(\tilde{C}_{t}\gamma_{c}^{t}\right)^{-\sigma}$$

$$1=\left(\sum_{j=1}^{J}\left(\mu_j\right)^{1/(1-\rho)}\left(\tilde{u}_{j,t}\gamma_{u,j}^{t}\right)^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$$

$$N_t=\left[\sum_{j=1}^{J}x_{j,t}\left(\frac{1}{\mu_j}\tilde{u}_{j,t}\gamma_{u,j}^{t}\right)^{1/(\rho-1)}\tilde{\psi}_{j,t}\gamma_{\psi,j}^{t}\right]\frac{\tilde{C}_t\gamma_c^t}{\tilde{Z}_{1,t}\gamma_1^t\tilde{\psi}_{1,t}\gamma_{\psi,1}^{t}}$$

$$1=\beta\left(1+i_t\right)E_t\left[\frac{1}{\pi_{t+1}}\left(\frac{\tilde{C}_{t+1}\gamma_c^{t+1}}{\tilde{C}_t\gamma_c^t}\right)^{-\sigma}\right]$$

$$\log\left(\frac{1+i_t}{1+\bar{\imath}}\right)=\chi_R\log\left(\frac{1+i_{t-1}}{1+\bar{\imath}}\right)+\left(1-\chi_R\right)\left(\chi_\pi\log\left(\frac{\tilde{\pi}_t}{\tilde{\pi}}\right)+\chi_c\log\left(\frac{\tilde{C}_t\gamma_c^t}{\bar{C}\gamma_c^t}\right)\right)+\varepsilon_t$$

$$\log\left(\tilde{\pi}_t\right)=\sum_{j=1}^{J}\varphi_j\log\left(\pi_{j,t}\right).$$

On the BGP the above equilibrium conditions imply the following restrictions:

$$1+\bar{\imath}=\frac{\left(\gamma_c\right)^\sigma}{\beta}\pi$$

$$\pi\gamma_u=\pi_j$$

$$\alpha\left(N\right)^\eta=\left(\gamma_{j,\psi}\right)^t\tilde{\psi}_j\left(\gamma_j\right)^t\tilde{Z}_j\left(\left(\gamma_c\right)^t\tilde{C}\right)^{-\sigma}$$

$$\left(\gamma_j\right)^t\tilde{Z}_j\left(\gamma_{j,\psi}\right)^t\tilde{\psi}_j=\left(\gamma_1\right)^t\tilde{Z}_1\left(\gamma_{1,\psi}\right)^t\tilde{\psi}_1$$

$$1=\left(\sum_{j=1}^{J}\left(\mu_j\right)^{1/(1-\rho)}\left(\left(\gamma_{u,j}\right)^t\tilde{u}_j\right)^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$$

$$\left(\gamma_{x,j}\right)^t\left(1-\frac{\omega_j\left(\pi_j\right)^\theta}{\gamma_{x,j}}\right)x_j=\left(1-\omega_j\right)\left(\frac{\left(\gamma_{q,j}\right)^t\tilde{q}_j}{\left(\gamma_{u,j}\right)^t\tilde{u}_j}\right)^{-\theta}$$

$$\omega_j\left(\pi_j\right)^{\theta-1}=1-\left(1-\omega_j\right)\left(\frac{\left(\gamma_{q,j}\right)^t\tilde{q}_j}{\left(\gamma_{u,j}\right)^t\tilde{u}_j}\right)^{1-\theta}$$

$$\left(\gamma_{\Lambda,j}\right)^t\left(1-\omega_j\beta\pi^\theta\gamma_{\Lambda,j}\right)\tilde{\Lambda}_j=\left(\left(\gamma_c\right)^t\tilde{C}\right)^{1-\sigma}\left(\left(\gamma_{j,\psi}\right)^t\tilde{\psi}_j\right)\left(\left(\gamma_{u,j}\right)^t\tilde{u}_j\right)^{1/(\rho-1)+\theta}$$

$$\left(\gamma_{\Gamma,j}\right)^t\left(1-\omega_j\beta\pi^{\theta-1}\left(\gamma_{\Gamma,j}\right)\right)\tilde{\Gamma}_j=\left(\left(\gamma_c\right)^t\tilde{C}\right)^{1-\sigma}\left(\left(\gamma_{u,j}\right)^t\tilde{u}_j\right)^{1/(\rho-1)+\theta}$$

$$\left(\gamma_{q,j}\right)^t\tilde{q}_j=\frac{\theta}{(\theta-1)}\frac{\left(\gamma_{\Lambda,j}\right)^t\tilde{\Lambda}_j}{\left(\gamma_{\Gamma,j}\right)^t\tilde{\Gamma}_j}$$

$$N=\left[\sum_{j=1}^{J}\left(\gamma_{x,j}\right)^t x_j\left(\frac{1}{\mu_j}\tilde{u}_j\left(\gamma_{u,j}\right)^t\right)^{1/(\rho-1)}\left(\gamma_{\psi,j}\right)^t\tilde{\psi}_j\right]\frac{\left(\gamma_c\right)^t\tilde{C}}{\left(\gamma_1\right)^t\tilde{Z}_1\left(\gamma_{\psi,1}\right)^t\tilde{\psi}_1}.$$

From these relationships we conclude the following:

$$\gamma_{q,j}=\gamma_{u,j},$$

which in turn implies:

$$\gamma_{x,j}=1.$$

Now, we make use of the definition of C_t and of x_t :

$$\left(\gamma_c\right)^t\tilde{C}=\left(\sum_{j=1}^{J}\mu_j\left(\left(\gamma_{c,j}\right)^t\tilde{C}_j\right)^{\rho}\right)^{1/\rho}$$

$$x_{j,t}=\frac{N_j\left(\gamma_j^t\right)\tilde{Z}_j}{\left(\gamma_{c,j}\right)^t\tilde{C}_j}.$$

The second equation implies:

$$\gamma_j=\gamma_{c,j}.$$

Using this in the first of the two equations before we obtain:

$$\left(\gamma_c\right)^t=\left(\sum_{j=1}^{J}\mu_j\left(\left(\gamma_j\right)^t\frac{\tilde{C}_j}{\tilde{C}}\right)^{\rho}\right)^{1/\rho}$$

$$1=\left(\sum_{j=1}^{J}\mu_j\left(\left(\frac{\gamma_j}{\gamma_c}\right)^t\frac{\tilde{C}_j}{\tilde{C}}\right)^{\rho}\right)^{1/\rho}.$$

This in general diverging sectoral trends are incompatible with a balanced growth path. For the special case of the Cobb-Douglas aggregator though, i.e. $\rho \to 0$, we are able to get:

$$\gamma_c=\prod\nolimits_{j=0}^{J}\left(\gamma_j\right)^{\mu_j}$$

and thus a balanced growth path is feasible. From here on, we focus on this special case.

Next we get the following relationships:

$$\left(\gamma_j\right)\left(\gamma_{j,\psi}\right)=\left(\gamma_1\right)\left(\gamma_{1,\psi}\right);\forall j$$

$$\left(\gamma_c\right)^{\sigma}=\left(\gamma_{j,\psi}\right)\left(\gamma_j\right);\forall j$$

$$\gamma_{\Lambda,j}=\left(\gamma_c\right)^{(1-\sigma)}\left(\gamma_{\psi,j}\right)\left(\gamma_{u,j}\right)^{(1/(\rho-1)+\theta)}$$

$$\gamma_{\Gamma,j}=\left(\gamma_c\right)^{(1-\sigma)}\left(\gamma_{u,j}\right)^{1/(\rho-1)+\theta}$$

$$\gamma_{q,j}=\frac{\gamma_{\Lambda,j}}{\gamma_{\Gamma,j}}$$

$$1=\left(\sum_{j=1}^{J}\left(\mu_j\right)^{1/(1-\rho)}\left(\left(\gamma_{u,j}\right)^t u_j\right)^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$$

$$1=\left[\sum_{j=1}^{J}\left(\gamma_{x,j}\right)^t\left(\left(\gamma_{u,j}\right)^t\right)^{1/(\rho-1)}\left(\gamma_{\psi,j}\right)^t\right]\frac{\left(\gamma_c\right)^t}{\left(\gamma_1\right)^t\left(\gamma_{\psi,1}\right)^t}.$$

From the last three equations we get:

$$\gamma_{q,j}=\gamma_{\psi,j}(=\gamma_{u,j}).$$

Next, we reduce the system of growth rate equations to:

$$\gamma_{j,u} = \frac{\gamma_1}{\gamma_j}\gamma_{1,u}; \forall j$$

$$\gamma_c = \left[\gamma_{j,u}\gamma_j\right]^{1/\sigma}; \forall j$$

$$\gamma_{\Gamma,j} = (\gamma_c)^{(1-\sigma)}\left(\gamma_{u,j}\right)^{1/(\rho-1)+\theta}$$

$$\gamma_{u,j} = \frac{\gamma_{\Lambda,j}}{\gamma_{\Gamma,j}}$$

$$1 = \left[\sum_{j=1}^{J}\left(\left(\gamma_{u,j}\right)^{\rho/(\rho-1)}\right)^t\right]\left(\frac{\gamma_c}{\gamma_1\gamma_{u,1}}\right)^t$$

$$1 = \prod\nolimits_{j=0}^{J}\left(\left(\gamma_{u,j}\right)^t\frac{u_j}{\mu_j}\right)^{\mu_j}$$

From the second equation we deduce:

$$\frac{(\gamma_c)^\sigma}{\gamma_j} = \gamma_{j,u}; \forall j.$$

From the last equation in the system:

$$1 = \prod\nolimits_{j=0}^{J}\left(\left(\frac{(\gamma_c)^\sigma}{\gamma_j}\right)^t\frac{u_j}{\mu_j}\right)^{\mu_j}$$

we obtain

$$1 = (\gamma_c)^\sigma\left(\prod\nolimits_{j=0}^{J}(\gamma_j)^{\mu_j}\right)^{-1}.$$

Finally, to check the correctness of the analysis, we check the only unused equation:

$$1 = \left[\sum_{j=1}^{J}\left(\left(\gamma_{u,j}\right)^{\rho/(\rho-1)}\right)^t\right]\left(\frac{\gamma_c}{\gamma_1\gamma_{u,1}}\right)^t.$$

This equation indicates, as before, that:

$$\gamma_c = \gamma_1 \gamma_{u,1}$$

which in turn confirms our conclusion that for the balanced growth path to exist requires that $\rho \rightarrow 0$ and $\sigma \rightarrow 1$. Furthermore, we can conclude that:

$$\gamma_{u,j} = \left(\prod\nolimits_{j=0}^{J} (\gamma_j)^{\mu_j} \right) / \gamma_j, \forall j.$$

Regarding the last two growth rates, we find that:

$$\begin{aligned} \gamma_{\Gamma,j} &= \left(\left(\prod\nolimits_{j=0}^{J} (\gamma_j)^{\mu_j} \right) / \gamma_j \right)^{1/(\rho-1)+\theta} \\ \gamma_{\Lambda,j} &= \left(\left(\prod\nolimits_{j=0}^{J} (\gamma_j)^{\mu_j} \right) / \gamma_j \right)^{\rho/(\rho-1)+\theta}. \end{aligned}$$

Using these findings, we can solve for the variables on the balanced growth path using the next equation system:

$$1 + \bar{\imath} = \frac{(\gamma_c)^{\sigma}}{\beta} \pi$$

$$\pi \gamma_{u,j} = \pi_j$$

$$\tilde{Z}_j \tilde{\psi}_j = \tilde{Z}_1 \tilde{\psi}_{1,t}$$

$$\alpha (N)^{\eta} = \tilde{\psi}_j \tilde{Z}_j \left(\tilde{C} \right)^{-\sigma}$$

$$N = \left[\sum_{j=1}^{J} x_j \left(\frac{1}{\mu_j} \tilde{u}_j \right)^{-1} \tilde{\psi}_j \right] \frac{\tilde{C}}{\tilde{Z}_1 \tilde{\psi}_1}$$

$$\frac{\tilde{q}_j}{\tilde{u}_j} = \left(\frac{1 - \omega_j \left(\pi_j\right)^{\theta - 1}}{1 - \omega_j} \right)^{1/(1-\theta)}$$

$$x_j = \frac{(1 - \omega_j)}{\left(1 - \omega_j \left(\pi_j\right)^{\theta}\right)} \left(\frac{\tilde{q}_j}{\tilde{u}_j} \right)^{-\theta}$$

$$\tilde{q}_j = \frac{\theta}{(\theta - 1)} \frac{\tilde{\Lambda}_j}{\tilde{\Gamma}_j}$$

$$\left(1 - \omega_j \beta \pi^{\theta} \left(\gamma_{\Lambda,j}\right)\right) \tilde{\Lambda}_j = \left(\tilde{C}\right)^{1-\sigma} \left(\tilde{\psi}_j\right) \left(\tilde{u}_j\right)^{1/(\rho-1)+\theta}$$

$$\left(1 - \omega_j \beta \pi^{\theta-1} \left(\gamma_{\Gamma,j}\right)\right) \tilde{\Gamma}_{j,t} = \left(\tilde{C}\right)^{1-\sigma} \left(\tilde{u}_j\right)^{1/(\rho-1)+\theta}$$

$$1 = \prod_{j=0}^{J} \left(\frac{\tilde{u}_j}{\mu_j} \right)^{\mu_j}$$

with

$$\gamma_c = \prod_{j=0}^{J} \left(\gamma_j\right)^{\mu_j}$$

$$\gamma_{u,j} = \gamma_{\psi,j} = \gamma_{q,j} = \gamma_c / \gamma_j, \forall j$$

$$\gamma_{\Gamma,j} = \left(\gamma_{u,j}\right)^{1/(\rho-1)+\theta}$$

$$\gamma_{\Lambda,j} = \left(\gamma_{u,j}\right)^{\rho/(\rho-1)+\theta}.$$

Next, we solve for the BGP allocation. Right away, we determine the following variables:

$$1 + \bar{\imath} = \frac{\gamma_c}{\beta} \bar{\pi}$$

$$\pi_j = \bar{\pi} \gamma_{u,j}$$

$$\tilde{Z}_j \tilde{\psi}_j = \tilde{Z}_1 \tilde{\psi}_1$$

$$\frac{\tilde{q}_j}{\tilde{u}_j}=\left(\frac{1-\omega_j\left(\pi_j\right)^{\theta-1}}{1-\omega_j}\right)^{1/(1-\theta)}$$

$$x_j=\frac{\left(1-\omega_j\right)}{\left(1-\omega_j\left(\pi_j\right)^{\theta}\right)}\left(\frac{\tilde{q}_j}{\tilde{u}_j}\right)^{-\theta}.$$

Then we can also determine $\frac{\tilde{q}_j}{\tilde{\psi}_j}$:

$$\frac{\tilde{q}_j}{\tilde{\psi}_j}=\frac{\theta}{(\theta-1)}\frac{\tilde{\Lambda}_j}{\tilde{\psi}_j\tilde{\Gamma}_j}$$

using the expressions

$$\left(1-\omega_j\beta\pi^{\theta}\left(\gamma_{\Lambda,j}\right)\right)\tilde{\Lambda}_j=\left(\tilde{C}\right)^{1-\sigma}\left(\tilde{\psi}_j\right)\left(\tilde{u}_j\right)^{1/(\rho-1)+\theta}$$

$$\left(1-\omega_j\beta\pi^{\theta-1}\left(\gamma_{\Gamma,j}\right)\right)\tilde{\Gamma}_{j,t}=\left(\tilde{C}\right)^{1-\sigma}\left(\tilde{u}_j\right)^{1/(\rho-1)+\theta}$$

for $\tilde{\Lambda}$ and $\tilde{\Gamma}$. We find:

$$\frac{\tilde{q}_j}{\tilde{\psi}_j}=\frac{\theta}{(\theta-1)}\frac{\left(1-\omega_j\beta\pi^{\theta-1}\left(\gamma_{\Gamma,j}\right)\right)}{\left(1-\omega_j\beta\pi^{\theta}\left(\gamma_{\Lambda,j}\right)\right)},$$

and thus,

$$\frac{\tilde{\psi}_j}{\tilde{u}_j}=\frac{\frac{\tilde{q}_j}{\tilde{u}_j}}{\frac{\tilde{q}_j}{\tilde{\psi}_j}}=\frac{\theta-1}{\theta}\frac{\left(\frac{1-\omega_j\left(\pi_j\right)^{\theta-1}}{1-\omega_j}\right)^{1/(1-\theta)}}{\frac{\left(1-\omega_j\beta\pi^{\theta-1}\left(\gamma_{\Gamma,j}\right)\right)}{\left(1-\omega_j\beta\pi^{\theta}\left(\gamma_{\Lambda,j}\right)\right)}}.$$

Defining

$$f_j=\frac{\theta-1}{\theta}\frac{\left(\frac{1-\omega_j\left(\pi_j\right)^{\theta-1}}{1-\omega_j}\right)^{1/(1-\theta)}}{\frac{\left(1-\omega_j\beta\pi^{\theta-1}\left(\gamma_{\Gamma,j}\right)\right)}{\left(1-\omega_j\beta\pi^{\theta}\left(\gamma_{\Lambda,j}\right)\right)}}$$

we can use the equalities $\tilde{Z}_j\tilde{\psi}_j=\tilde{Z}_1\tilde{\psi}_1$ to get:

$$\tilde{u}_j=\frac{\tilde{Z}_1 f_1}{\tilde{Z}_j f_j}\tilde{u}_1$$

which gives us $\tilde{u}_1$

$$\tilde{u}_1 = \left(\prod_{j=0}^{J} \left(\frac{1}{\mu_j}\frac{\tilde{Z}_1 f_1}{\tilde{Z}_j f_j}\right)^{\mu_j}\right)^{-1},$$

and thus, all the other $\tilde{u}_j$.Then we can reverse the process and obtain:

$$\begin{aligned} \tilde{\psi}_j &= \frac{\tilde{\psi}_j}{\tilde{u}_j}\tilde{u}_j \\ \tilde{q}_j &= \frac{\tilde{q}_j}{\tilde{u}_j}\tilde{u}_j. \end{aligned}$$

All this in hand, we are now ready to solve for the aggregate variables, $\tilde{C}, N$:

$$\alpha\,(N)^{\eta} = \tilde{\psi}_j \tilde{Z}_j \left(\tilde{C}\right)^{-\sigma}$$

$$N = \left[\sum_{j=1}^{J} x_j \left(\frac{\mu_j}{\tilde{u}_j}\right)\tilde{\psi}_j\right]\frac{\tilde{C}}{\tilde{Z}_1\tilde{\psi}_1}$$

which leads to:

$$\tilde{C} = \left(\tilde{\psi}_j \tilde{Z}_j/\alpha \left(\left[\sum_{j=1}^{J} x_j \left(\frac{\mu_j}{\tilde{u}_j}\right)\tilde{\psi}_j\right]\frac{1}{\tilde{Z}_1\tilde{\psi}_1}\right)^{\eta}\right)^{1/(\eta+\sigma)}$$

$$N = \left(\frac{\tilde{\psi}_j \tilde{Z}_j}{\alpha \tilde{C}^{\sigma}}\right)^{1/\eta}.$$

A4. Transforming annual frequency processes into quarterly

Both the sectoral productivity processes and the cost-push shocks are estimated using annual frequency data. In this appendix, we show how to derive the proper quarterly-frequency counterparts of the AR1 processes estimated from the annual data.

Suppose we have a quarterly-frequency stochastic process of the productivity in sector j, given by $Z_{j,t}$, which has a growth component growing at the rate γ_j and a cyclical component

$$\log \tilde{Z}_{j,t} = \rho_j \log \tilde{Z}_{j,t-1} + \varepsilon_{j,t}.$$

So that $Z_{j,t} = Z_{j,0}\left(\gamma_j^t\right)\tilde{Z}_{j,t}$. Taking the logarithm we get

$$\log Z_{j,t} = \log Z_{j,0} + t\log\gamma_j + \rho_j \log \tilde{Z}_{j,t-1} + \varepsilon_{j,t}. \tag{A1}$$

We allow for cross-sector correlation of the productivity innovations $\varepsilon_{j,t}$:

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{J,t} \end{bmatrix} \sim N\left(\mathbf{0}_{J\times 1}, \Omega_{J\times J}\right)$$

where the variance-covariance matrix is

$$\Omega_{J\times J} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1J}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2J}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{J1}^2 & \sigma_{J2}^2 & \cdots & \sigma_{JJ}^2 \end{bmatrix}.$$

Denote $z_t = \begin{bmatrix} \log Z_{1,t} \\ \log Z_{2,t} \\ \vdots \\ \log Z_{J,t} \end{bmatrix}$, $\tilde{z}_t = \begin{bmatrix} \log \tilde{Z}_{1,t} \\ \log \tilde{Z}_{2,t} \\ \vdots \\ \log \tilde{Z}_{J,t} \end{bmatrix}$, $\Gamma = \begin{bmatrix} \log\gamma_1 \\ \log\gamma_2 \\ \vdots \\ \log\gamma_J \end{bmatrix}$, and $R_{J\times J} = \begin{bmatrix} \rho_1 & 0 & \cdots & 0 \\ 0 & \rho_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \rho_J \end{bmatrix}$.
Then we can represent the stochastic process (A1) as follows

$$\begin{aligned} z_t &= z_0 + t\Gamma + \tilde{z}_t \\ \tilde{z}_t &= R\tilde{z}_{t-1} + \varepsilon_t \end{aligned}$$

Suppose we have only annual-frequency observations of this process, in quarters $t-1$, $t+3$, $t+7$,...

Iterating on the quarterly process we get

$$
\begin{aligned}
z_t &= z_0 + t\Gamma + R\tilde{z}_{t-1} + \varepsilon_t \\
z_{t+1} &= z_0 + (t+1)\,\Gamma + R\tilde{z}_t + \varepsilon_{t+1} \\
z_{t+2} &= z_0 + (t+2)\,\Gamma + R\tilde{z}_{t+1} + \varepsilon_{t+2} \\
z_{t+3} &= z_0 + (t+3)\,\Gamma + R\tilde{z}_{t+2} + \varepsilon_{t+3}.
\end{aligned}
$$

Thus

$$
\begin{aligned}
z_{t+3} &= z_0 + (t+3)\,\Gamma + R\tilde{z}_{t+2} + \varepsilon_{t+3} \\
&= z_0 + (t+3)\,\Gamma + R^2\tilde{z}_{t+1} + R\varepsilon_{t+2} + \varepsilon_{t+3} \\
&= z_0 + (t+3)\,\Gamma + R^3\tilde{z}_t + R^2\varepsilon_{t+1} + R\varepsilon_{t+2} + \varepsilon_{t+3} \\
&= z_0 + (t+3)\,\Gamma + R^4\tilde{z}_{t-1} + R^3\varepsilon_t + R^2\varepsilon_{t+1} + R\varepsilon_{t+2} + \varepsilon_{t+3}.
\end{aligned}
$$

Comparing this to $z_{t-1} = z_0 + (t-1)\,\Gamma + \tilde{z}_{t-1}$, we can see that the unconditional expectation of the annual (log) growth rate is

$$E\left[z_{t+3} - z_{t-1}\right] = 4\Gamma$$

This means we can take the average annual growth rates to the power $\frac{1}{4}^{th}$ to get the quarterly growth rate estimates γ_j. Comparing the cyclical parts we can see that

$$\tilde{z}_{t+3} = R^4\tilde{z}_{t-1} + R^3\varepsilon_t + R^2\varepsilon_{t+1} + R\varepsilon_{t+2} + \varepsilon_{t+3},$$

which implies, first, that we should take the annual estimate of the persistence matrix R^{annual} to the power $\frac{1}{4}^{th}$ to get the quarterly persistence matrix R; second, the variance covariance matrix of the annual process is

$$\Omega^{annual} = \left(R^3 \Omega \left(R^T \right)^3 + R^2 \Omega \left(R^T \right)^2 + R \Omega R^T + \Omega \right).$$

We solve the above equation for Ω by first having an initial guess Ω_0 and then iterating the following equation

$$\Omega_{n+1} = \Omega^{annual} - \left(R^3 \Omega_n \left(R^T \right)^3 + R^2 \Omega_n \left(R^T \right)^2 + R \Omega_n R^T \right)$$

until convergence of Ω_n.

With the simpler univariate process of the cost-push shocks, the exactly same logic as above gives rise to: $\rho_u = \left(\rho_u^{annual} \right)^{1/4}$ for the quarterly persistence and

$$\sigma_u^2 = \frac{\left(\sigma_u^{annual} \right)^2}{1 + \rho_u^2 + \rho_u^4 + \rho_u^6}$$

for the standard deviation of the quarterly frequency innovations.