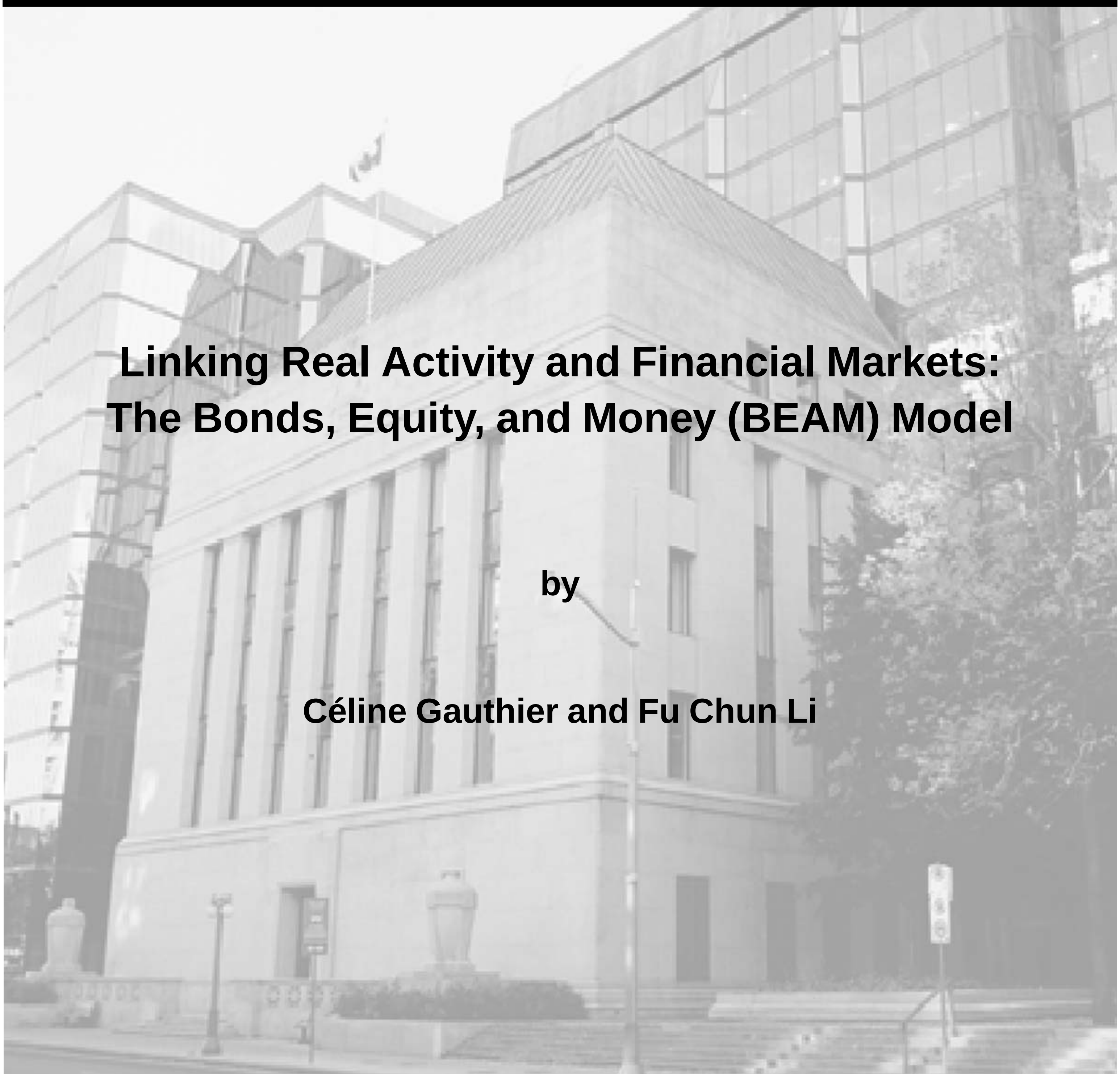
Bank of Canada Banque du Canada

Working Paper 2006-42/ Document de travail 2006-42



Linking Real Activity and Financial Markets: The Bonds, Equity, and Money (BEAM) Model

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ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2006-42

November 2006

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Acknowledgements

The authors wish to thank Allan Crawford, Ramdane Djoudad, Scott Hendry, Pierre St-Amant, David Tessier, and Carolyn Wilkins for very helpful comments, and Jim Day and Alejandro Garcia for their technical assistance. We would also like to thank Glen Keenleyside for his editorial assistance.

Abstract

The authors estimate a small monthly macroeconometric model (BEAM, for bonds, equity, and money) of the Canadian economy built around three cointegrating relationships linking financial and real variables over the 1975–2002 period. One of the cointegrating relationships allows the identification of a supply shock as the only shock that permanently affects the stock market, and a demand shock that leads to important transitory stock market overvaluation. The authors propose a monetary policy reaction function in which the impact of a permanent inflation shock on the overnight rate is simulated and the future path of the overnight rate adjusted accordingly, to prevent any forecast persistent deviation from the inflation target. They introduce a technical innovation by showing under which conditions permanent shocks can be identified in a vector error-correction model with exogenous variables.

JEL classification: C5, E4
Bank classification: Financial markets; Financial stability

Résumé

Les auteurs estiment pour le Canada un petit modèle macroéconomique mensuel (du nom de BEAM, sigle formé des initiales des mots *bonds*, *equity and money*) articulé autour de trois relations de cointégration mettant en rapport des variables financières et réelles pour la période de 1975 à 2002. Ils définissent par l'une de ces relations un choc d'offre, seul choc à exercer un effet permanent sur le marché boursier, et un choc de demande, qui donne lieu à une surévaluation passagère mais sensible des titres cotés. Les auteurs proposent une fonction de réaction des autorités monétaires qui simule l'effet d'un choc d'inflation permanent sur le taux du financement à un jour et où celles-ci ajustent en conséquence l'évolution de ce taux afin d'éviter tout écart persistant entre les prévisions et le taux d'inflation cible. Du côté de la méthodologie, ils innovent en énonçant les conditions d'identification des chocs permanents dans un modèle vectoriel à correction d'erreurs qui renferment des variables exogènes.

Classification JEL : C5, E4
Classification de la Banque : Marchés financiers; Stabilité financière

1. Introduction

As Garratt et al. (2003) mention, there are two main theoretical approaches to the derivation of long-run, steady-state relations of a core macroeconomic model. One is to start with the intertemporal optimization problems faced by "representative" agents and solve for the long-run relations. The strength of this approach lies in the explicit identification of macroeconomic disturbances as innovations (shocks) to processes generating tastes and technology. However, this is achieved at the expense of often strong assumptions concerning the form of the underlying utility and production functions. Consequently, despite the progress recently seen in the dynamic general-equilibrium (DGE) literature, there is still a lot of work to be done before a general-equilibrium model incorporates in a satisfying way the real and financial sectors of the economy. An alternative approach followed by Garratt et al. (2003) is to work directly with the arbitrage and long-run equilibrium conditions that provide intertemporal links between prices and asset returns in the economy as a whole. This latter approach, by focusing on long-run theory restrictions and leaving the short-run dynamics largely unrestricted, provides a much more flexible modelling strategy.

We propose a small model for Canada, combining Garratt et al.'s (2003) approach with King et al.'s (1991) methodology allowing the identification of permanent shocks in a cointegrated system. Crowder, Hoffman, and Rasche (1999), Dhar, Pain, and Thomas (2000), Jacobson et al. (2001), and Cassola and Morana (2002) all follow that route for, respectively, the United States, the United Kingdom, Sweden, and Europe, and show the degree of "structure" that may be assigned to a simple vector autoregression (VAR) framework characterized by cointegration if one embraces sufficient identifying restrictions. Following Dhar, Pain, and Thomas (2000) and Cassola and Morana (2002), we focus on the interactions between different marketable asset values and the real economy. One technical contribution of this paper is to include exogenous variables and to show under which conditions King et al.'s (1991) identification procedure can be applied to a vector-error-correction model (VECM) with weakly exogenous I(1) variables.

The building blocks of our model consist in three cointegrating relationships: (i) a money market equilibrium relation, (ii) an arbitrage relation between short- and long-term bonds, and (iii) a long-run relation between the stock market and real output. This last relation allows the identification of a supply shock as the only shock that permanently affects the stock market, and a demand shock that leads to important transitory stock market overvaluation.

A weakness of most models that purport to describe the transmission mechanism is their failure to pass the simple test of generating a different steady-state rate of inflation in response to a series of monetary policy actions.[1] Such models with a unique steady state-rate of inflation are very difficult to reconcile with the unit-root test results found in the empirical literature.[2] In this paper, we identify permanent shocks that cause inflation to reach a new steady-state rate of growth as the only shocks having a permanent impact on the level of inflation. We then propose a monetary policy reaction function that consists in reversing any identified nominal shock, causing inflation to permanently deviate from the target.

Our paper is organized as follows. The theoretical foundations of the model are presented in section 2. The results of the cointegration analysis and specification tests are given in section 3 and Appendix C. Section 4 analyzes the impulse-response functions. Section 5 proposes a monetary policy reaction function. Section 6 offers some conclusions.

2. The Model's Theoretical Foundations

In this section, we describe the long-run relations used as building blocks of our model. We "loosely" base our core model on Blanchard (1981), who develops a simple model of the determination of output, the stock market, and the term structure of interest rates. The model is an extension of the IS-LM model. However, whereas the IS-LM model emphasizes the interaction between "the interest rate" and output, Blanchard's model emphasizes the interactions between output and four marketable asset values. These are shares that are titles to the physical capital, private short- and long-term bonds issued and held by individuals, and money.

Linking the real economy and the stock market

We assume that there are two main determinants of spending.[3] The first is the value of shares in the stock market. It may affect spending directly through the wealth effect on consumers, or indirectly through its impact on the borrowing capacity of consumers and investors (the credit channel effect); determining the value of capital in place relative to its replacement costs, it affects investment. The second determinant of spending is current income, which may affect spending independently of wealth if consumers are liquidity constrained. Total spending is expressed as:

$$d_t = \alpha sm_t + \beta y_t; \quad \alpha > 0; \quad \beta > 0; \tag{1}$$

1. More details on this point are provided in Selody (2001).
2. This is also a very difficult issue, since inflation is expected to become stationary, or at least more stable, in a successful inflation-targeting environment.
3. Blanchard also includes a balanced budget change in public spending as a third determinant of total spending.

where all variables are real, d denotes spending, sm is the stock market value, and y is income.[4] We can consider equation (1) a forward-looking aggregate spending curve, with sm being a function of expected actualized future profits, the latter being a function of expected future output. Hence, aggregate spending is implicitly a negative function of actual and expected interest rates, and a positive function of actual and future expected output. Output adjusts to spending over time:

$$\dot{y}_t = \sigma(d_t - y_t) = \sigma(\alpha sm_t - by_t); \quad \sigma > 0; \quad b \equiv 1 - \beta; \tag{2}$$

where a dot denotes a time derivative. Since output growth is a stationary variable and the level of output and the stock market price are both I(1) variables, equation (2) can be seen as an error-correction equation linking positively the short-run dynamics of output to deviations of the stock market from the real economy. Such a long-run relation between output and the stock market implies that transitory changes in output (the stock market) cannot permanently affect the level of the stock market (output).

Money market equilibrium

Portfolio balance is characterized by a long-run relation between money, output, the interest rate, and inflation:

$$M_t - p_t = cy_t - hi_t - \beta\pi_t; \quad c > 0; \quad h > 0; \quad \beta > 0; \tag{3}$$

where i denotes the short-term nominal rate, y is real income, M and p denote the logarithms of nominal money and the price level, respectively, and π is the level of inflation. The parameter c is positive because an increase in output shifts upward the money demand for transactions purposes; an increase in the interest rate and an increase in inflation both increase the opportunity cost of holding money, which decreases the real balance. Given that all the variables in equation (3) are better characterized as I(1) variables, if deviations of real money from its determinants are transitory, then this equation represents a cointegrating relationship.

Arbitrage between short- and long-term bonds

The expectations hypothesis is perhaps the best known and most intuitive theory of the term structure of interest rates. If lr_t is the nominal yield to maturity of a discount bond and i_t is the period-*t* one-period rate, the expectations hypothesis in the absence of uncertainty implies that

4. No stochastic error terms are included in this section, to simplify the presentation.

$$(1 + lr_t)^n = \prod_{j=0}^{n-1} (1 + i_{t+j}) . \tag{4}$$

This is an arbitrage condition ensuring that the holding-period yield on the *n*-period bond is equal to the yield from holding a sequence of one-period bonds. Taking logs of both sides and recalling that $\ln(1 + x) \sim x$ for small x yields a common approximation:

$$lr_t = \frac{1}{n} \sum_{j=0}^{n-1} i_{t+j} . \tag{5}$$

The long-term yield is equal to the average of one-period yields. Hence, a permanent shock to the short-term yield will, in the long run, be reflected one-for-one in the long-term yield, once the shock is correctly perceived as permanent by the financial markets. Cointegration between short- and long-term interest rates is consistent with a stationary term premium.

3. Cointegration Analysis

We estimate a monthly VECM over the 1975–2002 period with six endogenous and one exogenous variables and two lags.[5] The endogenous variables are the following Canadian variables: real GDP at basic prices,[6] the over-10-year marketable bond rate, the overnight rate,[7] a broad money aggregate (real CPI deflated M2++), the real stock market price (real CPI deflated TSX), and the CPI year-over-year inflation rate. M2++ includes mutual funds, the importance of which increased continuously in consumer portfolios over the nineties, and which are relatively liquid. Using a broad aggregate like M2++ in the model avoids interpreting a precautionary portfolio adjustment from mutual funds to money as inflationary.[8] Given the strong economic links between Canada and the United States, we incorporate as an exogenous variable the real U.S. industrial production index, a monthly proxy for U.S. activity. This allows simulation of different U.S. scenarios. Unit-root tests indicate that all variables can be treated as I(1) variables.[9]

5. Two lags minimize the Hannan-Quinn and Schwartz information criteria and are sufficient to remove the correlation in residuals. We use monthly data because the Bank of Canada has adopted a schedule of eight fixed announcement dates per year regarding its decision on its key policy interest rate. Other specification tests are grouped in Appendix C, together with the forecasting performance of the model.
6. This series has been merged with real GDP at factor cost for the period 1975–80.
7. As Selody (2001) notes, a good monetary policy instrument must be under the direct or close control of the central bank.
8. Moreover, Longworth (2003) finds that, since 1992, both core inflation and M2++ have been remarkably stable.
9. Unit-root test results are available upon request.

We add a dummy equalling one from 1993 onward, and zero before, to capture the change in the trend of inflation apparent after the adoption of the inflation target in 1991. We are aware of the possibility that inflation might have become stationary since the adoption of an inflation-targeting regime in 1991. However, the evidence on that point, at least for the United States, is not clear-cut. Cogley and Sargent (2001) argue that there has been a downward shift in the degree of persistence in the inflation process in the United States. Others (see Stock 2002) counter that the statistical evidence in favour of such a break is weak. But even if there was no doubt that inflation has become stationary, the treatment of variables whose degree of integration changes over the estimated sample is still unknown. Moreover, Coenen (2002) and Angeloni, Coenen, and Smets (2003) show that when there is uncertainty about inflation persistence, it is better for monetary policy-makers to work under the assumption that the economy is characterized by a high degree of inflation persistence.

Based on the model's theoretical foundations described in section 2, we expect to find three cointegrating relations in the estimated VECM (as described by equations (2), (3), and (5)). The cointegration tests corrected for the presence of one exogenous variable, as proposed by Pesaran, Shin, and Smith (2000), are identified in Table 1. Both the L-max and the trace tests indicate the presence of two cointegration vectors, but the L-max test marginally rejects the presence of a third cointegration vector, which would support our a priori expectations.

Table 1: Cointegration Tests[a]

L-max	Trace	H0:r=	L-max (0.10)	Trace (0.10)
63.12	151.48	0	40.2	104.4
46.36	88.36	1	34.1	76.9
26.84	42.00	2	28.3	54.8
10.39	16.17	3	22.2	35.9
2.97	5.78	4	15.9	20.8
2.81	2.81	5	9.5	9.5

a. The critical values corrected for the presence of one exogenous variable are taken from Table T.3 in Pesaran, Shin, and Smith (2000).

Given the borderline results of our cointegration tests, we look at the *t*-values of the α coefficients for the third vector, as suggested in Hendry and Juselius (2000); when these are small, say less than 3.0, then one would not lose greatly by excluding that vector as a cointegrating relation in the model. Given that some of these t-values are greater than 3.0 for all three vectors, and that our theoretical model also suggests three vectors, we proceed under the assumption that there are three cointegration vectors in our model.

The Johansen (1992) procedure allows us to identify the number of cointegration vectors. However, in the case of multiple cointegration vectors, an interesting problem arises: α and β are determined only up to the space spanned by them. Thus for any non-singular matrix ζ comformable by product:

$$\Pi = \alpha\beta' = \alpha\zeta\zeta^{-1}\beta'.$$

In other words, β and $\beta'\zeta$ are two observationally equivalent bases of the cointegration space. The obvious implication is that, before solving such an identification problem, no meaningful economic interpretation of coefficients in the cointegration space can be proposed. The solution is to impose a sufficient number of restrictions on parameters that the matrix satisfying such restrictions in the cointegration space is unique. Such a criterion is derived in Johansen (1992). We base our restrictions on Blanchard's (1981) model, which suggests more than a sufficient number of constraints to the cointegration space. The overidentification restrictions can therefore be tested. The results are shown in Table 2.

Table 2: Testing Restrictions on the Cointegration Vectors[a]

	The LR test, $\chi^2(10) = 7.02$, p-value = 0.72					
inf	y	onr	m	sm	lr	y^{us}
2.41 (0.27)	-1.18 (0.08)	2.41 (0.27)	1	0	0	0
0	0	1	0	0	-1	0
0	-1	0	0	1	0	0

a. Standard errors are shown in parentheses.

The restricted core model is easily accepted with a p-value of 0.72. In comparison, Dhar, Pain, and Thomas (2000) do not find a significant core model, whereas Cassola and Morana (2002) just slightly accept theirs with a p-value of 0.11. Our results are consistent with the theoretical foundations presented in section 2. The first cointegrating relation corresponds to the money market equilibrium, the second corresponds to an approximation of the pure expectations hypothesis based on an arbitrage relation between short- and long-term bonds, and the third links real activity with the real stock market. The coefficients of the cointegrating relation cannot usually be interpreted as elasticities, even if the variables are in logs, since a shock to one variable implies a shock to all variables in the long run. Hence the coefficients do not, in general, allow for a *ceteris paribus* interpretation (see Lutkepohl 1994). Interpreting the coefficients in the first cointegrating relation is thus meaningless. However, since the last two cointegrating relations involve only two variables, we do not need the *ceteris paribus* interpretation. The second long-run relation specifies that a permanent 1 per cent increase in the overnight rate is associated with the equivalent increase in the long-run interest rate. This is consistent with a stationary term spread and the expectation hypothesis of the term structure of interest rates. The third cointegrating relation suggests that a 1 per cent permanent increase in output (or a 1 per cent increase in potential output) is associated with a permanent 1 per cent increase in the stock market. Since the ratio of the TSX to output has been hovering around a constant value for most of the past 25 years, unit coefficients in this cointegrating relationship are not surprising. Interestingly, this last relation also implies that transitory changes in real output can only lead to transitory changes in the level of the stock market. The second and third cointegration vectors are similar to those found in Cassola and Morana (2002). However, they find a Fisher relation, which was impossible to find over our sample, and their money-demand relationship includes only the level of real output, which is not standard.

The economy is in a long-run equilibrium when the three cointegrating relationships are respected; that is, when there is no persistent gap between money, output, inflation, and the overnight rate (or no money gap), the overnight rate is equal to the long-term rate up to the impact of transitory shocks to both variables and an unidentified constant (no interest rate gap), and the stock market level deviates only temporarily from potential output (no stock market gap).[10] In other words, because the three relationships may be respected but the economy is still affected by *transitory* shocks, a long-run equilibrium is attained only when the *permanent* components of the variables respect the three cointegration vectors.

Appendix C provides a detailed analysis of the stability of the model, serial correlation, and normality tests, and an evaluation of the forecasting performance of BEAM in terms of point forecast, conditional density forecast, and probability forecast. Because normality of the residuals is rejected, bootstrap methods are used to obtain the confidence bands around the impulse-response functions presented in the following section.

10. Notice that there are constant terms in the three cointegration vectors.

4. Shock Analysis

The impact of a change in U.S. industrial production

The response functions to a permanent 1 per cent increase in U.S. industrial production are shown in Figure 1. Small inflation pressures are generated as output is boosted by almost 0.2 per cent on impact. Interest rates are increased by around 25 basis points to keep demand in line with short-run supply. The Canadian stock market is temporarily hurt by the higher interest rate. It nevertheless increases by 0.12 per cent in the long run, in line with the permanent increase in output. Broad aggregate money is negatively affected in the short run by the slight increases in inflation and real interest rates. Only output is significantly affected in the long run.

Identification of the permanent shocks

Given the presence of three cointegration vectors and six endogenous variables, there are three stochastic trends or permanent shocks to be identified. Appendix A shows that King et al.'s (1991) identification methodology can be used, provided the exogenous variable does not cointegrate with the endogenous variables. The first permanent shock, ε_{π_t}, labelled an inflation shock, is the only shock that has a permanent impact on inflation. According to the "monetarist" view, the long-run money growth and inflation rate are ultimately set exogenously by monetary authorities. The inflation shock therefore relates to central bank monetary policy. A positive inflation shock reflects the central bank's decision to permanently increase the inflation rate. Hence, the structural inflation shock is identified by assuming that the long-run system has the following recursive structure:

$$\lim_{s \to \infty} \begin{bmatrix} inf_{t+s} \\ y_{t+s} \\ onr_{t+s} \\ m_{t+S} \\ sm_{t+S} \\ lr_{t+S} \end{bmatrix} = \begin{bmatrix} \tau_{11} & 0 & 0 \\ \tau_{21} & \tau_{22} & 0 \\ \tau_{31} & \tau_{32} & \tau_{33} \\ \tau_{41} & \tau_{42} & \tau_{43} \\ \tau_{51} & \tau_{52} & \tau_{53} \\ \tau_{61} & \tau_{62} & \tau_{63} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{\pi t} \\ \varepsilon_{yt} \\ \varepsilon_{dt} \end{bmatrix}$$

Note that τ_{ij} is the long-run response of the ith endogenous variable to the jth element in the vector of structural disturbances, ε_t. The restrictions $\tau_{12} = 0$ and $\tau_{13} = 0$ mean that only an inflation shock, ε_{π_t}, affects the long-run level of inflation. The mainstream view would predict that the decision to change inflation permanently has no permanent impact on real variables and therefore that $\begin{bmatrix} \tau_{21} & \tau_{41} & \tau_{51} \end{bmatrix} = 0$. However, economic theory provides no clear-cut predictions on that question. In several theoretical models, the superneutrality result due to Sidrausky (1967) breaks down, since inflation can have either positive or negative effects on real variables such as consumption and investment, depending on the exact assumptions concerning preferences. Additionally, in these models the real interest rate may or may not be independent of inflation in the long run. Some recent empirical results (see, for example, Rapach 2003; Gauthier and Pelgrin 2003) find support for the Mundell-Tobin effect, suggesting that an unexpected increase in inflation has a permanent negative impact on the real interest rate. We let the data talk on this point by leaving unconstrained the parameters in $\begin{bmatrix} \tau_{21} & \tau_{31} & \tau_{41} & \tau_{51} & \tau_{61} \end{bmatrix}$.

Most theoretical models define supply shocks as being governed by technology innovations that determine the technical capacity of the economy. We thus define a supply shock as a shock allowed to have a permanent effect on output but not on inflation. The long-run effects on all the other real variables are left unconstrained. Notice that all shocks are allowed to impact all the variables in the short run. In particular, a supply shock is expected to decrease inflation in the short run.

The third structural shock is a shock that has no permanent impact either on output or on inflation. This shock is labelled a demand shock. Our interpretation of disturbances with permanent effects as supply disturbances, and of disturbances with transitory effects as demand disturbances, is motivated by a traditional Keynesian view of fluctuations (see Blanchard and Quah 1989 for a simple model that delivers those implications).

The inflation shock

A positive inflation shock reflects the central bank's decision to permanently increase the inflation rate.[11] Given the instrument used by the central bank, this can be achieved only by decreasing the overnight rate. Figure 2 shows that our results are consistent with this view. To achieve a typical unexpected inflation increase of around 0.3 per cent in the long run, the central bank has to decrease the overnight rate by about 25 basis points. Given the expectations hypothesis of the term structure in our core model, the long rate is persistently depressed as well. The bank's intervention leads to a small output stimulus in the short run. The shock also hurts the stock market significantly and decreases *real* broad aggregate money in the short run.

The permanent significant negative effect of inflation on interest rates may be explained through the Mundell effect: an *unexpected* increase in inflation decreases real wealth, which increases savings. Real interest rates must then fall to restore good market equilibrium. Our results are in line with the need to increase the interest rate persistently in disinflation periods and in the first

11. Such a shock can always be reversed by a negative inflation shock of the same size, if the central bank decides to do so.

years of inflation targeting, in order to gain credibility. Rapach (2003) also finds that an unexpected permanent increase in inflation is associated with permanently lower long-run real interest rates in every industrialized country of a sample of 14, including Canada, Germany, France, and Italy.[12]

The supply shock

The typical supply shock increases the productive capacity of the economy by around 0.9 per cent in the long run. Inflation is pushed downward in the short run as production costs are decreased (Figure 3), but goes back to its initial level in the long run. The central bank has, over the sample, accommodated the shock by decreasing interest rates to eliminate the excess supply in the good market and bring inflation back to target.[13] Interestingly, interest rates are not affected in the long run. This is consistent with Ramsey's model, in which the interest rate is determined by the rate of time preferences and technology determines the level of capital such that the marginal product of capital is equal to the interest rate.

The stock market leads output and overshoots somewhat. Broad money is higher in the short run because of the accommodative stance of monetary policy, and remains higher in the long run because of both higher money demand for transaction purposes and the higher real value of the stock market. These results are similar to Cassola and Morana (2002), except that in their model output decreases in the short run, which is kind of a puzzle.

A demand shock[14]

The demand shock increases inflation, output, and the stock market in the short run (Figure 4). Short and long interest rates increase in the short run, as expected. This can be seen as the result of a standard textbook open market operation with a disinflationary objective. When inflation and output turn out to be higher than expected, an inflation-targeting central bank has to increase interest rates. It is interesting to notice that, since a demand shock has no permanent impact on output, the important stock market surge in the first months following the shock slowly dissipates as investors realize that higher profits cannot be sustained without a permanent increase in productivity.

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12. Notice that a permanent inflation shock represents an *unexpected* persistent deviation of inflation from its deterministic trend. This source of increase in inflation is associated in the long run with a decrease in interest rates. That, of course, does not mean that *expected* changes in inflation have the same effect on interest rates.
13. In some stochastic DGE (SDGE) models with adjustment costs on capital (see Neiss and Nelson (2001, 23), for example), productivity shocks would decrease the *neutral* rate in the short run. This provides further incentives to decrease the actual interest rate after a productivity shock.
14. Other demand shocks having only transitory effects may also be identified.

The permanent positive impact on the overnight rate implies that the so-called demand shock induces, on average, a higher equilibrium interest rate. According to Ramsey's model, this would correspond to a rate of time preference shock. King et al. (1991) estimate a significant cointegrating relationship that links negatively the ratio of investment over output and the real interest rate in the United States, and they identify what they call a "real interest rate shock" with long-run properties very similar to our "demand" shock. They also identify what they call a "balanced-growth" shock, which is very similar to our supply shock, increasing output permanently while leaving the ratios of investment and consumption over output and the real interest rate and level of inflation unchanged in the long run. For example, a fiscal shock that crowds out investment persistently would be associated with persistently higher interest rates.

5. BEAM's Proposed Reaction Function

When inflation is forecast to deviate permanently from the target, the central bank's reaction must differ from the historical estimated reaction function (the equation for the overnight rate) in order to prevent the unwanted deviation. Only permanent shocks to inflation can reverse a permanent deviation from target. We thus simply propose to simulate the impact of the necessary permanent inflation shock on the overnight rate and adjust the future path of the overnight rate accordingly. For example, if the difference between the long-run forecast of inflation and the target is 1 per cent, we know from the long-run matrix in Table 3 that an inflation shock of size -(1/0.32) times the typical inflation shock will bring inflation back to the target. We also know the overnight rate's response to such a shock, so we can adjust the forecast reaction function accordingly.

Table 3: Long-Run Impact of Permanent Shocks ($\Gamma(1)$)

	ε_{π}	ε_{y}	ε_{d}
inf	0.32	0	0
y	-0.05	0.89	0
onr	-0.24	0.01	0.25
m	-0.26	1.03	-0.61
sm	-0.05	0.89	0
lr	-0.24	0.01	0.25

Preliminary simulation exercises suggest that such a reaction function would have led to interest rate recommendations close to what the Bank of Canada chose to do over the sample.

6. Conclusion

We have estimated a small monthly VECM to study the interactions between the real and financial sectors of the Canadian economy. To take into account the high degree of economic integration between Canada and the United States, the U.S. industrial production index has been included as an exogenous variable. Identification of permanent shocks in a VECM with exogenous variables represents a technical contribution to the literature.

Our principal contributions are: (i) the identification of a long-run relation between the stock market and real output, which allows the identification of a supply shock as the only shock that permanently affects the stock market, and a demand shock that leads to important transitory stock market overvaluation; and (ii) a demonstration of the conditions under which permanent shocks can be identified in a VECM with exogenous variables.

An important remaining question is the impact on BEAM's reaction function of assuming that inflation is non-stationary in the actual inflation-targeting environment, which has rendered inflation at least more stable. Since BEAM's proposed reaction function is based on an average degree of persistence of inflation and an average level of credibility of the Bank of Canada over the sample, it should be seen as being more aggressive than what is probably needed in the actual environment.

The model could possibly be used to build a financial conditions index for Canada using the stock market and money gaps from the core model, together with the deviation of the actual real interest rate from the neutral interest rate recommended by the proposed reaction function. This index could eventually be completed with the deviation of the Canadian exchange rate from equilibrium, provided in Gauthier and Tessier (2002), and tested against those proposed in Gauthier, Graham, and Liu (2004). This is left for future research.

References

Angeloni, I., G. Coenen, and F. Smets. 2003, “Persistence, the Transmission Mechanism and Robust Monetary Policy. European Central Bank Working Paper No. 250.

Atkeson, A. and L. Ohanian. 2001. “Are Phillips Curves Useful for Forecasting Inflation?” *Federal Reserve Bank of Minneapolis Quarterly Review* 25(1): 2–11.

Blanchard, O.J. 1981. “Output, the Stock Market, and Interest Rates.” *American Economic Review* 71: 132–43.

Blanchard, O.J. and D. Quah. 1989. “The Dynamic Effects of Aggregate Demand and Supply Disturbances.” *American Economic Review* 79: 1146–64.

Cassola, N. and C. Morana. 2002. “Monetary Policy and the Stock Market in the Euro Area.” European Central Bank Working Paper No. 119.

Coenen, G. 2002. “Inflation Persistence and Robust Monetary Policy Design.” European Central Bank. Photocopy.

Cogley, T. and T. Sargent. 2001. “Evolving Post-World War II Inflation Dynamics.” *NBER Macroeconomics Annual* 16: 331–73.

Crowder, J., D.L. Hoffman, and R.H. Rasche. 1999. “Identification, Long-Run Relations, and Fundamental Innovations in a Simple Cointegrated System.” *Review of Economics and Statistics* 81: 109–21.

Dhar, S., D. Pain, and R. Thomas. 2000. “A Small Structural Empirical Model of the UK Monetary Transmission Mechanism.” Bank of England Working Paper No. 113.

Dupuis, D. and D. Tessier. 2003. “The U.S. Stock Market and Fundamentals: An Historical Decomposition.” Bank of Canada Working Paper No. 2003-20

Garratt, A., K. Lee, M. H. Pesaran, and Y. Shin. 2003. “A Long-Run Structural Macroeconometric Model of the U.K.” *Economic Journal* 113: 412–55.

Gauthier, C., C. Graham, and Y. Liu. 2004. “Financial Conditions Indexes for Canada.” Bank of Canada Working Paper No. 2004-22.

Gauthier, C. and F. Pelgrin. 2003. “The Long-Run Relationship Between Inflation and Real Stock Prices: Is There Neutrality?” Bank of Canada. Photocopy.

Gauthier, C. and D. Tessier. 2002. “Supply Shocks and Real Exchange Rate Dynamics: Canadian Evidence.” Bank of Canada Working Paper No. 2002-31.

Hendry, D.F. and K. Juselius. 2000. “Explaining Cointegration Analysis: Part II.” University of Copenhagen. Department of Economics. Discussion Paper 00–20.

Jacobson T., P. Jansson, A. Vredin, and A. Warne. 2001. “Monetary Policy Analysis and Inflation Targeting in a Small Open Economy: A VAR Approach.” *Journal of Applied Econometrics* 16: 487–520.

Johansen, S. 1992. “Cointegration in Partial Systems and the Efficiency of Single-Equation Analysis.” *Journal of Econometrics* 52: 389–402.

Kilian, L. and M.P. Taylor. 2001. “Why Is It So Difficult to Beat the Random Walk Forecast of Exchange Rates?” University of Michigan. European Central Bank. Photocopy.

King, R.G., C.I. Plosser, J.H. Stock, and M.W. Watson. 1991. “Stochastic Trends and Economic Fluctuations.” *American Economic Review* 81: 819–40.

Longworth, D. 2003. *Money in the Bank (of Canada).* Technical Report No. 93. Ottawa: Bank of Canada.

Lutkepohl, H. 1991. *Introduction to Multiple Time Series Analysis*. Berlin: Springer-Verlag.

———. 1994. “Interpretations of Cointegrating Relations.” *Econometric Reviews* 13: 391–4.

Lutkepohl, H. and H.-E. Reimers. 1992. “Impulse Response Analysis of Cointegrated System.” *Journal of Economic Dynamics and Control* 12: 53–78.

Neiss, K.S. and E. Nelson. 2001. “The Real Interest Rate Gap as an Inflation Indicator.” Bank of England Working Paper No. 130.

Pesaran, M.H., Y. Shin, and R.J. Smith. 2000. “Structural Analysis of Vector Error Correction Models with Exogenous I(1) Variables.” *Journal of Econometrics* 97: 293–343.

Ploberger, W., W. Krämer, and K. Kontrus. 1989. “A New Test for Structural Stability in the Linear Regression Model.” *Journal of Econometrics* 40: 307–18.

Rapach, D.E. 2003. “International Evidence on the Long-Run Impact of Inflation.” *Journal of Money, Credit and Banking* 35: 23–48.

Selody, J. 2001. “Principles for Building Models of the Monetary Policy Transmission Mechanism.” Bank of Canada. Photocopy.

Sidrausky, M. 1967. “Inflation and Economic Growth.” *Journal of Political Economy* 75: 796–810.

Stock, J. 2002. “Comment on ‘Evolving Post-World War II U.S. Inflation Dynamics’ by T. Cogley and T.J. Sargent.” *NBER Macroeconomics Annual 2001*.

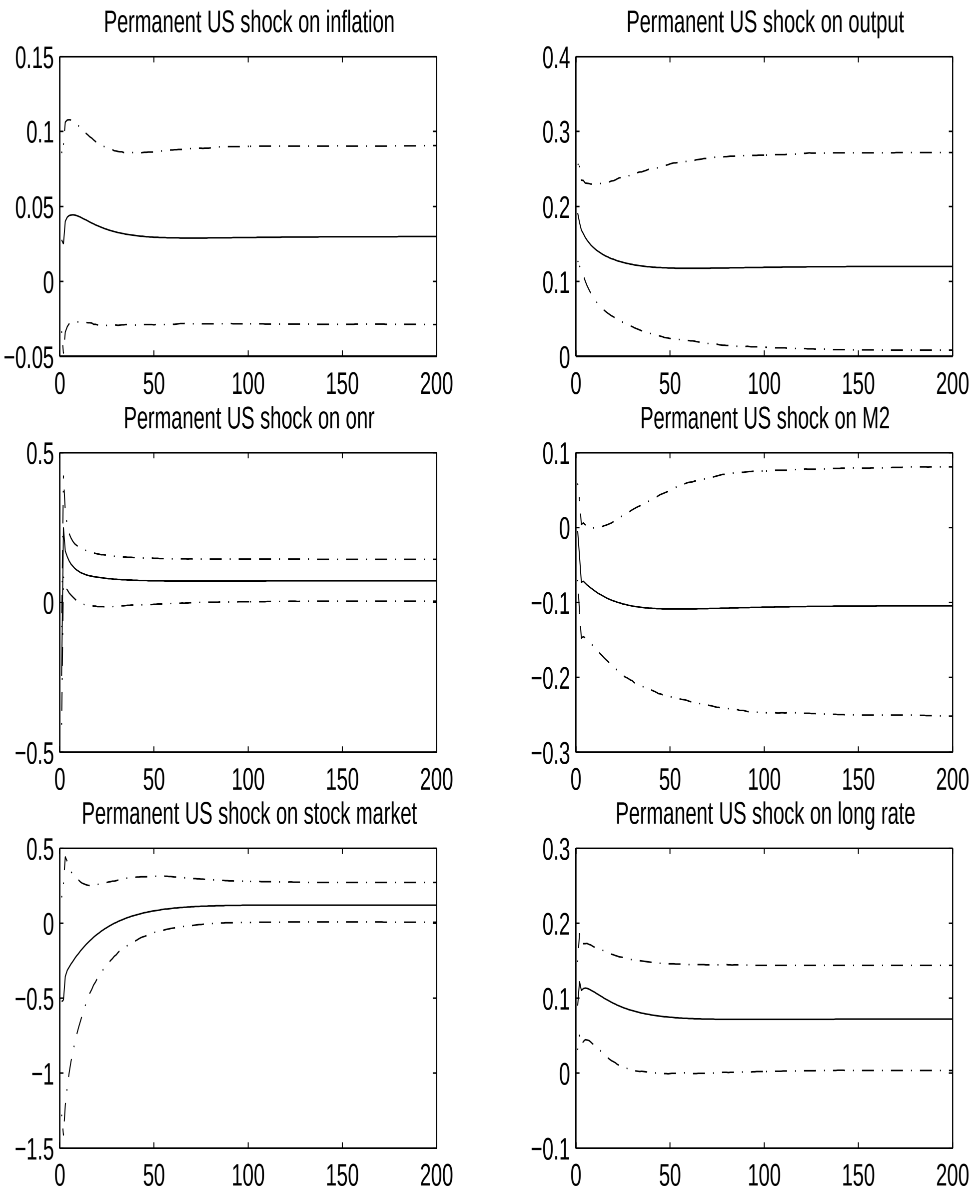
White, H. 2000. “A Reality Check for Data Snooping.” *Econometrica* 68: 1097–1126.

Wickens, M.R. 1996. “Interpreting Cointegrated Vectors and Common Stochastic Trends.” *Journal of Econometrics* 74: 255–71.

Wickens, M.R. and R. Motto. 2001. “Estimating Shocks and Impulse Response Functions.” *Journal of Applied Econometrics* 16: 371–87.

Yang, M. 1998. “On Identifying Permanent and Transitory Shocks in VAR Models.” *Economics Letters* 58: 171–75.

Figure 1. Responses to a permanent increase in U.S. industrial production[1]



1. The confidence bands are calculated by the nonparametric bootstrap method.

Figure 2. Impulse responses to an inflation shock

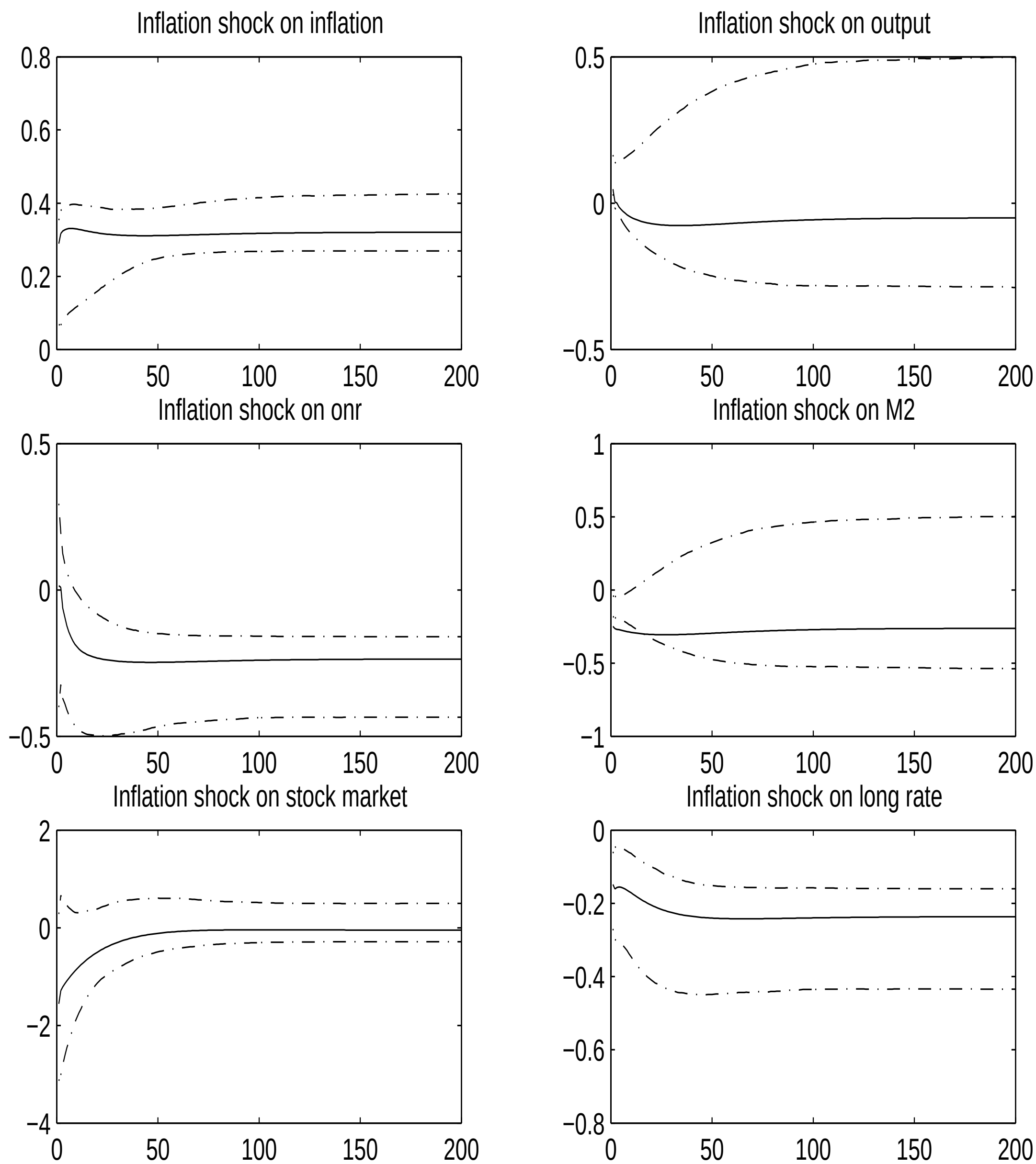


Figure 3. Impulse responses to a supply shock

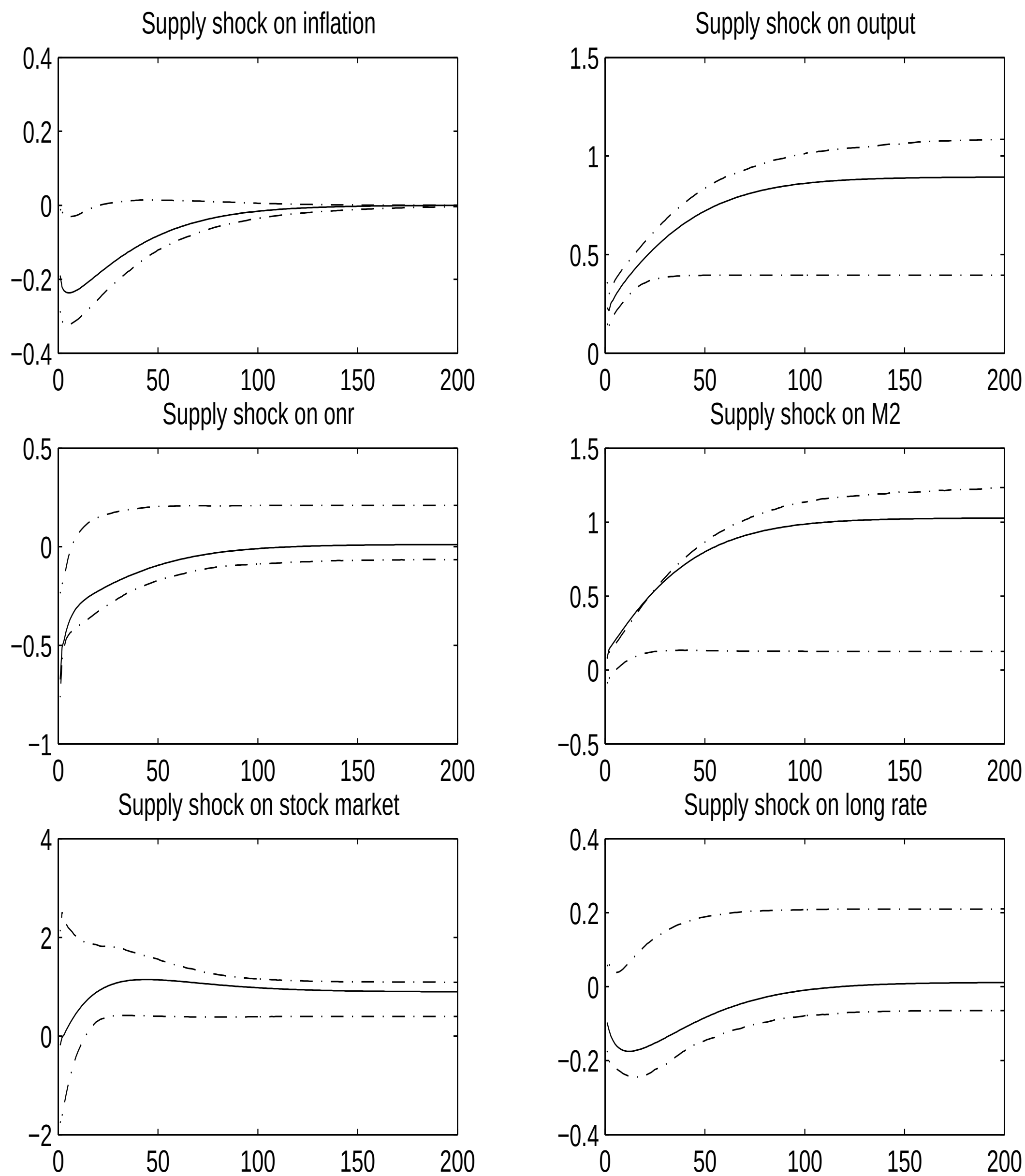
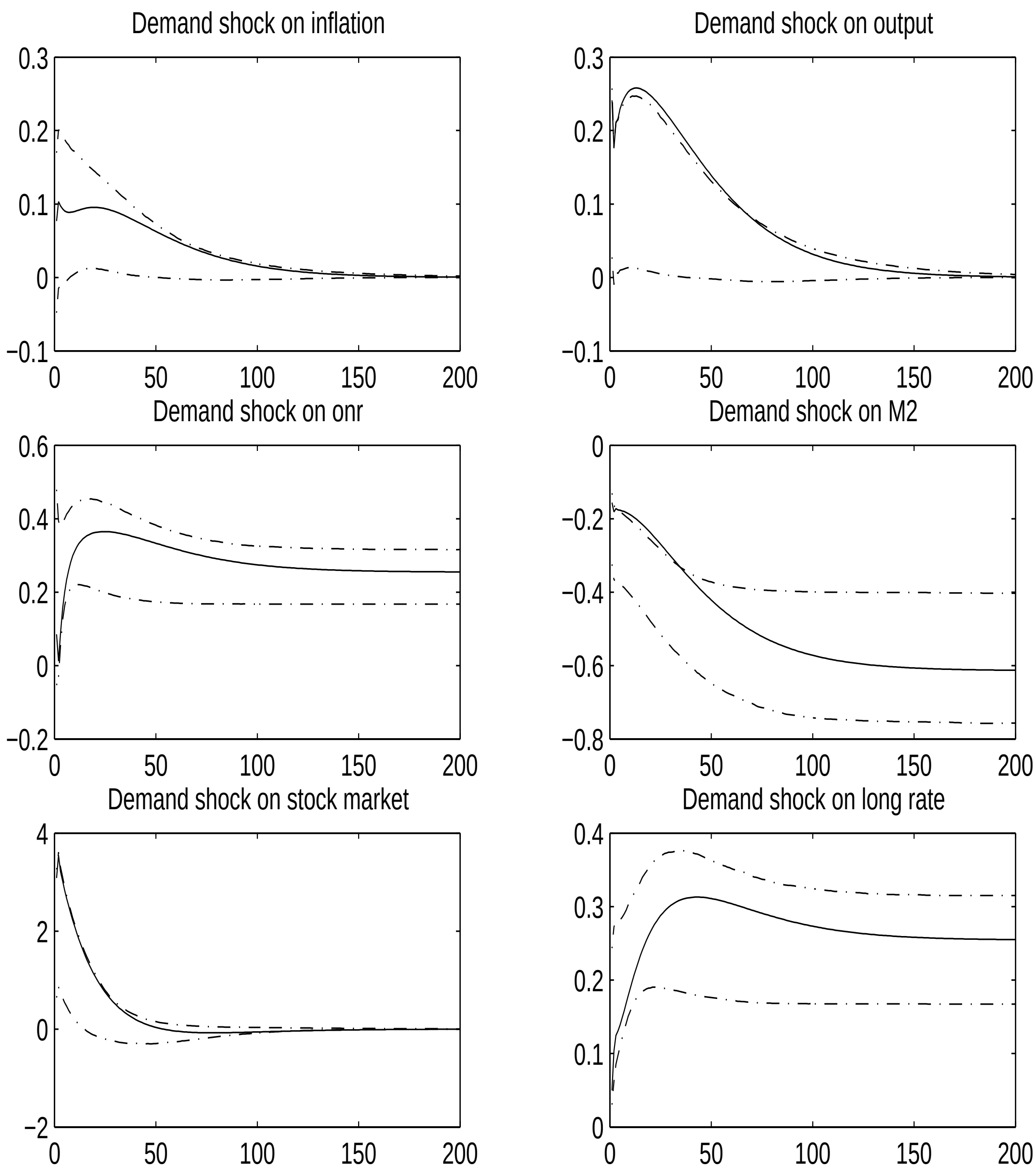


Figure 4. Impulse responses to a demand shock



Appendix A: Identification of Permanent Shocks in a Model with Exogenous Variables

We show that the identification procedure proposed in King et al. (1991) can be generalized to the case of a model with weakly exogenous I(1) variables, provided the exogenous variables do not cointegrate with the endogenous variables. Given the assumption of weak exogeneity, a partial model is efficiently estimated. A simple way to invert such a VECM estimated as a partial model is suggested in Appendix B.

A.1 Efficient estimation of a VECM with weakly exogenous variables

Economic systems often have so many potentially useful variables that the system gets extremely large. Johansen (1992) shows, however, that a partial model can be efficiently estimated when some of the variables are weakly exogenous. Consider an m-dimensional VAR(p) process $\{z_t\}_{t=1}^{\infty}$ expressed as the VECM:

$$\Delta z_t = a + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-1} + e_t, \qquad t = 1, 2, \ldots \tag{A1}$$

where $\Delta = 1 - L$ with L being the lag operator, the long-run multiplier Π and the short-run response matrices Γ_i are $m \times m$ constant coefficient matrices, a is a constant vector, and the m-dimensional disturbance $e_t \sim IN(0, \Omega)$.

We next partition the m-vector of random variables z_t into the n-vector y_t and the k-vector x_t, where $k \equiv m - n$; that is, $z_t = (y_t', x_t')'$, $t = 1, 2, \ldots$. By partitioning the error term e_t comformably with $z_t = (y_t', x_t')'$ as $e_t = (e'_{yt}, e'_{xt})'$ and its variance matrix as

$$\Omega = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{bmatrix},$$

we are able to express e_{yt} conditionally in terms of e_{xt} as

$$e_{yt} = \Omega_{yx}\Omega_{xx}^{-1} e_{xt} + u_t, \tag{A2}$$

where $u_t \sim IN(0, \Omega_{uu})$, $\Omega_{uu} \equiv \Omega_{yy} - \Omega_{yx}\Omega_{xx}^{-1}\Omega_{xy}$, and u_t is independent of e_{xt}. We also use a similar partitioning of the parameter vectors and matrices $a = (a'_y, a'_x)'$, $\Pi = (\Pi'_y, \Pi'_x)'$ and $\Gamma_i = (\Gamma'_{yi}, \Gamma'_{xi})'$, $i = 1, \ldots, p-1$. Following Johansen (1992), we make the following assumption:

Assumption 2.1. $\Pi_x = 0$.

Under Assumption 2.1 (i.e., the process $\{x_t\}_{t=1}^{\infty}$ is weakly exogenous with respect to the matrix of the long-run multiplier Π), the following conditional model in terms of z_{t-1}, Δx_t, Δz_{t-1}, Δz_{t-2}, $\ldots$ is efficiently estimated by maximum likelihood without using the equations for $\{x_t\}_{t=1}^{\infty}$:

$$\Delta y_t = c + \Lambda \Delta x_t + \sum_{i=1}^{p-1} \psi_i \Delta z_{t-i} + \Pi_y z_{t-1} + u_t, \qquad t = 1, 2, \ldots \tag{A3}$$

where $c \equiv a_y - \Omega_{yx}\Omega_{xx}^{-1}a_x$, $\Lambda \equiv \Omega_{yx}\Omega_{xx}^{-1}$, and $\psi_i \equiv \Gamma_{yi} - \Omega_{yx}\Omega_{xx}^{-1}\Gamma_{xi}$, $i = 1, \ldots, p-1$.

A.2 Identification of the permanent shocks

The identifying procedure documented in King et al. (1991) is based on the infinite moving average (MA) form obtained by inverting the estimated VECM. This inversion cannot be made directly, because of the presence of cointegration. An easier way to invert a VECM than those commonly suggested in the literature (see Yang 1998, for example) is proposed in Appendix B. The inverted reduced-form model obtained is:

$$\Delta y_t = \mu + C_x(L)\Delta x_t + C(L)u_t, \tag{A4}$$

where all the parameters are defined in Section A.1. Notice that, since u_t is independent of e_{xt}, u_t is independent of Δx_t.

Consider a structural model of the form:

$$\Delta y_t = \mu + C_x(L)\Delta x_t + \Gamma(L)\eta_t, \tag{A5}$$

where $\eta_t \sim IN(0, \Omega_\eta)$ is an $n \times 1$ vector of serially uncorrelated disturbances independent of Δx_t (being a linear combination of u_t), and where the endogenous variables' response to a change in the exogenous variables is given by $C_x(L)$.

The identifying problem consists in identifying the individual components in η_t from the estimated reduced-form model given by (A4), and can be described as follows. There are $s = n - r$ identifiable common stochastic trends driving the $n \times 1$ vector y_t where

$r = Rank[\Pi_y]$.[15] We express $\Pi_y = \alpha_y\beta'$, where the $n \times r$ loading matrix α_y and the $m \times r$ matrix of the cointegration vector β are each full column rank and identified up to an arbitrary $r \times r$ non-singular matrix.[16] Partition β comformably with z_t as $\beta = (\beta_y', \beta_x')'$, where β_y and β_x are, respectively, $n \times r$ and $k \times r$, and partition the vector of structural disturbances η_t into two components, $(\eta_t^{1'}, \eta_t^{2'})'$, where η_t^1 contains the s disturbances that have permanent effects on the components of y_t and η_t^2 contains $n - s$ elements that have only temporary effects.

Partition the matrix of long-run multipliers, $\Gamma(1)$, comformably with η_t as $\Gamma(1) = [\Theta, 0]$, where Θ is the $n \times s$ matrix of the long-run multipliers of η_t^1 and 0 is an $n \times (n-s)$ matrix of zeros corresponding to the long-run multipliers of η_t^2.

Assumption 3.1. $\beta_x' = 0$

Under Assumption 3.1, $\beta' z_t$ being stationary implies that $\beta'_y y_t$ is stationary, which implies that $\beta_y'\Gamma(1) = 0$. Hence the matrix of long-run multipliers is determined by the condition that its columns are orthogonal to β_y', and $\Theta\eta_t^1$ represents the innovations in the long-run components of y_t. While the cointegration restrictions identify the permanent innovations $\Theta\eta_t^1$, they fail to identify η_t^1, because $\Theta\eta_t^1 = (\Theta P)(P^{-1}\eta_t^1)$ for any non-singular matrix P. To identify the individual elements of η_t^1, we need the following identifying restrictions:

Assumption 3.2. $u_t = \Gamma_0\eta_t$ where Γ_0^{-1} exists.

Under assumption 3.2, the structural disturbances are in the space spanned by the current and lagged values of z_t, and there are no singularities in the structural model.

Assumption 3.3. Θ is assumed to be triangular, which permits us to write $\Gamma(1) = [\tilde{\Theta}\Pi, 0]$, where $\tilde{\Theta}$ is an $n \times s$ matrix with no unknown parameters, the columns of which are orthogonal to β_y', and Π is an $s \times s$ lower triangular matrix with full rank and 1's on the diagonal.[17]

The covariance matrix of the structural disturbances is partitioned comformably with $\eta_t = (\eta_t^{1'}, \eta_t^{2'})'$ and is assumed to be

Assumption 3.4. $\Omega_\eta = \begin{bmatrix} \Omega_{\eta^1} & 0 \\ 0 & \Omega_{\eta^2} \end{bmatrix}$ where Ω_{η^1} is diagonal.

15. We implicitly make the assumption that s is strictly positive. Wickens (1996) shows that if $rank(\Pi) = n$, then the full model has to be estimated and the common stochastic trends can be equated with the non-stationary component of the exogenous variables.
16. That is, $(\alpha_y K^{-1})(K\beta') = (\alpha_y\beta')$ for any (r,r) non-singular matrix K.
17. The diagonal elements of Π are normalized to unity without loss of generality, since the variances of η_t^1 are unrestricted.

That is, the permanent shocks, η_t^1, are assumed to be uncorrelated with the transitory shocks, η_t^2, and the permanent shocks are assumed to be mutually uncorrelated.

The permanent innovations, η_t^1, can be determined from the reduced form (A4) as follows. From equations (A4) and (A5) and Assumption 3.2, $C(L) = \Gamma(L)\Gamma_0^{-1}$ and $C(1) = \Gamma(1)\Gamma_0^{-1}$. Let D be any solution of $C(1) = \tilde{\Theta}D$. Thus, $\tilde{\Theta}Du_t = \tilde{\Theta}\Pi\eta_t^1$ and $D\Omega_u D' = \Pi\Omega_{\eta^1}\Pi'$. Let $\bar{\Pi} = chol(D\Omega_u D') = \Pi\Omega_{\eta^1}^{1/2}$. Since Π is a triangular matrix, and Ω_{η^1} is diagonal, there is a unique solution for Π and Ω_{η^1}. We can thus identify the permanent shocks $\eta_t^1 = \Pi^{-1}Du_t$. Defining $G = \Pi^{-1}D$, it is then easy to show that the dynamic multipliers associated with η_t^1 are $C(L)\Omega_u G'\Omega_{\eta^1}^{-1}$.

Appendix B: A Simple Way to Invert a VECM with Exogenous Variables

The identifying procedure documented in King et al. (1991) is based on the infinite moving average (MA) form obtained by inverting the estimated VECM. This inversion cannot be made directly because of the presence of cointegration. In this section, we propose an easier way to invert a VECM than those commonly suggested in the literature (see Yang 1998, for example).

By partitioning Π_y and Ψ_i comformably with $z_t = (y_t', x_t')'$ as $\Pi_y = (\Pi_y^{y}{}', \Pi_y^{x}{}')'$ and $\Psi_i = (\psi_i^{y}{}', \psi_i^{x}{}')'$, where Π_y^y and ψ_i^y are $n \times n$ and Π_y^x and ψ_i^x are $n \times k$ constant coefficient matrices, we can rewrite (A3) as:

$$y_t = c + B_0 x_t + \sum_{i=1}^{p} A_i y_{t-i} + \sum_{i=1}^{p} B_i x_{t-i} + u_t, \tag{B1}$$

where $B_0 = \Lambda$, $B_1 = -(\Lambda - \Pi_y^x - \psi_1^y)$, $B_i = (\psi_i^x - \psi_{i-1}^x)$ for $i = 2, \ldots, p-1$, $B_p = -\psi_{p-1}^x$, $A_1 = (\psi_1^y + \Pi^y + I_n)$, $A_i = (\psi_i^y - \psi_{i-1}^y)$ for $i = 2, \ldots, p-1$ and $A_p = -\psi_{p-1}^y$.

We then write (A4) as the following VARX(1):

$$\mathbf{y}_t = C + A\mathbf{y}_{t-1} + B\mathbf{x}_t + U_t, \tag{B2}$$

where $\mathbf{y}_t \equiv (y_t', y'_{t-1}, \ldots, y'_{t-p+1}, x_t', x'_{t-1}, \ldots, x'_{t-p+1})'$, $U_t \equiv (u_t', 0, 0, \ldots, 0)'$, and $C \equiv (c', 0, 0, \ldots, 0)'$ are $mp \times 1$ matrices. Matrices A and B, respectively, of dimensions $mp \times mp$ and $mp \times k$ are defined accordingly to Y and x following Luktepohl (1991, 335). Assuming that the process starts at a finite time $t = 0$, it is straightforward to obtain the inverted form[18]:

$$\mathbf{y}_t = A^t \mathbf{y}_0 + \sum_{i=0}^{t-1} A^i C + \sum_{i=0}^{t-1} A^i B x_{t-i} + \sum_{i=0}^{t-1} A^i U_{t-i}. \tag{B3}$$

18. In this unstable system, a one-time impulse may have a permanent effect, in the sense that it shifts the system to a new equilibrium, but the impulse responses may be calculated just as in the stable case. See Lutkepohl and Reimers (1992) for further details.

Taking the first difference of (B3), assuming for simplicity that $U_0 = x_0 = y_0 = 0$, and extracting the endogenous variables with the appropriate $nm \times p$ matrix $J = [I_n, 0, \ldots, 0]$, we get:

$$\Delta y_t = \mu + C_x(L)\Delta x_t + C(L)u_t, \tag{B4}$$

where $\mu = JA^{t-1}c$, $C_x(L) = \sum_{i=0}^{t-1} JA^iBL^i$, $C(L) = \sum_{i=0}^{t-1} C_iL^i$,

$C_i = J(A^i - A^{i-1})J'L^i$ for $i = 1, \ldots, t-1$ and $C_0 = I_n$.

Appendix C: Specification Tests and Forecasting Performance

C.1 Testing the stability of BEAM

It is necessary to test for the structural stability parameter constancy of economic models for both forecasting and policy analysis. Parameter non-constancy may have severe consequences on inference if it is undetected. We examine the parameter stability of every equation in BEAM by using the fluctuation test detailed below.

Suppose the linear regression model is as follows:

$$Z_t = W_t'\theta_t + U_t, \tag{C1}$$

where Z_t is the dependent variable, W_t is a $K \times 1$ vector of observations on the independent variables, θ_t is a $K \times 1$ vector of unknown regression coefficients, and U_t is an unobservable disturbance term.

The null hypothesis is that $\theta_t = \theta_0$ is the same for all time periods $t = 1, \ldots, T$. The fluctuation test S^T is,

$$S^T = max_{t \in \Lambda} \frac{t\left\|(W^{(T)\prime}W^{(T)})^{1/2}(\hat{\theta}_t - \hat{\theta}_T)\right\|_\infty}{T\hat{\sigma}}, \tag{C2}$$

where Λ represents $\{K_0, K_0 + 1, \ldots, T\}$, $K_0 \geq K$, $W^{(t)} = [W_1, \ldots W_t]'$, $\|.\|$ denotes the maximum norm, $\hat{\theta}_t = (W^{(t)\prime}W^{(t)})^{-1}(W^{(t)\prime})Z_t$, and $\hat{\sigma} = [\sum(Z_t - W_t'\hat{\theta}_T)^2/(T-k)]^{1/2}$.

Asymptotic critical values for the fluctuation test are presented in Table1 in Ploberger, Krämer, and Kontrus (1989). The critical values depend on the number of coefficients in the equation. They provide the asymptotic critical values up to the number of coefficients equal to 10.

We propose a bootstrap procedure to approximate the finite sample distribution of the test statistic S^T under the null hypothesis, and call the resulting test the bootstrap test. The bootstrap procedure consists of the following steps:

Step 1. Use the original sample to compute $\hat{\theta}_T$ and the associated residuals $\{\hat{U}_{K_0}, \ldots, \hat{U}_T\}$.

Step 2. Draw $\{\hat{U}^*_{K_0}, \ldots, \hat{U}^*_T\}$ by sampling with replacement from $\{\hat{U}_{K_0}, \ldots, \hat{U}_T\}$. Then generate the bootstrap sample $\{y_t^*\}$ from the model.

Step 3. Use the bootstrap sample to compute S^T and call it S^{T*}.

Step 4. Repeat Steps 2 and 3 a number of times, say B times, and obtain the empirical distribution of $S_1^{T*}, \ldots, S_B^{T*}$. This empirical distribution is called the bootstrap distribution and is used to approximate the finite sample distribution of S^T under H_0.

Let S_α^{T*} be the α percentile of the above bootstrap distribution. We will reject the null hypothesis at significant level α if $S^T > S_\alpha^{T*}$. This test is called the bootstrap test.

We want to test for structural change that occurs during the period from 1993 January to 2002 December. There are six equations in BEAM. We perform the fluctuation test for every equation. The test is carried out under the $\alpha = 0.05$ level and $B = 100$. The test results are provided in Table C1. For all six equations in BEAM, the null hypothesis that parameters keep constant is not rejected.

Table C1:

Testing the Structural Stability in the BEAM Model			
Equations in BEAM model	Test statistic	Bootstrap critical value	Result
Inflation equation	0.4108	1.2273	Not rejected
Output equation	0.4725	1.3868	Not rejected
Overnight rate equation	0.7331	1.0968	Not rejected
Money equation	0.7020	1.0453	Not rejected
Stock price equation	0.5726	1.2133	Not rejected
Long-term interest rate equation	0.6223	1.2395	Not rejected

C.2 Normality and serial correlation tests

In many economic models, distributional assumptions play an important role in the estimation, inference, and forecasting procedures. For example, given the assumption that the error term follows the normal distribution, the confidence interval for the impulse-response function and the probability forecast can be easily built up. However, as a practical matter, in the absence of any theoretical rationale for adopting one particular specification for the distribution of the error term, each specification must be applied to the data.

Testing for normality is a common procedure in much applied work and many tests have been proposed. We use the multivariate omnibus test suggested by Doornik and Hansen (1994), which is asymptotically χ^2 with $2n$ degrees of freedom, where n is the dimension of the error term.

The test statistic, 965.2307, is above the critical value from the chi-squared distribution with 12 degrees. Therefore, the multivariate normality hypothesis of the error term in BEAM is rejected.

For a time-series model, the common problem is serial correlation of the disturbances. Testing for serial correlation has long been a standard practice in applied economic analysis, because if the disturbances are serially correlated, it can be inconsistent if the regressors contain lagged dependent variables. Moreover, the serial correlation is often an indication of omitting important explanatory variables, or of functional form misspecification. In addition, it is important to test for serial correlation because the choice of an appropriate estimation procedure for a given model crucially depends on the error structure assumed by the model.

We use the Ljung-Box test. The Ljung-Box test statistic is asymptotically χ^2 with $n^2([T/4]-1)-nr$ degrees of freedom. The test statistic, 0.0371, is below the bootstrap critical value, 0.0452, and the asymptotic critical value from the chi-squared distribution with $n^2([T/4]-1)-nr$ degrees of freedom. Hence, we cannot reject the null hypothesis that no serial correlation is based on both the bootstrap test and the asymptotic test.

C.3 Evaluating BEAM forecast performance

We have performed point forecasts, probability forecasts, and density forecasts.

C.3.1 Point forecast

We can rewrite the model as,

$$x_t = C_0 + C_1 D_t + A_1 x_{t-1} + A_2 x_{t-2} + B_0 y_t^{us} + B_1 y_{t-1}^{us} + B_2 y_{t-2}^{us} + u_{xt}\,, \tag{C3}$$

where

$$C_0 = \left(a_x^0 + \alpha_x c_0\right),\ C_1 = a_x^1,$$

$$A_1 = (I_6 + \Gamma_1^y + \Pi^y),\ A_2 = -\Gamma_1^y,$$

$$B_0 = \Gamma,\ B_1 = (\Pi^x + \Gamma_1^y - \Gamma),\ B_2 = -\Gamma_1^x.$$

Let $$A = \begin{bmatrix} A_1 & A_2 & B_1 & B_2 \\ I_6 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} B_0 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ \dots \\ 1 \end{bmatrix}, U_t = \begin{bmatrix} u_{xt} \\ 0 \\ 0 \end{bmatrix}.$$

The optimal K-step-ahead forecast of x_t at time t is:

$$\hat{x}_{t+K} = JA^K x_t + \sum_{i=0}^{K-1} JA^i C + \sum_{i=0}^{K-1} JA^i CD_{(t+K-i)} + \quad + \sum_{i=0}^{K-1} JA^i By^{us}_{(t+K-i)}, \tag{C4}$$

where $J = [I_6, 0, \dots, 0]$ and I_6 is 6×6 unit matrix.

We consider two benchmark models: a random-walk model without drift for inflation forecast, and a random-walk model with drift for forecasts of Canadian gross domestic product. These are respectively specified as,

$$\pi_{t+K} = \pi_t + \varepsilon_t, \tag{C5}$$

$$y_{t+K} = \alpha + y_t + \zeta_t, \tag{C6}$$

where both ε_t and ζ_t are identically, independently distributed (i.i.d.) error terms and α in (C6) is the drift parameter to be estimated.

The forecasts we use in our study are K-step-ahead out-of sample forecasts from January 1998 to December 2002. An expanding window is utilized, where every observation prior to time t is used. We use the observations from 1975Q1 to 1997Q12 to estimate models to obtain K-step-ahead out-of-sample forecasts. Having done this, we re-estimate the parameters of the forecasting models by adding a new observation. We then use the estimated parameters to construct a new K-step-ahead out-of-sample forecast.

The forecast performance of our model uses standard summary statistics, such as root mean squared error (RMSE), mean absolute deviation (MAD), and mean absolute percent error (MAPE), which are, respectively, defined as,

$$\text{RMSE}=\sqrt{\frac{\sum(F_t-A_t)^2}{n}}\text{ , MAE= }\frac{\sum|F_t-A_t|}{n}\text{ , MAPE= }\frac{100\sum|F_t-A_t|}{n|A_t|}\text{ ,}$$

where A_t is the actual series and F_t is the forecast series.

The forecast performances of inflation and output by using BEAM along with random-walk models are presented in Tables C2 and C3, respectively.

Table C2: Inflation Forecast

Forecast horizon		Evaluation criteria	
	RMSE	MAD	MAPE
One month	0.4211 (0.4411)	0.3295 (0.3383)	19.8295 (20.0234)
One quarter	0.7060 (0.7198)	0.5491 (0.5581)	36.5149 (36.4705)
Half year	0.9218 (0.9420)	0.6296 (0.6650)	46.9206 (45.6795)
One year	1.2551 (1.2222)	0.9703 (0.9897)	72.5206 (66.8948)
Two years	1.2551 (1.2494)	1.0538 (1.0618)	72.1616 (64.6144)

Note: Forecast results of the random-walk models are shown in parentheses.

Table C3: Output Forecast

Forecast horizon		Evaluation criteria	
	RMSE	MAD	MAPE
One month	0.3519 (0.3748)	0.2858 (0.2928)	0.02085 (0.0213)
One quarter	0.6165 (0.6793)	0.5011 (0.5818)	0.0365 (0.0424)
Half year	0.9705 (1.1470)	0.8074 (0.9791)	0.0588 (0.0713)
One year	1.6635 (2.0661)	1.3523 (1.7693)	0.0984 (0.1289)
Two years	2.6349 (3.6039)	2.3706 (3.0899)	0.1725 (0.2251)

Note: Forecast results of the random-walk models are shown in parentheses.

From Table C2 we observe that in terms of RMSE, MAD, and MAPE, neither the BEAM nor the random-walk model dominates. However, for output forecasts, Table C3 clearly shows that BEAM yields a lower RMSE, MAD, and MAPE than the random-walk model across different forecast horizons.

The literature on inflation forecasting and the exchange rate (for example, Kilian and Taylor 2001; Atkeson and Ohanian 2001) shows that the forecast performance of the random-walk model often cannot be dominated by densely parameterized models with larger information sets. One reason is that the forecast-error variance has two components: one is the residual variance of the underlying model, and the other is an estimation error associated with using estimates of model parameters rather than their true values. Adding a right-hand-side variable to a specification improves forecasts only if the reduction in the residual uncertainty outweighs the increased estimation error. Based on the reduction of the two component errors, the predictability of the model with more economic theories tends to improve relative to the random-walk model.

We perform White's (2000) test to determine whether BEAM has predictive superiority that is statistically significant over the random-walk model in terms of RMSE, MAE, and MAPE.

Let $f_{RMSE} = B_{RMSE} - R_{RMSE}$, $f_{MAE} = B_{MAD} - R_{MAD}$, and $f_{MAPE} = B_{MAPE} - R_{MAPE}$, where B_{RMSE}, R_{RMSE} represent the RMSE from the BEAM and random-walk models, respectively, and similarly for B_{MAD}, R_{MAE}, and B_{MAPE} and R_{MAPE}. We compare the predictive accuracy between the BEAM and random-walk models. We want to test,

$$H_0^{RMSE}:\ E(B_{RMSE}) = E(R_{RMSE}),\ H_1^{RMSE}(L):\ E(B_{RMSE}) = E(R_{RMSE}) \text{ or}$$
$$H_1^{RMSE}(R):\ E(B_{RMSE}) > E(R_{RMSE});$$

$$H_0^{MAE}:\ E(B_{RMSE}) = E(R_{RMSE}),\ H_1^{MAD}(L):\ E(B_{MAD}) > E(R_{MAD}) \text{ or}$$
$$H_1^{MAD}(R):\ E(B_{MAD}) > E(R_{MAD})\ ;$$

and

$$H_0^{MAPE}:\ E(B_{MAPE}) = E(R_{MAPE}),\ H_1^{MAPE}(L):\ E(B_{MAPE}) < E(R_{MAPE})) \text{ or}$$
$$H_1^{MAPE}(R):\ E(B_{MAPE}) > E(R_{MAPE}).$$

For a given forecast horizon τ, where $\tau = 1, 3, 6, 12$, and 24, suppose a forecast function is $g(.)$, which can be f_{RMSE}, f_{MAD}, and f_{MAPE}; the test statistic is,

$$\bar{g} \equiv n^{-1} \sum_{t=R}^{T} \hat{g}_{t+\tau}, \tag{C7}$$

where $\hat{g}_{t+\tau} \equiv g(\hat{\theta}_t, Z_{t+\tau})$, $\hat{\theta}_t$ is the estimator of the model parameter θ formed using observations 1 through t.

We use the critical value from the stationary bootstrap method introduced in White (2000). The number of the bootstrap repetition is 100 at the significant level $\alpha = 0.05$. The test results are reported in Tables C4 and C5.

Table C4: Reality Check for Inflation Forecast

	Forecast horizon	Test statistic	Bootstrap 1	Bootstrap 2	Test results
RMSE	One month	0.1288	-0.1080	0.1173	Superiority
	One quarter	0.1364	-0.3489	0.2851	Equivalent
	Half year	0.2838	-0.5416	0.5950	Equivalent
	One year	-0.4633	-1.7101	1.1220	Equivalent
	Two years	0.3674	-2.8960	2.2676	Equivalent
MAD	One month	0.0683	-0.1217	0.0957	Equivalent
	One quarter	0.0753	-0.2860	0.2236	Equivalent
	Half year	0.2592	-0.3720	0.4671	Equivalent
	One year	0.2277	-0.7922	0.5003	Equivalent
	Two years	0.1939	-0.9220	0.8809	Equivalent
MAPE	One month	1.4057	-10.5028	9.3578	Equivalent
	One quarter	0.0753	-0.2860	0.2236	Equivalent
	Half year	-9.5284	-28.6313	29.5758	Equivalent
	One year	-37.1558	-49.7460	33.5703	Equivalent
	Two years	-48.0799	-66.4429	53.8402	Equivalent

Table C5: Reality Check for Inflation Forecast

	Forecast horizon	Test statistic	Bootstrap 1	Bootstrap 2	Test results
RMSE	One month	0.0938	-0.2206	0.2869	Equivalent
	One quarter	0.4749	-0.5277	0.4856	Equivalent
	Half year	2.3655	-2.0409	1.2250	Superiority
	One year	8.9174	-4.4019	3.0227	Superiority
	Two years	30.9962	-12.5977	9.2752	Superiority
MAD	One month	0.0414	-0.2960	0.2805	Equivalent
	One quarter	0.5151	-0.4852	0.4322	Superiority
	Half year	0.9512	-1.0992	0.7615	Superiority
	One year	2.8872	-1.7538	0.9875	Superiority
	Two years	5.4588	-2.4913	1.8029	Superiority
MAPE	One month	0.0030	-0.0216	0.0205	Equivalent
	One quarter	0.0375	-0.0353	0.0316	Superiority
	Half year	0.0691	-0.0805	0.0554	Superiority
	One year	0.2108	-0.1282	0.0520	Superiority
	Two years	0.3996	-0.1815	0.1314	Superiority

From Table C4, we find that BEAM forecast performances of inflation are not statistically significant from those for the random-walk model. Regarding the forecast performance for output, we find that BEAM dominates statistically the random-walk model.

C.3.2 Conditional density forecast

A conditional density forecast of a random variable at some future time given conditional variables is an estimate of the conditional probability distribution of the possible future values of that variable. It thus provides a complete description of the uncertainty associated with a forecast, and stands in contrast to a point forecast, which by itself contains no description of the associated uncertainty.

Below we describe our approach to evaluate the conditional distributional function from the six equations in BEAM.

Let $-\alpha_x\beta' = [\Pi_1, \ldots, \Pi_7]'$,

$$V_{t-1} = [\pi_{t-1}, y_{t-1}, r_{t-1}, m_{t-1}, s_{t-1}, lr_{t-1}, y^{us}_{t-1}]',$$

$$\Delta V_{t-1} = [\Delta\pi_{t-1}, \Delta y_{t-1}, \Delta r_{t-1}, \Delta m_{t-1}, \Delta s_{t-1}, \Delta lr_t, \Delta y^{us}_{t-1}]',$$

$$a^0_x + a_x c_0 = [\Gamma_{01}\ldots, \Gamma_{06}]', \Gamma = [\bar{\Gamma}_1\ldots, \bar{\Gamma}_6]', \Gamma_1 = [\Gamma_{11}\ldots, \Gamma_{17}]'.$$

Under the assumption that the error is from a normal distribution, the model-implied conditional distribution function of $\Delta\zeta_{t-1}$, given V_{t-1}, ΔV_{t-1}, and Δy^{us}_{t-1}, is

$$F_0(\Delta\zeta^i_{t-1} | V_{t-1}, \Delta V_{t-1}, \Delta Y^{us}_{t-1}) = \frac{1}{\sigma^i}\int_{-\infty}^{u} \phi[(v-u_i)/\sigma]dv,$$ where $\Delta\zeta^1_{t-1}$, $\Delta\zeta^2_{t-1}\ldots$, and $\Delta\zeta^6_{t-1}$, respectively, denote $\Delta\pi_{t-1}$, Δy_{t-1}, Δr_{t-1}, Δm_{t-1}, Δs_{t-1}, and Δlr_{t-1}, $\sigma^1, \ldots, \sigma^6$ denote, respectively, the standard error in the corresponding inflation equation, output equation, short-term interest rate equation, money equation, stock price equation, and long-term interest rate equation. Furthermore, $u_i = \Gamma_{0i} + \bar{\Gamma}_i\Delta Y^{us}_{t-1} + \Gamma_{1i}'\Delta V_{t-1} + \Pi_i'V_{t-1}$. Let $F_0(\Delta\zeta^i_{t-1} | V_{t-1}, \Delta V_{t-1}, \Delta Y^{us}_{t-1})$ express the data-implied conditional distribution; then the null hypothesis to be tested is:

$$H_0: \; F(\Delta\zeta^i_{t-1} | V_{t-1}, \Delta V_{t-1}, \Delta Y^{us}_{t-1}) = F_0(\Delta\zeta^i_{t-1} | V_{t-1}, \Delta V_{t-1}, \Delta Y^{us}_{t-1}). \qquad \text{(C8)}$$

Let $\pi(.)$ be the unknown marginal density function of $(V_{t-1}, \Delta V_{t-1}, \Delta Y^{us}_{t-1})$. We define,

$$I(u, \sigma) = \left\{\int E[F(\Delta\zeta_t | V_{t-1}, \Delta V_{t-1}, \Delta Y^{us}_{t-1}) - F_0(\Delta\zeta_t | V_{t-1}, \Delta V_{t-1}, \Delta Y^{us}_{t-1})]^2 \times \pi(V_{t-1}, \Delta V_{t-1}, \Delta Y^{us}_{t-1})\right\} dF(\Delta\zeta_t). \qquad \text{(C9)}$$

If (C8) is correctly specified, we have $I(u, \sigma) = 0$ for some $(u, \sigma) \in \Theta$, and if (C8) is not correctly specified, we have $I(u, \sigma) > 0$ for all $(u, \sigma) \in \Theta$. Hence, $I(u, \sigma)$, as a measure of departure from the correct hypothesis, can be used as an indicator for constructing a consistent test for parametric conditional distributions.

Denote $I(\Delta\zeta_t \le y)$ as the indicator function of the event $\{\Delta\zeta_t \le y\}$. Let $\varepsilon_t(y, u, \sigma) \equiv I(\Delta\zeta_t \le y) - F_0(y|V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us}, u, \sigma)$. Then, H_0 holds if and only if there exists $(u, \sigma) \in \Theta$ such that,

$$E(\varepsilon_t(y, u, \sigma)|x_i) = F(y|V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us}) - F_0(y|V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us}, u, \sigma) = 0.$$

We have,

$$I(u, \sigma) = \int E\{\varepsilon_t(y, \theta)E[\varepsilon_t(y, \theta)|V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us}]\pi(V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us})\}dF(y). \quad \text{(C10)}$$

Let $\hat{\theta}_n$ be an estimator of θ_0, and $\hat{\pi}(V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us})$ and $\hat{E}[\varepsilon_t(y, \hat{\theta}_n)|V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us}]$ the leave-one-out kernel estimators of $\pi(V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us})$ and $E[\varepsilon_i(y, \hat{\theta}_n)|V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us}]$, respectively. Then, the parametric conditional distribution functions $F_0(y|V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us}, \theta_0)$ and $E[\varepsilon_t(y, u, \sigma)|V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us}]\pi(V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us})$ can be respectively estimated by $F_0(y|V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us}, \hat{\theta}_n)$ and $\hat{E}[\varepsilon_i(y, \hat{\theta}_n)|x_i]\hat{\pi}(V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us})$, which is:

$$\hat{E}[\varepsilon_i(y, \hat{\theta}_n)|(V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us})]\hat{\pi}(V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us})$$

$$= \frac{1}{nh^p}\sum_{j \ne i} K\left(\frac{(V_{i-1}, \Delta V_{i-1}, \Delta Y_{i-1}^{us}) - (V_{j-1}, \Delta V_{j-1}, \Delta Y_{j-1}^{us})}{h}\right)\varepsilon_i(\Delta\zeta_k, \hat{\theta}_n),$$

where $K(\cdot)$ is a kernel function, and $h \equiv h_n$ is a sequence of smoothing parameters used in the nonparametric estimations of $E[\varepsilon_i(y, \hat{\theta}_n)|(V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us})]$ and $\pi(V_{t-1}, \Delta V_{t-1}, \Delta Y_{t-1}^{us})$.

Putting the above estimations into (C9) yields the following estimation of $J(\theta_0)$,

$$\hat{J}_n = n^{-3}h^{-p}\sum_{k, j \ne i} K_{ij}\varepsilon_i(\Delta\zeta_k, \hat{\theta}_n)\varepsilon_j(\Delta\zeta_k, \hat{\theta}_n), \quad \text{(C11)}$$

where $K_{ij} = K\left(\frac{(V_{i-1}, \Delta V_{i-1}, \Delta Y_{i-1}^{us}) - (V_{j-1}, \Delta V_{j-1}, \Delta Y_{j-1}^{us})}{h}\right)$. Our test statistic is,

$$T_n \equiv nh^{p/2} J_n / \hat{\sigma}_n \,, \tag{C12}$$

where

$$\hat{\sigma}_n^2 = \frac{2}{n^2 h^p} \sum_{j \neq i} K_{ij}^2 \left[\int \varepsilon_i(y, \hat{\theta}_n) \varepsilon_j(y, \hat{\theta}_n) d\hat{F}(y) \right]^2 . \tag{C13}$$

Under the assumption that (C8) is correctly specified, T_n converges to a standard normality distribution.

We use the product kernel $K(x_i) = \Pi_{t=1}^{14} k(x_{i,t})$, where $k(.)$ is a univariate standard normal density. The smoothing parameter is $h = cn^{-1/18}$. Then, c is chosen to minimize the integrated mean squared error of the estimator.

Table C6: Conditional Density Forecast

Equations in BEAM model	Test statistic	Result
Inflation equation	2.0285	Reject
Output equation	2.4396	Reject
Overnight rate equation	1.8272	Reject
Money equation	2.5176	Reject
Stock price equation	2.2919	Reject
Long-term interest rate equation	2.1191	Reject

We reject the null hypothesis (C8), indicating that the Bank of Canada should not use a normal conditional distribution function for the purposes of making probability statements about future inflation. To obtain forecasts of the probabilities at different horizons that inflation will fall into the Bank of Canada's target range, we use stochastic simulation methods by resampling techniques in which the simulated errors are obtained by the nonparametric bootstrap method.

The results from the conditional density forecast are shown in Table C6.

C.3.3 Probability forecast

Single-point forecasts, without specifying their accuracy, are usually inadequate in practice. We are interested in the use of probability forecasts in the characterization of the various sources of uncertainty that surround forecasts from the VECM. For example, we are interested in the forecast of probability that inflation will fall into the Bank of Canada's target range.

A common way of calculating probability forecasts is to assume the conditional distribution of the inflation variable, given conditional variables, as a given conditional distribution. For example, a predictive distribution of inflation represented by a two-piece normal distribution has been published by the Bank of England in its quarterly inflation report since February 1996. However, such probability forecasts may be quite misleading when the predictive conditional distribution is not the true conditional distribution. In particular, we reject the normality distribution.

Note that from equation (C3), the K-step-ahead value of x_{t+K} can be written as,

$$x_{t+K} = JA^K x_t + \sum_{i=0}^{K-1} JA^i C + \sum_{i=0}^{K-1} JA^i DD_{(t+K-i)} + \quad + \sum_{i=0}^{K-1} JA^i By^{us}_{(t+K-i)}$$

$$+ \sum_{i=0}^{K-1} JA^i By^{US}_{(t+K-i)} + \sum_{i=0}^{K-1} JA^i J' u_{x(t+K-i)} \,. \qquad \text{(C14)}$$

Now the exogenous variables $y^{us}_{t+1}, y^{us}_{t+2}, \ldots, y^{us}_{t+K}$ are given; however, $u_{t+1}, \ldots u_{t+K}$ are not available.

We use the nonparametric bootstrap method to simulate $u_{t+1}, \ldots u_{t+K}$, obtaining the simulating value of $u^*_{t+1}, \ldots u^*_{t+K}$. Then we obtain the simulation value of x^*_{t+K}. Thus for any event E, the probability of the event $\Phi(x^*_{t+K}) \in E$ can be computed as

$$\frac{1}{S}\sum_{i=1}^{S} I(\Phi(x^*_{t+K}) \in E)\,, \qquad \text{(C15)}$$

where S is the number of simulating the values of x_{t+K}. Therefore, the probability of the event $\Phi(x^*_{t+K}) \in E$ is calculated as the proportion of the S simulations in which the event is observed to occur.

We focus on the central events of interest to a central bank policy-maker: namely, keeping the rate of inflation within the announced target range of 1 per cent to 3 per cent.

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