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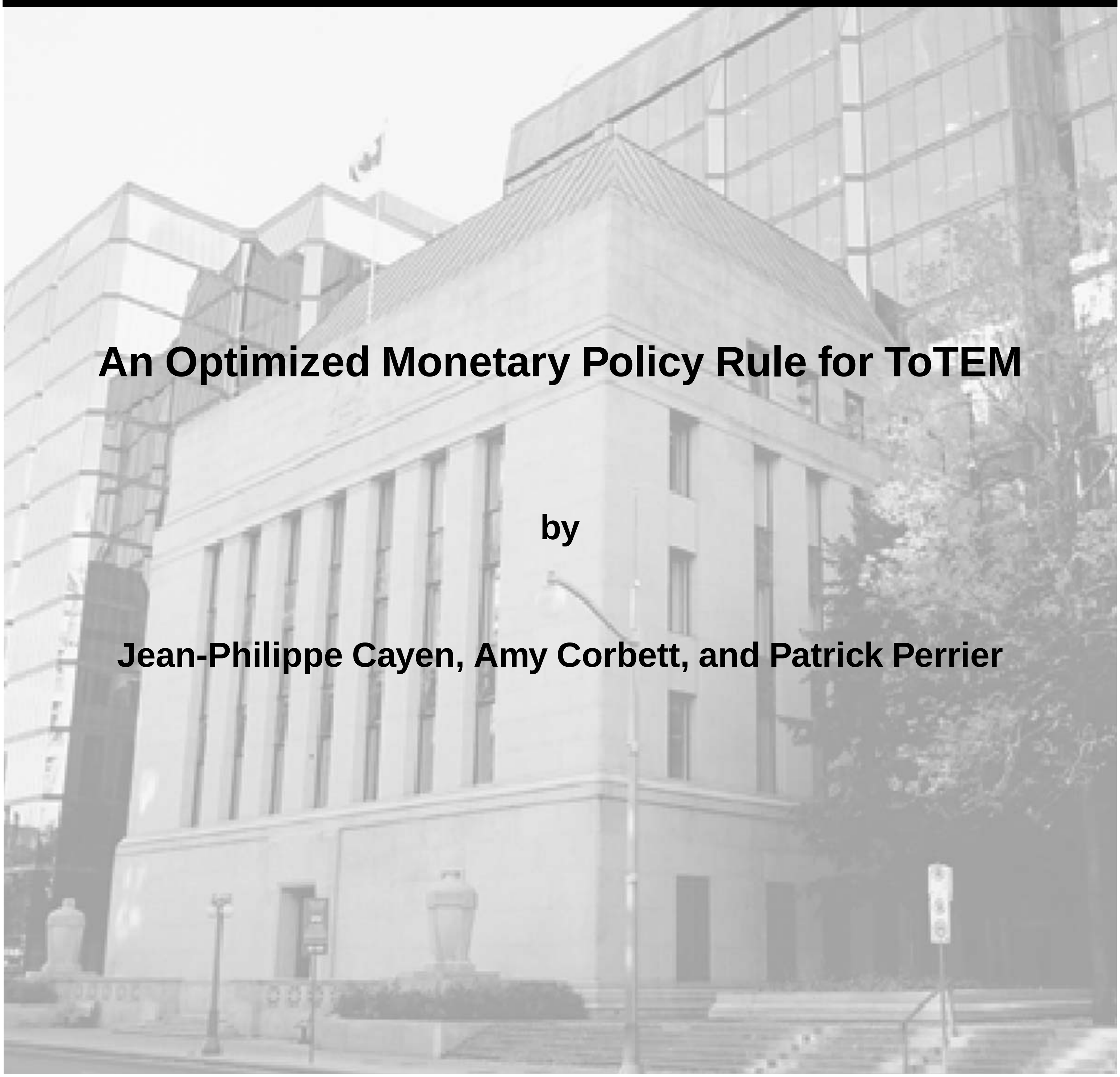

An Optimized Monetary Policy Rule for ToTEM

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Jean-Philippe Cayen, Amy Corbett, and Patrick Perrier

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Jean-Philippe Cayen, Amy Corbett, and Patrick Perrier

Research Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
jcayen@bankofcanada.ca
acorbett@bankofcanada.ca

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Abstract

The authors propose a monetary policy rule for the Terms-of-Trade Economic Model (ToTEM), the Bank of Canada's new projection and policy-analysis model for the Canadian economy. They consider simple instrument rules such as Taylor-type and inflation-forecast-based rules. The proposed rule minimizes a loss function that reflects the assumed preferences of the monetary authority over inflation and output, as well as over the variability of its instrument. The authors also investigate how robust the proposed rule is with respect to a particular realization of shocks that differs from the historical distribution used to find the optimized rule.

JEL classification: E5, E52
Bank classification: Economic models; Monetary policy framework; Transmission of monetary policy

Résumé

Les auteurs proposent une règle de politique monétaire adaptée à TOTEM, le nouveau modèle utilisé par la Banque du Canada pour l'analyse de politiques et l'élaboration de projections concernant l'économie canadienne. Ils examinent plusieurs règles d'instrument simples, telles que des règles à la Taylor et d'autres fondées sur l'inflation prévue. La règle qu'ils proposent permet de minimiser une fonction de perte qui reflète les préférences présumées de la banque centrale au sujet de l'inflation, de la production et de la variabilité de l'instrument de politique monétaire lui-même. Les auteurs cherchent aussi à déterminer la sensibilité de leur règle au choix d'une distribution des chocs différente de la distribution empirique qui leur a servi à établir la règle optimale.

Classification JEL : E5, E52
Classification de la Banque : Modèles économiques; Cadre de la politique monétaire; Transmission de la politique monétaire

1. Introduction

The goal of this paper is to propose an optimized monetary policy rule to be used in the Terms-of-Trade Economic Model (ToTEM), the Bank of Canada's new projection and policy-analysis model for the Canadian economy. For the purpose of this paper, we focus on a simple instrument rule that minimizes an assumed loss function of the monetary authority.[1] Indeed, two loss functions are considered. The first one is defined over inflation and output gap variability. The second allows us to examine the implications of allowing the monetary authority to put a weight on the variability of interest rate movements.

We consider two classes of simple instrument rules. First, we look at Taylor-type rules where the policy instrument reacts to a set of contemporaneous and lagged variables. Second, we consider the class of inflation-forecast-based (IFB) rules. For this class of rules, we treat as unknown the horizon at which the monetary authority should look at inflation to set its policy instrument, as in Batini and Nelson (2001). Thus, this horizon is determined jointly with the other parameters in the monetary policy rule.

We calculate an optimal parameterization for both the IFB and the Taylor-type rules given the assumed preferences of the monetary authority. For each type of rule, we use stochastic simulations to calculate the variability and the persistence of inflation, the output gap, and the policy instrument. In addition, we discuss several important issues such as the average and median horizon over which inflation returns to its targeted level. As well, we consider the implications of interest rate smoothing (in the preferences of the policy-maker) for the persistence and variability of inflation, the output gap, and interest rates. We also calculate confidence bands around inflation outcomes, which provide useful information about an appropriate width for the inflation target band in our modelled economy.

[1] Future work will investigate a specific targeting rule as advocated by Svensson (1999, 2003).

Finally, we examine how robust our results are to a realization of shocks that differs from the historical distribution used to find the proposed rule. We feel that this investigation is particularly relevant given that this model and its policy rule will be used to produce economic projections and provide recommendations for the policy instrument. When the monetary authority has to take a decision about its policy instrument, it often faces a realization of shocks that could be very different from the historical distribution. Our goal is to get a sense of how far the response of the proposed rule will be from the optimal response required for the specific realization of shocks at play.

This paper is organized as follows. In section 2, we provide a brief description of ToTEM. In section 3, we consider two loss functions and two classes of simple instrument rules. In section 4 we describe the approach used to determine the optimized rule within the class of simple instrument rules, and we present and defend our preferred rule. We perform a sensitivity analysis in section 5. In section 6 we offer some conclusions.

2. A Description of ToTEM

ToTEM is a fairly standard open-economy dynamic stochastic general-equilibrium (DSGE) model. However, since we intend to use ToTEM to perform both economic projections and policy-analysis experiments, much emphasis is being placed on getting the model to fit the historical data and to replicate several stylized facts of the Canadian economy. Therefore, the structure of ToTEM is more detailed than the structure of the open-economy DSGE models typically seen in the literature.

What follows is a brief non-technical description of the model. More details of the model are provided in Murchison and Rennison (2006). We also discuss briefly how the parameterization has been determined.

2.1 A non-technical description of the model

Figure 1 illustrates the production side of ToTEM. There are four types of final products produced by domestic firms: consumption goods and services, investment goods, government goods, and non-commodity export goods. To produce these, firms use a constant elasticity of substitution (CES) technology that combines capital with labour services, imported intermediate goods, and commodities. There is also a commodity-producing sector. Commodities are produced by domestic firms by combining labour with capital services and a fixed factor that we refer to as land. All firms are allowed to vary their capital utilization rate, but this comes at a cost in terms of foregone output. Firms also face adjustment costs on the level of employment and on the change in capital and investment, also measured in terms of foregone output.

ToTEM assumes that final-products firms are monopolistically competitive, which allows them to fix prices for more than one period (following the Calvo pricing framework). The Calvo pricing framework is also used to introduce wage rigidities and import price rigidities, as in Smets and Wouters (2002).

Figure 1: The production side of ToTEM

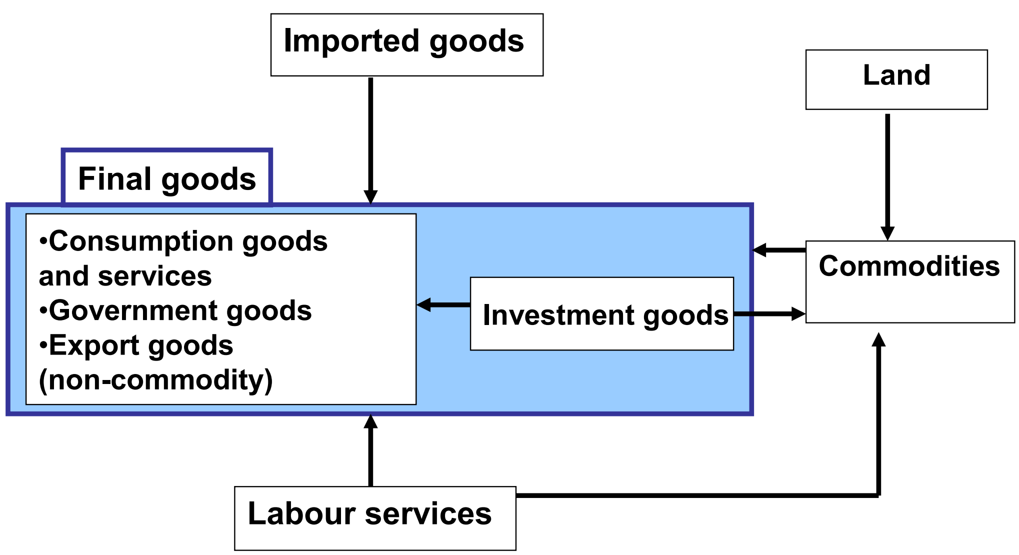
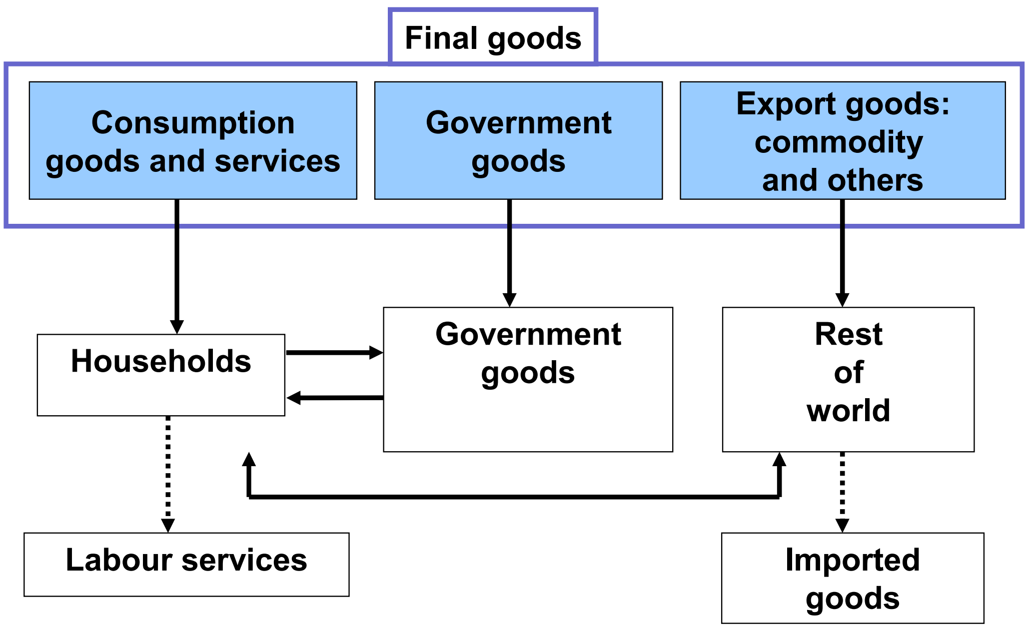


Figure 2 illustrates the demand side of ToTEM. Domestic households buy the final consumption goods as well as bonds from the (domestic) government and the rest of world. They earn (after-tax) labour income from the labour services that they provide to the domestic firms, and income from their holdings of domestic and foreign bonds, in the form of interest payments. They also receive transfers from the government. The government buys the final government goods from the domestic firms with tax revenues and distributes transfers to the domestic households. These expenditures are financed with the tax revenues from labour income and indirect taxes. We assume that the government targets a desired level for the debt-to-GDP ratio, with some smoothing, and uses the tax rate on labour income as the policy instrument. The rest of world buys the commodity exports as well as the final non-commodity export goods. They also sell intermediate imported goods to the domestic importers, and buy and sell bonds.

Figure 2: The demand side of ToTEM



It should be noted that the foreign variables in ToTEM are currently generated with a semi-structural model.[2] This model is exogenous with respect to the core of ToTEM, in that there is no feedback from domestic variables to the foreign variables. This is consistent with the fact that Canada is a small, open economy. The foreign variables that enter in ToTEM are output and the output gap, inflation rate, interest rates (real and nominal), and real commodity prices. This model allows us to perform temporary shocks to foreign demand, inflation, and monetary policy, as well as permanent shocks to foreign output and inflation (i.e., a change in the target inflation rate). We can also perform temporary and permanent shocks to real commodity prices.

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[2] Work is ongoing to improve the foreign side of ToTEM.

2.2 Parameterization of ToTEM

The parameters of ToTEM have been calibrated in order to mimic some key moments of the Canadian data.[3] A monetary policy rule was used for the parameterization of ToTEM, which is given by:

$$(1) \quad R_t = 0.8R_{t-1} + (1-0.8)R^* + (1-0.8)\left[2.5(\pi^q_{t+2} - \pi^*_t)\right],$$

where R and π^q are, respectively, the policy instrument and the quarterly inflation rate (annualized), and * denotes equilibrium values. The parameters of (1) have been determined simultaneously with the other parameters of the model. We refer to this policy rule as the historical rule.

3. The Central Bank's Problem

In this paper, we choose a monetary policy rule that minimizes an assumed loss function of the monetary authority. This section describes the loss function of the monetary authority, as well as the type of monetary policy rules that we consider.

3.1 Loss function

We assume that the monetary authority has preferences over inflation stability, with some concern for output stabilization. We also allow for the possibility that the authority cares about the volatility in the movements of its instrument. We formalize these preferences of the monetary authority with the following contemporaneous loss function:

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[3] The generalized method of moments (GMM) has also been used to obtain some information about the value of these parameters. Work is under way to estimate ToTEM using the Bayesian approach.

(2) $L_t = \left(\pi_t - \pi^*\right)^2 + \lambda_y \left(y_t - y_t^*\right)^2 + \lambda_{\Delta R} \left(\Delta R_t\right)^2$,

where B_t, y_t, and ΔR_t are, respectively, the (year-over-year) inflation rate in period t, the log-level of real output in period t, and the change in the level of the policy instrument between period t-1 and period t, and a * denotes equilibrium values.[4] The parameters λ_y and $\lambda_{\Delta R}$ are, respectively, the relative weight on output fluctuations and the movements of the policy instrument in the preferences of the monetary authority. Formally, we write the intertemporal loss function for the monetary authority as:

(3) $\mathcal{L}_t = E_t\left[(1-\beta)\sum_{i=0}^{\infty}\beta^i L_{t+i}\right] = (1-\beta)E_t\sum_{i=0}^{\infty}\beta^i\left[\left(\pi_{t+i} - \pi^*\right)^2 + \lambda_y\left(y_{t+i} - y_{t+i}^*\right)^2 + \lambda_{\Delta R}\left(\Delta R_{t+i}\right)^2\right]$,

where β is the rate at which the central bank discounts future losses and E_t is the conditional expectations operator, based on information available in period t. Under certain conditions, when $\beta \rightarrow 1$, the value of the intertemporal loss function will approach the unconditional mean of the period loss function, which is given by[5]:

(4) $\overline{\mathcal{L}} = \sigma_\pi^2 + \lambda_y \sigma_{ygap}^2 + \lambda_{\Delta R}\sigma_{\Delta R}^2$,

where σ^2_π, σ^2_{ygap}, and $\sigma^2_{\Delta R}$ are the unconditional variances of the deviations of the year-over-year inflation rate from its targeted level, of the output gap, and of the movements of the policy instrument, respectively. Because it is easier to compute, and because it is used more often in the literature, we will work with this unconditional loss function.

As for the weight given to λ_y and $\lambda_{\Delta R}$, we consider two situations. In the first, $\lambda_y = 1$ and $\lambda_{\Delta R} = 0$:

(5) $\overline{\mathcal{L}}_1 = \sigma_\pi^2 + \sigma_{ygap}^2$.

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[4] The equilibrium output definition here is conceptually related to the natural rate hypothesis, which implies the long-run neutrality of monetary policy. Therefore, there is no inflation bias resulting from the period loss function used here. Svensson (1999) discusses this point.

[5] This is true if $E(\pi_t)=\pi^*$ and if $E(y_t)=y^*$.

This is equivalent to saying that the monetary authority cares only about the inflation rate and the output gap. It does not put any weight on smoothing its policy instrument. Since we do not have any priors about the relative weight given to output and inflation by the monetary authority, we give the same weight to inflation and the output gap.

We also consider an alternative loss function where the monetary authority penalizes volatility in the policy instrument, but relatively less so than inflation deviation from the targeted level or output from its equilibrium. In this alternative loss function, we set $\lambda_y = 1$ and $\lambda_{\Delta R} = 0.5$:

$$(6) \quad \overline{\mathcal{L}}_2 = \sigma_\pi^2 + \sigma_{ygap}^2 + 0.5\sigma_{\Delta R}^2 .$$

We follow Rudebusch and Svensson (1999) and Batini and Nelson (2001) for the choice of $\lambda_{\Delta R}$. The policy instrument may enter as an argument of the loss function for three different reasons: (i) big and unexpected changes to interest rates may cause problems for financial stability (Cukierman 1990; Smets 2003), (ii) the policy-makers may be concerned about hitting the lower nominal bound on interest rates (Rotemberg and Woodford 1997; Woodford 1999; Smets 2003), or (iii) in reality, the monetary authority (and other agents) may be uncertain about the nature and the persistence of the shocks at play in the economy at the time it must take a decision about its policy instrument. We acknowledge that the third reason seems to be inconsistent with the assumption in ToTEM that all agents can identify the nature and the persistence of the shocks exactly. However, since we intend to use ToTEM to produce economic projections, we believe that the change in the policy rate in the loss function should help to generate a plausible path for the policy rate.[6]

3.2 Monetary policy rules

We consider two types of simple instrument rules for this study: Taylor-type rules and IFB rules. For Taylor-type rules, the instrument usually reacts to a lagged value of itself, and to

[6] As we will show in the next section, when we optimize a loss function that incorporates the change of the policy instrument, we get a monetary policy rule that puts a large weight on the lagged interest rate. Adding the change of the policy instrument in the loss function also leads to a substantial fall in the variability of the policy instrument, with essentially no increase in the variability of inflation and the output gap.

contemporaneous gaps between inflation and output relative to their targeted values. For the IFB rules, the instrument reacts to expected deviations of inflation from its targeted level at some horizon, usually in addition to the contemporaneous value of the output gap and lagged interest rates. In this type of rule, the forecast of inflation enters as a feedback variable for the setting of the policy instrument.

The inevitable lags between monetary policy actions and their effect on inflation make the IFB rules appealing. The IFB rules illustrate more naturally than the Taylor-type rules the problem faced by policy-makers in inflation-targeting countries. As discussed by Batini and Haldane (1999), since IFB rules include forecasts of inflation, they may be preferable to Taylor-type rules, because they implicitly allow for the use of all relevant information, including the judgment of the forecaster. Armour, Fung, and Maclean (2002) find that the IFB rules outperform the Taylor-type rules in QPM in terms of the volatility of inflation, output, and interest rates.[7,8] However, these authors note that the IFB rules may be less robust across models than the Taylor-type rules, since IFB rules include a model-consistent forecast of inflation, which could be affected by a change to the model.[9]

A generic form that nests the simple instrument rules considered in this study is given by[10]:

$$(7)\quad R_t = \theta_R R_{t-1} + (1-\theta_R)R^* + (1-\theta_R)\left[\theta_\pi(\pi^q_{t+h} - \pi^*_t) + \theta_y YGAP_t\right],$$

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[7] QPM stands for Quarterly Projection Model. It is of similar size as ToTEM. QPM includes two components, which are referred to as steady-state QPM and dynamic QPM. Steady-state QPM describes the long-run behaviour of successive generations of utility-maximizing households and profit-maximizing firms, given the choices of the fiscal authority and the links to the rest of the world. The dynamic model traces the adjustment path of the economy to this long-run equilibrium. The dynamics of QPM are driven by multiperiod contracts, costly adjustment, and a mix of backward-looking and model-consistent expectations. The latter comes from the assumption of incomplete knowledge by agents about the true structure of the economy. Please see Coletti et al. (1996) for more information about QPM.

[8] The analysis in Armour, Fung, and MacLean (2002) is done without the incorporation of judgment.

[9] Amano, Coletti, and Macklem (1999) document that IFB rules might not be so sensitive to changes in the behaviour of economic agents. However, it seems that a change in the level of the monetary authority's credibility could affect the coefficients and the horizon of the optimized IFB rule.

[10] We are working on improving the foreign part of ToTEM. Once this becomes satisfactory, we will investigate whether the addition of variables related to the open-economy feature of ToTEM would qualify as determinants of the monetary policy rule, following the work of Batini, Harrison, and Millard (2003).

where B^q is the quarterly inflation rate (annualized) and h is the feedback horizon as described in Batini and Nelson (2001). For the Taylor-type rules, h will be set to 0, while for the IFB rules this feedback horizon will come from the optimization exercise. It should be noted that, with this specification, the model will not solve when θ_R=1, so we exclude price-level targeting from the possible set of rules.

4. Optimized Simple Instrument Rules

In this section, we first describe the approach used to determine the optimized rule within the class of simple instrument rules. Second, we report the optimized rule for both the Taylor-type and the IFB rules for the loss functions $\mathcal{L}_1$ and $\mathcal{L}_2$. We then present and defend our preferred rule. We complete this section by showing the implications of using this rule on the optimal policy horizons (OPH) and on the possible confidence bands around inflation.

4.1 Methodology used to determine optimized simple instrument rules

We use the method of stochastic simulations to determine the vector of parameters, θ, and the feedback horizon, h, which minimize the loss function $\mathcal{L}$. Using this method, we submit the model at each period to realizations of structural shocks that we draw from the historical distribution of shocks. The number of periods for the stochastic simulations is 5,000. The historical distribution of shocks used for these simulations is taken from the 1991–2005 period. We use this short sample period because there seems to be a break in the distribution of shocks over a longer sample, and we prefer the distribution of shocks that corresponds to the actual inflation-targeting regime. We use 27 different shocks in these simulations. These shocks can be grouped into demand, supply, and markup shocks.[11]

With these stochastic simulations, it is possible to calculate the variance of inflation, the output gap, and the change in the level of the interest rate for each possible combination of the

[11] We do not consider shocks to the inflation target, or domestic monetary policy shocks, since our goal is to establish the monetary policy rule.

parameter vector, θ, and of the feedback horizon, h, considered in equation (7). This allows us to calculate the value of the loss function for each combination of θ, and h. The optimized rule is therefore the one that minimizes the loss function. This problem can be expressed formally as follows:

$$(8) \quad \Theta = \arg\min_{\Theta} \overline{\mathcal{L}} ,$$

where $\Theta = (\theta', h)'$. We perform the minimization of the loss function with a grid search over all the coefficients and the feedback horizon.

4.2 Optimized parameters for the different instrument rules

Table 1 reports the optimized parameters for the Taylor-type and the IFB rules under the loss functions $\mathcal{L}_1$ and $\mathcal{L}_2$ described in section 3.1, as well as the coefficients of the rule estimated over history. It also reports the standard deviation of year-over-year inflation, of the output gap, and of the nominal interest rate for each of the policy rules. It should be noted that, for the IFB rules, the feedback horizon (h) is optimized, while for the Taylor-type rule and the historical rule it is fixed to zero and two, respectively. This horizon is linked to the length of time it takes for monetary policy actions to have their maximum effect on inflation.

Before we discuss the results, it is important to note that the model is solved under commitment to a policy rule by the monetary authority. Therefore, agents in our modelled economy know the exact form and parameterization of the rule, which they take into account to form their expectations. This feature likely affects the variance of the key variables on the downside, in comparison with a situation where there is no commitment by the central bank.

Table 1: Results for the simple instrument rules

Rule	θ_R	θ_π	θ_y	h	σ_π	σ_{ygap}	σ_R	$\sigma_{\Delta R}$	$\mathcal{L}$
A: $\overline{\mathcal{L}}_1 = \sigma_\pi^2 + \sigma_{ygap}^2$									
Historical	0.8	2.5	0	2	0.47	2.15	0.48	0.15	4.84
Taylor	0	6.5	0.1	-	0.31	2.03	2.35	1.92	4.21
IFB	0	7	0.1	1	0.35	2.01	2.10	1.83	4.16
B: $\overline{\mathcal{L}}_2 = \sigma_\pi^2 + \sigma_{ygap}^2 + 0.5\sigma_{\Delta R}^2$									
Historical	0.8	2.5	0	2	0.47	2.15	0.48	0.15	4.85
Taylor	0.85	6.5	0.1	-	0.28	2.06	0.92	0.35	4.40
IFB	0.95	20	0.35	2	0.36	2.02	0.66	0.21	4.21

Notes: The results of the estimated rule are included as a benchmark. All series used to calculate the standard deviation are expressed in percentage points.

Table 1 displays some interesting results. First, we can see that the IFB rule performs slightly better than the Taylor-type rule for both loss functions, which means that the monetary authority gains by reacting to expected inflation. This result can be better explained by inspecting the impact of each type of rule on inflation and output variability. As expected, the variability of inflation under an IFB rule is larger than under a Taylor rule, because the Taylor rule reacts more quickly to every departure of inflation from target, whereas an IFB rule will react only if this departure is expected to be persistent. On the other hand, this allows the IFB rule to do a better job at keeping the output gap variability low relative to the Taylor rule, although the difference is small. Overall, the latter effect dominates in the case of the loss function that has a weight on interest rate movements ($\mathcal{L}_2$), whereas the performance of the IFB rule is almost the same as the Taylor-type rule in the case of the loss function $\mathcal{L}_1$.

This is consistent with the results in Armour, Fung, and Maclean (2002) who also conclude that the Canadian monetary authority gains by adopting an IFB rule over a Taylor-type rule, in the context of QPM. However, the gain in favour of the IFB rule is more important in QPM than it is for ToTEM. Also, the optimal feedback horizon is longer in QPM (between six and seven quarters) than in ToTEM.[12] This seems to be due to the fact that ToTEM is more forward looking and generates less inflation persistence than QPM.

Second, when the loss function does not incorporate interest rate variability, our optimized policy rule prescribes putting no weight on the lagged interest rate. As seen in Table 1, these rules will generate a lot of variability for the interest rate. However, when the monetary authority puts a weight on interest rate variability in its loss function, the coefficient on the lagged interest rate (θ_R) increases considerably, from 0.0 to 0.85 for the Taylor-type rule and to 0.95 for the IFB rule. Increasing interest rate smoothing in the policy rule helps to reduce the variability of interest rates. In fact, when the rules are optimized such that the monetary authority puts a weight on interest rate volatility in the loss function, the standard deviation of interest rates decreases by about 140 basis points for both rules. This reduction in interest rate volatility is consistent with other studies (Woodford 1999; Smets 2003; Batini, Harrison, and Millard 2003). What is unique with the current version of ToTEM is that reducing the interest rate volatility is not very costly in terms of the variability of inflation and the output gap. By including the objective of stabilizing the policy rate, we see only marginal changes in these variabilities for the IFB rule. The same conclusion holds for the Taylor-type rule. This result is linked to the important role played by expectations, and the assumption that these expectations are rational and that agents have a very low rate-of-time preference, or a high discount rate. Therefore, agents are essentially indifferent, for instance, between an immediate increase of 100 basis points in the policy rate and four increases of 25 basis points each.

Third, the optimal feedback horizon for the IFB rule is longer when the monetary authority cares about interest rate variability, although there is a difference of only one quarter compared with the case where the objective is absent from the loss function. With preferences defined over interest rate variability as well as over inflation and output, the policy-maker cannot react as quickly as before to offset the deviations of inflation from its target. Hence, it has to be more

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[12] For QPM, the optimal feedback horizon reported is in terms of the year-over-year inflation rate. For ToTEM, the optimal feedback horizons (h) reported in Table 1 are in terms of the quarterly inflation rate. However, if they were expressed in terms of the year-over-year inflation rate, they would add about two quarters to the value of h reported in Table 1.

forward looking than in the case with no concern for interest rate smoothing. Putting a weight on the change in the policy instrument in its preferences induces the monetary authority to be more forward looking, since it introduces stickiness in its instrument.

Fourth, the fact that the historical rule indicates a large smoothing coefficient (θ_R = 0.8) is consistent with the idea that the monetary authority had some concerns about interest rate volatility for financial stability reasons, or that it wanted to reduce the probability of hitting the zero bound of the interest rate. It could also be that uncertainty about the nature of the shocks that have hit the economy over history was limiting the reaction of the monetary authority to these shocks.

4.3 Preferred rule

Based on the results of this section, we tend to prefer the IFB rule with a smoothing parameter of 0.95. This is the rule that optimizes the second loss function ($\mathcal{L}_2$). For both loss functions, the IFB specification performs modestly better than the Taylor specification, which means that the monetary authority benefits from being forward looking.[13] Also, using the rule consistent with minimizing $\mathcal{L}_2$ does not cause $\mathcal{L}_1$ to increase greatly. In fact, in Table 1 we can see that this specification significantly reduces interest rate volatility with only marginal effects on the variability of inflation and output gap.

4.4 The optimal policy horizon

For countries that target inflation, it is important to know how long it takes for inflation to return to target. This is what Batini and Nelson (2001) refer to as the optimal policy horizon (OPH). The OPH is largely dependent on the specific realization of shocks prevailing at the time the monetary authority must decide on its policy instrument, since some shocks have a more persistent effect on inflation than others.

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[13] However, forward-looking rules are potentially less robust to model misspecification than Taylor-type rules, since they include a model-consistent forecast of the inflation rate.

Given that the number of quarters it takes for inflation to go back to its targeted value is dependent on the realization of shocks, we run another set of stochastic simulations with 500 draws from the historical distribution of the structural shocks. For each draw, we calculate how many quarters of inflation are required to get back on target. Using the outcomes of these simulations, we can calculate the average and median OPH, as well as the empirical distribution. However, given that the inflation rate returns only asymptotically to its targeted value, we need to impose a practical criterion to assume that inflation is back to the target. We use Batini and Nelson's (2001) proposed absolute criterion, where the year-over-year inflation rate is deemed to be back to its targeted level when it falls inside a band with a width of 0.1 percentage points on either side of the target. Since the variance of inflation over the 1991–2005 period has been relatively low, there is a risk that inflation returns to target very fast simply because the chosen criterion is not restrictive enough. For this reason, we also use a more restrictive version of this criterion, where the width of the band is 0.05.

Figure 3: The number of quarters required for inflation to return to target

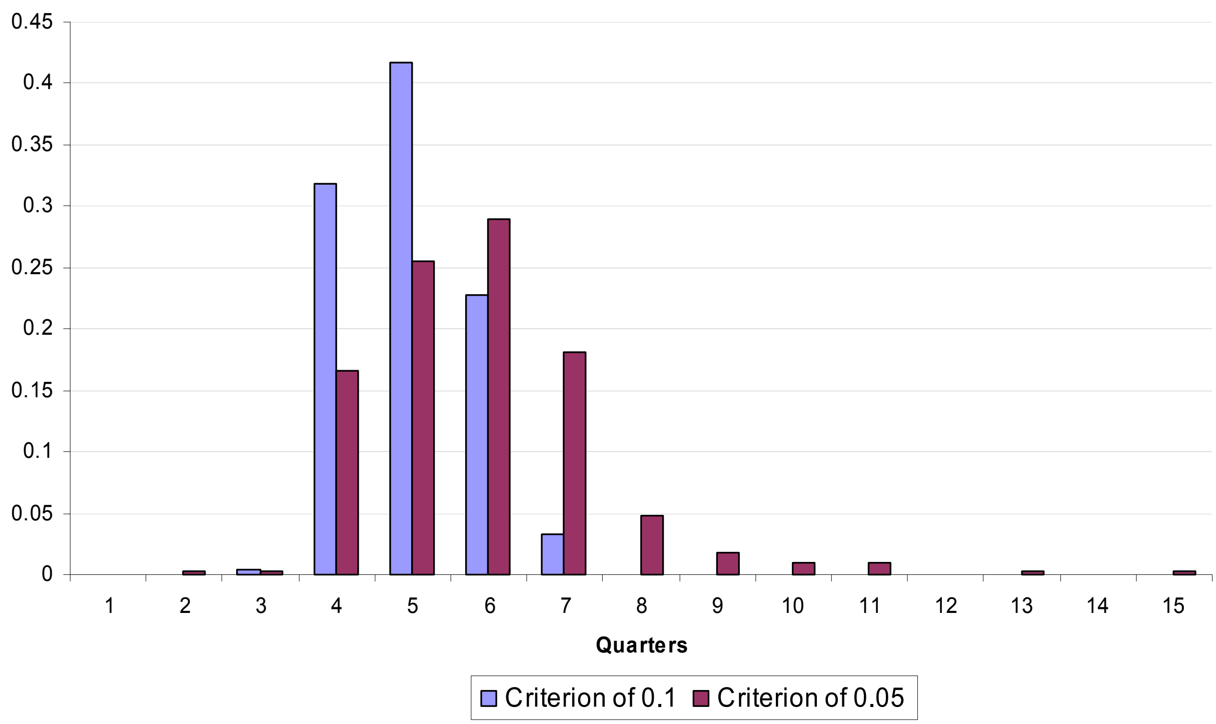


Figure 3 shows a histogram of the OPHs for our preferred policy rule for each criterion we consider. For the first criterion, where the band is 0.1 percentage points, the average and median

OPH is five quarters, and year-over-year inflation is back to the target within four to seven quarters 95 per cent of the time. For the more restrictive criterion (with a band of 0.05 percentage points), the average and median OPH is six quarters, and inflation is back to the target within four to eleven quarters 95 per cent of the time.

4.5 Confidence bands around the inflation rate

The stochastic simulations performed in the previous sections provide the moments of key endogenous variables. Among other things, they can be used to calculate confidence bands around the inflation rate. These confidence bands can be used to obtain some information regarding the appropriate width of the inflation target band. However, we hasten to add that the standard deviations of inflation and the other endogenous variables do not incorporate model uncertainty (of both the parameters and the model's structure).

Table 2 presents the 90, 95, and 99 per cent confidence bands around the year-over-year inflation rate for each of the policy rules associated with the loss function $\mathcal{L}_2$. We can see that these confidence bands are all contained within the actual inflation target band in Canada of 1–3 per cent. As well, the probability of inflation being within the band for Canada is 0.999 in the case of the optimized Taylor rule and 0.995 in the case of the optimized IFB rule. As an indication, the core inflation rate has been outside the official inflation target band only once since 1991, or, alternatively, it has been inside 98 per cent of the time.

Table 2: Confidence intervals surrounding the outcome for inflation

	90%	95%	99%	Prob. (1%–3%)
Taylor	1.5%–2.5%	1.4%–2.6%	1.3%–2.7%	99.9%
IFB	1.4%–2.6%	1.3%–2.7%	1.1%–2.9%	99.5%

5. Robustness to the Distribution of Shocks

An important disadvantage of simple instrument rules is that they are not immune to changes in the distribution of shocks. In other words, the parameters of these rules depend on the relative variances of the structural shocks. This can have non-trivial implications in a model used to produce economic projections and monetary policy recommendations. If the economy is facing a distribution of shocks over a certain period that differs significantly from the one used to determine our proposed rule, the parameters will not be optimal over this specific period. Svensson (2003) uses this argument to justify the use of targeting rules, since they do not depend on the distribution of shocks. This guarantees an optimal reaction for the monetary authority for any realization of shocks.

In this section, we investigate how different the optimized rule would be if the model was hit by only one type of shock. We then use a different sample period to estimate the historical distribution of shocks, and reoptimize the monetary policy rule based on this new distribution of shocks.

5.1 Robustness to specific realization of shocks

We first investigate what the optimized IFB rule would be if the model was subject to only one type of shock.[14] We apply this exercise to three specific shocks: a temporary exchange rate shock, a permanent technology shock, and a wage markup shock. The first two shocks do not create any tension between the objectives of monetary policy, but the last one does.

Table 3 reports the optimized coefficients for each type of shock, while Figures 4, 5, and 6 show the impulse-response functions (IRF) of key variables to these shocks. In these figures, the solid lines represent the IRF using the IFB rules optimized for the specific shocks (the rules given in Table 3), and the dotted lines represent the IRF using our preferred IFB rule optimized over the average historical distribution of shocks, as reported in Table 1.

[14] For this exercise, we assume that the monetary authority's preferences are given by the loss function $\mathcal{L}_2$.

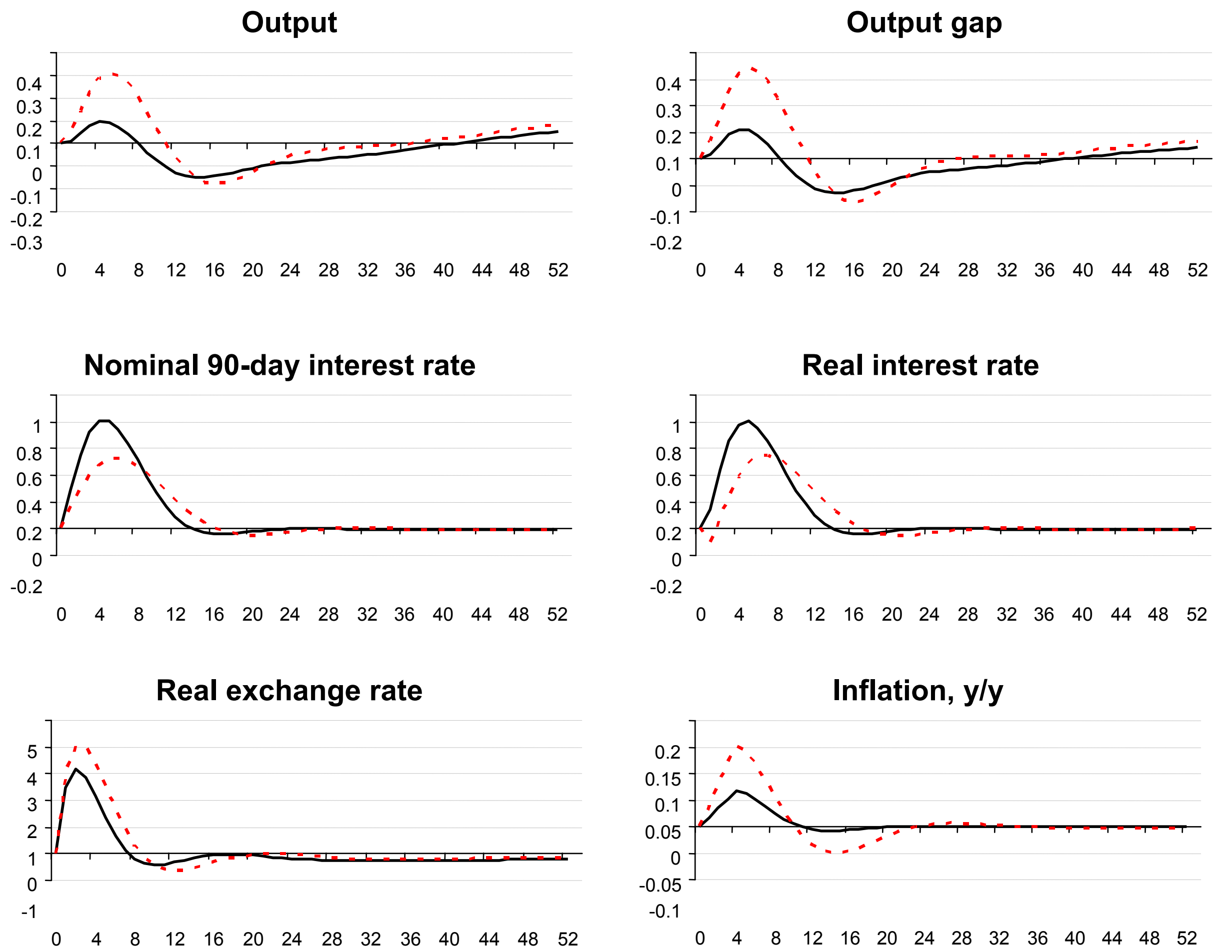
Table 3: Optimized rules based on the loss function $\mathcal{L}_2$ for different shocks

Rule	θ_R	θ_π	θ_y	h	$(1-\theta_R)\cdot\theta_\pi$	$(1-\theta_R)\cdot\theta_y$
Temporary exchange rate shock	0.75	18	0	0	4.5	0
Permanent technology shock	0.90	14	0.2	2	1.4	0.02
Wage markup shock	0.95	2	0.3	4	0.1	0.015
Preferred IFB rule using average distribution of all shocks	0.95	20	0.35	2	1.0	0.0175

We can see in Figures 4 and 5 that the exchange rate shock and the permanent technology shock cause inflation and the output gap to move initially in the same direction. Since a change in interest rate makes the output gap and inflation move in the same direction, the monetary authority's optimal reaction to these shocks is to move interest rates aggressively to the deviations of output and inflation from their equilibrium. This implies that the weights attributed to inflation and the output gap in the policy rule must be relatively large. In Table 3, we can see that the optimized IFB rule specific to these shocks puts a larger weight in the short run and in the long run on inflation (and on the output gap for the technology shock) than does the IFB rule optimized over the average historical distribution of shocks.[15] This explains why, in Figures 4 and 5, interest rate deviations from equilibrium are larger for the IFB rules that optimize these two specific shocks. The aggressiveness of these monetary policy rules also explains why they generate smaller deviations of inflation and of the output gap from equilibrium than for the IFB rule optimized over the average historical distribution of shocks.

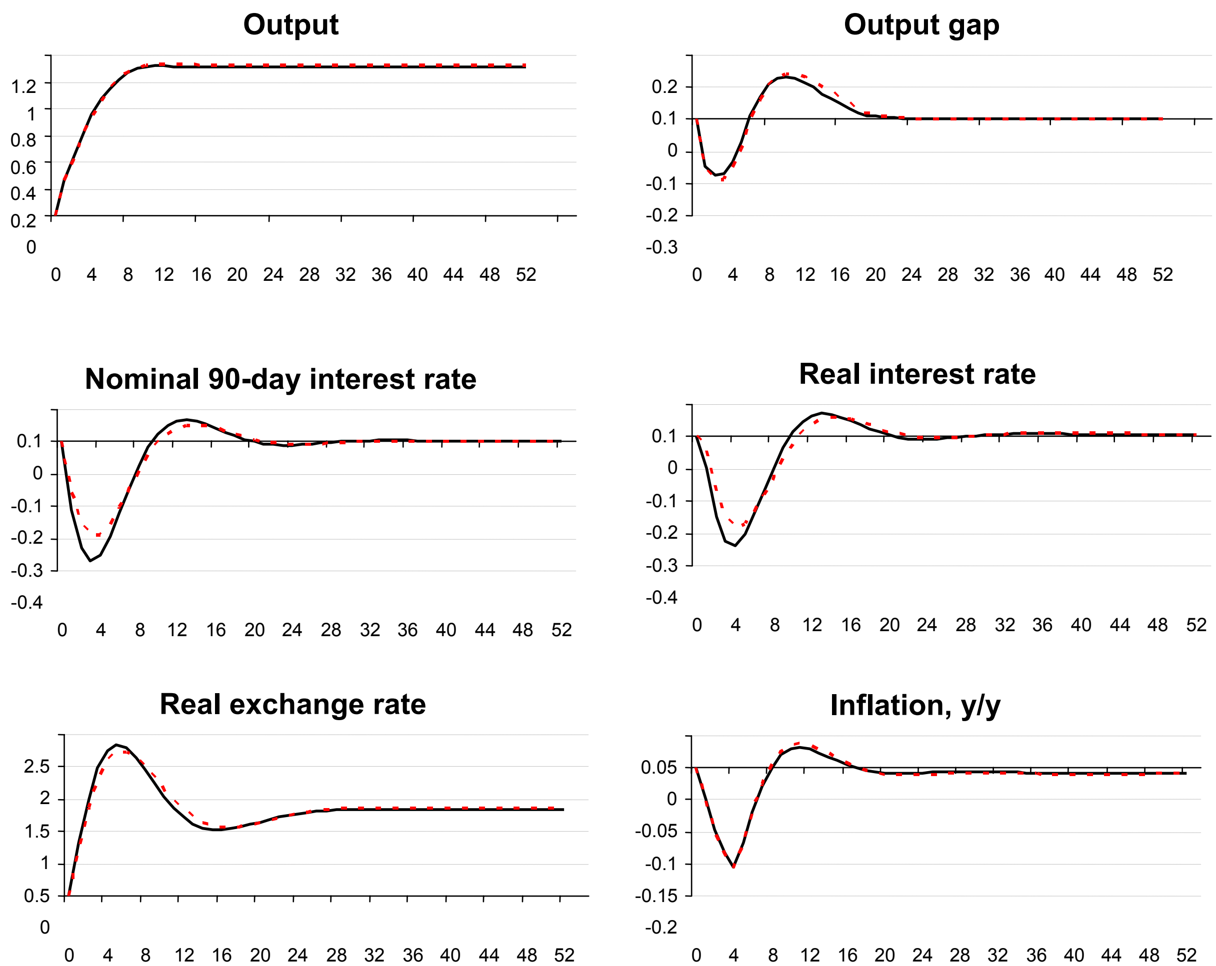
[15] The parameters θ_π and θ_y represent the long-run responses of the interest rate to movements in inflation and the output gap, respectively. The short-run responses of the interest rate to movements in inflation and the output gap are given by the terms $(1-\theta_R)\cdot\theta_\pi$ and $(1-\theta_R)\cdot\theta_y$. Both short-run and long-run responses are presented in Table 3.

Figure 4: Impulse-response functions to a temporary exchange rate shock



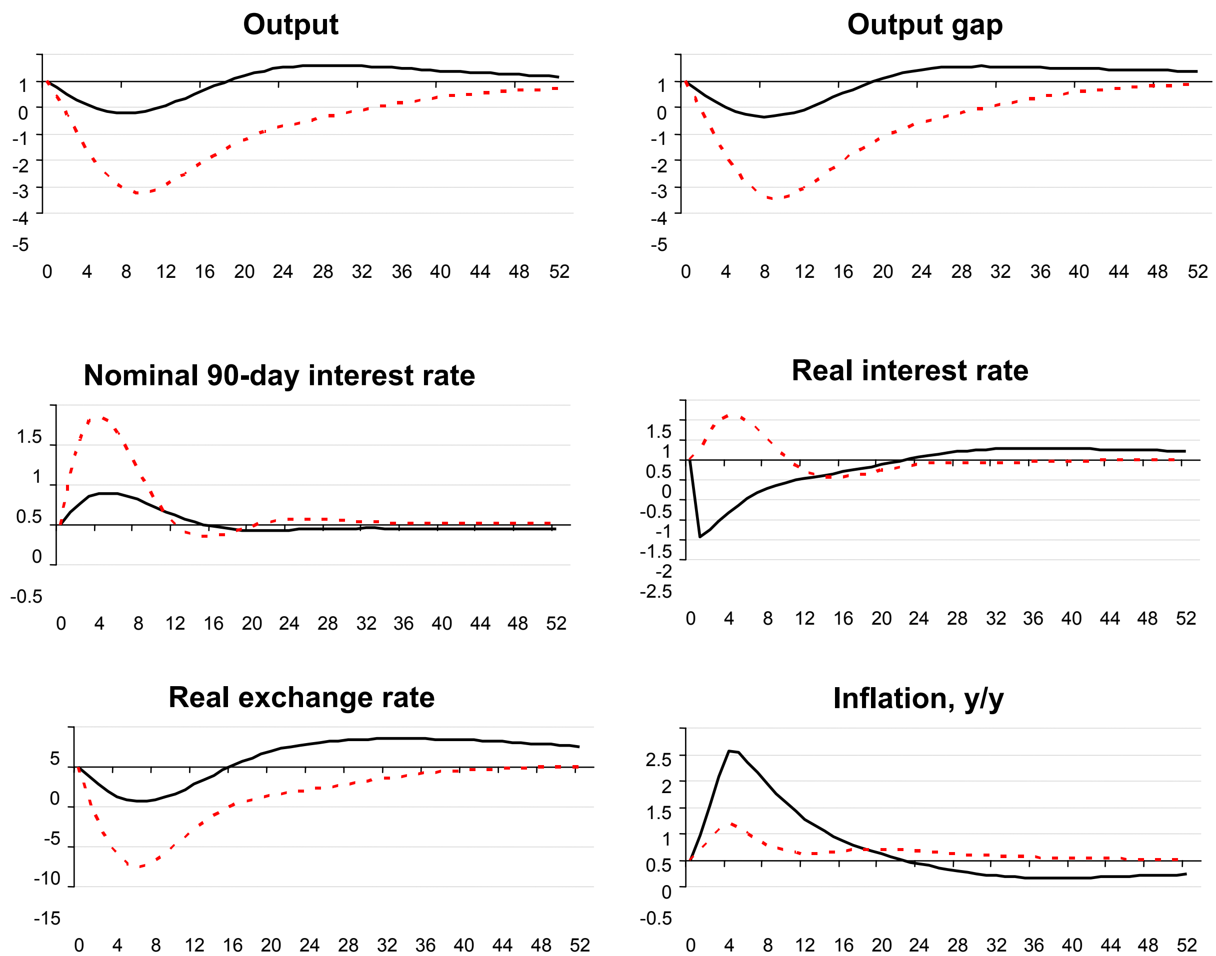
Note: An increase in the exchange rate represents a depreciation of the currency.

Figure 5: Impulse-response functions to a permanent technology shock



Note: An increase in the exchange rate represents a depreciation of the currency.

Figure 6: Impulse-response functions to a wage markup shock



Note: An increase in the exchange rate represents a depreciation of the currency.

Contrary to the technology and the exchange rate shocks, the wage markup shock causes inflation and the output gap to move initially in the opposite direction (Figure 6). Monetary policy must therefore be less aggressive than for the previous two shocks, since moving interest rates in one direction will necessarily increase either the output gap or the inflation gap. As seen in Table 3, this explains why the weight put on inflation (θ_π) in the policy rule for this shock is small relative to the weight given in the IFB rule optimized over the average historical distribution of shocks. The tensions between inflation and the output gap also mean that the monetary authority must be more forward looking, which explains the larger value of the feedback horizon (h) for the optimized rule specific to the wage shock. In Figure 6, we can see that the IFB rule that is optimized for the wage markup shock prescribes a less aggressive reaction for the interest rate than for the rule that is optimized for the average distribution of shocks. In fact, the IFB rule that is optimized for this specific shock causes the real interest rate to be negative, to help close the negative output gap. As a result, the deviation of inflation from equilibrium is much more pronounced, and the deviation of the output gap from equilibrium is less pronounced, when the IFB rule is optimized for this specific shock.

5.2 Robustness of the rule to a different sample period

Our preferred IFB rule presented in section 4 has been optimized based on the average distribution of the shocks for the 1991–2005 period. A legitimate experiment is to see how robust this optimized rule is to the sample period used to estimate the average distribution of the shocks. Normally, we would expect the distribution of the shocks to be relatively constant over time. But ToTEM seems to have difficulties in explaining the change in the volatility of some key variables, such as GDP growth and the inflation rate gap (i.e., the level of inflation minus its target) between the 1980s and the 1990s. Consequently, the estimated volatility of some shocks declines in the 1990s relative to the 1980s. This change in the volatility also means that the average distribution of the shocks changes between these two periods. In this section, we investigate how this change in the distribution of the shocks affects the parameters of our preferred IFB rule by reoptimizing the IFB rule based on the distribution of shocks for the 1980–2004 period.[16]

Table 4 reports the optimized parameters of the IFB rule that uses the distribution of shocks estimated over the 1980–2004 period.[17] It is compared with our preferred rule presented in section 4.3 that uses the distribution of shocks estimated over the 1991–2005 period. We can see that the rule that uses the longer sample puts a smaller weight on inflation both in the short run ($(1-\theta_R)\cdot\theta_\pi$) and in the long run ($\theta_\pi$). However, it puts a larger weight in the short run on the output gap $(1-\theta_R)\cdot\theta_y$. The rule that uses the longer sample also has a smaller coefficient on the lagged interest rate (θ_R), which means that the monetary authority smooths the interest rate less under this rule.

Table 4: Results for the simple instrument rules

Rule	θ_R	θ_π	θ_y	h	σ_π	σ_{ygap}	σ_R	$\sigma_{\Delta R}$	$\mathcal{L}$
IFB, 1980–2004	0.80	2	0.15	2	0.90	5.14	2.54	1.01	27.74
IFB, 1991–2005	0.95	20	0.35	2	0.92	5.18	1.85	0.57	27.79

Notes: All series used to calculate the standard deviation are expressed in percentage points. The simulations used to calculate the standard deviations are based on the average distribution of shocks over the 1980–2004 period.

Table 4 also reports the standard deviations of the simulated series. First, we can see that these standard deviations are very large relative to the ones presented in Table 1. This comes from the fact that we recalibrate the variance of the shocks to match the observed variance of our key variables for the 1980–2004 period.[18]

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[16] We do not include the data for the four quarters of 2005, as in the other sections, so that our results can be compared with those presented in Coletti, Selody, and Wilkins (2006). It should be noted, however, that adding the four quarters of 2005 in the sample used to calculate the average distribution of the shocks would not modify the results presented in this section.

[17] We report only the rule that optimizes the loss function $\mathcal{L}_2$.

[18] This recalibration affects the absolute variance of all the variables by the same proportion. Therefore, it does not affect the relative variances of the variables, or any other moments of the key variables.

Second, there is little difference in the standard deviation of the key variables between the two rules, which explains the very close values for the two loss functions. However, even though the optimized rule base for the 1980–2004 period is less aggressive on inflation, it still does a better job at minimizing the variability of inflation. One possible explanation for this result is that the 1980–2004 period is dominated by price and wage markup shocks, which penalizes more rules that are aggressive on inflation. We can also see that the smaller coefficient on the lagged interest rate (θ_R) in the optimized rule base on the 1980–2004 period causes interest rates to be more volatile.

Figure 7: The number of quarters required for inflation to return to target with the policy rule optimized over the 1980–2004 period

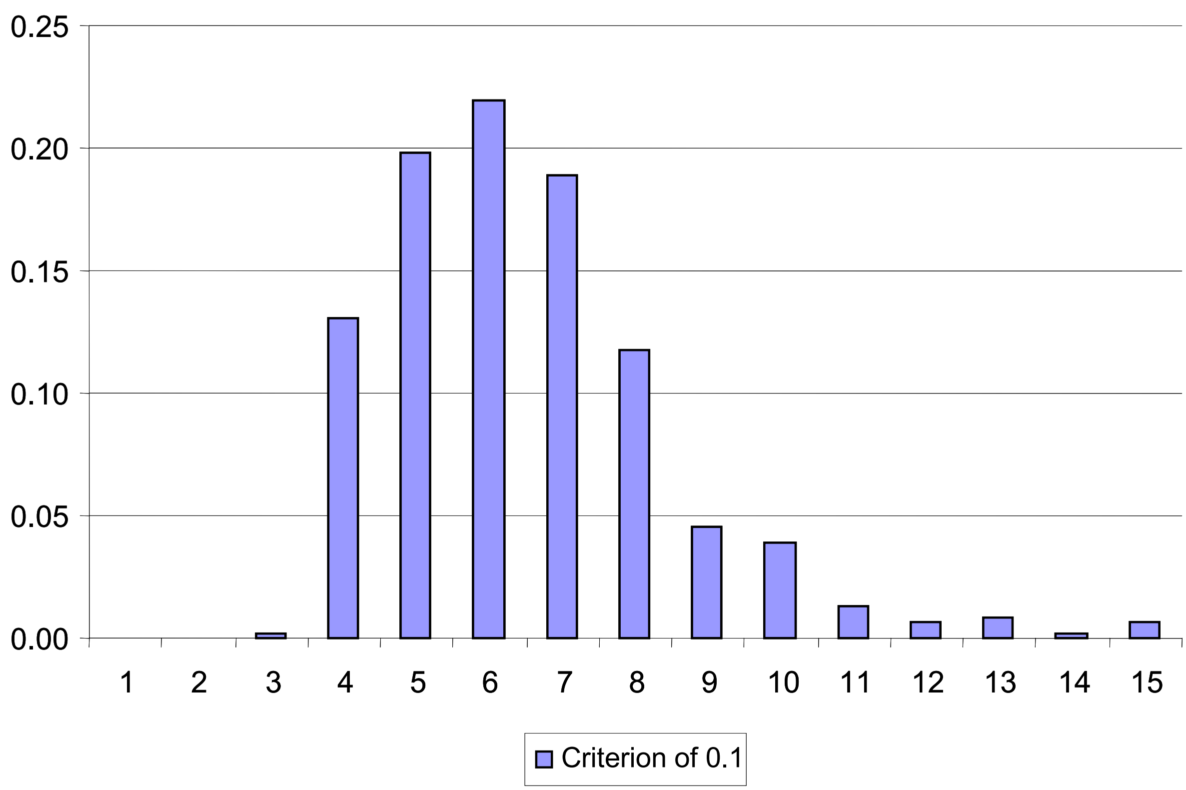

Figure 7 shows a histogram of the OPH for the policy rule optimized over the 1980–2004 period. The median OPH is six quarters, while the average OPH is seven quarters. Also, year-over-year inflation is back to the target within four to eleven quarters 90 per cent of the time. This is longer than what we report in Figure 3 and in section 4.4 for the 1991–2005 period, and is mainly explained by the fact that the variance of inflation in the 1980s was higher than in the 1990s and the 2000s.

6. Conclusion

We propose a monetary policy rule for ToTEM. The proposed rule minimizes a loss function that reflects the assumed preferences of the monetary authority over inflation and output fluctuations, as well as over the variability of its policy instrument. We propose an inflation-forecast-based (IFB) rule with a feedback horizon of two quarters (for the quarterly inflation rate), a relatively large coefficient on inflation (20.0), a much smaller coefficient on the output gap (0.35), and a relatively large coefficient on the lagged interest rate (0.95).

Our results show that allowing the central bank to be concerned by the volatility of interest rate movements significantly reduces the variance of the policy instrument, with no perceptible increases in the variance of inflation and the output gap. Also, the fact that we obtain an optimal feedback horizon of two quarters indicates that IFB rules outperform Taylor-type rules in the context of ToTEM. This means that the monetary authority gains by reacting to expected inflation. Our results also show that the number of quarters required for inflation to return to target can vary considerably, in accordance with the composition of shocks that affect the economy.

It should be noted that the simple instrument rule that we propose in this study is optimized only for the specific distribution of shocks that characterize the 1991–2005 period; simple instrument rules are not immune to changes in the distribution of structural shocks. Our results show that the optimal policy may differ significantly for different types of shocks, and this seems to have a non-trivial effect on the impulse-response function of the key variables of the model. Also, we show that the parameters of the policy rule are different if we use a longer sample period (1980 to 2004).

The lack of robustness of simple instrument rules to the distribution of shocks is an important problem for a model used to perform projection scenarios, given that, for a specific quarter, the economy may be affected by a realization of shocks that differ from the one used to optimize the simple rule. Our results therefore support the need to examine alternative policy decision strategies that are optimal under any particular realization of shocks. Future work will examine the possibility of implementing a specific targeting rule in ToTEM, as advocated by Svensson (1999, 2003), among others, since this type of rule is robust to different realizations of shocks.

The lack of robustness of simple instrument rules to the distribution of shocks does not mean that they do not have any value. More complex rules are often highly model dependent and less transparent, whereas simple instrument rules are generally recognized to be more robust across different models. Given the high uncertainty regarding model specification and parameterization, the simplicity of these small instrument rules can be viewed as an advantage.

Finally, it is important to note that one important feature of this model is that agents have a perfect knowledge of the model and of the shocks that affect the economy. This, of course, has a direct effect on our results. An interesting avenue for future research would be to investigate the extent to which model uncertainty affects our results.

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