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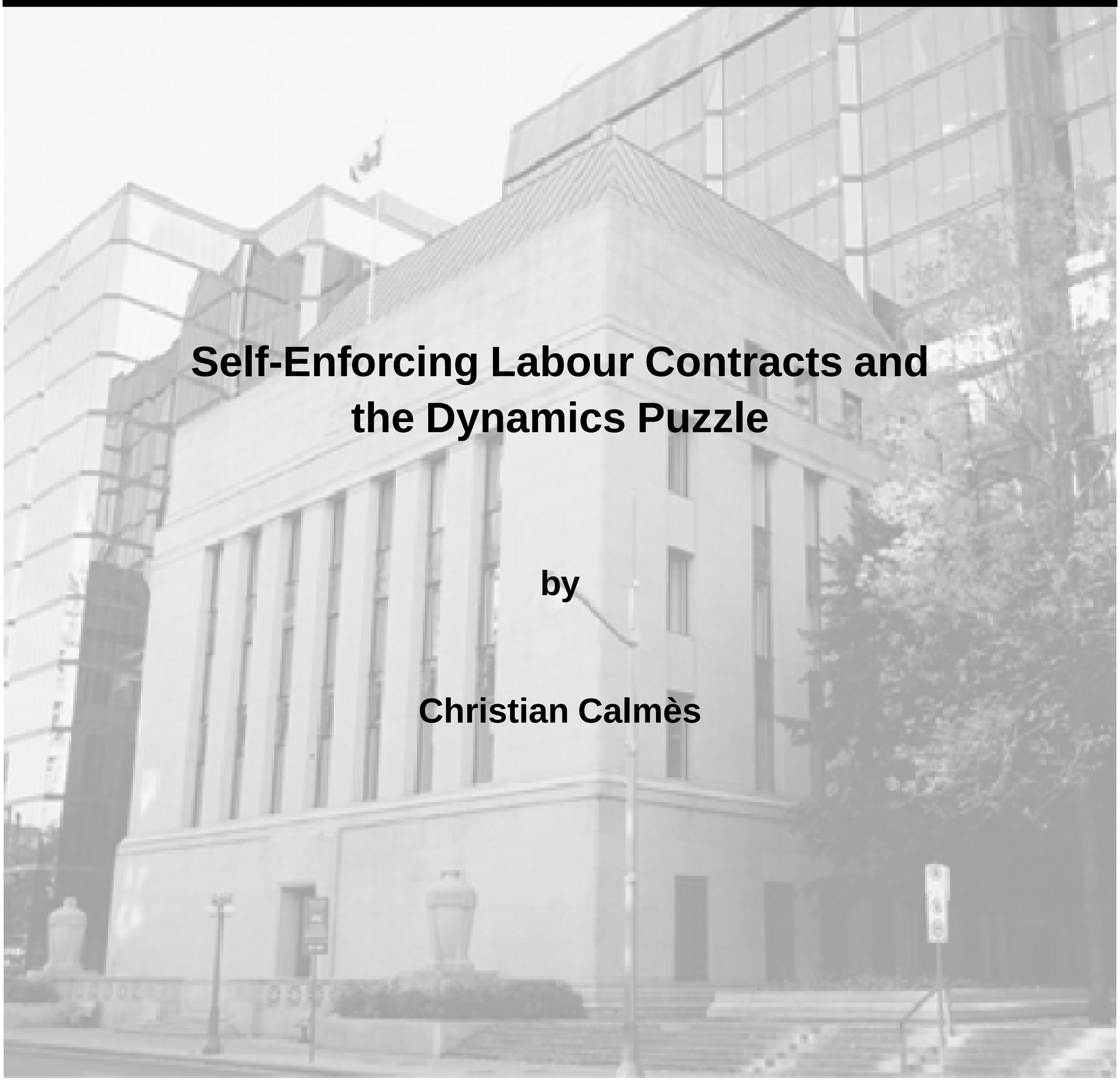

Self-Enforcing Labour Contracts and the Dynamics Puzzle

by

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Abstract

To properly account for the dynamics of key macroeconomic variables, researchers incorporate various internal-propagation mechanisms in their models. In general, these mechanisms implicitly rely on the assumption of a perfect equality between the real wage and the marginal product of labour. The author proposes a theoretical validation of a micro-founded internal-propagation mechanism: he builds a model that features a limited-commitment economy, and derives endogenous self-enforcing labour contracts that produce a different linkage between the real wage and the marginal product of labour. The risk-sharing between the entrepreneur and the worker, both faced with enforcement problems, provides an admissible explanation of the prolonged co-movements observed between consumption and labour. Since these co-movements are at the core of the persistence of the impulse response of output to exogenous technology shocks, this persistence can, in turn, be rationalized with the endogenous real rigidity emerging from the economy. The author shows that, in this framework, the persistence ultimately depends on the initial bargaining power and the magnitude of the risk-sharing.

JEL classification: E12, E49, J30, J31, J41
Bank classification: Business fluctuations and cycles; Economic models; Labour markets

Résumé

Afin de bien rendre compte de la dynamique des variables macroéconomiques clés, les chercheurs intègrent divers mécanismes de propagation interne à leurs modèles. En règle générale, ces mécanismes reposent de façon implicite sur l’hypothèse d’égalité parfaite entre le salaire réel et la productivité marginale du travail. L’auteur propose un cadre théorique pour la validation d’un mécanisme de propagation interne reposant sur des fondements microéconomiques. Il élabore un modèle représentant une économie à engagement partiel dans laquelle des contrats de travail auto-exécutoires endogènes ont pour effet de découpler le salaire réel et la productivité marginale du travail. Le partage du risque entre l’entrepreneur et le travailleur, tous deux confrontés à des problèmes d’engagement, permettrait d’expliquer les covariations prolongées observées entre la consommation et le travail. Comme ces covariations sont à la source de la persistance de la réaction de la production aux chocs technologiques exogènes, cette persistance pourrait à son tour être générée par les rigidités réelles endogènes propres à l’économie. L’auteur démontre que, dans ce cadre, la persistance dépend en définitive du pouvoir initial de négociation et du degré de partage du risque.

Classification JEL : E12, E49, J30, J31, J41
Classification de la Banque : Cycles et fluctuations économiques; Modèles économiques; Marchés du travail

1 Introduction

This paper examines two well-known shortcomings of the macroeconomic literature. The first shortcoming is that some real business cycle (RBC) models have insufficient internal-propagation mechanisms. They have difficulties in replicating some key elements of the economy's dynamics. For example, they sometimes fail to completely replicate the observed permanent and transitory characteristics of the impulse-response functions of output, and the observed persistence of output growth (Cogley and Nason 1995). Watson (1993) reports that the variances of output, consumption, and investment are not well matched over the 6- to 32-quarter range; spectral decomposition of these aggregates does not necessarily coincide with the observed decompositions. One could therefore conclude that a number of macroeconomic models face the challenge of properly explaining the trade cycle and its persistence; this can be called the dynamics puzzle.

The second shortcoming is that, despite some partially successful attempts (for example, Hansen 1985), some RBC models have trouble explaining the aggregate wage dynamics observed in the data. The usual difficulty is to mimic simultaneously (i) the discrepancy between the wage and the marginal product of labour (MPL); (ii) the fact that the real wage does not exhibit a clear cyclical pattern in the data (i.e., the Dunlop-Tarshis observation: Christiano and Eichenbaum 1992, Gomme and Greenwood 1995, Collard and de la Croix 2000); and (iii), more importantly, the wage stickiness empirically observed (Dutkowsky and Atesoglu 1993). Sometimes, the models are challenged to account for individual downward nominal-wage rigidity (Altonji and Devereux 1999, Burda et al. 1998), and the sensitivity of wages to labour market conditions, as estimated by Beaudry and DiNardo (1991) and McDonald and Worswick (1999), or its high persistence—see the argument in Boldrin and Horvath (1995).

Some ongoing research tries to cope with the dynamics puzzle while ignoring the wage dynamics issue. Within a flexible-wage framework, one common approach used to overcome the insufficient dynamics in the models is to introduce internal-propagation mechanisms that operate through the technical rate of substitution, as in Wen (1998a) or Huang and Liu (1998), among others. One can force an increase in the marginal product of labour relative to the marginal productivity of capital to replicate the observed dynamics of output, by making the flow of capital larger than in the standard case. Variable capital utilization (Wen 1998a) or home production (Perli 1998) are two mechanisms that will work in that regard. The same result can be achieved by making labour flows smaller than in the standard case. To do this, a labour adjustment cost serves as a good internal-propagation mechanism (Cogley and Nason 1995, Ambler, Guay, and Phaneuf 1999, Lettau and Uhlig 2000). This consensual approach often relies on the equality of the MPL and the fully flexible wage.

In this paper, I conjecture that the dynamics puzzle and the wage dynamics issue are linked, and that a contract formulation of the wage might help solve the puzzle, regardless of any other internal-propagation mechanism. In particular, I show how a class of micro-founded labour contracts can plausibly explain the persistence of the economy's response to exogenous technology shocks.

To correctly replicate most stylized facts regarding the wage, or its precise link to the MPL, it is often useful to depart from the usual assumption of wage flexibility, and rely instead on a more general form of wage determination: wage contracts. Beaudry and DiNardo (1995) note that the deviation between the wage and the MPL is at odds with the flexible-wage model, but consistent with contracting models. Implicit wage contract models à la Azariadis (1975), Baily (1974), and Gordon (1974) have already been shown to be relatively effective in generating wage stickiness (Hart and Holmström 1987).

In this paper, the shortcomings of flexible-wage models vis-à-vis dynamics motivate the introduction of labour contracts. I consider a theoretical framework based on implicit labour contracts, where risk-sharing initiates a long-term relationship between a worker and an entrepreneur. The original form of the risk-sharing hypothesis comes from the seminal papers of Azariadis (1975), Baily (1974) and Gordon (1974) (ABG hereafter), which were concerned with implicit contracts only, and therefore with full-commitment allocations. I consider a two-sided limited commitment economy that features self-enforcing contracts. These contracts are characterized by imperfect risk-sharing that results from the existence of outside opportunities. More precisely, I consider contracts not legally enforceable (or enforceable with a cost), where all agents are unable to precommit. This hypothesis implies that the relationship's continuation is subject to the constraint that no agent is better off taking advantage of an external opportunity (MacLeod and Malcomson 1989).

The model is partly inspired by the formulation of Boldrin and Horvath (1995), who show how one-period contracts with one-sided limited commitment can help explain the contemporaneous volatilities of the key macroeconomic variables. To expand upon their argument, I consider long-term dynamic contracts with two-sided limited commitment and focus on the persistence issue. Unlike Thomas and Worrall (1988), I also provide a numerical computation of contracts that specifies the wage and the hours worked, and a study of their macroeconomic implications for the dynamics puzzle. The endogenous self-enforcing contracts are used to analyze the implied dynamics of labour and consumption, and to examine the extent to which these dynamics differ from the ones implied by a flexible-wage model.

I show that this type of theoretical framework generates an endogenous real rigidity from the income insurance provided by risk-sharing. More specifically, I find that it is possible, in theory, to create extreme persistence even when the exogenous technology shock is assumed to be purely transitory. In the model, the persistence is driven by the initial bargaining power and the magnitude of the risk-sharing domain.

This paper sheds light on the labour-income dynamics implied by a model of infinite-term self-enforcing contracts, and aids in the study of the potential implications of those contracts for macroeconomic modelling and the dynamics puzzle. I investigate whether these contracts can constitute a theoretically valid internal-propagation mechanism. The positive answer is encouraging.

In section 2, I discuss the rationale for self-enforcing contracts, and describe their key dynamic properties. In section 3, I define the theoretical framework, detail the assumptions upon which I build the model, and define a self-enforcing labour contract. I also discuss the choice of contracts with two-sided limited commitment, provide a justification for the assumption of an infinite-term risk-sharing relationship, and sketch the underlying dynamic bargaining game and its internal-propagation properties. In section 4, I fully describe the economic environment, composed of three elements: the flexible-wage model, the full-commitment benchmark, and the limited-commitment economy. The section includes some basic results that characterize the contracts, and a description of the structure of the contracting economy. It also defines the contract set, describes the endogenous real rigidity, and provides additional definitions and propositions. It includes a qualitative characterization of the economy's dynamics. In section 5, I explain the computation of the flexible-wage model, the full-commitment benchmark, and the limited-commitment economy; define the recursive self-enforcing equilibrium; and detail some key numerical results. In section 6, I offer some conclusions.

2 Why Use Self-Enforcing Labour Contracts?

The rationale for using self-enforcing labour contracts is that workers are relatively more risk-averse than entrepreneurs. This is the central hypothesis of the ABG model. The assumption that entrepreneurs are less risk-averse is based on the Knightian argument of self-selection presented by Rosen (1985) in the general-equilibrium case (see also a complementary explanation in Romer 1996), which states that occupational choice is determined by the degree of risk aversion (Kihlstrom and Laffont 1979). Indeed, according to Keynes (1936), sanguine temperament and the propensity to gamble are important characteristics of entrepreneurship: "Business men play a mixed game of skill and chance, the average results of which to the players are not known by those who take a hand. If human nature felt no temptation to take a chance, no satisfaction (profit apart) in constructing a factory, a railway, a mine or a farm, there might not be much investment merely as a result of cold calculation."

This difference in the level of risk aversion gives rise to a mutually advantageous contract. The worker, owing to the risk-sharing hypothesis, is assured of having a less-fluctuating labour income within the relationship. Hence, while it is equally possible to study economies with homogeneous risk aversion, I study a case of heterogeneous risk aversion. I consider a risk-averse worker type and a risk-neutral entrepreneur type.

The risk-sharing hypothesis has several advantages. First, it provides an explicit microeconomic foundation for a specific form of wage stickiness (especially if the supply of hours is assumed to be inelastic, Rosen 1985). In this type of contract model, the stickiness is an artifact of the risk-sharing hypothesis; it has nothing to do with considerations of costs or anticipation errors regarding the fixed-duration contracts, à la Fischer (Rosen 1985). Furthermore, the dynamic properties that self-enforcing contracts add to the models are not strictly dependent on the preference specification. For example, the results in Kydland and Prescott (1982) and Hansen (1985), among others, rely on precise assumptions about preference to properly account for the dynamics of the key macroeconomic variables. In other respects, Rosen (1985) notes that one can always write a contract and exploit its dynamic properties, regardless of the preference specification. One advantage of using this type of contract, then, is that it constitutes a potential internal-propagation mechanism compatible with a wide range of utility functions.

Since I focus on the dynamics puzzle in this paper, the theory of self-enforcing labour contracts provides another, more beneficial, feature. The contract insures the worker against fluctuations caused by shocks (Hart and Holmström 1987). It is therefore reasonable to think that the income effect, which arguably plays a neutralizing role in flexible-wage models, is reduceable with the risk-sharing hypothesis. This effect can still act through the wage when a model assumes heterogeneous contracts: Beaudry and DiNardo (1995) establish the empirical plausibility of the self-enforcing contracts hypothesis by observing that, in this context, the wage has mainly this type of income effect (whereby productivity changes are controlled for). Overall, the income transfers through states permitted by the risk-sharing device allow workers to be almost unconcerned by the variability of consumption.

In other words, only the (opportunistic) substitution effect may play its role fully. In that case, two important corollaries can apply. First, even if there were savings in the model, hours could still be quite volatile. In that respect, the risk-sharing hypothesis provides a plausible alternative to the intertemporal substitution hypothesis (Rosen 1985). Second, despite being smoothed, consumption can be more volatile than in regular models, because it is tied to hours. In fact, in the literature, it is often assumed for tractability that a worker consumes all of their wage. Therefore, it is possible to have too much consumption volatility. To overcome this problem, one needs to relax the assumption of no savings by introducing a cash-in-advance constraint or any asset that helps to smooth consumption over time. Boldrin and Horvath (1995) provide a good example of this by allowing the entrepreneur to save a share of his or her revenue. These two important corollaries derive from the fact that the insurance device smooths consumption over time *and* over states.

Consequently, in such a framework, it is theoretically conceivable to tie consumption and hours together, using the risk-sharing hypothesis, and—unless one relies on extremely separable preferences—have both variables potentially more volatile than in the standard case (see the detailed argument in Calmès 2005). This is the theory that inspires macroeconomic study of the dynamics implications of self-enforcing contracts. The introduction of risk-sharing at the core of the worker-entrepreneur relationship is of key importance: it almost removes the income effect, and uncovers a joint behaviour of consumption and hours absent from the flexible-wage model. To illustrate the intuition behind this joint behaviour, imagine the occurrence of a good period. On the one hand, wages and consumption tend to rise due to the substitution effect. On the other hand, a weak income effect limits the decrease in hours after the occurrence of the shock. This is possible because of the quasi-absence of the usual (almost complete) impediment caused by the income effect. Thus, in the theoretical framework I consider, hours can co-move with consumption for a longer period, which can lead, ceteris paribus, to a more persistent response from the economy.

This is an interesting qualitative property, because the behaviour of consumption and hours is imperfectly reproduced by a number of RBC models (see, for example, Wen 1998b). In particular, the theoretical spectral density of consumption is often at odds with its true spectral density, as is the case for labour. In fact, the spectra of these two real variables seem to be the most difficult to explain within the RBC paradigm. Interestingly, Perli (1998) also reports that, in the standard RBC model, consumption and labour move in the same direction *only* at impact, but move in *opposite* directions afterwards. This is an issue already emphasized by Summers (1986). Rotemberg and Woodford (1996) suggest that this shortcoming is ultimately responsible for the insufficient macroeconomic dynamics associated with the intertemporal substitution hypothesis of the RBC paradigm (see also Basu, Fernald, and Kimball 1998, and Galí 1999).

To summarize, even if the dynamics puzzle and the wage dynamics problem encountered in RBC models are linked (Ambler, Guay, and Phaneuf 1999), it is common to deal with the former while ignoring the latter. Nevertheless, I argue that self-enforcing contracts can magnify hours and consumption volatilities and their co-movements. A model that features self-enforcing labour contracts can display dynamic properties that are quite different from the usual ones. Such a model can persistently prolong the co-movement between consumption and hours. The central mechanism is risk-sharing under limited commitment: it provides a specific form of insurance to the worker, which works like a "natural" real rigidity in the model. This rigidity can lead to persistence.[1] Hence, self-enforcing contracts potentially constitute an endogenous internal-propagation mechanism.

Conjecture 1 *For a given self-enforcing labour relationship, economic dynamics (persistence) might partly result from the embedded weakness of the dampening income effect.*

This paper investigates this conjecture.

3 The Theoretical Framework

3.1 Description of the assumptions

In the usual general-equilibrium framework, agents interact mainly through the price system. In this paper, I introduce another, yet central source of interaction: the risk-sharing requirement between the players (Salanié 1994). The worker must consider that the entrepreneur may renege at any moment during bad periods. The entrepreneur must consider that the worker may also renege, as his or her opportunity cost increases during good periods, when the outside wage is more attractive. The problem is that there is no possible precommitment, at least not an explicit one, possibly because of the dynamic inefficiency of commitment, the high legal costs of making the contract enforceable, or the infeasibility of making it enforceable (the latter being particularly relevant for developing economies).

Assuming only a limited commitment from the worker could have at least two drawbacks. First, a priori, there is no good reason why only the worker would consider external opportunities when deciding on whether to renege. Second, this assumption would restrict the analysis of the dynamics to situations where almost all the initial bargaining power

[1] In another context, Gauthier, Poitevin, and González (1997) refer to this phenomenon as "rating experience."

is given to the entrepreneur. Indeed, in that case, as in the full-commitment case, risk-sharing holds true for almost all the admissible initial bargaining powers. Hence, insurance would be almost complete (one-sided commitment), or complete (full commitment) and there would not be much to say. That is, while the risk-sharing hypothesis is essential to generate persistence, the assumption of two-sided limited commitment is crucial for creating non-trivial transitory dynamics. Without this assumption, only the initial state of nature might have an impact on the relationship. As a result of the hypothesis of two-sided commitment problems, the theoretical framework I consider displays a real rigidity characterized by a "sluggish" transition, before reaching the perfect insurance state. For that reason, hypothesizing a truly imperfect insurance is justified in principle, given the conjecture being investigated.

While the hypothesis regarding the nature of the enforcement problem is central to this study, the assumption regarding the contract's duration is not. To guarantee that the worker does not renege on the infinite-term relationship, Beaudry and DiNardo (1995), following Thomas and Worrall (1988), assume that, in case of defection, the worker receives reservation utility payoff on every contingency forever. The assumption of an infinite-term relationship is usually required, as Thomas and Worrall (1988) explain, because if the term of the relationship were finite, the self-enforcing contract would always pay the spot market wage: in the last period, there would be nothing to offset the short-term incentive to renege. By backward induction, only a standard back-loading effect could prevail. Furthermore, in that case, the contract set would be reduced to a singleton, with the contract economy coinciding with the flexible-wage economy.

Following these authors, I hypothesize that the contract's duration is infinite, even if the theoretical framework I consider might potentially allow for finite durations. This choice does not lead to a loss of generality in investigating Conjecture 1. It amounts to thinking of an infinite-term risk-sharing relationship with two-sided limited commitment. To investigate Conjecture 1, I only have to focus on the admissibility of the self-enforcing contracts framework. With variable duration, one could regulate the *degree* of persistence. But I am interested in determining whether the self-enforcing contracts can generate any persistence, and, if so, characterizing it. I aim to determine the conditions under which persistence exists within the theoretical framework I analyze.

The set of additional assumptions used in the model are standard:

Assumption 1: The worker is risk-averse, whereas the entrepreneur is risk-neutral in the presence of the exogenous technology shock, z_t, where z_t can take N different values at any time t; i.e., $\left\{z_t^1, ..., z_t^N\right\}$, with $\forall t, z_t = \cup_{i=1}^N \left\{z_t^i\right\}$, and z_t^i is i.i.d. A particular sequence of past realizations up to time t is a history, $h_t = (z_1, ..., z_t)$.

Assumption 2: The worker and the entrepreneur are infinitely lived agents.

Assumption 3: The worker cannot save their labour-income (wage, w, times hours, n), and consumption, c, itself is the only insurance device available; i.e., $c(h_j) = w(h_j)n(h_j)$.

Assumption 4: If default was occurring, both agents would remain on the spot market.

Assumption 5: Within a relationship, a history-dependent contract, $\delta_{h_j} = \{w(h_j); n(h_j)\}_{j=1}^{\infty}$, specifies a wage and labour input pair for every realization of nature. The risk-averse worker and the risk-neutral entrepreneur share the generated optimal surplus with a common intertemporal discount factor, $\beta \in (0, 1)$.

Assumption 6: Information is symmetric, and agents have perfect foresight, with E being the expectation operator.

These assumptions are common in the literature. Sometimes, it is also assumed that the surplus sharing is such that the worker is systematically left with minimum bargaining power, or the converse. In the framework I consider, I require self-enforcing constraints (SEC) for both types of agents, and do not make any particular assumption regarding the bargaining power: it is an endogenous outcome of the model, once the relationship starts. Note, however, that when a firm (i.e., a pair that consists of a worker and an entrepreneur) emerges, there is a continuum of (feasible) initial bargaining power values; e.g., depending on the beliefs of the agents. Each value corresponds to a unique contract, δ, of the contract set. In other words, I treat the bargaining power as a vector of endogenous state variables.

3.2 Definition of a self-enforcing contract

In general terms, regardless of the specifics of the model, but within the theoretical framework just described, an infinite-term self-enforcing contract can be defined as follows:

Definition 1 *For any standard pair of utility, u, and production function, F, an infinite-term implicit labour contract, δ_{h_j}, is a contract where legally enforceable precommitment is absent. It is defined as a self-enforcing contract if no party wants to renege on the relationship. For this to hold, at any time τ during a contract begun at any time t, the following constraints (i.e., the SEC) must be verified:*

$$\forall \tau, \; E_\tau \sum_{j=\tau}^{\infty} \beta^j u(c(h_j); n(h_j)) \geq E_\tau \sum_{j=\tau}^{\infty} \beta^j u(c^s(z_j); n^s(z_j)), \tag{1}$$

where the superscript s denotes spot market variables, other notations being conventional. The expected overall profit is at least zero at any possible time τ (the SEC of the entrepreneur):

$$\forall \tau,\ E_\tau \sum_{j=\tau}^{\infty} \beta^j \left[F(z_j; k; n(h_j)) - w(h_j)n(h_j) - r(h_j)k\right] \geq 0, \tag{2}$$

where capital, k, is held constant for tractability,[2] *and other notations are conventional.*

For the worker, the spot market could, for instance, correspond to casual jobs always available on a continuous basis; e.g., home production. On the spot market, there is no risk-sharing. Hence, wages can fluctuate substantially, depending on the state of the economy. The contract provides insurance against these fluctuations: within a long-term risk-sharing relationship between the worker and the entrepreneur, labour income is insured conditional on the SEC being satisfied. Without any loss of generality, I assume that the entrepreneur has no strong incentive to renege, since they would get a zero spot market profit in every future period. In that respect, the SEC of the entrepreneur can be interpreted as a pseudo-form of reputation.

If capital, k, was a storable fraction of the consumption good, then the set of contracts would need to be made convex by randomization over k. This complication, however, would not add anything to the argument.

The above definition can be rewritten in a Bellman form as follows:

Definition 2 *Definition 1, in its Bellman form, noting that v and π are two objects such that $\forall j \in (1, \infty)$:*

$$\begin{aligned} v(\delta, h_j) &= u(c(h_j), n(h_j)) - u(c^s(z_j), n^s(z_j)) \\ &+ E_j \sum_{i=j+1}^{\infty} \beta^{i-1}[u(c(h_i), n(h_i)) - u(c^s(z_i), n^s(z_i))] \\ \pi(\delta, h_j) &= F(z_j, k, n(h_j)) - w(h_j)n(h_j) - r(h_j)k \\ &+ E_j \sum_{i=j+1}^{\infty} \beta^{i-1}[F(z_i, k, n(h_i)) - w(h_i)n(h_i) - r(h_i)k], \end{aligned}$$

is expressed as:

$$v(\delta, h_j) \geq 0$$
$$\pi(\delta, h_j) \geq 0.$$

[2] I thank Víctor Ríos-Rull for pointing out the variability of the interest rate; see also Sigouin (2004a).

In these definitions, the first inequality states that the worker cannot be attracted by the spot market. Hence, the contract is self-enforcing with respect to the spot market. The second inequality states that the worker wants to prevent the entrepreneur from reneging. The constraints used are the SEC, and they represent the continuation value of the contract.

These definitions imply that the variables agreed upon through the contract depend on h_j (i.e., the whole past history up to time j), and on all possible future histories, $h_i \in H_{j+1}$. As shown below, however, it is still possible to compute a recursive equilibrium by relying on a promise-keeping utility approach.

4 The Model

4.1 Description of the model

The economic environment is composed of the flexible-wage benchmark, the full-commitment benchmark, and the limited-commitment economy itself. This economy is populated by two types of infinitely lived agents: the risk-averse worker and the risk-neutral entrepreneur. The matching between the two agents does not need to be modelled to study Conjecture 1. I assume that the initial bargaining power is exogenous. At each period, the firm (i.e., a worker and an entrepreneur) has to combine k units of capital with labour for the stochastic production of a unique, non-durable good to be consumed by the worker. The surplus generated by the risk-sharing is optimally shared within the firm. It guarantees a positive expected value to any feasible implicit contract, δ_{h_j}, given the presence of outside opportunities (i.e., the SEC). Otherwise, on the spot market, the entrepreneur has no gain, and the worker cannot enjoy more surplus than would be possible within the relationship. The worker suffers an exogenous dead loss. In this environment, imperfect contract competition leads to a symmetric equilibrium, where all firms are similar and competition is not perfect enough to purge the economy from all its risk-sharing gains.

This economy has an RBC flavour, in that preference, technology, and the driving process are fully specified. Within the theoretical framework that has just been described, the unfolding dynamic properties are compatible with a wide range of functions, u and F (see the discussion in section 2). For the numerical computation, however, I use the same kind of functions as Boldrin and Horvath (1995), choosing the three fundamentals of preference, technology, and driving process:

Preference: The worker has a time-separable Bernouilli utility function of the constant elasticity of substitution class, as given by:

$$\forall t,\ u(c_t;n_t) = \frac{1}{1-\gamma}c_t^{1-\gamma} + \frac{\theta}{1-\gamma}(T-n_t)^{1-\gamma}, \gamma \in (0,1), \tag{3}$$

where all notations are conventional, T represents the total of non-sleeping hours, and γ captures the risk aversion of the worker. The worker consumes all their wage earnings, so that:

$$\forall t,\ c_t = w_t n_t. \tag{4}$$

Technology: The entrepreneur uses the usual Cobb-Douglas production technology. Capital is held constant for simplicity, as in McCallum and Nelson (1997), Rotemberg (1999), and Sigouin (2004b).[3] To operate, the firm requires k units of capital, no more and no less. k is not a choice variable; it is set to be compatible with the spot market, notably the spot steady-state level of hours (n_{ss}). Defining r as the marginal product of capital associated with n_{ss} and k,

$$\forall t,\ F(z_t;k;n_t) = z_t k^{\alpha} n_t^{1-\alpha}, \tag{5}$$

with

$$\forall t, r(z_t) = \alpha F(z_t;k;n_{ss})/k. \tag{6}$$

Driving process: The toy economy is perturbed by a driving process, z_t, with discrete distribution. At any time t, z_t can take N different values $\left\{z_t^1, ..., z_t^N\right\}$, with p_{sj} representing the transition probability to be in state z^j, knowing that the previous state was z^s, and $\forall s = 1, ..., N, \quad \forall j = 1, ..., N, \quad p_{sj} = 1/N.$

To get a more tractable model, imperfect contract competition is assumed to be exogenous (Burda et al. 1998). For example, the worker could be thought of as being too specialized (or endowed with firm-specific human capital) to easily find an equivalent position when reneging on the contract. Hence, the worker is unable to perfectly bid up their labour income vis-à-vis other contracts. It is possible to consider a model with purely competing contracts, as in Krueger and Uhlig (2000). That would be done to introduce the possibility that the firm might fire the worker in order to hire a cheaper one, for instance. To avoid the degenerate result of a quasi-empty contract set, however, some friction needs to be added to ensure that contract competition does not drive allocations towards their fully flexible levels, where risk-sharing is totally squeezed (for the use of frictions, see

[3] Cooley (1995, chapter 1), states that about two-thirds of aggregate fluctuations are attributable to labour input.

Sigouin 2004b). It is precisely to avoid this singleton solution set that perfect contract competition is beyond the scope of this paper.

While the assumption of labour market friction is essential, the actual type of the competitive imperfection is of minor relevance for the analysis, as well as for addressing the dynamics puzzle. More precisely, to take advantage of the dynamic properties of the self-enforcing contracts, I only need to compute the whole contract set (that is, solving for all feasible bargaining power levels), based on a certain degree of imperfect competition. To capture this, let P be a positive constant real number belonging to the $(0, 1)$ interval. P represents the exogenous degree of market imperfection.

More formally, given the theoretical framework I consider, I define the contract set as follows:

Definition 3 $\forall j \in (1, \infty)$, *define* $\omega(h_j)$ *as the set of allocations such that, for the sequence of productivity shocks* h_j*, and any subsequent histories* $h_i \in H_{j+1}$ *(the set of all possible histories), equations (1) and (2) hold for a given* P*. The contract set* $\omega(h_j)$ *is the convex family of contracts* δ_{h_j} *over all the feasible initial bargaining power levels (BP) associated with* P*:* $\omega(h_j) = \{\delta_{h_j}\}_{bp}$*.*

Remark 1 *For any given initial bargaining power, there exists a unique associated contract* δ_{h_j} *in* $\omega(h_j)$*.*

This comes directly from the convexity of $\omega(h_j)$. The contract set defines a compact family associated with P. One interpretation of the model is that, to operate its unit, the firm has to employ k, the capital stock, and a corresponding worker who works n hours. Only one pair consisting of a worker and an entrepreneur needs to be considered: the firm requires at least zero profit to operate, and the worker must be promised a surplus superior or equal to the reservation surplus, the bargaining power evolving inside the corresponding range dictated by the SEC.

4.2 Endogenous rigidity of the labour-income dynamics

Within the theoretical framework, I introduce the following definition of the full-commitment Pareto frontier:

Definition 4 *The full-commitment Pareto frontier,* Π^**, is characterized by* $(P1)$*:*

$$\Pi^*(z_s, U) = \max F(z_s, k, n) - wn - r(z_s)k + \beta \textstyle\sum_{j=1}^{N} \Pi^*(z_j, U_j)p_{sj}$$

subject to constraints (3)-(5), and:

$$u(wn, n) + \beta \textstyle\sum_{j=1}^{N} U_j p_{sj} \geq U,$$

where U denotes the expected utility of the worker.

Remark 2 *Since u and F are concave functions, so is Π^*.*

I solve the full-commitment benchmark for all feasible full-commitment contracts. In other words, I search the decision rules associated with the control variables w and n, for all the initial bargaining power values compatible with the utility domain restriction corresponding to the participation constraint.

In this definition, the U_j are promise-keeping utility levels compatible with the participation constraint. Under full commitment, this menu of U_j is fixed, and corresponds to the initial bargaining power vector, in keeping with the fact that the full commitment case represents a situation where the labour income is perfectly insured. As Proposition 1 establishes, the risk-sharing domain is "large" (i.e., unconstrained) in the full-commitment case, and there is a perfect smoothing of the marginal utility of consumption. It is the benchmark that, along with the spot market, allows the computation of the limited-commitment economy. Given the framework, U characterizes the bargaining power of the worker:

Remark 3 *In $(P1)$, the expected utility of the worker, U, is a vector of state variables. Under full commitment, its domain is constant and maps into all the bargaining power levels consistent with the participation constraint. Under limited commitment, the domain is further constrained and varies with the SEC; U becomes a vector of endogenous state variables.*

I also introduce the following principles:

Proposition 1 *Under full commitment, any feasible implicit contract is such that consumption is completely smoothed over states and time.*

Proof: In the previous program, let λ be the Lagrange multiplier associated with the participation constraint, and consider that $c = wn$. Then the first-order condition with respect to U_j is given by $\Pi^*_{U_j}(z_j, U_j) + \lambda = 0$, and the envelope condition gives $\Pi^*_U(z_s, U) + \lambda = 0 \Rightarrow \Pi^*_{U_j}(z_j, U_j) = -\lambda'$. This, together with the first-order condition for consumption, $-1 + \lambda u_c(c, n) = 0$, yields $\lambda = \lambda' \Leftrightarrow c = c'$ (where the primes denote future period variables).

One interpretation of Proposition 1 is to consider that, if agents were able to commit, with lock-in (no outside opportunity), any initial shock to the economy would have a purely persistent effect on labour income. More precisely, after the initial shock, consumption would remain at the same level, regardless of future history, even when considering a purely transitory shock. Obviously, such an environment generates infinite persistence. Introducing the outside opportunities mitigates this property somehow, because perfect insurance is no longer achievable. This corresponds to a more reasonable assumption that still has the potential to deliver a substantial amount of persistence, which is the reason the no-commitment case is being investigated.

Following Thomas and Worrall (1988, 1994), and Sigouin (2004a, b), I define the Pareto frontier of the no-commitment economy (consistent with $(P1)$) as the following form:

Definition 5 *Given a degree of labour market imperfection, P, a capital stock level, k, any realization of z_s, and any feasible expected utility level, U, the Pareto frontier without commitment is the function solving $(P2)$:*

$$\Pi(z_s, U) = \max_{\{w,n,U_j\}} F(z_s, k, n) - wn - r(z_s)k + \beta \sum_{j=1}^{N} \Pi(z_j, U_j)p_{sj}$$

Subject to constraints (3)-(5), and:

$$u(wn, n) + \beta \textstyle\sum_{j=1}^{N} U_j p_{sj} \geq U, \tag{7}$$

$$\forall j = 1, ..., N \qquad \Pi(z_j, U_j) \geq 0, \tag{8}$$

$$\forall j = 1, ..., N \qquad U_j \geq V^s(z_j), \tag{9}$$

where s indicates a spot variable.

The last constraint of $(P2)$ is the SEC of the worker. It states that the entrepreneur must promise at least a contingent menu $\{U_j\}_{j=1}^{N}$ to the worker. Depending on which state of nature is realized, U_j is then set as the new U. This is the main feature of the promise-keeping utility approach. It is designed to account for the history dependence of $\delta_{(h_j)}$.

The expected utility value associated with the spot market, V^s, can be chosen after finding V^s_{max}; i.e., the value function associated with the maximum feasible surplus under autarky. More precisely:

Definition 6 *Given a value for P (the labour-market imperfection), for ω (the contract set) to be non-empty, V^s (the spot market surplus) has to be inferior to the value that corresponds to the pure contract competition case: $V^s = PV^s_{max}$, with*

$$V^s_{max}(z_s) = \max_{\{n^s_{max}\}} u(F(z_s, k, n^s_{max}) - r(z_s)k, n^s_{max}) + \beta \sum_{j=1}^{N} V^s_{max}(z_j)p_{sj}. \tag{10}$$

The spot market is derived from an economy where the worker lives alone forever, not saving but consuming the good they produce. It is directly inspired by the flexible-wage assumption.

I have defined Π^* and V^s to solve for Π. It is then possible to qualitatively characterize the endogenous rigidity of the labour income. I first introduce the worker's full bargaining power with U^{max}:

Definition 7 *Let* $(U^{max}(z_j))_{j=1}^N$ *be the endogenous state variable of the vector* U *such that:*

$$\Pi(z_j, U^{max}(z_j); U_j) = 0 \qquad \forall j = 1, ..., N.$$

Then, within the theoretical framework, I consider the following proposition:

Proposition 2 *Under limited commitment, consumption is bounded by the* SEC*. The upper bound of the interval corresponds to the consumption associated with the worker's full bargaining power. The lower bound is reached for the full bargaining power of the entrepreneur.*

Proof: Let λ, $\{\beta p_{sj}\psi_j\}_{j=1}^N$, and $\{\beta p_{sj}\phi_j\}_{j=1}^N$ be the multipliers respectively associated with the last three constraints of $(P2)$. Then the first-order conditions of this program are,

with respect to $U_j \;\; \forall j = 1 \dots N$:

$$(1 + \psi_j)\Pi_{U_j}(z_j, U_j) + \lambda + \phi_j = 0, \tag{11}$$

with respect to wage and labour input,

$$-n + \lambda u_1(wn, n)n = 0, \tag{12}$$
$$F_3(z_s, k, n) - w + \lambda(u_1(wn, n)w + u_2(wn, n)) = 0, \tag{13}$$

with respect to the Kuhn-Tucker conditions, $\quad \forall j = 1 \dots N$:

$$\Pi(z_j, U_j) \geq 0, \;\; \psi_j \geq 0, \;\; \Pi(z_j, U_j)\psi_j = 0, \tag{14}$$
$$U_j \geq V^s(z_j), \phi_j \geq 0, [U_j - V^s(z_j)]\phi_j = 0, \tag{15}$$

with respect to the following equation, corresponding to the saturation of the participation constraint:

$$U \geq V^s(z_s) \Rightarrow u(wn, n) + \beta \sum_{j=1}^{N} U_j p_{sj} = U, \tag{16}$$

and, with respect to the envelope condition:

$$\Pi_U(z_s, U) = -\lambda. \tag{17}$$

Equation (12) implies that $u(wn, n) = 1/\lambda$. Equation (12) allows (13) to simplify to $F_3(z_s, k, n) = -u_2(wn, n)$. Whenever $\forall j = 1...N, \quad \phi_j = \psi_j = 0$, it is easy to show, by combining (12) and (17) together with (11), that consumption is equal to that obtained under full commitment. This case coincides with the full-commitment case, since no SEC is binding. In more regular situations, at any time, and depending on the realization of the shock, one SEC or another might be binding. Then it can be shown that, given the functional forms I consider:

$$U^{max}(z_j) \geq U_j \geq V^s(z_j) \qquad \forall j = 1, ..., N,$$

for every element of $(U^{max}(z_j))_{j=1}^{N}$, where U_j is an element of the menu $(U_j)_{j=1}^{N}$ solving $(P2)$. Equations (12) and (17) yield $u_1(wn, n) = -1/\Pi_U(z_s, U)$. Hence, $\forall t \in (0, \infty)$:

$$c \in \left[(u_1)^{-1}(-(\Pi_U(z_s, u(w^s n^s, n^s))^{-1}), (u_1)^{-1}(-(\Pi_U(z_s, U^{max}(z_s)))^{-1})\right].$$

Depending on which z_t^s is realized, U takes its value in $[V^s(z_s), U^{max}(z_s)]$, which in turn determines the interval of the remuneration $c = wn$. Define c_l and c_u as the extremities of the interval to which consumption belongs at a given date, and define $c^{'}$ as next-period consumption.

Dropping function arguments and noting that $\Pi_U(.)$, the derivative of Π with respect to its second argument, is negative and decreasing in consumption, the following proposition about labour-income dynamics can be made:

Proposition 3 *Within a self-enforcing labour contract, labour income always changes by the least amount compatible with the SEC.*

Proof:

If $\psi_j = \phi_j = 0$, then (11) becomes $\Pi_{U_j} = \Pi_U$. This is the full-commitment case, which equivalently states that $c^{'} = c$ (cf. Proposition 1).

If $\psi_j = 0$ and $\phi_j > 0$, then (11) implies that $\Pi_U = \Pi_{U_j} + \phi_j$, and $\Pi_U > \Pi_{U_j}$; therefore, we have $c < c^{'}$. Since $\phi_j > 0$, we also have that $U_j = V^s(z_j)$ or, equivalently, that $c^{'} = c_l^{'}$. If labour income increases, it does so by the smallest incrementation compatible with the constraints, setting its level at the lower bound of the interval associated with the future contingent menu of promised utilities. In this situation, the worker's surplus is at its lowest level.

If $\psi_j > 0$ and $\phi_j = 0$, then (11) implies that $c > c^{'}$. With $\psi_j > 0$, we have that $\Pi(U_j, z_j) = 0$, hence $c^{'} = c_u^{'}$. When the technology shocks decrease the labour income, the decrease is minimized. This corresponds to the least decrementation compatible with the SEC: the level of labour income is thus set equal to the upper bound of the interval associated with the highest utility level. In this situation, the employer's surplus is at its lowest level.

If $\psi_j > 0$ and $\phi_j > 0$, then $U_j = V^s(z_j)$ and $\Pi(U_j, z_j) = 0$. This is possible only if $c^{'} = c_l^{'} = c_u^{'}$, which would correspond to an interval reduced to a singleton, and a degenerated contract set.

I cast the result in terms of internal propagation. As Proposition 3 suggests, the behaviour of the economy displays an endogenous real rigidity in the labour income. More specifically, in the presence of the outside opportunities, initial shocks no longer have a purely permanent effect on the economy, as in the full-commitment case. With full commitment, the effect would persist despite future shocks, whereas here, however long-lived, the effect dissipates with incoming shocks, until the ergodic set is reached. Consecutive shocks have a long-lasting effect both because of the risk-sharing, and because labour income is adjusted only by the smallest amount: this is the property that ultimately motivates investigation of the dynamics implications of self-enforcing contracts.

Since I focus on the dynamics puzzle, I interpret the proposition as follows. When matching to form a firm, the worker and the entrepreneur initiate a long-term relationship given the state of the economy, as characterized by the initial shock and the initial bargaining power. As long as consecutive shocks make one of the SEC binding, there is an endogenous adjustment in the bargaining power. After a number of periods, the ergodic set is reached. At this point, the economy is in its perfect insurance state. The risk-sharing fully insures the worker and therefore the labour income is constant, as in the full-commitment case. In other words, before reaching the steady state, there is a "sluggish" labour-income dynamics; then, at a certain point in time, $\phi = \psi = 0$ and Proposition 1 holds. Hence, any shock that hits the economy in its transition has a permanent effect, captured by the steady-state bargaining power, consistent with its corresponding initial bargaining power value. In section 5, I explicitly define such a dynamics.

5 Computation and Results

In this section, I compute the self-enforcing contracts associated with all the optimal bargaining power levels compatible with the SEC. In practice, since I cannot compute the infinity of contracts belonging to ω, I discretize the utility domain that corresponds to the whole range of bargaining power levels. I need to use only a few points for the grid because I rely on an interpolant to compute the value function Π. One of the important aspects of this approach is precisely that it enables the simultaneous determination of all the feasible contracts.

Since the model is in essence a stylized model designed to thoroughly characterize a qualitative property of the risk-sharing hypothesis, and for the sake of clarity, the computation is based on two states of nature only, z_t being equal to either 0.99 or 1.01. Generalizing by assuming a more refined stochastic process does not add anything crucial to the analysis. Similarly, studying the matching stage and the initial bargaining game is also beyond the scope of this study, since I focus on the investigation of the economy's dynamics, not its fit with the data. The set of structural parameters used for the resolution is $\Lambda = \{\gamma, \theta, T, \beta, \alpha\}$. γ, the risk-aversion parameter, is set to 0.32, as in Boldrin and Horvath (1995). In fact, since this paper expands upon Boldrin and Horvath's (1995) argument, I choose the same type of preference and technology, and consider almost the same values for the parameters, including θ, the weight on leisure (1.195); T (1369 being the non-sleeping hours per average person per quarter); and β (0.99). The calibration used by Boldrin and Horvath (1995) is standard, with all the parameters being broadly consistent with empirical regularities. While other types of calibration can be considered, in fine, the dynamics of the economy does not crucially depend on the choice of Λ (cf. section 2). Instead, it is mainly driven by the strategic and dynamic interactions between the worker and the entrepreneur within the firm they compose (i.e., the dynamic bargaining game). In other words, the general properties of this toy economy are not related to the calibration in any critical way. This is one of the most advantageous features of the framework (Rosen 1985).

5.1 Computation of V^s_{max}

Before solving for the limited-commitment value function, and the associated economy's optimal allocations, I first need to solve the spot market "shadow" economy. Note that the associated model can be interpreted as representing a virtual economy because of the self-enforcement of the toy economy—once formed, the firm never dies. The only role of the spot market is to reflect the presence of the SEC; for the rest, it is essentially fictitious.

The spot market is formalized by assuming that the worker consumes the production and enjoys all the surplus. This is the well-known form of a flexible-wage model. Thus, the maximization problem in (10) can be solved with a regular dynamic programming approach. The first-order condition is:

$$(1-\alpha)(z_s k^\alpha (n^s)^{(1-\alpha)} - r(z_s)k)^{-\gamma} z_s k^\alpha (n^s)^{-\alpha} - \theta(1-n^s)^{-\gamma} = 0,$$

where T has been normalized to one. Capital is set at a level compatible with this equation taken at its steady state, where hours (n_{ss}) equal 0.33. More precisely, k solves:

$$(1-\alpha)(k^\alpha n_{ss}^{(1-\alpha)} - r_{ss}k)^{-\gamma} k^\alpha n_{ss}^{-\alpha} - \theta(1-n_{ss})^{-\gamma} = 0$$

with a Newton algorithm that starts with an initial root value, k_0, of 0.0157 corresponding to a steady-state output-capital ratio of 3, $k_0 = (3n_{ss}^{(\alpha-1)})^{(1/(\alpha-1))}$, leading to $k = 1.488$. Once V^s_{max} is found, it is possible to fix V^s arbitrarily lower, such that there is risk-sharing; i.e., ω is non-empty. If P was close to 1, then ω would be reduced to a singleton, and the unique contract would correspond to that of the flexible-wage economy. I assume for the moment that imperfect contract competition is associated with a 10 per cent welfare loss. Then, $P = 0.9$ and $V^s = 0.9V^s_{max}$.

Table 1: The Spot Market

	n^s_{max}	w^s_{max}	V^s_{max}	V^s
Good state	0.333	0.963	199.943	179.949
Bad state	0.332	0.911	199.918	179.926

As discussed in section 2, given that the spot market is derived from a flexible-wage framework, it displays no persistence. After a one-time shock, consumption and hours come back to their steady-state values immediately. Moreover, the n^s_{max} deviation from steady state is small compared with the shock support (Table 1), and consumption dynamics is driven by the procyclical flexible wage. As argued, the lack of amplitude of the response of hours can be attributed to the dampening income effect.

5.2 The full-commitment Pareto frontier

The full-commitment benchmark is also needed to solve the model. As for the spot market, the full-commitment Pareto frontier solving ($P1$) can be found using the standard contraction-mapping approach. Given that no SEC are attached to the problem, the utility domain is constant and "large." Consistent with Proposition 1, the slope of the value function, $-\lambda$, is equal across states throughout the risk-sharing domain. It is also increasingly downward-trended with higher utility levels. In Table 2, λ_{inf} corresponds to the lowest absolute value of the slope. It is only admissible under full commitment, when the initial bargaining power is all given to the entrepreneur (for example, if the matching occurs during a period where agents perceive it has to be so). λ^{sup} corresponds to the highest value of the slope. In this area of the frontier Π^{*}, the full-commitment contracts are associated with a large stake of the surplus going to the worker, who then has the highest bargaining power levels.

Table 2: Slope of the Full-Commitment Frontier

	Bad state	Good state
λ_{inf}	0.46949	0.47473
λ^{sup}	2.26230	2.26259

Figure 1 displays such a slope for the whole U range. Not surprisingly, as the bargaining power of the worker increases, the remaining share of the surplus enjoyed by the entrepreneur decreases. Given that the support of the shock is small, and consistent with Proposition 1, the frontiers associated with the two states are almost indistinguishable. They share the same curvature; i.e., the same slope for any full-commitment initial bargaining power.

Note that the second line crosses the horizontal zero-axis (i.e., $\Pi^{*}(.) = 0$) at the utility level equal to $V^{s}_{max}(.)$. In other words, under full commitment, when the firm has a zero gain, the worker enjoys the maximum feasible surplus compatible with the fundamentals under autarky (first best).

5.3 Solving the limited-commitment case

In Figure 1, the vertical lines indicate the initial risk-sharing domain of the limited-commitment economy; i.e., a subset of the unconstrained (full-commitment) utility range. This domain is fixed at its lower bound, which corresponds to V^{s}. The first line depicts precisely the fixed utility level where the SEC of the worker binds under limited commitment. When solving the limited-commitment case, however, the upper bound will change throughout the iterations. It corresponds to a jumping bargaining power. Only under full commitment is this bound fixed.

The solution of the limited-commitment case requires the determination of the allocations associated with the endogenous state variable, U^{max}. All other feasible δ_{h_j} are simultaneously found once this contract is found. As Figure 1 shows, the starting value associated with U^{max} corresponds to $V^s_{max}(.)$ for all states, because this is the initial value that satisfies the entrepreneur's SEC: equation (8). For each subsequent iteration, I need to determine U^{max} such that $\Pi(.) = 0$, for every state of nature, which is not a trivial task.

Regarding the computation, Marcet and Marimon (1992) note that, in the class of problems that I analyze, the expectational commitment constraints are not conventional, since they depend on the whole stochastic process and not just on past history. This explains why a modification of the optimization programming is used. As Thomas and Worrall (1988) demonstrate, the introduction of a state-contingent menu of utilities enables one to account for future shocks and the history-dependent issue. Using this approach, and dropping the arguments, the system of first-order conditions of $(P2)$ to solve for can be rearranged as:

$$
\begin{gathered}
(1-\alpha) z_s k^{\alpha} n^{-\alpha} - \theta\lambda(1-n)^{-\gamma} = 0, \\
\tfrac{1}{1-\gamma}(wn)^{1-\gamma} + \tfrac{\theta}{1-\gamma}(1-n)^{1-\gamma} + \beta \textstyle\sum_{j=1}^{N} U_j p_{si} - U = 0, \\
\lambda - (wn)^{\gamma} = 0, \\
\forall j = 1, ..., N,\ (1+\psi_j)\Pi_{U_j}(z_j, U_j) + \lambda + \phi_j = 0, \\
\forall j = 1, ..., N,\ \psi_j(U - U_j) = 0, \\
\forall j = 1, ..., N,\ \phi_j(U_j - V(z_j)) = 0.
\end{gathered}
$$

Definition 8 *The recursive self-enforcing equilibrium is defined with the solution to* $(P2)$*, including* Π*, and* $\{w, n, c, \lambda, U_j, \psi_j, \phi_j\}$*, as well as* P*,* Λ*, the initial conditions (*h_0*, bp), and the driving process,* z*.*

Following Sigouin (2004a, b), I rely on a special iteration-based numerical method, starting with Π^* as the initial guess for the limited-commitment value function. The problem is that, as the system shows, the set of constraints incorporates the (unknown) value function. Furthermore, the first-order conditions of $(P2)$ are based on derivatives of this unknown function. Hence, I need to approximate Π. This is done based on its previous value and slope; i.e., based on the values Π and Π_U associated with the optimal utility domain of the previous iteration. Relying on this and equation (17) (the envelope condition that gives the first derivative of Π), I compute a bicubic Hermite interpolant of the value function Π that preserves its shape. More specifically, let $\tilde{U}$ be a new utility level to be found, and let U_i and U_{i+1} be the two consecutive solutions of the previous iteration, such that $\tilde{U} \in [U_i; U_{i+1}]$. Since I know $\Pi(., U_i), \Pi(., U_{i+1}), \Pi_U(., U_i), \Pi_U(., U_{i+1})$, dropping the first argument of Π, then $\Pi(\tilde{U})$ can be expressed as:

$$\begin{aligned}\Pi(x(\tilde{U})) &= \Pi(U_i)H_0(x) + \Pi(U_{i+1})H_1(x) \\ &+ (U_{i+1} - U_i)\Pi_U(U_i)K_0(x) \\ &+ (U_{i+1} - U_i)\Pi_U(U_{i+1})K_1(x),\end{aligned}$$

with

$$x = \frac{\tilde{U} - U_i}{U_{i+1} - U_i}, \tag{18}$$

where $x \in [0, 1]$, and the associated bases functions

$$\begin{aligned}H_0(x) &= 2x^3 - 3x^2 + 1, \\ H_1(x) &= -2x^3 + 3x^2, \\ K_0(x) &= x^3 - 2x^2 + x, \\ K_1(x) &= x^3 - x^2.\end{aligned}$$

Using this type of interpolant is quite helpful, since, from one iteration to the next, the utility domain might change considerably until convergence. Moreover, I need to find Π for the whole span of the U_j control variables (and not just at one point); thus, the fact that the interpolant is piecewise shape-preserving reduces the computation time substantially.

Also, to use a Newton algorithm to solve the system, I need to compute its gradient and Jacobian. Another advantage of this type of interpolant is that it is twice differentiable.

To twice differentiate the bicubic Hermite function, note that $\Pi(x(\tilde{U}))$ has the following form:

$$\Pi = Ax^3 + Bx^2 + Cx + D, \tag{19}$$

where A, B, C, and D come from the following derivation:

$$\begin{aligned}\Pi(x(\tilde{U})) &= [2(\Pi(U_i) - \Pi(U_{i+1})) + (U_{i+1} - U_i)(\Pi_U(U_i) + \Pi_U(U_{i+1}))]x^3 \\ &+ [3(\Pi(U_{i+1}) - \Pi(U_i)) + (U_{i+1} - U_i)(2\Pi_U(U_i) + \Pi_U(U_{i+1}))]x^2 \\ &+ [\Pi_U(U_i(U_{i+1} - U_i))]x \\ &+ \Pi(U_i).\end{aligned} \tag{20}$$

Π can be written as a simple trinomial transform in each interval, and, as a consequence, it is straightforward to show that the derivatives are:

$$\Pi_U(x(\tilde{U})) = (3Ax^2 + 2Bx + C)\frac{1}{h}, \tag{21}$$

and

$$\Pi_{UU}(x(\tilde{U})) = (6Ax + 2B)\frac{1}{h^2}, \tag{22}$$

with

$$\frac{1}{h} \equiv \frac{dx}{d\tilde{U}} = \frac{1}{U_{i+1} - U_i}. \tag{23}$$

A Newton algorithm cannot directly handle inequality equations (the Kuhn-Tucker constraints corresponding to the SEC). The variables of the system are thus expressed in terms of trigonometric transformations. For the constrained variables (w, n, c, U_j), the form of the transformation is

$$Y = \underline{Y} + \frac{1}{2}(1 + \cos(\tilde{Y}))(\bar{Y} - \underline{Y}). \tag{24}$$

For the unconstrained variables (λ, ψ, ϕ), the equivalent form is

$$Y = \frac{1}{2}(1 + \cos(\tilde{Y}))\Gamma,$$

where Γ denotes an arbitrary large number.

For each iteration, U^{max} has to be found, to delimit the utility domain over which Π will be determined. The lower bound is set at V^s, and the upper bound is computed using the value of its former iteration, the former utility domain, and the approximate value function. This value function is an interpolant of the true (previous) function, adjusted for concavity to eliminate the eventual jumps between the previous U_j's.

Once the problem is solved, the dynamics of the economy can be summarized as in Figure 2. A horizontal band starts at the point in time, τ, when the firm emerges. The boundaries of this band delimit the ergodic set that corresponds to the steady state of perfect insurance. For any infinite-term contract, this band goes to infinity. Before reaching that region, however, the economy can display a transitory dynamics, as shown by the triangular area in Figure 2. This surface corresponds to situations where the initial bargaining power levels are such that the shocks are imperfectly insured against. These situations occur when the initial bargaining power level does not belong to the perfect risk-sharing utility domain.

Figure 3 shows the result of a numerical experiment where the contract is initiated during a good period, with the economy thereafter hit by a bad shock and then a good one. As conjectured, giving the initial full bargaining power to the entrepreneur leads to a non-trivial persistent response of consumption and hours. In theory, it is indeed possible to generate situations where there are (infinitely) prolonged co-movements between these two variables, even with purely transitory shocks.

Furthermore, and consistent with Proposition 1, if I set the initial bargaining power of the worker higher than in the previous experiment, where it was at its lowest level, then the persistence of the economy "increases." Table 3 shows how persistence evolves as the initial bargaining power of the worker departs from its lower bound. As BP increases, the difference narrows between the bad state of consumption and the good state of consumption, due to the smoothing of the marginal utility. Eventually, for high enough initial bargaining power levels, consumption is constant through states, as when the BP corresponds to any of its steady-state values (i.e., the band in Figure 2). In that case, only the initial shock can have an impact, and it is infinitely lived. In other words, in the limited-commitment economy, there is a region of initial bargaining power levels where persistence is pure in that, from the start of the contract, the worker is perfectly insured and fluctuation disutility is at its lowest.

Table 3: Response Persistence of Consumption with Worker's Varied BP

	Bad state	Good state
zero BP	0.2161	0.2184
BP_1	0.2222	0.2224
BP_2	0.2255	0.2257
BP_3	0.2270	0.2272
BP_4	0.2285	0.2286

Two additional observations can be made. First, when varying P, hours and consumption remain procyclical, whereas wages do not. This should not be surprising since, albeit imperfect, there is a smoothing of the marginal utility of the labour income. Second, and quite interestingly, as P increases, so does the triangular region where the transition dynamics takes place (Figure 4).

As the utility domain where the pure risk-sharing is feasible shrinks with increasing P, the shocks are more likely to have an impact on the economy. In fact, the utility domain shrinks asymetrically. The upper bound, where the initial bargaining power is high, is the contraction of the full-commitment value of U generating a zero profit for the entrepreneur. This bound is thus rather stable. The lower bound, however, shrinks more during the good state of nature. As a consequence, there are more initial bargaining power levels compatible with the limited-commitment economy. One interpretation of this result is quite simple. As the economy converges towards pure contract competition, more limited-enforcement situations occur (Table 4). Table 4 describes, for each state, and as the degree of market imperfection decreases, the magnitude of the utility domain where there is risk-sharing under limited commitment. As V^s converges to V^s_{max}, the utility domain shrinks asymetrically, and there is an increased likelihood of imperfect risk-sharing.

Table 4: Magnitude of the Utility Domain with P Increasing

States	P=0.89	P=0.90	P=0.91	P=0.92	P=0.93
Good state	0.0371	0.0292	0.0154	0.0077	0.0047
Bad state	0.0381	0.0315	0.0187	0.0132	0.0105

6 Conclusion

This paper has aimed to verify that a self-enforcing contracts framework could deliver a persistent response to exogenous technology shocks. This conjecture (section 2) derives directly from Proposition 3.

Instead of relying on traditional propagation mechanisms, this study has indicated that it is possible to generate a significant amount of persistence by removing the income effect from the economy. In the model, persistence is attributable to the risk-sharing hypothesis. The commitment problems prevent the economy from being perfectly insured against uncertainty. In this context, a real rigidity emerges, the properties of which are used to rationalize the dynamics puzzle.

Although the proposed framework is unique, the idea of introducing limited commitment and studying its dynamic properties is not. Sigouin (2004b) and Cooley, Marimon, and Quadrini (2003) share similar views, which suggests that the avenue is worth pursuing.

The toy economy described in this paper should be extended in several ways: for example, there is an ad-hoc treatment of imperfect contract competition, the amplitude of risk-sharing is arbitrarily set, the matching stage and the initial bargaining power game are not modelled, and contract duration is assumed to be infinite. Likewise, the capital is exogenous in the model. While these shortcomings are not detrimental to the main argument, it would be interesting to address them. That is left for future work.

References

Altonji, J.G. and P.J. Devereux. 1999. "The Extent and Consequences of Downward Nominal Wage Rigidity." NBER Working Paper No. 7236.

Ambler, S., A. Guay, and L. Phaneuf. 1999. "Wage Contracts and Labor Adjustment Costs as Endogenous Propagation Mechanisms." CIRPEE Working Paper No. 69.

Azariadis, C. 1975. "Implicit Contracts and Underemployment Equilibria." *Journal of Political Economy* 63: 1183–202.

Baily, M.N. 1974. "Wages and Employment under Uncertain Demand." *Review of Economics Studies* 161: 37–50.

Basu, S., J.G. Fernald, and L. Kimball. 1998. "Are Technology Improvements Contractionary?" U.S. Federal Reserve Board's International Finance Discussion Paper No. 625.

Beaudry, P. and J. DiNardo. 1991. "The Effect of Implicit Contracts on the Movement of Wages over the Business Cycle: Evidence from Microdata." *Journal of Political Economy* XCIX: 665–88.

———. 1995. "Is the Behavior of Hours Worked Consistent with Implicit Contract Theory?" *The Quarterly Journal of Economics* 110(3): 743–68.

Boldrin, M. and M. Horvath. 1995. "Labor Contract and Business Cycles." *Journal of Political Economy* 103(5): 972–1004.

Burda, M., W. Güth, G. Kirchsteiger, and H. Ulhig. 1998. "Employment Duration and Resistance to Wage Reductions: Experimental Evidence." CEPR Working Paper No. 9873.

Calmès, C. 2005. "La poignée de main invisible et la persitance des cycles d'affaire : un survol." *Actualité Économique*, forthcoming.

Christiano, L. and M. Eichenbaum. 1992. "Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations." *American Economic Review* 82(3): 430–50.

Cogley, T. and J.M. Nason. 1995. "Output Dynamics in Real-Business-Cycle Models." *American Economic Review* 85: 495–511.

Collard, F. and D. de la Croix. 2000. "Gift Exchange and the Business Cycle: The Fair Wage Strikes Back." *Review of Economic Dynamics* 3: 166–93.

Cooley, T. 1995. *Frontiers of Business Cycle Research.* Edited by T. Cooley. Princeton: Princeton University Press.

Cooley, T., R. Marimon, and V. Quadrini. 2003. "Aggregate Consequences of Limited Contracts Enforceability." NBER Working Paper No. 10132.

Dutkowsky, D. and S. Atesoglu. 1993. "Wage Contracting in the Macroeconomy." *Journal of Money Credit and Banking* 25(1): 62–78.

Galí, J. 1999. "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?" *American Economic Review* 89(1): 249–71.

Gauthier, C., M. Poitevin, and P. González. 1997. "Ex Ante Payments in Self-Enforcing Risk-Sharing Contracts." *Journal of Economic Theory* 76(1): 106–44.

Gomme, P. and J. Greenwood. 1995. "On the Cyclical Allocation of Risk." *Journal of Economic Dynamics and Control* 19: 91–124.

Gordon, D.F. 1974. "A Neoclassical Theory of Contracts." *Economic Inquiry* 12: 431–59.

Hansen, G.D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics* 16: 309–27.

Hart, O.D. and B. Holmström. 1987. "The Theory of Contracts." In *Annal of Economic Theory: Fifth World Congress*, edited by T. Bewley, 71–155. Cambridge: Cambridge University Press.

Huang, K.X.D. and Z. Liu. 1998. "Staggered Contracts and Business Cycle Persistence." Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis, Discussion Paper No. 127.

Keynes, J.M. 1936. "The General Theory of Employment, Interest and Prices." London: MacMillan.

Kihlstrom, R. and J.J. Laffont. 1979. "A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion." *Journal of Political Economy* 87(4): 719–48.

Krueger, D. and H. Uhlig. 2000. "Competitive Risk Sharing Contracts with One-Sided Commitment." Stanford and CEPR. Photocopy.

Kydland, F.E. and E. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50: 1345–70.

Lettau, M. and H. Uhlig. 2000. "Can Habit Formation Be Reconciled with Business Cycle Facts?" *Review of Economic Dynamics* 3: 79–99.

MacLeod, W.B. and J. Malcomson. 1989. "Implicit Contracts, Incentive Compatibility, and The Involuntary Unemployment." *Econometrica* 57: 447–80.

Marcet, A. and R. Marimon. 1992. "Communication, Commitment, and Growth." *Journal of Economic Theory* 58(2): 219–49.

McCallum, B.T. and E. Nelson. 1997. "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis." NBER Working Paper No. 5875.

McDonald, J.T. and C. Worswick. 1999. "Wages, Implicit Contracts, and the Business Cycle: Evidence from Canadian Micro Data." *Journal of Political Economy* 107(4): 884–92.

Perli, R. 1998. "Increasing Returns, Home Production and Persistence of Business Cycles." *Journal of Economic Dynamics and Control* 22: 519–43.

Romer, D. 1996. "Implicit Contracts." In *Advanced Macroeconomics*, edited by D. Romer, 461–65. McGraw-Hill Companies Inc.

Rosen, S. 1985. "Implicit Contracts: A Survey." *Journal of Economic Literature* 23: 1144–75.

Rotemberg, J. 1999. "Wages and Labor Demand in an Individualistic Bargaining Model with Unemployment." MIT. Photocopy.

Rotemberg, J. and M. Woodford. 1996. "Real Business Cycle Models and the Forecastable Movements in Output, Hours, and Consumption." *American Economic Review* 86: 71–89.

Salanié, B. 1994. "Théorie des contrats." Éd. Economica, collection économie et statistiques avancées.

Sigouin, C. 2004a. "Investment Decisions, Financial Flows, and Self-Enforcing Contracts." *International Economic Review* 44(4): 1359–83.

———. 2004b. "Self-Enforcing Employment Contracts and Business Cycle Fluctuations." *Journal of Monetary Economics* 51(2): 339–74.

Summers, L. 1986. "Some Skeptical Observations on Real Business Cycle Theory." *Federal Reserve Bank of Minneapolis Quarterly Review* 10: 23–27.

Thomas, J. and T. Worrall. 1988. "Self-Enforcing Wage Contracts." *Review of Economic Studies* 55: 541–54.

———. 1994. "Foreign Direct Investment and the Risk of Exportation." *Review of Economic Studies* 61: 81–108.

Watson, M. 1993. "Measures of Fit for Calibrated Models." *Journal of Political Economy* 101: 1011–41.

Wen, Y. 1998a. "Capacity Utilization under Increasing Returns to Scale." *Journal of Economic Theory* 81: 7–36.

———. 1998b. "Can a Real Business Cycle Model Pass the Watson Test?" *Journal of Monetary Economics* 42: 185–203.

Figure 1: Full-Commitment Pareto Frontier

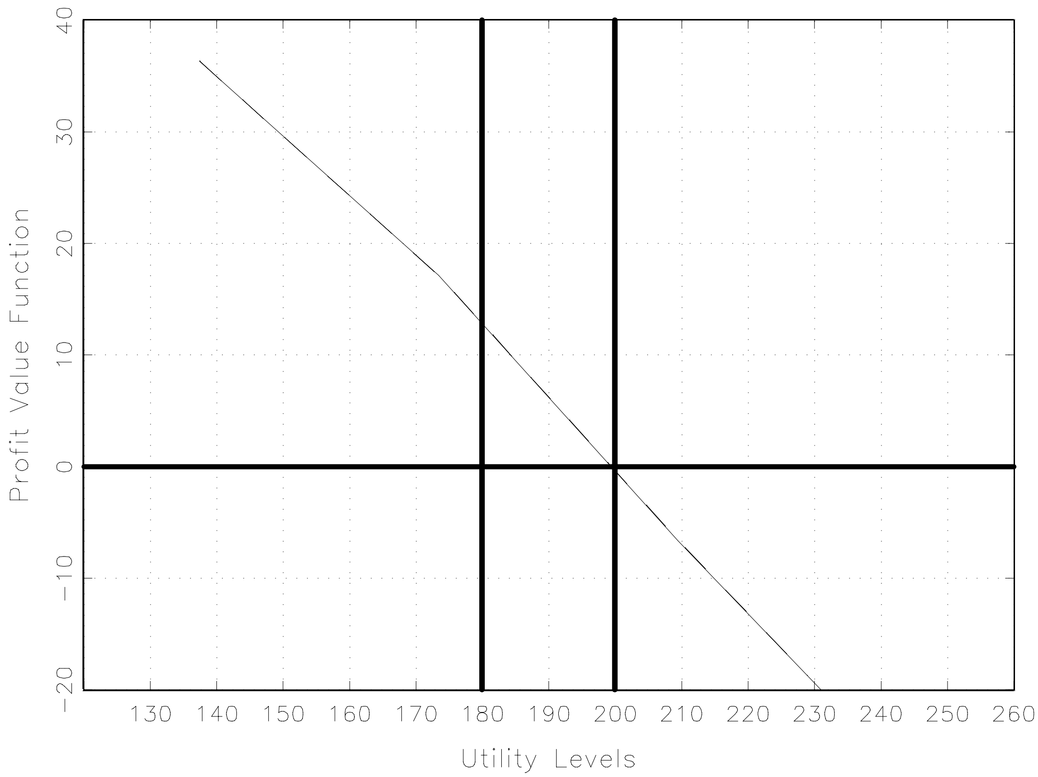


Figure 2: Economy's Dynamics

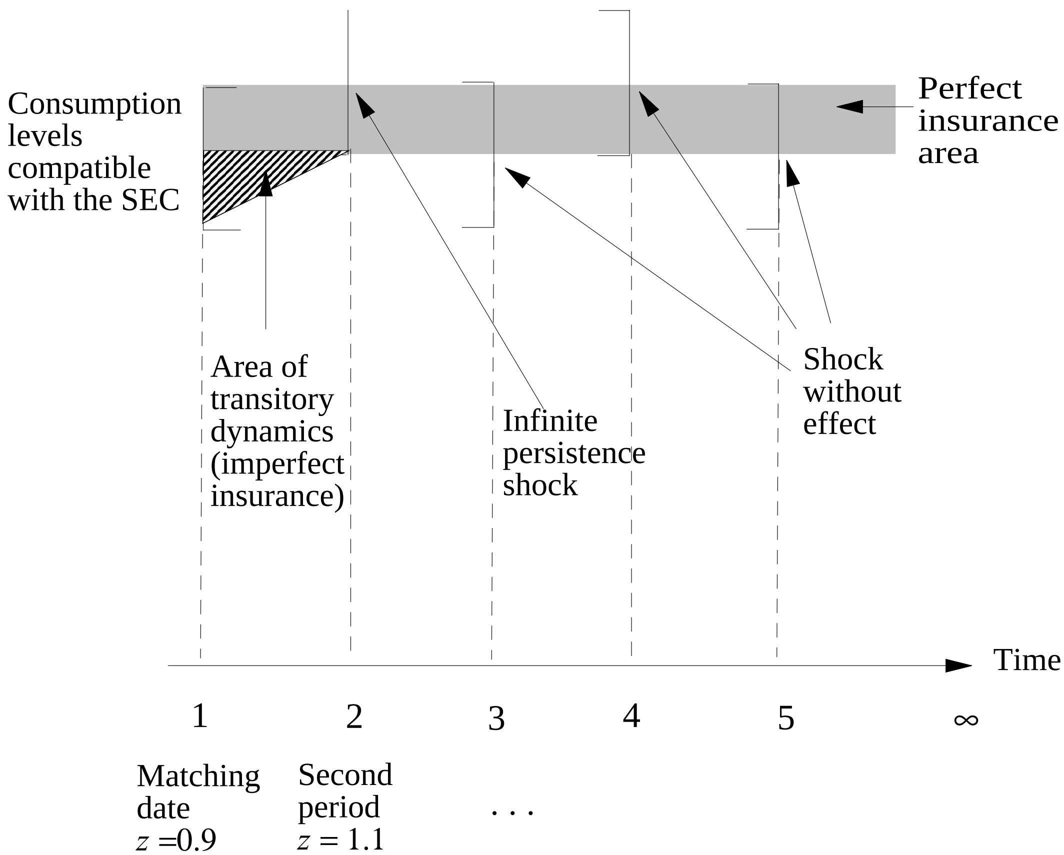


Figure 3: Dynamics of Consumption and Hours

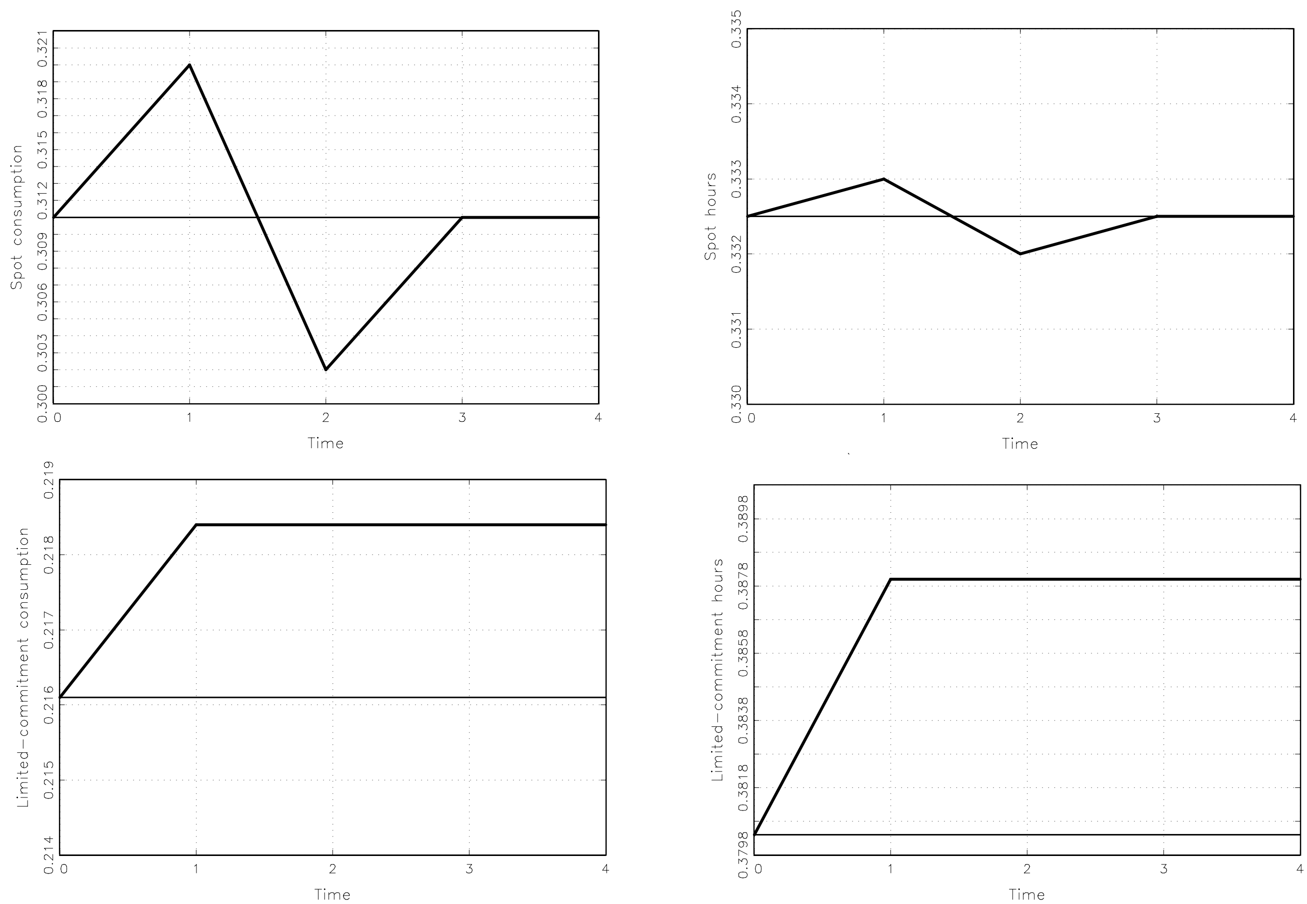
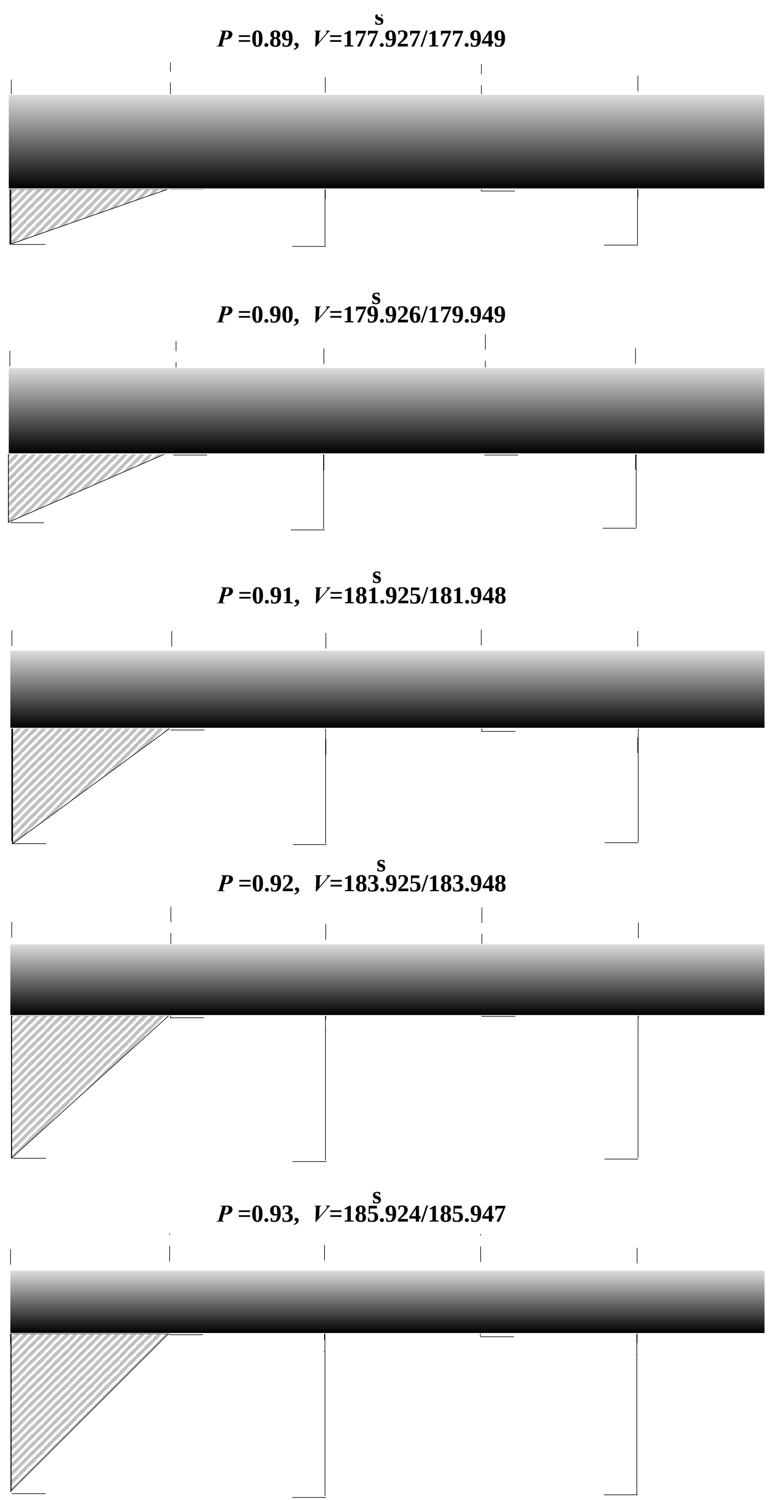


Figure 4: Persistence and P



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