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## Abstract

Using a rich sample of Canadian government securities auctions, we estimate the structural parameters of a share-auction model accounting for asymmetries across bidders. We find little evidence of asymmetries between participants at Canadian government nominal bond auctions. A counter-factual analysis also suggests that given the assumptions underlying the model used, including that participation and quality of information are exogenous, the discriminatory format currently in place is superior to the uniform-price format in terms of the revenue generated by the Canadian government. Both auction formats, however, are found, under the same assumptions, to be dominated by the so-called "Spanish auction" format.

*JEL classification: D44, G28, D63*
*Bank classification: Debt management; Financial markets; Market structure and pricing*

## Résumé

À l'aide d'un riche échantillon d'adjudications de titres du gouvernement canadien, les auteurs estiment les paramètres structurels d'un modèle d'enchère pluri-unitaire qui tient compte de la présence éventuelle d'asymétries dans le comportement des soumissionnaires. Leurs résultats ne mettent en évidence aucune asymétrie majeure entre les participants aux adjudications d'obligations à rendement nominal. En outre, une analyse contrefactuelle donne à penser que, compte tenu des hypothèses qui sous-tendent le modèle des auteurs, dont celle voulant que le degré de participation et la qualité de l'information soient des variables exogènes, le mode actuel d'adjudication à prix multiples rapporte au gouvernement canadien des revenus supérieurs à ceux que l'on obtiendrait au moyen d'enchères à prix uniforme. Il semble toutefois qu'à ce chapitre, la forme d'adjudication « à l'espagnole » dominerait les deux autres formes, toujours selon les mêmes hypothèses.

*Classification JEL : D44, G28, D63*
*Classification de la Banque : Gestion de la dette; Marchés financiers; Structure de marché et fixation des prix*

# 1 Introduction

Beginning with Wilson (1979), most Treasury auctions have been modeled under the assumption that bidders are symmetric with respect to resources, information available, and aversion to risk. The symmetry assumption however is empirically questionable in most Treasury auctions around the world. For instance, Armantier and Sbaï (2005) (hereafter A&S) find that in France a small group of large banks submits most bids, and obtains roughly 60% of the securities allocated. Likewise, comparable asymmetries have been observed in Treasury auctions in Mexico (Umlauf 1993), in the U.S. (Malvey and Archibald 1998), in Norway (Bjonnes 2001), and in Turkey (Hortaçsu 2002). As demonstrated by A&S (2005), the presence of such asymmetries may have major consequences for the ranking of auction formats. The object of the present paper is to estimate a structural model with a sample of Canadian government securities auctions to test whether participants are symmetric, and to determine which auction format generates the highest revenues in this context.

At a Treasury auction, a specific type of security is sold to several accredited financial institutions. The bidders simultaneously submit sealed bids consisting of the number of units of the security requested at each possible price. The market-clearing price, also known as the stop-out price, matches aggregate demand with the available supply of the security. Two basic formats are typically considered: at discriminatory auctions, the most frequently used, the highest bids are filled at the price bid until supply is exhausted; at uniform-price auctions, bidders pay the stop-out price for all units they requested at prices exceeding the stop-out price. These two auction formats, however, are not the only payment mechanisms to have been considered in practice. For instance, the so-called "Spanish auction" format has been employed since January 1987 in Spain to sell Treasury securities.[1]

Beginning with Friedman (1960), the choice between Treasury auction mechanisms has been often debated among economists.[2] Both theoretical

[1]The Spanish auction format is a hybrid system of discriminatory and uniform price auctions. See Álvarez and Mazón (2002), Álvarez, Cerdá and Mazón (2003), and A&S (2006) for theoretical analyses of the Spanish auction format, or Abbink, Brandts and Pezanis-Christou (2005) for a laboratory experiment.

[2]For surveys of the literature on Treasury auctions, see Bikchandani and Huang (1993), Das and Sundaram (1996), Nandi (1997), or Klemperer (2000), or A&S (2005).

and empirical analyses however, have yielded ambiguous results, and it appears that the ranking of the two auction formats may only be established on a case-by-case basis.[3] As demonstrated by A&S (2005), the presence of asymmetries across participants is an important factor in ranking auction formats in terms of the revenues they generate. Indeed, A&S show that risk averse and/or less-informed bidders may become relatively more aggressive at uniform-price auctions, since they do not have to pay their bids. In their study, A&S find that the increased competition would be sufficient to raise (on average) the revenues of the French Treasury compared to the discriminatory format. A&S's conclusions, however, are not directly relevant to the Canadian government, because they are only valid for the set of structural parameters estimated with the French data.

To determine which format dominates in Canada, we apply a similar methodology as in A&S (2005) to estimate the specific structural parameters fitting the Canadian government securities auctions. Observe that although the questions addressed and the methodology adopted are similar, the present paper may be distinguished in several dimensions from A&S: i) the sample of data from the Canadian government securities auctions is significantly richer; ii) participants in Canadian government securities auctions cannot be naturally separated into groups of homogenous bidders, and therefore several partitions will be tested; iii) in addition to the uniform-price and discriminatory format, we will evaluate the revenues the Spanish auction format would have generated for the Canadian government.

The paper is structured as follows: section 2 briefly describes the market for Canadian government securities, as well as our sample of data; in section 3, we summarize the main features of the structural approach used by A&S (2005); the estimation results are presented in section 4, and the counterfactual analysis is conducted in section 5; finally, section 6 concludes.

[3]See e.g. Back and Zender (1993), Wang and Zender (2002) or Ausubel and Cramton (2002) for theoretical analyses. Empirical studies based on the reduced form approach include Umlauf (1993), Tenorio (1993), Simon (1994), Mester (1995), Nyborg and Sundaresan (1996), Malvey and Archibald (1998), and Berg, Boukai and Landsberger (1999); while empirical studies based on the structural approach include Hortaçsu (2002), Fevrier, Preguet and Visser (2004), Castellanos and Oviedo (2005), as well as A&S (2005, 2006).

# 2 The Market and the Data

## 2.1 The Market

In this section, we present a summary of the institutional characteristics of Government of Canada (GoC) nominal bond auctions. The Bank of Canada, on behalf of the Minister of Finance, sells through auctions bonds which pay coupons twice a year. The GoC bond auctions are sealed and discriminatory yield auctions in which bidders can submit multiple bids for different amounts at different yields.

Under the rules of participation in GoC securities auctions, bidders are categorized as follows. Only firms designated as Government Securities Distributors (GSDs) may bid directely. Other firms can only submit bids through GSDs, and so are categorized as "Customers". GSDs are further subdivided into two groups: Primary Dealers (PDs) are a subgroup of GSDs whose activity is above a certain threshold and who meet some other criteria. GSDs that are not PDs are referred to as "other GSDs" or OGSDs.

GSDs' bidding limits in GoC bond auctions are tiered based on their performance in the primary market and their trading activity in the secondary market. The primary dealers have maximum and minimum bidding requirements, consistent with their market shares in the primary and secondary markets, while OGSDs and customers have no minimum bidding requirements. The primary dealers in GoC bond auctions are required to ensure a minimum participation at each auction at a reasonable price defined in terms of participation, and to support secondary markets by making two-sided markets (bid and offer) under normal market conditions. They are, on the other hand, granted higher bidding limits on their own behalf and on behalf of the customers than those allotted to OGSDs.

In GoC bond auctions, the government securities distributors (PDs and OGSDs) and their customers can participate competitively (i.e. a bidder specifies the yield he or she will be willing to pay for a given quantity) or noncompetitively (i.e. a bidder agrees to pay the average yield set at auction). The Bank of Canada typically participates at the auction by bidding non-competitively for balance sheet purposes. Bidders cannot bid to acquire more than 25% of the issue and a government securities distributor's share in the auction cannot exceed 40% of the securities being auctioned, whether the distributor is bidding on its own behalf or on behalf of customers. Participants in GoC auctions submit tenders electronically before the bidding deadline.

The Bank of Canada first accepts all non-competitive bids, and then competitive bids in ascending order in terms of their yields (lowest to highest), until the issue size is reached. When the allotment process of an auction is completed, the auction results are immediately announced on the Bank of Canada website and on Bloomberg and Reuters screens to ensure that all market participants have equal access to the auction results. The results consist of the following: the average, low and cut-off yield (which is the highest yield alloted at auction), the total amount of the securities to be issued, the total amount of bids submitted competitively and noncompetitively by distributors, the Bank of Canada's purchase, and the allotment ratio at the cut-off yield. During the period of this study, the auction results were announced within a maximum of 10 minutes after the deadline for submission of bids.[4]

The schedule for each bond auction is as follows. The announcement of a bond auction starts when the Bank of Canada, on behalf of the Minister of Finance, publishes the quarterly bond auction schedule (at the start of each quarter). In the days leading up to the auction, the Bank of Canada releases the call for tenders on its website, typically on a Thursday afternoon, and the auction takes place the following Wednesday. The call for tenders includes the time and the date of the auction, the details of the upcoming issue, the amount to be auctioned, the outstanding amount of the security, the settlement date, the maturity date, and the minimum purchase of the Bank of Canada[5]. The when-issued trading of the securities to be auctioned begins following the release of the call for tenders and ends once the deadline for the submission of bids arrives. The payment and delivery of the securities takes place on the third business day following the auction (the settlement date for the 2-year bond takes place two business days after the auction).

The institutional aspects of GoC bond auctions have important implications for this study. First, the number of auction participants has declined

[4] In March 2006, the maximum turnaround time for auctions (the time from the bidding deadline to the release of auction results and settlement details) was reduced from 10 minutes to 5 minutes and the Bank of Canada targets an average turnaround time of less than 3 minutes.

[5] Since June 1996, the Bank has announced on each bond a call for tenders indicating the minimum amount that it would acquire. The Bank purchases a fixed and pre-determined percentage at each nominal bond auction at a non-competitive price. These purchases are for balance-sheet purposes only.

progressively since 1998. There were 12 primary dealers, 5 other government securities distributors, and 14 customers participating in GoC nominal bond auctions in 1998 compared to 11, 3 and 5 respectively in 2005. Second, the winning share in bond auctions of primary dealers has increased in recent years when compared with those of other government securities distributors and customers.

## 2.2 The Data

Our sample covers a total of 100 discriminatory auctions of the 2-, 5-, 10- and 30-year GoC nominal bonds, of which 29 were auctions of new bonds (bonds issued for the first time) and 71 were re-openings (bonds being auctioned that have the same coupon, maturity and International Securities Identification Number as a previously issued bond). The sample period is October 14, 1998 to September 1, 2005. Tables 1 and 2 present summary information on Government of Canada bond auctions and a breakdown of auctions across different maturities.

Our sample consists of detailed information by auction. For each auction, we observe the characteristics of the bond being auctioned, the auction size, the number of bidders by each bidder's status (PD, OGSD or customer), the quantity allotted to competitive and non-competitive bidders, the net position of each institutional group in the securities to be auctioned, and the auction results. In addition, we have individual bidders' bids, their net position in the bond being auctioned prior to the auction, and the amount allotted to each bidder. We do not, however, include information on bidders' names; they are identified only by number for reasons of confidentiality. This detailed dataset is composed of data gathered from the Bank of Canada and Bloomberg.

It is important to highlight the fact that primary dealers play a key role in providing coverage and distribution in GoC bond auctions. On average, they accounted for 76.9% of winning bids in GoC bond auctions during the sample period. Their winning share increased from 74.7% in 1999 to 82.1% in 2004. Other government securities distributors winning bids declined to 2.7% in 2005 from 4.2% in 1999. Customers' winning shares also declined over the entire sample period and accounted for only 1.7% in 2005.

Table 3 reports the average volume bid and the average amount allotted to the three types of participants in GoC bond auctions. The winning share of foreign dealers in GoC bond auctions decreased when compared with those of domestic primary dealers, reflecting the departure of three U.S. dealers from the Canadian fixed-income market in recent years.

# 3 The Structural Approach

## 3.1 A Share Auction Model

We start by summarizing the main features of the structural model developed by A&S (2005). This model is a generalization of the common-value share auction model with supply uncertainty of Wang and Zender (2002), to account for informational and risk aversion asymmetries across bidders.

At a given auction, a specific quantity of a perfectly divisible good is for sale to $N$ competitive bidders ($N \geq 2$) each maximizing his ex-ante expected utility.[6] A bidder's decision to participate in the auction (i.e., to submit a competitive bid) is assumed to be exogenous and common knowledge. The quantity supplied to competitive bidders by the auctioneer is unknown at the time of the auction, and it is represented by a random variable $Q \in \Theta_Q$, with cumulative distribution function (c.d.f.) $G(Q)$.[7] The actual value of the good, $V \in \Theta_V$, is random with a c.d.f. $F_0(V)$. This true value is assumed to be the same for each bidder, but unknown at the time of the auction.

To test for the possible presence of asymmetry across bidders we must first partition the participants in $L$ different groups. Let us denote by $N_l$ $\left(\sum_{l=1}^{L} N_l = N\right)$ the number of bidders in group $l$.[8] Bidders within a given group are symmetric, but bidders are asymmetric across groups. Bidder $i$

[6] As previously mentioned, accredited bidders may also submit a "non-competitive" bid prior to the auction. Non-competitive bids consist only in a (limited) quantity that will be filled systematically by the auctioneer. The price paid by all non-competitive bidders is the same, but it is unknown at the time they submit their bids. Depending on the auction rule, the price is either equal to the stop-out price, or a function of the prices paid by the competitive bidders in the auction. We assume here that non-competitive bids result from exogenous decisions made prior to the auction. Therefore, non-competitive bidding will not be modelled, and it is assumed to affect only the quantity available to competitive bidders. The competitive bidders will be referred, in the remainder, as the "bidders", the "players", or the "participants" unless mentioned otherwise

[7] The quantity available to competitive bidders is uncertain because non-competitive bids are only revealed after the auction.

[8] In the data, $(N_1, ..., N_l)$ and $N$ vary from one auction to the next.

in group $l = 1, ..., L$ receives a signal, $s_{i,l} \in \Theta_S$ containing some private information about the value of the good. This signal is generated from a conditional distribution with c.d.f. $F_l(s_{i,l}|V)$. After bidder $i$ in group $l$ receives the private signal $s_{i,l}$, she submits a sealed bid. This bid consists of a schedule specifying the share of the good demanded $\varphi_{i,l}(p, s_{i,l})$ for any price $p > 0$.[9] The demand schedules are assumed to be (piecewise) continuously differentiable. The stop-out price $p^0$ is defined as the non-negative price at which aggregate competitive demand $\Phi(.)$ equals total supply:

$$\Phi\left(p^0, \varphi, s\right) = \sum_{l=1}^{L} \sum_{i=1}^{N_l} \varphi_{i,l}\left(p^0, s_{i,l}\right) = Q \qquad \text{given} \quad p^0 \geq 0 \quad , \tag{1}$$

where $\varphi$ and $s$ are the vectors of bid functions and signals. Winning bids are those submitted for prices greater than the stop-out price. In other words, bidder $i$ in group $l$ receives a quantity $\varphi_{i,l}(p^0, s_{i,l})$.

The GoC securities auctions are conducted under the discriminatory pricing rule. In other words, for the quantity $\varphi_{i,l}(p^0, s_{i,l})$ received, a bidder $i$ in group $l$ is asked to pay

$$p^0 \varphi_{i,l}\left(p^0, s_{i,l}\right) + \int_{p_0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l})\, dp \quad , \tag{2}$$

where $p_{i,l}^{\max}$ is the highest price for which the demand for bidder $i$ in group $l$ is strictly positive. Under this pricing rule, the profit of bidder $i$ in group $l$ may be written as

$$\Pi_{i,l}\left(\varphi_{i,l}(\cdot), p^0, V, s_{i,l}\right) = \left(V - p^0\right) \varphi_{i,l}\left(p^0, s_{i,l}\right) - \int_{p_0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l})\, dp \quad . \tag{3}$$

Following A&S (2005), we account for the possibility of risk aversion by assuming that bidder $i$ in group $l$ exhibits risk aversion in the form of a

[9] As explained in section 2, bids at GoC securities auctions are expressed in terms of "yield" instead of "price". To remain consistent with A&S (2005), and to compare our results with previous studies, we will mostly (but not exclusively) refer to bids in terms of prices. Note also that, to simplify, we do not model the bidding constraints faced by participants at GoC securities auctions. Indeed, there is no clear evidence that these constraints are in fact binding.

CARA utility function:

$$U_{i,l}\left(\varphi_{i,l}(\cdot), p^0, V, s_{i,l}, \lambda_l\right) = -\exp\left[-\lambda_l \Pi_{i,l}\left(\varphi_{i,l}(\cdot), p^0, V, s_{i,l}, Q\right)\right] \quad , \qquad (4)$$

where $\lambda_l > 0$ is the constant level of absolute risk aversion for players in group $l$.[10]

To conclude this section, note that the asymmetric share auction model we just presented cannot be solved analytically. Therefore, we rely on the Constrained Strategic Equilibrium technique developed by Armantier, Florens and Richard (2005) to approximate intractable Bayesian Nash Equilibria. This approximation technique will also be used in the counter-factual analysis in section 5 to solve the Bayesian Nash equilibrium under the uniform-price and the Spanish auction format. To avoid redundancy, we refer the reader to A&S (2005) for further details on the application of the Constrained Strategic Equilibrium approach to solve the share auction model presented in this section.

## 3.2 The Empirical Methodology

To test for the presence of asymmetries across bidders and compare auction formats, we adopt an empirical methodology consisting of two steps. In the first step, we estimate the structural parameters characterizing the model presented in section 3 using GoC securities auctions data; in the second step, using Monte Carlo simulations and the structural parameters estimated in step 1, we conduct a counter-factual analysis to compare the revenues that the discriminatory, uniform-price and Spanish auction formats would have generated for the Canadian government during the 100 auctions in our sample.

*Step 1: Estimation of the Structural Model*

Before we turn to the specification of the structural model, we must estimate the distribution of $Q_t$, the quantity actually supplied at auction $t$ to

[10] According with part of the financial literature, we suspect that, more than the financial institution itself, it is the manager(s) in charge of bidding in Treasury auctions that may exhibit risk aversion, as his job or his remuneration may depend on how he performs. Note however, that risk aversion is not an assumption imposed in the model, it is an hypothesis that will be tested. For additional information relative to the specification of the model (e.g. the choice of the CARA utility function, or the common value assumption) see A&S (2005).

competitive bidders. To specify this distribution, we assume that $NC_t$, the percentage of non-competitive bids at auction $t$, is exogenously determined by the following relationship

$$NC_t = \alpha_0 + \alpha_1 M\_duration_t + \alpha_2 Yield\_Curve\_Slope_t + \alpha_3 Numb\_Customer_t + \alpha_4 Numb\_GSD_t + \alpha_5 Quantity\_Deviation_t + \alpha_6 PD\_Net\_Position + \alpha_7 Spread\_Two\_Years\_CAN/US + \alpha_8 CAN/US\_\$ + \alpha_9 S\&PTSX\_Volatility + \upsilon_t \quad , \tag{5}$$

where $\upsilon_t$ is an identically and independently normally distributed error term with mean zero and variance $\sigma^2_{NC}$.[11] The results of the regression presented in Table 4 indicate that the extent of non-competitive bidding depends on the characteristics of the security for sale (i.e., the maturity, the number of each type of participants, the relative quantity supplied), as well as the overall current economic context (e.g. the volatility of the S&PTSX, or the slope of the yield curve). Interestingly, we find that two variables reflecting the financial conditions in Canada relative to the U.S. (i.e. the exchange rate, and the Canadian/U.S. benchmark two-year bond ratio) appear to play a role in non-competitive bidding. In the remainder, it is assumed that bidders know the distribution of $NC_t$, and therefore the distribution of $Q_t$ when they derive their equilibrium strategy.

To complete the econometric specification, we assume that $V_t$, the true value of the security at auction $t$, is normally distributed with mean

$$\begin{aligned} \mu_{V_t} &= \delta_0 + \delta_1 Pre\_Auction\_Yield_t + \delta_2 M\_duration_t \\ &\quad + \delta_3 30\_Year\_bond + \delta_4 New\_issue + \delta_5 BOC\_\% \\ &\quad + \delta_6 Yield\_2\_Year + \delta_7 Yield\_Curve\_Slope + \delta_8 CAN/US\_\$ \end{aligned} \tag{6}$$

and variance $\sigma^2_V \left(S\&PTSX\_Volatility\right)^{\zeta}$. Finally, the conditional distribution of $s^t_{i,l}$, the private signal at auction $t$ of bidder $i$ in group $l$, is assumed to be normal with mean $V_t$ and variance $\sigma^2_l$.

The structural parameters underlying our Treasury auction model are then estimated using an application of the method of simulated moments

[11] A formal definition of the variables may be found in Appendix. Note also that we only present here the regression that generated the best fit. Other variables, as well as other specifications have been tested. In particular, the number of primary dealers has been included in an alternative regression model, but since this number is very stable from one auction to the next, it did not improve the fit of the model.

to the estimation of games of incomplete information. This method enables us in particular to take advantage of all the information available in the sample. The objective of the inference method is to estimate the unknown structural parameter $\theta = (\beta, \sigma, \lambda)$, where $\beta = (\delta_0, ..., \delta_8, \sigma_V, \zeta)$ characterizes the distribution of the true value of the security, while $\sigma = (\sigma_1, ..., \sigma_L)$ and $\lambda = (\lambda_1, ..., \lambda_L)$ are vectors of dimension $L$ whose $l^{th}$ component $(\sigma_l, \lambda_l)$ represents the standard deviation of the private signals and the coefficient of relative risk aversion of the bidders in group $l$. To do so, we apply the method of simulated moments (MSM) as originally introduced by McFadden (1989), and Pakes and Pollard (1989). The basic principle behind the estimation technique is to compare the quantity actually demanded by each bidder in our sample at each possible price, with the equilibrium expected quantity demanded at that price, as determined by our theoretic model presented in section 3.1. Again, we refer the reader to A&S (2005) for a more detailed presentation of the properties and the practical implementation of this structural estimator.

*Step 2: The Counter-Factual Analysis*

To compare the revenues the Canadian government would have generated under different auction formats during the 100 auctions in our sample, we conduct a counter-factual analysis based on Monte Carlo simulations. To do so, we simulate 1,000 times each of these 100 auctions based on i) $\widehat{\theta}$, the vector of structural parameters estimated in step 1, and ii) the exogenous variables corresponding to that specific auction. More specifically, conditional on the exogenous variables observed at auction $t$, each of the 1,000 simulations consists of drawing three sequences of random variables: first, a quantity available to competitive bidders, $\widetilde{Q}_t$, is drawn from the distribution characterized in (5); second, a true value, $\widetilde{V}_t$, is generated randomly from a normal distribution with mean defined in (6) and variance $\widehat{\sigma}_V^2 \left(S\&PTSX\_Volatility\right)^{\widehat{\zeta}}$; and finally, for each bidder $i$ in group $l$ who participated at auction $t$, a private signal, $\widetilde{s}_{i,l}^t$, is drawn from a normal distribution with mean $\widetilde{V}_t$ and variance $\widehat{\sigma}_l^2$. Note that the Monte Carlo simulations rely on the common random number technique. In other words, the simulations under each payment mechanism are conducted with the same exogenous variables, the same vector of estimated parameters $\widehat{\theta}$, and the same sequence of pseudo-random numbers (i.e. $\widetilde{Q}_t$, $\widetilde{V}_t$, $\widetilde{s}_{i,l}^t$, $\forall i, l, t$). As a result, the Monte Carlo simulations may be directly compared across payment mechanisms.

Finally, observe that when comparing auction formats, we implicitly assume that the variables considered exogenous, such as the quantity available to competitive bidders or the number of competitive bidders, remain unaffected after switching to a different auction format. This assumption may be questioned since participation and/or non-competitive bids could be expected to differ between, for example a discriminatory and uniform-price auction. To the best of our knowledge, however, the empirical analyses conducted after natural experiments did not find any conclusive evidence suggesting that the pricing rule significantly affects the variables that have been assumed exogenous in the present paper (see e.g. Malvey and Archibald 1998). This result may be partially explained by the fact that the number of bidders, and participation in non-competitive bidding, are often somewhat inelastic in the short term.

# 4 Estimation results

In contrast with the French Treasury auctions analyzed by A&S (2005), there is no obvious way to partition the participants at GoC securities auctions in distinct groups of homogeneous bidders. Indeed, as we will see below, the nomenclature presented in section 2 (i.e. a partition in PDs, OGSDs and customers) is essentially institutional. Therefore, the structural model will be estimated successively under different partitions of participants. A test for non-nested hypotheses will then allow us to determine which partition is better able to organize the data. We consider four partitions: Model 1 consists in the three institutional groups (i.e. the PDs, the OGSDs and the customers). Model 2 consists of a slightly modified version of the institutional groups, in which a PD and an OGSD switch groups. Note that this switch is natural since the first participant moved from being a PD to an OGSD during the sample period, while the other moved from being an OGSD to a PD during the same period. Model 3 is similar to Model 2, except that the PDs are separated in two groups. The rationale behind this separation is that all primary dealers do not necessarily behave in the same way. In particular, the bid functions submitted by some PDs tend to be more aggressive and/or more detailed (i.e. composed of more price/quantity pairs) than others. Two groups of PDs have been distinguished with a cluster analysis along these and other dimensions including the average share allocated per auction, or the probability of being a winning bidder. Finally, Model 4 is similar to Model 2 except that all non-Canadian institutions have been assembled to constitute their own group.

The results of the structural estimations under the four models are presented in Table 5. Observe first that the variables explaining the mean and the standard deviation of the true value are all significant (at either a 5% or a 10% significance level) independently of the model adopted. As one may have expected, the pre-auction yield on the secondary market plays a significant role in explaining the true value. It appears, however, that the pre-auction yield does not summarize all the information about the bond's true value. Indeed, other variables reflecting the economic environment (e.g. the slope of the yield curve or the Canadian/U.S. exchange rate) also influence the mean of the true value. Likewise, the characteristics of the bond seem to affect its value. For instance, after controlling for the maturity, we find that long term bonds (i.e. 30-year bonds) are more valuable than issues with shorter terms. In contrast, we find that, all else equal, a new issue of a bond is of lesser value. Finally, note that although statistically significant, the volatility of the S&PTSX has in fact only a moderate economic impact on the standard deviation of the true value.

Let us now turn to the estimations of the parameters specific to each group: the standard deviation of the private signals ($\sigma_l$), and the CARA parameter ($\lambda_l$). In each of the four models, we find the pair $(\sigma_l, \lambda_l)$ to be significantly higher (at a 5% significance level) for the customers than for the other groups.[12] In other words, it appears that the customers are relatively more risk averse and not as well informed about the true value of the bond.[13] The differences in information and risk aversion between the other groups are more nuanced. Indeed, although the parameters $\sigma_l$ and $\lambda_l$ are found in Model 1 to be smaller for the PDs than for the OGSDs, the differences are not statistically significant. Even after a readjustment of these two groups in Model 2, an informational asymmetry between PDs and OGSDs can only be detected at a 10% significant level, while the risk aversion parameters still cannot be statistically distinguished. It is only when we differentiate the PDs into two separate groups, that we are able to single out statistically significant

[12] To test for informational or risk aversion symmetry between two groups $l$ and $l'$ (i.e., $\sigma_l = \sigma_{l'}$ or $\lambda_l = \lambda_{l'}$) we adopt the extension to the general method of moment framework of the Wald test (see e.g., Newey and West 1987).

[13] The estimated absolute risk aversion parameters may appear rather low, if not compared to the bidders potential profits. Indeed, when calculated with the estimated potential profits presented in section 5, the value of the average coefficient of relative risk aversion is around 1, which may be considered reasonable.

asymmetries. Indeed, the first group of PDs (group 1 in Table 5) is found to be significantly (at a 5% significance level) better informed, and significantly less risk averse than both the second group of PDs (group 2) and the group of OGSDs (group 3). As for the remaining PDs and the OGSDs in groups 2 and 3, they can once again be distinguished (at a 10% significance level) solely on the basis of their information. Note, however, that although statistically significant, the asymmetries identified are relatively modest compared to A&S (2005). Indeed, the standard deviation of the private signals of the well informed group ($\sigma_1$) is roughly 50% lower than its nearest competitor ($\sigma_2$), while the difference identified in France by A&S exceeds 140%.

Finally, the results in model 4 suggest a surprising informational asymmetry between non-Canadian and Canadian dealers. Indeed, $\sigma_l$ is found to be significantly larger (at a 5% significance level) for bidders in group 3 (the non-Canadian dealers) than for the Canadian PDs and OGSDs (groups 1 and 2). The asymmetry appears to be essentially informational, since the risk aversion parameters of bidders in the first three groups are not statistically different (at a 5% significance level). The source of this informational advantage cannot be established from the present study. We can however conjecture that Canadian institutions may obtain better information from i) a clearer understanding of the Canadian environment; or ii) inferences from the flow of pre-auction orders submitted by their own customers (as suggested by Bikchandani and Huang 1993).

Observe that the CARA specification we adopted does not include risk neutrality as a special case. Therefore, although the CARA parameters have been found to be significantly different from zero, we have not yet established statistically whether the different groups of bidders may be considered risk averse. To do so, we adopt the non-nested approach proposed by Singleton (1985). In other words, we create a more general structural Treasury auction model nesting as special cases both the CARA (specification S1) and risk neutrality (specification S2) hypotheses.[14] The implementation of the test however, only requires the re-estimation of the model under the risk neutrality assumption. Across the different models and groups, the various tests of S1 against S2 (respectively S2 against S1) yielded $P$-values of the order 0.116 (respectively, 7.626E-3). Therefore, we are led to conclude in favor of

[14] The risk neutral model is derived along the same lines as the CARA model, except that bidders are now assumed to maximize their expected profits in equation (3), instead of their expected utility in equation (4).

the presence of risk aversion for all groups in all models.

An assessment of the ability of the estimated structural model to fit the data may be obtained by looking at the distribution of $\omega_{i,l}^{j,t}$, the error term representing the difference between the bids observed in the sample and the expected equilibrium bids predicted by our model. Table 5 indicates that the estimated mean of $\omega_{i,l}^{j,t}$ is not significantly different from zero in all four models. In addition, the standard deviations of $\omega_{i,l}^{j,t}$ are modest in comparison with the average range of bids submitted during an auction by the bidders in our sample (i.e. 0.071). The bids predicted by our Bayesian Nash equilibrium are therefore tightly distributed around the actual bids observed in our sample, thereby providing support to the estimated theoretic model. Finally, observe that the estimated standard deviations of $\omega_{i,l}^{j,t}$ are the smallest under Model 3, which suggests that this model is better able to organize the data. Following A&S (2005), this observation is confirmed by a sequence of non-nested specification tests based on Singleton (1985). Therefore, the counter-factual analysis conducted in the subsequent section will rely on the group structure, and on the parameters estimated under Model 3.

An additional measure of fit is provided in Table 6 where the outcomes of the simulated auctions may be compared to the descriptive statistics in our sample.[15] Regardless of the model estimated, the simulated stop-out yield, and average winning yield are very similar to those observed in the data. Note, however, that Model 3 appears to be the best at replicating the data. We also report in Table 6, that the estimated true value of the yield ranges, depending on the model, from 4.902 to 4.909, or roughly 0.025 basis point below the stop-out-yield. When expressed in terms of price, the average true value of a bond is found to be 0.40% above the stop-out price.[16] This figure is comparable in magnitude with A&S (2005) (0.60%) but smaller than Fevrier et al. (2004) (1.844%).[17]

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[15] The standard deviations reported in Tables 6, 7 and 8 are calculated across simulations and across auctions in our sample. In other words, these standard deviations should not be confused for a measure of the accuracy of an estimate (as typically presented along with the results of a regression), and therefore they do not reflect any notion of "significance".

[16] Previous studies by Hortacsu (2002), Fevrier et al. (2004), and A&S (2005, 2006), were all conducted in terms of price, instead of yield. Therefore, to compare our results with these previous studies, we express our results in terms of prices in the remainder of the paper.

[17] Note, however, that a direct comparison may not be appropriate since the securities for sale in France and in Canada do not have the same characteristics.

To summarize, we find that except for a small subset of PDs, the main participants at Canadian government securities auctions (i.e. the PDs and the GSDs) do not exhibit major informational and risk aversion asymmetries. This result contrasts sharply with A&S (2005) who found large asymmetries between participants at French Treasury auctions. A&S also found that the informational and risk aversion asymmetries had dissimilar consequences on the revenues of the French Treasury. While a reduction of the risk aversion asymmetry would have virtually no effect, a reduction of the informational asymmetry would greatly benefit the French Treasury. Because of the lack of major asymmetries, we find that the Canadian government's gains would be negligible if all participants had access to the same type of information or if they had a similar level of risk aversion. In particular, if all participants were to receive private signals generated from the same distribution as the first group of PDs, then, all things being equal otherwise, this would lower the average winning yield by a factor of 0.5%. Alternatively, if all participants had the same CARA parameter as the first group of PDs (but still receive noisier information), then the average yield paid to the Canadian government's would fall by 0.2%. These results may be explained by the fact that competition among bidders is fiercer when they all have access to better information, and/or when they all exhibit a lower degree of risk aversion.

# 5 The Counter-factual Analysis

We start by presenting briefly the three auction formats that will be compared in the counter-factual analysis. At this point, it is important to note that the different auction formats leave the allocation mechanism unaffected. In particular, the stop-out price $p^0$ is still determined by equation (1), and the security is still divided among the participants who submit bids above $p^0$. In other words, if the bidders submit the same bid functions, then they receive the same share of the security under every payment mechanism, and only the amount they are required to pay will differ.

Following, Viswanathan and Wang (2000), we can generalize equation (2) to define simultaneously the discriminatory, uniform-price and Spanish auction formats. For the quantity $\varphi_{i,l}\left(p^0, s_{i,l}\right)$ received, a bidder $i$ in group $l$

is asked to pay

$$p^0 \varphi_{i,l}\left(p^0, s_{i,l}\right) + \alpha \int_{p_0}^{\overline{p}} \varphi_{i,l}\left(p, s_{i,l}\right) dp \quad . \tag{7}$$

Within this formulation, the price paid under the discriminatory format corresponds to $\left(\alpha = 1, \overline{p} = p_{i,l}^{\max}\right)$, where, as previously defined, $p_{i,l}^{\max}$ is the highest price for which the demand for bidder $i$ in group $l$ is strictly positive; the price paid under the uniform-price format corresponds to $\alpha = 0$; and finally, the price paid under the Spanish format corresponds to $(\alpha = 1, \overline{p} = p_a)$, where $p_a$ is the (weighted) average winning bid

$$p_a = \frac{p^0 Q + \int_{p^0}^{p^{\max}} \Phi\left(p, s\right) dp}{Q} \quad ,$$

$Q$ is the quantity available to competitive bidders, $\Phi\left(p, s\right)$ represents the aggregate demand as defined in (1), and $p^{\max} = \max_{i,l}\left(p_{i,l}^{\max}\right)$ is the highest price announced across all bidders.

To illustrate how the payment mechanisms differ, consider an auction with two bidders in which the Treasury sells $Q$ units of a fully divisible bond. Let us assume that bidders 1 and 2 submit two bid functions $\varphi_1$ and $\varphi_2$. We plot in Figures 1, 2 and 3 these two bid functions, along with the bidders payments (denoted by the green shaded area) under the discriminatory (Figure 1), uniform-price (Figure 2), and Spanish auction formats (Figure 3). Observe in particular, that, as previously mentioned, the stop-out price $p^0$, the average winning bid $p_a$, and the quantity allocated to each bidder (i.e. $q_1$ and $q_2$) are the same under each auction format. Only the amounts paid by the bidders to receive their shares vary across formats. Intuitively, for each unit received, a bidder is asked to pay his bid under the discriminatory format, while he only has to pay the stop-out-price $p^0$ under the uniform-price format. As illustrated in Figure 3, the Spanish auction may be seen as a compromise between the discriminatory and uniform-price formats. Indeed, a bidder must pay his bid for each unit he demanded at a price below the average price $p_a$; however, then the bidder is only asked to pay the average price $p_a$ when his bid price exceeds $p_a$.

At first glance, Figures 1, 2 and 3 seem to indicate that the discriminatory format is clearly superior in terms of the revenues it generates for the Treasury. Indeed, one may wonder why the Treasury would limit payments to $p^0$

or $p_a$, when it could ask participants to pay up to their bids. This argument, however, ignores the strategic components of the auction. Indeed, there is no reason to believe that, like in Figures 1, 2 and 3, the bidding behaviour of the participants will remain unchanged under the three auction formats. In particular, the Bayesian Nash equilibrium bid functions will be clearly different since the payment mechanisms differ. In other words, a ranking of the auction formats based on a simple comparison of Figures 1, 2 and 3 would be misleading.

When evaluating the superiority of an auction format in the presence of asymmetries, A&S (2006) find that, in addition to the unit-price paid by a bidder, a key feature is the degree of control that each bidder has on the determination of this unit-price. For instance, since at a discriminatory auction one pays his own bids, a bidder has virtually full control of his payments through his bid function.[18] In contrast, since at a uniform-price auction everyone pays the stop-out-price $p^0$, a slight change in his bid function has little bearing on the price paid by a bidder. As a result, bidders, and in particular small or less informed bidders, can afford to be more aggressive at uniform-price auctions, which may benefit the Treasury.

We now turn to the counter-factual analysis. The estimated (per-auction) revenues generated by the Canadian government under the three different payment mechanisms may be found in Table 7.[19] We find that, had the Canadian government conducted the 100 auctions in our sample under the uniform pricing rule instead of the discriminatory pricing rule, it would have decreased its revenues significantly by 1.47%. In addition, we can see in Table 8 that the Canadian government revenues would have been lower in 69% of the auctions if it had conducted them under the uniform-price format. Table 8 also indicates that the proportion of auctions yielding a lower revenue under the uniform-price format is roughly stable across maturities.

At first glance, this result may seem difficult to reconcile with studies conducted previously in different countries. Indeed, it contrasts on one hand with A&S (2005) who found that the uniform-price format would have increased the French Treasury revenues by 4.8%. On the other hand however,

[18] A bidder does not have full control over his payments at a discriminatory auction since the determination of the stop-out price depends also on the other bidders' behavior.

[19] Again, to compare our results with previous studies, we present the simulation outcomes in terms of revenues. The Canadian Treasury revenues at a given auction are defined as the average price paid by a winning bidder multiplied by the quantity allocated by the Canadian Treasury.

the direction (but not the magnitude) of the effect is consistent with Hortacsu (2002) and Fevrier et al. (2004), who estimated that, in terms of the revenues generated, by respectively, the Turkish and French Treasuries, the discriminatory format dominates the uniform-price format by roughly 11% and 8%. This apparent contradiction has in fact a simple explanation. Indeed, recall that the distinction between these previous studies is that there are major asymmetries across bidders in A&S (2005), while Hortacsu (2002) and Fevrier et al. (2004) work under the symmetry assumption. Since we did not identify major asymmetries in Canadian government securities auctions, it is not surprising to find results consistent with Hortacsu (2002) and Fevrier et al. (2004), instead of A&S (2005). In other words, because of the apparent lack of major asymmetries in GoC securities auctions, the slightly less informed and more risk averse bidders have a lesser incentive to become more aggressive under the uniform-price auction. As a result, and in contrast with A&S (2005), the increase in the aggressiveness of bidding by less informed and more risk averse bidders is not strong enough to compensate for the loss of revenue when bidders are no longer required to pay their own bids.

Table 7 also indicates that, had the Canadian government conducted the 100 auctions in our sample under the Spanish format instead of the discriminatory format, it would have significantly increased its revenues by an average of 2.34%, or close to 52.71 million dollars, per auction. Furthermore, we can see in Table 8 that, given the assumptions underlying the model, Canadian government revenues would have been higher in roughly 62% of the auctions if it had conducted them under the Spanish format. Observe also that the Spanish format dominates in an additional dimension. Indeed, we can see in Table 7 that the standard deviation of the revenues generated across the 100 auctions is the smallest under the Spanish format. In other words, the stream of revenues generated by the Canadian government from one auction to the next would have been more stable than under the current pricing rule. Finally, Table 7 indicates that the additional revenues the Canadian government would generate by switching from the discriminatory to the Spanish format, would be almost equally spread across maturities. Indeed, we are unable to detect any clear pattern in the additional revenues generated at auctions for 30, 10, 5 or 2 years bonds.

These results are consistent with A&S (2006) who also find that the Spanish format provides an appropriate compromise between asking bidders to pay up to their bids, and promoting aggressive behaviour by offering participants the guarantee that they will not have to pay more than the average winning bid. A&S (2006) also explore a possible alternative to the Spanish format in which the highest price paid by a bidder is not the average winning bid $p_a$, but a pre-determined fraction $k \cdot p_a$. In the present study, we tested two such fractions, $k = 0.75$ and $k = 1.25$. Compared to the traditional Spanish auction (i.e. $k = 1$), we find that the variation in the Canadian government revenues would only be modest (i.e. $-0.04\%$ when $k = 0.75$ and $+0.02\%$ when $k = 1.25$).

We report in Table 9 the "gross potential profits" (per auction) for each of the four groups of participants. The gross potential profits of a bidder at an auction is defined here as the difference between the estimated true value of the bond and the average price paid by this bidder, multiplied by the quantity allocated to the bidder. In other words, the gross potential profits represent the revenues of a bidder if he were to immediately sell the quantity he is allocated at the true value. Although informative, these figures should be interpreted with caution because they are not expected to reflect the actual profits of the participants. Indeed, the gross potential profits do not take into consideration i) the fact that the true value is a somewhat abstract concept; ii) the possibility that a bidder may have an incentive to hold some of the securities it is allocated at an auction; iii) the fact that at an auction a bidder may be executing an order on behalf of a client at a pre-determined price; iv) the research or administrative costs typically incurred while participating in Treasury auctions; and finally, v) the value of bonds in the firm's market-making activity. Table 9 indicates that switching from the discriminatory to the Spanish format would contract only moderately the groups' gross potential profits. In addition, observe that the effect would be almost equally shared across groups. The change in auction format, however, would also have advantageous aspects for the bidders. Indeed, as indicated by the standard deviations in Table 9, the bidders' gross potential profits would be more stable from one auction to the next. In other words, the Treasury auction participants would benefit from the added stability provided by the Spanish auction format.

Finally, we present in Table 10 the allocation of the bonds across bidders as a function of the group to which they belong. According with A&S, we find that switching from the discriminatory to either the uniform-price or Spanish format, lowers the share allocated to the well-informed and less risk-averse bidders. As mentioned earlier, this variation may be explained by the fact that less informed bidders can afford to become more aggressive under the uniform-price or Spanish format. Indeed, they know they are unlikely to pay the highest prices they submit, since those have almost no weight in deciding the stop-out price. The effect of the auction format on the allocation process, however, is small compared to the case of France. Indeed, A&S found the allocation ratio to be roughly 60/40 in favor of the small group of well informed bidders under the discriminatory format, while the ratio is reversed (i.e. 40/60) under the uniform-price auction. This difference between the two studies may again be explained by the lack of major asymmetries in Canadian government securities auctions.

# 6 Conclusions

The object of the present analysis was twofold: first, we estimated a structural model to test whether participants at Canadian government nominal bond auctions may be considered symmetric with respect to information and risk aversion; second, we conducted a counter-factual analysis to evaluate which of the discriminatory, uniform-price or Spanish auction formats would generate the highest revenues for the Canadian government.

We did not identify any major asymmetry across participants at Canadian government securities auctions, in contrast with the findings of Armantier and Sbaï (2005). Yet this paper provides empirical evidence for the ranking of auction formats in terms of the revenues they generate in the Canadian context. In this context, we find that the discriminatory format is superior to the uniform-price format. Both payment mechanisms, however, appear to be dominated by the Spanish auction format. In other words, as found by Armantier and Sbaï (2006), the Spanish format appears to provide an appropriate compromise between asking bidders to pay up to their bids, and promoting aggressive behaviour by offering participants the guarantee that they will not have to pay more than the average winning bid.

However, we must acknowledge some of the limitations of the approach we adopted. In particular, the counter-factual analysis assumes that the participation at Canadian government securities auctions, the composition of the different groups, and the quality of the information available to bidders about the true value of the bonds, are all exogenous. However, it is possible that the number of bidders, or the incentives to invest in information gathering, may change depending on the auction format adopted. The analysis of such a model is significantly more challenging, as it would require the estimation of a model with endogenous participation and endogenous investment in information. To the best of our knowledge, such a model does not exist in the literature on Treasury auctions.

# 7 References


Abbink K, Brandts J, Pezanis-Christou P. 2005. Auctions for Government Securities: A Laboratory Comparison of Uniform, Discriminatory and Spanish Designs. Forthcoming in the *Journal of Economic Behavior and Organization.*

Alvarez F, Mazon C. 2002. Treasury Auctions: the Spanish Format. Working paper, Universidad Computense de Madrid.

Alvarez F, Cerda E, Mazon C. 2003. Treasury Bill Auctions in Spain: a Linear Approach. *Spanish Economic Review*, 5, 1, p. 25-48.

Armantier O, Florens JP, Richard JF. 2005. Approximation of Bayesian Nash Equilibria. Working Paper, Universite de Montreal, available at http://www.sceco.umontreal.ca /liste_personnel/armantier/.

Armantier O, Sbai E. 2005. Estimation and Comparison of Treasury Auction Formats when Bidders are Asymmetric. Forthcoming in the *Journal of Applied Econometrics*. Available at http://www.sceco.umontreal.ca /liste_personnel/armantier/.

Armantier O, Sbai E. 2006. Comparison of Alternative Payment Mechanisms for French Treasury Auctions. Journal of Applied Econometrics, Vol 26, Issue 6, 2006.

Ausubel LM, Cramton P. 2002. Demand Reduction and Inefficiency in Multi-Unit Auctions. Working Paper, University of Maryland.

Back K, Zender J. 1993. Auctions of Divisible Goods: On the Rationale for the Treasury Experiment. *Review of Financial Studies,* 6, 733-764.

Berg SA, Boukai B, Landsberger M. 1999. Bid Functions for Treasury Securities; Across Countries Comparison. Working Paper, Haifa University.

Bikhchandani S, Huang C. 1993. The Economics of Treasury Security Markets. *Journal of Economic Perspectives*, 7, 117-134.

Bjonnes GH. 2001. Bidder Behavior in Uniform Price Auctions: Evidence from Norwegian Treasury Bond Auctions. Working Paper, Norvegian School of Management.

Castellanos S, Oviedo M. 2005. Optimal Bidding in the Mexican Treasury Securities Primary Auctions: A Structural Econometrics Approach. Working Paper, Yale University.

Das S, Sundaram R. 1996. Auction Theory: A Summary with Applications and Evidence from the Treasury Markets. *Financial Markets, Institutions and Instruments*, v5(5), 1-36.

Fevrier P, Preguet R, Visser M. 2004. Econometrics of Share Auctions. Working Paper, CREST, Paris.

Friedman M. 1960. *A Program for Monetary Stability*. New York, NY : Fordham University Press.

Hortaçsu A. 2002. Bidding Behavior in Divisible Good Auctions: Theory and Evidence from the Turkish Treasury Auction Market. Working Paper, Stanford University.

Klemperer P. 2000. *The Economic Theory of Auctions*. (ed.), Edward Elgar (pub.), Cheltenhaum, UK.

McFadden D. 1989. A Method of Simulated Moments for Estimation of Discrete Response Models without Numerical Integration. *Econometrica*, 57, 995-1026.

Malvey PF, Archibald CF. 1998. *Uniform-Price Auctions: Update of the Treasury Experience*. U.S. Treasury.

Mester LJ. 1995. There's More than One Way to Sell a Security: The Treasury's Auction Experiment. *Federal Reserve Bank of Philadelphia Business Review*, July/August, p.3-17.

Nandi S. 1997. Treasury Auctions : What Do the Recent Models and Results Tell Us? *Federal Reserve Bank of Atlanta Economic Review*, Fourth Quarter.

Newey WK, West K. 1987. Hypotheses Testing with Efficient Method of Moments Testing. *International Economic Review*, 28: 777-787.

Nyborg KG, Sundaresan S. 1996. Discriminatory versus Uniform Treasury Auctions: Evidence from When-Issued Transactions. *Journal of Financial Economics*, Vol 42:1, 63-105.

Pakes A, Pollard D. 1989. Simulation and the Asymptotics of Optimization Estimators. *Econometrica*, 57, 1027-1057.

Simon D. 1994. The Treasury's Experiment with Single-Price Auctions in the Mid 1970's: Winner's or Taxpayer's curse? *Review of Economics and Statistics*, 76, 754-760.

Singleton K. 1985. Testing Specifications of Economic Agents' Intertemporal Optimum Problems in the Presence of Alternative Models. *Journal of Econometrics,* 30, 391-413.

Tenorio R. 1993. Revenue Equivalence and Bidding Behavior in a Multi-Unit Auction Market: An Empirical Analysis. *The Review of Economics and Statistics*, Vol. 75, No. 2., 302-314.

Umlauf S. 1993. An Empirical Study of the Mexican Treasury Bill Auctions. *Journal of Financial Economics*, 33, 313-40.

Viswanathan S, Wang J. 2000. Auctions with When-Issued Trading : A Model of the U.S. Treasury Markets. Working Paper, Duke University.

Wang J, Zender J. 2002. Auctioning Divisible Goods. *Economic Theory*, vol. 19, 673-705.

Wilson R. 1979. Auctions of Shares. *The Quarterly Journal of Economics*, 93, 675-689.

# 8 Appendix : Description of the Variables

$30\_Year\_bond_t$ : is an indicator function taking the value 1 when the bond for sale at auction $t$ is a 30 years bond.

$BOC\_\%_t$ : is the percentage of the total amount issued by the government at auction $t$ directly allocated to the Bank of Canada.

$CAN/US\_\$_t$ : is the U.S./Canadian Dollars exchange rate (expressed in Canadian Dollars) at noon the day of auction $t$.

$M\_duration_t$ : is the modified duration of the bond sold at auction $t$.

$NC_t$ : is the percentage of non-competitive bids submitted at auction $t$ with respect to the total amount allocated in that auction to competitive and non-competitive bidders.

$New\_issue_t$ : is an indicator function taking the value 1 when the bond is issued for the first time at auction $t$.

$Numb\_Customer_t$ : is the number of customers submitting a competitive bid at auction $t$.

$Numb\_GSD_t$ : is the number of GSDs submitting a competitive bid at auction $t$.

$PD\_Net\_Position_t$ : is the PDs' total net position on the bond sold at auction $t$.

$Pre\_Auction\_Yield_t$ : is the yield observed prior to the auction on the secondary market for the bond sold at auction $t$.

$Quantity\_Deviation_t$ : is the percentage deviation of the total amount issued by the Canadian government at auction $t$ with respect to the average amount issued in all 100 auctions in our sample.

$S\&PTSX\_Volatility_t$ : is the volatility of the $S\&PTSX$ index on the day of auction $t$.

$Spread\_Two\_Years\_CAN/US_t$ : is the spead between the yield of Canadian and U.S. benchmark two years bonds on the day of auction $t$.

$Yield\_2\_Year_t$ : is the yield of the Canadian benchmark two years bond on the day of auction $t$.

$Yield\_Curve\_Slope_t$ : is the slope of the yield curve on the day of auction $t$.

**Table 1**

**Government of Canada (GoC) Nominal Bond Auctions**

| **Year** | **Number of Auctions** | **Amount Issued (billion dollar)** |
|---|---|---|
| 2005 | 10 | 20.90 |
| 2004 | 14 | 32.30 |
| 2003 | 14 | 34.40 |
| 2002 | 14 | 36.90 |
| 2001 | 14 | 37.80 |
| 2000 | 15 | 41.60 |
| 1999 | 15 | 41.45 |
| 1998 | 4 | 10.00 |
| All issues | 100 | 255.35 |
| New issues | 29 | 82.10 |
| Reopenings | 71 | 173.25 |

**Table 2**
**Government of Canada nominal bond auctions:**
**Summary Statistics**

| Nominal Bonds | Number of Auctions | | Amount Issued | Coverage Ratio % | | Bids dispersion (bps) | | Spreads (bps) | |
|---|---|---|---|---|---|---|---|---|---|
| | New Issues | Reopenings | (billion dollar) | Avg. | Std. Dev. | Avg. | Std. Dev. | Avg. | Std. Dev. |
| 2-year Bond Auctions | 14 | 13 | 89.30 | 2.32 | 0.13 | 0.56 | 0.20 | 0.21 | 4.32 |
| 5-year Bond Auctions | 7 | 22 | 70.55 | 2.35 | 0.10 | 0.57 | 0.29 | 1.34 | 4.11 |
| 10-year Bond Auctions | 6 | 23 | 69.50 | 2.33 | 0.17 | 0.74 | 0.57 | 0.31 | 2.42 |
| 30-year Bond Auctions | 2 | 13 | 26.00 | 2.31 | 0.18 | 1.19 | 1.11 | -0.27 | 2.75 |

Coverage ratio is the ratio of total competitive bids in an auction to the total amount issued at the auction. Bids dispersion in basis points is the difference between the cut-off yield and the weighted average yield in the auction. Spread in basis points is the difference between the weighted average yield in the auction and the pre-auction yield in the secondary market.

**Table 3**
**The average and the standard deviation of the amount bid and allotted**
**to the three types of participants in Government of Canada nominal bond auctions (% of the amount issued)**

| Nominal Bonds | PDs | | OGSDs | | Customers | |
|---|---|---|---|---|---|---|
| | Amount Bid % | Amount Allotted % | Amount Bid % | Amount Allotted % | Amount Bid % | Amount Allotted % |
| 2-year Bond Auctions | 217.19<br>*13.19* | 82.76<br>*4.67* | 9.23<br>*2.39* | 3.94<br>*2.21* | 8.27<br>*7.56* | 4.09<br>*4.12* |
| 5-year Bond Auctions | 220.65<br>*10.12* | 77.25<br>*5.59* | 8.40<br>*2.20* | 3.44<br>*2.52* | 7.12<br>*7.32* | 3.54<br>*4.26* |
| 10-year Bond Auctions | 213.55<br>*15.33* | 74.54<br>*7.21* | 6.38<br>*3.06* | 2.23<br>*1.64* | 14.55<br>*12.64* | 7.81<br>*6.00* |
| 30-year Bond Auctions | 209.09<br>*13.24* | 70.27<br>*9.05* | 5.92<br>*3.22* | 1.78<br>*1.15* | 16.23<br>*11.46* | 10.85<br>*8.35* |

**Table 4**
**Non-Competitive Bids (in %)**

| | |
|---|---|
| $\alpha_0$ (Constant) | 3.493* (0.541) |
| $\alpha_1$ (Modified Duration) | 0.075* (0.010) |
| $\alpha_2$ (Yield Curve Slope) | -0.147* (0.050) |
| $\alpha_3$ (Number of Customer) | -0.008** (0.004) |
| $\alpha_4$ (Number of GSD) | 0.045* (0.021) |
| $\alpha_5$ (Quantity Issue in deviation wrt the mean) | -0.361* (0.042) |
| $\alpha_6$ (PD Total Net Position in %) | 0.116* (0.047) |
| $\alpha_7$ (Spread Two Years Bonds CAN/US) | -0.125* (0.040) |
| $\alpha_8$ (Can/US Exchange Rate) | -0.022* (0.006) |
| $\alpha_9$ (S&PTSX Volatility) | -0.747** (0.391) |
| $\sigma_{NC}$ | 0.011* (0.002) |
| $\overline{R}^2$ | 0.989 |

Number of Observations: 100
* Indicates parameters significant at a 5% significance level.
** Indicates parameters significant at a 10% significance level.
Standard deviations are in parenthesis.

**Table 5**
**Structural Parameter Estimates**

| | | Model 1 (Institutional Groups) | Model 2 (Adjusted Institutional) | Model 3 (4 Groups) | Model 4 (CAN/Non-CAN Groups) |
|---|---|---|---|---|---|
| **Mean of the True value** | $\delta_0$ (Constant) | -0.717* (0.340) | -0.731* (0.351) | -0.683* (0.322) | -0.692* (0.330) |
| | $\delta_1$ (Pre-auctionYield) | 0.936* (0.038) | 0.919* (0.032) | 0.948* (0.040) | 0.927* (0.036) |
| | $\delta_2$ (Modified Duration) | 0.031* (0.010) | 0.027* (0.012) | 0.035* (0.009) | 0.032* (0.013) |
| | $\delta_3$ (30 Year Bond) | -0.155* (0.067) | -0.166* (0.073) | -0.147* (0.062) | -0.147* (0.067) |
| | $\delta_4$ (New Issue) | 0.028** (0.016) | 0.031* (0.014) | 0.030* (0.012) | 0.034** (0.017) |
| | $\delta_5$ (BOC Share) | -0.611** (0.347) | -0.597** (0.313) | -0.632* (0.291) | -0.607** (0.336) |
| | $\delta_6$ (Yield 2 Years) | 0.117* (0.041) | 0.125* (0.036) | 0.122* (0.032) | 0.114* (0.040) |
| | $\delta_7$ (Yield Curve Slope) | 0.120* (0.052) | 0.115* (0.050) | 0.124* (0.047) | 0.121* (0.055) |
| | $\delta_8$ ($Can/$US) | 0.439** (0.231) | 0.432** (0.228) | 0.444* (0.208) | 0.450* (0.217) |
| **Standard Deviation of the True Value** | $\sigma_V$ | 0.051* (0.011) | 0.051* (0.010) | 0.048* (0.009) | 0.053* (0.012) |
| | $\xi$ (S&PTSX Volatility) | 0.005** (0.003) | 0.005* (0.002) | 0.004* (0.002) | 0.005** (0.003) |
| **Standard Deviation of Private Signal** | $\sigma_1$ (Group 1) | 0.019* (0.005) | 0.017* (0.005) | 0.013* (0.003) | 0.015* (0.004) |
| | $\sigma_2$ (Group 2) | 0.025* (0.007) | 0.025* (0.006) | 0.020* (0.004) | 0.025* (0.005) |
| | $\sigma_3$ (Group 3) | . | . | 0.025* (0.007) | 0.033* (0.003) |
| | $\sigma_4$ (Customer) | 0.039* (0.010) | 0.040* (0.010) | 0.039* (0.009) | 0.041* (0.011) |
| **Coefficient of Relative Risk Aversion** | $\lambda_1$ (Group 1) | 9.423E-7* (3.735E-7) | 8.083E-7* (3.100E-7) | 4.162E-7* (1.934E-7) | 5.850E-7* (2.301E-7) |
| | $\lambda_2$ (Group 2) | 1.531E-6* (4.865E-7) | 1.462E-6* (4.284E-7) | 9.235E-7* (4.284E-7) | 1.792E-6* (6.382E-7) |
| | $\lambda_3$ (Group 3) | . | . | 1.574E-6* (4.156E-7) | 9.163E-7* (3.246E-7) |
| | $\lambda_4$ (Customer) | 7.635E-4* (6.735E-5) | 7.278E-4* (4.092E-5) | 7.455E-4* (4.831E-5) | 6.991E-4* (5.328E-5) |
| **Error Term** | $E\left[\omega_{i,l}^{j,t}\right]$ (Mean) | 6.178E-4 (2.083E-3) | -2.399E-4 (9.731E-4) | 4.009E-4 (9.454E-4) | 5.742E-5 (1.878E-3) |
| | $Std\left[\omega_{i,l}^{j,t}\right]$ (Standard Deviation) | 1.360E-3* (4.901E-4) | 1.043E-3* (3.853E-4) | 8.871E-4* (1.930E-4) | 1.088E-3* (4.404E-4) |

Number of Observations: 37,651. Standard deviations are in parenthesis.
* Indicates parameters significant at a 5% significance level. ** Indicates parameters significant at a 10% significance level.

| Table 6<br>Discriminatory Treasury Auctions<br>(Average per-auction) | | | | | |
|---|---|---|---|---|---|
| | **Observed In the Data** | **Model 1** (Institutional Groups) | **Model 2** (Adjusted Institutional) | **Model 3** (4 Groups) | **Model 4** (CAN/ Non-CAN Groups) |
| True Value | . | 4.903<br>(0.934) | 4.905<br>(0.933) | 4.909<br>(0.929) | 4.902<br>(0.941) |
| Stop-Out-Yield | 4.931<br>(0.940) | 4.928<br>(0.937) | 4.930<br>(0.937) | 4.932<br>(0.938) | 4.928<br>(0.938) |
| Average Winning Yield | 4.923<br>(0.938) | 4.925<br>(0.945) | 4.924<br>(0.943) | 4.925<br>(0.941) | 4.923<br>(0.942) |

**Pre-auction Yield Observed in the Data: 4.918 (0.937)**
**Post-Auction Yield Observed in the Data: 4.905 (0.931)**

| Table 7<br>Treasury Auction Formats Comparison<br>Canadian Treasury's Revenues<br>(Average per-auction) | | | | | |
|---|---|---|---|---|---|
| | **Value**† | | | **% Difference with Discriminatory Format** | |
| | **Discriminatory Format** | **Uniform-Price Format** | **Spanish Format** | **Uniform-Price Format** | **Spanish Format** |
| All Maturities | 2.248E9<br>(5.380E8) | 2.215E9<br>(5.523E8) | 2.300E9<br>(5.124E8) | -1.468*<br>(0.653) | 2.341*<br>(0.588) |
| 2-year Bond Auctions | 2.990E9<br>(3.298E8) | 2.953E9<br>(3.398E8) | 3.077E9<br>(3.244E8) | -1.237**<br>(0.675) | 2.910*<br>(0.587) |
| 5-year Bond Auctions | 2.088E9<br>(2.047E8) | 2.033E9<br>(2.103E8) | 2.115E9<br>(2.019E8) | -2.634*<br>(0.681) | 1.269**<br>(0.649) |
| 10-year Bond Auctions | 2.083E9<br>(9.989E7) | 2.055E9<br>(1.034E8) | 2.126E9<br>(9.779E7) | -1.344*<br>(0.639) | 2.064*<br>(0.563) |
| 30-year Bond Auctions | 1.540E9<br>(1.629E8) | 1.520E9<br>(1.678E8) | 1.565E9<br>(1.578E8) | -1.299<br>(0.697) | 1.643*<br>(0.603) |

**Per-Auction Canadian Treasury Revenue in the Data: 2.248E9 (5.378E8)**

† The numbers in parenthesis in the three columns below refer the standard deviations of revenues across the 100 auctions.
* Indicates estimates significant at a 5% significance level.
** Indicates estimates significant at a 10% significance level.

| Table 8<br>Treasury Auction Formats Comparison<br>Percentage of Auctions Yielding a Higher Revenue than the Discriminatory Format | | |
|---|---|---|
| | **Uniform-Price Format** | **Spanish Format** |
| All Maturities | 31.0% | 61.8% |
| 2-year Bond Auctions | 35.1% | 63.8% |
| 5-year Bond Auctions | 27.7% | 57.2% |
| 10-year Bond Auctions | 29.1% | 61.9% |
| 30-year Bond Auctions | 33.8% | 66.9% |

**Table 9**
**Treasury Auction Formats Comparison**
**Gross Potential Profits**
**(Average per-Auction and per-Group)**

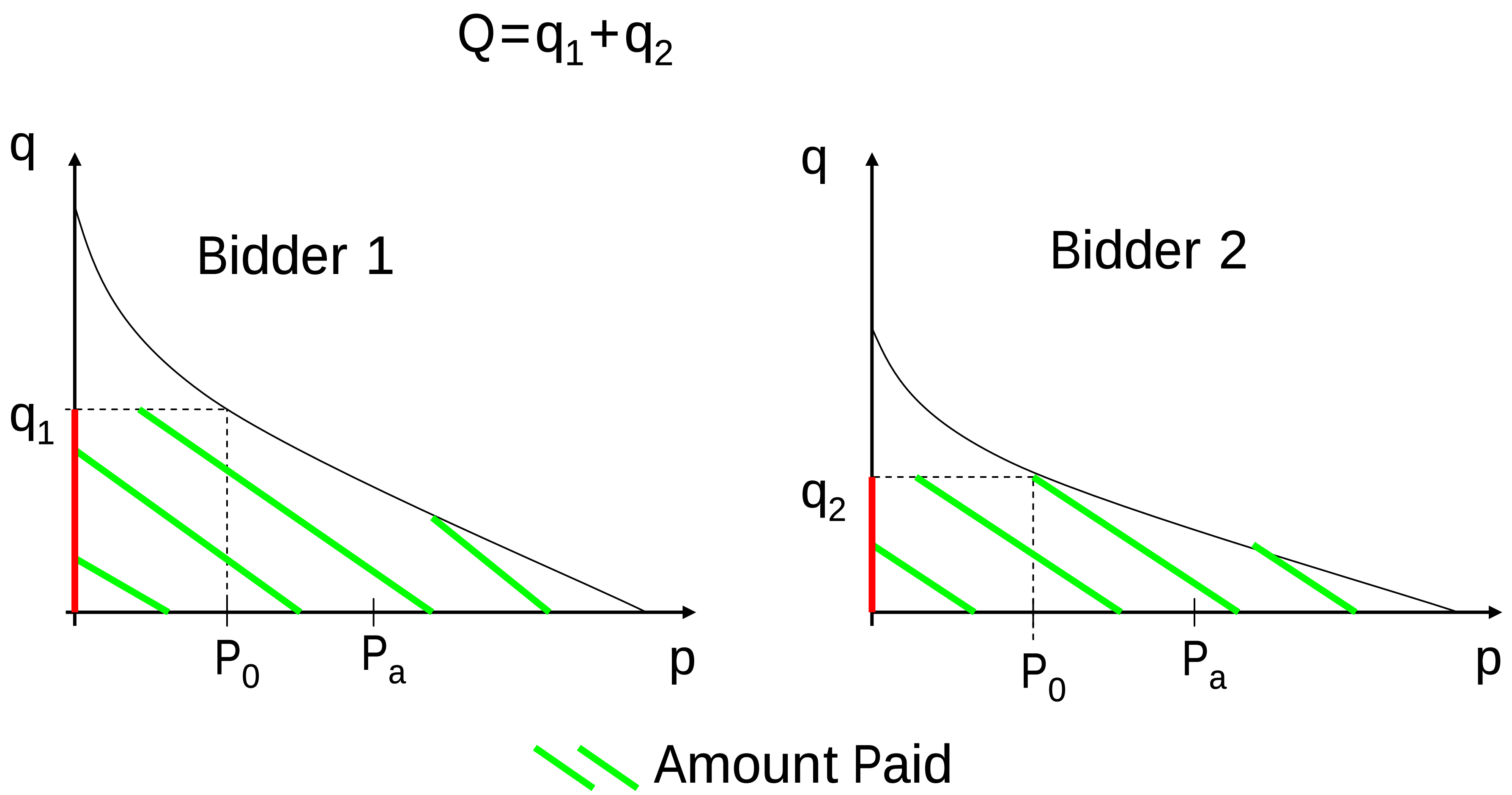
| | Value† | | | % Difference with Discriminatory Format | |
|---|---|---|---|---|---|
| | **Discriminatory Format** | **Uniform-Price Format** | **Spanish Format** | **Uniform-Price Format** | **Spanish Format** |
| Group 1 | 1.308E6 (3.575E5) | 1.297E6 (3.721E5) | 1.262E6 (3.367E5) | -0.841** (0.454) | -3.518* (0.572) |
| Group 2 | 5.788E6 (1.112E6) | 5.740E6 (1.172E6) | 5.610E6 (1.037E6) | -0.829 (0.525) | -3.074* (0.682) |
| Group 3 | 4.698E5 (9.686E4) | 4.900E5 (9.173E5) | 4.609E5 (9.134E5) | 4.301* (1.213) | -1.895 (1.487) |
| Customers | 5.9574E5 (1.563E4) | 6.664E5 (1.580E4) | 6.366E5 (1.544E4) | 11.863* (3.069) | 6.859** (3.602) |

† The numbers in parenthesis in the three value columns refer the standard deviations of revenues across the 100 auctions.
* Indicates estimates significant at a 5% significance level.
** Indicates estimates significant at a 10% significance level.

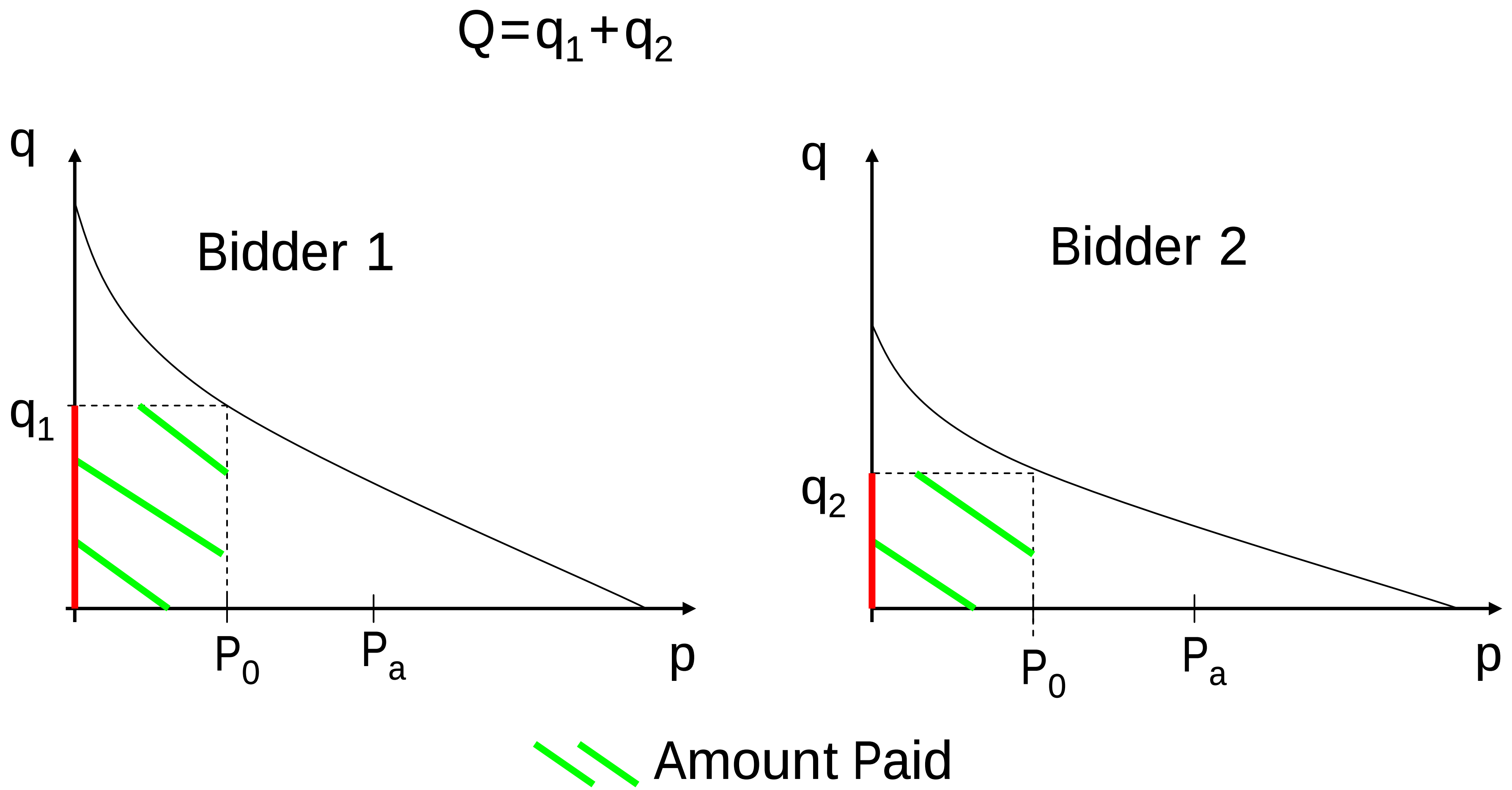
**Table 10**
**Average Share Allocated Per Group**
**(Average per-Auction and per-Bidder)**

| | **Discriminatory Format** | **Uniform-Price Format** | **Spanish Format** |
|---|---|---|---|
| Group 1 | 15.781% (2.621) | 15.170% (2.930) | 14.983% (2.508) |
| Group 2 | 6.491% (2.768) | 6.427% (3.172) | 6.468% (2.556) |
| Group 3 | 0.669% (0.456) | 0.784% (0.567) | 0.712% (0.448) |
| Customers | 1.963% (2.844) | 2.568% (4.063) | 2.445% (3.086) |

# Figure 1
# Discriminatory Format



# Figure 2
# Uniform-Price Format



# Figure 3
# Spanish Auction Format