BANK OF CANADA
BANQUE DU CANADA

Working Paper/Document de travail
2008-15

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by Gino Cateau

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Bank of Canada Working Paper 2008-15

May 2008

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Gino Cateau

Research Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
gcateau@bankofcanada.ca

ISSN 1701-9397

Acknowledgements

I thank Bob Amano, Stephen Doxey, Sharon Kozicki, Oleksiy Kryvtsov, Rhys Mendes, Stephen Murchison, and seminar participants at the Bank of Canada and ECB for comments and suggestions. Errors are mine.

Abstract

The purpose of this paper is to make a quantitative contribution to the inflation versus price level targeting debate. It considers a policy-maker that can set policy either through an inflation targeting rule or a price level targeting rule to minimize a quadratic loss function using the *actual* projection model of the Bank of Canada (ToTEM). The paper finds that price level targeting dominates inflation targeting, although it can lead to much more volatile inflation depending on the weight assigned to output gap stabilization in the loss function. The price level targeting rule is also found to mimic the full-commitment solution quite well. There is, however, an important difference: the full-commitment solution *does not* require stationarity in the price-level. The paper then analyzes the extent to which the results are sensitive to Hansen and Sargent (2004) model uncertainty. The paper finds the price level targeting rule to be robust; its performance deteriorates slower than the inflation targeting rule and the absolute decline in performance is small in magnitude.

JEL classification: E5, E58, D8, D81
Bank classification: Uncertainty and monetary policy

Résumé

La présente étude apporte une contribution d'ordre quantitatif au débat entourant le choix d'une cible formulée en fonction du taux d'inflation ou du niveau des prix. On y examine le cas d'une banque centrale qui peut poursuivre soit une cible d'inflation, soit une cible de niveau des prix afin de minimiser une fonction de perte quadratique dans le cadre du modèle que la Banque du Canada utilise en pratique pour l'élaboration de ses projections (TOTEM). L'auteur constate qu'un régime ciblant le niveau des prix est supérieur à un régime de cibles d'inflation, même s'il peut s'accompagner d'une bien plus grande volatilité de l'inflation selon l'importance accordée à la stabilité de l'écart de production dans la fonction de perte. Un tel régime produit des résultats similaires à la solution de Ramsey. Une importante différence demeure cependant : la solution de Ramsey ne requiert pas la stationnarité du niveau des prix. L'auteur analyse ensuite le degré de sensibilité des résultats à l'incertitude du modèle (au sens de Hansen et Sargent, 2004). Le régime fondé sur une cible de niveau des prix s'avère robuste; son efficacité baisse plus lentement que celle du régime de cibles d'inflation, et elle ne diminue pas autant en termes absolus.

Classification JEL : E5, E58, D8, D81
Classification de la Banque : Incertitude et politique monétaire

1 Introduction

What is a better monetary policy framework: inflation targeting or price level targeting? Since its introduction in New Zealand in 1990, inflation targeting has been adopted by more than twenty economies (Svensson 2008), including U.K. and Canada. Yet, recent *theoretical* results in the literature suggest that price level targeting can dominate inflation targeting. Indeed, using a Lucas-type Phillips curve, Svensson (1999) finds that price level targeting delivers lower inflation variability than inflation targeting and comparable output gap variability.

Vestin (2006) arrives at a similar conclusion using a simple forward-looking NKPC model (Clarida, Gali, and Gertler 1996, Woodford 1999). He further emphasizes the reason why price level targeting dominates inflation targeting in a forward-looking world: under price level targeting, expectations act like automatic stabilizers. That is, under price level targeting, if there is a shock that pushes up the price level, private agents expect the price level to fall in the near future because the price level must eventually go back to target. These lower expectations of inflation for the near future are something that the policy-maker can exploit to bring desired changes in the economy.

There are at least three caveats to keep in mind when evaluating those results for *practical* purposes. First, the fact that price level targeting can theoretically dominate inflation targeting does not necessarily mean that the benefits are quantitatively important. Second, those results have typically been derived in small-scale models that may be over-simplified versions of models that policy-makers rely on in practice. Third, those results are derived in an environment where the policy-maker and economic agents face no uncertainty about how the economy works. If the policy-maker's model was mis-specified, however, would the properties of price-level and inflation targeting rules be unaffected?

This paper makes a quantitative contribution to the inflation versus price-level targeting debate, but avoids the above three criticisms by (i) using the *actual* projection model of the Bank of Canada (ToTEM)[1], and (ii) using

[1]The paper focuses on ToTEM as model of the economy because the inflation versus price level targeting debate is of considerable practical importance to the Bank of Canada. Prior to its 2011 "renewal of the inflation-control" meetings with the Government of Canada, the Bank of Canada is currently leading an ambitious research program to understand whether it should (i) target a lower inflation rate than 2 per cent, or (ii)

robust control to allow for the possibility of model misspecification.

Specifically, the paper considers a policy-maker that sets policy either through an inflation or a price-level targeting rule to minimize a quadratic loss function, using ToTEM as reference model of the economy. The paper first finds that price-level targeting dominates inflation targeting. But *by what magnitude* and *in what manner* depend *crucially* on the weight assigned to output gap stabilization in the loss function. When the weight is zero, price-level targeting achieves lower inflation, output gap and change in interest rate volatility than inflation targeting. On the other hand, when the weight is increased, price-level targeting leads to less volatility in the output gap and change in the interest rate than inflation targeting, but those lower volatilities come at the expense of higher volatility in inflation.

The paper then compares outcomes achieved under simple price level and inflation targeting rules to those that would be achieved if the policy-maker could implement the first best full-commitment solution. I find that the optimized price-level targeting rule performs almost as well as the first-best full commitment solution. Yet, there is an important difference between the two: while the price level targeting rule induces price level stationarity, the full commitment solution does not.

Woodford (2003) found that in a basic NKPC model, full-commitment optimal policy led to price level stationarity. He, however, argued that since welfare did not depend on the range of variation in the level of prices, the results were likely to be unique to the NKPC model. Recently, Gaspar, Smets, and Vestin (2007) argued that price level stationarity could in fact be a feature of optimal full-commitment policy in more general models. They use the Smets and Wouters (2003) model - a model that embodies many more frictions than the basic NKPC model - to illustrate that the price level was stationary under cost-push shocks. The result above shows that price-level stationarity is indeed not a general feature of the full-commitment solution. For DSGE models like ToTEM, the reason why the price-level targeting rule can replicate the properties of the full-commitment solution is not because it leads to a stationary price level. Rather, it is because the full-commitment solution, like the price-level targeting solution, induces history dependence in policy. That feature allows expectations to play an important role in stabilizing the economy.

The paper then analyzes the extent to which the good performance of

target the price level rather than the inflation rate.

the price-level targeting rule depends on the assumption that the policy-maker knows how the economy functions? Specifically, it analyzes how the price-level and inflation targeting rules optimized for the reference version of ToTEM would perform in a world where the correct model is not the reference version of ToTEM but a robust control version (Hansen and Sargent 2004, Dennis, Leitemo, and Söderström 2006) that economic agents use to form expectations. Using detection probabilities as a measure of the statistical distance between the reference model and the alternative model, the paper finds that the performance of the optimized price-level targeting rule deteriorates slower than the performance of the optimized inflation targeting rule. Moreover, in alternative models that would be statistically plausible, the absolute decline in performance is small in magnitude.

The paper is organized as follows: section 2 briefly introduces ToTEM, section 3 describes the problem of the policy-maker and presents the results, section 4 discusses the policy-maker's problem under model uncertainty and presents the results and section 5 concludes.

2 Policy analysis in ToTEM

ToTEM is an open-economy DSGE model in which micro-foundations are used to describe the interactions between various economic agents: households, firms, government, and central bank. Optimizing behavior from these agents yield a set of first-order conditions that dictate how these agents behave. This set of first-order conditions combined with market clearing conditions yield a system of dynamic non-linear equations that characterize the behavior of the economy (see Murchison and Rennison 2006). Since ToTEM is used not only for policy analysis but also projections at the Bank of Canada, the model is more elaborate than the typical open economy model of the literature. What follows is a brief non-technical summary borrowed from Cayen, Corbett, and Perrier (2006).

The production side of ToTEM is as follows: There are four types of final goods produced by domestic firms: consumption, investment, government and non-commodity export goods. To produce these goods, firms use a CES technology that combines capital with labor services, imported intermediate goods, and commodities. There is also a commodity sector. The commodities are produced by domestic firms by combining labor services with capital goods and a fixed factor that we refer to as land. All firms are allowed to vary their utilization rate, but this comes at a cost in terms of foregone output. The firms also face adjustment costs on the level of employment and on the change in investment, also in terms of foregone output. ToTEM assumes that final good producers are monopolistically competitive, which allow them to fix prices for more than one period following Calvo (1983). The Calvo pricing framework is also used for introducing wage rigidities and import prices rigidities as in Smets and Wouters (2002).

The demand side of ToTEM can be summarized as follows. Domestic households buy the final consumption goods as well as bonds from the (domestic) government and foreigners. They earn (after-tax) labor income from the labor services that they provide to the domestic firms and income from their holding of domestic and foreign bonds in the form of interest payments. They also receive transfers from the government. The government buys the final government goods from the domestic firms with tax revenues and distributes transfers to the domestic households. These expenditures are financed with the tax revenues from labor income and indirect taxes. The model assumes that the government targets a desired level for the debt-to-GDP ratio, with some smoothing, and uses the tax rate on labor income as the policy instrument. Foreigners buy the commodities exports as well as the final non-commodity export goods. They also sell intermediate imported goods to the domestic importers, and they buy and sell bonds.

Foreign variables in ToTEM are presently generated with a semi-structural model. This model is exogenous with respect to the core of ToTEM in the sense that there is no feedback from domestic variables to the foreign variables. This is consistent with the fact that Canada is a small open-economy. The foreign variables that enter in ToTEM are output and the output gap, inflation rate, interest rates (real and nominal) and real commodity prices.

Following Cayen, Corbett, and Perrier (2006), projections using ToTEM assume that monetary policy is implemented through the generalized Taylor rule

$$i_t = \rho_\pi i_{t-1} + B_\pi \mathrm{E}_t \pi_{t+h} + \phi_\pi y_t, \tag{1}$$

By minimizing a quadratic loss function in inflation, output gap and the change in interest rate, Cayen, Corbett, and Perrier (2006) obtain the optimized values $(\rho_\pi, B_\pi, \phi_\pi, h) = (0.95, 1, 0.0175, 2)$.

In this paper, I will work with a first-order linearized version of ToTEM.

Once linearized, ToTEM can be written in structural form as

$$H_1 X_{t-1} + H_2 X_t + H_3 X_{t+1} + B i_t + C\epsilon_{t+1} = 0, \tag{2}$$

where X_t is the time t structural vector, i_t is the interest rate, and ϵ_t is a vector of shocks. In the set-up above, the non-zero columns of H_3 determine the forward-looking variables that the policy-maker needs to solve for when setting i_t.

3 Inflation Targeting or Price-level Targeting for ToTEM

In this paper, I assume a policy-maker that has preferences for inflation stability, output stabilization relative to potential, as well as some concern for the volatility of the interest rate. The policy-maker can credibly commit either to an inflation targeting rule[2]

$$i_t = \rho_\pi i_{t-1} + B_\pi \mathrm{E}_t \pi_{t+2} + \phi_\pi y_t, \tag{3}$$

or to a price-level targeting rule

$$i_t = \rho_p i_{t-1} + B_P \mathrm{E}_t P_{t+2} + \phi_P y_t. \tag{4}$$

to minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (\pi_t - \pi^*)^2 + \omega y_t^2 + \nu (i_t - i_{t-1})^2 \right\}, \tag{5}$$

subject to the forward-looking model (2).

If the policy-maker sets policy using the inflation targeting rule, he chooses the coefficients $(\rho_\pi, B_\pi, \phi_\pi)$ of (3) to minimize the loss function (5) subject to the model (2). On the other hand, if he chooses policy through the price-level targeting rule, he optimally chooses (ρ_p, B_p, ϕ_P) of (4) to minimize (5) subject to (2).

My objective in this section is to compute and compare the performance of optimized inflation and price-level targeting rules. Since I assume an ad-hoc loss function, I will present results for various choices of the weights to output gap (ω) and interest rate stabilization (ν).

[2]An inflation targeting rule like (1) is currently used for projections and policy analyis in ToTEM at the Bank of Canada. Cayen, Corbett, and Perrier (2006) finds that 2 quarter-ahead expected inflation is optimal for ToTEM.

Table 1: Optimized simple rules: $0 \leq \rho_\pi, \rho_P \leq 1$ and $\phi_\pi, \phi_P \geq 0$

		IT			PLT			
ω	ν	ρ_π	B_π	ϕ_π	ρ_P	B_P	ϕ_P	$\frac{\text{loss IT}}{\text{loss PLT}}$
0	0.1	1	3.94	0.02	0.94	0.94	0.01	1.5
0	0.5	1	1.49	0	0.92	0.330	0.00	1.5
0	1	1	0.99	0	0.92	0.21	0.00	1.4
0.1	0.1	1	12.57	0	0.73	0.07	0.98	4.8
0.1	0.5	1	5.50	0	0.76	0.03	0.38	3.9
0.1	1	1	3.72	0	0.78	0.02	0.25	3.6
0.5	0.1	1	21.05	0	0.77	0.04	2.44	15.8
0.5	0.5	1	11.72	0	0.77	0.02	0.94	9.8
0.5	1	1	8.42	0	0.78	0.01	0.62	8.0
1	0.1	1	24.27	0	0.78	0.03	3.63	26.3
1	0.5	1	15.48	0	0.77	0.01	1.40	14.6
1	1	1	11.62	0	0.77	0.01	0.93	11.4

3.1 Results

I first impose the constraint that the degree of inertia, for both rules, lie between 0 and 1 i.e. $0 \leq \rho_\pi, \rho_P \leq 1$ and the response to the output gap non-negative i.e. $\phi_\pi, \phi_P \geq 0$. Table 1 shows the optimized coefficients of the inflation targeting rule (3), of the price-level targeting rule (4), and their relative performance across different policy objectives, obtained by varying the weight to output gap stabilization, ω, between 0, 0.1, 0.5 and 1 and the weight to interest rate stabilization, ν, between 0.1, 0.5 and 1. Three observations are in order: (i) price-level targeting dominates inflation targeting significantly - the value of the loss function for inflation targeting ranges from 1.5 times to 26.3 times higher relative to price-level targeting, (ii) the optimized price-level targeting rule requires a relatively high degree of inertia (> 0.77) and a strong response to the output gap, and (iii) the optimized inflation targeting rule is only constrained optimal; the optimal degree of inertia is 1 and the optimal response to the output gap is 0 across most preference configurations

A degree of inertia equal to one in the inflation targeting rule (and an optimized response to the output gap equal to 0 for most of our considered

Table 2: Optimized rules with no inertia i.e. $\rho_\pi = \rho_P = 0$ and $\phi_\pi, \phi_P \geq 0$

		IT		PLT		
ω	ν	B_π	ϕ_π	B_P	ϕ_P	$\frac{\text{loss IT}}{\text{loss PLT}}$
0	0.1	4.29	0.07	1.26	0.02	3.00
0	0.5	2.11	0.03	0.49	0.03	3.28
0	1	1.60	0.02	0.33	0.03	3.51
0.1	0.1	12.83	0	0.10	1.64	5.33
0.1	0.5	5.06	0	0.05	0.74	4.59
0.1	1	3.47	0	0.04	0.54	4.29
0.5	0.1	27.09	0	0.04	3.80	14.06
0.5	0.5	11.96	0	0.02	1.66	9.23
0.5	1	8.01	0	0.02	1.18	7.81
1	0.1	35.02	0	0.03	5.47	21.84
1	0.5	17.32	0	0.02	2.37	12.66
1	1	11.85	0	0.01	1.67	10.26

preference configurations) makes it a policy rule where the *first-difference* of the interest rate responds to inflation. That rule is equivalent to a price-level targeting rule. Since my purpose in this paper is to compare the properties of inflation and price-level targeting rules, the rest of this paper will consider *non-inertial* inflation and price-level targeting rules except where noted. Table 2 reports the optimized coefficients when the degree of inertia for both rules are constrained to zero i.e. $\rho_\pi = \rho_P = 0$. I find that the non-inertial optimized price-level targeting rule dominate the non-inertial optimized inflation targeting rule across all configurations.[3]

How do price-level targeting and inflation targeting differ and why does price-level targeting dominate inflation targeting so significantly? Table 3 provides a first clue to the answer. Table 3 displays the standard deviation of inflation, output gap, and change in the interest rate under the optimized non-inertial inflation targeting and price-level targeting rules. Two results

[3]Table 5 in the appendix computes the unrestricted optimized coefficients. The unrestricted optimized inflation targeting rule still performs significantly worse than the optimized price-level targeting rule. However, the large values of the unrestricted optimized coefficients make them not sensible for practical applications.

Table 3: Standard deviation of inflation, output gap, change in interest rate under non-inertial IT and PLT

		σ_{π_t}		σ_{y_t}		$\sigma_{\Delta i_t}$	
ω	ν	PLT	IT	PLT	IT	PLT	IT
0	0.1	0.40	0.68	2.03	2.43	0.19	0.34
0	0.5	0.50	0.85	2.09	2.67	0.10	0.20
0	1	0.54	0.93	2.13	2.79	0.07	0.17
0.1	0.1	0.89	0.51	0.53	2.09	0.43	0.73
0.1	0.5	0.88	0.68	0.74	2.25	0.24	0.39
0.1	1	0.87	0.75	0.85	2.33	0.19	0.30
0.5	0.1	0.94	0.41	0.31	2.01	0.71	1.16
0.5	0.5	0.94	0.53	0.49	2.10	0.43	0.70
0.5	1	0.93	0.59	0.58	2.16	0.34	0.54
1	0.1	0.95	0.37	0.24	1.99	0.87	1.34
1	0.5	0.95	0.47	0.40	2.05	0.54	0.88
1	1	0.94	0.53	0.49	2.10	0.43	0.70

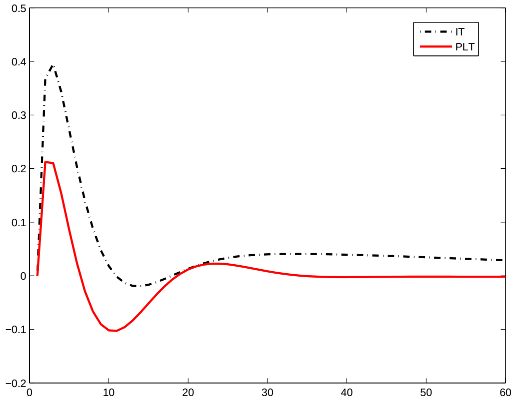
stand out: (i) when $\omega = 0$, price-level targeting dominates inflation targeting both in stabilizing inflation, output gap and the interest rate, and (ii) when $\omega > 0$, inflation targeting works by stabilizing inflation at the expense of stabilizing the output gap. As a result, it does very well in stabilizing inflation relative to price-level targeting but only at the expense of letting the output gap become quite volatile.

To further understand how price-level targeting differs from inflation targeting, I now compare how the two rules would respond to a demand shock[4]- an exogenous decline in the discount factor that pushes up consumption under two benchmark preference configurations: $(\omega, \nu) = (0, 0.5)$ and $(\omega, \nu) = (1, 0.5)$.

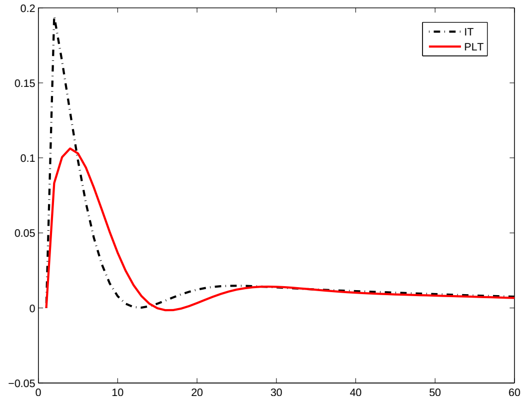
Figure 1 depicts the $(\omega, \nu) = (0, 0.5)$ case. The positive consumption shock pushes up consumption by 2.2% after one year. Following the increase in consumption, firms in the economy want to increase output. In ToTEM,

[4]In the appendix, Figure 6 and Figure 7 display the impulse responses to a technology shock for $\omega = 0$ and $\omega = 1$ respectively. The same type of analysis done for the demand shock carries over to the technology shock.

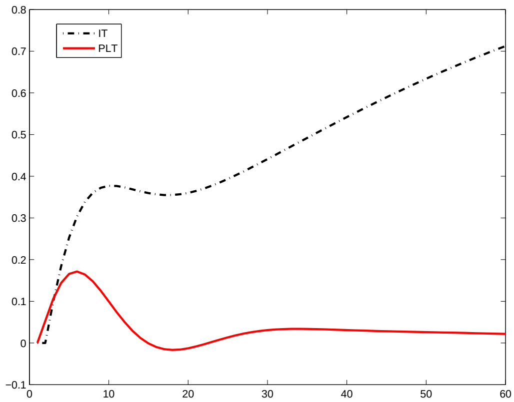
Figure 1: impulse responses to a positive consumption shock ($\omega = 0$)



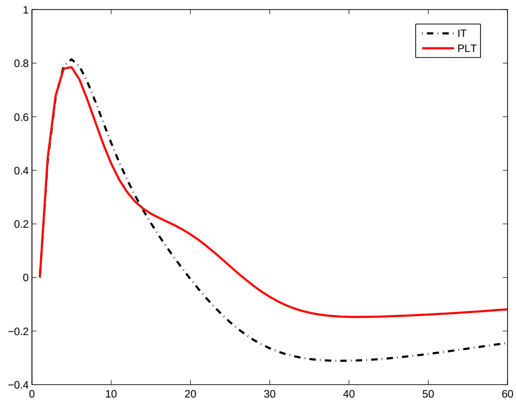
(a) inflation



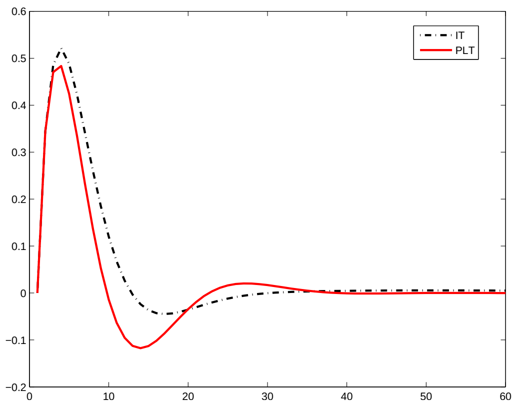
(b) interest rate



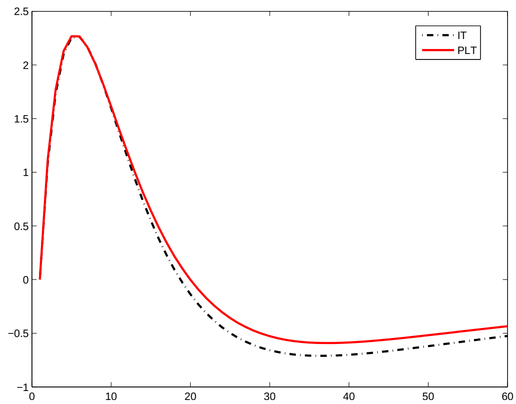
(c) price level



(d) output gap



(e) marginal cost



(f) consumption

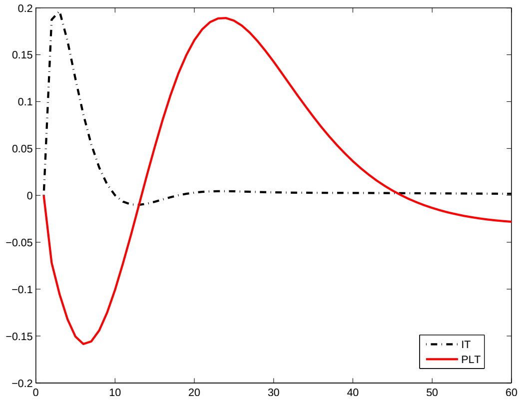
however, an increase in output leads to an increase in marginal cost. With higher marginal cost, firms that increase output also want to increase their prices (although not all of them can). Notice that under price-level targeting, the initial impact of the consumption shock on inflation is smaller than under inflation targeting (0.2 pp vs 0.4 pp). Why does this happen? Under price-level targeting firms know that the policy-maker is committed to bringing the price-level back to target. Hence, they anticipate that an initial increase in prices will eventually be followed by falling prices in the near future. The expectation of falling prices in the near future makes it optimal for firms to increase their prices by less in response to the shock. Therefore, under price-level targeting, the expectation that the policy-maker will bring the price-level back to its anticipated path helps to spread the effects of shocks over time and hence reduce volatility.

Figure 2 considers the $(\omega, \nu) = (1, 0.5)$ case. The behavior of the inflation targeting rule is not markedly different compared to the $(\omega, \nu) = (0, 0.5)$ case. In response to the consumption shock, some firms increase their prices leading to aggregate inflation of 0.2 pp. The policy-maker reacts to the positive inflation by increasing the interest rate by 0.7 pp. Since not all firms can adjust their prices in response to the interest rate and consumption changes, they adjust by changing their output. This accounts for the high volatility of output under inflation targeting.

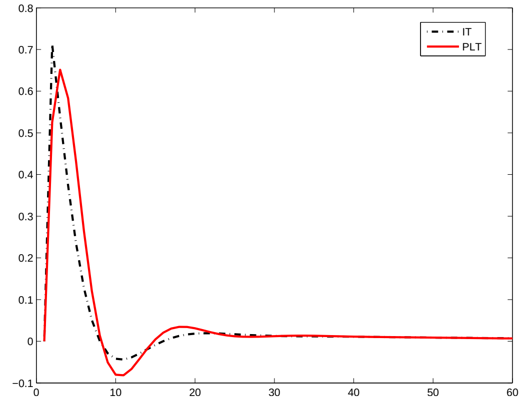
Under price-level targeting, the policy-maker reacts very strongly to the output gap. When firms increase output in response to the positive consumption shock, the policy-maker increases the interest rate more than proportionately (0.65 pp). The interest rate increase is so high that it causes firms to decrease their prices in response to the consumption shock. The decline in the price-level is then followed by a period of rising prices to bring the price-level back to target. Since it takes time for the effect of the shock to vanish, the price-level targeting rule leads to higher volatility in inflation relative to the inflation targeting regime.

To the extent that the ad hoc quadratic loss function is a good representation of the criterion that policy-makers use to set policy, the analysis above illustrates the importance of correctly gauging what the weight to output gap stabilization is in deciding between inflation and price-level targeting rules. While the qualitative behavior of the inflation targeting rule does not vary much with that parameter, how the price-level targeting rule stabilizes the economy differs markedly depending on whether that parameter is high or low.

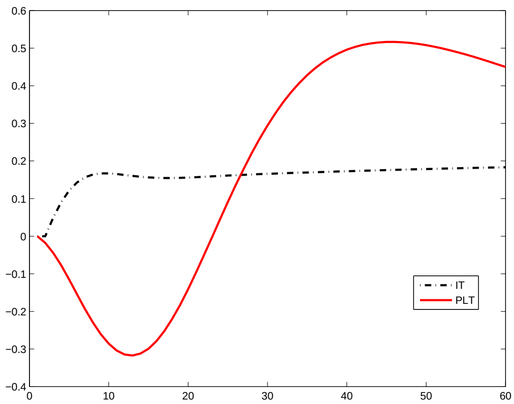
Figure 2: impulse responses to a positive consumption shock ($\omega = 1$)



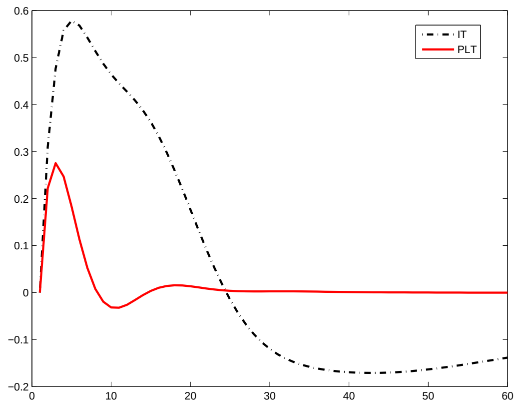
(a) inflation



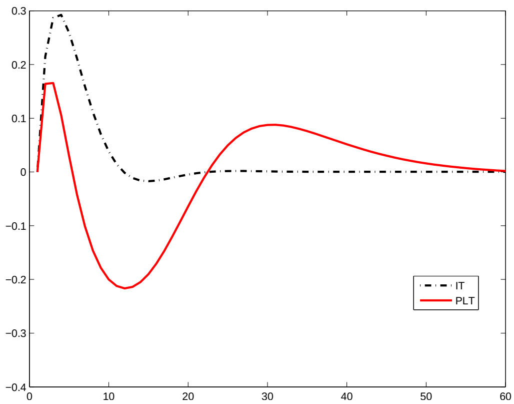
(b) interest rate



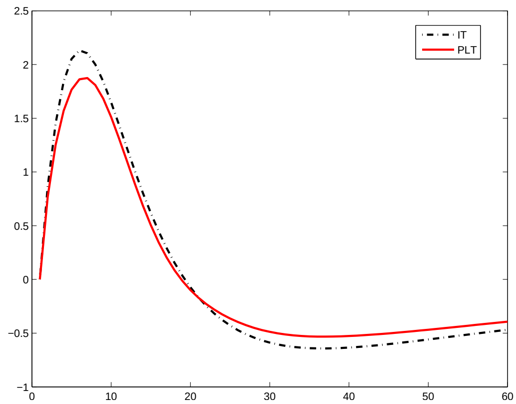
(c) price level



(d) output gap



(e) marginal cost



(f) consumption

The result also points to the importance of doing proper model-consistent welfare calculations. In a model as complex as ToTEM, social welfare will in general be of a much more complicated functional form than the assumed quadratic loss function. For accurate social welfare comparisons, we should in fact use second-order approximation techniques (see Kim and Kim 2003, Schmitt-Grohé and Uribe 2004 and references therein). This is, however, presently not feasible in ToTEM.

3.2 Price-level targeting versus full-commitment

Table 4: Standard deviation of inflation, output gap, change in interest rate under the optimized inertial PLT and the full commitment solution

ω	ν	σ_{π_t} FC	PLT	PLT $\rho_P = 0$	σ_{y_t} FC	PLT	PLT $\rho_P = 0$	$\sigma_{\Delta i_t}$ FC	PLT	PLT $\rho_P = 0$
0	0.1	0.32	0.32	0.40	2.05	2.04	2.03	0.14	0.16	0.19
0	0.5	0.40	0.41	0.50	2.08	2.07	2.09	0.08	0.08	0.10
0	1	0.44	0.45	0.54	2.10	2.10	2.13	0.06	0.06	0.07
0.1	0.1	0.79	0.90	0.89	0.45	0.45	0.53	0.30	0.37	0.43
0.1	0.5	0.80	0.90	0.88	0.63	0.63	0.74	0.17	0.21	0.24
0.1	1	0.81	0.90	0.87	0.71	0.72	0.85	0.13	0.16	0.19
0.5	0.1	0.87	0.96	0.94	0.24	0.25	0.31	0.47	0.60	0.71
0.5	0.5	0.88	0.96	0.94	0.40	0.40	0.49	0.30	0.36	0.43
0.5	1	0.88	0.96	0.93	0.48	0.49	0.58	0.24	0.29	0.34
1	0.1	0.88	0.96	0.95	0.19	0.20	0.24	0.54	0.72	0.87
1	0.5	0.89	0.97	0.95	0.32	0.33	0.40	0.37	0.46	0.54
1	1	0.89	0.97	0.94	0.40	0.40	0.49	0.30	0.36	0.43

This section compares the performance and behavior of optimized price-level targeting rules to the full commitment solution. Given the quadratic loss function (5) and the model (2), section A.1 of the appendix shows that the full-commitment solution for the interest rate is

$$i_t = F_X X_{t-1} + F_\mu \mu_{t-1}, \qquad (6)$$

where X_{t-1} are predetermined state variables and μ_{t-1}, the lagrange multipliers associated with the forward-looking variables X_{t+1}. Therefore, the full-commitment solution is in general much more complex than a simple policy rule, in that it requires the interest rate to depend not only on the set of predetermined variables, but also on the shadow price of the forward-looking variables. What this implies for a policy-maker using the full-commitment solution is that he is tied to the promise he makes when optimizing at time 0 i.e. he must make choices consistent with the value at which he initializes μ_{-1} at time 0. The full-commitment solution is said to be time-inconsistent since at any time $t > 0$, there is a temptation for the policy-maker to re-optimize and reset the lagrange multipliers.

Table 4 displays the standard deviation of inflation, output gap and change in interest rate under the full-commitment solution, optimized non-inertial price-level targeting rule and optimized inertial price-level targeting rule. I find that both the non-inertial and inertial perform reasonably well with respect to the full-commitment solution. In particular, similar to the full commitment solution, both rules emphasize stabilizing the output gap at the expense of inflation for higher weights to output gap stabilization. The inertial rule in particular performs only slightly worse than the full commitment solution in stabilizing inflation and the output gap.

Figures 4 and 5 in the appendix compares the impulse responses of different variables to the consumption shock under the non-inertial rule, inertial rule, and full-commitment solution. I find that the paths of variables under the price-level targeting rules follow closely the paths of the first-best full-commitment solution to one exception: while the price-level eventually gets back under control under price-level targeting, the first-best solution does not. In fact, given our loss function, the first-best solution *does not* require stationarity in the price-level.[5]

Why then do the price-level targeting rules mimic the full-commitment solution so well? The price-level targeting solution is characterized by two important features: (i) stationarity in the price level, and (ii) history dependence. It induces a dependence on the past, in that the policymaker is committed to correct past deviations of the price-level from target. It is that commitment to correct past deviations that allow expectations to play a cru-

[5]Cateau (2008) derives a formula for determining whether price level stationarity is a characteristic of full commitment optimal policy in three different forward-looking DSGE models. He finds that, in general, it is not.

cial role in price-level targeting. That history dependence, rather than the stationarity in the price-level, is the feature that introduces a connection to the full commitment solution.

Indeed, the full-commitment solution is also characterized by history dependence. The full-commitment solution depends on the lagrange multipliers associated with the forward-looking variables. When optimizing at time $t = 0$, the policymaker initializes those shadow prices to zero and makes a promise that at any time $t > 0$, it will not re-optimize and reset those shadow prices to zero. The commitment to the time 0 promise implies that at any time $t > 0$, optimal policy must be consistent with the time $t = 0$ lagrange multiplier initializations. Thus, the optimal full commitment policy is also history dependant. That history dependence allows for a role for expectations in the same way that it does under price-level targeting.

4 Price-level targeting versus inflation targeting under model uncertainty

The previous section showed that in ToTEM, with a quadratic loss function, an optimized price-level targeting rule would dominate an optimized inflation targeting rule across a number of the policy-maker's preference configurations. To what extent do those results depend on the assumption that the policy-maker knows how the economy works? If ToTEM was a misspecified representation of the economy, there are at least two reasons why the performance of the price-level targeting rule could be affected. First, the policy prescribed in the reference model could be misleading. Thus, in practice, the policy-maker may find it difficult to control the price-level. Second, price-level targeting works by exploiting the expectations of economic agents. Therefore, if the expectations of economic agents differ in an important way from the model-consistent expectations, the mechanism through which price-level targeting works may be hampered.

The policy-maker of this paper addresses his concerns about model misspecification by analyzing how the price-level and inflation targeting rules, optimized for his reference version of ToTEM, would perform in a world where the correct model is not the reference version of ToTEM, but a robust control version that economic agents use to form expectations.

More formally, the distorted model is obtained as follows: given an optimized simple rule

$$i_t = F_1^* X_{t-1} + F_2^* X_t + F_3^* X_{t+1} \tag{7}$$

an evil agent distorts the reference model (2)

$$H_1 X_{t-1} + H_2 X_t + H_3 X_{t+1} + B i_t + C w_t + C \epsilon_{t+1} = 0, \tag{8}$$

by choosing a distortion w_t to maximize the loss function (5) subject to the distortions being bounded by

$$E_0 \sum_{t=0}^{\infty} \beta^t w_t w'_t < \zeta. \tag{9}$$

Following Hansen and Sargent (2004) and Dennis, Leitemo, and Söderström (2006), the robust control problem can be conveniently written as

$$\max_{w_t} \sum_{t=0}^{\infty} \beta^t \left\{ X_t' \tilde{Q} X_t - \theta w_t w_t' \right\}, \tag{10}$$

subject to

$$\tilde{H}_1 X_{t-1} + \tilde{H}_2 X_t + \tilde{H}_3 X_{t+1} + \tilde{C} w_t + \tilde{C} \epsilon_t = 0, \tag{11}$$

where $\tilde{Q}, \tilde{H}_1, \tilde{H}_2, \tilde{H}_3$ and $\tilde{C}$ are obtained by substituting (7) in (5) and (8) respectively and simplifying. Following Hansen and Sargent (2004), it can also be shown that θ^{-1} is directly related to the size of the distortion ζ.

The distorted model will have dynamics given by the transition equation

$$\begin{bmatrix} X_t \\ \mu_t \end{bmatrix} = N^*(\theta^{-1}) \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix} + C^*(\theta^{-1}) \epsilon_t, \tag{12}$$

and the optimal distortion that the evil agent will pick will be given by

$$w_t = K_X(\theta^{-1}) X_{t-1} + K_\mu(\theta^{-1}) \mu_{t-1}. \tag{13}$$

The distortion that the evil agent introduces in the reference model will, therefore, influence both the dynamics and volatility of variables in the economy. How much the model is distorted by the evil agent will depend on the bound on the distortion, and hence θ^{-1}. In practical applications, it is important to pick θ^{-1} sensibly. I use the statistical detection probability theory to discipline the choice of θ^{-1}.

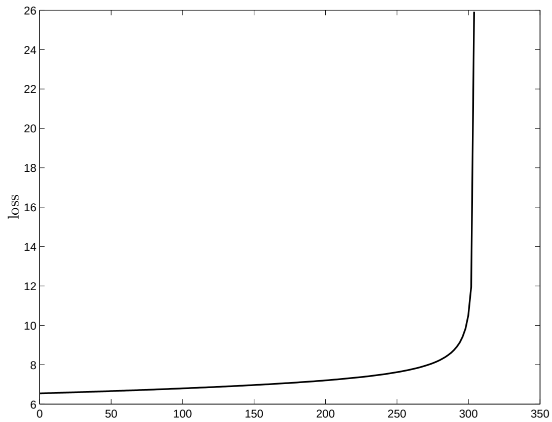
To understand what detection probabilities are, consider the following example. Suppose that a decision-maker faces two models, A and B, and has a finite data set to determine which one of the two models is the data generating model. Now suppose that model A generates the data. In a finite data set, if the models are not too far apart there is a positive probability for the decision-maker to conclude that model B is the data-generating model even though model A generates the data. The detection probability is the average probability that the decision-maker erroneously concludes that model B generates the data when it is in fact model A or that model A generate the data when it is in fact model B (see Dennis, Leitemo, and Söderström 2006 for more details).

Why are detection probabilities suitable for disciplining the choice of θ^{-1}? They give a measure of the statistical distance between the reference model and the distorted model. If $\theta^{-1} = 0$, the detection probability is equal to 0.5. Why? Because if $\theta^{-1} = 0$, the reference model and distorted model are the same, and hence they are both equally likely to generate the data. However, as we increase θ^{-1}, the distorted model grows further apart from the reference model. The detection probability will fall below 0.5 since it becomes easier to distinguish between the two models even in a finite data set. Hansen and Sargent (2004) argue that θ^{-1} should be chosen to correspond to a detection probability between 10-20%.

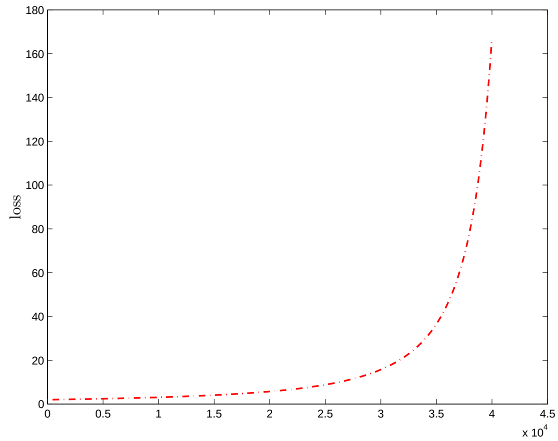
4.1 Results

In this section, I answer two questions: (i) how much does model misspecification affect the absolute level of the losses under price-level targeting and inflation targeting, and (ii) how fast does the performance of the price-level (and inflation) targeting rule deteriorate with misspecification.

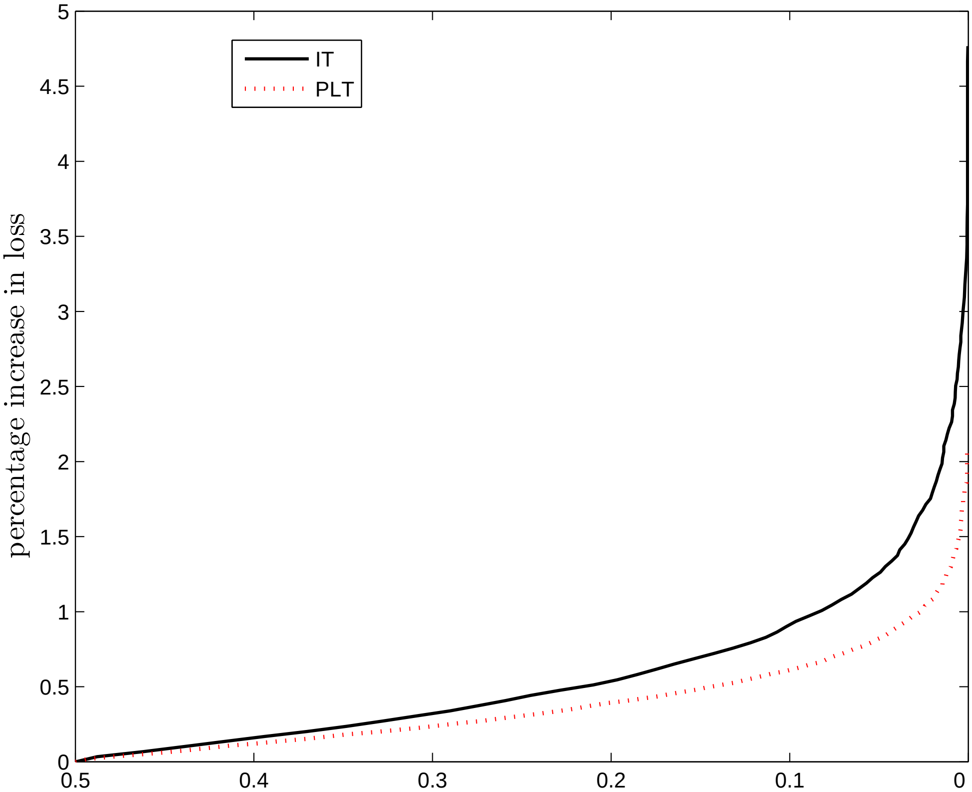
Figure 3 considers the benchmark case $(\omega, \nu) = (0, 0.5)$. The upper left panel displays how the inflation targeting rule optimized for the reference ToTEM ($\theta_{IT}^{-1} = 0$) would perform if the true model was in fact a distorted model indexed by some $\theta_{IT}^{-1} > 0$. The upper right panel repeats that exercise for the price-level targeting rule optimized for the reference model ($\theta_{PT}^{-1} = 0$). The two figures convey a similar conclusion: rules optimized for the reference model can perform very badly for large values of $\theta_j^{-1}, j = IT,\ PT$, the bound on the size of the distortion. Intuitively, a large θ_j^{-1} implies a distorted model that is quite far from the reference model and hence, it is not surprising to find that the rule optimized for the reference model does not have enough



(g) θ_{IT}^{-1}



(h) θ_{PT}^{-1}



(i) detection probability

Figure 3: performance of non-inertial inflation and price-level targeting rule under misspecification

flexibility to perform well in the distorted model.

To compare the performance of price-level targeting and inflation targeting under model uncertainty, I compute the detection probabilities implied by θ_{IT}^{-1} and θ_{PT}^{-1}. The lower panel of figure 3 plots the percentage increases in losses, $\Delta_j = 100\left(\frac{\text{loss}_\text{j}(\theta_j^{-1}>0)}{\text{loss}_\text{j}(\theta_j^{-1}=0)} - 1\right)$, $j =$ IT, PT against the detection probability. I find that for models that are statistically close i.e. a detection probability range between 0.5-0.1, the performance of the price-level targeting rule deteriorates more slowly than that of the inflation targeting rule. Furthermore, for the same set of statistically close models, the consequence of model uncertainty does not seem very important in absolute terms. Indeed, for the price-level targeting rules, the performance deteriorates by about 0.5% while for the inflation targeting rule, it deteriorates by 0.7%.

What can we conclude from the above exercise? If the assumption that ToTEM is a good reference model holds, the optimized price-level targeting rule is robust. If agents form expectations according to an alternative model within a reasonable distance from the reference model, it's performance does not deteriorate very fast. Further, the absolute increase in the level of the loss is relatively small. Therefore, allowing for model mis-specification à la Hansen and Sargent (2004) does not affect the conclusion that the price-level targeting rule dominates the inflation targeting rule significantly.

We should keep in mind however that the result above does not imply that model uncertainty does not matter for policy-making. First, using a different metric than detection probabilities may allow model mis-specification to have a greater impact. The upper panels of figure 3 show that for large values of θ_{IT}^{-1} and θ_{PT}^{-1} the increase in losses is high. Therefore, a different metric that admits higher values of θ_{IT}^{-1} and θ_{PT}^{-1} would allow for model uncertainty to have bigger impact. Moreover, we allow for model mis-specification à la Hansen and Sargent (2004). Their approach is a simple and computationally convenient way to allow for unstructured model mis-specification in a linear-quadratic set-up. Different types of unstructured model uncertainty, e.g. Onatski and Williams (2003), or more structured model uncertainty, e.g parameter uncertainty, may yield different results.

5 Conclusion

This paper considers a policy-maker that uses ToTEM as the model of the Canadian economy and sets policy through a simple rule to minimize a weighted average of the variance of inflation, output gap, and changes in the interest rate. I first compare the performance of inflation-targeting and price-level targeting rules for a number of configurations of the policy-maker's loss function.

I find that across all configurations, the optimized price-level targeting rule dominates the inflation targeting rule significantly. How the price-level targeting achieves that superiority, however, depends importantly on how much weight the policy-maker assigns to stabilizing the output gap. If that weight is zero, the optimized price-level targeting rule achieves lower volatility in both inflation, output gap and the change in the interest rate than the optimized inflation targeting rule. However, when the weight is positive, there is a trade-off between stabilizing inflation and output gap. The optimized price-level targeting rule dominates the inflation targeting rule by stabilizing the output gap much better even though it allows the inflation rate to be more volatile.

A stable output gap and a relatively more volatile inflation rate (when the weight to output-gap stabilization is positive) is in fact what would be fully optimal in a full-commitment solution. I show that the optimal price-level targeting rule yields responses that are very similar to the full commitment solution to one exception: the full commitment solution does not require the price-level stationarity. I argue from that result that the critical reason why price-level targeting behaves very similarly to the full commitment solution is not because it yields stationarity in the price-level but because both solutions induce a dependance on the past and rely on expectations to spread the effects of shocks over time.

Finally, I verify whether my conclusion that price-level targeting dominates inflation targeting significantly is sensitive to model uncertainty. I consider how a price-level targeting rule optimized for the reference ToTEM would perform in a world where the correct model is not the reference version of ToTEM but a robust control version that economic agents use to form expectations. I find that in models that are at a reasonable distance from the reference model (based on detection probabilities), the price-level targeting rule is robust. It's performance does not deteriorate very rapidly and the absolute increase in the loss is also relatively small.

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A Optimal and robust control

A.1 Full commitment solution

The full commitment solution is obtained by

$$\min_{i_t} \sum_{t=0}^{\infty} \beta^t \left\{ X_t' Q X_t + i_t' R i_t \right\}, \tag{14}$$

subject to the model

$$H_1 X_{t-1} + H_2 X_t + H_3 X_{t+1} + B i_t = 0. \tag{15}$$

Notice that since the loss function (14) is quadratic and the model linear, I can solve the non-stochastic version of the policy-maker's problem owing to certainty equivalence.

The Lagrangian for this problem is

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ X_t' Q X_t + i_t' R i_t + 2\mu_t' \left(H_1 X_{t-1} + H_2 X_t + H_3 X_{t+1} + B i_t \right) \right\}. \tag{16}$$

The first-order conditions are

$$\begin{aligned} i_t &: \quad i_t = -R^{-1} B' \mu_t & (17) \\ X_t &: \quad Q X_t + H_1' \beta \mu_{t+1} + H_2' \mu_t + H_3' \beta^{-1} \mu_{t-1} = 0. & (18) \end{aligned}$$

By substituting the f.o.c.'s for i_t into the constraint (15), we obtain

$$H_1 X_{t-1} + H_2 X_t + H_3 X_{t+1} - B R^{-1} B' \mu_t = 0. \tag{19}$$

From (19) and (18), I can construct a system of difference equations in X_t and μ_t :

$$\begin{aligned} \begin{bmatrix} H_1 & 0 \\ 0 & H_3' \beta^{-1} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix} &+ \begin{bmatrix} H_2 & -BR^{-1}B' \\ Q & H_2' \end{bmatrix} \begin{bmatrix} X_t \\ \mu_t \end{bmatrix} \\ &+ \begin{bmatrix} H_3 & 0 \\ 0 & H_1' \beta \end{bmatrix} \begin{bmatrix} X_{t+1} \\ \mu_{t+1} \end{bmatrix} = 0, \end{aligned} \tag{20}$$

which can be rewritten as

$$A_1 \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix} + A_2 \begin{bmatrix} X_t \\ \mu_t \end{bmatrix} + A_3 \begin{bmatrix} X_{t+1} \\ \mu_{t+1} \end{bmatrix} = 0. \tag{21}$$

It can be shown that given the transversality conditions and appropriate initial conditions X_{-1} and μ_{-1}, the solution to the difference equation (21) is

$$\begin{bmatrix} X_t \\ \mu_t \end{bmatrix} = N \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix}. \tag{22}$$

The matrix N can be solved for using invariant subspace methods (e.g. Dennis 2003) or iterative methods. The full commitment decision rule for i_t is then obtained from (17) and (22). From (22),

$$\mu_t = \begin{bmatrix} 0 & I \end{bmatrix} N \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix}. \tag{23}$$

From (17), it follows that

$$i_t = -R^{-1}B' \begin{bmatrix} 0 & I \end{bmatrix} N \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix}. \tag{24}$$

I will write the full commitment solution as $i_t = F_X X_{t-1} + F_\mu \mu_{t-1}$.

A.2 Dynamics in a stochastic system

In this section I consider the problem recast as a stochastic system. Beginning with

$$H_1 X_{t-1} + H_2 X_t + H_3 X_{t+1} + B i_t + C \epsilon_{t+1} = 0, \tag{25}$$

and performing similar substitutions and manipulations as in section A.1, I obtain the difference system

$$A_1 \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix} + A_2 \begin{bmatrix} X_t \\ \mu_t \end{bmatrix} + A_3 \begin{bmatrix} X_{t+1} \\ \mu_{t+1} \end{bmatrix} + \begin{bmatrix} C \\ 0 \end{bmatrix} \epsilon_{t+1} = 0. \tag{26}$$

Using (22), I get

$$\begin{bmatrix} X_t \\ \mu_t \end{bmatrix} = N \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix} + D \epsilon_{t+1}. \tag{27}$$

where $D = (A_2 + A_3 N)^{-1} \begin{bmatrix} C \\ 0 \end{bmatrix}$.

A.3 Robust control

In the robust control problem, the policy-maker chooses policy through the policy instrument i_t while an evil agent distorts the model by choosing w_t i.e.,

$$\min_{i_t} \max_{w_t} \sum_{t=0}^{\infty} \beta^t \left\{ X_t' Q X_t + i_t' R i_t - \theta w_t w_t' \right\}. \tag{28}$$

subject to

$$H_1 X_{t-1} + H_2 X_t + H_3 X_{t+1} + B i_t + C^* w_t = 0. \tag{29}$$

It can be shown (see Hansen and Sargent 2004) that θ^{-1} indexes the size of the distortion that the evil agent is allowed to introduce in the policy-maker's reference model. For my purpose, I want to analyze how a particular policy rule

$$i_t = F_1^* X_{t-1} + F_2^* X_t + F_3^* X_{t+1} \tag{30}$$

that the policy-maker chooses on the basis on an undistorted reference model fares in a model distorted by the evil agent. Therefore, given the rule (30), the evil agent distorts the model, by choosing w_t to

$$\max_{w_t} \sum_{t=0}^{\infty} \beta^t \left\{ X_t' \tilde{Q} X_t - \theta w_t w_t' \right\}. \tag{31}$$

subject to

$$\tilde{H}_1 X_{t-1} + \tilde{H}_2 X_t + \tilde{H}_3 X_{t+1} + \tilde{C} w_t = 0. \tag{32}$$

where $\tilde{Q}, \tilde{H}_1, \tilde{H}_2, \tilde{H}_3$ and $\tilde{C}$ are obtained by substituting (30) in (28) and (29) respectively.

The Lagrangian for this problem is

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ X_t' \tilde{Q} X_t - \theta w_t w_t' + 2\mu_t' \left(\tilde{H}_1 X_{t-1} + \tilde{H}_2 X_t + \tilde{H}_3 X_{t+1} + \tilde{C} w_t \right) \right\}. \tag{33}$$

The first-order conditions are

$$w_t \; : \; w_t = \theta^{-1} \tilde{C}' \mu_t \tag{34}$$

$$X_t \; : \; \tilde{Q} X_t + \tilde{H}_1' \beta \mu_{t+1} + \tilde{H}_2' \mu_t + \tilde{H}_3' \beta^{-1} \mu_{t-1} = 0. \tag{35}$$

By repeating the procedure in section (A.1), the solution to the evil agent problem is obtained by solving a system of difference equations

$$\tilde{A}_1 \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix} + \tilde{A}_2 \begin{bmatrix} X_t \\ \mu_t \end{bmatrix} + \tilde{A}_3 \begin{bmatrix} X_{t+1} \\ \mu_{t+1} \end{bmatrix} = 0. \tag{36}$$

The solution to (36) is

$$\begin{bmatrix} X_t \\ \mu_t \end{bmatrix} = \tilde{N} \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix}. \tag{37}$$

The decision rule for w_t is obtained from 37 and (34). It is given by

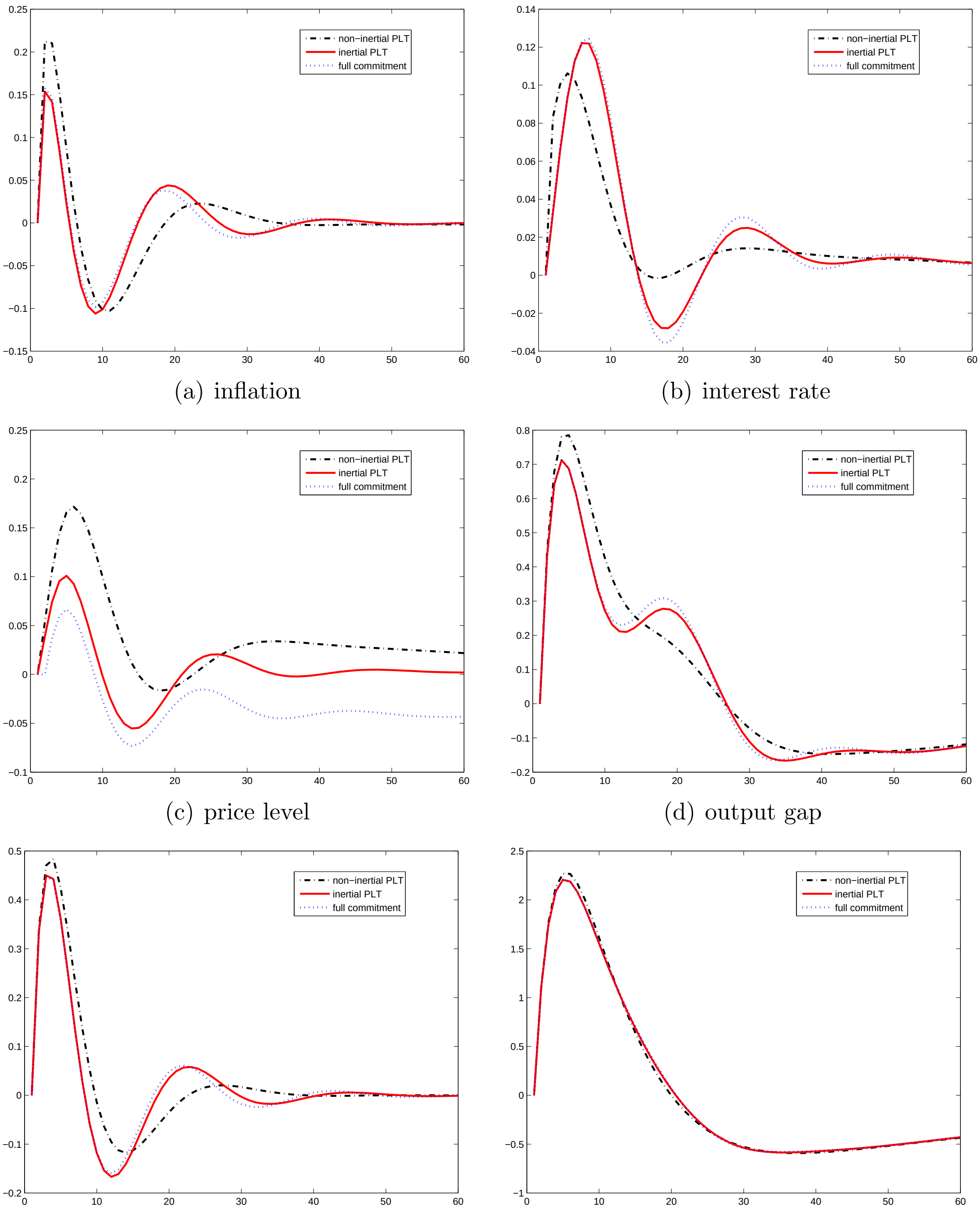
$$w_t = \theta^{-1} C' \begin{bmatrix} 0 & I \end{bmatrix} N(\tilde{\theta}^{-1}) \begin{bmatrix} X_{t-1} \\ \mu_{t-1} \end{bmatrix}. \tag{38}$$

I will write the decision rule for w_t as $w_t = K_X X_{t-1} + K_\mu \mu_{t-1}$.

Table 5: Unrestricted optimized inflation targeting rule

ω	ν	ρ_π	B_π	ϕ_π	$\frac{\text{loss IT}}{\text{loss PLT}}$
0	0.1	2.19	10.90	-0.02	1.15
0	0.5	1.56	2.75	-0.01	1.15
0	1	1.43	1.65	0.00	1.16
0.1	0.1	3.46E+05	8.60E+06	-1.91E+06	2.96
0.1	0.5	2.97E+05	2.06E+06	-3.60E+05	2.65
0.1	1	6.68E+04	3.02E+05	-4.90E+04	2.50
0.5	0.1	1.74E+05	9.56E+06	-4.75E+06	4.75
0.5	0.5	9.20E+05	1.68E+07	-7.29E+06	3.99
0.5	1	3.35E+05	3.40E+06	-1.32E+06	3.69
1	0.1	1.81E+00	1.28E+02	-8.14E+01	5.96
1	0.5	4.19E+05	1.13E+07	-6.70E+06	4.44
1	1	3.32E+06	4.96E+07	-2.73E+07	4.06

Figure 4: impulse responses to a positive consumption shock; full commitment versus price-level targeting ($\omega = 0$)



(a) inflation

(b) interest rate

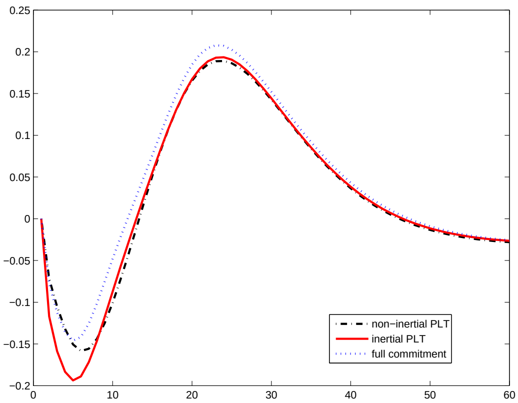
(c) price level

(d) output gap

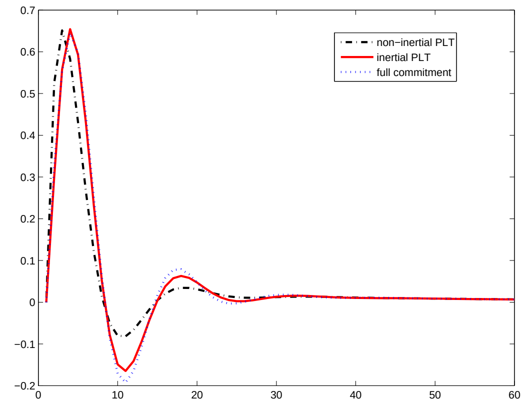
(e) marginal cost

(f) consumption

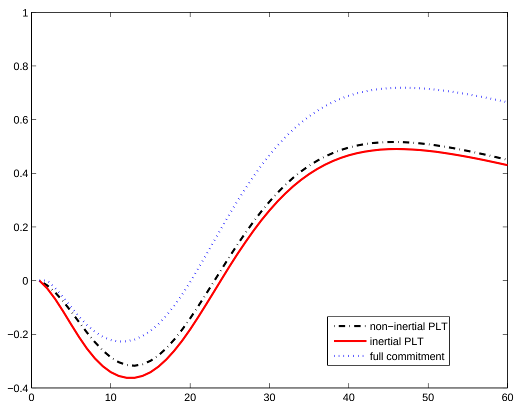
Figure 5: impulse responses to a positive consumption shock; full commitment versus price-level targeting ($\omega = 1$)



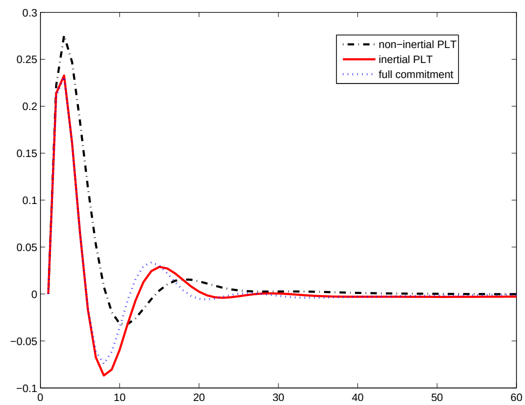
(a) inflation



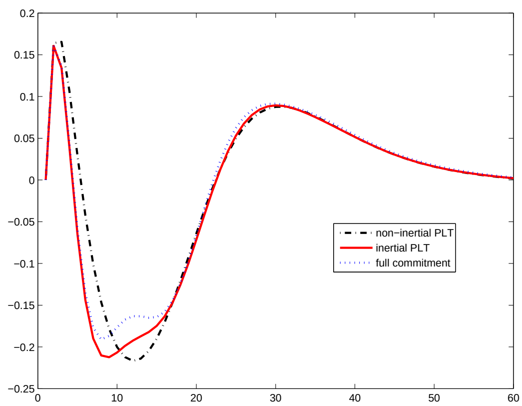
(b) interest rate



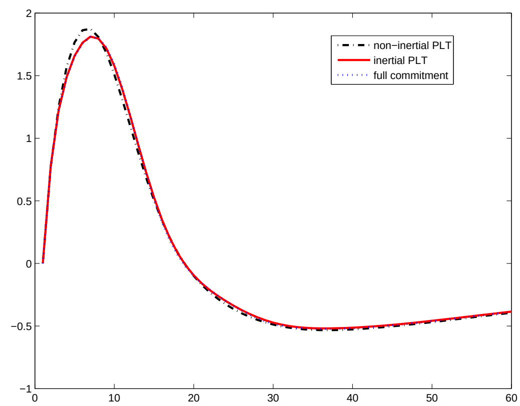
(c) price level



(d) output gap

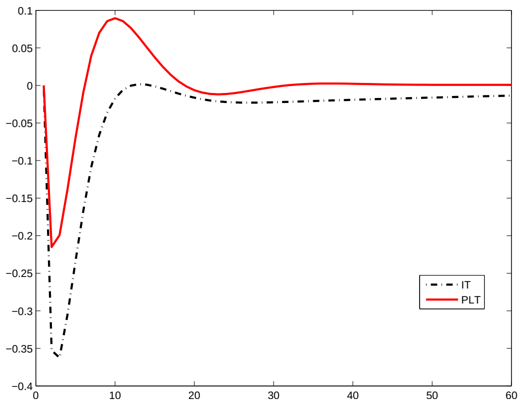


(e) marginal cost

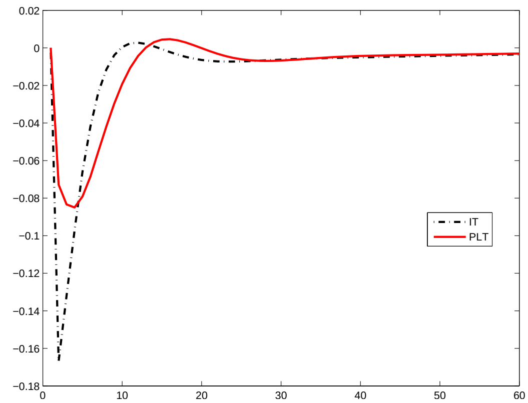


(f) consumption

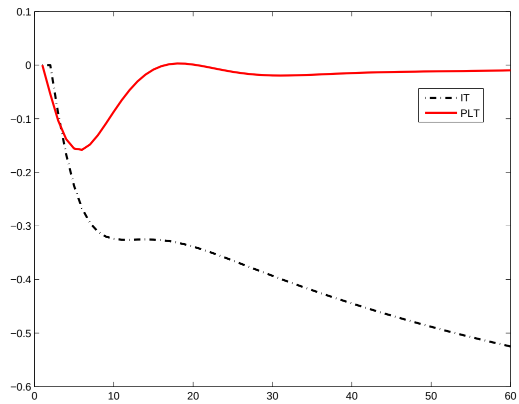
Figure 6: impulse responses to a positive technology shock ($\omega = 0$)



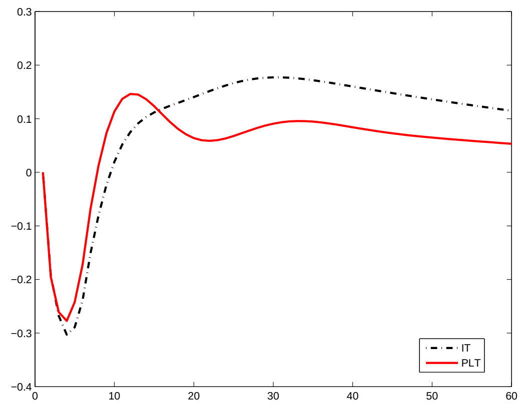
(a) inflation



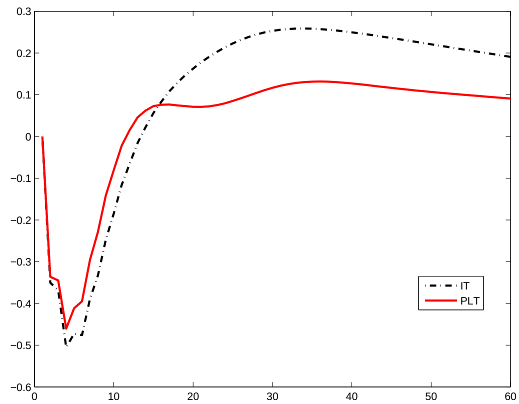
(b) interest rate



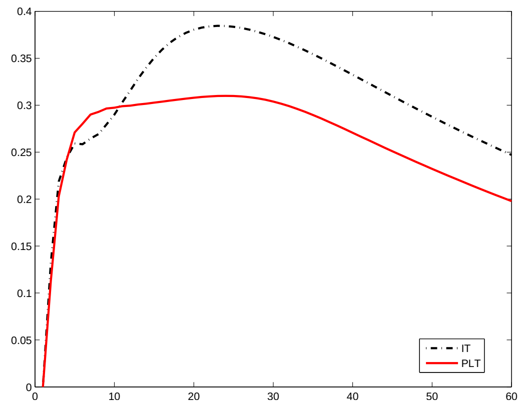
(c) price level



(d) output gap

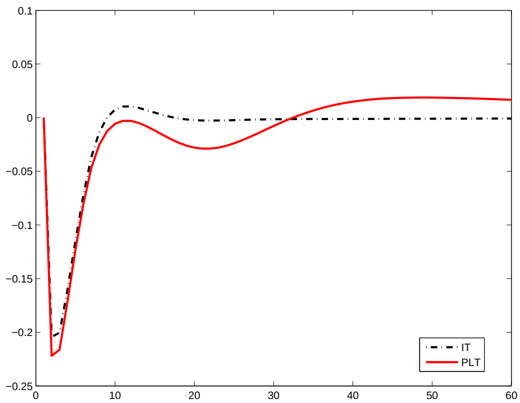


(e) hours

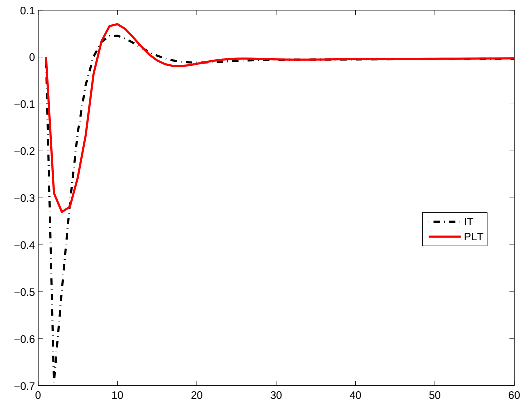


(f) consumption

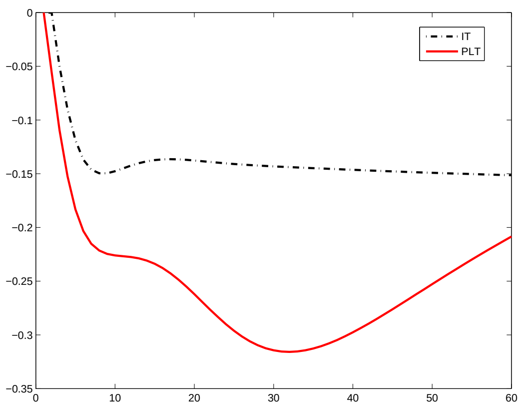
Figure 7: impulse responses to a positive technology shock ($\omega = 1$)



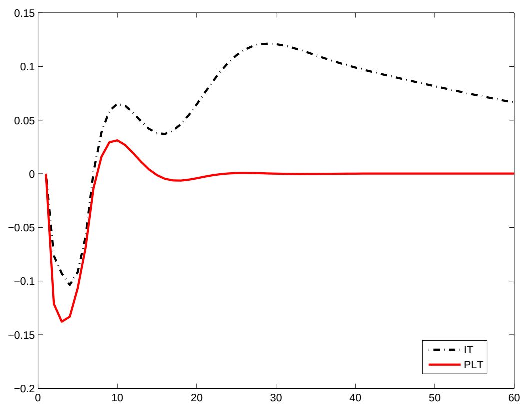
(a) inflation



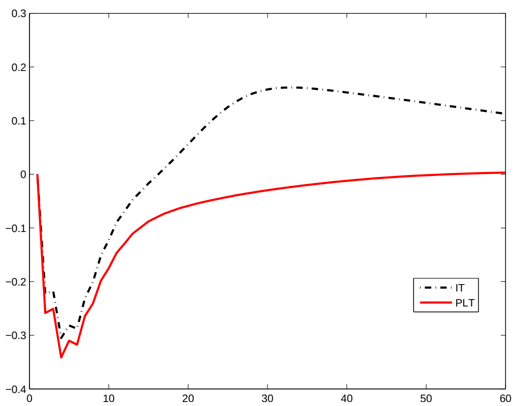
(b) interest rate



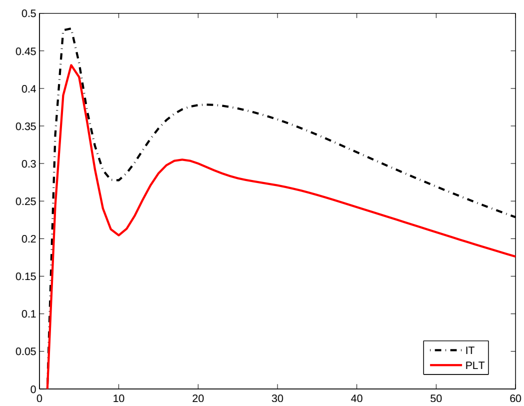
(c) price level



(d) output gap



(e) hours



(f) consumption