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Abstract

Using identification-robust methods, the authors estimate and evaluate for Canada and the United States various classes of inflation equations based on generalized structural Calvo-type models. The models allow for different forms of frictions and vary in their assumptions regarding the type of price indexation adopted by firms. Point and confidence-set parameter estimates are obtained based on the inversion of identification-robust test statistics. Focus is maintained on the structural aspect of the model with formal imposition of the restrictions that map the theoretical model into the econometric one. The results show that there is some statistical merit to using indexation-based Calvo-type models for inflation. However, some identification difficulties are also uncovered with considerable uncertainty associated with estimated parameter values. In particular, we find that implausibly-high frequency of price re-optimization values cannot be ruled out from our identification-robust confidence sets.

JEL classification: C13, C52, E31
Bank classification: Inflation and prices; Econometric and statistical methods

Résumé

À l'aide de méthodes d'inférence robustes sur le plan de l'identification, les auteurs estiment et évaluent, pour le Canada et les États-Unis, plusieurs classes d'équations d'inflation fondées sur des modèles structurels généralisés comportant un mécanisme de révision des prix à la Calvo. Ces modèles autorisent des frictions diverses et définissent le type d'indexation des prix adopté par les entreprises selon différentes hypothèses. Les auteurs obtiennent une estimation ponctuelle des paramètres et déterminent une région de confiance en inversant le résultat de tests d'inférence robustes. Les restrictions qu'implique le modèle théorique sont imposées au modèle économétrique afin de maintenir la dimension structurelle du modèle. Les résultats révèlent une certaine légitimité statistique des modèles à la Calvo avec indexation des prix aux fins de la prévision de l'inflation. Ces modèles présentent cependant certains problèmes d'identification puisqu'une forte incertitude entache les valeurs estimées des paramètres. Les auteurs n'arrivent notamment pas à exclure des régions de confiance calculées des fréquences de révision des prix trop élevées pour être vraisemblables.

Classification JEL : C13, C52, E31
Classification de la Banque : Inflation et prix; Méthodes économétriques et statistiques

1. Introduction

In this paper we use identification-robust methods to assess the statistical performance of indexation-based Calvo models of inflation. Calvo-type sticky price models have been popular because of two main reasons: First, the so-called Calvo assumption (where the number of firms that change their prices at any given time is given exogenously) make working with these time-dependent models substantially easier than working with more intuitive state-dependent models. Second, because statistical support has been claimed for inflation equations based on these models (see, for example, Galí and Gertler 1999; Galí, Gertler, and Lopez-Salido 2001, Sbordone 2002, Eichenbaum and Fisher (2007)).

Early model versions have since been criticised for issues related to specification bias, the use of limited-information setups, and the appropriateness of instrumental-variables-based inference (see, for example, Rudd and Whelan 2005; Linde 2005; Dufour, Khalaf, and Kichian 2006). Thus, present-day Calvo-type models incorporate various generalizations that try to address some of these criticisms. In particular, they incorporate various nominal or real rigidities in labour or capital markets and account for a non-zero steady-state for inflation.

For example, the study by Christiano, Eichenbaum, and Evans (2005) presents a model that integrates features such as frictions in the labour market, variable capital utilization, and dynamic indexation, and the model is shown to have economic support based on partly-calibrated and partly-estimated parameter values.[1] Another example is the study of Eichenbaum and Fisher (2007) that presents a dynamic indexation model based on Christiano, Eichenbaum, and Evans (2005) and that is estimated with generalized method of moments (GMM). Using the J-test, Eichenbaum and Fisher (2007) find statistical support for indexation-based Calvo-style pricing models *in general*, and suggest that two particular extensions (namely, firm-specific capital and an elasticity of demand for intermediate goods that is increasing in firms' prices) are necessary in order to obtain plausible estimates of the average frequency of price re-optimization.

Clearly, the usefulness of existing variants of Calvo-type models for empirical or policy analysis depends importantly on their statistical identifiability, i.e., whether reliable econometric methods permit the estimation of underlying model parameters with measurable pre-

[1]Matching moments methods are used for the estimation. More precisely, a measure of the distance between the model's impulse response functions and the empirical impulse response functions is minimized.

cision. The theoretical frameworks of the above models typically yield Euler equations that lead to orthogonality conditions amenable to estimation by instrumental variables (IV) or GMM. When taken to the data, these models are often confronted with two central concerns: (i) endogeneity, which stems, in particular, from the presence of expectations-based regressors and from errors-in-variables issues, and (ii) parameter nonlinearity, that results from the connection between the key parameters of the underlying theoretical model and the parameters of the estimated econometric model, and that can impose discontinuous parameter restrictions.[2]

Both, endogeneity and nonlinear parameter constraints complicate statistical analysis in a non-trivial way. Furthermore, in many cases, models are heavily parametrized so that some of the parameters are calibrated, and direct estimation is typically feasible for transforms of the remaining parameters of interest. Estimates of the latter are then "backed-out", and confidence intervals are constructed using the *delta*-method or alternative projection techniques; see, for instance, Eichenbaum and Fisher (2007).

All of these difficulties, in conjunction with possibly-weak instruments, lead to the eventuality of *weak identification*. Weak-identification causes the breakdown of standard asymptotic procedures based on estimated standard errors [including IV-based t-tests, usual J-tests, and Wald-type confidence intervals of the form: estimate $\pm$ (asymptotic standard error) $\times$ (asymptotic critical point)], and a heavy dependence on unknown nuisance parameters. As a result, standard and even bootstrap-based tests and confidence intervals can be unreliable, and spurious model rejections occur frequently, even with large data sets.[3]

It is important to understand the fundamental reason behind such failures. When parameters are not identifiable on a subset of the parameter space, or when the admissible set of parameter values is unbounded (which occurs with nonlinear parameter constraints such as ratios), valid methods for the construction of confidence sets should allow for possibly-unbounded outcomes (Dufour 1997). Wald-type intervals are "bounded" by construction, and are thus inappropriate in a fundamental way. They cannot be saved nor improved. Even if maximum likelihood (ML) is used for the estimation, resorting to usual t-type significance

[2]For a discussion of both problems, see, for example, Galí, Gertler, and Lopez-Salido (2005).

[3]The so-called *weak instruments* theoretical literature is now considerable; see, for example, Dufour (1997), Dufour (2003), Staiger and Stock (1997), Wang and Zivot (1998), Zivot, Startz, and Nelson (1998), Dufour and Jasiak (2001), Kleibergen (2002), Kleibergen (2005), Stock, Wright, and Yogo (2002), Moreira (2003), Dufour and Taamouti (2005b), Dufour and Taamouti (2007), and Andrews, Moreira, and Stock (2006).

tests, or reliance on the *delta*-method, will lead to the same problems that plague GMM and linear or nonlinear IV. On recalling that identifying restrictions typically imply nonlinearity, we see that weak identification is indeed inherent to the definition of structural models. This is true even with a single linear simultaneous equation, which is identified via "exclusion" restrictions.[4] Despite the huge associated theoretical literature, these problems remain somewhat misunderstood, and confused with issues such as very large estimated standard errors or poorly-approximated cut-off points. We thus emphasize that usual point and interval estimation methods (whether based on ML, on matching moments methods, or on IV, and whether one considers a single structural equation or a multi-equation structural system), are flawed and should not be used. Instead, one has to rely on different methods that, by construction, allow for unbounded outcomes.[5]

The works of Christiano, Eichenbaum, and Evans (2005), and Eichenbaum and Fisher (2007) rely on standard approaches, and thus are prone to the danger of drawing wrong conclusions because of the concerns mentioned above. The pitfalls of weak instruments are quite subtle, as demonstrated by Dufour, Khalaf, and Kichian (2006). The latter study re-examines the Galí and Gertler (1999) model using methods that are robust to weak instruments and finds clear evidence of identification difficulty. In particular, although the point estimates of the deep parameters yield a fairly large forward-looking component for inflation, the identification-robust confidence set associated with the parameter estimates is quite large, and includes the case where the backward-looking component of inflation is more important than the forward-looking part. Furthermore, when survey expectations are used instead of rational expectations, identification difficulties remain, and both the point estimates and the identification-robust confidence set indicate a larger role for the backward-looking component for inflation.

With this backdrop in mind, in this paper we use Canadian and U.S. data and identification-robust methods to examine alternative dynamic indexation-based inflation models that are based on generalized Calvo setups.[6] In all cases, structural estimation is carried out. One category of models that we examine makes use of the full-indexation assumption, whereby

[4] This is easy to see when one derives the reduced-form or the structural likelihood function.

[5] See Dufour (1997) for further analysis of the unbounded parameter case.

[6] Although other types of indexation schemes such as static or rule-of-thumb approaches have also been used in the literature, we focus on the dynamic-indexation class of models because they seem to be more routinely used.

all the firms that cannot re-optimize prices index them to lagged aggregate inflation. The equations that we estimate and test in this category were presented and judged to be statistically acceptable according to GMM-based criteria, and for US data, in Eichenbaum and Fisher (2007). Another category of models that we consider allow for partial indexation, where only a proportion of those firms that cannot re-optimize their prices index the latter to lagged inflation. Such an assumption is made, for example, in Smets and Wouters (2003). Other than the extent of indexation, the models we examine also differ in their assumptions regarding the type of capital market (i.e., whether capital is homogenous or firm-specific) and the nature of the elasticity of intermediate goods demand that firms face.

In the next section we present the New Keynesian Phillips Curve (NKPC) inflation models under full and partial dynamic indexation. Section 3 discusses our methodology. Section 4 presents the empirical results, and section 5 offers some conclusions.

2. NKPC Models with Indexation

We follow the modelling setup in Eichenbaum and Fisher (2007). Firms evolve in a monopolistically-competitive environment but face constraints on the adjustment of their prices. A Calvo-type assumption is used for this purpose: at any given time t, a firm faces an exogenous probability of adjusting its price. When it can adjust the price, it re-optimizes. The rest of the time, the firm's price can be indexed to some measure of aggregate inflation.

Two forms of price indexation have specially been considered in the recent literature: full dynamic indexation, where all non-optimizing firms' prices are indexed to previous period's aggregate inflation level, and partial dynamic indexation, where only some firms' prices are indexed to lagged aggregate inflation.

With full dynamic indexation, the aggregate inflation process, π_t, evolves according to the equation:

$$\Delta\hat{\pi}_t = \beta E_{t-\tau}\Delta\hat{\pi_{t+1}} + \lambda E_{t-\tau}\hat{s}_t. \tag{1}$$

Assuming rational expectations, the econometric model can be written as:

$$\Delta\hat{\pi}_t = \beta\Delta\hat{\pi}_{t+1} + \lambda\hat{s}_t + u_{t+1}, \tag{2}$$

where the error term u_{t+1} is a moving average of order τ, and where the parameter λ is given by:

$$\lambda = \frac{A.D.(1-\theta)(1-\beta\theta)}{\theta}. \tag{3}$$

In the above, Δ is the first difference operator, $\hat{x}$ is the variable x in deviation from its steady-state value, s_t is real marginal costs, τ refers to the implementation delay (that is, the number of periods between the time the re-optimization decision is taken and the actual implementation of the changes), β is the subjective discount rate, and θ is the Calvo probability of not adjusting prices. The corresponding average frequency of price re-optimization is given by the expression $1/(1-\theta)$.

The parameters A and D in the λ term are defined according to different assumptions regarding the price elasticity of intermediate goods' demand that firms face, and the type of capital market, respectively. The possibilities are:

1. the standard version of sticky price Calvo models, where capital is homogeneous and firms face a constant price elasticity of demand. In this case, $A = D = 1$.

2. capital is homogeneous ($D = 1$), but firms face a variable price elasticity of demand ($A < 1$).

3. Firms face a variable price elasticity of demand ($A < 1$), and capital is firm-specific ($D < 1$). In the latter case, capital adjustment costs may also intervene.

The parameters A and D have fundamental structural implications: A governs the degree of pass-through from a rise of marginal cost to prices, or, alternatively,

$$A = \frac{1}{\zeta\epsilon + 1} \tag{4}$$

where ϵ is the per cent change in the elasticity of demand for a given intermediate good due to a one per cent change in the relative price of the good at steady state, and ζ denotes the firm's steady state mark-up. D is a nonlinear function of β, θ, A, and other deep parameters. It is defined as:

$$D = \frac{(1-\beta\theta\kappa_1)}{(1+\overline{\eta}\xi A)(1-\beta\theta\kappa_1)+\xi A\beta\theta\kappa_2}, \tag{5}$$

where $\overline{\eta}$ is the steady state elasticity of demand, related to ζ according to the equation

$$\zeta = \overline{\eta}/(\overline{\eta}-1) - 1. \tag{6}$$

The parameter ξ is defined as:

$$\xi = \overline{\alpha}/(1-\overline{\alpha}), \tag{7}$$

where $\overline{\alpha}$ is the share of capital in the production function. Finally, κ_1 and κ_2 are the solutions of the 3-equation system that solve for κ_1, κ_2 and ν subject to the constraint that $|\kappa_1| < 1$. The system is given by:

$$\begin{aligned} 1 - \left[\phi + (1-\theta\nu)\left(\beta\kappa_2 - \Xi\right)\right]\kappa_1 + \beta\kappa_1^2 &= 0 & (8)\\ \Xi\theta + \left[\phi - \beta\left(\theta + \kappa_1\right) - (1-\theta)\,\Xi\nu\right]\kappa_2 + \beta\left(1-\theta\right)\nu\kappa_2^2 &= 0 & (9)\\ \frac{\xi A(1-\beta\theta)}{(1+\overline{\eta}\xi A)(1-\beta\theta\kappa_1) + \xi A\beta\theta\kappa_2} - \nu &= 0 & (10) \end{aligned}$$

with

$$\begin{aligned} \Xi &= \left(1-\beta\left(1-\delta\right)\right)\overline{\eta}\frac{1}{1-\overline{\alpha}}\frac{1}{\psi}, & (11)\\ \phi &= 1 + \beta + \left(1-\beta\left(1-\delta\right)\right)\frac{1}{1-\overline{\alpha}}\frac{1}{\psi}. & (12) \end{aligned}$$

In this set-up, ψ is the capital adjustment cost parameter; and δ is defined such that the elasticity of the investment-to-capital ratio with respect to Tobin's q (evaluated at steady-state) is given by $1/(\delta\psi)$. When $\psi = 0$,

$$\begin{aligned} \kappa_1 &= 0, \quad \kappa_2 = -\widetilde{\Xi}/\widetilde{\phi}, \quad \nu = \xi A(1-\beta\theta)/\left[(1+\overline{\eta}\xi A) + \xi A\beta\theta\kappa_2\right], \\ \widetilde{\Xi} &= \left(1-\beta\left(1-\delta\right)\right)\overline{\eta}/(1-\overline{\alpha}), \quad \widetilde{\phi} = 1 + \beta + \left(1-\beta\left(1-\delta\right)\right)/(1-\overline{\alpha}), \end{aligned}$$

in which case

$$D = \frac{1}{(1+\overline{\eta}\xi A) - \xi A\beta\theta\widetilde{\Xi}/\widetilde{\phi}}. \tag{13}$$

Note that the structural parameters β and θ that we will be estimating enter the definition of the calibrated D parameter. In order to facilitate the exposition in the methodology section (Section 3), we thus introduce the following notation: let ω represent the calibrated parameters of the model. We can then express D as:

$$D = d(\omega, \text{estimated parameters}). \tag{14}$$

The function $d(.)$ is then defined according to the various considered assumptions.

Instead of full indexation, it is also possible to allow only a fraction of firms to index their prices to lagged inflation. Such a partial dynamic indexation assumption was made, for example, by Smets and Wouters (2003). Let the partial indexation parameter be given by ν_2. The above general structure is then modified as follows:

$$\hat{\pi}_t = \frac{\beta}{(1+\beta\nu_2)}\hat{\pi}_{t+1} + \frac{\nu_2}{(1+\beta\nu_2)}\hat{\pi}_{t-1} + \frac{\lambda_2}{(1+\beta\nu_2)}\hat{s}_t + e^p_{2,t+1}. \quad (15)$$

For convenience, we denote the coefficients on $\hat{\pi}_{t+1}$, $\hat{\pi}_{t-1}$, and $\hat{s}_t$ as γ_{2f}, γ_{2b}, and λ_2^p, respectively. Thus, $\gamma_{2f} = \beta/(1+\beta\nu_2)$, $\gamma_{2b} = \nu_2/(1+\beta\nu_2)$, and $\lambda_2^p = \lambda/(1+\beta\nu_2)$. Note that when $\nu_2 = 1$, the full-indexation model obtains.

3. Methodology

Identification-robust methods make use of inference procedures where error probabilities can be controlled in the presence of endogeneity and nonlinear parameter constraints, even in the presence of identification difficulties. Our approach differs from the usual IV-based one in that it avoids: (i) standard t-type confidence intervals, and (ii) reliance on the *delta*-method. Rather, we propose confidence set (CS) parameter estimates based on "inverting" identification-robust test statistics. The general theory underlying this approach is developed in Dufour and Taamouti (2005a), [see also Dufour and Taamouti (2007)]. Inverting a test produces the set of parameter values that are not rejected by this test, and the least-rejected parameters are the so-called Hodges-Lehmann point estimates (see Hodges and Lehmann 1963, 1983, and Dufour, Khalaf, and Kichian 2006). In contrast to the usual t-type confidence intervals, confidence sets formed by inverting a test lead (by construction) to possibly-unbounded solutions, a prerequisite for ensuring reliable coverage (see Dufour 1997).

The tests that we invert not only ensure identification-robustness, but they also maintain the structural aspect of the model by formally imposing the restrictions that map the theoretical model into the econometric one. Indeed, the analyses of Galí, Gertler, and Lopez-Salido (2005) and Fernandez-Villaverde, Rubio-Ramirez, and Sargent (2005) emphasize the fact that any econometric method should formally take into account the constraints on the parameters and/or error terms, as implied by the underlying theoretical model, whether inference is based on a single structural equation, on the closed form, or on a structural system. Our structural analysis is also carried out respecting the moving-average error structure and the calibration exercise (as will be shown later, the latter involves repeatedly solving a system of nonlinear equations). Furthermore, we avoid the *delta*-method altogether so that we do not need to "back-out" the structural parameters of interest from estimated transforms, in contrast to Eichenbaum and Fisher (2007).[7]

We deal with all such irregularities by making use of simple F-type procedures (with or without standard autocorrelation-robust corrections), for which standard finite-sample and asymptotic distributional theory applies. This exercise is extremely simple, despite the complexity of the nonlinear model under consideration. Our procedure has two further "built-in" advantages. First, extremely wide confidence sets provably reveal identification difficulties. Second, if all economically-sound values of the model's deep parameters are rejected at some chosen significance level, the confidence set will be empty and we can infer that the model is soundly rejected. This provides an identification-robust alternative to the standard GMM-based J-test. For all cases, we estimate the price re-optimization parameter (θ) and the subjective discount rate (β), and focus on the uncertainty of their estimates. For the partial indexation models, we also estimate the partial indexation parameter.

We hereby describe the details of our methodology as it applies for one illustrative case. Suppose that we would like to estimate the deep parameter θ in the context of the full indexation model while maintaining the calibrated parameters ω at their values.[8] The equation under consideration is thus given by:

$$\Delta\widehat{\pi}_t = \beta\Delta\widehat{\pi_{t+1}} + \lambda\ \hat{s}_t + u_{t+1}, \qquad t = 1, ..., T, \tag{16}$$

with ω described by the following vector:

$$\omega = \begin{pmatrix} \beta & \overline{\alpha} & \psi & \delta & \zeta & \epsilon \end{pmatrix}'.$$

Alternatively, we can write

$$y_t = Y_t'\gamma + u_{t+1}, \tag{17}$$

where $y_t \equiv \Delta\hat{\pi}_t$, $Y_t = (\Delta\widehat{\pi_{t+1}}, \hat{s}_t)'$, $\gamma = (\beta,\ \lambda)'$, and where the error term reflects the rational expectations hypothesis. An instrument set, X_t, of dimension $k \times 1$ is also available at time t. Finally, u_{t+1} follows an MA(τ) structure; an implication of Eichenbaum and Fisher (2007)'s theoretical model.

[7]The projection technique used by Eichenbaum and Fisher (2007) is valid in principle when the underling transformation is monotonic; since the model at hand is highly nonlinear, monotonicity is not granted.

[8]In this example, the subjective discount rate parameter β is calibrated. It is straightforward to extend the methodology to a joint estimation setup. A joint confidence set is obtained (as will be discussed below) from which projections for each component are obtained.

To simplify presentation, we further adopt the following notation: y is the T dimensional vector of observations on $\Delta\widehat{\pi}_t$, Y is the $T \times 2$ matrix of observations on $\Delta\widehat{\pi_{t+1}}$ and $\hat{s}_t$, X is the $T \times k$ matrix of the instruments, and u is the T dimensional vector of error terms. We also denote by $M[V]$ the projection matrix $I - V(V'V)^{-1}V'$.

To obtain a confidence set with level $1 - \alpha$ for the deep parameter θ, we invert an identification-robust test (see below) associated with the null hypothesis

$$H_0 : \theta = \theta_0 \text{ and } \omega = \omega_0, \tag{18}$$

where ω_0 and θ_0 are known values. Formally, this implies collecting the values of θ_0 that, given the calibrated ω_0, are not rejected by the test (i.e. for which the test is not significant at level α).

Using a grid search over the economically-meaningful set of values for θ, we sweep the choices for θ_0 given ω_0. For each parameter choice considered, we compute test statistics and their associated p-values (the tests are described below). The parameter vectors for which the p-values are greater than the level α thus constitute a confidence set with level $1 - \alpha$. In addition, the values of θ_0 (and knowing ω_0) that lead to the largest p-value formally yield the set of "least-rejected" models, i.e., models that are most compatible with the data. This method underlies the principles of the Hodges-Lehmann estimation method; see Hodges and Lehmann (1963); Hodges and Lehmann (1983). Whereas uniqueness (as obtained through the usual point estimation approach) is not granted, analyzing the economic information content of these least rejected models provides very useful model diagnostics.

Thus, given ω_0, for each choice of θ_0:

1. Solve (14) for values of A and D associated with ω_0, and θ_0.[9] Using (3), obtain the corresponding value for λ. Denote the latter λ_0.

2. Conduct the test in the context of the following regression (which we denote the AR-regression in reference to Anderson and Rubin 1949) of

$$\{\Delta\widehat{\pi}_t - \beta_0\Delta\widehat{\pi_{t+1}} - \lambda_0\ \hat{s}_t\} \text{ on } \{\text{the } instruments\ X_t\}. \tag{19}$$

[9]Note that solving this equation is numerically complex, as it involves solving systems such as (4)-(12). The model examined in Dufour, Khalaf, and Kichian (2006), though structural, did not raise such numerical challenges.

Under the null hypothesis [specifically (16)-(18)], the coefficients of the latter regression should be zero. Hence testing for a zero null hypothesis on the coefficients of X_t in (19) provides a test of (18).

Our approach maps the structural equation (16) that faces identification difficulties into the standard regression (19). The latter provides a regular framework (because the right-hand side regressors are not "endogenous"), where identification constraints are no longer needed. Therefore, usual statistics that test for the exclusion of X_t can be applied in a straightforward manner. For instance, under the *i.i.d.* error assumption for (19) (i.e., the case of $\tau = 0$), the usual F statistic can be used:

$$AR(\omega_0, \theta_0) = \frac{(y - Y\gamma_0)'(I - M[X])(y - Y\gamma_0)/(k)}{(y - Y\gamma_0)' M[X](y - Y\gamma_0)/(T-k)}, \tag{20}$$

with the $F(k, T-k)$ or $\chi^2(k)$ null distribution. To correct for departures from the *i.i.d.* error hypothesis (i.e., when $\tau \geqslant 0$), we consider a Wald-type statistic with Newey-West autocorrelation-consistent covariance estimator for the coefficient of the AR regression (19):

$$\begin{aligned} AR\text{-HAC}(\omega_0, \theta_0) &= (y - Y\gamma_0)' X(X'X)^{-1}\widehat{Q}^{-1}(X'X)^{-1}X'(y - Y\gamma_0) \qquad (21) \\ \widehat{Q} &= \frac{1}{T}\sum_{t=1}^{T} \widehat{u}_t^2 X_t X_t' + \frac{1}{T}\sum_{l=1}^{L}\sum_{t=l+1}^{T} w_l \widehat{u}_t \widehat{u}_{t-l}\left(X_t X_{t-l}' + X_{t-l}X_t'\right) \\ w_l &= 1 - \frac{l}{L+1} \end{aligned}$$

where $\widehat{u}_t$ is the OLS residual associated with (19) and L is the number of allowed lags.[10]

To conclude, despite the complex underlying nonlinearities (recall the definitions of A, D and λ), the approaches proposed in this section are tractable, and they are identification-robust (in the sense that they are statistically valid whether the model is identified or not).

4. Empirical Results

We conduct our estimations on quarterly U.S. and Canadian data. The U.S. sample extends from 1982Q3 to 2006Q4.[11] We use the GDP deflator for the price level, P_t, the compensation

[10]In our applications, we use the $\chi^2(k)$ null distribution, and allow 4 quarters for L.

[11]This sample includes a few more years than the second subsample examined by Eichenbaum and Fisher (2007). We did not consider the earlier dates because of the change in monetary policy that occurred at the end of the seventies and early eighties and that likely generated a structural break in the inflation series around those dates.

per hour in the non-farm business sector for wages, W_t, and we define the labour share of income as total compensation paid to employees divided by nominal GDP. The Canadian data are from Statistics Canada and span the 1982Q2–2007Q2 range. The GDP deflator is used for the price level, P_t, wage is given by total compensation per hour in the business sector, and labour share is defined as wages, salaries and supplementary labour income of persons and unincorporated businesses divided by nominal output.

Taking the log of these series (which we represent by the corresponding small letters), we define inflation, π_t, as gross inflation, and real marginal cost, s_t as the logarithm of the labour share of income. The instrument set contains lags of price inflation, real marginal cost, wage inflation, and quadratically-detrended real GDP.[12] These are the same variables as those used by Eichenbaum and Fisher (2007), except that we do not include a lagged Euler error term.

We choose the lag order of the variables that form the instrument set depending on the considered value for τ. We set the latter to one, similar to Eichenbaum and Fisher. Thus, the structure of the error term is $MA(1)$ and, as a result, the AR-HAC test is used. For the latter, significance refers to a five per cent test level. In addition, in all the estimations, four lags are used in the Newey-West heteroskedasticity and autocorrelation-consistent covariance estimator.[13] Finally, all variables are taken in deviation from the sample mean, which, in our methodological context, implies that instead of fixing steady-state values to specific parameters we allow them to be free constants.[14]

We first repeat the exercise conducted by Eichenbaum and Fisher (2007) on our U.S. data, except that we apply the identification-robust methodology described in the previous section. Thus, we use the dynamic indexation model and we calibrate all parameters to the values considered by Eichenbaum and Fisher (2007) estimating only θ.[15] The grid search is conducted using increments of 0.01 for θ over the economically-plausible range of values

[12]Our output gap measure is real-time, in the sense that the gap value at time t does not use information beyond that date. Thus, as in Dufour, Khalaf, and Kichian (2006), we proceed iteratively: to obtain the value of the gap at time t, we detrend GDP with data ending in t. The sample is then extended by one observation and the trend is re-estimated. The latter is used to detrend GDP, and yields a value for the gap at time $t+1$. This process is repeated until the end of the sample. A quadratic trend is used for this purpose.

[13]We also experimented with $L = 12$ with generally qualitatively-similar results.

[14]See Sbordone (2007) for a discussion on the importance of doing so in empirical contexts.

[15]Recall that our sample is longer than the 1982Q3–2001Q4 subsample considered by Eichenbaum and Fisher (2007), and note that we do not include the lagged Euler error in our instrument set.

[0.01,0.99].

The calibration assumes that the share of labour in the production function is $\frac{2}{3}$, that the quarterly depreciation rate of capital is 2.5 per cent, that the quarterly discount rate β is 0.99, and that the markup is 10 per cent. Three values of ϵ are considered: 0, 10 and 33, and that imply values of 1, 0.50 and 0.23, respectively, for the A parameter. Where applicable, the values 0 and 3 are considered for the capital cost adjustment parameter ψ. Finally, a price implementation lag delay of one period is assumed.

Table 1 below summarizes our results in Panel A and reports the values obtained by Eichenbaum and Fisher (2007) in Panel B for ease of comparison. Immediately, four clear features stand out. First, our point estimates are higher than those of Eichenbaum and Fisher (EF). Second, though higher in value, these move in the same direction as those of EF, in that they decline when the elasticity is higher and as capital markets change from homogeneous to firm-specific. Third, under identification-robust conditions, the results do not change whether capital adjustment costs are considered or not. This is not the case in EF. Finally, the identification-robust confidence sets of θ that we obtain (and, consequently, also of the average frequency of price re-optimization Fq) are markedly larger than those reported in the study by EF, indeed, hitting in all cases the upper bound of this parameter space.

We next turn to the cases where β, and, where relevant, ν_2, are also estimated. The indexation-based models found in equations (2) and (15) are estimated under each of the two hypotheses $A = D = 1$, and $A < 1$, $D < 1$, having imposed all of the appropriate structural constraints as described in Section 2. For the latter hypothesis, we calibrate the elasticity parameter to 33, which is the highest value considered by Eichenbaum and Fisher, and thus $A = 0.23$.[16] We also allow for positive adjustment costs and, as in Eichenbaum-Fisher, set Ψ to a value of 3.

For the model with full indexation, the estimated structural parameters are θ and β. In the case of equation (15), the partial indexation parameter ν_2 is also estimated. The search

[16]Eichenbaum and Fisher also consider an elasticity value of 10. Since we conduct estimations under the limit values of 0 and 33 for the elasticity parameter, outcomes for values of the elasticity falling within this range can be guessed by extrapolation. Note, also, that we considered a set-up where the parameter A was estimated along with θ and β. However, in this case, the uncertainty around the estimated A parameter covered all of its considered space, meaning that the data could not provide any information on this parameter. Accordingly, we instead used calibrated values for it.

space for θ is [0.02,0.98] and for ν_2 it is [0.02,1.00], with grid increments of 0.02. In the case of β, we consider the [0.90,0.99] grid, moving through the grid with increments of 0.01.

Tables 2A and 3A report the results for the U.S. and Canada, respectively, pertaining to the full-indexation models, while Tables 2B and 3B show the corresponding outcomes for the partial-indexation models. Overall, we find that none of the models for the two countries yield empty confidence sets for the estimated parameters at the 5 per cent level. This implies that there is some statistical merit to using these type of models.[17] However, it is also clear that the sets are fairly wide (these cover all of the parameter space for some parameters), indicating that there are identification difficulties. These are examined more closely in the following sections.

Looking first at the results for the U.S., we notice that under the full indexation assumption, both instrument sets yield similar conclusions. Thus, when capital is homogeneous, the point estimates for θ reveal implausibly-high price stickiness, with average price durations of 12.5 to 16.5 quarters. At the same time, point estimates of the subjective discount rate are 0.99; a number very much in line with conventional wisdom on the value of this parameter. Upon allowing for firm-specific capital, and for both instrument sets, the point estimates for θ drop substantially, translating into durations of 4 and a half to 5 and a half quarters of average price stickiness in the economy. These results are qualitatively similar to the outcomes of the Eichenbaum-Fisher study, though our point estimates are higher regardless of the type of capital market and calibrated elasticity value assumed.

The numbers reported in brackets and located under every point estimate refer to the projected confidence intervals around those point estimates. They thus represent the smallest and highest values in the joint identification-robust confidence set of a given estimated parameter. Note that, with both structural parameters, the intervals include the upper or lower limit of their admissible parameter spaces (the interval for θ includes the uppermost value of 0.98, while for β, all of the admissible values are found in its interval).

Accordingly, with regard to estimate uncertainty, it is possible to assert that, when all firms follow a lag-inflation-indexing pricing strategy, price stickiness is, at minimum, just over 3 and a half quarters with homogenous capital markets, and just over one and a half quarters

[17] Recall that our procedure automatically executes a model specification test, and that an empty confidence set would mean a rejection of the tested specification according to our identification-robust version of the J-test.

with firm-specific capital markets. However, no further information is obtained from the data to help to narrow the range of the uncertainty at the upper end of this estimate's confidence interval. Similarly, for the β parameter, the data does not provide any information to narrow the range of the uncertainty for the estimate at either end of the confidence interval.

The interesting feature of the two models is that, despite the important identification difficulties associated with the structural parameters of the model, it is possible to obtain useful information on the coefficient of the real marginal cost parameter, λ. In particular, we find that the estimate for this parameter is significant, though its point value is fairly small under both capital market assumptions.

The results for the U.S. under partial indexation are qualitatively similar to those obtained under the full-indexation scenario with the exception of two things. One is that point estimates obtained using the instrument set Z_1^{EF} yield implausibly-high amounts of price stickiness even when capital is assumed to be firm-specific. The other is that point estimates for β are 0.90 regardless of the considered model. In addition, we find that the indexation parameter has a point estimate value of one with the samller instrument set, and a value of 0.56 with Z_2^{EF}, while the uncertainty associated with this parameter is extensive, covering the whole range of its admissible parameter space.

As for the implications of these results on the implied coefficients of the regressors, we see that, except for one case, point estimates show a more forward-looking curve, though the confidence intervals indicate that we cannot say so conclusively (the intervals include cases where the backward-looking component of inflation is more important). In addition, and similar to the case of the full-indexation models, we find that the coefficient on marginal costs is small but significant, even though relativel larger values are also included in its confiedence interval.

The results for Canada under full-indexation are quite similar to those for the U.S. except for two main features. One is that point estimates for the subjective discount rate are 0.90 regardless of the model considered. Another is that point estimates with the smaller instrument set yield implausible price-stickiness even with firm-specific capital. Apart from these two things, we find that the point estimate for θ drops to 0.76 with instrument set Z_2^{EF} and in a world with firm-specific capital market, translating into a price stickiness duration of about 4 quarters in the economy. At the same time, confidence intervals hit one or both boundries of the considered parameter space for the structural parameters (and under all possible model configurations) and we can at best assert that, at minimum, price stickiness is of the order of 3 and a half quarters when capital is assumed to be homogeneous, and one and a half quarters, when capital is assumed firm-specific. Finally, again we find that the point estimate on the marginal cost parameter is small, though it is significant and although its confidence set includes values as large as 0.1.

A comparison between Table 2B for the U.S. and Table 3B for Canada shows, again, qualitatively similar outcomes. We find, in particular, that with instrument set Z_1^{EF} point estimates for θ are much too high and for β they are much too low. In addition, minimum price stickiness durations are the same as those obtained with the full-indexation case. A difference exists with respect to the partial indexation parameter in that the confidence interval for its estimate is smaller with Canadian data than with U.S. data. Finally, results are similar with respect to the implied coefficients on the regressors.

4. Conclusion

In sum, the fact that the confidence sets based on identification-robust methods are non-empty (i.e., that the specifications were not rejected altogether) for both countries implies that there is some merit to using Calvo and indexation-based NKPC models for inflation; recall that our confidence set estimation method includes a built-in specification check which provides an identification-robust alternative to the GMM-based J-test. However, we find that there are also identification difficulties leading to non-negligible uncertainty around point estimates. In particular, the fact that the econometric models cannot rule out high or implausibly-high values of θ from the obtained confidence sets renders assertions about model fit based on the obtained θ values tenuous.

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Appendix: Tables

Table 1: Estimates of θ, U.S. data, Calibrated Full Dynamic Indexation Model

	Panel A: AR-HAC Test Results								
	Rental Capital Market			Firm-Specific Capital Market					
				$\Psi = 0$			$\Psi = 3$		
Elasticity	$\hat{\theta}$	Fq	$p-val$	$\hat{\theta}$	Fq	$p-val$	$\hat{\theta}$	Fq	$p-val$
0	0.94	16.67	0.9044	0.84	6.25	0.9044	0.85	6.67	0.9044
	(0.74, 0.99)			(0.46, 0.99)			(0.46, 0.99)		
10	0.91	11.11	0.9044	0.83	5.88	0.9044	0.84	6.25	0.9044
	(0.65, 0.99)			(0.44, 0.99)			(0.44, 0.99)		
33	0.87	7.69	0.9044	0.81	5.26	0.9044	0.81	5.26	0.9044
	(0.53, 0.99)			(0.39, 0.99)			(0.39, 0.99)		
	Panel B: The EF GMM-based Results								
0	0.83	5.9	-	0.75	4.0	-	0.63	2.7	-
	(0.73, 0.93)			(0.56, 0.92)			(0.45, 0.85)		
10	0.77	4.4	-	0.70	3.3	-	0.60	2.5	-
	(0.64, 0.90)			(0.51, 0.90)			(0.43, 0.84)		
33	0.68	3.1	-	0.62	2.6	-	0.56	2.3	-
	(0.52, 0.86)			(0.43, 0.85)			(0.38, 0.81)		

Panel A reports our AR-HAC test results and Hodges-Lehmann point estimates ($\hat{\theta}$). The implementation lag τ is one. The cases where $\psi = 0$ and $\psi = 3$ refer to firm-specific market cases (where $D < 1$) with, or without adjustment costs, respectively. Instruments include time $t - \tau - 1$ lags of each of inflation, marginal costs, wage inflation, and one-sided quadratically-detrended output gap. Fq is average frequency of price re-optimization (in quarters), and p-val denotes p-values. Panel B reproduces the Table 4 Panel B results reported in Eichenbaum and Fisher (2006).

Table 2A: U.S. Full-Indexation Model: Estimation and Test Results

Inst.	θ	β	Fq	D	λ	Max P-val
			$A = D = 1$			
Z_1^{EF}	0.94	0.99	16.7	1.00	0.0044	0.9044
	(0.74,0.98)	(0.90,0.99)	(3.85,50)		(0.0006,0.0992)	
Z_2^{EF}	0.92	0.99	12.5	1.00	0.0078	0.8991
	(0.72,0.98)	(0.90,0.99)	(3.57,50)		(0.0006,0.1173)	
			$A = 0.23,\ D < 1$			
Z_1^{EF}	0.82	0.99	5.56	0.48	0.0046	0.9044
	(0.40,0.98)	(0.90,0.99)	(1.67,50)		(0.0001,0.0964)	
Z_2^{EF}	0.78	0.99	4.55	0.47	0.0069	0.8991
	(0.36,0.98)	(0.90,0.99)	(1.56,50)		(0.0001,0.1186)	

The applied test is the AR-HAC test. Four lags are used in the Newey-West heteroskedasticity and autocorrelation-consistent covariance estimator. Hodges-Lehmann point estimates are reported with the corresponding p-value under the heading 'Max P-val', while $Fq = 1/(1-\theta)$ refers to the implied price stickiness (in quarters). The numbers in parentheses reported underneath a parameter estimate correspond to the projection-based confidence interval for that parameter. Instrument sets are as follows: Z_1^{EF} includes the second lag of each of: inflation, marginal costs, output gap, and change in nominal wages. Z_1^{EF} includes the second and third lags of each of inflation, marginal costs, output gap, and change in nominal wages.

Table 2B: U.S. Partial-Indexation Model: Estimation and Test Results

Inst.	ν_2	θ	β	Fq	D	γ_{2f}	γ_{2b}	λ_2^P	Max P-val
					$A = D = 1$				
Z_1^{EF}	1.00	0.98	0.90	1.00	50.0	0.60	0.37	0.0098	0.7058
	(0.02,1.00)	(0.72,0.98)	(0.90,0.99)		(3.57,50)	(0.47,0.97)	(0.02,0.53)	(0.0006,0.1117)	
Z_2^{EF}	0.56	0.94	0.90	1.00	16.7	0.84	0.15	0.0038	0.9552
	(0.02,1.00)	(0.72,0.98)	(0.90,0.99)		(3.57,50)	(0.47,0.97)	(0.02,0.53)	(0.0006,0.1117)	
					$A = 0.23,\ D < 1$				
Z_1^{EF}	1.00	0.98	0.90	0.64	50.0	0.47	0.53	0.0004	0.7109
	(0.02,1.00)	(0.36,0.98)	(0.90,0.99)		(1.62,50)	(0.47,0.97)	(0.02,0.53)	(0.0001,0.1085)	
Z_2^{EF}	0.56	0.78	0.90	0.49	4.55	0.60	0.37	0.0094	0.9552
	(0.02,1.00)	(0.72,0.98)	(0.90,0.99)		(1.56,50)	(0.47,0.97)	(0.02,0.53)	(0.0001,0.1186)	

Refer to the table notes under Table 2A for details.

Table 3A: Canadian Full-Indexation Model: Estimation and Test Results

Inst.	θ	β	Fq	D	λ	Max P-val
			$A = D = 1$			
Z_1^{EF}	0.98	0.90	50.0	1.00	0.0024	0.7058
	(0.72,0.98)	(0.90,0.99)	(3.57,50)		(0.0006,0.1173)	
Z_2^{EF}	0.94	0.90	16.7	1.00	0.0098	0.4418
	(0.74,0.98)	(0.90,0.99)	(3.85,50)		(0.0006,0.1044)	
			$A = 0.23,\ D < 1$			
Z_1^{EF}	0.98	0.90	50.0	0.64	0.0004	0.7109
	(0.38,0.98)	(0.90,0.99)	(1.63,50)		(0.0001,0.0964)	
Z_2^{EF}	0.76	0.90	4.17	0.47	0.0110	0.4421
	(0.38,0.98)	(0.90,0.99)	(1.63,50)		(0.0001,0.1051)	

Refer to the table notes under Table 2A for details.

Table 3B: Canadian Partial-Indexation Model: Estimation and Test Results

Inst.	ν_2	θ	β	Fq	D	γ_{2f}	γ_{2b}	λ_2^P	Max P-val
					$A = D = 1$				
Z_1^{EF}	1.00	0.98	0.90	1.00	50.0	0.47	0.53	0.0024	0.7058
	(0.02,1.00)	(0.72,0.98)	(0.90,0.99)		(3.57,50)	(0.47,0.97)	(0.02,0.53)	(0.0006,0.1117)	
Z_2^{EF}	1.00	0.94	0.90	1.00	16.7	0.47	0.53	0.0098	0.4418
	(0.12,1.00)	(0.74,0.98)	(0.90,0.99)		(3.85,50)	(0.47,0.83)	(0.11,0.53)	(0.0006,0.1044)	
					$A = 0.23,\ D < 1$				
Z_1^{EF}	1.00	0.98	0.90	0.64	50.0	0.47	0.53	0.0004	0.7109
	(0.02,1.00)	(0.38,0.98)	(0.90,0.99)		(1.61,50)	(0.47,0.97)	(0.02,0.53)	(0.0001,0.1085)	
Z_2^{EF}	1.00	0.76	0.90	0.48	4.17	0.47	0.53	0.0110	0.4421
	(0.12,1.00)	(0.38,0.98)	(0.90,0.99)		(1.61,50)	(0.47,0.83)	(0.11,0.53)	(0.0001,0.1051)	

Refer to the table notes under Table 2A for details.