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B. Ravikumar[1] and Enchuan Shao[2]

[1]University of Iowa
ravikumar@uiowa.edu

[2]Currency Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
eshao@bankofcanada.ca

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Abstract

We examine the quantitative effect of search frictions in product markets on asset price volatility. We combine several features from Shi (1997) and Lagos and Wright (2002) in a model without money. Households prefer special goods and general goods. Special goods can be obtained only via a search in decentralized markets. General goods can be obtained via trade in centralized competitive markets and via ownership of an asset. There is only one asset in our model that yields general goods. The asset is also used as a medium of exchange in the decentralized market to obtain the special goods. The value of the asset in facilitating transactions in the decentralized market is determined endogenously. This transaction role makes the asset pricing implications of our model different from those in the standard asset pricing model. Our model not only delivers the observed average rate of return on equity and the volatility of the equity price, but also accounts for most of the spectral characteristics of the equity price.

JEL classification: E44, G12
Bank classification: Financial markets; Market structure and pricing

Résumé

Les auteurs analysent l'incidence quantitative des frictions liées à l'acquisition d'information sur la volatilité du prix des actifs. Ils intègrent au sein d'un modèle sans monnaie plusieurs éléments empruntés aux modèles de Shi (1997) et de Lagos et Wright (2002). Les ménages ont le choix entre biens spécialisés et biens généraux : les premiers s'acquièrent au moyen de recherches sur des marchés décentralisés, et les seconds, au moyen de transactions sur des marchés concurrentiels centralisés ou grâce à la détention d'un actif. Dans le modèle des auteurs, un seul actif produit des biens généraux. Cet actif peut également servir de moyen d'échange sur le marché décentralisé où se négocient les biens spécialisés, et sa valeur en tant que telle est établie de façon endogène. En raison du rôle que joue cet actif dans les échanges, les implications du modèle en matière d'évaluation sont différentes de celles du modèle type d'évaluation des actifs financiers. Le modèle permet non seulement de reproduire l'évolution du taux de rendement moyen des actions et la volatilité de leur prix, mais aussi la plupart des caractéristiques spectrales de ce dernier.

Classification JEL : E44, G12
Classification de la Banque : Marchés financiers; Structure de marché et fixation des prix

1 Introduction

LeRoy and Porter (1981) and Shiller (1981) calculated the time series for asset prices using the simple present value formula – the current price of an asset is equal to the expected discounted present value of its future dividends. Using a constant interest rate to discount the future, they showed that the variance of the observed prices for U.S. equity exceeds the variance implied by the present value formula (see figure 1). This is the *excess volatility puzzle*.

Equilibrium models of asset pricing deliver a generalized version of the present value formula. In Lucas (1978), for instance, the discount factor is stochastic and depends on the intertemporal marginal rate of substitution (IMRS) of the representative consumer. In models such as Lucas (1978) the asset yields a flow of dividends over time and its value is determined by the IMRS of the representative consumer. We develop a model with search frictions where the asset is valued not only for the flow of dividends, but also for its usefulness as a medium of exchange. We use the model to examine the extent to which the dual role of the asset can explain the excess volatility puzzle.[1]

We combine several features from Shi (1997) and Lagos and Wright (2002) in a model *without* money. Households prefer special goods and general goods. Special goods can be obtained only via trade in decentralized markets. This trading process involves search and bargaining. Similar to Shi and Lagos-Wright, the search frictions make intertemporal trade infeasible in our model. General goods can be obtained via trade in centralized competitive markets and via ownership of an asset. There is only one asset in our model. The asset is similar to a Lucas tree that yields stochastic fruits that can be consumed directly. The asset is also used as a medium of exchange in the decentralized market to obtain the special goods. The value of the asset in facilitating transactions in the decentralized market is determined endogenously.[2] If

[1] There have been several attempts to explain the excess volatility puzzle. LeRoy and LaCivita (1981) and Michener (1982) examine the role of risk aversion. Flavin (1983) and Kleidon (1986) examine whether small sample bias can statistically account for violations of the variance bound. Marsh and Merton (1986) try to resolve the puzzle with different statistical assumptions on the dividend process. West (1988) develops a volatility test that circumvents the above small sample bias and dividend process criticisms and shows that the observed stock prices are indeed too volatile. Shiller (1984) and Ingram (1990) explore whether the existence of rule-of-thumb traders can account for the excess volatility.

[2] See Bansal and Coleman (1996) for a reduced form model of the transaction role of assets and its implications for asset returns.

we shut down the decentralized trading process (i.e., eliminate the special goods and search frictions), then our model is identical to that of Lucas (1978). If we shut down the centralized markets (i.e., eliminate the general goods and frictionless competitive markets), then our model is essentially the same as the monetary model of Shi (1997).

With only one asset, our model cannot address the equity premium puzzle posed by Mehra and Prescott (1985), but given the stochastic process for dividends we provide parameters for which the model delivers the average rate of return on equity and the volatility of equity price. (See Lagos (2006) for a search model of the equity premium.) We calibrate the two parameters in our model (matching probability and risk aversion) to deliver the mean S&P 500 return and the standard deviation of S&P 500 price in the annual data from 1871-1995.

The model does a reasonable job of accounting for the time series properties of asset prices. A substantial amount of the variance in the S&P 500 price is in the low frequencies. With the calibrated parameters, the benchmark price sequence in the model also displays similar characteristics – the low frequencies account for 86% of the total variance in the data and 85% in the model. Furthermore, both the model and the data spectra peak at the same frequency.

As we increase the matching probability the search friction in the decentralized market is reduced and the asset's value as the medium of exchange is diminished. We define the medium-of-exchange value of the asset as its "liquidity value." We compute the liquidity value as the percentage difference between the asset price in our model and the asset price generated by a variant of our model where the liquidity value is designed to be zero. For the latter, we set the discount factor and the risk aversion at the calibrated values, eliminate the search friction and then calculate the resulting asset price time series. In this time series, the liquidity value of the asset is zero, by construction, since we have eliminated the medium of exchange role for the asset. The difference between this time series and the benchmark sequence of asset prices generated by our model is the time series of the liquidity value of asset. The average percentage difference between the two time series is 17.5%.[3]

The rest of paper is organized as follows. In the next section we set up the economic environment and derive the equilibrium asset pricing equation. In section

[3] Vayanos and Wang (2007), Duffie, Garleanu and Pedersen (2005) and Weill (2008) consider search frictions in the asset market and present models of liquidity premium based on the ease of transactions with some assets relative to others.

3, we study the quantitative implications of the model.

2 The Environment

Consider a discrete-time non-monetary economy with special goods and general goods, decentralized day markets and centralized night markets, and aggregate uncertainty. There is a continuum of types of households and a continuum of households in each type. The measure of types and the measure of households in each type are both normalized to one. It is convenient to imagine the types to be points on a unit circle. A type h household produces only good h but consumes the good at a small but finite distance away in the circle (at $h+\Delta$). The utility from consuming c units of the special good is $u(c)$. The utility function is increasing and strictly concave, and satisfies $u'(0)=\infty$ and $u'(\infty)=0$. To produce q units of the special good, households incur q units of disutility. The special goods are non-storable between periods.

There is an infinitely lived asset (tree) in this economy that yields dividends (fruits) each period. Fruits are general goods and they follow an exogenous stationary stochastic process. The utility from consuming d units of fruits is $U(d)$, where $U(\cdot)$ is increasing and strictly concave. Note that there is no cost to produce the fruits. The fruits are also perishable. Each household is initially endowed with one (divisible) tree.

Special goods are exchanged in a decentralized market in daytime where agents meet in pairs, as in standard search theory. Random pairs are formed with probability α. The random matching technology combined with the household preferences rules out barter in pairwise meetings. Furthermore, there is no public record of transactions to support any credit arrangements. Thus, in pairwise meetings special goods are exchanged for trees. General goods are available for trade only in the centralized market at night. The night market is frictionless and trees are exchanged for general goods at the competitive equilibrium price p.

Time is indexed by $t=0,1,...$ The discount factor between periods is β. There is no discounting between day and night.

Random matching during the day will typically result in non-degenerate distributions of asset holdings. In order to maintain tractability, we use the device of large households along the lines of Shi (1997). Each household consists of a continuum of worker-shopper (or, seller-buyer) pairs. Buyers cannot produce the special good, only sellers are capable of production. We assume the fraction of buyers = fraction of sellers = $\frac{1}{2}$. Then, the probability of single coincidence meetings during the day is $\frac{1}{4}\alpha$. Each household sends its buyers to the decentralized day market with *take-it-or-leave-it* instructions (q, s) – accept q units of special goods in exchange for s trees. Each household also sends its sellers with "accept" or "reject" instructions. There is no communication between buyers and sellers of the same household during the day. After the buyers and sellers finish trading in the day, the household pools the trees and shares the special goods across its members each period. By the law of large numbers, the distributions of trees and special goods are degenerate across households. This allows us to focus on the representative household. The representative household's consumption of the special good is $\frac{\alpha}{4}q$.

2.1 Timing of events in each period

- The representative household starts the period with a trees.
- It observes the aggregate state d (fruits per tree), but the fruits are not available for trade during the day.
- The household determines the take-it-or-leave-it offer (q, s). It allocates s trees to each buyer in the household and provides trading instructions to its sellers and buyers.
- The sellers and buyers from households of *all* types are randomly matched in the decentralized market. In single coincidence meetings, the sellers produce the special good in exchange for trees from the buyers.
- Each household then pools its purchases and consumes the special goods.
- Next, each household enters the centralized market at night with its new asset balance and fruits. Households trade fruits and trees in the centralized competitive asset market (much like the standard consumption based asset pricing model) at price p.
- Then, they consume the fruits and end the period with a' trees.

2.2 Optimization

We begin with the representative household's instructions to its buyers and sellers. Clearly, if a member of the household is not in a single coincidence meeting, the instruction is not to trade. The instruction to the buyers in single coincidence meetings is a take-it-or-leave-it offer (q, s). For another household's seller to be indifferent between accepting and rejecting the buyer's offer in the random match, (q, s) has to satisfy the seller's participation constraint:

$$\Omega s - q = 0, \tag{1}$$

where Ω is the other household's valuation of the asset. The first term on the left hand side is the gain to the seller from obtaining s trees in the trade. The second term is the disutility from q units of the special good. The take-it-or-leave-it offer will leave no surplus for the seller, so the right hand side is 0. We will assume that the seller will accept the offer whenever he is indifferent. An additional restriction on the offer is that the total number of trees allocated to the buyers by the representative household cannot exceed the number of trees that the household started the period with:

$$\frac{1}{2}s \leq a. \tag{2}$$

This is because (i) the decentralized market does not support credit arrangements, so the buyer cannot short-sell the asset and (ii) the buyer is temporarily separated from other members of the household, so he cannot borrow from the other members of the household. We can eliminate s by combining the two constraints (1) and (2):

$$\frac{1}{2}\left(\frac{q}{\Omega}\right) \leq a.$$

The representative household's instruction to its sellers in single coincidence meetings are straightforward. Suppose that the buyer from the other household offers (Q, S). The instruction is, if the surplus from (Q, S) is non-negative, accept the offer and produce Q units of the special good; otherwise, reject the offer and do not trade.

The representative household's problem then is described by the following dynamic program:

$$v(a, d) = \max_{q,x,a'} u\left(\frac{\alpha}{4}q\right) - \frac{\alpha}{4}Q + U(x) + \beta E_{d'|d} v(a', d') \tag{3}$$

$$\text{s. t. } \frac{1}{2}\left(\frac{q}{\Omega}\right) \leq a \tag{4}$$

$$x + pa' = \left\{a + \frac{\alpha}{4}S - \frac{\alpha}{4}\left(\frac{q}{\Omega}\right)\right\}(p + d), \tag{5}$$

where Q is the amount of the special good obtained by the buyers from other households and S is the number of trees obtained by the sellers from other households. The second constraint is the wealth constraint for the household. Note that p is the relative price a tree in terms of the fruits in the centralized night market.

Remark 1. If we eliminate the part of our model that has the search friction, special goods, etc., then our model is identical to that of Lucas (1978):

$$v(a, d) = \max_{a'} U\left(a\left(p + d\right) - pa'\right) + \beta E_{d'|d} v(a', d').$$

In this case, the asset has positive value since it yields dividends. The presence of search frictions implies an additional "liquidity" value to the asset.

Remark 2. If we eliminate the part of our model that has the dividends, centralized frictionless competitive markets etc., then we have a monetary model.

$$v(a) = \max_{q,a'} u\left(\frac{\alpha}{4}q\right) - \frac{\alpha}{4}Q + +\beta v(a')$$

$$\text{s. t. } \frac{1}{2}\left(\frac{q}{\Omega}\right) \leq a;\; a' = a + \frac{\alpha}{4}S - \frac{\alpha}{4}\left(\frac{q}{\Omega}\right).$$

The asset is now intrinsically useless and its value is determined by its role as the medium of exchange.

Uniqueness, concavity and differentiability of $v(\cdot)$ follows from theorems 9.6, 9.7, and 9.8 in Stokey, Lucas and Prescott (1989).

2.3 Equilibrium

Definition 1. An equilibrium consists of a sequence $\{q_t, x_t, s_t, a_{t+1}\}_{t=0}^{\infty}$, given initial asset holdings, such that

1. Given other households' offers and valuations, each household's choices solve the dynamic program (3);
2. The choices and the asset valuations are the same across households;

3. The centralized markets clear for all t: $x_t = d_t$, $a_{t+1} = 1$.

Let $\frac{\alpha}{2}\lambda$ be the multiplier on the constraint (4). The first order conditions for the representative household with respect to q and a' are as follows.

$$u'(\frac{\alpha}{4}q) = \frac{1}{\Omega}\left\{(p+d)\,U'(x) + \lambda\right\} \tag{6}$$

$$pU'(x) = \beta E_{d'|d}\frac{\partial v(a',d')}{\partial a'} \tag{7}$$

In these conditions, we have used the wealth constraint (5) to substitute for x. Note that if the no-short-sales constraint (4) does not bind, then $\lambda = 0$. The envelope condition for a implies that

$$\frac{\partial v(a,d)}{\partial a} = (p+d)\,U'\,(x) + \frac{\alpha}{2}\lambda \tag{8}$$

Using (6) to substitute for λ in (8), we get

$$\frac{\partial v(a,d)}{\partial a} = \left(1-\frac{\alpha}{2}\right)(p+d)\,U'\,(x) + \frac{\alpha}{2}u'\left(\frac{\alpha}{4}q\right)\Omega.$$

We can rewrite (7) using the above expression for $\frac{\partial v}{\partial a}$:

$$pU'(x) = \beta E_{d'|d}\left\{\left(1-\frac{\alpha}{2}\right)(p'+d')\,U'\,(x') + \frac{\alpha}{2}u'\left(\frac{\alpha}{4}q'\right)\Omega'\right\}. \tag{9}$$

We have to now impose the equilibrium conditions on (9). The valuation of the asset, Ω, by other households in the decentralized market during the day, has to equal the valuation, ω, by the representative household, in equilibrium. We can determine ω as follows. An additional unit of asset obtained in the decentralized market yields d fruits at night; the asset can also be sold for p fruits in the centralized market at night. At the margin these additional fruits are valued at $U'(x)$. In equilibrium, the general goods market clearing condition at night implies $x = d$. Hence,

$$\omega = \Omega = (p+d)\,U'\,(d)\,.$$

Using the equilibrium values for Ω and x, we can write (9) as

$$pU'(d) = \beta E_{d'|d}\left\{(p'+d')\,U'\,(d')\left[1-\frac{\alpha}{2}+\frac{\alpha}{2}u'\left(\frac{\alpha}{4}q'\right)\right]\right\}.$$

Hence, the equilibrium sequence of asset prices satisfy

$$p_t U'(d_t) = \beta E_t \left\{ (p_{t+1} + d_{t+1}) U'(d_{t+1}) \left[1 - \frac{\alpha}{2} + \frac{\alpha}{2} u' \left(\frac{\alpha}{4} q_{t+1} \right) \right] \right\}. \tag{10}$$

In the presence of search frictions, the price of the asset in the competitive market accounts for the future liquidity value of the asset as well.

To solve for the equilibrium sequence $\{q_t\}$, we have to account for two possible scenarios. If the constraint (4) does not bind in period t, then λ_t equals zero and $u'(\frac{\alpha}{4}q_t) = 1$. Denote the solution to this equation as q^*. Note that the solution does not depend on the aggregate state and, hence, is time-invariant. Furthermore, if $q_t = q^*$ for all t, then the search frictions are irrelevant for the asset pricing implications and the price sequence in our model is the same as in Lucas (1978). If the constraint (4) binds in period t, then

$$q_t = 2 (p_t + d_t) U'(d_t). \tag{11}$$

3 Quantitative Implications

To examine the quantitative implications of our model, we restrict the utility functions to be of the CRRA class,

$$\begin{aligned} u(c) &= \frac{c^{1-\delta}}{1-\delta} \\ U(x) &= \frac{x^{1-\delta}}{1-\delta} \end{aligned}$$

where $0 < \delta < \infty$ (with the usual assumption that if $\delta = 1$ then we will interpret the utility function as logarithmic). Hence, q^* is the unique solution to $\left(\frac{\alpha}{4}q\right)^{-\delta} = 1$.

When the no-short-sales constraint (4) binds, $q = 2(p+d)d^{-\delta}$. Thus, we can write the asset pricing equation (10) for these functional forms as

$$p_t d_t^{-\delta} = \beta E_t \left\{ (p_{t+1} + d_{t+1}) d_{t+1}^{-\delta} \left[1 - \frac{\alpha}{2} + \frac{\alpha}{2} \left(\frac{\alpha}{4} q_{t+1} \right)^{-\delta} \right] \right\}. \tag{12}$$

3.1 A simple example

Suppose that $\delta = 1$ and that the dividend follows an i.i.d. process. Assume that α, β and the parameters of stochastic process are such that the no-short-sales constraint binds in all states i.e., $q = 2\frac{p+d}{d}$ for all d. Then, we can derive an analytical expression of for the equilibrium asset price:

$$p_t = \frac{\beta\left(2 - \frac{\alpha}{2}\right)}{1 - \beta\left(1 - \frac{\alpha}{2}\right)} d_t.$$

In an otherwise similar environment with no search frictions, special goods or decentralized exchange, the asset is valued only for its dividend payoff and the equilibrium asset price $p_t = \frac{\beta}{1-\beta} d_t$. Since the no-short-sales constraint binds in all states, it must be case that

$$\begin{aligned} \frac{\beta\left(2 - \frac{\alpha}{2}\right)}{1 - \beta\left(1 - \frac{\alpha}{2}\right)} &> \frac{\beta}{1-\beta} \\ \text{or, } 1 - \frac{\alpha}{2} &> \beta. \end{aligned}$$

Furthermore, the model with search frictions will generate a higher volatility in the asset price relative to a model without search frictions.

3.2 Numerical method

To compute the price sequence, we modify the version of Parameterized Expectation Approach (PEA), originally proposed by Den Haan and Marcet (1990), and add some features of the Monte Carlo simulation method proposed by Judd (1998). In essence, the algorithm iteratively approximates the *future* conditional expectations that appear in the equilibrium conditions with flexible functions of a set of parameters and of the vector of state variables and approximates the current conditional expectation via Monte Carlo simulation. As part of the computation we have to deal with the possibility that the no-short-sales constraint may binding in some states but not in others.

Recall the equilibrium conditions are determined by the following set of equations:

$$p_t = E_t\left\{\beta \frac{(p_{t+1} + d_{t+1})\, U'(d_{t+1}) + \frac{\alpha}{2}\lambda_{t+1}}{U'(d_t)}\right\} \tag{13}$$

$$\lambda_t = \left[u'\left(\frac{\alpha}{4} q_t\right) - 1\right] (p_t + d_t)\, U'(d_t) \tag{14}$$

$$\lambda_t \left[1 - \frac{q_t}{2U'(d_t)(p_t + d_t)}\right] = 0 \tag{15}$$

The last equation is the Kuhn-Tucker condition. There is only one state variable d_t in this economy. Observe that the conditional expectation in (13) is a time-invariant function ξ of state variable d_t. Therefore, to find a solution for prices $\{p_t\}$, we need to find an approximation of ξ, i.e. we want to choose a class of functions that can approximate ξ arbitrarily well, and fix the degree of approximation. The functional form we choose is the exponential function of a polynomial

$$\xi(\theta, d) = \exp\left[\theta_0 + \theta_1 \ln d + \theta_2 (\ln d)^2 + \ldots + \theta_n (\ln d)^n\right],$$

where $\theta = [\theta_0\ \theta_1\ \theta_2\ \ldots\ \theta_n]$. Since the price in the model is positive, the exponential function guarantees the positive value for the expectation. We can also increase the degree of accuracy by increasing the order n of polynomials. For the calculations below, we fix $n = 3$.

The problem now is to find a parameter-vector θ that is consistent with the true expectation. The detailed procedure is described as follows. Denote the initial guess of θ as θ^0; the superscript denotes the iteration number.

1. Assume that the log of dividends follows a trend stationary process:

$$\ln d_{t+1} = b_0 + b_1 \ln d_t + b_3 t + \eta_{t+1} \tag{16}$$

   where η_{t+1} is the disturbance with mean 0.[4]

2. For each sample period t, use the observed dividend in period t to simulate the next period dividend level. That is, generate d_{t+1} using the coefficients in (16) and drawing the disturbances η_{t+1} from its empirical distribution. (An alternative is to draw these disturbances under the assumption that η is normally distributed.)

3. At iteration j, given $\theta = \theta^j$, use the simulated dividend in step 2 to calculate the price at $t+1$ using the function $\xi(\theta^j, d_{t+1})$.

4. Set $q_{t+1} = q^*$ under the assumption that $\lambda_{t+1} = 0$. Then test whether $q^* \leq$

[4]See DeJong and Whiteman (1991) for evidence on trend stationarity. Our results below are robust to a trend stationary AR(2) process as well.

$2U'(d_{t+1})(\xi(\theta^j, d_{t+1}) + d_{t+1})$. If it is the case, set $\lambda_{t+1} = 0$, otherwise set

$$\begin{aligned} q_{t+1} &= 2U'(d_{t+1})\left(\xi\left(\theta^j, d_{t+1}\right) + d_{t+1}\right), \\ \lambda_{t+1} &= \left[u'\left(\frac{\alpha}{4}q_{t+1}\right) - 1\right]\left(\xi\left(\theta^j, d_{t+1}\right) + d_{t+1}\right)U'(d_{t+1}). \end{aligned}$$

5. Calculate the price at period t

$$\widetilde{p}_t = \frac{\beta}{U'(d_t)}\left[U'(d_{t+1})\left(\xi\left(\theta^j, d_{t+1}\right) + d_{t+1}\right) + \frac{\alpha}{2}\lambda_{t+1}\right].$$

6. Repeat step 2 to 5 many times. The number of replications we use is 3000. The average value of these 3000 calculations of $\widetilde{p}_t$ is p_t.

7. Repeat above steps for periods $t+1$, $t+2, ...,$ until the end of sample period. And we have a vector of prices $\{p_t\}$ over the whole sample period.

8. Using the observed dividend sequence, run the regression

$$\ln p = \theta_0 + \theta_1 \ln d + \theta_2 (\ln d)^2 + \ldots + \theta_n (\ln d)^n$$

and obtain the OLS estimate $\widehat{\theta}$. Update the parameter-vector used in the next iteration by $\theta^{j+1} = \gamma\widehat{\theta} + (1-\gamma)\theta^j$, where $\gamma \in [0,1]$ controls the smoothness of convergence.

9. If $\|\theta^{j+1} - \theta^j\|$ is less than some tolerance value then stop, otherwise go back to step 2. We use a tolerance value of 10^{-6}.

Using the time series of p calculated from above steps, we can compute the rate of return sequence $\{R_{t+1}\}$ for the whole sample period by $R_{t+1} = (p_{t+1} + d_{t+1})/p_t$. This will allow us to calculate the unconditional moments of prices and returns. One of key problems in implementing PEA algorithm is to select initial conditions for θ. In our calculation, we first solve the standard Lucas asset pricing model using Judd's algorithm and get the θ vector. We then use the θ vector in the Lucas tree model as our initial value. It turns out that the convergence is fast with this initial guess.[5] One

[5] We use Monte-Carlo simulation instead of numerical integration to solve the expectation. This method helps to improve the convergence speed a lot comparing to the standard PEA method. For example, in the benchmark case it takes only 100 seconds to get the results on the Centrino 1.5GHz laptop.

advantage of our algorithm is that it can handle the occasionally binding constraint easily, as described in step 4.

3.3 Data and Parameters

The data are all in real terms and obtained from Shiller's website. The sample period is 1871-1995. We measure the asset prices and dividends by the S&P 500 prices and per capita dividends. We measure the volatility of a variable by the standard deviation of the *detrended* time series of the variable. The average rate of return on equity in this sample is 8% and the standard deviation of the equity price is 81. The mean growth rate of dividend is 1.91% and the standard deviation of detrended dividend is 1.61.

Other than the coefficients in the trend stationary process, we have two preference parameters, δ and β, and one parameter α that describes the matching friction. The estimates of the coefficients are $b_0 = 0.308$, $b_1 = 0.802$, $b_2 = 0.002$ and the variance of η is 0.0136. We set $\beta = 0.96$. We searched for α and δ to match the observed average rate of return on equity and standard deviation of the asset price.

Table 1. Benchmark Parameters

β	α	δ
0.96	0.42	1.8

For the benchmark parameters in Table 1, the average rate of return on the asset in our model is 8% and the standard deviation of the asset price is 80. Recall that the corresponding values in the data are 8% and 81. In figure 2, we illustrate the equilibrium price sequence implied by the model, which is clearly more volatile relative to the simple present value formula in figure 1.

3.4 Results

Our model also accounts for the spectral properties of the asset price. Figures 3 and 4 illustrate the asset price spectra for the model and the data. We used covariances up to lag 20 and the Bartlett kernel to estimate these spectra. In the data, most of the volatility in the S&P 500 price is in the low frequencies – 86% of the volatility is in frequencies below $\frac{\pi}{4}$ (or cycles of length greater than 8 years). In the model, the corresponding figure is 85%.

Table 2 presents a summary of the comparative dynamics associated with changes in δ and α. (The other parameter β is fixed at its benchmark value 0.96.) Changes in δ affect the curvature of the utility function. As δ increases, the asset price volatility increases along with the average rate of return. As we decrease the search friction (increase in α), the average rate of return increases. This is because the asset's value as the medium of exchange decreases. The asset price volatility, however, decreases with α.

Table 2. Comparative dynamics ($\beta = 0.96$)

Average rate of return (%)					
$\alpha \diagdown \delta$	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.05	5.15	5.37	4.8	5.8	7.8
0.2	5.15	6.16	7.02	6.77	8.24
0.4	5.15	6.16	7.24	8	8.78
0.6	5.15	6.16	7.24	8.44	9.32
0.75	5.15	6.17	7.24	8.50	9.66

Std. dev. of the asset price					
$\alpha \diagdown \delta$	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.05	29.5	47.7	173	799	3647
0.2	29.5	38.8	66.4	274	1174
0.4	29.5	38.8	49.1	148	624.8
0.6	29.5	38.8	49.1	92.3	394.5
0.75	29.5	38.8	49.1	80.1	343.9

The standard asset pricing model (no search friction, special goods etc.) delivers the observed average rate of return on equity for risk aversion $\delta = 1.85$. The standard deviation of the asset price, however, is 56 while the observed volatility is 81. As α approaches 1 in our model, the search frictions become smaller and the average rate of return and the volatility in our model approach the values in the standard asset pricing model.

To compute the "liquidity value" of the asset, we set β and δ set at their benchmark values (Table 1) and calculate the price sequence for a standard asset pricing model such as Lucas (1978). This is easily done by setting $u'(\frac{\alpha}{4}q_t) = 1$ for all t in equation (10). Since the standard asset pricing model does not assign any medium of exchange role to the asset, the difference between the prices implied by the standard model and ours would be the liquidity value of the asset. We compute the liquidity value as a fraction of the price implied by the standard model i.e., liquidity value $= \frac{P_{\text{model}} - P_{\text{Lucas}}}{P_{\text{Lucas}}}$. The mean liquidity value implied by our model is 17.5%.

Table 3. Liquidity Value ($\beta = 0.96$)

Average liquidity value (%)					
$\alpha \diagdown \delta$	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.05	0.2	23	203	844	2849
0.2	0.2	0.1	18	226	850
0.4	0.2	0.07	0.1	75	405
0.6	0.2	0.04	0.08	25	243
0.75	0.2	0.02	0.06	9.2	177

As noted in Table 3, the liquidity value is sensitive to the model parameters.

4 Conclusion

In this paper, we consider an environment with search frictions in the goods market. The asset in our model yields positive dividends and is also used to facilitate trading in the goods market. This transaction role makes the asset pricing implications of our model different from those in the standard asset pricing model. We show that this "small" departure from the standard asset pricing model can simultaneously deliver the observed average rate of return on equity, the volatility of the asset price and the spectral properties of the asset price.

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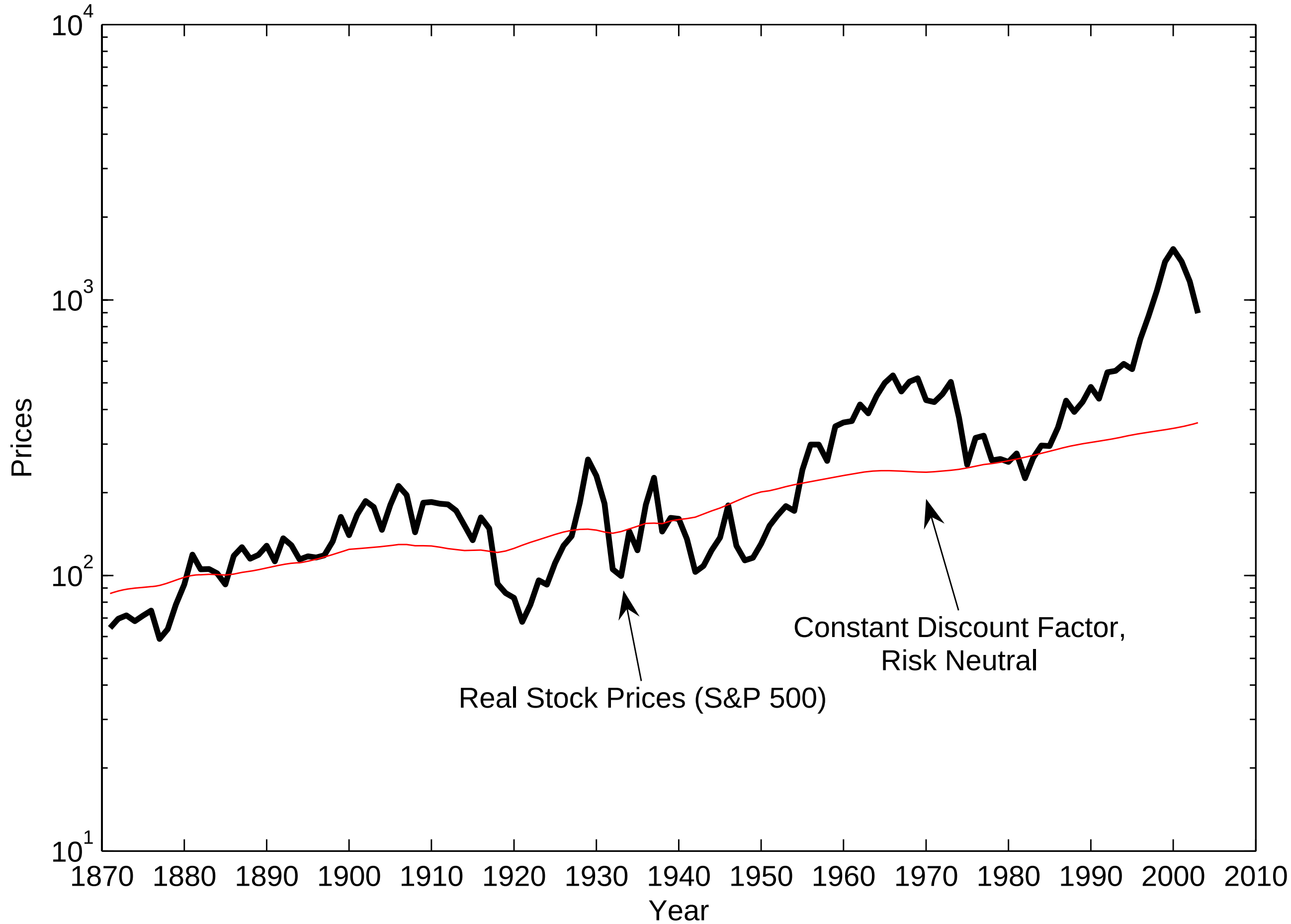

Figure 1: Asset Prices: Data and Shiller's Constant Discount Factor Model

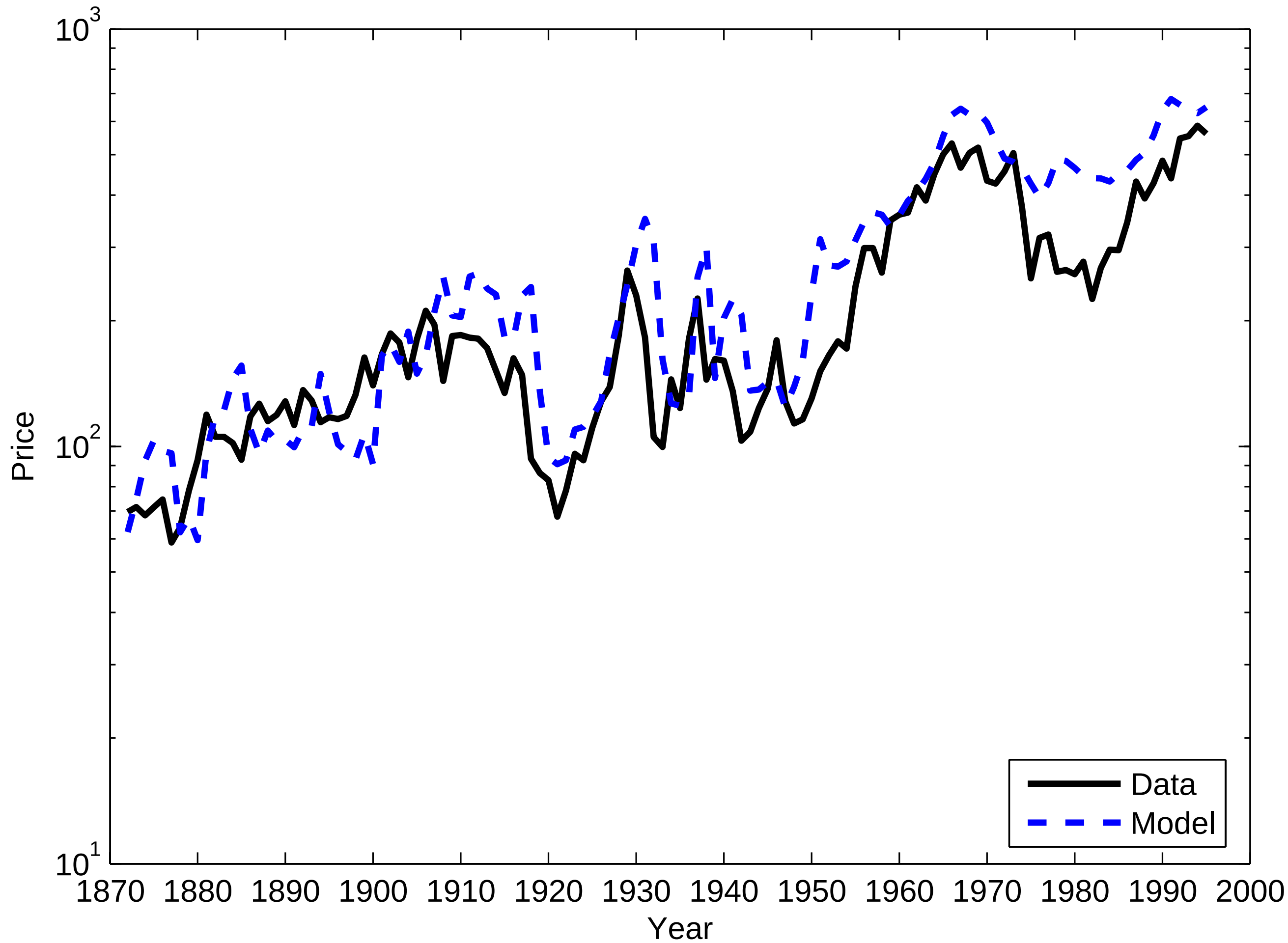

Figure 2: Asset Prices: Data and Model

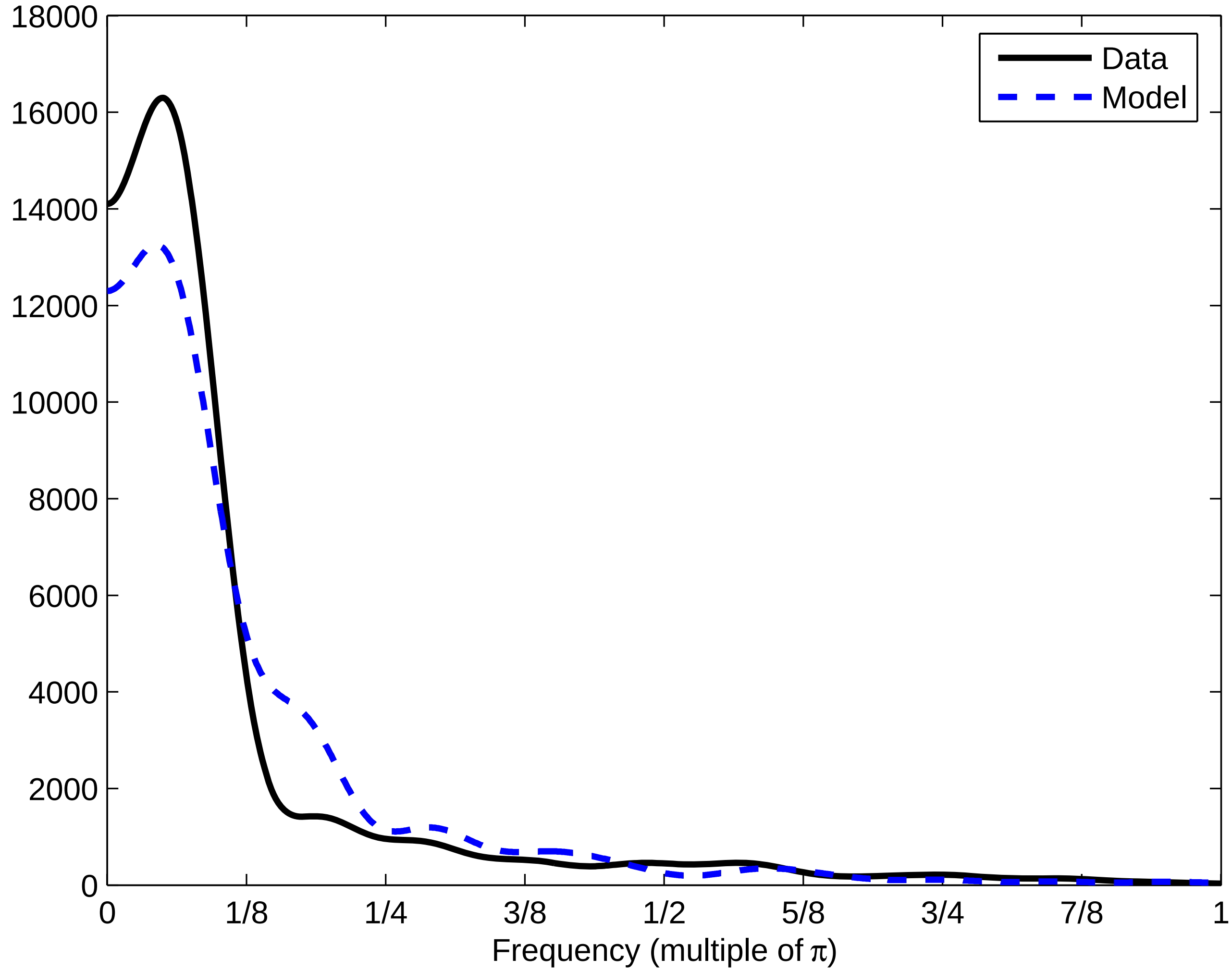

Figure 3: Spectra of Asset Prices: Data and Model

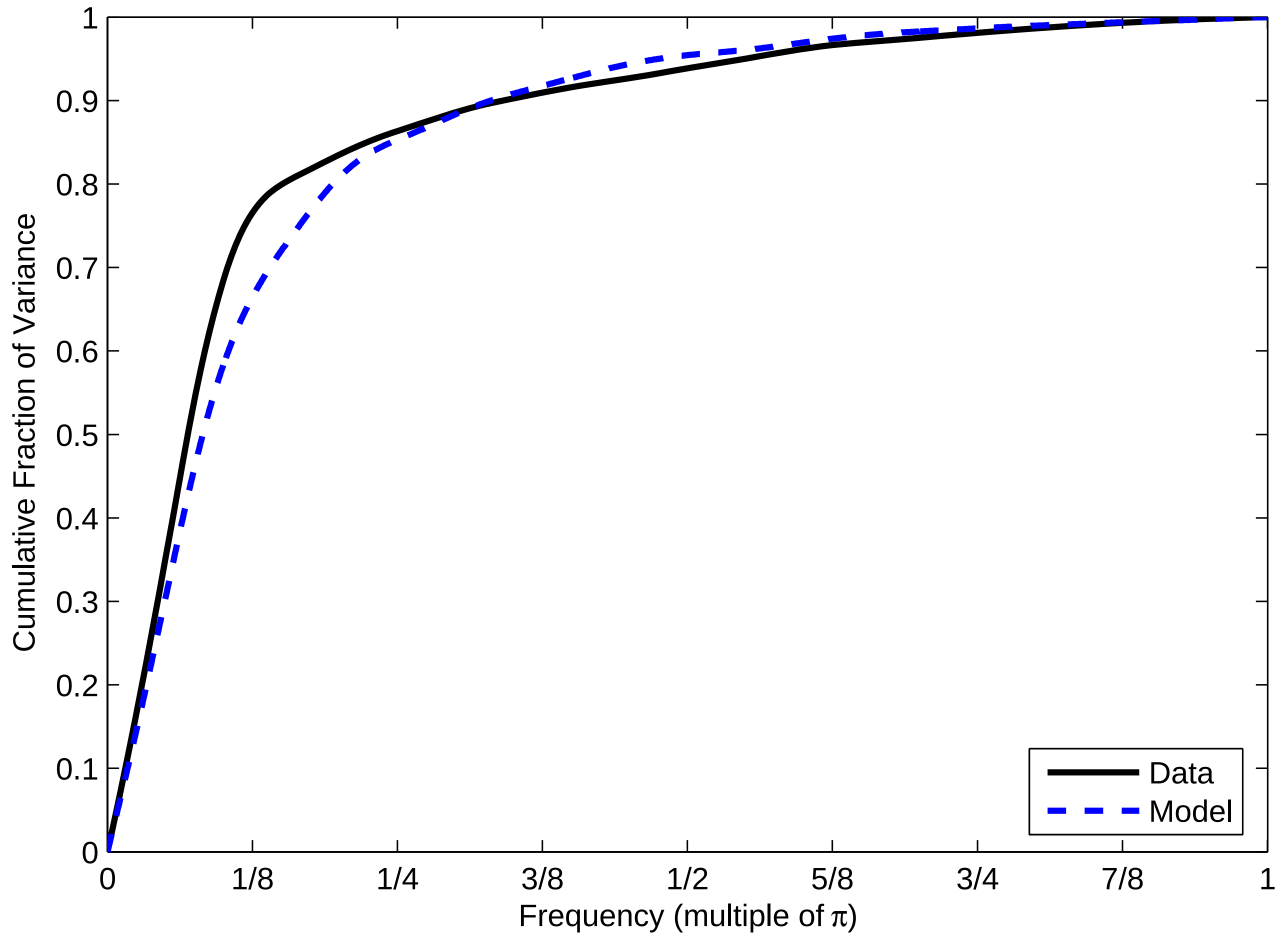

Figure 4: Cumulative Variance: Data and Model