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Abstract

Real wage rigidities have recently been proposed as a way of building intrinsic persistence in inflation within the context of New Keynesian Phillips Curves. Using two recent illustrative structural models, we evaluate empirically the importance of real wage rigidities in the data and the extent to which such models provide useful information regarding price stickiness. Structural estimation and testing is carried out using Canadian data and identification-robust methods.

Results based on one of the models are relatively uninformative. Our tests reveal important identification difficulties and considerable estimate uncertainty, as can be seen from the wide projections for the estimates. However, we obtain economically reasonable ranges for estimates of average frequency of price changes and some evidence for rigidity in real wages (as measured by a rigidity index) based on the other model we examine. In addition, our specification for the latter model yields significant [at usual levels] and correctly-signed reduced-form coefficient estimates, showing a trade-off between unemployment and inflation in the New Keynesian Phillips curve. From a methodological perspective, these results derive from our treatment of the productivity term as observable although with error, which seems to capture vital information and improve overall identification. From a substantive perspective, our findings suggest that wage-rigidity based New Keynesian Phillips Curves hold promise empirically and provide interesting research directions.

JEL classification: C13, C52, E31
Bank classification: Inflation and prices; Labour markets; Econometric and statistical methods

Résumé

Des études récentes proposent que l’on introduise des rigidités des salaires réels dans les modèles fondés sur la nouvelle courbe de Phillips keynésienne pour générer une persistance intrinsèque dans la dynamique de l’inflation. En prenant pour illustration deux modèles structurels récents, les auteurs évaluent empiriquement l’importance de la rigidité des salaires réels ainsi que la mesure dans laquelle ces modèles fournissent de l’information utile sur la rigidité des prix. Pour réaliser l’estimation structurelle des modèles et les tests, les auteurs ont recours à des données canadiennes et appliquent des méthodes qui permettent de surmonter, s’il y a lieu, les problèmes d’identification.

Les résultats du premier modèle donnent relativement peu d’information. Les tests révèlent d’importantes difficultés d’identification et un degré d’incertitude très élevé des coefficients estimés, qui affichent un large éventail de valeurs. Le second modèle, en revanche, produit une fourchette d’estimations raisonnablement étroite – du point de vue économique – de la fréquence moyenne de révision des prix et fait ressortir certains signes de rigidité (mesurée par un indice) des salaires réels. De plus, les coefficients estimés de forme réduite issus de la spécification de ce modèle sont significatifs aux seuils habituels et du signe attendu et montrent qu'il existe un arbitrage entre chômage et inflation dans la nouvelle courbe de Phillips keynésienne. Vus sous un angle méthodologique, ces résultats sont attribuables au choix des auteurs de considérer le terme de la productivité comme observable, bien qu'avec erreur; ce traitement semble permettre de recueillir de l'information cruciale et d'améliorer l'identification de façon générale. Sur le fond, les résultats portent à croire que l'intégration de la rigidité des salaires à la nouvelle courbe de Phillips keynésienne est prometteuse sur le plan empirique et ouvre des avenues de recherche intéressantes.

Classification JEL : C13, C52, E31
Classification de la Banque : Inflation et prix; Marchés du travail; Méthodes économétriques et statistiques

1. Introduction

The existence of real wage rigidities in labor markets has been the focus of recent work on inflation models; see, for example, Christoffel and Linzert (2006), Rotemberg (2006), Gertler, Sala, and Trigari (2007), Blanchard and Gali (2007), Blanchard and Gali (2008) and Krause, Lopez-Salido, and Lubik (2008). One reason for this is that modeling approaches based on real wage rigidities can generate theoretical inflation inertia in Calvo-based New Keynesian Phillips Curve (NKPC) equations. A second reason is that the existence of such frictions allows the New Keynesian framework to present a trade-off between the stabilization of inflation and output by Central Banks, thereby breaking the unrealistic so-called 'divine coincidence' in otherwise standard New Keynesian setups.

In this paper we consider two illustrative structural inflation models with real wage rigidities that lend themselves well to estimation and evaluate statistically (i) the extent to which they provide useful information regarding price stickiness, and (ii) the importance of real wage rigidities in the data. Canadian quarterly data is used for the structural estimations. The two studies that we consider, namely Blanchard and Gali (2007) and Blanchard and Gali (2008), differ in the way they capture labor market conditions in the model structure but they are similar in the Calvo-type assumptions made in firms' price-setting behavior. Examining whether reliable estimates can be obtained for the structural measure of inflation persistence in the data thus amounts to looking at the precision of Calvo parameter estimates from these models, since it is this parameter that is used to calculate the average frequency of price adjustments in the economy. As for the extent of real rigidities present, in both models we look at the value of the parameter estimate representing the real wage rigidity index.

Additional contributions in this paper are that we provide structural estimates of the two models that we examine; to our knowledge, Blanchard and Gali (2007) has so far only been estimated in reduced-form while the structural equation we consider from Blanchard and Gali (2008) has not yet been estimated. As a matter of fact, the latter model is not directly amenable to estimation. The equation we consider from Blanchard and Gali (2008) explains inflation as a function of current and lagged unemployment as well as a productivity shock, where the coefficients on each of these variables, including the productivity shock, is a non-linear function of the structural parameters. Consequently, vital information on the structural constraints coming from the productivity term needs to be captured for estimation purposes. We address this difficulty by using an observable proxy for productivity and by accounting for the fact that it is observed with error.

A number of econometric challenges arise in our analysis. The chosen NKPC specifications require, among other things, building proxies for some regressors as argued above, finding valid instrumental variables to conduct estimations with, and accounting for specification and estimation uncertainties. Thus, errors-in-variables, under-identification, weak instruments, and specification issues are important concerns. To deal with these difficulties we resort to estimation and testing using identification-robust methods.[1] Such methods are valid irrespective of the identification status of the examined model, which is an advantage not shared, for example, by standard method-of-moments-based approaches. As a result, we can find out how well a particular structural parameter is identified, and, what the "true" (i.e, the reliably-assessed) uncertainty associated with its estimate is if the parameter is weakly-identified. In addition, the methods provide further advantages, such as formally accounting for the integration of calibration with estimation, and correcting for errors-in-variables. The latter property is specially appealing in the case of the Blanchard and Gali (2008) model for which estimation in a structural manner is not straightforward, and which, as mentioned, we address by introducing a variable that is observed with error.

Results based on the Blanchard and Gali (2007) model are overall not very informative. Our tests reveal important identification difficulties and considerable estimate uncertainty, as can be seen from the wide projections for the estimates. However, we obtain economically reasonable ranges for estimates of average frequency of price changes and some evidence for rigidity in real wages (as measured by a rigidity index) based on the Blanchard and Gali (2008) model. In addition, our specification of the latter model yields significant and correctly-signed reduced-form coefficient estimates, showing a trade-off between unemployment and inflation in the New Keynesian Phillips curve.

Recent econometric methods that carter to weak-instruments problems are gaining credibility in macro-economics. Studies having examined identification issues in inflation models

[1] For comprehensive surveys on accounting for some of these issues in the presence of identification problems, see Stock, Wright, and Yogo (2002) and Dufour (2003). Additional references include Dufour (1997), Staiger and Stock (1997), Wang and Zivot (1998), Zivot, Startz, and Nelson (1998), Dufour and Jasiak (2001), Kleibergen (2002), Kleibergen (2005), Dufour and Taamouti (2005), Dufour and Taamouti (2007), Andrews, Moreira, and Stock (2006), Hoogerheide, Kaashoek, and van Dijk (2007), Joseph and Kiviet (2005), Kiviet and Niemczyk (2007), Bolduc, Khalaf, and Moyneur (2008), Beaulieu, Dufour, and Khalaf (2008).

previously include Ma (2002), Mavroeidis (2004), Khalaf and Kichian (2005), Mavroeidis (2005), Dufour, Khalaf, and Kichian (2006), Canova and Sala (2006), Nason and Smith (2008), Kleibergen and Mavroeidis (2008) as well as Dufour, Khalaf, and Kichian (2008). These works seem to suggest that data may be weakly informative on key NKPC parameters, which raises serious concerns. In contrast, our results provide evidence on the empirical worth of the Blanchard and Gali (2008) model that entices further work with wage-rigidity based NKPCs. Results derive from our treatment of the productivity term in this model as observable although with error, which seems to capture vital information from the data and improve overall identification and fit.

In the next section we present the structural forms of the models that we examine. Section 3 explains the methodology applied. Section 4 presents the empirical results, and Section 5 offers some conclusions.

2. The Models

It has recently been suggested that one way of building intrinsic persistence into NKPC models is to allow for stickiness in real wages. A variety of modeling assumptions capturing different aspects of labor market search and matching frictions have been proposed for this purpose.[2] We focus on the models by Blanchard and Gali (2007) and Blanchard and Gali (2008) for our analysis given that they lend themselves relatively easily to structural estimation.[3]

Blanchard and Gali (2007) propose a Calvo (1983) staggered price setting mechanism where, in any given period, each firm has a probability $(1-\theta)$ of re-setting its price. That is, a fraction $(1-\theta)$ of firms can adjust their prices. Another assumption made in the model is that, as a result of some market imperfection, real wages respond sluggishly to labor demand conditions. An index of real wage rigidity, γ_1, is proposed such that the higher its value the more wages depend on lagged wages. Furthermore, an inflation-unemployment relationship is derived implying the following inflation equation:

$$\pi_t = \gamma_{1f} E_t \pi_{t+1} + \gamma_{1b} \pi_{t-1} + \chi_1^u U_t + \chi_1^v \Delta v_t + \epsilon_{1,t+1}.$$

[2]Examples include Christoffel and Linzert (2006), Gertler, Sala, and Trigari (2007), Rotemberg (2006), Blanchard and Gali (2007), Blanchard and Gali (2008) as well as Krause, Lopez-Salido, and Lubik (2008).

[3]For instance, the Krause, Lopez-Salido, and Lubik (2008) approach constructs a measure of real marginal costs reflecting underlying labor conditions and thus requires data that is not as readily available.

In the above, π_t is the inflation rate, U_t is the rate of unemployment, Δv_t is the change in the real price of the non-produced good in the economy, and the error term is an independently identically distributed process. Finally, β is the subjective discount rate.

Under rational expectations, and imposing the structural constraints on the coefficients of the equation, the econometric version of the above model is given by:

$$\pi_t = \frac{\beta}{1+\beta}\pi_{t+1} + \frac{1}{1+\beta}\pi_{t-1} - \frac{\lambda_1(1-\alpha_1)(1-\gamma_1)\phi_1}{\gamma_1(1+\beta)}U_t + \frac{\alpha_1\lambda_1}{(1+\beta)}\Delta v_t + e_{1,t+1}. \tag{1}$$

Here, parameter α_1 is the share of the non-produced good in total output, ϕ_1 is the slope of labour supply, the error term now reflects rational expectation error, and λ_1 is defined as:

$$\lambda_1 = \frac{(1-\theta)(1-\beta\theta)}{\theta}. \tag{2}$$

The second model that we examine is the one proposed in Blanchard and Gali (2008). In this case, staggered price and nominal wage setting is combined with an articulated set of assumptions regarding frictions in the labor market, along the lines of the search and matching model of Diamond-Mortensen-Pissarides. Again, log-linearization around a zero steady-state inflation, and making a theoretical link between inflation and the unemployment rate, yields the following inflation equation:

$$\pi_t = \chi_2^u\hat{U}_t + \chi_2^b\hat{U}_{t-1} + \chi_2^a\overline{a}_t. \tag{3}$$

Here, $\hat{x}$ is the variable x in deviation from its steady-state value, $\overline{a}_t$ is log deviations of productivity from its steady-state and it is assumed to follow a stationary autoregressive process with a parameter ρ_2, while the variable $\hat{U}_t$ is the unemployment rate in deviation from U_2 (the steady-state value of unemployment). The coefficients χ_2^u, χ_2^b as well as χ_2^a are non-linear functions [as shown below] of the model's "deep parameters"; these include a Calvo parameter denoted θ [we retain the same notation as in the previous model], and an index of real wage rigidities, denoted γ_2, that intervenes only via χ_2^a. Thus presented, this model is not immediately amenable to estimation. In particular, the specification of the productivity term affects the way in which vital information coming from the structural constraints on the productivity term would be accounted for.

We address this challenge by using an observable proxy, a_t, for the variable $\overline{a}_t$, and by accounting for the fact that it is observed with error. In this new context, all of the model's structural constraints can now be imposed and the following econometric model is obtained:

$$\pi_t = -\kappa_2\hat{U}_t + \kappa_2(1-\delta_2)(1-x_2)\hat{U}_{t-1} - \Psi_2\gamma_2 a_t + e_{2,t}. \tag{4}$$

Due to the hypothesized autoregressive nature of the productivity variable, when we substitute the proxy in (3), we allow for time dependence in the error term $e_{2,t}$. In addition, the coefficients κ_2 and Ψ_2 are defined as:

$$\kappa_2 = \frac{\alpha_2 \lambda_1 m_2 g_2}{\delta_2 (1 - U_2)}, \tag{5}$$

and

$$\Psi_2 = \frac{\lambda_1 \phi_2}{(1 - \beta \rho_2)}, \tag{6}$$

with ϕ_2 imposed to be less than one and defined as $\phi_2 = 1 - (1 - \beta(1 - \delta_2)) m_2 g_2$. In the above, δ_2 is an exogenous separation rate in the labor market, x_2 is the job finding rate, α_2 is a parameter related to hiring costs, m_2 is the gross steady-state mark-up, defined as $\epsilon/(\epsilon - 1)$ with ϵ being the price elasticity of demand, g_2 equals $B_2({x_2}^{\alpha_2})$ where B_2 is a parameter related to the level of hiring costs, and the steady-state unemployment rate is given by $U_2 = (\delta_2(1 - x_2))/(x_2 + \delta_2(1 - x_2))$. Finally, as defined before,

$$\lambda_1 = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}. \tag{7}$$

For later reference, we denote the reduced-form coefficients on $\hat{U}_t$, $\hat{U}_{t-1}$, and a_t in the econometric model as χ_2^a, χ_2^b, and χ_2^c, respectively.

3. Weak Identification and Inference

In this section, we briefly re-visit the intuition for the use of identification-robust methods and present the specific procedure we adopt. An illustration of the application of this method to the Blanchard and Gali (2007) model is also provided. The reader may consult the above cited econometric literature for insights and further references; for macroeconomic applications, see Mavroeidis (2004), Mavroeidis (2005), Dufour, Khalaf, and Kichian (2006), Canova and Sala (2006), Nason and Smith (2008), Dufour, Khalaf, and Kichian (2008), and Kleibergen and Mavroeidis (2008).

When taken to the data, the models described in the previous section (as well as most optimization-based models) are often confronted with two central concerns: (i) endogeneity, that stems, in particular, from the presence of expectations-based regressors and from errors-in-variables issues, and (ii) parameter nonlinearity, that results from the connection between the key parameters of the underlying theoretical model and the parameters of the estimated econometric model.[4]

Although the models lead to orthogonality conditions that lend themselves well to instrumental variable (IV) or GMM estimation methods, endogeneity and non-linear parameter constraints, in conjunction with weak instruments, lead to the eventuality of *weak identification.* The latter causes the breakdown of standard asymptotic procedures such as IV-based t-tests and Wald-type confidence intervals of the form: [estimate $\pm$ (asymptotic standard error) $\times$ (asymptotic critical point)], and a heavy dependence on unknown nuisance parameters. As a result, standard and even bootstrap-based tests and confidence intervals can be unreliable and spurious model rejections can occur even with large data sets. Indeed, non-identification should, in principle, lead to diffuse confidence sets that can alert the researcher to the problem. Unfortunately, if traditional Wald-type methods are applied when estimating weakly-identified parameters, the expected diffuse intervals often do not obtain. Rather, traditional Wald-type are likely to yield very tight confidence intervals that are focused on "wrong" values. For practitioners, this problem is doubly-misleading. On the one hand, estimated intervals would severely understate estimation uncertainty. On the other hand, and perhaps more importantly, intervals will fail to cover the true parameter value, which, in view of their tightness, will go unnoticed.

These problems are averted if one applies an inference method that does not require identification. Formally, identification-robust methods are inference procedures where error probabilities [e.g. test size, confidence level] can be controlled in the presence of endogeneity, nonlinear parameter constraints and identification difficulties. From the confidence set perspective, when parameters are not identifiable on a subset of the parameter space, or when the admissible set of parameter values is unbounded (which occurs, for example, with nonlinear parameter constraints such as ratios), it is rarely possible to ensure proper coverage unless the set construction method allows for unbounded outcomes. Our methodology can be described as follows.

Consider a nonlinear equation of the form

$$\mathcal{F}_t(\mathcal{Y}_t, \vartheta) = \mathcal{U}_t, \quad t = 1, \ \dots \ , T, \tag{8}$$

[4] For a discussion of these problems, see, for example, Galí, Gertler, and Lopez-Salido (2005) and Sbordone (2005).

where $\mathcal{F}_t$, $t = 1, \ldots, T$ are scalar functions that may have a different form for each observation, ϑ is an $m \times 1$ vector of unknown parameters of interest, $\mathcal{Y}_t$ is the $n \times 1$ vector of observed variables and $\mathcal{U}_t$ is a disturbance with mean zero. Conformably with the GMM literature, our notation for $\mathcal{Y}_t$ includes the exogenous and endogenous variables. The objective is to invert an identification-robust test of the hypothesis:

$$H_0 : \vartheta = \vartheta_0.$$

Inverting a test produces the set of parameter values that are not rejected by this test; furthermore, the least-rejected parameters are the so-called Hodges-Lehmann point estimates (see Hodges and Lehmann 1963, 1983, and Dufour, Khalaf, and Kichian 2006).

Clearly, if H_0 holds true, then $\mathcal{F}_t(\mathcal{Y}_t, \vartheta_0) = \mathcal{U}_t$. Thus, if Z_t is a $k \times 1$ vector of exogenous or predetermined variables such that $k \geq m$, then the coefficients of the regression

$$\mathcal{F}_t(\mathcal{Y}_t,\ \vartheta_0) = Z_t'\varpi + \varepsilon_t \tag{9}$$

should be close to zero. Hence, H_0 in the context of (8) can be tested by assessing

$$H_0' : \varpi = 0 \tag{10}$$

in the context of (9). Z_t can be viewed as a vector of instruments, which may include the exogenous variables in $\mathcal{Y}_t$; (9) may be viewed as an auxiliary or artifical regression and the test of H_0' in the context of (9) an auxiliary or artificial regression test for H_0. Rewriting the latter in matrix form where

$$\mathcal{F}(\mathcal{Y}, \vartheta_0) = [\mathcal{F}_1(\mathcal{Y}_t,\ \vartheta_0), \ldots, \mathcal{F}_2(\mathcal{Y}_t,\ \vartheta_0)]', \tag{11}$$

$$\mathcal{Y} = [\mathcal{Y}_1, \mathcal{Y}_2, \ldots, \mathcal{Y}_T]', \tag{12}$$

$$Z = [Z_1, Z_2, \ldots, Z_T]', \tag{13}$$

the F-statistic for H_0' is given by

$$\mathcal{T}(\vartheta_0) = \frac{\mathcal{F}(\mathcal{Y},\ \vartheta_0)'\,(I - M\,[Z])\,\mathcal{F}(\mathcal{Y}, \vartheta_0)/k}{\mathcal{F}(\mathcal{Y},\ \vartheta_0)'M\,[Z]\,\mathcal{F}(\mathcal{Y}, \vartheta_0)/(T-k)} \tag{14}$$

$$M\,[Z] = I - Z(Z'Z)^{-1}Z'. \tag{15}$$

If Z and $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T$ are independent, the matrix Z has full column rank and $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T$ are *i.i.d.* homoscedastic normal, under the null hypothesis (10), $\mathcal{T}(\vartheta_0)$ follows a central Fisher distribution with degrees of freedom k and $T-k$. The latter exact result may be relaxed leading to the standard χ^2 based distribution compatible with classical least-squares.

To illustrate the above, consider the model in equation (1) which is reproduced here for convenience:

$$\begin{aligned}
\pi_t &= \frac{\beta}{1+\beta}\pi_{t+1} + \frac{1}{1+\beta}\pi_{t-1} - \frac{\lambda_1(1-\alpha_1)(1-\gamma_1)\phi_1}{\gamma_1(1+\beta)}U_t + \frac{\alpha_1\lambda_1}{(1+\beta)}\Delta v_t + e_{1,t+1}, \quad (16)\\
\lambda_1 &= \frac{(1-\theta)(1-\beta\theta)}{\theta}. \quad (17)
\end{aligned}$$

Our aim is to estimate the structural parameters θ and γ_1, given $\Omega = (\beta, \alpha_1, \phi_1)'$ which we will calibrate, conforming with common practice. The model can be rewritten as in (8), with $\mathcal{Y}_t = (y_t, Y_t')'$, $y_t = \pi_t$, $Y_t = (\pi_{t+1}, \pi_{t-1}, U_t, \Delta v_t)'$, $\vartheta = (\theta, \gamma_1)'$

$$\mathcal{F}_t(\mathcal{Y}_t,\ \vartheta) = y_t - Y_t'\bar{\Gamma}\left(\vartheta; \Omega\right) \quad (18)$$

and $\bar{\Gamma}\left(\vartheta; \Omega\right)$ is the four-dimensional function of ϑ

$$\bar{\Gamma}\left(\vartheta; \Omega\right) = \begin{bmatrix} \beta/(1+\beta) \\ 1/(1+\beta) \\ \left(\lambda_1(1-\alpha_1)(1-\gamma_1)\phi_1\right)/\left(\gamma_1(1+\beta)\right) \\ \alpha_1\lambda_1/(1+\beta) \end{bmatrix} \quad (19)$$

implied by (16)-(17). If the *i.i.d.* error hypothesis is maintained, the test associated with (14) using (18) can be inverted. Depending on whether chosen instruments are strongly or weakly exogenous, the associated procedure will be either exact or asymptotically valid. In the latter case, regular least-squared-based asymptotics would hold, in contrast to usual IV methods that require rank restrictions to identify ϑ. Allowing for departures from the *i.i.d.* error hypothesis, the test we invert is based on a Wald-type statistic with Newey-West autocorrelation-consistent covariance estimator given by:

$$\begin{aligned}
AR\text{-HAC}\left(\vartheta_0\right) &= \mathcal{F}(\mathcal{Y},\ \vartheta_0)'Z(Z'Z)^{-1}\hat{Q}^{-1}(Z'Z)^{-1}Z'\mathcal{F}(\mathcal{Y},\ \vartheta_0) \quad (20)\\
\hat{Q} &= \frac{1}{T}\sum_{t=1}^{T}\widehat{u}_t^2 Z_t Z_t' + \frac{1}{T}\sum_{l=1}^{L}\sum_{t=l+1}^{T} w_l \widehat{u}_t \widehat{u}_{t-l}\left(Z_t Z_{t-l}' + Z_{t-l}Z_t'\right)\\
w_l &= 1 - \frac{l}{L+1}
\end{aligned}$$

where $\widehat{u}_t$ is the OLS residual associated with the artificial regression (9), and L is the number of allowed lags. Although, for simplicity, our notation may not clearly reflect this fact, it is worth emphasizing that $\hat{Q}$ is a function of ϑ_0 so the minimum-distance based test we consider involves continuous updating of the weighting matrix.

It is easy to see [refer e.g. to Dufour (2003)] that if (8) is a linear Limited Information Simultaneous Equation, then $\mathcal{T}(\vartheta_0)$ reduces to the test proposed by Anderson and Rubin (1949) for hypotheses specifying the full vector of the left hand side endogenous variables coefficients. In addition, the non-linear test statistics (14) and (20) corresponds closely to Stock and Wright (2000)'s asymptotic GMM-based test. The statistical foundations which lead to the test's identification robustness are the following: whereas traditional set estimation and testing in the context of (8) [via GMM or even with regular FIML] is inappropriate and cannot be salvaged under weak identification, inverting the auxiliary regression test of (10) in the context of (9) translates the problem into the regular regression framework while maintaining its structural foundations. The modification that we perform in this paper in order to correct for non-*i.i.d.* errors exploits the fact that (9) is indeed a regular regression where routine heteroscedasticity- and autocorrelation-consistent (HAC) corrections can be applied. Our procedure has two further "built-in" advantages. First, extremely-wide confidence sets reveal identification difficulties. Second, if all economically-sound values of the model's deep parameters are rejected at some chosen significance level, the confidence set will be empty and we can then infer that the model is soundly rejected. This provides an identification-robust alternative to the standard GMM-based J-test.

In practice, test inversion is performed numerically. A $1-\alpha$ level confidence based set is constructed by collecting the couples (θ_0, γ_0) that, given the calibrated Ω_0, are not rejected by the above tests at level α. For this purpose we conduct a grid search over the economically-meaningful set of values for the structural parameters, sweeping the choices for θ_0, γ_0, given Ω_0. For each parameter combination choice, equation (19) is used in order to obtain $\bar{\Gamma}(\vartheta_0; \Omega_0)$. The appropriate test statistic is applied, and the associated p-value is calculated from the $\chi^2(k)$ null distribution. Collecting those vector choices for which the p-values are greater than a test level α constitute a joint confidence region with level $1-\alpha$. Individual confidence intervals for each parameter can then be obtained by projecting the latter region (i.e. by computing, in turn, the smallest and largest values for each parameter included in this region). A point estimate can also be obtained from the joint confidence set. This corresponds to the model that is most compatible with the data, or, alternatively, that is least-rejected, and is given by the vector of parameter values with the largest p-value.

4. Empirical Results

4.1 The Data

We conduct our estimations on quarterly Canadian data which extends from 1982Q2 to 2007Q2. We use the GDP deflator for the price level, P_t and to obtain the real price of the non-produced good in the economy, V_t, we deflate the producer price of crude materials by the relevant GDP deflator.

Taking the log of these series (which we represent by the corresponding small letters), we define inflation, π_t, as gross inflation, and the change in the price of the non-produced good, Δv_t, as the log difference in V_t. In addition, we use the quarterly unemployment rate for the variable U_t, and define productivity, a_t, as the first difference of the log of the ratio of GDP to employment, where total non-farm employment is used for employment, and where the first difference of the ratio is taken to render the series stationary.

A number of additional variables are used as instruments. These include the yield spread, defined as the 10-year bond yield minus the yield on 3-month Treasury bill, the log difference in total commodity prices, and the log difference of employment. Finally, we use a quadrically-detrended measure of the output gap, defined in a real-time sense so that the gap value at time t does not use information beyond that date. Thus, as in Dufour, Khalaf, and Kichian (2006), we obtain the value of the gap at time t by detrending GDP with data ending in t. Then the sample is extended by one observation and the trend is re-estimated. The latter is used to detrend GDP, yielding a value for the gap at time $t+1$. The process is repeated in this fashion until the end of the sample.

4.2 Estimations and Results

The test applied in all cases is the AR-HAC test, and significance refers to a five per cent test level. All variables are taken in deviation from the sample mean, which is in accordance with not fixing steady-state values to specific parameters, but allowing them to be free constants.[5] Four lags are used in the Newey-West heteroscedasticity and autocorrelation-consistent covariance estimator.

Estimation and test results are reported in the tables found in the Appendix. In the

[5]See Sbordone (2007) for a discussion on the importance of doing so in empirical contexts.

case of each model, we report the point estimates of the structural and selected reduced-form parameters, the average frequency of price adjustment, denoted by Fq and given by $1/(1-\theta)$, as well as the test p-value associated with the vector of point estimates (i.e., the maximal p-value). In addition, for each estimated parameter, we report in parentheses its smallest and highest values in the confidence set.

We conduct estimations for each model using two different instrument sets, one set associated more directly with the two models and that includes lags of only variables found in our two equations, and another set that, to account for a more general serial dependence structure, includes only variables outside of the models. Thus, the first instrument set, Z_1, includes lags of each of: inflation, the unemployment rate, and productivity. The second set, Z_2, includes lags of each of: output gap, change in employment, the yield spread, and change in total commodity price.[6] For the Blanchard and Gali (2007) model, we report results using the second and third lag of these variables, in line with the original study; results are qualitatively unchanged with third and fourth lags. With the Blanchard and Gali (2008) we consider third and fourth lags to account for possible time dependence in measurement errors. Relying on the optimal instrument set which corresponds to Kleibergen (2002)'s method [see also Dufour et al. (2006)] yields qualitatively similar results.

In the case of the Blanchard and Gali (2007) model, we structurally estimate θ and the real wage rigidity index, γ_1, calibrating the remaining parameters. As in the original study, we set the subjective discount rate, β, to 0.99, the Frisch labor supply elasticity, ϕ_1, to 1, and α_1 to 0.025. For sensitivity analysis, we also consider the value 0.33 for the latter parameter, which is in line with the Chari, Kehoe, and McGrattan (2000) study. The search space for θ is (0.02, 0.98), and for γ_1 it is (0.02,1.00). For both parameters, the grid increments are 0.02.

Table 1 reports structural estimation and test results for this model. Overall, we see that point estimates of the parameters are fairly similar for both α_1 values considered, and for either instrument set. These indicate that there is very high real wage rigidity (the lowest value is 0.78), but that prices are fully flexible ($\theta = 0.02$; the corresponding average frequency of price adjustment is one quarter). However, there are important differences across instrument sets and α_1 values regarding the uncertainty associated with each estimate.

Specifically, in the case of $\alpha_1 = 0.33$, Z_1 yields utterly uninformative outcomes for the

[6] Except for the change in employment, these extra-model instruments were also used in the original Galí and Gertler (1999) study.

structural parameters, as projections cover all of the admissible search space for both estimates. There is some improvement when a lower calibrated value is considered for α_1, as it is possible to rule out some values from the identification-robust projections. Thus, it can be affirmed that wage rigidity index is at least as high as 0.82 (implying a dominant backward-looking component in real wages), and that average frequency of price changes are at most 1.06 quarters (which, for all practical purposes, indicates fully-flexible prices). As for the implied reduced-form parameters, these are generally insignificant, with the exception of the coefficient estimate on the inflation rate in non-produced good when the lower α_1 value is considered.

With instrument set Z_2, projections are not much different across the two α_1 values for the wage rigidity index, but the calibration seems to matter somewhat for the upper bound of the Calvo parameter projection (it is 0.52 in the highest case, implying a more reasonable average price adjustment frequency of 2 quarters). In addition, projection ranges are quite wide for the reduced-form parameters. However, while with $\alpha_1 = 0.33$, the coefficient on unemployment is still insignificant, it is significant when the calibrated parameter has a much lower value.

Overall, and judging based on results from the more successful instrument set (Z_2), the inflation model of Blanchard and Gali (2007) indicates important sluggishness in real wages (as captured by high values of the index) but relatively little stickiness in nominal prices (given that average prices change fairly frequently) despite the explicit assumption in the model for generating nominal price stickiness in the NKPC. At the same time, changes in the price of crude materials seem to play an important role for the dynamics of Canadian inflation, while the effect of the real side on the latter is less clear.

We next turn to the results for the Blanchard and Gali (2008) model. In this case, we estimate structurally the parameters θ (the Calvo parameter) and γ_2 (the real wage rigidity index), calibrating β again to 0.99. The search ranges are again (0.02, 0.98) for θ and (0.02, 1.00) for γ_2, while the grid search increment is 0.02. As explained previously, it is assumed that the autoregressive term of the productivity shock is known and fixed. We consider two calibrated values for this parameter: 0.90 and 0.95.

Table 2 reports the estimation results of this model. We find that outcomes are fairly similar across the two calibrated values for the productivity autoregressive term, although projections with instrument set Z_2 are tighter. Interestingly, and unlike with the previous specification, projections for the Calvo parameter are bounded at both ends with this model. At the same time, while point estimates indicate high values for the wage rigidity index, the projections for this parameter estimate are fairly wide. What is also interesting is that all of the reduced-form parameter estimates are significant and have the right signs, specially in the case of instrument set Z_2.

Taking a closer look at the case for $\rho_a = 0.90$ and for instrument set Z_2, we find that the projected range for the Calvo parameter estimate suggests that prices adjust on average every one and a half to three quarters, which is largely in line with micro-based evidence on price adjustments. There is more uncertainty regarding the importance of real wages as captured by the rigidity index in the model. Nonetheless, it is possible to affirm that there is at minimum a one-third weight attributable to lags in real wages. Furthermore, both unemployment and productivity are found to play significant roles in the dynamics of Canadian inflation.

The motivation in Blanchard and Gali (2008) for introducing real wage rigidities through a rigidity index into a dynamic stochastic general equilibrium model was twofold: to generate intrinsic inflation persistence and to create inflation-output tradeoff. In this sense, the results of our structural estimations are supportive of the model. We find on the one hand that the model implies average frequencies of price changes that are economically-reasonable, and on the other hand, we find that there is a significant trade-off between inflation and unemployment, with higher rates of unemployment leading (over two quarters) to lower inflation in Canada.

From a statistical perspective, our treatment of the productivity term seems empirically vital for the Blanchard and Gali (2008) model. Indeed, the observable proxy we use for productivity seems to capture crucial information on the structural constraints which improves overall identification yet maintains the structural foundations of the model. Our results underscore the identifying role of this variable which motivates its use beyond our specific setting.

5. Conclusion

We apply identification-robust methods to structurally estimate two recent inflation models based on real wage rigidities, with Canadian quarterly data. Both models (Blanchard and

Gali (2007) and Blanchard and Gali (2008)) attempt to build intrinsic inflation persistence in inflation and to generate a real-nominal trade-off. To do so, they make similar Calvo assumptions on nominal prices but they model labor market frictions differently. We aim to assess the importance of real wage rigidities in the data and the extent to which such models provide useful information regarding price stickiness.

Results from the Blanchard and Gali (2007) model are relatively uninformative on both questions. Our tests reveal important identification difficulties and considerable uncertainty, as can be seen from the wide projections on parameter estimates. In contrast, using our specification for the Blanchard and Gali (2008) model, we obtain economically reasonable ranges for average frequency of price changes and some evidence for rigidity in real wages (as measured by a rigidity index). The model also yields significant and correct signs on the reduced-form coefficients, in particular showing a trade-off between unemployment and inflation in the New Keynesian Phillips curve. These findings underscore the informational content of the productivity variable which we introduce to formulate the empirically testable implications arising from the Blanchard and Gali (2008) model. More generally, our findings suggest that wage-rigidity based NKPCs hold promise empirically and provide interesting research directions.

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Table 1: Blanchard-Gali (2007) Model, Estimation and Test Results

Inst.	γ_1	θ	Fq	χ_1^u	χ_1^v	Max P-val
			$\underline{\alpha_1 = 0.33}$			
Z_1	1.00 (0.02,1.00)	0.02 (0.02,0.98)	1.02 (1.02,50.0)	0.0000 (-6.2887,0.0000)	7.96 (0.0001,7.96)	0.5167
Z_2	0.88 (0.64,1.00)	0.02 (0.02,0.52)	1.02 (1.02,2.08)	-2.2051 (-9.0961,0.0000)	7.96 (0.0743,7.96)	0.7268
			$\underline{\alpha_1 = 0.025}$			
Z_1	1.00 (0.82,1.00)	0.02 (0.02,0.06)	1.02 (1.02,1.06)	0.0000 (-3.5497,0.0000)	7.96 (2.44,7.96)	0.2469
Z_2	0.78 (0.62,0.98)	0.02 (0.02,0.20)	1.02 (1.02,1.25)	-4.5610 (-9.0961,-0.2067)	7.96 (0.5320,7.96)	0.6590

Instrument set Z_1 includes second and third lags of each of: inflation, the unemployment rate, and productivity. Instrument set Z_2 includes second and third lags of each of: output gap, change in employment, the yield spread, and change in total commodity price.

Table 2: Blanchard-Gali (2008) Model, Estimation and Test Results

Inst.	γ_2	θ	Fq	χ_2^a	χ_2^b	χ_2^c	Max P-val
				$\underline{\rho_a = 0.90}$			
Z_1	0.96	0.70	3.33	-0.1065	0.028	-1.145	0.4139
	(0.10,1.00)	(0.46,0.84)	(1.85,6.25)	(-0.52,-0.03)	(0.006,0.137)	(-2.02,-0.22)	
Z_2	0.96	0.64	3.33	-0.1667	0.044	-1.794	0.1943
	(0.34,1.00)	(0.40,0.68)	(1.67,3.13)	(-0.73,-0.12)	(0.033,0.194)	(-2.79,-1.39)	
				$\underline{\rho_a = 0.95}$			
Z_1	1.00	0.78	4.55	-0.052	0.014	-1.067	0.4458
	(0.06,1.00)	(0.46,0.88)	(1.85,8.33)	(-0.52,-0.01)	(0.004,0.137)	(-2.10,-0.22)	
Z_2	0.92	0.72	3.57	-0.090	0.024	-1.706	0.2466
	(0.20,1.00)	(0.42,0.76)	(1.72,4.17)	(-0.65,-0.06)	(0.017,0.172)	(-2.68,-1.27)	

Instrument set Z_1 includes third and fourth lags of each of: inflation, the unemployment rate, and productivity. Instrument set Z_2 includes third and fourth lags of each of: output gap, change in employment, the yield spread, and change in total commodity price.