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**The Welfare Implications of Inflation versus  
Price-Level Targeting in a Two-Sector,  
Small Open Economy**

by

**Eva Ortega and Nooman Rebei**

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The views expressed in this paper are those of the authors.  
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## Abstract

The authors analyze the welfare implications of simple monetary policy rules in the context of an estimated model of a small open economy for Canada with traded and non-traded goods, and with sticky prices and wages. They find statistically significant heterogeneity in the degree of price rigidity across sectors. They also find welfare gains in targeting only the non-traded-goods inflation, since prices are found to be more sticky in this production sector, but those gains come at the cost of substantially increased aggregate volatility.

The authors look for the welfare-maximizing specification of an interest rate reaction function that allows for a specific price-level target. They find, however, that, overall, the higher welfare is achieved, given the estimated model for the Canadian economy, with a strict inflation-targeting rule where the central bank reacts to the next period's expected deviation from the inflation target and does not target the output gap.

*JEL classification: E31, E32, E52*

*Bank classification: Economic models; Exchange rates; Inflation targets*

## Résumé

Les auteurs analysent les implications pour le bien-être de règles simples de politique monétaire, en estimant un modèle de petite économie ouverte pour le Canada qui compte deux secteurs (biens échangeables et non échangeables) et où les prix et les salaires sont rigides. Ils observent que le degré de rigidité des prix diffère de façon statistiquement significative entre les deux secteurs. Ils constatent également que la poursuite d'une cible fondée exclusivement sur l'inflation des prix des biens non échangeables permet un gain de bien-être, car les prix de ces biens sont plus rigides que ceux des biens échangeables. Ce gain est toutefois obtenu au prix d'une augmentation substantielle de la volatilité globale.

Les auteurs cherchent à établir s'il existe une spécification de la fonction de réaction aux taux d'intérêt qui admet une cible précise pour le niveau des prix et qui permet de maximiser le bien-être. Ils remarquent cependant que le bien-être s'accroît davantage, dans le modèle estimé de l'économie canadienne, lorsque la banque centrale prend strictement pour cible l'inflation et qu'elle réagit aux écarts qu'elle s'attend à observer par rapport à la cible au cours de la période suivante, sans tenir compte de l'écart de production.

*Classification JEL : E31, E32, E52*

*Classification de la Banque : Modèles économiques; Taux de change; Cibles en matière d'inflation*



# 1 Introduction

This paper analyzes the welfare implications of simple monetary policy reaction functions in the context of a New Keynesian, small open economy model with a traded-goods and a non-traded-goods sector and with imperfect competition and staggered prices in the product and labour markets, estimated for the case of Canada. The model belongs to the class of dynamic stochastic general-equilibrium models with explicit microfoundations that constitute the so-called New Open Economy Macroeconomics (NOEM), pioneered by Obstfeld and Rogoff (1995), that has become a substantial literature, the results of which are partly summarized in Lane (2001), among others. Several such models have been estimated for Canada (for example, Ambler, Dib, and Rebei 2003 and Bergin 2003), none of which is in a multisectoral setting.

In this paper, we have two main objectives. First, we want to characterize the simple, Taylor-type monetary policy reaction function that would deliver higher welfare, given the estimated model. Throughout the paper, we consider simple reaction functions only. We do not compute the optimal monetary policy; i.e., we do not solve for the instrument value needed to bring inflation to target at each period, given all models' responses to realized shocks, but rather derive the proportional reaction of interest rates to deviations of inflation from target, and to the other arguments in the specified Taylor-type rule. We therefore compare the welfare gain of the welfare-maximizing standard Taylor rule with alternative specifications of the nominal interest rate feedback rule that allow for different coefficients on wage inflation as well as on price inflation in the traded-goods and non-traded-goods sectors, since the preferences of households may favour one sector over another.

Second, we evaluate the welfare gain or loss of using a monetary policy rule that reacts to deviations from the target of the price level. If willing to acknowledge that households would like to reduce uncertainty regarding the long-run purchasing power of money, a monetary authority that optimizes social welfare may want to target the price level on top of, or instead of, the inflation-rate level. However, many issues arise when a price-level target is introduced, such as the implications for the volatility of the main macro variables, not the least of which is inflation itself (see, for example, Bank of Canada 1998). With an inflation target, the initial increase in the price level after a shock that pushes inflation above its target would not be reversed, so there would be a permanent rise in the price level. In contrast, with a price-level target, a shock that pushed the price level above its target path would initially cause inflation to rise above its long-run average, but as the central bank took action to return the price level to its target path, the inflation rate would have to decline below its long-run average for some time to unwind the effect of the initial positive shock on the price level.

To the best of our knowledge, neither of these two issues—i.e., characterizing the welfare-maximizing simple inflation-targeting rule and evaluating the welfare gain of alternative specifications of the monetary policy reaction function, including price-level targeting—has been explored in the context of a multisector, small open economy NOEM model.<sup>1</sup>

The model economy aims at representing the main features needed for conducting monetary policy

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<sup>1</sup>Papers by Kollmann (2002) and Smets and Wouters (2002) are recent examples of where the welfare implications of monetary policy are investigated for small open economy NOEM models.

analysis in a tractable characterization of the Canadian economy. The main features of our model economy are that (i) there is monopolistic competition and staggered prices in the labour market, as well as in all product markets (domestic non-traded goods, domestic traded goods—for domestic consumption or for exports—and imports); the degree of price rigidity can differ across sectors and with respect to wages; (ii) labour and capital are mobile across sectors and each sector has its own technology process; (iii) traded goods are priced to market; and (iv) the systematic behaviour of the monetary policy is represented by the standard Taylor rule, where nominal interest rates respond to deviations of overall inflation from target and to the output gap. The economy is subject to eight shocks: three common domestic shocks (monetary policy shocks, shocks to the money demand, and shocks to the risk premium), two sector-specific technology shocks (to the non-traded-goods sector and to the domestic traded-goods sector), and three foreign shocks (output, inflation, and nominal interest rate). The model is estimated using Bayesian techniques for quarterly Canadian data. Our estimates seem reasonable and are compatible with other small open economy estimated models in the NOEM literature for the Canadian case. We find statistically significant heterogeneity in the degree of nominal rigidity across sectoral prices, but wages are the stickiest prices of all.

We evaluate the welfare gains of alternative specifications of a simple inflation-targeting rule using a second-order approximation of the expected permanent utility in each case compared with that of the estimated rule. We also compare monetary policy rules according to their implications in terms of aggregate fluctuations. In particular, we compute the unconditional volatility they imply for the utility and its arguments, as well as the unconditional volatility they imply for some crucial macro variables, such as output, inflation, and the nominal interest rate. We also compute the long-run variance decomposition under each monetary policy rule, the impulse responses to different shocks, and the prediction for the time series of the inflation deviations with respect to target, in order to gauge the amount of time in which inflation would be out of a certain range, given the monetary policy reaction function and the type of shock.

We find that there would have been some welfare improvement with respect to the estimated rule for the past three decades in Canada had the central bank been a slightly more aggressive inflation targeter, i.e., with no reaction to the output gap. Despite the fact that the nominal wage is the stickier price, we find welfare losses if the central bank were to target wage inflation rather than consumer price index (CPI) inflation. Impulse-response functions show that pure CPI inflation targeting brings the main macroeconomic variables (particularly aggregate demand) closer to their reaction in the case of flexible prices and wages than targeting wage inflation. However, a substantial welfare gain is made from targeting sectoral rather than aggregate inflation, particularly from targeting only inflation deviations from target in the stickier sector, i.e., non-traded goods. But this higher welfare comes at the expense of higher volatility in the main macro variables, including inflation and output (while inflation in the non-traded-goods sector is stabilized), than when targeting aggregate inflation.

Finally, we compute the welfare implications of moving away from strict inflation targeting to pure price-level targeting. We find that there is no noticeable welfare gain in doing so. A hybrid rule is preferable to strict inflation targeting only when the reaction to price and inflation deviations from target is very low, i.e., when monetary policy is not aggressive and therefore takes far longer to bring about price and inflation stabilization, but the welfare gain is still virtually unnoticeable and comes from the lower volatility induced

by the mild reaction of the monetary policy. When exploring the welfare implications of price-level or hybrid rules for sectoral prices, it is always preferable to target only the non-traded-goods sector, as was the case under strict inflation targeting.

Still, strict inflation targeting with moderate nominal interest rate smoothing and no output-gap targeting is the simple rule that delivers higher welfare, particularly when the central bank reacts to expected future deviations from target inflation instead of to contemporaneous inflation deviations.

The remainder of the paper is organized as follows. In section 2, we describe the model. In section 3, we describe the estimation method and discuss the parameter estimates. We outline the more relevant quantitative implications of the model in section 4. In section 5, we discuss the optimized parameterization for the monetary policy rule under alternative specifications of inflation-targeting Taylor-type rules. In section 6, we analyze the effect of the size of the non-traded-goods sector on the choice of the inflation rate to target. In section 7, we explore the welfare implications of considering price-level and hybrid targeting rules. In section 8, we consider forward-looking monetary policy reaction functions, and we offer conclusions in section 9.

## 2 The Model

### 2.1 Households

The  $i^{th}$  household chooses consumption  $c_t(i)$ , investment  $i_t(i)$ , money balances  $M_t(i)$ , hours worked  $h_t(i)$ , local riskless bonds  $Bd_t(i)$ , and foreign bonds  $Bd_t^*(i)$  that maximize its expected utility function, and it sets the wage rate constrained to a Calvo-type nominal rigidity in wages.

The preferences of the  $i^{th}$  household are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left( c_t(i), \frac{M_t(i)}{P_t}, h_t(i) \right), \quad (1)$$

where  $\beta \in (0, 1)$ ,  $E_0$  is the conditional expectations operator,  $M_t$  denotes nominal money balances held at the end of the period, and  $P_t$  is a price index that can be interpreted as the CPI. The functional form of time  $t$  utility is given by

$$U(\cdot) = \frac{\gamma}{\gamma - 1} \log \left( c_t(i)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} \left( \frac{M_t(i)}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right) + \eta \log(1 - h_t(i)), \quad (2)$$

where  $\gamma$  and  $\eta$  are positive parameters. Total time available to the household in the period is normalized to one. The  $b_t$  term is a shock to money demand. It follows the first-order autoregressive process given by

$$\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}, \quad (3)$$

with  $0 < \rho_b < 1$  and where the serially uncorrelated shock  $\varepsilon_{bt}$  is normally distributed with zero mean and

standard deviation  $\sigma_b$ . The household's budget constraint is given by

$$P_t c_t(i) + P_t [i_t(i) + CAC_t(i)] + M_t + \frac{Bd_t(i)}{R_t} + \frac{e_t Bd_t^*(i)}{\kappa_t R_t^*} \leq \\ W_t(i) h_t(i) + R_t^k k_t(i) + M_{t-1}(i) + Bd_{t-1}(i) + e_t Bd_{t-1}^*(i) + T_t + D_t, \quad (4)$$

where  $CAC_t(i) = \frac{\chi}{2} \left( \frac{i_t(i)}{k_t(i)} - \delta \right)^2 k_t(i)$  is the cost faced each time the household adjusts its stock of capital  $k_t(i)$ ,  $i_t(i)$  is the investment,  $W_t(i)$  is the nominal wage rate,  $R_t^k$  is the nominal interest on rented capital,  $Bd_t^*(i)$  and  $Bd_t(i)$  are foreign-currency and domestic-currency bonds purchased in  $t$ , and  $e_t$  is the nominal exchange rate. Domestic-currency bonds are used by the government to finance its deficit.  $R_t$  and  $R_t^*$  denote, respectively, the gross nominal domestic and foreign interest rates between  $t$  and  $t+1$ . The household also receives nominal lump-sum transfers from the government  $T_t$ , as well as nominal profits  $D_t = D_t^T + D_t^{NT} + D_t^M$  from domestic producers of traded and non-traded goods and from importers of intermediate goods.

We assume that each household  $i$  sells in a monopolistically competitive market their labour supply,  $h_t(i)$ , to a representative, competitive firm that transforms it into aggregate labour input,  $h_t$ , using the following technology:

$$h_t = \left[ \int_0^1 h_t(i)^{\frac{\vartheta^h - 1}{\vartheta^h}} di \right]^{\frac{\vartheta^h}{\vartheta^h - 1}}, \quad (5)$$

where  $\vartheta^h > 1$  is defined as the constant elasticity of substitution (CES) between differentiated labour skills. The demand for individual labour by the labour aggregator firm is

$$h_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\vartheta^h} h_t, \quad (6)$$

where  $W_t$  is the aggregate wage rate that is related to individual household wages,  $W_t(i)$ , via the relationship

$$W_t = \left[ \int_0^1 W_t(i)^{1-\vartheta^h} di \right]^{\frac{1}{1-\vartheta^h}}. \quad (7)$$

Households face a nominal rigidity coming from a Calvo-type contract on wages. When allowed to do so, with probability  $(1 - d_h)$  each period, the household chooses the nominal-wage contract,  $\tilde{W}_t(i)$ , to maximize its utility.<sup>2</sup>

$\kappa_t$  is a risk premium that reflects departures from uncovered interest rate parity. It depends on the ratio of net foreign assets to domestic output:

$$\log(\kappa_t) = \varphi \left[ \exp \left( \frac{e_t Bd_t^*}{P_t y_t} \right) - 1 \right] + \log(\varpi_{\kappa t}) \quad (8)$$

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<sup>2</sup>There will thus be a distribution of wages  $W_t(i)$  across households at any given time  $t$ . We follow Christiano, Eichenbaum, and Evans (2001, 2005) and assume that there exists a state-contingent security that insures the households against variations in households' specific labour income. As a result, the labour component of households' income will be equal to aggregate labour income, and the marginal utility of wealth will be identical across different types of households. This allows us to suppose symmetric equilibrium and proceed with the aggregation.

with  $Bd_t^* = \int_0^1 Bd_t^*(i)di$ . By following this functional form, the risk premium ensures that the model has a unique steady state.<sup>3</sup> We allow for an exogenous shock on the risk premium whose law of motion is

$$\log(\varpi_{\kappa t}) = \rho_{\kappa} \log(\varpi_{\kappa t-1}) + \varepsilon_{\kappa t}, \quad (9)$$

with serially uncorrelated disturbance  $\varepsilon_{\kappa t}$  normally distributed with zero mean and standard deviation  $\sigma_{\kappa}$ , and with  $0 < \rho_{\kappa} < 1$ .

The foreign nominal interest rate,  $R_t^*$ , is exogenous and evolves according to the following stochastic process:

$$\log(R_t^*) = (1 - \rho_{R^*}) \log(R^*) + \rho_{R^*} \log(R_{t-1}^*) + \varepsilon_{R^* t}, \quad (10)$$

with  $0 < \rho_{R^*} < 1$  and where the serially uncorrelated shock,  $\varepsilon_{R^* t}$ , is normally distributed with zero mean and standard deviation  $\sigma_{R^*}$ .

Households also face a no-Ponzi-game restriction:

$$\lim_{T \rightarrow \infty} \left( \prod_{t=0}^T \frac{1}{\kappa_t R_t^*} \right) Bd_T^*(i) = 0.$$

The first-order conditions are as follows:

$$\frac{c_t(i)^{-\frac{1}{\gamma}}}{c_t(i)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t(i)^{\frac{\gamma-1}{\gamma}}} = \lambda_t(i) \quad (11)$$

$$\frac{b_t^{\frac{1}{\gamma}} m_t(i)^{-\frac{1}{\gamma}}}{c_t(i)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t(i)^{\frac{\gamma-1}{\gamma}}} = \lambda_t(i) \left( 1 - \frac{1}{R_t} \right) \quad (12)$$

$$\frac{\lambda_t(i)}{R_t} = \beta E_t \lambda_{t+1}(i) \frac{1}{\pi_{t+1}} \quad (13)$$

$$s_t E_t \frac{\pi_{t+1}^*}{\kappa_t R_t^*} = E_t \frac{s_{t+1}}{R_t} \pi_{t+1} \quad (14)$$

$$\lambda_t(i) \left[ 1 + \chi \left( \frac{i_t(i)}{k_t(i)} - \delta \right) \right] = \beta E_t \lambda_{t+1}(i) \left[ 1 + r_{k_{t+1}(i)} + \chi \left( \frac{i_{t+1}(i)}{k_{t+1}(i)} - \delta \right) - \delta + \frac{\chi}{2} \left( \frac{i_{t+1}(i)}{k_{t+1}(i)} - \delta \right)^2 \right] \quad (15)$$

$$\tilde{w}_t(i) = \frac{\vartheta^h}{\vartheta^h - 1} \frac{E_t \sum_{\tau=0}^{\infty} \beta^{\tau} d_h^{\tau} \frac{\eta}{(1-h_{t+\tau}(i))} h_{t+\tau}(i)}{E_t \sum_{\tau=0}^{\infty} \beta^{\tau} d_h^{\tau} \lambda_{t+\tau}(i) h_{t+\tau}(i) \prod_{k=1}^{\tau} \pi_{t+k}^{-1}}, \quad (16)$$

where lower-case letters are the real counterparts of the nominal variables explained before, except for  $s_t$ , which stands for the real exchange rate.  $\tilde{w}_t$  is the real wage contract.

<sup>3</sup>If domestic and foreign interest rates are equal, the time paths of domestic consumption and wealth follow random walks. For an early discussion of this problem, see Giavazzi and Wyplosz (1984). Our risk-premium equation is similar to the one used by Senhadji (1997). For alternative ways of ensuring that stationary paths exist for consumption in small open economy models, see Schmitt-Grohé and Uribe (2003).

## 2.2 Firms

Monopolistically competitive firms produce traded and non-traded goods. The traded goods are either imported or produced domestically, which in turn can either be sold at home or exported.

### 2.2.1 Non-traded-goods sector

A continuum of firms is indexed by  $j \in [0, 1]$  in the non-traded-goods sector. There is monopolistic competition in the market for non-traded goods, which are imperfect substitutes for each other in the production of the composite good  $y_t^N$ , produced by a representative competitive firm. Aggregate non-traded-goods output is defined using the Dixit and Stiglitz aggregator function

$$y_t^N = \left( \int_0^1 y_t^N(j)^{\frac{\vartheta^N - 1}{\vartheta^N}} dj \right)^{\frac{\vartheta^N}{\vartheta^N - 1}}, \quad (17)$$

where  $\vartheta^N$  is the elasticity of substitution between differentiated non-traded goods. Given the aggregate and individual prices  $P_t^N$  and  $P_t^N(j)$ , respectively, the non-traded final-good-producing firm chooses the production,  $y_t^N$ , that maximizes its profits. The first-order condition corresponds to the demand constraint for each intermediary firm  $j$

$$y_t^N(j) = \left( \frac{P_t^N(j)}{P_t^N} \right)^{-\vartheta^N} y_t^N, \quad (18)$$

where the price index for the composite imported goods is given by

$$P_t^N = \left( \int_0^1 P_t^N(j)^{1-\vartheta^N} dj \right)^{\frac{1}{1-\vartheta^N}}. \quad (19)$$

Each monopolistically competitive firm has a production function given by

$$y_t^N(j) = A_t^N [k_t^N(j)]^{\alpha^N} [h_t^N(j)]^{1-\alpha^N},$$

where  $A_t^N$  is the non-traded-goods sector-specific total-factor productivity that follows the stochastic process

$$\log(A_t^N) = (1 - \rho_{AN}) \log(A^N) + \rho_{AN} \log(A_{t-1}^N) + \varepsilon_{ANt} \quad (20)$$

with  $\varepsilon_{ANt}$  a non-serially correlated technology shock normally distributed with zero mean and standard deviation  $\sigma_{AN}$ .

Firms face a nominal rigidity coming from a Calvo-type contract on prices. When allowed to do so, with probability  $(1 - d_N)$  each period, the producer of non-traded good  $j$  sets the price  $\tilde{P}_t^N(j)$  to maximize its weighted expected profits. Therefore, each firm chooses  $k_t^N(j)$ ,  $h_t^N(j)$ , and  $\tilde{P}_t^N(j)$  through solving

$$\max_{\{k_t^N(j), h_t^N(j), \tilde{P}_t^N(j)\}} E_t \left[ \sum_{l=0}^{\infty} (\beta d_N)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \frac{D_{t+l}^N(j)}{P_{t+l}} \right], \quad (21)$$

where  $\lambda_t$  is the marginal utility of wealth for a representative household, and time  $t + l$  profits of the firm changing price at time  $t$  are

$$D_{t+l}^N(j) \equiv \tilde{P}_t^N(j) y_{t+l}^N(j) - W_{t+l} h_{t+l}^N(j) - R_{t+l}^k k_{t+l}^N(j).$$

The first-order conditions are

$$\frac{W_t}{P_t} = \xi_t(j)(1 - \alpha^N) \frac{y_t^N(j)}{h_t^N(j)} \quad (22)$$

$$\frac{R_t^k}{P_t} = \xi_t(j) \alpha^N \frac{y_t^N(j)}{k_t^N(j)} \quad (23)$$

$$\tilde{P}_t^N(j) = \left( \frac{\vartheta^N}{\vartheta^N - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_N)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \xi_{t+l}(j) y_{t+l}^N(j)}{E_t \sum_{l=0}^{\infty} (\beta d_N)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) y_{t+l}^N(j) \frac{1}{P_{t+l}}}, \quad (24)$$

where  $\xi_t(i)$  is the Lagrange multiplier associated with the production function constraint. It measures the real marginal cost of the firm in the non-traded-goods sector.

## 2.2.2 Traded-goods sector

Domestic firms producing goods in the traded sector have to solve a similar problem, except that each monopolistically competitive firm  $k$  produces two types of goods:  $y_t^{Td}(k)$ , which will be consumed in the domestic market, and  $y_t^X(k)$ , which will be exported, for  $k \in [0, 1]$ .

The production function is as follows:

$$y_t^T(k) = A_t^T [k_t^T(k)]^{\alpha^T} [h_t^T(k)]^{1-\alpha^T},$$

where  $A_t^T$  is the traded-goods sector-specific total-factor productivity

$$\log(A_t^T) = (1 - \rho_{A^T}) \log(A^T) + \rho_{A^T} \log(A_{t-1}^T) + \varepsilon_{A^T t} \quad (25)$$

and  $\varepsilon_{A^T t}$  is the serially uncorrelated shock, which is normally distributed with zero mean and standard deviation  $\sigma_{A^T}$ .

Each firm chooses  $k_t^T(k)$ ,  $h_t^T(k)$ ,  $P_t^{Td}(k)$ , and  $P_t^X(k)$ . We assume complete pricing to market for exports; i.e.,  $P_t^X(k)$  is labelled in U.S. dollars.<sup>4</sup> In addition, once the firm has the chance to update its price (with probability  $(1 - d_T)$  each period) it will choose simultaneously  $\tilde{P}_t^{Td}(k)$  and  $\tilde{P}_t^X(k)$ . The problem of each firm can be summarized by

$$\max_{\{k_t^T(k), h_t^T(k), \tilde{P}_t^{Td}(k), \tilde{P}_t^X(k)\}} E_t \left[ \sum_{l=0}^{\infty} (\beta d_T)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \frac{D_{t+l}^T(k)}{P_{t+l}} \right],$$

where time  $t + l$  profits of the firm changing price at time  $t$  are

$$D_{t+l}^T(k) \equiv \tilde{P}_t^{Td}(k) y_{t+l}^{Td}(k) + e_t \tilde{P}_t^X(k) y_{t+l}^X(k) - W_{t+l} h_{t+l}^T(k) - R_{t+l}^k k_{t+l}^T(k)$$

<sup>4</sup>There is substantial evidence in favour of the pricing-to-market hypothesis in the Canada-U.S. case. Engel and Rogers (1996) use CPI data for U.S. and Canadian cities and find that deviations from the law of one price are much higher for two cities located in different countries than for two equidistant cities in the same country. Also, there is evidence suggesting the prevalence of invoicing in U.S. dollars by foreign firms selling in the U.S. market. Indeed, according to the ECU Institute (1995), over 80 per cent of U.S. imports were invoiced in U.S. dollars.

under the constraints dictating the local and foreign demand for traded goods:

$$y_t^{Td}(k) = \left( \frac{P_t^{Td}(k)}{P_t^{Td}} \right)^{-\vartheta^T} y_t^{Td} \quad (26)$$

and

$$y_t^X(k) = \left( \frac{P_t^X(k)}{P_t^X} \right)^{-\vartheta^T} y_t^X, \quad (27)$$

where  $\vartheta^T$  is the elasticity of substitution between differentiated traded goods.

The first-order conditions are

$$\frac{W_t}{P_t} = \zeta_t(k)(1 - \alpha^T) \frac{y_t^T(k)}{h_t^T(k)} \quad (28)$$

$$\frac{R_t^k}{P_t} = \zeta_t(k) \alpha^T \frac{y_t^T(k)}{k_t^T(k)} \quad (29)$$

$$\tilde{P}_t^{Td}(k) = \left( \frac{\vartheta^T}{\vartheta^T - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_T)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \zeta_{t+l}(k) y_{t+l}^{Td}(k)}{E_t \sum_{l=0}^{\infty} (\beta d_T)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) y_{t+l}^{Td}(k) \frac{1}{P_{t+l}}} \quad (30)$$

$$\tilde{P}_t^X(k) = \left( \frac{\vartheta^T}{\vartheta^T - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_T)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \zeta_{t+l}(k) y_{t+l}^X(k)}{E_t \sum_{l=0}^{\infty} (\beta d_T)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) e_{t+l} y_{t+l}^X(k) \frac{1}{P_{t+l}}}, \quad (31)$$

where  $\zeta_{t+l}(k)$  is the real marginal cost of the firm in the traded-goods sector.

Similarly, the sector that produces final traded goods has the following aggregate functions:

$$y_t^{Td} = \left( \int_0^1 y_t^{Td}(k)^{\frac{\vartheta^T-1}{\vartheta^T}} dk \right)^{\frac{\vartheta^T}{\vartheta^T-1}} \quad (32)$$

and

$$y_t^X = \left( \int_0^1 y_t^X(k)^{\frac{\vartheta^T-1}{\vartheta^T}} dk \right)^{\frac{\vartheta^T}{\vartheta^T-1}} \quad (33)$$

with

$$y_t^T = y_t^{Td} + y_t^X, \quad (34)$$

where  $y_t^T$  is total production in the traded-goods sector, and  $y_t^{Td}$  and  $y_t^X$  are traded goods, respectively, for domestic and foreign markets.

The price indexes for domestically consumed traded goods and exports are as follows:

$$P_t^{Td} = \left( \int_0^1 P_t^{Td}(k)^{1-\vartheta^T} dk \right)^{\frac{1}{1-\vartheta^T}} \quad (35)$$

$$P_t^X = \left( \int_0^1 P_t^X(k)^{1-\vartheta^T} dk \right)^{\frac{1}{1-\vartheta^T}} \quad (36)$$

The foreign demand for locally produced goods is as follows:

$$y_t^X = \left( \frac{P_t^X}{P_t^*} \right)^{-\mu} y_t^*, \quad (37)$$

where  $\frac{\mu-1}{\mu}$  captures the elasticity of substitution between the exported goods and foreign-produced goods in the consumption basket of foreign consumers, and  $y_t^*$  and  $P_t^*$  are, respectively, foreign output and the price index. Both variables are exogenously given, and foreign output and inflation follow the stochastic processes

$$\begin{aligned} \log(y_t^*) &= (1 - \rho_{y^*}) \log(y^*) + \rho_{y^*} \log(y_{t-1}^*) + \varepsilon_{y^*t} \\ \log(P_t^*) &= (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log(\pi_{t-1}^*) + \varepsilon_{\pi^*t}, \end{aligned} \quad (38)$$

with  $0 < \rho_{y^*}, \rho_{\pi^*} < 1$  and where the serially uncorrelated shocks,  $\varepsilon_{y^*t}$  and  $\varepsilon_{\pi^*t}$ , are normally distributed with zero mean and standard deviation  $\sigma_{y^*}$  and  $\sigma_{\pi^*}$ , respectively.

### 2.2.3 Imported-goods sector

Finally, there is a continuum of intermediate-goods-importing firms indexed by  $i \in [0, 1]$ . Monopolistic competition takes place in the market for imported intermediate goods, which are imperfect substitutes for each other in the production of the composite imported good,  $y_t^M$ , produced by a representative competitive firm. We also assume Calvo-type staggered price setting in the imported-goods sector to capture the empirical evidence on incomplete exchange rate pass-through into import prices.<sup>5</sup> Thus, when allowed to do so (with probability  $(1 - d_M)$  each period), the importer of good  $i$  sets the price,  $\tilde{P}_t^M(i)$ , to maximize its weighted expected profits. It solves

$$\max_{\{\tilde{P}_t^M(i)\}} E_t \left[ \sum_{l=0}^{\infty} (\beta d_M)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) \frac{D_{t+l}^M(i)}{P_{t+l}} \right],$$

where time  $t + l$  profits of the firm changing price at time  $t$  are

$$D_{t+l}^M(i) = \left( \tilde{P}_t^M(i) - e_{t+l} P_{t+l}^* \right) \left( \frac{\tilde{P}_t^M(i)}{P_{t+l}} \right)^{-\vartheta^M} y_{t+l}^M \quad (39)$$

with  $\vartheta^M$  representing the elasticity of substitution across differentiated imported goods. Note that the marginal cost of the importing firm is  $e_t P_t^*$ <sup>6</sup> and thus its real marginal cost is the real exchange rate  $s_t \equiv \frac{e_t P_t^*}{P_t}$ . The first-order condition is

$$\tilde{P}_t^M(i) = \left( \frac{\vartheta^M}{\vartheta^M - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_M)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) y_{t+l}^M(i) e_{t+l} P_{t+l}^* / P_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta d_M)^l \left( \frac{\lambda_{t+l}}{\lambda_t} \right) y_{t+l}^M(i) / P_{t+l}}. \quad (40)$$

<sup>5</sup>Campa and Goldberg (2002) find that they can reject the hypothesis of complete short-run pass-through in 22 of the 25 Organisation for Economic Co-operation and Development (OECD) countries of their study for the period 1975–99, but they find complete long-run pass-through. Ghosh and Wolf (2001) argue that sticky prices or menu costs are a preferable explanation for imperfect pass-through since they are compatible with complete long-run pass-through, while that is not the case for explanations based on international product differentiation. The evidence of incomplete exchange rate pass-through in Canada is well documented and seems to conclude that zero pass-through has almost been reached in the recent past. See, for example, Bailliu and Bouakez (2004), Kichian (2001), and Leung (2003).

<sup>6</sup>For convenience, we assume that the price in foreign currency of all imported intermediate goods is  $P_t^*$ , which is also equal to the foreign price level.

As in the other cases, aggregate imported output is defined using the Dixit and Stiglitz aggregator function

$$y_t^M = \left( \int_0^1 y_t^M(i)^{\frac{\vartheta^M-1}{\vartheta^M}} di \right)^{\frac{\vartheta^M}{\vartheta^M-1}}$$

and the price index for the aggregated good is

$$P_t^M = \left( \int_0^1 P_t^M(i)^{1-\vartheta^M} di \right)^{\frac{1}{1-\vartheta^M}}. \quad (41)$$

## 2.2.4 Final-goods aggregators

The final domestically consumed good,  $y_t^d$ , is produced by a competitive firm that uses non-traded goods,  $y_t^N$ , and domestically consumed traded goods,  $y_t^{Td}$ , as inputs subject to the following CES technology:

$$y_t^d = \left[ n^{\frac{1}{\phi}} (y_t^N)^{\frac{\phi-1}{\phi}} + (1-n)^{\frac{1}{\phi}} (y_t^{Td})^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad (42)$$

where  $n > 0$  is the share of non-traded goods in the domestic goods basket at the steady state, and  $\phi > 0$  is the elasticity of substitution between non-traded and non-exported traded goods. Profit maximization entails

$$y_t^N = n \left( \frac{P_t^N}{P_t^d} \right)^{-\phi} y_t^d \quad (43)$$

and

$$y_t^{Td} = (1-n) \left( \frac{P_t^{Td}}{P_t^d} \right)^{-\phi} y_t^d. \quad (44)$$

Furthermore, the domestic final-good price,  $P_t^d$ , is given by

$$P_t^d = \left[ n(P_t^N)^{1-\phi} + (1-n)(P_t^{Td})^{1-\phi} \right]^{1/(1-\phi)}. \quad (45)$$

Finally, we aggregate domestic and imported goods using a CES function as follows:

$$z_t = \left[ m^{\frac{1}{\nu}} (y_t^d)^{\frac{\nu-1}{\nu}} + (1-m)^{\frac{1}{\nu}} (y_t^M)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \quad (46)$$

where  $m > 0$  is the share of domestic goods in the final-goods basket at the steady state, and  $\nu > 0$  is the elasticity of substitution between domestic and imported goods. The first-order conditions are

$$y_t^d = m \left( \frac{P_t^d}{P_t} \right)^{-\nu} z_t \quad (47)$$

and

$$y_t^M = (1-m) \left( \frac{P_t^M}{P_t} \right)^{-\nu} z_t. \quad (48)$$

The final-good price,  $P_t$ , which corresponds to the CPI, is given by

$$P_t = \left[ m(P_t^d)^{1-\nu} + (1-m)(P_t^M)^{1-\nu} \right]^{1/(1-\nu)}. \quad (49)$$

Aggregate output is used for consumption, investment, and for covering the cost of adjusting capital

$$z_t = c_t + i_t(1 + CAC_t). \quad (50)$$

The gross domestic product is  $y_t = z_t + y_t^X - y_t^M$ . Finally, sectoral hours and capital simply sum to the aggregate hours and capital offered by households, i.e.,  $h_t^N + h_t^T = h_t$  and  $k_t^N + k_t^T = k_t$ .

### 2.3 The government

The government budget constraint is given by

$$T_t + Bd_{t-1} = M_t - M_{t-1} + \frac{Bd_t}{R_t}, \quad (51)$$

which is combined with the no-Ponzi-game restriction:

$$\lim_{T \rightarrow \infty} \left( \prod_{t=0}^T \frac{1}{R_t} \right) Bd_T = 0.$$

We consider a simple decision rule for the nominal interest rate, such as the standard Taylor rule,

$$\log(R_t/R) = \rho_R \log(R_{t-1}/R) + \rho_\pi \log(\pi_t/\pi) + \rho_y \log(y_t/y) + \varepsilon_{Rt}, \quad (52)$$

where  $R$ ,  $\pi$ , and  $y$  are the steady-state values of the gross nominal interest rate, CPI inflation, and real gross domestic output, and where  $\varepsilon_{Rt}$  is a zero-mean, serially uncorrelated monetary policy shock with standard deviation  $\sigma_R$ .

## 3 Estimation

The above model is estimated using Bayesian estimation techniques that update prior distributions for the deep parameters of the model, which are defined according to a reasonable calibration, using the actual data. The estimation is done using recursive simulation methods, particularly the Metropolis-Hastings algorithm, which has been applied to estimate similar dynamic stochastic general-equilibrium models in the literature, such as Smets and Wouters (2003).

The model has eight shock processes: three common domestic shocks—to the monetary policy,  $\varepsilon_{Rt}$ , to the money demand,  $\varepsilon_{bt}$ , and to the risk premium,  $\varepsilon_{\kappa t}$ ; two sector-specific technology shocks—to the non-traded sector,  $\varepsilon_{ANt}$ , and to the traded one,  $\varepsilon_{ATd t}$ ; and three foreign shocks—to the foreign output,  $\varepsilon_{y^*t}$ , to the foreign inflation,  $\varepsilon_{\pi^*t}$ , and to the foreign nominal interest rate,  $\varepsilon_{R^*t}$ . To identify them in the estimation process, we need to use the same number of actual series. We choose them to be as informative as possible. We use HP-filtered and seasonally adjusted quarterly series for Canada for the period 1972Q1–2003Q4. The series are real exchange rate (against the U.S. dollar), real output, nominal interest rate on three-month T-bills, real M2 per capita (deflated with the CPI), CPI inflation, U.S. real output per capita, U.S. CPI inflation, and nominal U.S. interest rate on three-month T-bills.

Table 1 shows the prior distributions we have imposed for the deep parameters of the model, as well as the median and 90 per cent confidence interval for the posterior distributions. Figures 1 and 2 convey the

same information by drawing the prior distributions, in thick lines, together with the posterior ones, in thin lines.

We have borrowed some of the prior distributions from the literature, but for those for which we had no references, we used common sense, while trying to construct small restrictive priors. We selected beta distributions for those coefficients that we wanted to restrict to lie between 0 and 1, such as the autocorrelation coefficients of the shock processes or the share parameters. Gamma and Inverted Gamma distributions are imposed, when required, to guarantee real positive values.

All three sectors—domestic traded goods, imports, and non-traded goods—are treated symmetrically a priori. They are given the same degree of nominal rigidity, in the form of an average prior probability of not changing prices of 0.67, which corresponds to changing prices every three quarters on average. The priors for the elasticities of substitution between differentiated goods are also equal across sectors, corresponding to equal steady-state markups across sectors.

Some parameter values are taken as fixed rather than given a prior distribution that will be updated with the data; we calibrate them to values similar to those found in the literature. We performed sensitivity analysis on their calibrated values and observed that the estimates of the remaining model parameters were unchanged. These parameters are: the subjective discount rate,  $\beta = 0.99$ , which implies an annual real interest rate of 4 per cent; the weight of leisure in the utility function,  $\eta$ , which is calibrated to yield a steady-state share of time devoted to market activities of 30 per cent; the quarterly depreciation rate of capital,  $\delta = 0.025$ ; the gross steady-state markups in all sectors,  $\frac{\vartheta}{\vartheta-1} = 1.14$ , which lie between the estimates of the empirical literature (see, for example, Basu 1995); and the preference parameter governing the elasticity of substitution between consumption and real balances,  $\gamma = 0.1$ , for which we have taken a value that lies between values estimated for Canada by Dib (2003),  $\gamma = 0.03$ , and Ambler, Dib, and Rebei (2004),  $\gamma = 0.2$ .

We find that data are most informative for the adequate parameterization of the price stickiness, the monetary policy reaction function, and the shocks processes.

The prior of equal nominal rigidity across sectors does not hold, which is consistent with the findings of Bils and Klenow (2004), who document a high degree of heterogeneity in the frequency of price changes across retail goods and services. Indeed, we find significant heterogeneity in the degree of price stickiness across sectors; import prices are the more flexible (with posterior median duration for prices of two quarters), and non-traded-goods prices are more sticky (posterior median of almost three quarters). The prices of domestic traded goods are estimated to have a posterior median duration of two and one-half quarters.<sup>7</sup> Table 1 shows that the 90 per cent posterior confidence interval for  $d_M$  does not even overlap with those for  $d_N$  and  $d_T$ . Similarly, Figure 2 shows how the equal prior distribution barely overlaps with the posterior distributions for  $d_M$ . However, and consistent with virtually any study that examines wage and price rigidities, the highest nominal stickiness of all is found for wages, with an estimated posterior duration of five quarters. In fact, one

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<sup>7</sup>Our sectoral estimates bridge the gap between the usual estimates of around four quarters for the aggregate price level and the microeconomic evidence of average duration of prices at the individual firm level of around one quarter. In a back-of-the-envelope calculation, if we weight the sectoral posterior median durations by the posterior median estimates of the steady-state weights of the sectoral outputs in final consumption, we obtain an overall economy duration of prices of two and one-third quarters, i.e., seven months. Those estimated weights in the final consumption basket are 0.29 for non-tradable goods, 0.25 for tradable goods produced domestically, and 0.46 for imports.

of the possible reasons behind the higher stickiness of prices for non-traded goods versus prices for traded goods can be the higher weight of wages in the cost of production of non-traded goods.

This heterogeneity in the nominal rigidity is an important finding and will condition many of the model implications for the dynamics as well as for the welfare improvement of alternative specifications of the monetary policy reaction function. This is especially so when the central bank is willing to weight differently inflation stabilization in different sectors, owing to the consideration that agents may derive more utility from consumption from one particular sector than from another.

The posterior estimates of the Taylor rule almost halve the prior degree of interest rate smoothing (posterior median  $\rho_R = 0.46$ ), somewhat reduce the reaction to deviations of inflation from target to  $\rho_\pi = 1.19$ , and find a significant but low reaction to the output gap, with a posterior median coefficient  $\rho_y = 0.3$ . The historical estimated Taylor rule, therefore, is an inflation-targeting one with moderate concern for output stabilization and with some sluggishness in the monetary policy instrument.

The actual data are also found to be very informative for estimating the volatility of shocks, which were given equal priors. Posterior estimates indicate that aggregate demand shocks, represented by the money demand, are the more volatile—although the variance decomposition in the next section shows that they play a very small role in explaining aggregate fluctuations in this model—followed by shocks to the non-traded technology.

Data, however, found little informative for a number of parameters whose posterior distributions are coincident with their priors. Particularly, this is the case for the parameter governing risk-premium dynamics,  $\varphi$ , or those governing the steady-state shares of traded and non-traded goods in the domestic final composite good and those of domestic and imported content of the final consumption good,  $n$  and  $m$ , respectively.

## 4 Quantitative Implications of the Model

This section discusses the dynamics of the estimated model in terms of the variance decomposition of its main endogenous variables and in terms of their impulse responses to the shocks contemplated in the model. We discuss only the responses to the three shocks that are found to be more important in explaining the variability of consumption (the main determinant of utility and, hence, welfare), inflation, and output. These responses are the technology shock in the non-traded-goods sector, the monetary policy shock, and the foreign monetary policy shock.

### 4.1 Variance decomposition

Table 2 shows the decomposition of the long-run variance of the main endogenous variables of the model into the contribution of each of the eight shocks.

The business cycle volatility of the output in each production sector, traded and non-traded, is explained mainly by its corresponding sector-specific technology shock, but there is a substantial role for the monetary policy shocks as well, the domestic policy shocks on domestic traded production, and foreign shocks on exports and imports. Aggregate inflation is found to be better explained by technology shocks (through their impact on the non-traded inflation) and by foreign interest rate and risk-premium shocks (through

the impact of both on imports inflation) than by monetary policy shocks in the past three decades. Final spending, i.e., consumption and investment, are explained mainly by the non-traded-goods technology shock, which is one of the shocks with higher estimated volatility, although the steady-state share of the non-traded-goods sector in the final good is only one-third. Hours worked are substantially explained by technology shocks in the two sectors, but are also clearly affected by monetary policy shocks. Finally, the volatility of the real exchange rate is explained by shocks to technology, foreign monetary policy, and the risk premium.

## 4.2 Responses to a foreign shock

Figure 3 represents the responses in terms of percentage deviations with respect to the steady state to a one-period increase of 100 basis points in the monetary policy instrument of the foreign economy, the United States.

The uncovered interest rate parity yields a nominal and real impact depreciation of the Canadian dollar (2 per cent posterior median depreciation on impact of the real exchange rate,  $s_t$ ). The real depreciation causes a direct rise in the marginal cost of the importing firms and is therefore translated into higher import prices and fewer imports,  $y_t^M$ . It is important to note, however, that as result of the estimated sluggishness of import prices, the exchange rate pass-through is not complete, and imports inflation rises by only 50 basis points.

Exports benefit from the depreciation. Because exports are priced in the foreign currency but traded-sector firms maximize their profits in Canadian dollars, the depreciation by itself increases the benefits from the part of the production that is exported. As a result, producers in the traded-goods sector lower export prices and increase their exports on impact.

The increase of imports inflation makes aggregate inflation rise, which causes a monetary policy contraction. That, in turn, decreases demand ( $c_t$  and  $i_t$ ) that further reduces imports demand but also decreases demand of non-traded and of traded goods produced domestically. The monetary policy contraction also helps undo the initial depreciation.

## 4.3 Responses to a sectoral shock

Figure 4 represents the responses to a positive one-period technology shock of 1 per cent in the non-traded-goods sector only.

Increased production in the non-traded-goods sector<sup>8</sup> raises demand throughout the economy and therefore increases output in the traded and imports sectors, as well.

Prices in the non-traded-goods sector fall on impact, leading to a mild fall in overall inflation, which in turn causes an expansionary reaction of the monetary policy that feeds into a further increase of demand and causes a nominal and real depreciation on impact.

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<sup>8</sup>As is well known in the literature, sticky prices prevent the 1 per cent increase in total-factor productivity from being fully transformed into a 1 per cent increase in  $y_t^{NT}$ . Since capital is predetermined, the only way to generate that lower output increase is by reducing hours worked on impact, which is observed in Figure 4.  $h_t^{NT}$  falls on impact but increases after four quarters.

Growth in demand increases imports as well as imports inflation, which helps to quickly undo the fall in aggregate inflation.

As before, the depreciation increases the profits of the exported production in the traded sector, but demand in exports does not rise (foreign output being exogenous). Thus, maximization of the profit in the traded-goods sector makes firms lower export prices fixed in U.S. dollars (pricing to market) and increase exports.

#### 4.4 Responses to a common domestic shock

Figure 5 represents the responses to a temporary monetary policy contraction. The nominal interest rate shock increases by 100 basis points for one period. On impact, the monetary policy instrument rises by less than 1 per cent because of the immediate drop in inflation and because of significant interest rate smoothing. In fact, nominal interest rates rise by only one-half of the 1 per cent shock. Inflation falls on impact because of the impact decrease in demand and consequently in activity in every sector—traded goods, non-traded goods, and imports.

The monetary policy contraction causes a nominal and real impact appreciation of the Canadian dollar. Since export prices are being set in U.S. dollars, the appreciation reduces exporters' profits, and export prices consequently rise, which causes a drop in exports.

### 5 Simple Inflation-Targeting Rules

In this section, we search for the parameterization of feedback Taylor-type interest rate rules, similar to equation (52), that maximize household welfare given our estimated model. We evaluate the welfare gain they represent with respect to the estimated monetary policy reaction function (or “historical rule” in the tables), as well as their implications in terms of aggregate fluctuations.

The welfare implications are displayed in Table 3. Table 4 reports another dimension for comparing alternative monetary policy reaction functions: the unconditional volatility they imply for the utility and its arguments as well as for a number of crucial macro variables, i.e., output, inflation, and the nominal interest rate. We also compute the following business cycle implications for each different monetary policy rule discussed: the long-run variance decomposition (Tables 5, 6, and 7), the impulse responses to different shocks (Figures 6, 7, and 8), and the prediction for the time series of the inflation deviations with respect to target—to gauge the proportion of time inflation would be out of a certain range given the monetary policy reaction function and the type of shock (Figures 9 and 18 to 25).

The search for the welfare-maximizing feedback monetary policy rules is set out as follows. We maximize the unconditional expectation of lifetime utility<sup>9</sup> of households over the parameters of the Taylor rule. This

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<sup>9</sup>Schmitt-Grohé and Uribe (2004b) adopt the conditional welfare optimization in their framework and they consider the non-stochastic steady state as an initial state of the economy. By computing the unconditional long-run utility, we do not consider the effect of the initial state. Transition costs are crucially dependent on that initial state, especially if the real state of the economy is never at the deterministic level. In addition, Schmitt-Grohé and Uribe (2004b) show that the optimal rule is robust to these definitions of welfare, but that the welfare improvement could be different in the sense that it is higher in the case of unconditional welfare given that no short-term transition costs are incurred.

implies:

$$\max_{\rho_\pi, \rho_y} E \{u(c_t, m_t, h_t)\}.$$

We measure the welfare gain associated with a particular monetary policy in terms of its compensating variation. That is, we calculate the percentage of lifetime consumption that should be added to that obtained under the estimated Taylor rule in order to give households the same unconditional expected utility as under the new monetary policy rule scenario:

$$E \{u(c_t(1 + \text{welfare gain}), m_t, h_t)\} = E \{u(\tilde{c}_t, \tilde{m}_t, \tilde{h}_t)\},$$

where variables without tildes are obtained under the estimated rule described before, and variables with tildes are under the optimized Taylor rule. Based on the results found in Kim and Kim (2003) and subsequent literature, we compute the long-run average utility by means of a second-order approximation around the steady-state utility. In particular, we follow the approach of Schmitt-Grohé and Uribe (2004a):

$$E \left( u \left( \tilde{c}_t, \tilde{m}_t, \tilde{h}_t \right) \right) = u(c, m, h) + u' E \left( \hat{c}_t, \hat{m}_t, \hat{h}_t \right) + \frac{1}{2} E \left( \hat{c}_t, \hat{m}_t, \hat{h}_t \right)' u'' \left( \hat{c}_t, \hat{m}_t, \hat{h}_t \right),$$

where  $u'_t$  and  $u''$  are the first and second derivatives, respectively, of the utility function with respect to its arguments, evaluated at their deterministic steady-state values, and variables with hats measure deviations from their levels in the deterministic steady state. The compensating variation in consumption can therefore be decomposed into a first-level effect and a second-level or stabilization effect, i.e., into the welfare gains of the new parameterization of the monetary policy owing to its effect on the average levels of consumption, real balances, and leisure and its effect on their volatilities. The first-level effect is defined as:

$$E \{u(c_t(1 + \text{1st-level effect}), m_t, h_t)\} = u(c, m, h) + u' E \left( \hat{c}_t, \hat{m}_t, \hat{h}_t \right),$$

and the second-level effect as:

$$E \{u(c_t(1 + \text{2nd-level effect}), m_t, h_t)\} = u(c, m, h) + \frac{1}{2} E \left( \hat{c}_t, \hat{m}_t, \hat{h}_t \right)' u'' \left( \hat{c}_t, \hat{m}_t, \hat{h}_t \right).$$

The overall effect in all cases is such that, approximately,  $(1 + \text{welfare gain}) \approx (1 + \text{1st-level effect})(1 + \text{2nd-level effect})$ . Table 3 reports the welfare gains, together with the unconditional long-run average values of the arguments of the utility function, as well as that of the log utility itself.

In what follows, we limit our attention to the Taylor-type rules that guarantee the existence of a unique and stable equilibrium in the neighbourhood of the deterministic steady state. We also restrict our search

to monetary policy reactions to price and output deviations from target; we do this by keeping the degree of nominal interest smoothing unchanged and equal to the posterior median of the estimated value, i.e.,  $\rho_R = 0.46$ .<sup>10</sup>

Our reference interest rate feedback rule is the estimated one where, on top of that moderate nominal interest rate smoothing, the monetary authority has targeted inflation but not very aggressively (the posterior median estimate for the reaction to deviations of the aggregate CPI inflation from target is slightly above 1,  $\rho_\pi = 1.19$ ) and there has been a significant although weak response of the monetary policy to the output gap (posterior median of  $\rho_y = 0.31$ ).

## 5.1 CPI inflation rate targeting

First, we consider the case where the central bank targets the same variables as in the historical rule, i.e., aggregate CPI inflation and the output gap. The welfare-maximizing Taylor rule implies a very similar level of aggressiveness with respect to inflation deviations from target to that of the estimated historical rule,  $\rho_\pi = 1.20$ , but, contrary to the historical case, there is no response to the output gap,  $\rho_y = 0$ .

The historical rule entails a welfare cost of 0.08 per cent of the lifetime consumption associated with the optimized CPI inflation-targeting rule (see second row in Table 3). Most of the welfare improvement of choosing  $\rho_\pi = 1.20$  and  $\rho_y = 0$  rather than the estimated parameters comes from the first-level effect or improvement in long-run average utility, which amounts to a 0.11 per cent increase in lifetime consumption. This welfare-maximizing monetary policy reaction function implies slightly higher volatility in the utility arguments (see second row of Table 4), which is captured by a negative second-order effect, as well as in output, while it only very marginally stabilizes inflation.

As Table 4 shows, not only consumption and the other arguments in the utility function show higher volatility; so do output and the monetary policy instrument. Instead, inflation remains with similar levels of volatility. Table 5 shows the medians of the long-run variance decomposition of model variables under this new monetary policy rule. It does not differ much from that in Table 2. However, it is worth noting that consumption variability is better explained by domestic shocks, including the monetary policy shock, and less by foreign shocks than under the historical rule. Inflation variability owes much more to monetary policy shocks than under the historical rule, but the explanatory power of foreign shocks has not substantially decreased. In general, monetary policy shocks are more responsible for aggregate variability under this

<sup>10</sup>Several reasons motivate the choice of fixing  $\rho_R$ . One is that without interest rate smoothing, there would be indeterminacy for values of the coefficient on inflation smaller than one. By keeping  $\rho_R$  at its estimated value, we can compute the welfare gains of a wider range of values for  $\rho_\pi$ , including those smaller than one.

Another important reason is that because the optimized rule would aim at maximizing inflation stabilization rather than instrument smoothing, the welfare-maximizing value of  $\rho_R$  is very likely going to be zero. Indeed, Schmitt-Grohé and Uribe (2004b) find that the optimal degree of interest rate smoothing for Taylor rules in the Christiano, Eichenbaum, and Evans (2001) model is zero. However, they also look, as we do, for the parameterization of the Taylor rule that delivers higher utility for degrees of interest rate smoothing closer to the observed ones. Keeping our frame of analysis of alternative monetary policy reaction functions close to the observed features of monetary policy as it is implemented in practice constitutes a further reason for keeping  $\rho_R$  fixed as well as for sticking to simple Taylor rules. A final reason is that maximizing welfare over several parameters is computationally expensive.

optimized strict inflation-targeting rule than under the historical one.

In terms of the responses to shocks, the impulse responses obtained by replacing the historical rule with this new optimized CPI inflation-targeting rule are quite similar. The median responses are displayed in Figures 6 to 8, together with those that would have been obtained in the case of flexible prices and wages<sup>11</sup> and with an alternative monetary policy reaction function that will be explained later, i.e., targeting wage inflation.

What is the effect of this alternative monetary policy specification on the likelihood of inflation being out of target? The top two panels of Figure 9 show the time series of deviations of annualized inflation from target simulated out of the estimated model under the historical rule (top left panel) and under the CPI inflation stabilization rule (top right panel). The graphs show the median and 90 per cent bands of simulating 500 times inflation series of 200 observations each. Figure 9 reports the resulting series when all estimated shocks are occurring, with their respective estimated standard deviations. This figure reads as follows. Under the historical rule and given the observed shocks in the past three decades, 90 per cent of the times inflation has been around target plus or minus 5 per cent, i.e., if the target is a 2 per cent inflation rate, it says it has been between 7 per cent and  $-3$  per cent. Instead, had the monetary policy reaction function been the CPI stabilization function suggested in this subsection, the annual inflation rate would have been between 0.04 under or above the target 90 per cent of the time, e.g., between 1.96 per cent and 2.04 per cent inflation if the target is 2 per cent.

Figures 18 to 25 report the same simulated annual CPI inflation series under different monetary policy reaction functions when only one of the eight estimated shocks occurs at one time. These figures show that the shocks that are found to be responsible for the deviations of CPI inflation from target are, in order of importance, the total-factor productivity shock in the non-traded-goods sector, the shock to the foreign interest rate, the shock to the risk premium, the monetary policy shock, and the total-factor productivity shock in the traded-goods sector. Of less importance are the shocks to foreign inflation and output; the least important is the shock to real money demand. The relative degree of importance of each shock is explained by the combination of their estimated standard deviations (see Table 1) and their relevance in explaining the variance of inflation (see variance decomposition in Tables 2 and 5 to 7). In that sense, the shock to non-traded total-factor productivity has both high estimated volatility and high explanatory power of aggregate fluctuations, while the shock to real money demand has very high estimated volatility but almost no effect on inflation variability, as shown in the variance decomposition.

## 5.2 Targeting other inflation rates: CPI versus wage inflation

Our model has different degrees of nominal inertia in the different sectoral prices and in wages. A welfare-maximizing central bank may prefer to target just one sectoral inflation instead of aggregate CPI inflation, or it may prefer to target wage inflation or combinations of specific price inflations, depending on the sensitivity of households' utility to specific price and wage developments.

In fact, several recent papers have found that the optimal monetary policy may entail such choices

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<sup>11</sup>However, we keep the rest of the estimated parameters of Table 1, including those referring to the monetary policy reaction function. Suppressing the latter would mean not being able to solve the model.

in the context of sticky-price, dynamic stochastic general-equilibrium models with different sectors. Aoki (2001) shows that in a closed economy with a flexible-price sector and a sticky-price sector, the optimal monetary policy is to target sticky-price inflation only. In a closed economy but where labour and product markets exhibit staggered prices, Erceg, Henderson, and Levin (2000) find that strict price-inflation targeting generates relatively large welfare losses with respect to the optimal flexible-price, flexible-wage monetary policy, while combinations of wage- and price-inflation targeting or of price-inflation and output-gap targeting or even strict output-gap targeting perform nearly as well as the optimal one. In a similar economy but with two sectors, durables and non-durables, Erceg and Levin (2002) find it near-optimal to target a weighted average of aggregate price and wage inflation. Similarly, Huang and Liu (2004) find near-optimal an interest rate rule that targets a combination of CPI and producer price index (PPI) inflation when there are nominal rigidities in markets for both finished goods and intermediate goods.

In an open economy setting, Benigno (2004) shows in a model with different regions rather than sectors, that the monetary policy is near-optimal when the region with the higher nominal rigidity receives the higher weight in the inflation-targeting strategy. Finally, Smets and Wouters (2002) estimate different degrees of domestic and import price stickiness and find the optimal monetary policy minimizing a weighted average of both domestic and import-price inflations.

We have applied our welfare criterion specified above to optimize over the parameters of varieties of the Taylor rule. First, we have considered aggregate CPI inflation,  $\pi$ , wage inflation,  $\pi^W$ , and output-gap targeting, as in

$$\log(R_t/R) = \varrho_R \log(R_{t-1}/R) + \varrho_\pi \log(\pi_t/\pi) + \varrho_{\pi^W} \log(\pi_t^W/\pi^W) + \varrho_y \log(y_t/y). \quad (53)$$

Figure 10 represents the welfare surfaces with respect to  $\rho_\pi$  and  $\rho_{\pi^W}$  for different values of  $\rho_y$ , while holding constant the estimated degree of policy inertia,  $\rho_R = 0.46$ . As explained above, the welfare measure corresponds to a second-order approximation of  $E\left(u\left(\tilde{c}_t, \tilde{m}_t, \tilde{h}_t\right)\right) = E\left\{\frac{\gamma}{\gamma-1}\left(c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t^{\frac{\gamma-1}{\gamma}}\right) + \eta \log(1 - h_t)\right\}$ . The welfare surfaces appear to be piecewise smooth in  $\rho_\pi$ ,  $\rho_{\pi^W}$ , and  $\rho_y$ , except when approaching the zero-inflation-targeting area, where the decline in welfare is abrupt.<sup>12</sup> Figure 10 shows clearly that reacting aggressively to the output gap can be very damaging in terms of welfare losses. This is especially the case when the reaction to inflation deviations from target is low and where the welfare cost of the suboptimal rule is increasing in  $\rho_y$ .

The welfare-maximizing parameterization is the one explained above: strict CPI inflation targeting with coefficient  $\rho_\pi = 1.2$  ( $\rho_{\pi^W} = 0$  and  $\rho_y = 0$ ). It is interesting to note that moving to a higher  $\rho_\pi$  coefficient or to strict wage-inflation targeting with  $\rho_{\pi^W} > 1$  practically does not diminish welfare.

In the same spirit as Erceg, Henderson, and Levin (2000), rows 4, 5, and 6 of Tables 3 and 4 demonstrate the welfare and macroeconomic volatility implications of completely stabilizing one argument at a time in

<sup>12</sup>Due to the possible flatness of the welfare function in some areas of the parameter space, we search for the welfare-maximizing interest rate rule using a grid-search method over the policy parameters rather than relying on local optimizing routines. The intervals of the grid search on the coefficients are of size 0.2. The values for which there is indeterminacy, typically  $\rho_\pi = 0$  and  $\rho_{\pi^W} = 0$ , are not plotted.

Moreover, we restrict the search to values within the  $[0, 5]$  interval, and with the  $[0, 4]$  interval, when we search the coefficients for several sectoral inflations at the same time.

the above interest rate reaction function.<sup>13</sup> We find that strict output-gap stabilization reduces welfare with respect to the historical rule. Aggressive, strict CPI inflation stabilization can improve welfare only very marginally with respect to  $\rho_\pi = 1.2$  (a welfare gain of 0.085 versus 0.08 in the optimized CPI inflation targeting rule), while it significantly increases consumption and output volatility.

Strict wage-inflation stabilization substantially increases all volatilities except consumption and hence cannot improve welfare with respect to CPI inflation targeting. This result seems to contradict part of the reported previous research, which found that targeting the inflation rate of the stickier price would improve welfare.

To further understand the relatively poor performance in terms of welfare in wage-inflation targeting, we have simulated the impulse responses of the main macro variables in our model using a strict CPI inflation-targeting rule, a strict wage-inflation-targeting rule (with the same reaction to deviations of wage inflation from target as the optimized CPI inflation-targeting rule, i.e.,  $\rho_{\pi w} = 1.2$ ), and the responses under flexible prices and wages. Figures 6 to 8 show the impulse-response functions to the most important shocks given their contribution to explain the variability of the main macro variables, i.e., the non-traded technology shock and domestic and foreign monetary policy shocks. The magnitudes and signs of the shocks are the same as in section 4.

In the case of the main shock driving the dynamics of our model, the technology shock in the non-traded-goods sector, Figure 6 shows how the different impulse responses under optimized CPI inflation targeting are closer to the flexible-price scenario than the strict wage-inflation-targeting monetary policy rule. In particular, consumption increases to a far lesser degree under wage-inflation targeting than under CPI inflation targeting or than under the flexible-price case, and therefore welfare improvement is smaller. This is because of the different reaction of the monetary policy. Under wage inflation, interest rises after the increase in wages owing to the increase in non-traded-goods production. Instead, the fall in non-traded-goods and aggregate inflation after the positive technology shock makes interest rates fall under CPI inflation targeting, enhancing the increase in consumption and welfare.

In the case of a depreciation induced by an unexpected increase in U.S. interest rates, Figure 7 shows again how wage-inflation targeting yields responses that lie farther apart from those of the flexible-price scenario than in the case of targeting CPI inflation. Under wage-inflation targeting, interest rates do not rise after increased imports inflation. Because the import sector does not use labour in this model, the fall in imports due to the depreciation does not lower labour demand and hence does not lower wages, so the monetary policy does not react to the shock and as a result consumption falls far less than in the flexible-prices case.

Figure 8 shows the responses to a rise in domestic interest rates. In the case of flexible prices, the fall in activity translates into an immediate drop in all prices and hence an immediate offsetting fall of interest rates (due to the Taylor rule reacting to deviations from CPI inflation from target, as well as to the output gap) so that interest rates and all real variables and relative prices remain almost unchanged, consumption and utility included. With sticky prices and wages, monetary policy shocks obviously have an effect on real

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<sup>13</sup>An exceptional case is when we compare our welfare results with the case of the historical estimated rule rather than with the flexible-price optimal rule. To guarantee complete stabilization of the target variable, we impose very high coefficients one at a time:  $\rho_\pi = 100$ ,  $\rho_{\pi w} = 100$ , and  $\rho_y = 2$ . The latter is not that high, because higher values for  $\rho_y$  would cause indeterminacy unless they are coupled with high-inflation-reaction coefficients, which, by definition, is impossible in this exercise.

variables. However, this effect is more intense in the case of wage-inflation targeting, which again helps to explain the poor welfare performance of such a rule in this model relative to CPI inflation targeting. The reason for that is that all prices fall after the contractionary shock, but since wages are the stickiest of all prices, they fall by less and therefore the immediate offsetting drop in interest rates is smaller. This leads to a deeper fall in consumption and hence in utility and welfare than in the case of CPI inflation targeting.

We can conclude from these impulse responses that CPI inflation targeting, and not wage-inflation targeting, achieves similar responses to the flexible-prices case in crucial variables, especially consumption, which is the variable driving utility and hence welfare.

In terms of other business cycle properties under wage-inflation targeting, Table 6 shows that the variance decomposition in this case differs from that of the historical monetary policy rule or the optimized strict CPI inflation-targeting rule mainly in the substantially higher impact of domestic monetary policy shocks on all variables. The rest is similar.

In terms of the likelihood of aggregate inflation deviations from target, the lower right plot in Figures 9 and 18 to 25 shows the simulated CPI inflation series under wage-inflation targeting (again, with  $\rho_{\pi^w} = 1.2$ ). Obviously, in the case of monetary policy focusing on dampening wage-inflation deviations from target and not on CPI inflation deviations, the latter is found more likely than under wage-inflation targeting.

### 5.3 Targeting other inflation rates: Sectoral inflation rates

We now explore the case where the monetary authority can react differently to the different sectoral inflation rates: imports, traded goods, and non-traded goods in an interest rule of the type

$$\begin{aligned} \log(R_t/R) = & \varrho_R \log(R_{t-1}/R) + \varrho_{\pi^m} \log(\pi_t^m/\pi^m) \\ & + \varrho_{\pi^N} \log(\pi_t^N/\pi^N) + \varrho_{\pi^{Td}} \log(\pi_t^{Td}/\pi^{Td}). \end{aligned} \quad (54)$$

We compare the welfare gain and the volatility implications for different combinations of the monetary authority reactions to  $\pi_t^m$ ,  $\pi_t^N$ , and  $\pi_t^{Td}$ . Again, policy inertia is set to the estimated value, and we set  $\rho_y = 0$ , corresponding to the optimized value with the CPI inflation Taylor rule. We do so to diminish considerably the time of optimizing the monetary rule over various coefficients for the different inflation rates.

Figures 11 and 12 show a very clear result in two different ways: aiming to stabilize to a greater degree the inflation rate in the imports sector or in the domestically produced traded-goods sector does not increase welfare in any noticeable way. Only non-traded-goods inflation targeting does. Row 7 in Tables 3 and 4 shows the results for a coefficient on  $\pi_t^N$  of 4, which is higher in Figures 11 and 12. The welfare gain of such a rule is far superior to the others explored thus far, and that is so because of the higher long-run average consumption and real money balances, despite the much higher macroeconomic instability it causes. In fact, aggregate inflation and output volatilities are both more than five times those of the optimized CPI inflation-targeting rule. Pushing to the limit non-traded inflation stabilization (row 8 in Tables 3 and 4) doubles the welfare gain but at the cost of macroeconomic volatility that is twice as great.

Consequently, the central bank should react more aggressively to non-traded-goods inflation and not at all to the other sectors. This is consistent with previous findings in the literature whereby the optimal

monetary policy is to target exclusively the inflation rate of the sector that has more nominal inertia, which in our case is the non-traded-goods sector.

Table 7 shows the variance decomposition when the monetary policy reacts only to deviations of non-traded-goods inflation from target, with a coefficient  $\varrho_{\pi^N} = 4$ . Virtually no other shock other than a technology shock to non-traded goods explains the variability of all aggregate variables.

That shock has substantially higher estimated volatility than any other shock (except for the shock to money demand, which explains virtually nothing of the variability of model variables in all cases). This explains the high volatility observed in all variables in Table 4, as well as the higher likelihood of CPI inflation to diverge from target (bottom left plot in Figures 9 and 18 to 25).

This disconnect of the economy from shocks other than those to the non-traded-goods sector may also be behind the need of big movements of the monetary policy instrument to bring non-traded-goods inflation back to target. This would explain the high  $\varrho_{\pi^N}$  parameter value required to increase welfare as well as the high volatility of the nominal interest rate reported in Table 4. The high volatility of the interest rates translates into high volatility in real money balances, as can be seen in Table 4 as well. More important, however, it can include episodes of zero nominal interest rates.<sup>14</sup>

## 6 Effect of Size of Non-Traded Sector on Inflation Targeting

As we reported earlier, the idea behind targeting inflation in the non-traded-goods sector is motivated by the importance of this sector in the economy we estimate for Canada. In this section, we do some counterfactual exercises—first, to understand to which extent the non-traded-goods sector exhibits the most distortionary market, and second, to relate our findings to previous studies, particularly the one by Erceg, Henderson, and Levin (2000).

To understand to what extent the non-traded-goods sector is the most important one for the monetary authority when establishing policy, we set different values for the weight of non-traded goods in the consumption basket,  $n$ , keeping the rest of the parameters unchanged. Figure 13 reports cases under  $n = 0.00, 0.20, 0.40,$  and  $0.60$ , where we report the utility level as a function of the degree of reaction of the monetary authority to the price- versus wage-inflation gap. Note that for the one-sector model, where only tradable goods can be consumed domestically, we retrieve the result found by Erceg, Henderson, and Levin (2000). Particularly, when wages and prices are sticky, monetary policy cannot achieve the Pareto-optimal welfare level, and with the same duration for wage and price contracts, the monetary policy rule should focus more on stabilizing the wage-inflation fluctuations. We show clearly that this result remains the same when we consider an open economy model with one production sector; consequently, the exchange rate effect doesn't have a big impact on the optimal rule. This is clearly illustrated in the two top panels and the lower left-hand panel in Figure 13. Nevertheless, we notice that the gap between the performance of stabilizing either price or wage inflation tends to decrease when the share of non-traded goods increases. Moreover, the gap vanishes at a value of  $n$  between 0.40 and 0.60, and the result is then reversed as the lower right-hand panel shows: a rule of price-inflation targeting performs better than one that targets wage inflation.

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<sup>14</sup>We thank Pierre Duguay for this observation.

We also notice that CPI inflation stabilization can become welfare improving for higher values of  $n$ , even if the optimal rule is to target only inflation in the non-traded-goods sector; this is obviously explained by the high relative share of the non-traded goods in the final consumption basket of households.<sup>15</sup>

## 7 Price-Level Targeting

As stated in the introduction, many issues arise when a price-level target is introduced, such as the implications for the price level and responses of inflation to shocks, as well as for the volatility of the main macro variables, not the least of which is inflation itself.

Starting with Wicksell (1907), many authors have considered aggregate price-level stability as the main goal of central banks, and this is reflected in the mandates of many central banks. How to achieve price stability has more often been interpreted as targeting at an explicit inflation rate or range than as targeting a specific price-level path. Still, some recent research has shown that there can be substantial gains in including a specific price-level target in the monetary policy reaction function. In Bank of Canada (1998), Coulombe (1998) shows that there is a clear information gain under an explicit price-level-targeting regime: the price level itself conveys in that case useful information about future inflation, because past shocks to prices must be reversed in the future. Under strict inflation targeting, however, where all shocks to the price level are permanent, the price level reveals no useful information. In Bank of Canada (1998), Black, Macklem, and Rose (1998) show that, when comparing simple monetary policy rules in a calibrated small open economy one-good model of the Canadian economy, and provided the price-level target is credible and that private sector expectations of inflation adjust accordingly, the economy performs better with a price-level target than with an inflation target, in the sense that the variability of both inflation and output are lower with the price-level target. These potential benefits of price-level targeting are not without risks, however. How to communicate such policy is a challenge. It could be difficult to justify why, following an increase in inflation above its long-run average, inflation had to be reduced below this long-run average for some time to drive the price level back to its target. Also, that reduction in inflation after the monetary policy takes action can lead to sharper initial declines in economic activity than under a strict inflation-targeting regime.

Giannoni (2000) argues that simple price-level-targeting rules,<sup>16</sup> while as simple as standard inflation-targeting Taylor rules, have received considerably less attention in recent studies of monetary policy. It is widely believed that such rules would result in greater variability of inflation (and, under nominal rigidity, of the output gap), since the policy-maker would respond to an inflationary shock by generating a deflation in subsequent periods. Studies such as Lebow, Roberts, and Stockton (1992) and Haldane and Salmon (1995) support this conventional view. However, Giannoni (2000) shows that when agents are forward looking

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<sup>15</sup>We undertake the same exercise with elasticity of substitution between non-traded goods and the tradable goods sold domestically,  $\phi$ , and the conclusions are the same. Particularly, with a higher degree of substitution, the non-traded-goods sector is less important in the model; consequently, the monetary authority should stabilize wage inflation rather than price inflation.

<sup>16</sup>In those rules, the nominal interest rate deviates from a constant in response to the output gap and to deviations of the price level from a prespecified path of constant inflation. Giannoni (2000) follows Woodford (1998, 2003) in referring to such rules as “Wicksellian.” Wicksell (1907) argued that “price stability” could be obtained by allowing the interest rate to respond positively to fluctuations in the price level.

and the monetary authority credibly commits to a price-level-targeting rule, such a Wicksellian rule yields lower variability of inflation and of nominal interest rates. Agents' expectation of a future deflation after an inflationary shock dampens the initial increase in inflation, lowers the variability of inflation, and increases welfare. Williams (1999) confirms this result using the FRB/US model.

More recently, and closer to our approach, Batini and Yates (2003) also challenge the established view that price-level targeting entails lower price-level variance at the expense of higher inflation and output variance. They investigate monetary policy regimes that combine price-level and inflation targeting in a variety of models and conclude that the relative merits of each regime depend on several modelling and policy assumptions, and do so in a non-monotonic fashion when moving from one regime to another.

In this section, we conduct the same calculations of welfare gains and implied macroeconomic volatility as before, but we consider a different type of monetary policy reaction function, i.e., where the central bank is concerned with returning the price level to its target path as well as or instead of bringing the inflation rate to target.<sup>17</sup>

We follow Batini and Yates (2003) and encompass price-level and inflation targeting using the following specification of the monetary policy reaction function:

$$\log(R_t/R) = \rho_R \log(R_{t-1}/R) + \rho_P [\log(P_t/\bar{P}_t) - \eta_P \log(P_{t-1}/\bar{P}_{t-1})] + \rho_y \log(y_t/y), \quad (55)$$

where  $\bar{P}_t$  is the target or steady-state value for the price level at period  $t$ , compatible with the established inflation target. Note that for  $\eta_P = 1$ , we have the exact case of the Taylor rule defined for the inflation rate, while  $\eta_P = 0$  means pure price-level targeting. For  $0 < \eta_P < 1$ , the rule is a hybrid one where the central bank is concerned about reaching the inflation target rate but also about the evolution of prices on the way to the inflation target. As before, we keep  $\rho_R = 0.46$  and  $\rho_y = 0$  fixed while jointly optimizing over  $0 \leq \eta_P \leq 1$  and over  $\rho_P$ .

Figure 14 shows the utility surface of this optimization exercise, while further welfare implications and the implied volatility are shown in row 9 of Tables 3 and 4. Two results emerge from this exercise. First, it is almost impossible to establish a clear ranking of combinations of parameters in this case; the long-run utility level associated with the depicted parameter surface is virtually the same. Pure approximation errors embedded in our procedure could be behind the plotted differences. Second, for the central bank to give a non-zero weight to the deviations of the price level from its target path, i.e., for  $\eta_P < 1$ , the monetary policy reaction to price and inflation deviations from target has to be very low,  $\rho_P = 0.2$ . In that case, welfare is maximized for the hybrid rule with  $\eta_P = 0.25$ , i.e., where 25 per cent of the price-stability concern of the monetary authority takes the form of inflation targeting and the rest is pure price-level targeting. Still, the welfare gain is almost unnoticeable and comes from the lower volatility induced (smaller negative second-level effect) by the mild reaction of the monetary policy.

It is interesting that gains from an explicit price-level target come only with low policy reactions, causing a far longer time to bring about price and inflation stabilization than in strict inflation-targeting regimes.

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<sup>17</sup>We have computed the simulated impulse responses of the main macro variables after all shocks in the economy and find very similar reactions under pure inflation targeting as under pure price-level targeting for the same degree of price stabilization (same  $\rho_P$  coefficient).

This result is in line with the findings of Smets (2003).

We have explored the issue of whether wages or a particular sectoral price is a better target for the monetary policy than aggregate CPI in the context of strict price-level targeting, that is, with  $\rho_y = 0$  and  $\eta_P = 0$ . Figure 15 shows that, consistent with the findings in the case of pure inflation targeting, welfare improves only and substantially if the monetary authority targets only the price on the non-traded-goods sector. Row 10 in Tables 3 and 4 shows that the gain doesn't reach the level of the pure non-traded-goods inflation stabilization, but it does cause higher inflation and output volatility.

Finally, we jointly optimize again over  $0 \leq \eta_P \leq 1$  and over  $\rho_P$  for the case where only the price level and inflation of the non-traded-goods sector are taken into account. Figure 16 shows the results. For low levels of monetary policy response to deviations from price stability, there is virtually no welfare difference between price-level and inflation targeting, but as the central bank becomes more concerned with price stability (higher  $\rho_{PN}$ ), there is a clear welfare gain in moving towards strict inflation targeting ( $\eta_{PN} = 1$ ) in this sector-specific scenario.

## 8 Targeting Future Price Developments

To conclude these optimization exercises for simple monetary policy rules, we explore the impact of targeting expected future deviations of the inflation rate or the price level rather than targeting contemporaneous deviations. In their analysis of price-level versus inflation targeting under different model specifications, policy rules, and loss functions of the central bank, Batini and Yates (2003) find that the more forward looking the model, the less noticeable the difference between the reaction functions of inflation and price-level targeting.

The top panel of Figure 17 shows the welfare surfaces for the cases in which we optimize over the coefficients for CPI and wage-inflation stabilization for different values of the coefficient for output-gap stabilization. In this case, all deviations from target to which the monetary policy reacts are one-period-ahead expected future deviations, that is, in the next quarter.

The two main results are: (i) the welfare-maximizing parameter set is exactly the same as when the central bank is not forward looking, i.e.,  $\rho_\pi^{+1} = 1.2$ ,  $\rho_{\pi W}^{+1} = 0$ , and  $\rho_y^{+1} = 0$ , and (ii) the welfare attained with a forward-looking monetary policy rule is noticeably higher. Row 3 of Table 3 shows that the welfare gain is now 0.11 per cent of the lifetime consumption versus 0.08 per cent when optimizing a contemporary monetary policy rule. And this welfare gain comes with increased output and inflation volatility but with lower volatility in households' utility (see row 3 in Table 4).

The bottom panel of Figure 17 shows the welfare surface for the case when the monetary authority follows a forward-looking, one-period-ahead strict price-level-targeting rule and optimizes over its reaction to the next quarter's expected deviation of the CPI price level,  $\rho_P^{+1}$ , and the wage level,  $\rho_W^{+1}$ . Again, the values found for the coefficients are the same as in the contemporaneous price-level-targeting rule, i.e.,  $\rho_P^{+1} = 0.2$  and  $\rho_W^{+1} = 0$ , but the welfare attained is higher than in the non-forward-looking case. Still, the welfare gain is smaller than in the forward-looking strict inflation-targeting rule.

Of all the possible specifications explored in this paper, the one that achieves a higher welfare given the estimated model for the Canadian economy without causing substantial excess macroeconomic volatility is a strict inflation-targeting rule where the central bank reacts to the next period's expected deviation from the inflation target, and does not target the output gap but allows for a moderate degree of nominal interest rate smoothing.

## 9 Conclusion

We analyze welfare-improving monetary policy reaction functions in the context of a New Keynesian small open economy model with a traded-goods and a non-traded-goods sector and with sticky prices and wages. The model is estimated for the case of Canada and is used to evaluate the welfare gains of alternative specifications of the feedback nominal interest rate rule.

The model is estimated using Bayesian techniques for quarterly Canadian data. We find statistically significant heterogeneity in the degree of price rigidity across sectors. We explore what would have been the optimal parameterization of a Taylor rule such as the estimated one, where the central bank targets aggregate inflation. We find welfare gains in responding slightly more aggressively to aggregate inflation deviations from target than has been the case in the past three decades, and of not responding to the output gap, as opposed to what the Bank of Canada has done. We find further welfare gains in targeting sectoral rather than aggregate inflation. In particular, the gains are highest if the monetary authority reacts more aggressively to non-traded-goods inflation, since prices are stickier in that production sector. But the implications in terms of business cycle fluctuations of such a policy rule are discouraging—high volatility is induced in the system, including a high probability of large deviations of CPI inflation from target.

We then consider recent literature that has questioned the optimality of aiming at a stable inflation rate instead of a stable price level in a world where households would prefer to reduce uncertainty about the long-run purchasing value of money. We look for the welfare-maximizing specification of an interest rate reaction function that targets a combination of price-level and inflation targets or just one of the two, the price levels being the aggregate CPI, wages, and sectoral prices. We find no clear welfare gain in moving towards price-level targeting, unless the monetary authority is willing to accept very long horizons for prices and inflation to return to target.

We find that the higher welfare without inducing excess macroeconomic volatility is achieved with a strict inflation-targeting rule where the central bank reacts to next period's expected deviation from the inflation target. Moreover, the central bank should not target the output gap but should allow for a moderate degree of nominal interest rate smoothing.

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Table 1: Parameter Estimation Results

Parameter distribution					
Parameter	Type	Prior		Posterior	
		Mean	Std. error	Median	90 per cent interval
$\rho_{AN}$	Beta	0.85	0.1	0.7976	[0.7419 , 0.8404]
$\rho_{AT}$	Beta	0.85	0.1	0.5850	[0.5018 , 0.6746]
$\rho_b$	Beta	0.85	0.1	0.8128	[0.7359 , 0.8712]
$\rho_{R^*}$	Beta	0.8	0.1	0.7175	[0.6672 , 0.7913]
$\rho_{y^*}$	Beta	0.85	0.1	0.7486	[0.6419 , 0.8470]
$\rho_{\pi^*}$	Beta	0.8	0.1	0.5330	[0.4515 , 0.6044]
$\rho_{\kappa}$	Beta	0.8	0.1	0.6289	[0.5698 , 0.6727]
$\sigma_{AN}$	Inv. gamma	1.5	2	6.1442	[5.8442 , 6.5318]
$\sigma_{AT}$	Inv. gamma	1.5	2	1.5003	[1.3487 , 1.6095]
$\sigma_R$	Inv. gamma	1.5	2	0.9983	[0.9187 , 1.1228]
$\sigma_b$	Inv. gamma	1.5	2	12.3049	[12.1777 , 12.4786]
$\sigma_{R^*}$	Inv. gamma	1.5	2	0.8421	[0.7618 , 0.9330]
$\sigma_{y^*}$	Inv. gamma	1.5	2	1.1208	[1.0466 , 1.2398]
$\sigma_{\pi^*}$	Inv. gamma	1.5	2	0.4429	[0.4017 , 0.5006]
$\sigma_{\kappa}$	Inv. gamma	1.5	2	0.9846	[0.8981 , 1.1067]
$d_M$	Beta	0.67	0.05	0.5101	[0.4453 , 0.5585]
$d_N$	Beta	0.67	0.05	0.6243	[0.5790 , 0.6604]
$d_T$	Beta	0.67	0.05	0.5951	[0.5622 , 0.6296]
$d_h$	Beta	0.67	0.05	0.8027	[0.7519 , 0.8453]
$m$	Beta	0.6	0.05	0.5447	[0.5130 , 0.5845]
$n$	Beta	0.5	0.05	0.5355	[0.4825 , 0.5967]
$\mu$	Gamma	1.2	0.2	1.2496	[1.1320 , 1.3439]
$\nu$	Gamma	1.2	0.2	0.7140	[0.5915 , 0.8440]
$\phi$	Gamma	1.2	0.2	2.2653	[2.1644 , 2.3529]
$\varphi$	Normal	-0.02	0.005	-0.0238	[-0.0307 , -0.0166]
$\chi$	Gamma	20	5	10.1331	[10.0299 , 10.6912]
$b$	Gamma	0.4	0.1	0.2715	[0.1643 , 0.4142]
$\rho_R$	Beta	0.8	0.1	0.4612	[0.4077 , 0.5082]
$\rho_{\pi}$	Gamma	1.5	0.2	1.1888	[1.0624 , 1.3432]
$\rho_y$	Normal	0.2	0.1	0.3142	[0.2570 , 0.3937]
$\alpha_N$	Beta	0.34	0.05	0.1982	[0.1570 , 0.2453]
$\alpha_T$	Beta	0.36	0.05	0.4764	[0.4457 , 0.4964]

Table 2: Variance Decomposition, Historical Taylor Rule

Variable	$A_t^N$	$A_t^T$	$R_t$	$b_t$	$R_t^*$	$y_t^*$	$\pi_t^*$	$\omega_t$
$y_t$	14.60 [11.61, 17.72]	24.46 [21.50, 27.43]	34.63 [32.24, 38.42]	0.01 [0.00, 0.01]	9.31 [6.52, 12.68]	9.66 [8.135, 11.57]	0.47 [0.36, 0.58]	6.83 [4.75, 8.90]
$y_t^N$	93.00 [91.65, 94.74]	1.02 [0.85, 1.30]	1.69 [1.22, 2.35]	0.00 [0.00, 0.00]	2.48 [1.77, 3.32]	0.05 [0.035, 0.09]	0.01 [0.00, 0.02]	1.70 [1.11, 2.38]
$y_t^T$	20.37 [14.54, 28.94]	28.58 [24.27, 32.40]	28.75 [25.39, 32.07]	0.00 [0.00, 0.01]	6.98 [4.67, 9.48]	9.75 [8.171, 11.37]	0.41 [0.30, 0.51]	5.12 [3.56, 6.70]
$y_t^c$	38.17 [30.25, 47.77]	3.91 [3.03, 4.80]	2.21 [1.64, 2.95]	0.00 [0.00, 0.00]	26.00 [19.04, 34.87]	12.29 [9.690, 16.49]	0.11 [0.03, 0.23]	17.27 [13.07, 21.82]
$y_t^m$	16.79 [9.94, 24.49]	0.22 [0.13, 0.37]	8.80 [5.71, 12.56]	0.00 [0.00, 0.00]	41.06 [30.21, 50.40]	4.85 [2.972, 7.03]	0.39 [0.23, 0.58]	27.85 [21.46, 33.28]
$c_t$	52.15 [42.82, 60.85]	0.36 [0.28, 0.47]	8.45 [6.53, 11.12]	0.04 [0.03, 0.06]	21.67 [15.24, 30.10]	2.42 [1.650, 3.40]	0.15 [0.07, 0.23]	14.70 [10.45, 19.54]
$h_t$	17.53 [12.55, 24.91]	27.23 [21.59, 30.41]	25.63 [22.92, 29.47]	0.01 [0.01, 0.02]	13.00 [9.51, 17.28]	6.83 [5.508, 8.46]	0.35 [0.24, 0.43]	9.37 [6.58, 12.01]
$h_t^N$	43.23 [38.96, 48.19]	9.33 [7.97, 11.04]	15.24 [11.55, 19.87]	0.01 [0.00, 0.01]	18.51 [13.52, 23.55]	0.69 [0.367, 1.17]	0.18 [0.10, 0.26]	12.76 [9.80, 16.60]
$h_t^T$	30.98 [24.51, 41.80]	25.32 [20.03, 28.82]	20.04 [16.91, 23.10]	0.01 [0.00, 0.01]	9.30 [6.77, 12.22]	7.28 [5.769, 8.97]	0.29 [0.19, 0.37]	6.74 [4.68, 8.77]
$i_t$	44.76 [34.68, 52.82]	0.31 [0.22, 0.46]	8.78 [6.45, 11.67]	0.00 [0.00, 0.00]	26.72 [18.98, 36.07]	1.14 [0.773, 1.59]	0.20 [0.10, 0.31]	18.05 [12.92, 23.42]
$s_t$	51.13 [45.58, 57.00]	0.49 [0.38, 0.64]	3.84 [3.04, 4.80]	0.00 [0.00, 0.00]	21.95 [18.22, 26.48]	2.92 [2.220, 3.88]	0.83 [0.64, 0.98]	18.80 [16.16, 21.83]
$\pi_t$	27.40 [21.67, 34.18]	13.29 [11.70, 15.02]	7.66 [6.27, 9.75]	0.00 [0.00, 0.00]	27.15 [22.28, 33.24]	2.24 [1.461, 3.30]	0.83 [0.61, 1.07]	21.40 [17.38, 25.19]
$\pi_t^N$	98.05 [97.09, 98.95]	0.29 [0.18, 0.39]	0.11 [0.07, 0.14]	0.00 [0.00, 0.00]	0.90 [0.39, 1.51]	0.10 [0.056, 0.17]	0.01 [0.00, 0.02]	0.51 [0.22, 0.87]
$\pi_t^d$	14.49 [11.75, 17.36]	60.50 [55.31, 67.17]	4.94 [4.11, 6.31]	0.00 [0.00, 0.00]	11.07 [6.40, 16.60]	2.00 [1.339, 2.77]	0.19 [0.11, 0.25]	6.78 [4.20, 9.49]
$\pi_t^m$	19.34 [15.02, 23.34]	3.90 [3.17, 4.54]	6.05 [4.77, 7.70]	0.00 [0.00, 0.00]	36.23 [30.88, 41.38]	3.33 [2.429, 4.63]	1.25 [1.00, 1.52]	29.86 [26.00, 33.51]
$\pi_t^v$	13.12 [10.23, 16.58]	11.57 [9.58, 13.94]	0.94 [0.73, 1.23]	0.00 [0.00, 0.00]	31.57 [26.33, 37.17]	4.58 [3.503, 5.94]	12.86 [11.14, 14.45]	25.32 [22.18, 28.69]
$R_t$	26.18 [21.26, 30.10]	3.25 [2.82, 3.75]	27.18 [21.58, 34.16]	0.00 [0.00, 0.00]	24.52 [17.75, 32.11]	0.25 [0.126, 0.46]	0.39 [0.24, 0.54]	18.20 [13.32, 22.76]

Table 3: Welfare Implications of Alternative Monetary Policy Rules

Interest rate rules	Average $c_t$	Average $m_t$	Average $h_t$	Average $u_t$	Welfare gain	1st-level effect	2nd-level effect
<b>Historical rule</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 1.19\hat{\pi}_t + 0.31\hat{y}_t$	<b>0.5337</b>	<b>0.2497</b>	<b>0.3005</b>	<b>-0.7929</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
<b>CPI inflation targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 1.20\hat{\pi}_t$	<b>0.5345</b>	<b>0.2558</b>	<b>0.3013</b>	<b>-0.7921</b>	<b>0.0799</b>	<b>0.1112</b>	<b>-0.0311</b>
<b>Future CPI inflation targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 1.20\hat{\pi}_{t+1}$	<b>0.5349</b>	<b>0.2572</b>	<b>0.3018</b>	<b>-0.7918</b>	<b>0.1136</b>	<b>0.1549</b>	<b>-0.0410</b>
CPI inflation stabilization $\hat{R}_t = 0.46\hat{R}_{t-1} + \infty\hat{\pi}_t$	0.5345	0.2618	0.3008	-0.7921	0.0847	0.1606	-0.0755
Wage-inflation stabilization $\hat{R}_t = 0.46\hat{R}_{t-1} + \infty\hat{\pi}_t^w$	0.5345	0.2817	0.3012	-0.7923	0.0609	0.1719	-0.1104
Output-gap stabilization $\hat{R}_t = 0.46\hat{R}_{t-1} + \infty\hat{y}_t$	0.5333	0.2462	0.3001	-0.7933	-0.0415	-0.0551	0.0136
<b>Non-tradables inflation targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 4.00\hat{\pi}_t^N$	<b>0.5413</b>	<b>0.7278</b>	<b>0.2929</b>	<b>-0.7833</b>	<b>0.9779</b>	<b>2.8793</b>	<b>-1.8403</b>
Non-tradables inflation stabilization $\hat{R}_t = 0.46\hat{R}_{t-1} + \infty\hat{\pi}_t^N$	0.5502	1.1039	0.2820	-0.7723	2.0948	6.1086	-3.7616
<b>CPI level targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 0.20\hat{P}_t$	<b>0.5345</b>	<b>0.2500</b>	<b>0.3012</b>	<b>-0.7921</b>	<b>0.0834</b>	<b>0.0952</b>	<b>-0.0117</b>
<b>Non-tradables price-level targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 4.00\hat{P}_t^N$	<b>0.5422</b>	<b>0.5695</b>	<b>0.2943</b>	<b>-0.7845</b>	<b>0.8547</b>	<b>2.4680</b>	<b>-1.7398</b>

Note: The welfare gain is expressed as a permanent percentage increase of consumption compared to the historical consumption mean.

Table 4: Aggregate Volatility Induced by Alternative Monetary Policy Regimes

Interest rate rules	$\sigma_c$	$\sigma_m$	$\sigma_h$	$\sigma_u$	$\sigma_y$	$\sigma_\pi$	$\sigma_R$
<b>Historical rule</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 1.19\hat{\pi}_t + 0.31\hat{y}_t$	<b>0.0133</b>	<b>0.0552</b>	<b>0.0112</b>	<b>0.0226</b>	<b>0.0173</b>	<b>0.0077</b>	<b>0.0098</b>
<b>CPI inflation targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 1.20\hat{\pi}_t$	<b>0.0163</b>	<b>0.0596</b>	<b>0.0128</b>	<b>0.0301</b>	<b>0.0301</b>	<b>0.0076</b>	<b>0.0126</b>
<b>Future CPI inflation targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 1.20\hat{\pi}_{t+1}$	<b>0.0158</b>	<b>0.0595</b>	<b>0.0205</b>	<b>0.0277</b>	<b>0.0440</b>	<b>0.0140</b>	<b>0.0128</b>
CPI inflation stabilization $\hat{R}_t = 0.46\hat{R}_{t-1} + \infty\hat{\pi}_t$	0.0212	0.0624	0.0114	0.0357	0.0345	0.0007	0.0137
Wage-inflation stabilization $\hat{R}_t = 0.46\hat{R}_{t-1} + \infty\hat{\pi}_t^w$	0.0169	0.0678	0.0302	0.0197	0.0503	0.0204	0.0189
Output-gap stabilization $\hat{R}_t = 0.46\hat{R}_{t-1} + \infty\hat{y}_t$	0.0120	0.0525	0.0115	0.0245	0.0097	0.0084	0.0077
<b>Non-tradables inflation targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 4.00\hat{\pi}_t^N$	<b>0.0725</b>	<b>0.1718</b>	<b>0.0645</b>	<b>0.0974</b>	<b>0.1579</b>	<b>0.0389</b>	<b>0.0608</b>
Non-tradables inflation stabilization $\hat{R}_t = 0.46\hat{R}_{t-1} + \infty\hat{\pi}_t^N$	0.1068	0.2281	0.1031	0.1381	0.2480	0.0615	0.0801
<b>CPI level targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 0.20\hat{P}_t$	<b>0.0150</b>	<b>0.0564</b>	<b>0.0102</b>	<b>0.0276</b>	<b>0.0268</b>	<b>0.0065</b>	<b>0.0108</b>
<b>Non-tradables price-level targeting</b> $\hat{R}_t = 0.46\hat{R}_{t-1} + 4.00\hat{P}_t^N$	<b>0.0758</b>	<b>0.1467</b>	<b>0.0762</b>	<b>0.0959</b>	<b>0.1914</b>	<b>0.0474</b>	<b>0.0493</b>

Note:  $\sigma$  denotes the unconditional standard deviation for the listed variables.

Table 5: Variance Decomposition, Optimized CPI Inflation-Targeting Rule

Variable	$A_t^N$	$A_t^T$	$R_t$	$b_t$	$R_t^*$	$y_t^*$	$\pi_t^*$	$\omega_t$
$y_t$	15.19	33.73	31.40	0.01	7.05	6.97	0.50	5.14
$y_t^N$	90.56	0.17	3.71	0.00	3.17	0.25	0.05	2.08
$y_t^T$	10.27	40.69	30.68	0.01	5.91	7.56	0.50	4.37
$y_t^z$	37.49	9.77	9.25	0.01	19.77	10.94	0.25	12.53
$y_t^n$	22.12	1.07	12.86	0.01	36.44	3.36	0.71	23.44
$c_t$	54.83	1.29	11.91	0.03	18.02	1.93	0.30	11.69
$h_t$	15.50	6.23	40.48	0.03	16.60	8.82	0.69	11.64
$h_t^N$	32.52	0.79	27.97	0.02	21.57	2.41	0.42	14.29
$h_t^T$	26.46	8.23	34.15	0.02	12.59	9.03	0.61	8.92
$i_t$	48.94	1.74	13.23	0.00	21.06	1.19	0.39	13.45
$s_t$	57.85	0.64	9.06	0.00	16.19	0.65	0.71	14.89
$\pi_t$	22.25	2.48	25.56	0.00	26.44	0.12	0.71	22.44
$\pi_t^N$	97.76	0.06	0.60	0.00	0.93	0.05	0.01	0.58
$\pi_t^{Td}$	15.97	45.91	13.56	0.00	13.09	2.67	0.28	8.52
$\pi_t^n$	29.52	0.35	16.47	0.00	27.75	0.24	1.03	24.64
$\pi_t^z$	16.25	18.61	2.87	0.00	25.16	1.87	14.65	20.60
$R_t$	29.29	1.93	22.15	0.00	26.02	0.18	0.55	19.89

Table 6: Variance Decomposition, Optimized Non-Traded Inflation-Targeting Rule

Variable	$A_t^N$	$A_t^T$	$R_t$	$b_t$	$R_t^*$	$y_t^*$	$\pi_t^*$	$\omega_t$
$y_t$	98.42	0.74	0.64	0.00	0.08	0.07	0.01	0.04
$y_t^N$	99.46	0.02	0.38	0.00	0.07	0.01	0.00	0.05
$y_t^T$	98.04	1.01	0.68	0.00	0.11	0.09	0.01	0.06
$y_t^x$	94.46	0.83	0.43	0.00	2.14	0.85	0.04	1.25
$y_t^m$	97.13	0.03	0.71	0.00	1.18	0.17	0.02	0.75
$c_t$	98.62	0.04	0.64	0.00	0.37	0.09	0.00	0.23
$h_t$	99.01	0.18	0.70	0.00	0.02	0.07	0.01	0.01
$h_t^N$	99.07	0.04	0.72	0.00	0.09	0.02	0.00	0.06
$h_t^T$	98.88	0.26	0.70	0.00	0.04	0.09	0.01	0.02
$i_t$	98.59	0.04	0.56	0.00	0.46	0.04	0.01	0.30
$s_t$	96.36	0.02	0.47	0.00	1.62	0.11	0.07	1.36
$\pi_t$	97.37	0.13	0.59	0.00	1.03	0.02	0.03	0.82
$\pi_t^N$	98.94	0.14	0.88	0.00	0.00	0.04	0.00	0.00
$\pi_t^{Td}$	96.69	2.75	0.50	0.00	0.01	0.05	0.00	0.00
$\pi_t^m$	96.91	0.00	0.44	0.00	1.40	0.06	0.05	1.12
$\pi_t^x$	86.65	2.61	0.31	0.00	4.42	0.44	2.18	3.39
$R_t$	98.73	0.06	1.17	0.00	0.00	0.03	0.00	0.00

Table 7: Variance Decomposition, Optimized Wage-Inflation-Targeting Rule

Variable	$A_t^N$	$A_t^T$	$R_t$	$b_t$	$R_t^*$	$y_t^*$	$\pi_t^*$	$\omega_t$
$y_t$	12.48	30.67	46.72	0.04	3.92	3.80	0.40	1.97
$y_t^N$	89.58	0.05	9.31	0.01	0.59	0.04	0.03	0.38
$y_t^T$	25.13	29.52	35.68	0.03	3.88	3.44	0.33	1.99
$y_t^z$	21.31	12.29	8.77	0.02	28.89	11.48	0.46	16.77
$y_t^n$	5.02	0.74	42.83	0.03	28.68	3.96	0.75	18.00
$c_t$	38.18	1.28	43.39	0.11	9.25	1.76	0.26	5.78
$h_t$	35.12	8.29	50.80	0.05	1.10	3.80	0.33	0.52
$h_t^N$	63.20	0.44	33.83	0.03	1.19	0.40	0.13	0.78
$h_t^T$	35.20	10.73	46.08	0.05	2.00	4.59	0.34	1.02
$i_t$	30.03	2.09	43.59	0.00	14.21	0.65	0.42	9.01
$s_t$	27.03	0.51	15.75	0.01	29.65	1.73	0.91	24.41
$\pi_t$	44.37	1.53	18.54	0.01	19.39	0.67	0.38	15.11
$\pi_t^N$	99.57	0.04	0.35	0.00	0.01	0.03	0.00	0.00
$\pi_t^{Td}$	12.03	61.50	23.63	0.02	0.37	2.08	0.14	0.22
$\pi_t^n$	0.65	0.09	22.69	0.01	41.42	1.52	1.07	32.55
$\pi_t^z$	3.48	19.06	3.74	0.00	31.81	2.93	14.80	24.18
$R_t$	2.99	0.20	96.15	0.00	0.19	0.34	0.03	0.08

Figure 1: Prior and Posterior Distributions of the Shock Parameters

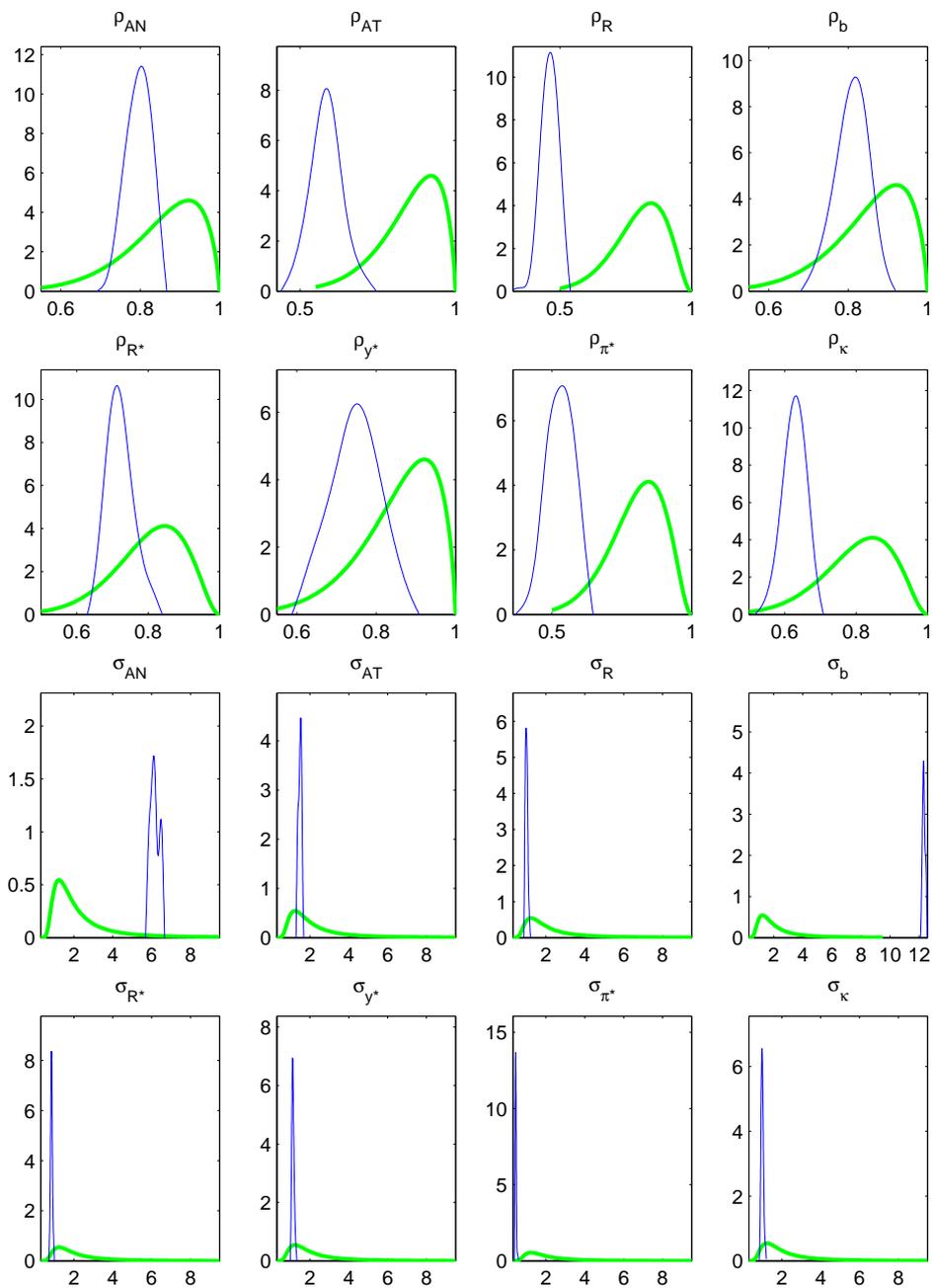


Figure 2: Prior and Posterior Distributions of the Behavioural Parameters

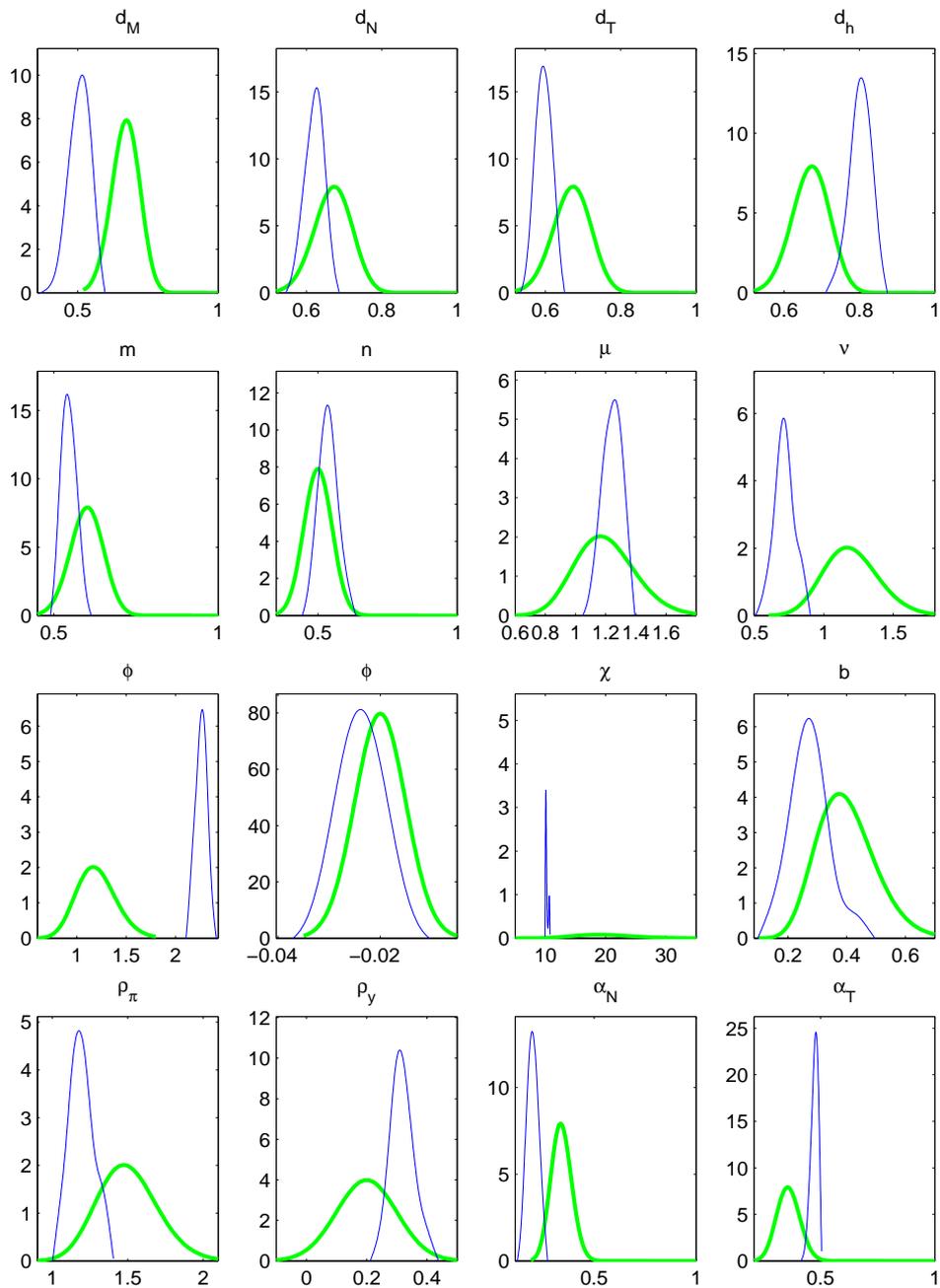


Figure 3: Foreign Nominal Interest Rate Shock

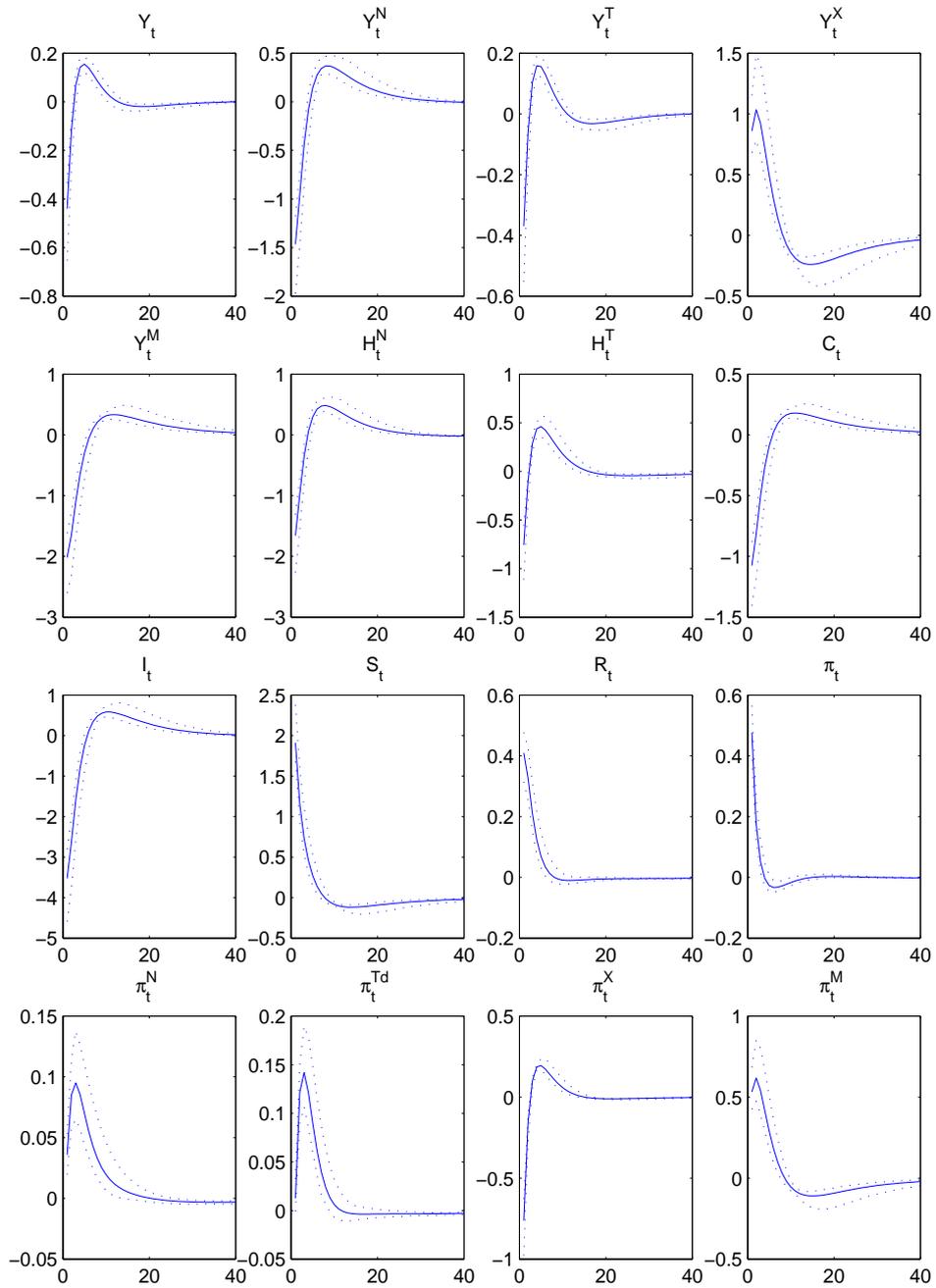


Figure 4: Non-Tradable-Goods Technology Shock

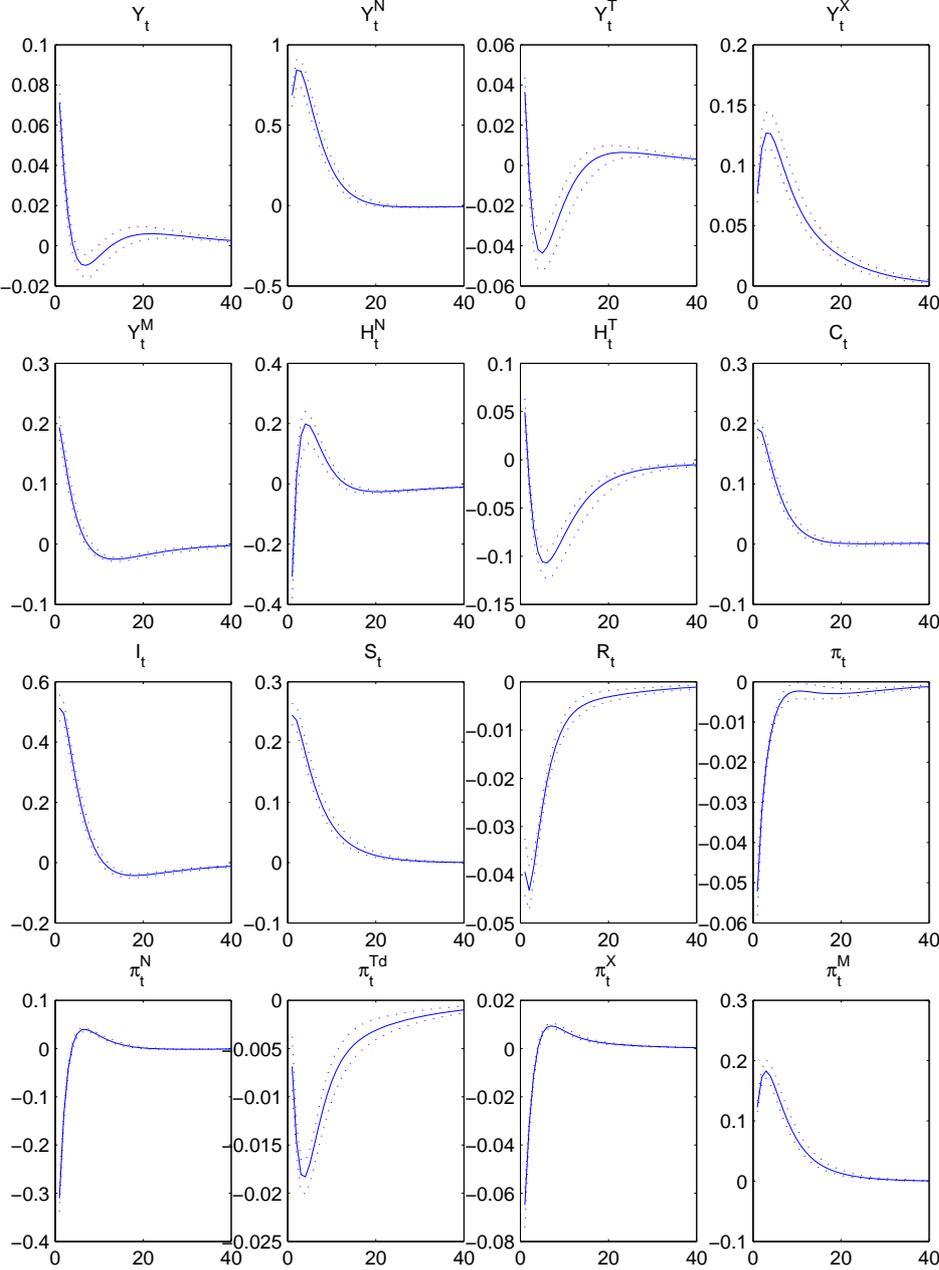


Figure 5: Local Nominal Interest Rate Shock

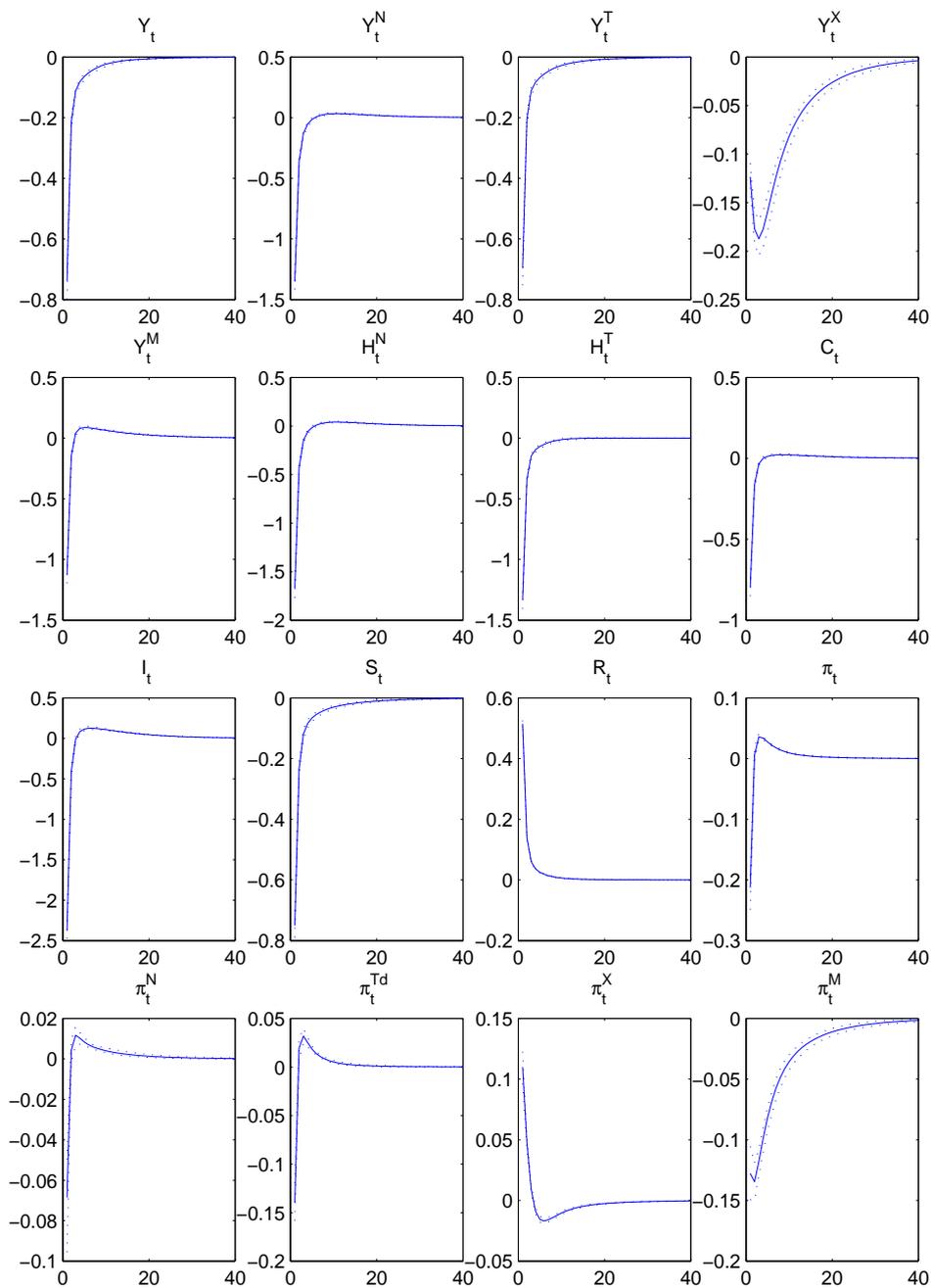


Figure 6: Non-Tradable-Goods Technology Shock

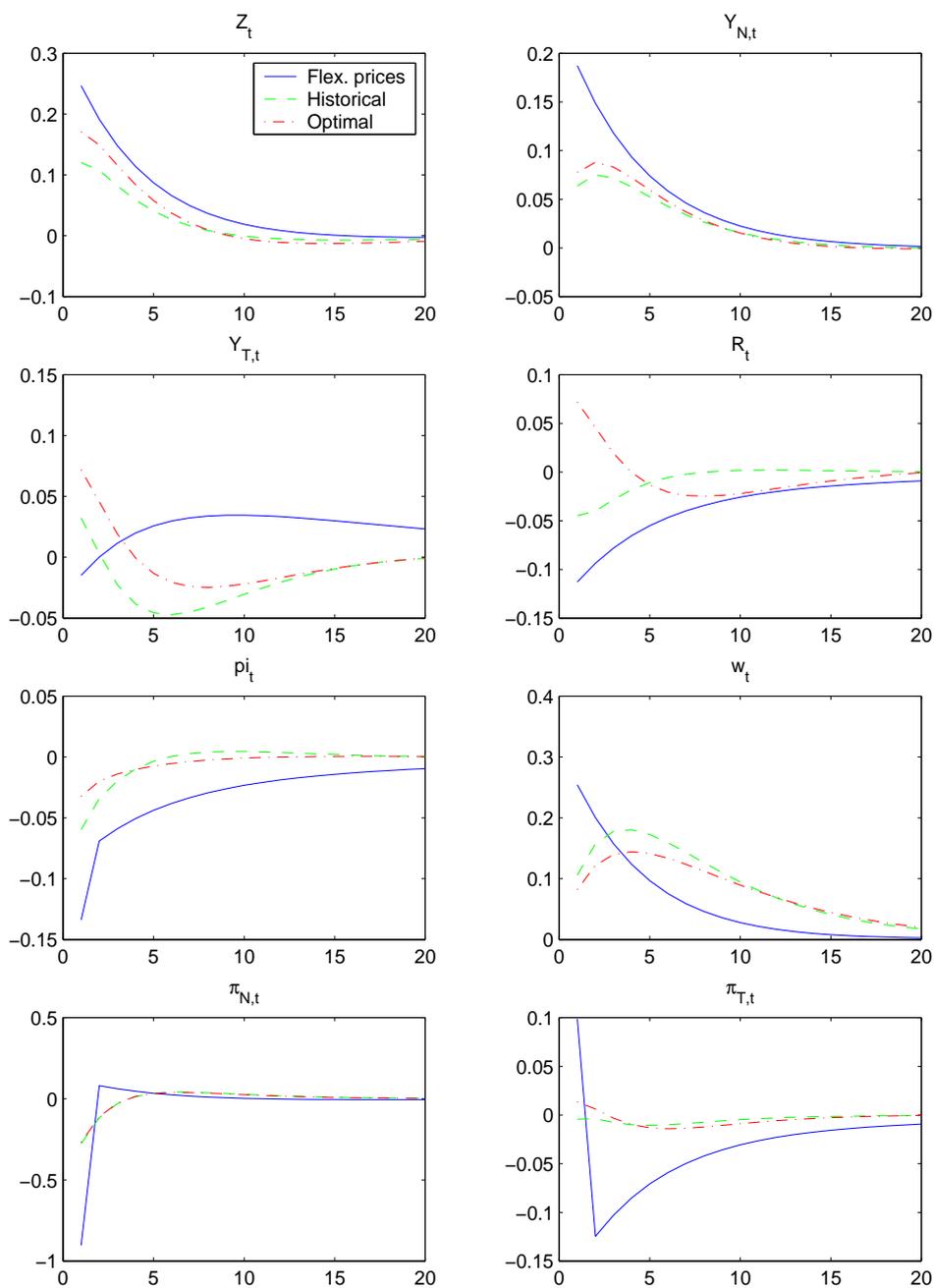


Figure 7: Foreign Nominal Interest Rate Shock

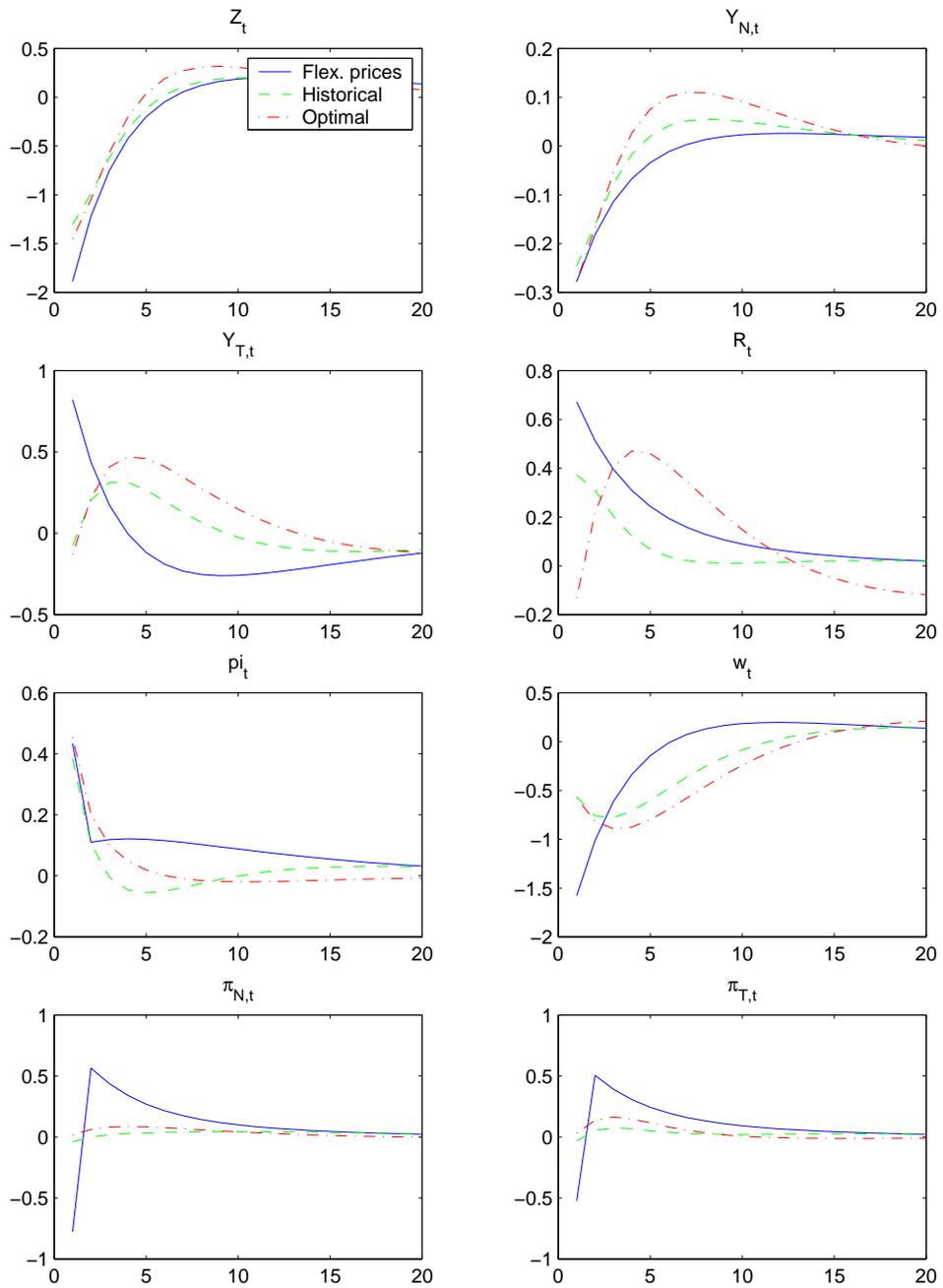


Figure 8: Local Nominal Interest Rate Shock

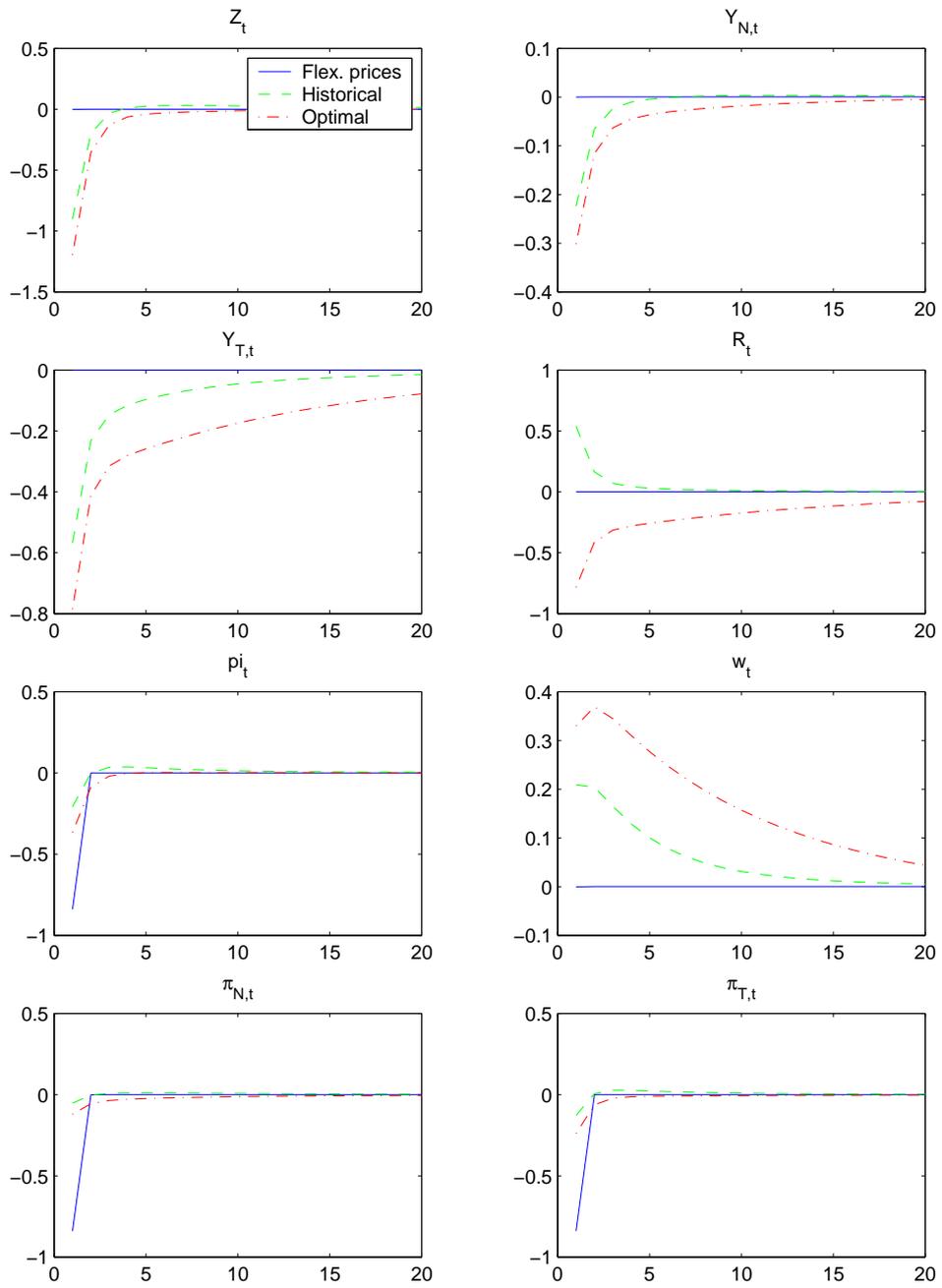


Figure 9: Simulated Time Series of CPI Inflation, All Shocks

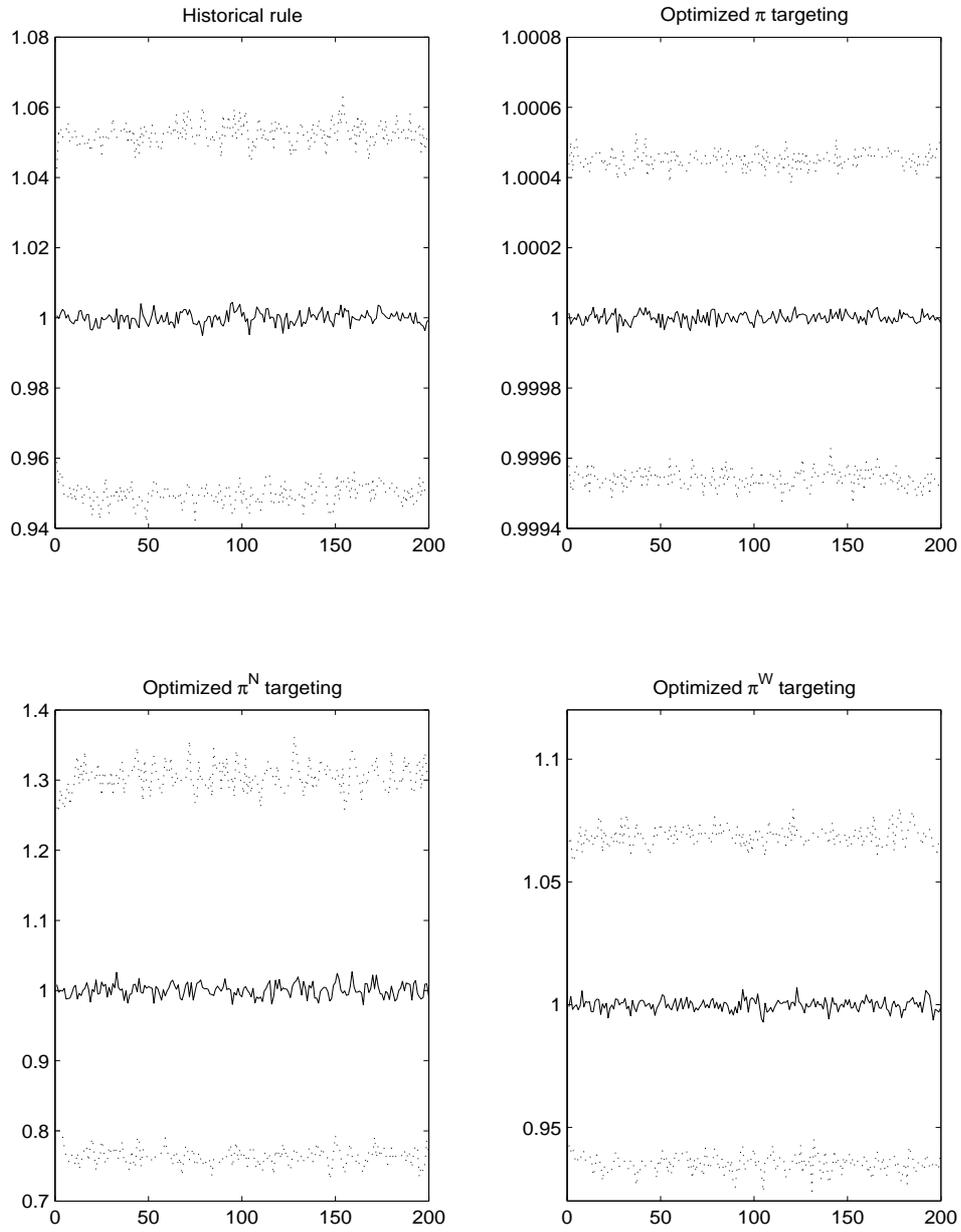


Figure 10: The Average Unconditional Utility with Respect to  $\rho_\pi$  and  $\rho_{\pi^w}$  ( $\rho_y$  Changing)

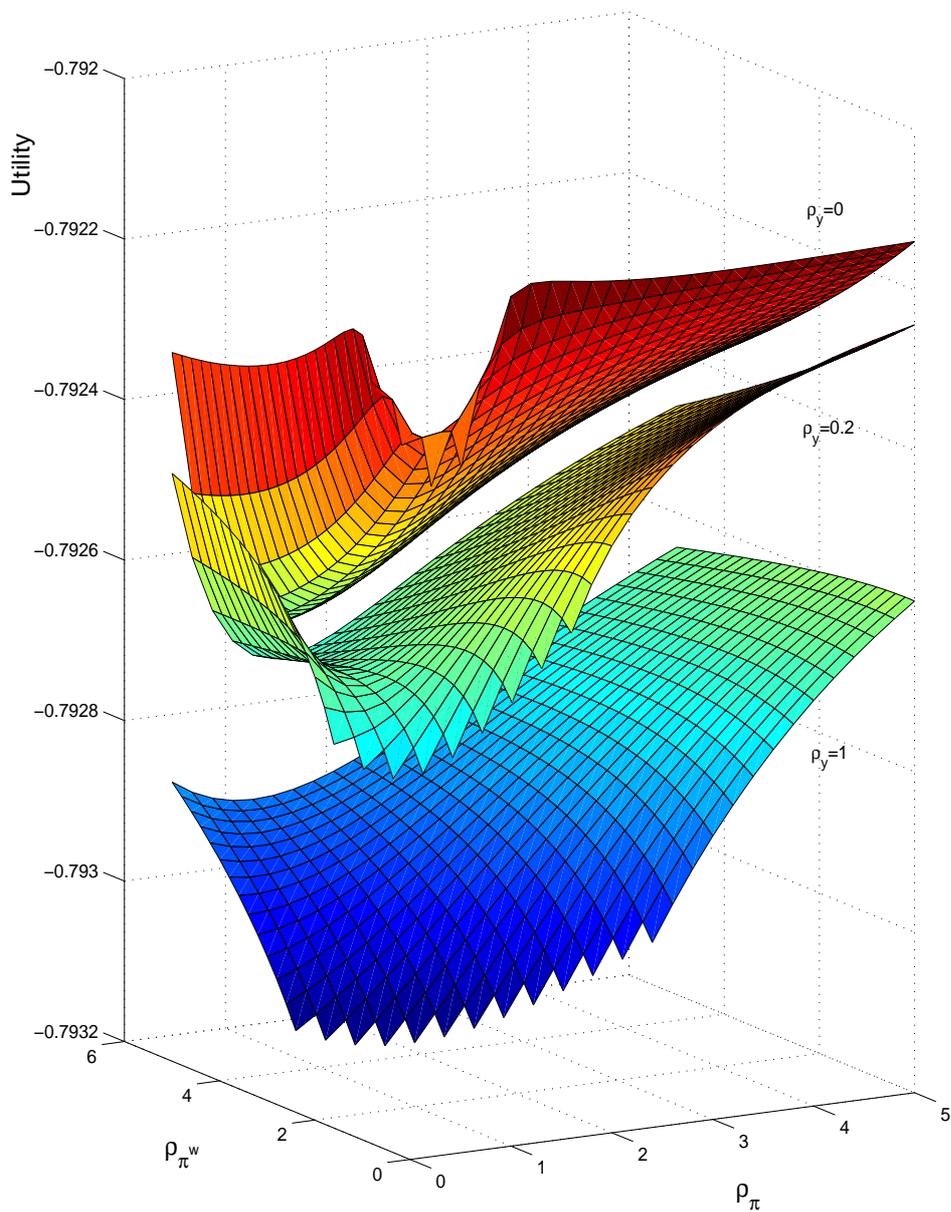


Figure 11: The Average Unconditional Utility with Respect to  $\rho_{\pi^N}$  and  $\rho_{\pi^M}$  ( $\rho_{\pi^{Td}}$  Changing)

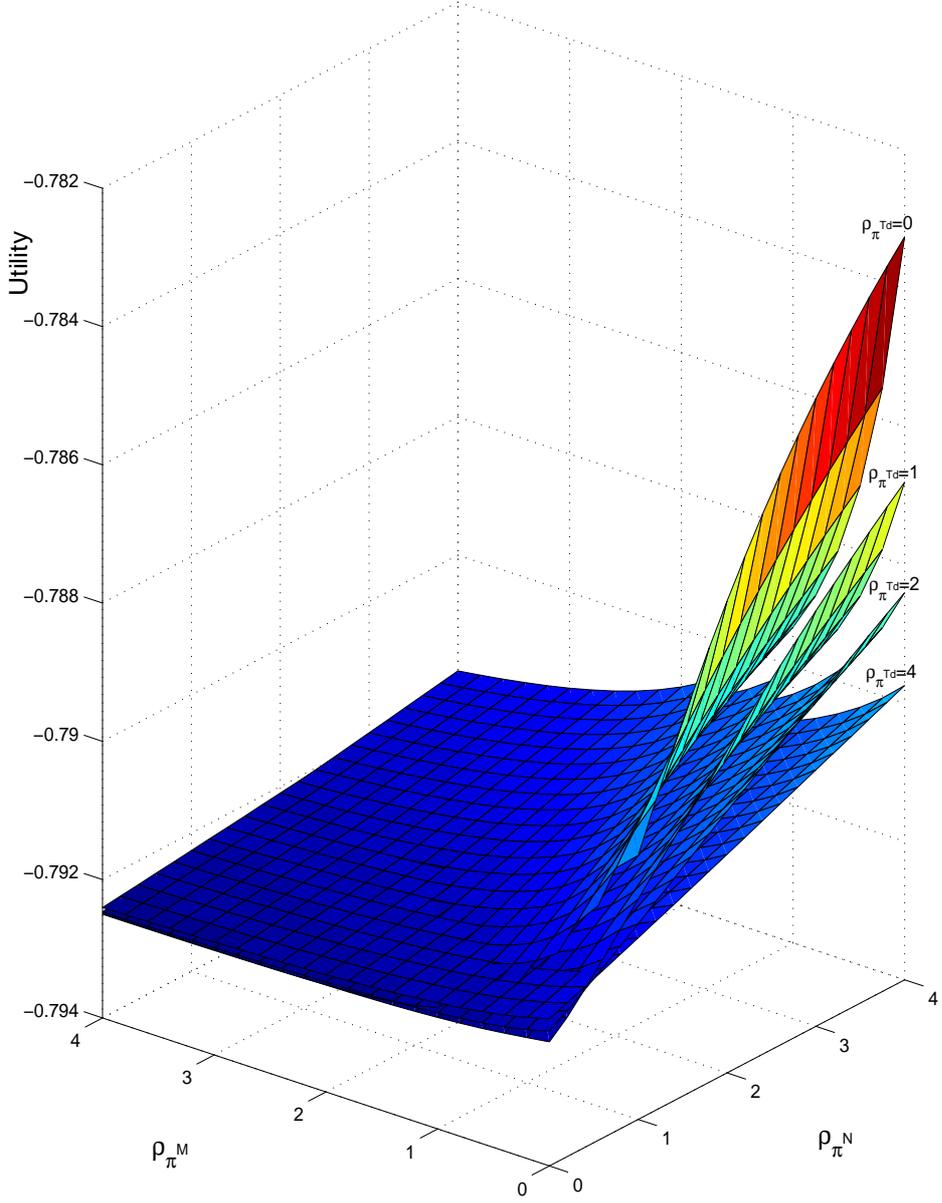


Figure 12: The Average Unconditional Utility with Respect to  $\rho_{\pi^N}$  and  $\rho_{\pi^{Td}}$  ( $\rho_{\pi^M}$  Changing)

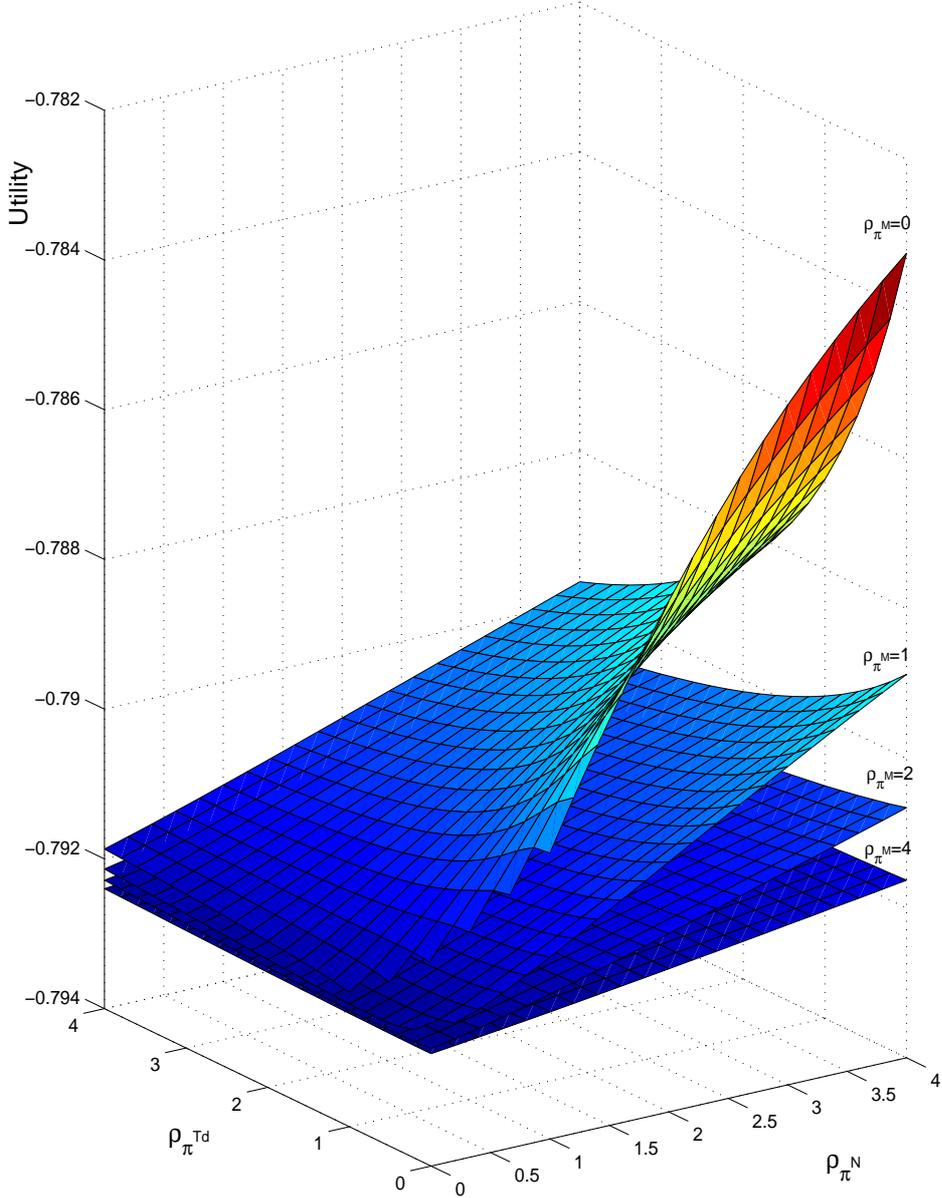


Figure 13: Price-Level Targeting versus Wage-Inflation Targeting

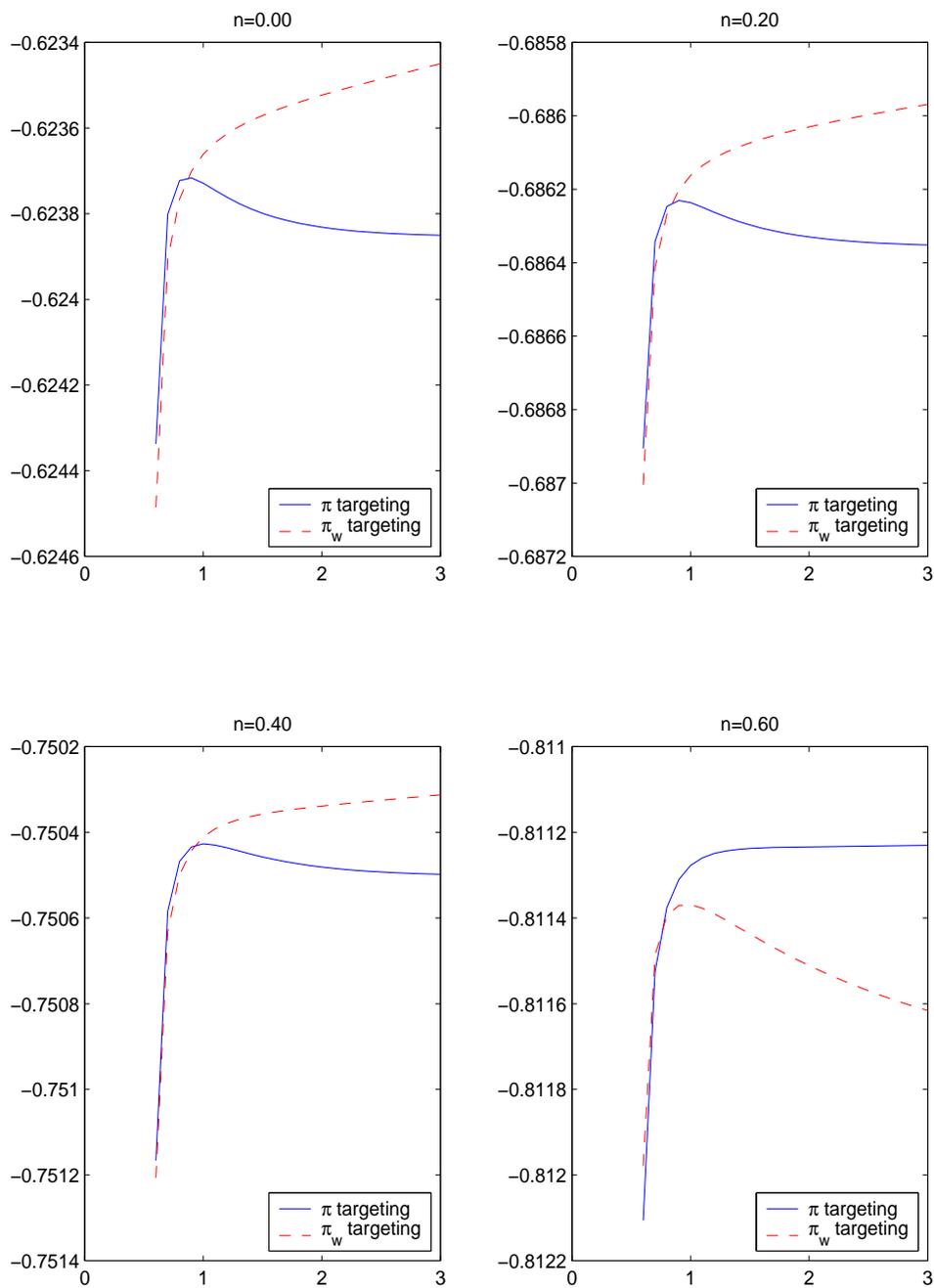


Figure 14: Inflation versus Price-Level Targeting

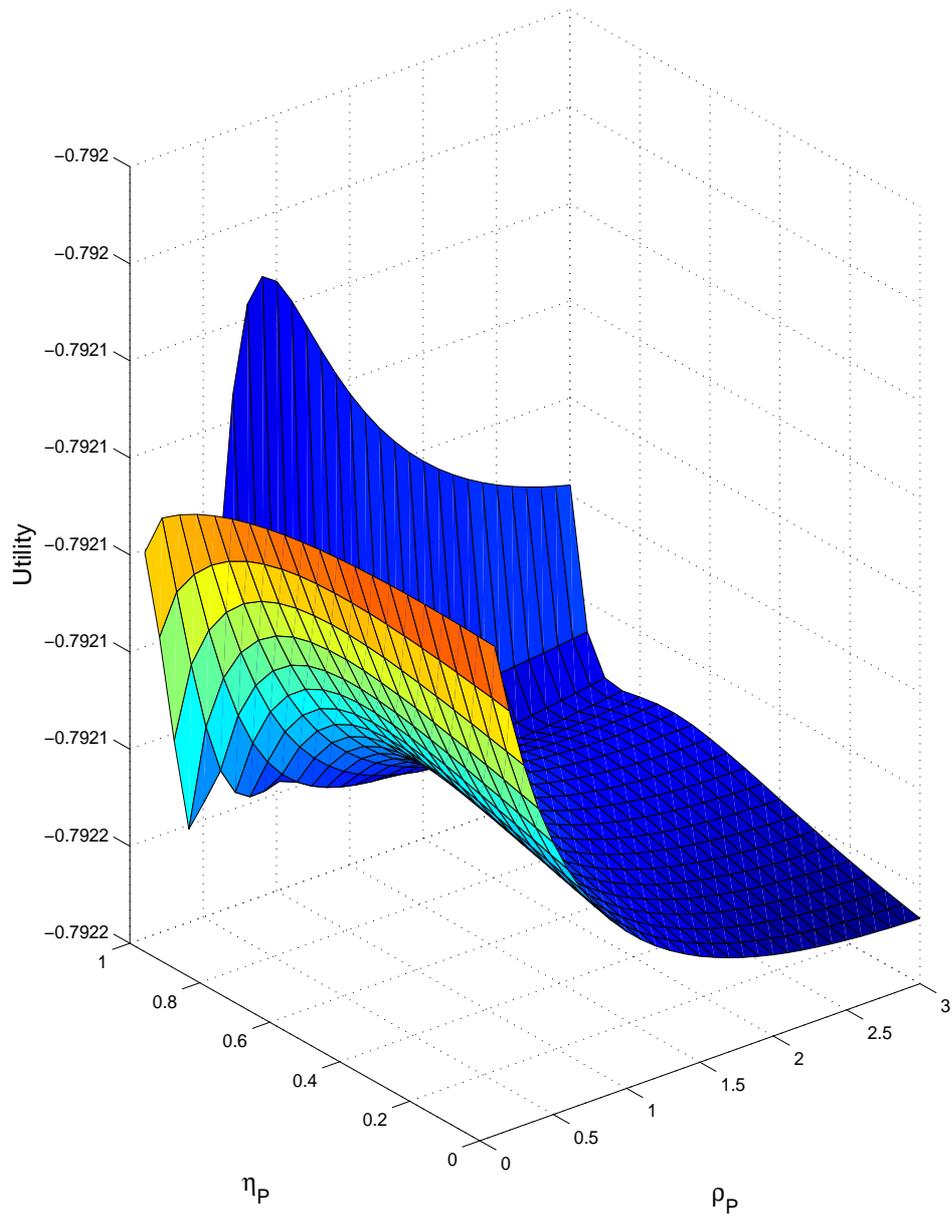


Figure 15: Which Price Level to Target?

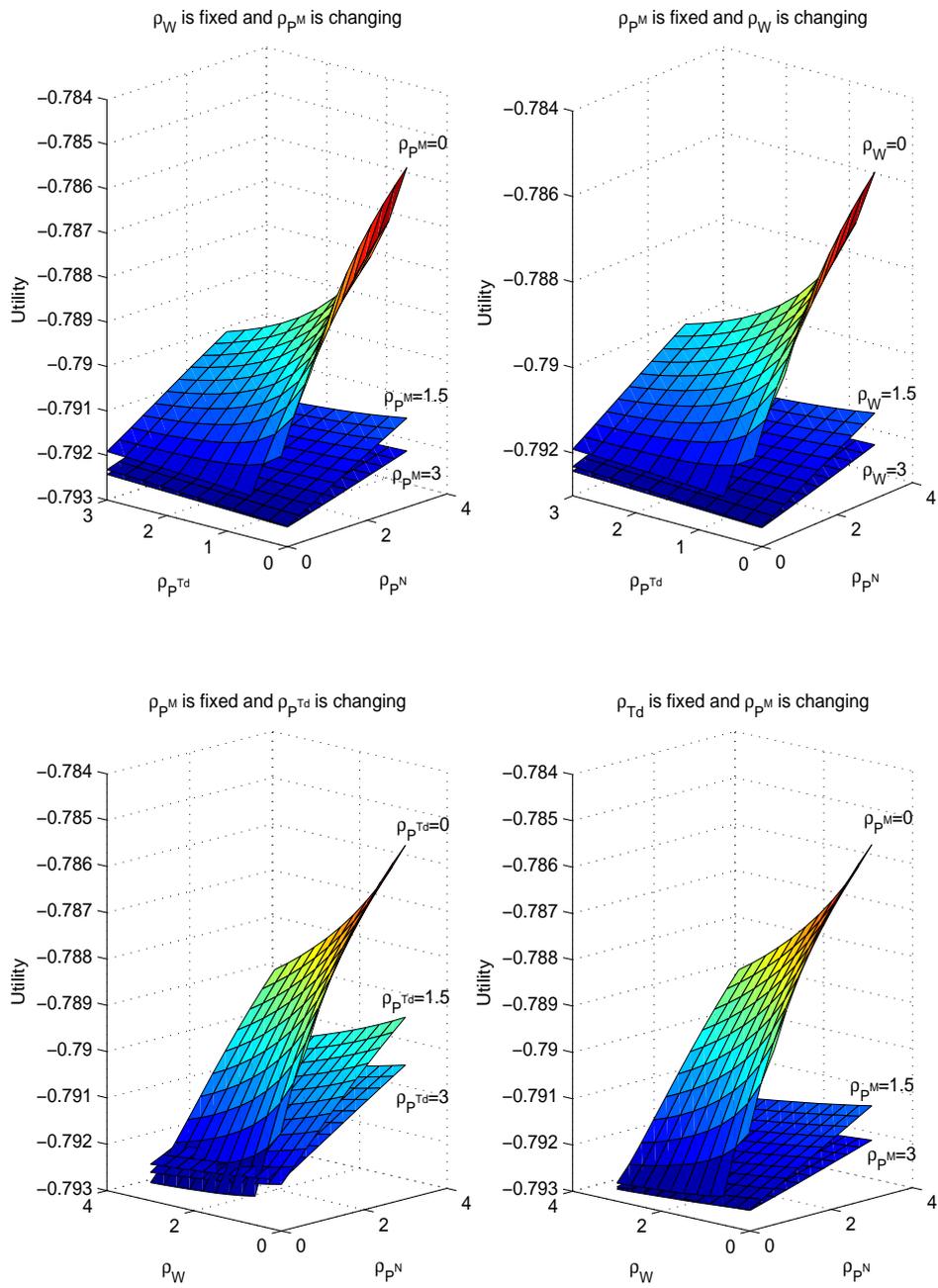


Figure 16: Non-Traded-Goods Inflation versus Non-Traded-Goods Price-Level Targeting

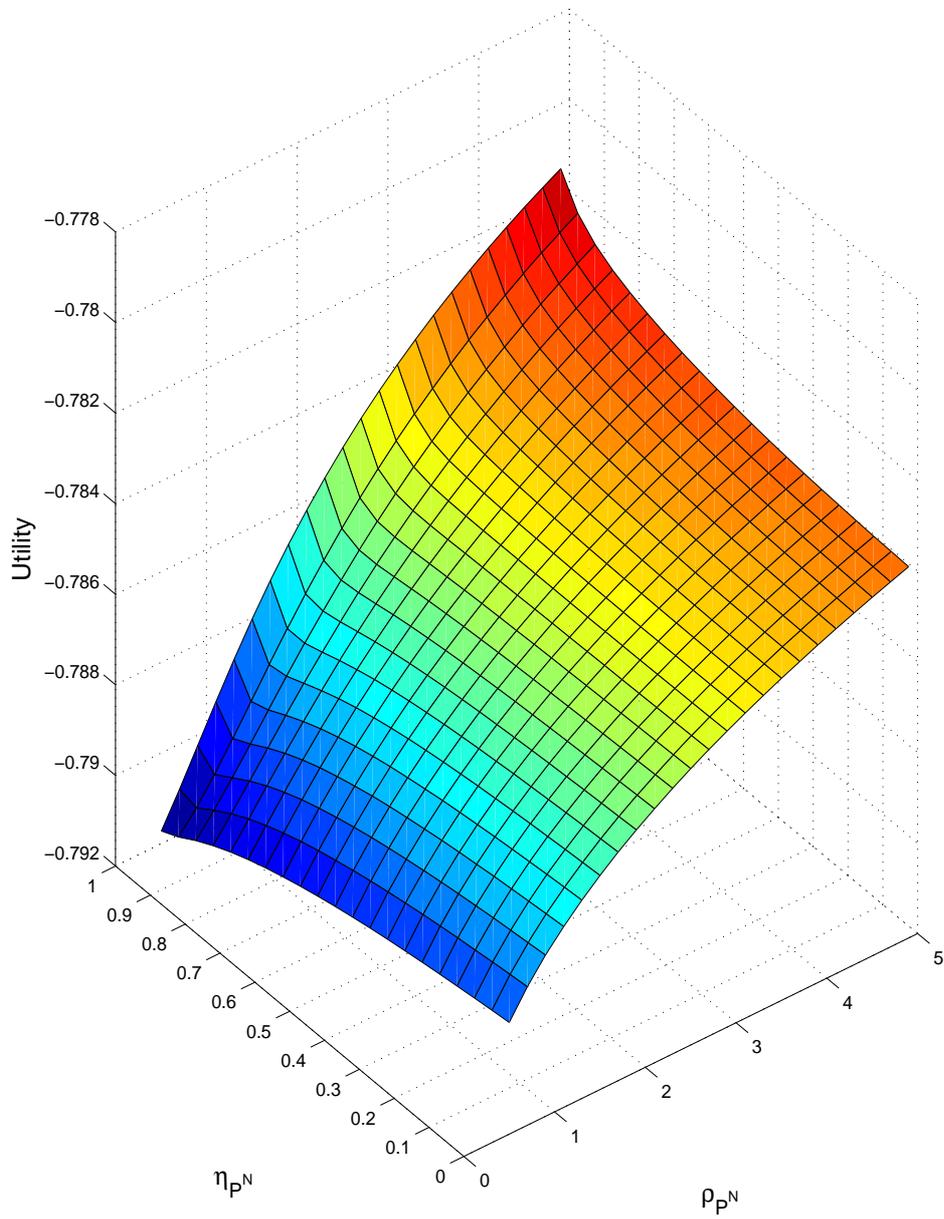


Figure 17: Period  $t + 1$  Optimized Rules

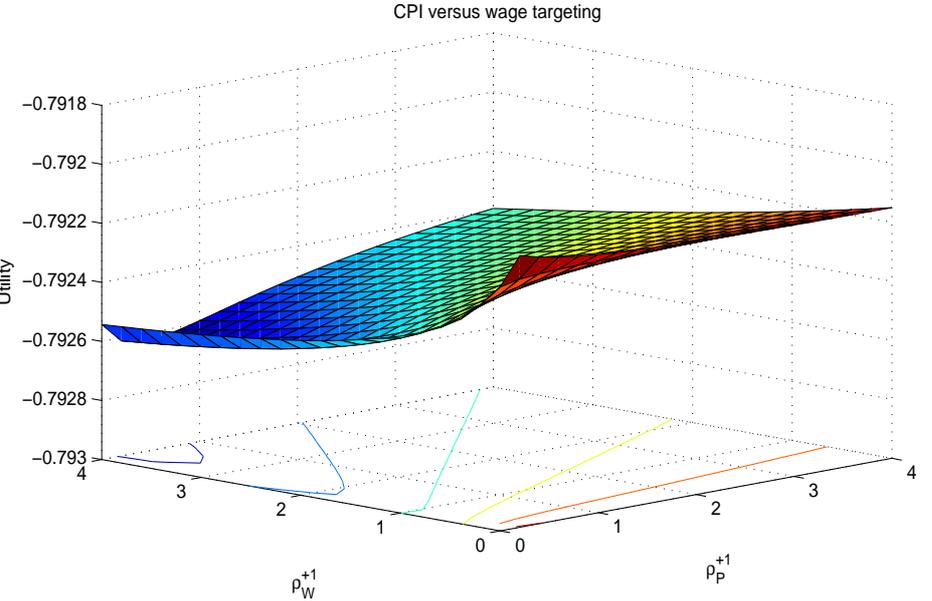
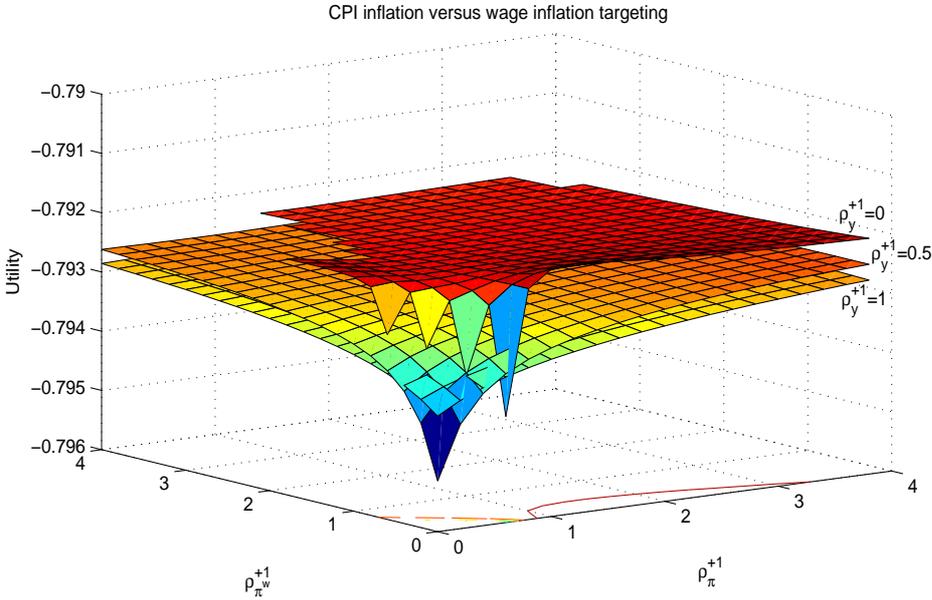


Figure 18: Simulated Inflation,  $A^N$  Shock Only

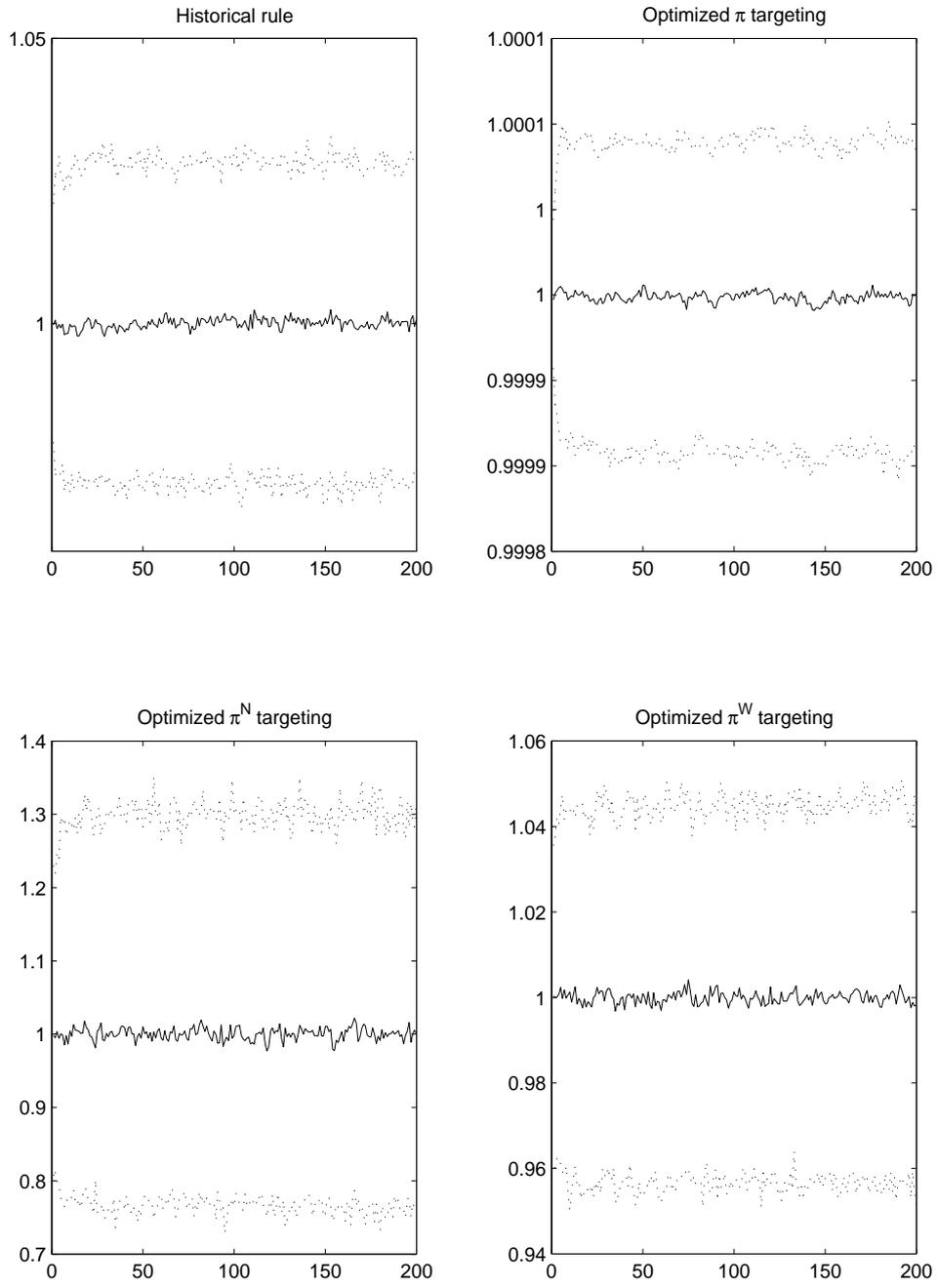


Figure 19: Simulated Inflation,  $A^T$  Shock Only

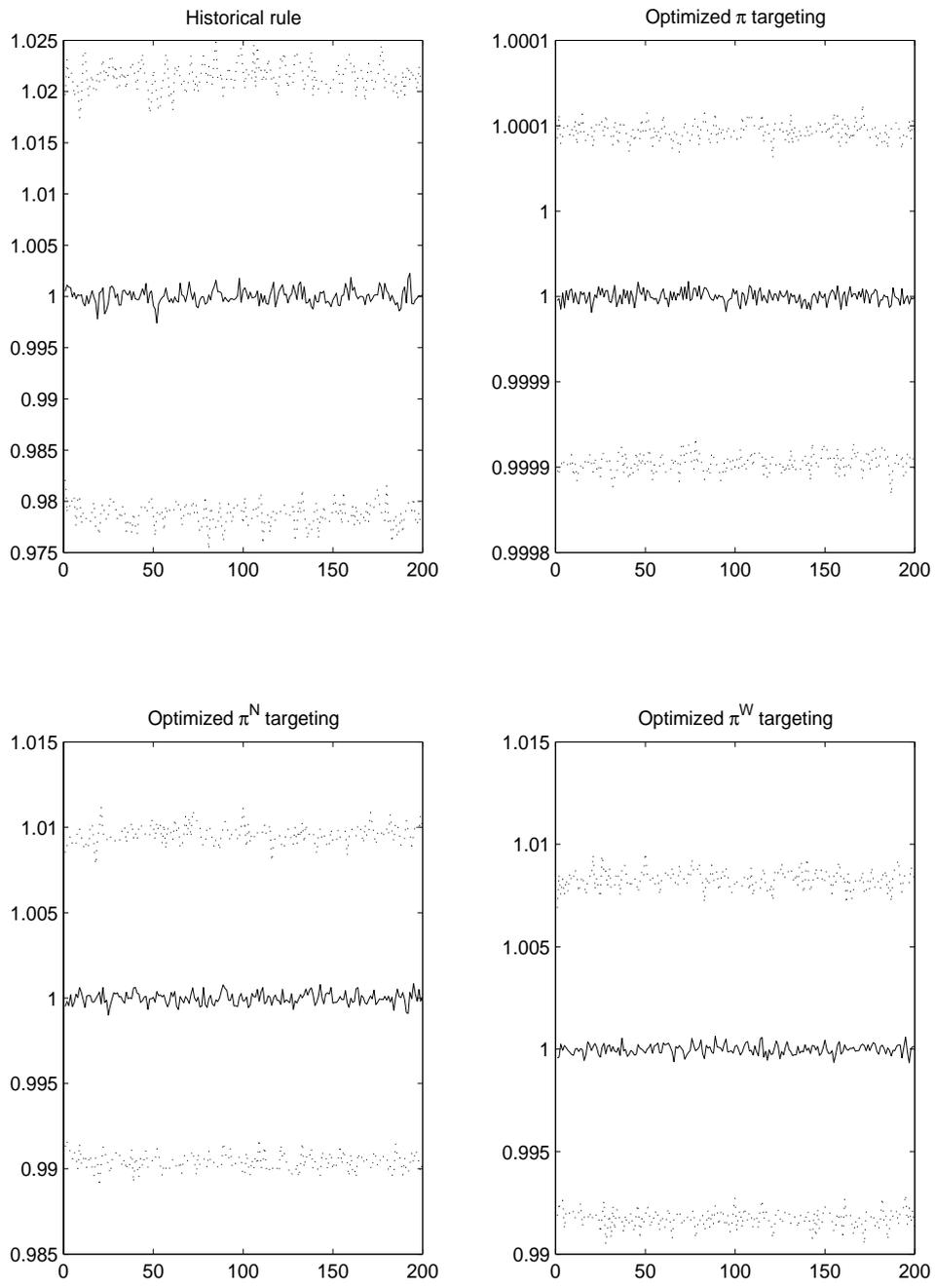


Figure 20: Simulated Inflation, Monetary Policy Shock Only

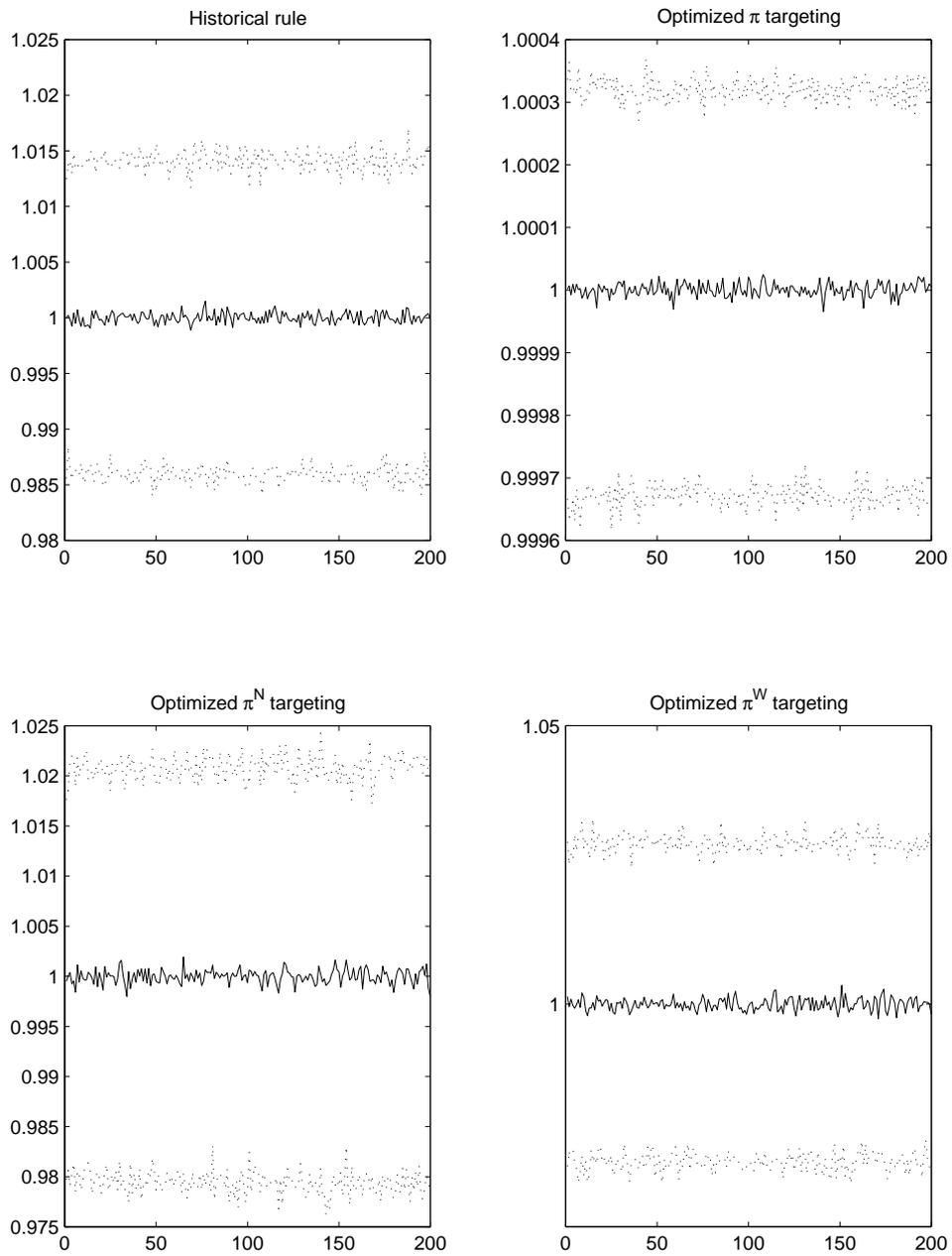


Figure 21: Simulated Inflation, Money-Demand Shock Only

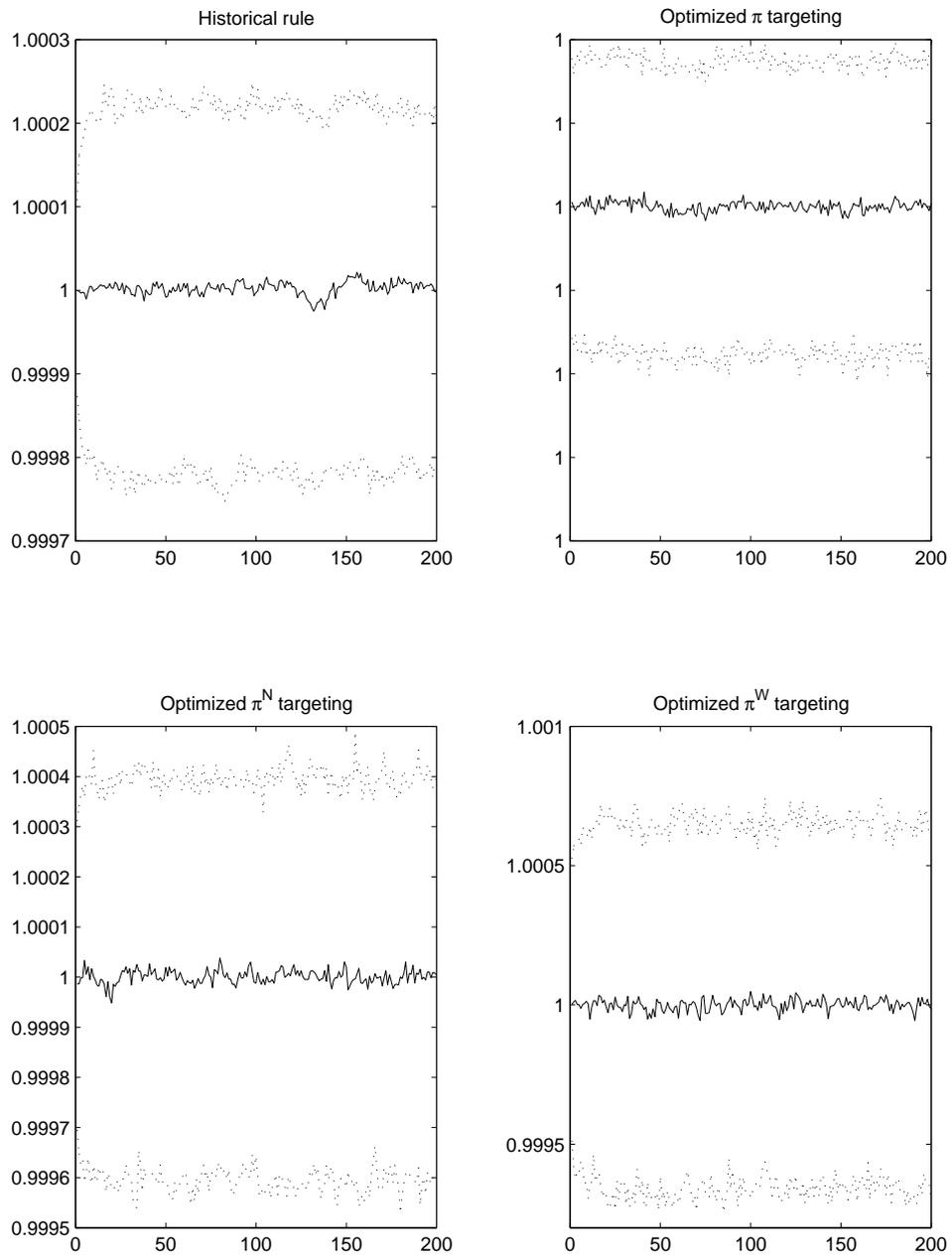


Figure 22: Simulated Inflation, U.S. Monetary Policy Shock Only

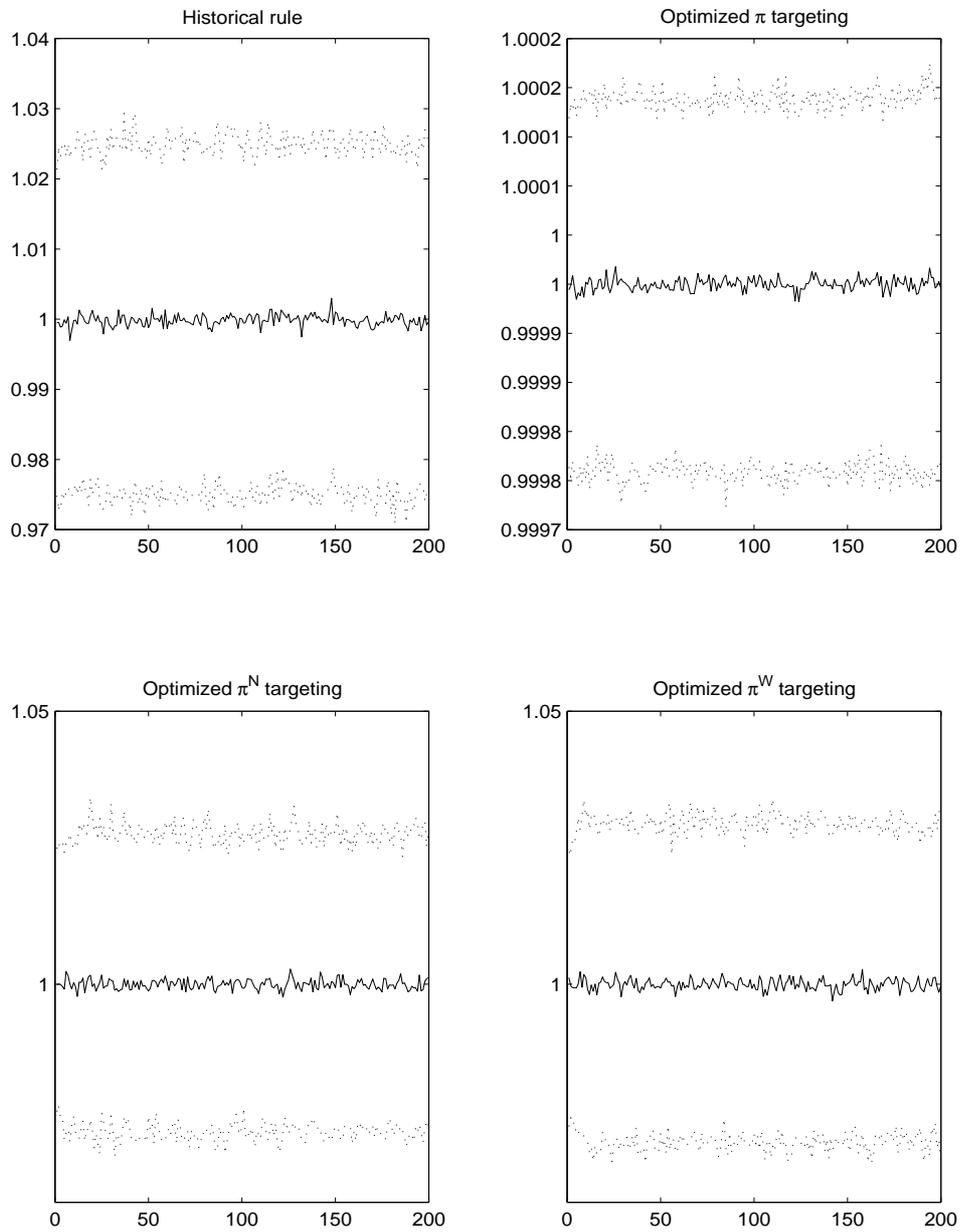


Figure 23: Simulated Inflation, U.S. Output Shock Only

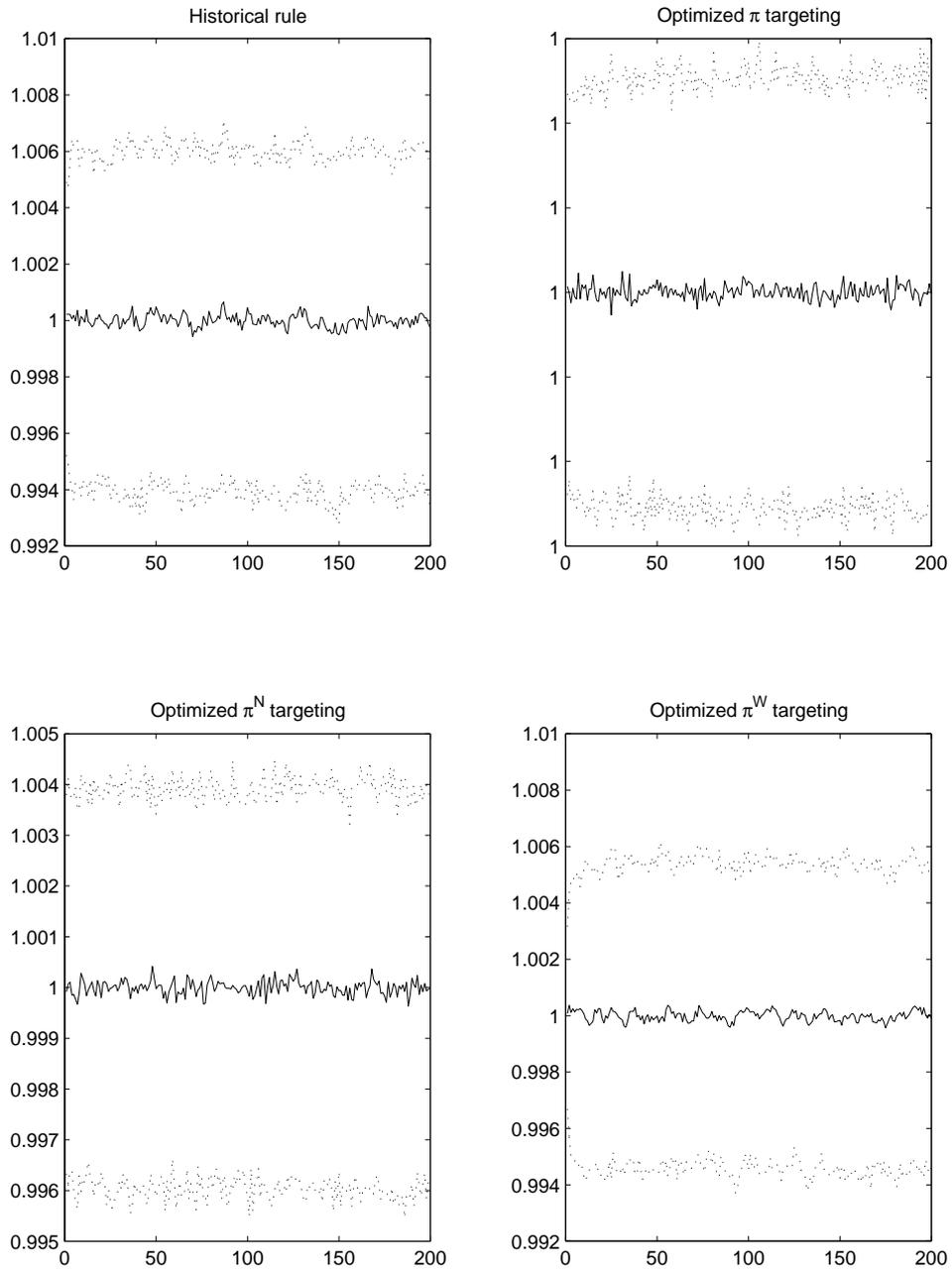


Figure 24: Simulated Inflation, U.S. Inflation Shock Only

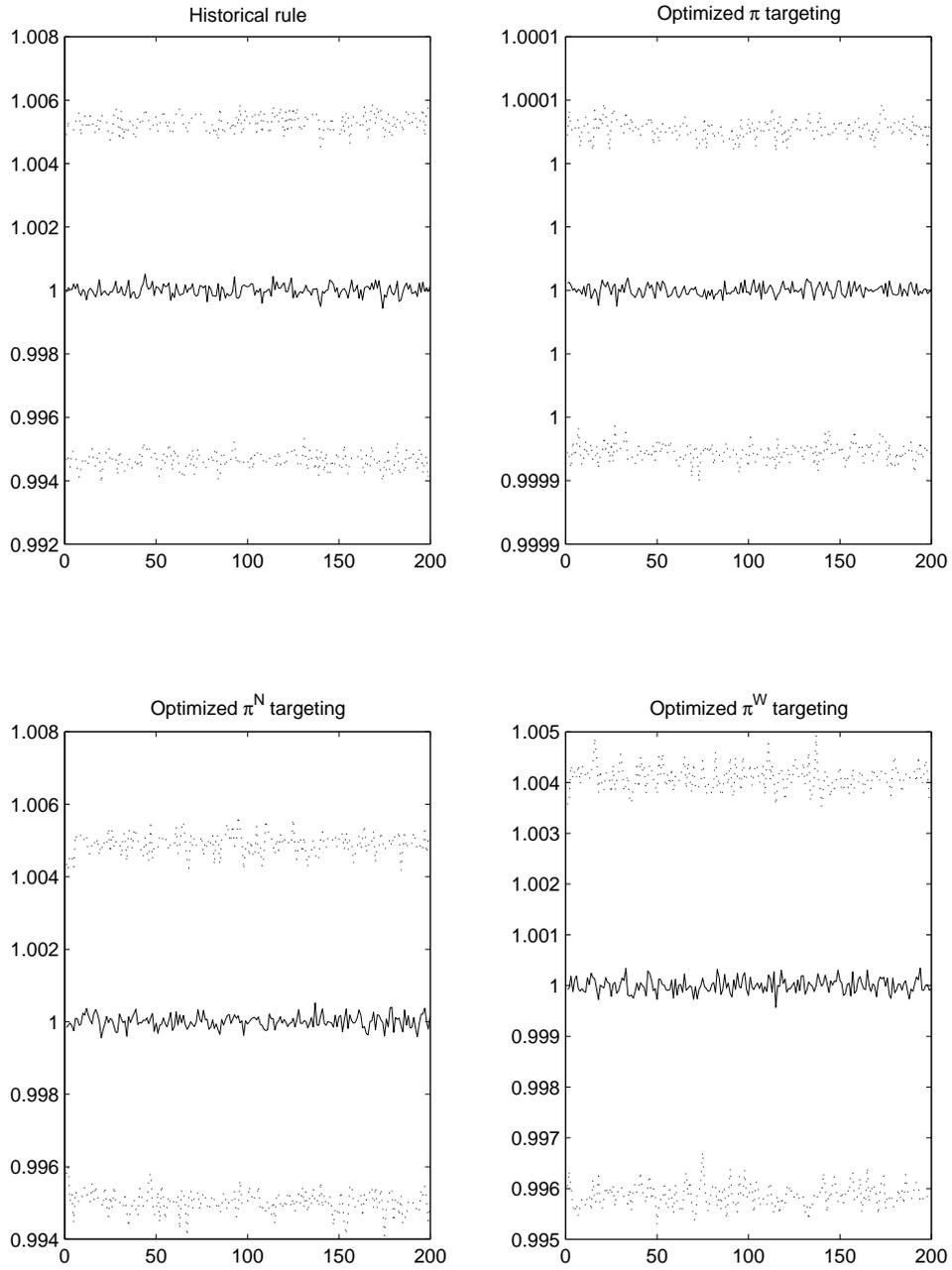
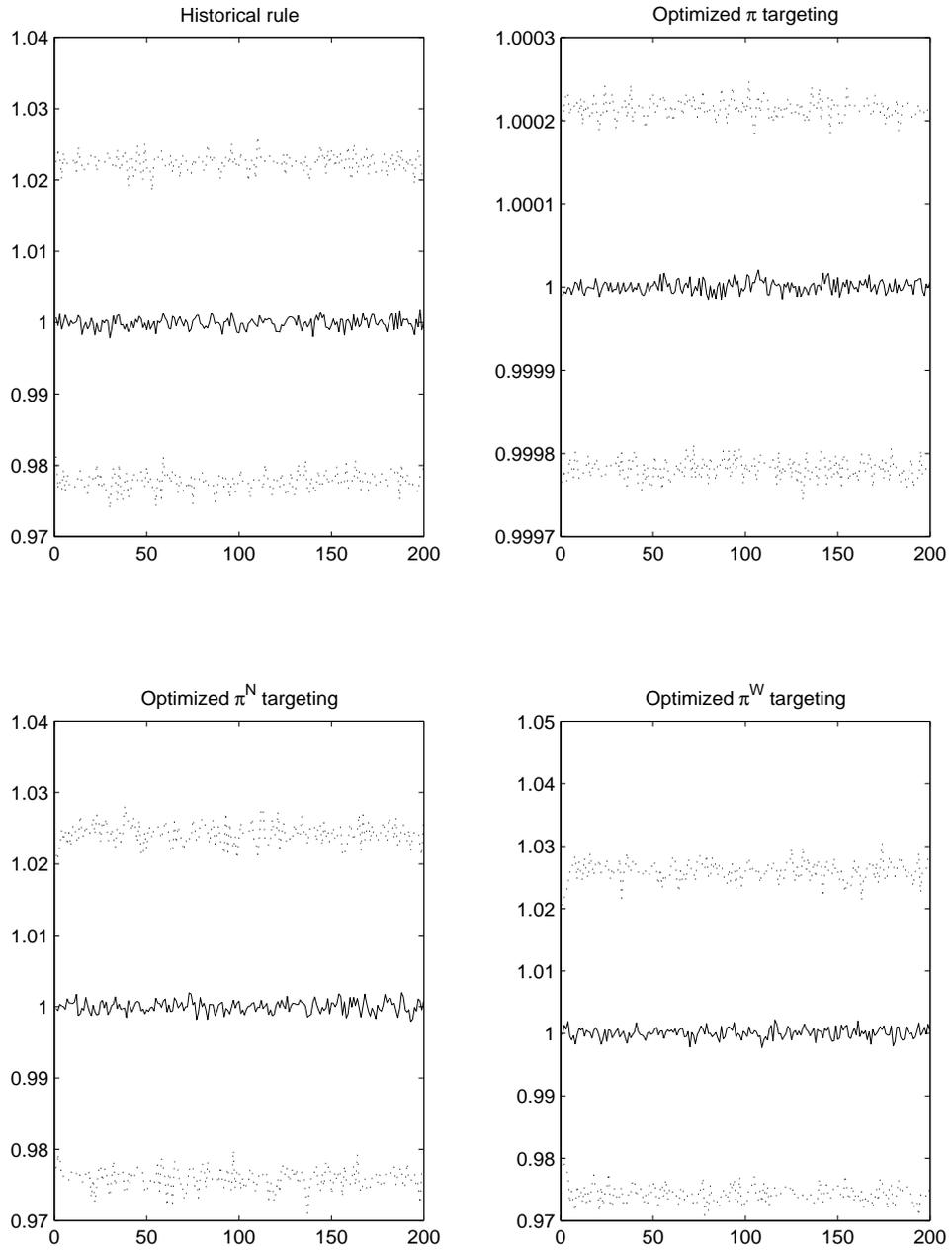


Figure 25: Simulated Inflation, Risk Premium Shock Only



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