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by Kimberly Beaton

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Abstract

This article investigates the stability of Okun's law for Canada and the United States using a time varying parameter approach. Time variation is modeled as driftless random walks and is estimated using the median unbiased estimator approach developed by Stock and Watson (1998). Okun's law exhibits structural instability in both countries, with the sensitivity of the unemployment rate to movements in output growth increasing recently over time in both Canada and the United States.

JEL classification: E24, J00
Bank classification: Business fluctuations and cycles; Labour markets

Résumé

L'auteure étudie la stabilité de la loi d'Okun au Canada et aux États-Unis au moyen d'un modèle à paramètres variables dans le temps. La variation temporelle est modélisée sous forme de marches aléatoires sans dérive et estimée à l'aide de la méthode d'estimation sans biais de la médiane élaborée par Stock et Watson (1998). La loi d'Okun affiche une instabilité structurelle dans les deux pays, la sensibilité du taux de chômage aux fluctuations de la croissance de la production s'étant graduellement accrue ces derniers temps tant au Canada qu'aux États-Unis.

Classification JEL : E24, J00
Classification de la Banque : Cycles et fluctuations économiques; Marchés du travail

1 Introduction

In the global economic crisis that began in the fall of 2008, unemployment appeared to rise much more rapidly than in previous economic downturns. This study determines whether the data supports this observation and provides updated estimates of the relationship between changes in the unemployment rate and output growth. One reason why this issue is important is because, if employment has become more cyclical, then the unemployment rate may decline more rapidly with the recovery than in the recent past. This paper addresses these questions by estimating Okun's law on data for Canada and the United States using a time-varying parameter (TVP) model.

Several authors have previously examined whether the empirical relationship between the unemployment rate and output has evolved since Okun's (1962) original estimates that found that a 3 percentage point decline in U.S. output is typically associated with a 1 percentage point increase in the unemployment rate. To date, however, no consensus has been reached regarding the extent to which the unemployment rate and output tend to move together. The results of some studies suggest that the Okun relationship is asymmetric over the course of the business cycle with unemployment generally found to move by more in recessions than in expansions (e.g. Crespo-Cuaresma 2003, Huang and Chang 2005, Silvapulle et. al 2004), which would be consistent with the larger response observed in the current recession. Other studies suggest that Okun's law has undergone structural change over time (e.g. Lee 2000, Huang and Chang 2005, Sögner and Stiassny 2002). Moreover, Okun's law has been shown to differ across countries, with a stronger relationship generally found in Canada and the United States compared to Europe and Japan, which have more rigid labour markets (Kaufmann 1988, Lee 2000, Moosa 1997).[1]

If the Okun relationship has changed over time and does differ over the course of the business cycle and across countries, it is important for policymakers to understand the magnitude and direction of these changes and differences. For example, stabilization policies designed to offset the effect of output on unemployment during recessions could benefit from an understanding of asymmetries and nonlinearities in Okun's law. Moreover, if there are asymmetries in Okun's law, as Harris and Silverstone (2001) point out, these asymmetries would strengthen the case for an asymmetrical version of the output gap-based Phillips curve, which could affect the policy decisions taken by central bankers in their attempts to control inflation. Moreover, ignoring asymmetries in the relationship could lead to forecasting errors. Finally, Okun's law provides policymakers with a benchmark to measure the cost associated with a given loss of output in terms of higher unemployment. If this benchmark is estimated incorrectly, it again could lead to policy errors.

[1] This difference in the observed response of the unemployment rate to output across countries has also been attributed to the inflexibility of real wages in Europe and Japan due to high unionization and generous long-term unemployment benefits (Nickell and Bell 1996, Nickell 1997).

The goal of this paper is to contribute to the literature on Okun's law by assessing its stability in Canada and the United States using a TVP model. Previous research generally treats the instability of Okun's law over time and asymmetries in the law over the course of the business cycle as separate issues. Ideally, however, these issues should be examined in a unified framework. This paper thus models Okun's law using a TVP approach that is flexible enough to capture structural changes over time as well as differences over the course of the business cycle. In this way, the TVP approach employed in this paper encompasses and is potentially able to reconcile previous lines of work on the evolution of Okun's law.

This study is most directly comparable to Sögner and Stiassny (2002) who also estimate Okun's law with a TVP model. In their approach, the TVPs of Okun's law are estimated using Maximum Likelihood Estimation (MLE) and the Kalman filter for 15 OECD economies. However, the downside of this approach is that when the period-to-period variation in the parameters is small, the MLE estimates of the parameter variance tend to be biased towards zero. Indeed, they find limited evidence of time variation in Okun's law for Canada and the United States. This paper thus reassesses the stability of Okun's law for these countries using the median unbiased estimator approach developed by Stock and Watson (1998). This approach is explicitly designed to deal with the small variance problem and thus should capture time variation in Okun's law, even when the period-to-period parameter variance is small. To the best of our knowledge, this is the first paper to assess the stability of Okun's law using this approach. This TVP approach is able to simultaneously capture long-run and cyclical changes in Okun's law.

Using this approach, I find that Okun's law exhibits instability in both Canada and the United States. For both countries, the results suggest that the relationship between the unemployment rate and output has strengthened over time. In particular, using quarterly data from 1961:I to 2009:II for Canada and 1948:I-2009:II for the United States, I find that a one percentage point decline in the unemployment rate is now typically associated with a 2.6 and 2.0 per cent decline in output in Canada and the United States, respectively. For both countries this estimated relationship is larger than was originally estimated by Okun. Nevertheless, the magnitude of the increase in the unemployment rate compared to the fall in output in the current recession in both countries has been larger than would be suggested by our TVP model. In the current recession, a 1.6 and a 0.8 percentage point fall in output has been coupled with a 1 percentage point increase in the unemployment rate in Canada and the United States, respectively.[2] However, the stronger observed co-movement between unemployment and output is consistent with the fact that I find evidence of asymmetric behaviour in Okun's law over the business cycle. In particular, I find that the unemployment rate typically increases by more during recessions than it falls during expansions.

[2] For the United States, I assume that the current recession began in 2007Q4, in line with the NBER recession dates. For Canada, I assume that the current recession began in 2008Q3.

This paper proceeds as follows. Section 2 specifies the TVP model and describes its estimation, while Section 3 discusses the data used in the study and presents the results. Section 4 concludes.

2 Empirical Model

In this section, I present the empirical model used to examine Okun's law in Canada and the United States. In particular, Okun's law is generally estimated by:

$$\dot{u}_t = \alpha_0 + \alpha_1 \dot{y}_t + \varepsilon_t \tag{1}$$

Where $\dot{u}_t$ and $\dot{y}_t$ are the quarterly changes in the unemployment rate and output, respectively and ε_t is an error term. Given the time lags involved in the relationship, past studies (e.g. Weber 1995, Sögner and Stiassny 2002) have shown that it is preferable to include additional dynamics in the equation and thus to estimate:

$$\begin{aligned} \dot{u}_t &= \alpha_0 + \alpha_1 \dot{y}_t + \alpha_2 \dot{y}_{t-1} + \varepsilon_t \\ &= \alpha_0 + \alpha_1 \Delta \dot{y}_t + (\alpha_1 + \alpha_2)\dot{y}_{t-1} + \varepsilon_t \end{aligned} \tag{2}$$

which allows the unemployment rate to respond with a delay to output.[3] The short-run effect of output on unemployment is captured by the coefficient α_1, while the total effect is captured by $\alpha_1 + \alpha_2$. These coefficients are referred to as the short-run and total Okun effect, respectively. As pointed out by Sögner and Stiassny (2002) this distributed lag specification also reduces the simultaneous equation bias for the total effect of output on unemployment when output growth is positively autocorrelated. In particular, the bias in the estimated short-run effect, $\hat{\alpha}_1$, is reduced by an opposite bias in $\hat{\alpha}_2$.

Although a model such as (2) can be used to assess whether Okun's law has undergone discrete changes over time, it may have instead evolved slowly over time. For example, structural changes in labour markets may have taken place slowly, suggesting that movements in Okun's law have also been of a more gradual nature than suggested by a discrete break model. Moreover, split-sample estimates used to assess the evolving nature of Okun's law implicitly suggest that all parameters in the model are simultaneously affected by the estimated discrete change. In practice, these parameters may evolve differently.

To overcome these weaknesses of the discrete break model, this paper estimates a TVP version of Equation 2:

$$\dot{u}_t = \alpha_0(t) + \alpha_1(t)\Delta \dot{y}_t + (\alpha_1(t) + \alpha_2(t))\dot{y}_{t-1} + \varepsilon_t \tag{3}$$

which can be simplified to:

$$\dot{u}_t = \phi(t) X_t + \varepsilon_t \tag{4}$$

[3] In practice, additional lags of output may be important for the relationship. This was tested for and the specification in equation 2 was found to be optimal.

The TVPs are jointly captured by ϕ_t in Equation 4, while the corresponding regressors are captured by X_t. Moreover, following Sögner and Stiassny (2002) the TVPs are assumed to follow driftless random walks:

$$\phi(t)_t = \phi(t)_{t-1} + w_t \tag{5}$$

where $E[w_t] = 0$.

As previously mentioned, the appeal of the TVP model is its ability to uncover changes in Okun's law that may have occurred slowly over time. In practice, the TVP model applies weights to all observations, with observations close to time period t having a larger effect than those further away.[4] Therefore, the mid-sample estimates of the parameters are less affected by observations at the beginning and end of the sample than the mid-sample observations. Moreover, unlike a discrete change model, the TVP model is able to separately analyze the evolution of each parameter. Thus, it can distinguish between changes in Okun's law associated with changes in the short-run and total Okun effects. Of course, as pointed out by Boivin (2006), if the change in the relationship is due to a single discrete jump, the TVP model would be mis-specified; however, even in this case the TVP model would provide a useful approximation of the discrete change. In this case, the estimated TVPs would change smoothly over time with the change beginning before the date of the discrete change. Thus, the results should still be consistent with those of a discrete break model.

The TVP model has another advantage with respect to a discrete break model in that it can provide a useful approximation of a discrete break model characterized by multiple discrete changes. Its advantage lies in the fact that estimating a multiple break model with a large number of breaks relative to the sample size or with break dates close to each other may be infeasible. Moreover, considerable uncertainty is involved when estimating the break date in a discrete change model. Thus, it may be advantageous to approximate the multiple break model with a TVP model capable of dealing with these issues.

The parameters of the TVP model, including the variance of w_t can be jointly estimated with MLE using the Kalman filter to construct the likelihood function from its forecast error decomposition representation. The downside to this approach is that when the variance of w_t is small, the MLE has a large point mass at zero. This is referred to as the pile-up problem and causes the MLE estimate of w_t to be biased toward zero.[5] Thus, although statistically significant time variation in the parameters may be present, the parameter variation from one period to the next may be very small and not easily captured by this approach.

[4] This compares to OLS which applies equal weights to all observations.
[5] See Stock and Watson (1998) for more information.

Confronted with this problem, an alternative estimation approach, followed by this paper, is to use the median-unbiased variance estimator approach proposed by Stock and Watson (1998). Designed to overcome the small variance problem, this approach pre-estimates the variance of w_t using the realization of a given stability test and uses the variance estimate in the MLE estimation of the TVPs. This is accomplished by exploiting the fact that, under the alternative of a TVP model, a stability test's distribution depends on the variance of the parameters therefore, if all other parameters can be consistently estimated, an estimate of the variance of the parameters can be inferred from the realization of a given stability test.[6]

Specifically, under this approach I follow Boivin (2006) and Kozicki and Tinsley (2006) and pre-estimate $\widetilde{Q}$, the $\widetilde{k}\ x \widetilde{k}$ variance matrix of the TVPs where k is the number of regressors, using the median-unbiased estimator:

$$\widetilde{Q} = (\hat{\lambda}/T)^2 (\frac{1}{T}\widetilde{X}'\widetilde{X})^{-1} \Omega (\frac{1}{T}\widetilde{X}'X)^{-1}, \tag{6}$$

where $\widetilde{X}$ denotes the full column stack of $\widetilde{X}_t$ and T is the sample size. The parameter λ governs the size of the parameter variance. Following Stock and Watson (1998) it is estimated by inverting the heteroskedasticity-robust version of the Quandt Likelihood Ratio (QLR_T) (Andrews 1993) test for observations in the middle (70 per cent) of the sample. This is accomplished using the look-up table in Stock and Watson (1998).

As the format of potential heteroskedasticity in the residuals due to the TVPs is not known prior to the model estimation, a heteroskedasticity consistent estimate of Ω is used:

$$\Omega = \frac{1}{T}\widetilde{X}'D\widetilde{X}\ , \tag{7}$$

where the non-zero elements of the $T\ x\ T$ diagonal matrix, $D_{\tau,\tau}$, are: $(T/(T-k))\,\hat{u}_t^2$ and $\hat{u}$ are the residuals of the fixed coefficient regression (Equation 2).

Finally, I estimate the remaining parameters of Equation 4 including the time-varying intercept, and the short-run and total Okun effects using the Kalman smoother conditional on the median-unbiased estimate of $\widetilde{Q}$.

3 Empirical Results

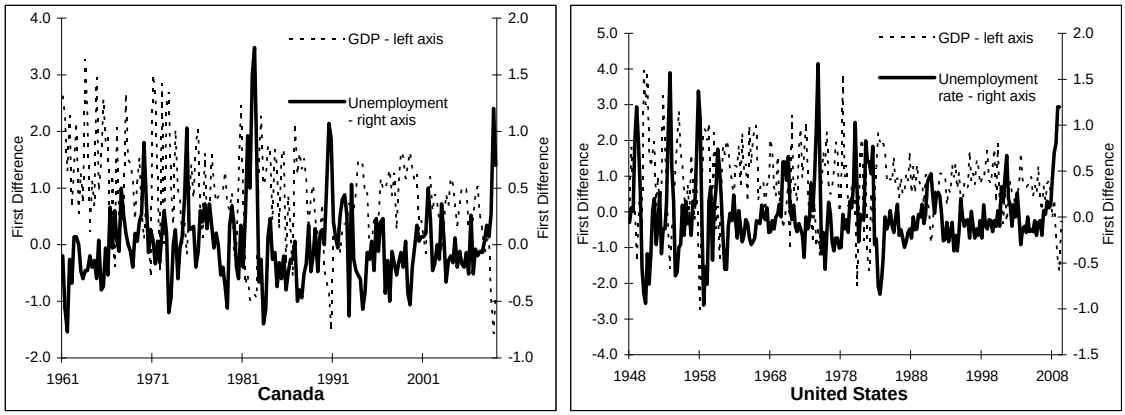
3.1 Data Description

The methodology discussed in the previous section is used to estimate Okun's law for Canada and the United States. For Canada, quarterly data on output and unemployment over 1961:I-2009:II is

[6] For more information on the median-unbiased estimator approach see also Boivin (2006).

obtained from Statistics Canada. For the United States, the data is available over a longer sample period (1948:I to 2009:II) and is obtained from the Bureau of Economic Analysis and the Bureau of Labor Statistics for output and unemployment, respectively. Figure 1 depicts the change in log real GDP and the change in the unemployment rate for Canada on the right and for the United States on the left. As seen, the behavior of the series is consistent with the theory suggested by Okun's law: the two series are highly negatively correlated. The contemporaneous correlation between the variables in Canada is -0.52, while it is -0.72 in the United States.

Figure 1: Real GDP and Unemployment in Canada and the United States



3.2 Results

To compare the results of the TVP analysis of Okun's law in Canada and the United States with the results of the existing literature, Equation 2 is first estimated using ordinary least squares (OLS). I then test the stability of Okun's law using Quandt likelihood ratio (QLR_T) tests (Andrews 1993), split-sample estimation of (2), and rolling regressions. I then consider the TVP model (3) and reconcile its results with those from the other stability tests.

3.2.1 Estimating Okun's law: OLS Results

The results of the fixed coefficient estimation of Okun's law (2) are reported in Table 1. The standard errors are corrected for possible heteroskedasticity and autocorrelation using the Newey-West (1987) covariance matrix. In Table 1, the short-run and total Okun effects are captured by α_1 and (α_1 + α_2), respectively.

The results for Canada suggest that the short-run Okun coefficient is -0.16. In the long-run there is a stronger relationship between the unemployment rate and output with a total Okun effect of -0.31. These results compare quite favorably to Schnabel (2002) and Huang and Chang (2005) who estimate total Okun effects of -0.33 over 1962-2000, and -0.28 over 1960-2002, respectively.

Over the full sample period, the results for the United States suggest that the unemployment rate and output move more closely together than in Canada. As seen in Table 1, the results suggest that a 1 percentage point fall in U.S. output is associated with a 0.23 percentage point increase in the unemployment rate in the short-run and a total increase of 0.39 percentage points. This larger response is statistically different than the Canadian response; however, this difference may be related to the different sample period used in the U.S. estimation. Thus, results for Equation 2 estimated over 1961:I to 2009:II are presented in Table 1 to be more comparable to the Canadian estimates. These results confirm that the U.S. unemployment rate has generally moved more closely together with output than the Canadian unemployment rate; however, the difference is no longer statistically different. The U.S. results in Table 1 compare favorably to Schnabel (2003) who estimates a total Okun coefficient of -0.42 over 1954-2000 and Lee (2000) who estimates a value of -0.54 over 1955-1996.

3.2.2 Testing for Structural Breaks in Okun's law

As a second step in the analysis, I provide preliminary evidence of instability in Okun's law that would not be captured by a typical fixed coefficient regression. In particular, I test for instability using a heteroskedasticity-robust version of the QLR_T test, applied jointly to all the coefficients in Equation 2. Results from the QLR_T test (Andrews (1993)), with 15 per cent trimming, for both countries are seen in Figure 2. For both countries, the results suggest that there have been important changes in Okun's law over time. For Canada, the QLR_T test suggests at least two structural breaks: one in 1979 and one in 1985.[7] The results also suggest at least two structural breaks in Okun's law in the United States: one in 1974 and one in 1983. Thus results from both countries suggest that a structural break in Okun's law may have been related to the oil shocks of the 1970s as hypothesized by previous authors (e.g. Gordon 1984, Evans 1989, Weber 1995).[8] These results provide a first indication that the structural instability in Okun's law is inconsistent with the one-time discrete change models that are often assumed.

As a further test of structural stability, the model is estimated over two different sample periods, choosing 1990:I as the break date to investigate more recent structural changes following the perceived Great Moderation. Structural stability is assessed using the Chow break point test (Chow 1960). Estimation results and the Chow test results are shown in Table 1. In the case of Canada, the results suggest that the relationship between unemployment and output has strengthened since 1990. In contrast, the results for the United States suggest that the relationship between the unemployment

[7] These periods are associated with the maximum QLR_T tests.
[8] Previous authors have suggested that the observed change in Okun's law following the oil price shocks may be associated with a change in employer perception regarding the depth of future recessions. In particular, it has been suggested that employers may have become more willing to incur the costs associated with layoffs and terminations following the oil price shocks as they felt that post oil shock recessions would be more severe than previous recessions. This explanation presumes that the cost of layoffs does not increase proportionally with the duration of layoff whereas the cost of labour hoarding does (Kaufman 1988).

rate and output has not changed since 1990.[9] Although not investigated in the context of this paper, this difference could be related to differences in institutional changes affecting the labour market. Sweeping reforms to employment insurance in Canada over the 1990s that restricted access to benefits, for example, may have contributed to the stronger relationship between unemployment and output since 1990. For Canada, the results from the Chow test are consistent with evidence from the QLR_T test that suggests instability in Okun's law over time, while for the United States, no structural break is detected. Moreover, following 1990 the differences observed in Okun's law across the two countries are no longer statistically significant.

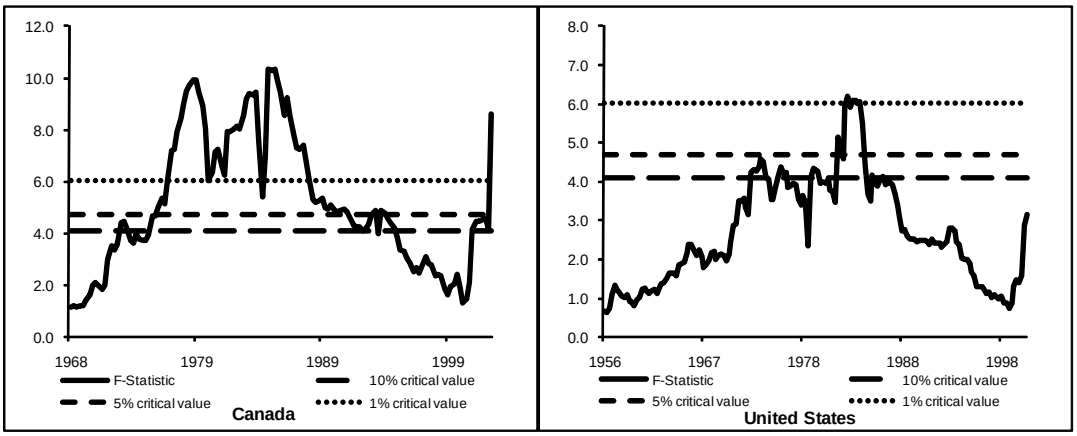
Table 1: OLS Regression Results: Canada and the United States

	α_0	α_1	$\alpha_1 + \alpha_2$	$\overline{R}^2$	Chow test F-Statistic
Canada					
1961:I-2009:II	0.2627***	-0.1598***	-0.3096***	0.3727	4.2042***
	(0.0581)	(0.0325)	(0.0524)		
1961:I-1989:IV	0.3148***	-0.1560***	-0.3096***	0.3406	
	(0.0978)	(0.0433)	(0.0746)		
1990:I-2009:II	0.2365***	-0.2294***	-0.3978***	0.5340	
	(0.0407)	(0.0485)	(0.0461)		
United States					
1948:I-2009:II	0.3371***	-0.2284***	-0.3938***	0.6508	1.8838
	(0.0348)	(0.0154)	(0.0310)		
1948:I-1989:IV	0.3551***	-0.2325***	-0.3920***	0.6667	
	(0.0427)	(0.0172)	(0.0345)		
1990:I-2009:II	0.3244***	-0.2099***	-0.4399***	0.6465	
	(0.0533)	(0.0356)	(0.0633)		
1961:I-2009:II	0.3030***	-0.2168***	-0.3665***	0.5868	
	(0.0587)	(0.0305)	(0.0518)		
1961:I-1989:IV	0.3042***	-0.2172***	-0.3492***	0.5694	
	(0.0435)	(0.0241)	(0.0410)		

Note: *, **, and *** represent significance at the 10, 5 and 1 per cent levels, respectively. The Chow test statistic presented tests whether there has been a structural change in the Okun relationship over 1961:I-1989:IV to 1990:I-2009:II in Canada and 1948:I-1989:IV to 1990:I-2009:II in the United States. Standard errors are in parentheses.

[9] This result is also obtained if I examine whether a structural change occurred from 1961:I-1989:IV to 1990:I-2009:II as was examined with the Canadian data set.

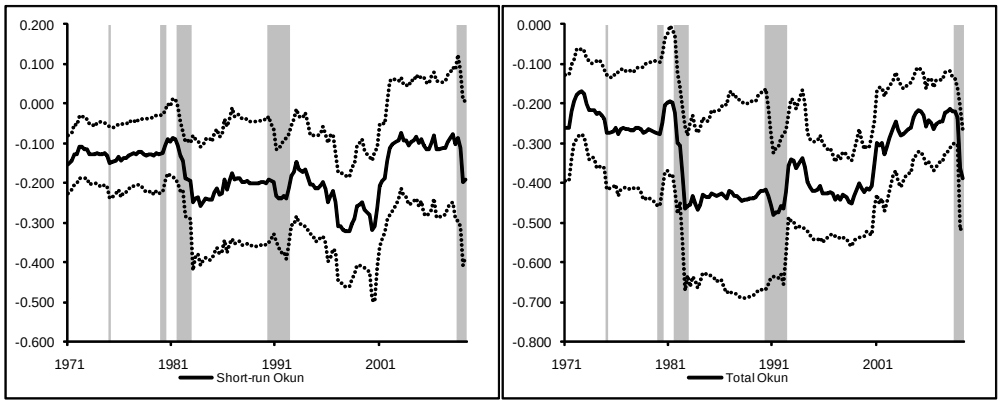
Figure 2: QLR test results for Canada and the United States



3.2.3 Testing for Instability in Okun's law: Rolling Regression Results

As a final test of the stability of Okun's law, rolling OLS regressions of Equation 2 are estimated. Figures 3 and 4 present the results along with standard error bands representing the 5 per cent confidence intervals for the rolling estimates. For both countries, the rolling regressions are estimated using a window size of ten years. For Canada, the results begin in 1971:I and in the United States they begin in 1957:III.

Figure 3: Rolling Okun Coefficients: Canada



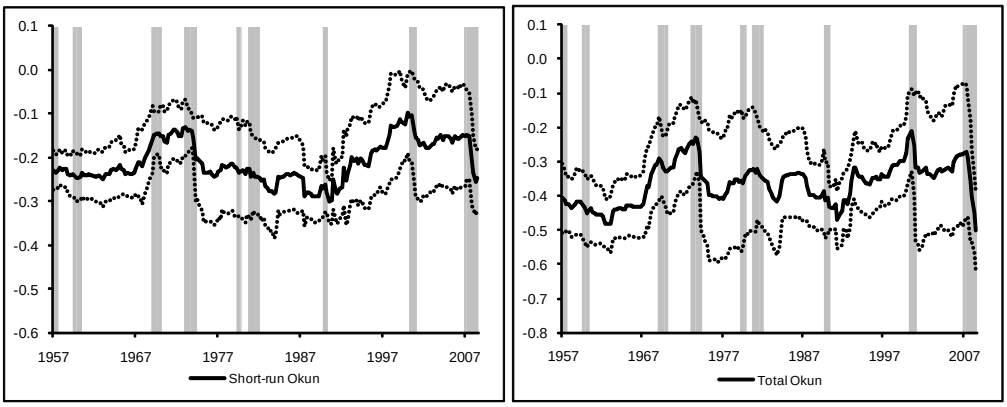
Note: Shaded areas depict recessionary periods. Canadian recessions are defined as two consecutive periods of negative real GDP growth.

The results for Canada confirm that there have been important changes in Okun's law over time. In particular, the absolute value of the short-run and total Okun effects appears to have increased from the late 1970s to the early 1980s before remaining relatively stable until the early 2000s. From the beginning of the 2000s up until the beginning of the current recession, the parameters appear to have then fallen in absolute value to levels comparable to those observed in the 1970s. With the onset of the current recession; however, the Okun coefficients appear to have increased again, returning to

levels similar to those observed over the 1980s and 1990s. However, the confidence intervals surrounding these estimates are quite wide, suggesting that although substantial time variation is observed, a large share of the variation is not statistically significant. What is also noteworthy is that the rolling regression results provide some limited evidence of asymmetry in Okun's law over Canadian business cycles (Figure 2), with the response of the unemployment rate to output tending to increase (decrease) in recessions (expansions).

For Canada, it is difficult to reconcile these results from the rolling regressions with those from the split-sample estimation. While the split-sample estimation suggests an increase in the absolute value of the Okun coefficients since 1990, the rolling regressions show that, up until the recession began in late 2008, they actually declined over the 2000s. This may be associated with the fact that the 10-year rolling window used in the estimation after 2002 and up until 2008:III would not include data from any Canadian recessions given that the last Canadian recession prior to the current recession ended in 1992.[10] Nevertheless, these results highlight the fact that discrete change models can easily mask changes in economic relationships of a more complicated nature.

Figure 4: Rolling Okun Coefficients: the United States



Note: Shaded areas depict recessionary periods. National Bureau of Economic Research (NBER) recession dates are shown.

For the United States, the rolling regressions suggest that the relationship between output and unemployment has been more volatile than in Canada. In particular, the Okun coefficients suggest that the relationship between unemployment and output weakened over the late 1960s and early 1970s before strengthening over the late 1970s, 1980s, and the early 1990s. The absolute value of the coefficient then appears to have fallen before increasing again in the current downturn. However, as in Canada, the confidence intervals surrounding these estimates are quite wide, suggesting that a large share of the observed time variation is not statistically significant. Moreover, the volatility in the U.S. Okun coefficients indicates asymmetry in Okun's law over recessions and expansions with the response of the unemployment rate to output tending to be larger during recessions than in

[10] The Canadian recession of 2008/2009 is assumed to have begun in 2008:III.

expansions. Finally, results from the rolling regressions capture the stronger co-movement between the unemployment rate and output observed in the current recession relative to previous recessions.

3.2.4 Testing for Instability in Okun's law: TVP Model Results

As mentioned previously, there is no guarantee that the changes in Okun's law suggested by the stability tests were of a discrete nature. Moreover, the rolling regressions, which are estimated over very short horizons, may also not be an optimal method to detect changes in economic relationships. This section thus presents results from the TVP model, which is more suitable to identifying changes in Okun's law of a general nature.

Specifically, the results from the QLR_T test are used to obtain an estimate of λ, which governs the size of the variance of the parameters. The results suggest that λ = 8.8 in Canada and 5.5 in the United States. These estimates are consistent with relatively small period-to-period variation in the Okun's law parameters (3). Moreover, Stock and Watson (1998) show that these values are within the range of values for λ for which MLE estimates are biased towards zero.[11] These estimates are then used to estimate $\widetilde{Q}$, which is used in the MLE estimation of the TVP model. Figure 5 presents the TVPs for the short-run and total Okun effects for both countries.

For Canada, the results suggest that since the early 1970s, the unemployment rate and output have moved more closely together; however, the change has been relatively minor. These results are comparable to those from the split-sample model discussed earlier; however, the TVP model suggests that Okun's law is more stable than suggested by the rolling regressions. This difference may be associated with the fact that rolling regressions, due to the small sample size used in their estimation, may overstate the effect of recent observations in their estimation and thus overstate the time variation present.[12] For instance, in Figure 3, it can be seen that the Okun coefficients tend to increase (in absolute value) when the dataset used in the estimation includes a larger number of recessionary quarters. This is consistent with previous evidence that finds that the effect of output on unemployment tends to be larger in recessions than in expansions. Nevertheless, results from both the rolling regressions and the TVP model are consistent in that they confirm that the behavior of Okun's law is more complex than suggested by typical split-sample estimates. Finally, although the TVP estimates find relatively small time variation in Okun's law in Canada, the estimates are able to capture time variation not captured by Sögner and Stiassny's (2002) TVP model. As explained earlier, this difference is likely associated with the fact that they employ the Kalman filter and MLE estimation

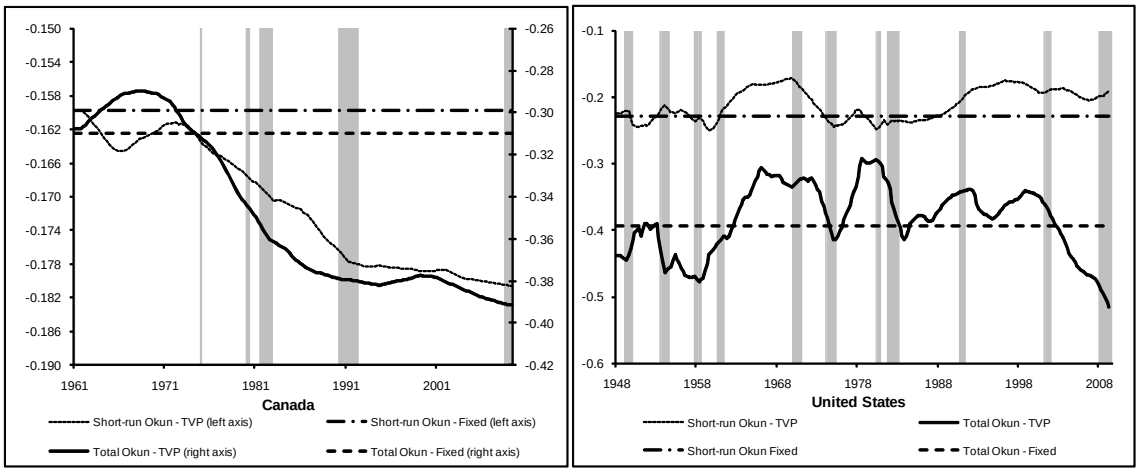
[11] Specifically, Stock and Watson (1998) show that for λ = 9 the probability of observing $\hat{\lambda}$ = 0 using the median-unbiased estimator approach is only 12 per cent versus 37 per cent for MLE. Likewise, they show that for λ = 6 the probability of observing $\hat{\lambda}$ = 0 using the median-unbiased estimator approach is only 19 per cent versus 56 per cent for MLE. In other words, the probability of a zero pileup problem using MLE varies inversely with λ.

[12] Although the median-biased estimator approach places more weight on recent observations as well, it takes into account all observations when estimating the TVPs.

to obtain their TVP estimates, which, when the variance of the TVP parameters is small, can lead to the MLE estimate of the variance of the parameters being biased towards zero. The median-unbiased estimator approach employed in this paper overcomes this problem, leading to additional, although minor, time-variation in the Okun's law parameter estimates.

For the United States, the results suggest that the unemployment-output relationship has evolved substantially more than in Canada over time. As suggested earlier, this may be associated with the fact that the United States has experienced more business cycles than Canada. In particular, the results suggest that the relationship between unemployment and output has strengthened over recent years, particularly since the late 1990s. However, it is interesting to note that the current estimates of Okun's law are quite comparable to what was observed in the 1950s. This result is somewhat at odds with the results from the split-sample estimations, which suggested that the Okun relationship remained stable over time, and with the results from the rolling regressions, which depicted a decrease in the effect of output on unemployment over the late 1980s up until the current recession. Moreover, Figure 5 suggests that the relationship between unemployment and output is stronger during recessions than in expansions. As is the case with Canada, the results suggest greater time variation in Okun's law than results from Sögner and Stiassny (2002). Moreover, the time variation present in the Okun coefficients evolves in line with the variation observed by Huang and Lin (2008).

Figure 5: Time-Varying Okun Coefficients: Canada and the United States



4 Conclusions

Okun's law, the negative relationship between movements of the unemployment rate and real economic activity, has been widely accepted as an empirical regularity in macroeconomics. Okun's (1962) original observation suggested that a one percentage point increase in the unemployment rate is associated with a three percentage point decrease in real output growth. More recently, however, economists have begun to question whether the Okun relationship differs across the state of the business cycle (e.g. Crespo-Cuaresma 2003, Silvapulle et al. 2004) and whether the Okun relationship

has undergone structural change over time (e.g. Sögner and Stiassny 2002 and Huang and Lin 2008). This paper contributes to this debate by providing estimates of Okun coefficients with recent data and by assessing the stability of Okun's law for Canada and the United States using a TVP model. Time variation is modeled as driftless random walks and is estimated using the median unbiased estimator approach developed by Stock and Watson (1998).

For both countries, the TVP model is able to unmask important developments in Okun's law that have not been detected in previous empirical analysis. For both Canada and the United States, I find evidence of asymmetric behavior in Okun's law over the business cycle. In particular, the results suggest that the unemployment rate tends to respond by more to changes in output in recessions than in expansions. Moreover, I find evidence of structural change in Okun's law in both countries. While Okun's original estimates (for the United States) suggested that a 3 per cent decline in output leads to a 1 percentage point increase in the unemployment rate, the results in this paper suggest that a 2.6 and 2.0 per cent decrease in output is now typically associated with a 1 percentage point increase in the unemployment rate in Canada and the United States, respectively. To conclude, given the instability of Okun's law over time and across the state of the business cycle, this rule of thumb should be used with caution, particularly with respect to forecasting.

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