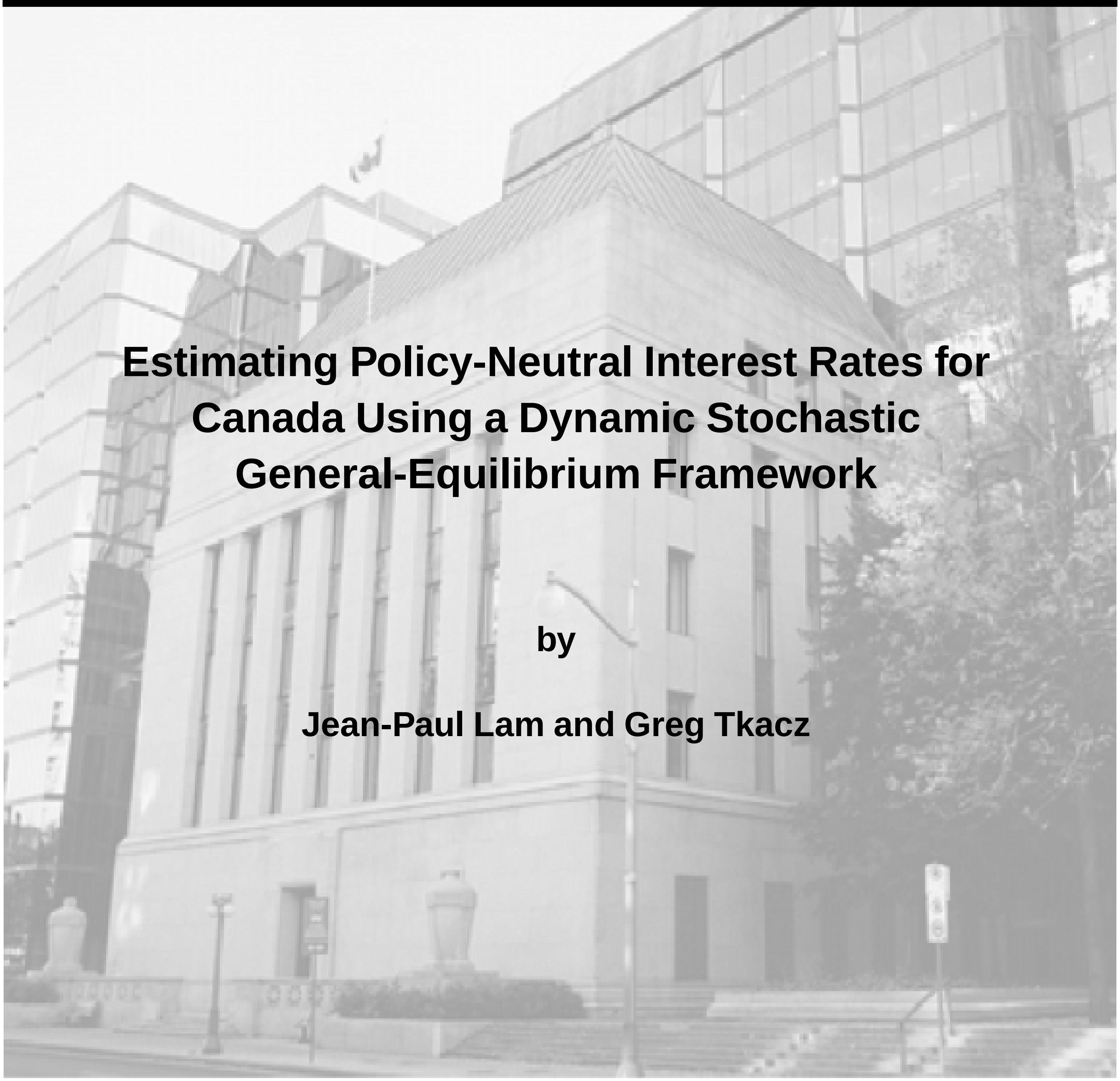
Bank of Canada   Banque du Canada

Working Paper 2004-9 / Document de travail 2004-9



Estimating Policy-Neutral Interest Rates for Canada Using a Dynamic Stochastic General-Equilibrium Framework

by

Jean-Paul Lam and Greg Tkacz

ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2004-9

March 2004

Estimating Policy-Neutral Interest Rates for Canada Using a Dynamic Stochastic General-Equilibrium Framework

by

Jean-Paul Lam and Greg Tkacz

Monetary and Financial Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
jplam@bankofcanada.ca
gtkacz@bankofcanada.ca

Contents

Acknowledgements

Thanks to Ed Nelson for valuable discussions on the neutral rate and to Bank of Canada seminar participants for several valuable comments and suggestions. Thank you also to Glen Keenleyside for editorial comments.

Abstract

In an era when the primary policy instrument is the level of the short-term interest rate, a comparison of that rate with some equilibrium rate can be a useful guide for policy and a convenient method to measure the stance of monetary policy. The real interest rate gap—the difference between the real equilibrium rate and the rate set by the central bank—can thus serve as a leading indicator of future inflationary or deflationary pressures in the economy. The authors estimate equilibrium interest rates for Canada using a sticky-price dynamic stochastic general-equilibrium model. They follow closely the methodology of Neiss and Nelson (2003) and derive measures of the interest rate gap for Canada. Their results indicate that the interest rate gap can be a useful guide for policy and is a good indicator of future output and inflation. The authors also find that their measures of the interest rate gap perform as well as the yield spread, a typical measure of policy stance that is assumed to contain significant information about future economic activity.

JEL classification: C32, E37
Bank classification: Interest rates

Résumé

En comparant le taux d’intérêt à court terme — qui est l’outil privilégié d’intervention de la banque centrale — à une estimation de sa valeur d’équilibre, il est possible d’obtenir des indications utiles pour la conduite de la politique monétaire de même qu’une mesure commode de son orientation. L’écart de taux d’intérêt réels, soit la différence entre le taux d’équilibre réel et le taux établi par la banque centrale, peut ainsi jouer le rôle d’indicateur avancé des pressions inflationnistes ou déflationnistes à venir. À l’aide d’un modèle dynamique d’équilibre général stochastique à prix rigides, les auteurs estiment le niveau d’équilibre du taux d’intérêt canadien. Ils suivent rigoureusement la méthodologie de Neiss et Nelson (2003) pour mesurer l’écart de taux observé au Canada. D’après leurs résultats, cet écart peut apporter aux autorités monétaires un précieux complément d’information et est un bon indicateur de l’évolution future de la production et de l’inflation. Les auteurs constatent également que leurs mesures de l’écart de taux réels se comportent aussi bien que l’écart de rendement, un indicateur courant de l’orientation de la politique monétaire qui est présumé bien renseigner sur le niveau futur de l’activité économique.

Classification JEL : C32, E37
Classification de la Banque : Taux d’intérêt

1. Introduction

Through the ages, some of the greatest economic thinkers (e.g., Thornton 1802, Wicksell 1898, and Meade 1933) have forwarded the proposition that there exists some unobservable interest rate that equilibrates supply and demand conditions in the economy. When observable market rates equal the equilibrium rate, which is roughly consistent with the marginal productivity of capital, price stability ensues. On the other hand, when observable market interest rates deviate from the unknown equilibrium rate, adjustments such as price increases or decreases occur in the economy to compensate.

In recent years, the notion of equilibrium interest rates has regained prominence, both in policy circles (e.g., Julius 1998, Meyer 2000, and Neiss and Nelson 2003) and academia (e.g., Woodford 2000, Bomfim 2001, and Laubach and Williams 2003). In an era when the primary policy instrument is the level of the short-term interest rate, a comparison of that rate with some equilibrium, or policy-neutral, rate is a convenient method to measure the stance of monetary policy. The real interest rate gap—the difference between the real equilibrium rate and the rate set by the central bank—can thus serve as a leading indicator of future inflationary or deflationary pressures in the economy.[1]

The objective of this paper is to construct measures of equilibrium interest rates, variables that are not directly observed. To derive the equilibrium natural rates, we follow Neiss and Nelson (2003) very closely and calibrate their sticky-price dynamic stochastic general-equilibrium (DSGE) model for Canada. Neiss and Nelson (2003) show that their derived stance measure for the United Kingdom (the difference between the actual and equilibrium rates) leads actual inflation relatively well, and thus can be useful for policy-makers.

As in Woodford (2000) and Neiss and Nelson (2003), we define the equilibrium rate of interest as the real rate that prevails in an economy characterized by fully flexible prices, or, equivalently, the real interest rate that returns output to its potential level in a sticky-price

1. This is especially true in models where the output gap reacts with a lag to the interest rate gap. The latter will give advance information on both the output gap and inflation.

economy.[2] Although this definition focuses on a short horizon for price stability and thus differs from the usual medium-run definition of the equilibrium rate—the rate that corresponds to the intermediate dynamics of the economy—it is worth remembering that in the model the "averaging" of the natural rate into a medium-run concept can be implemented by private agents, since their pricing and spending decisions are forward looking.[3]

In our model, the equilibrium real rate (and potential output) is obtained by eliminating all nominal rigidities; that is, we assume fully flexible prices in all markets. When prices are completely flexible all markets clear and, by definition, output and the real interest rate are at their equilibrium levels. On the other hand, when markets do not clear instantaneously, output and the real rate diverge from their equilibrium values. Any attempt on the part of policy-makers to close the output and/or real interest rate gap will reduce inflationary or deflationary pressures in the economy.

Once our measure of the equilibrium rate is derived, we provide several tests to evaluate the quality of our real interest rate gap series, particularly its forecasting abilities. We find that the real interest rate gap can be a useful measure of policy stance, as fluctuations in the real interest rate gap lead real activity, with the explanatory power peaking at about 4 to 5 quarters ahead. This is consistent with the view that monetary policy takes about 4 to 6 quarters to affect output. Our results also indicate that the empirical links of the real interest rate gap with real activity and inflation are strong. Our derived measure of the real equilibrium rate, for a given specification of the model, predicts output as accurately as the yield spread.

Moreover, our results indicate that our measure of the natural rate is not as volatile as in previous studies (Woodford 2000). Past studies have found the natural rate to be very volatile mostly because the parameter that governs the degree of intertemporal substitution in consumption was too highly calibrated. Since we choose less extreme values for this calibration, we do not obtain such a volatile response in the natural rate and therefore we obtain a smoother series for the equilibrium interest rate.

---

2. The equilibrium interest rate can be computed directly from any DGE structure.
3. The "averaging" of the natural rate into a medium-run concept can also be done in the definition itself. What matters in the model is not the actual output gap but also expected output gaps.

The remainder of this paper is organized as follows. Section 2 reviews some of the research on equilibrium rates performed by current and former central bankers in both the United States and the United Kingdom. Section 3 describes our DSGE model, and section 4 explains how it is calibrated. Section 5 describes the impulse-response functions of the model and how the measure of the equilibrium rate is obtained. The forecasting ability of our equilibrium series for output and inflation is discussed in section 6. Section 7 concludes.

2. Equilibrium Interest Rates and Monetary Policy

In virtually all developed countries, monetary policy is conducted through the control of a short-term interest rate that represents the rate at which commercial banks borrow to settle their overnight balances. The absolute level of this rate, however, cannot be viewed as an adequate gauge of the stance of monetary policy, because it ignores supply and demand conditions in product and credit markets. Instead, an appropriate measure of policy stance would be to compare the short-term interest rate with some equilibrium, or policy-neutral, rate. If the short-term interest rate is above equilibrium, then policy would be considered tight; if it is below equilibrium, policy would be considered loose. The challenge is to estimate such an equilibrium rate.

2.1 United States

Although the equilibrium rate is difficult to estimate and identify, the concept of such a rate is embraced and firmly entrenched in the thinking of some modern policy-makers. Most notably, in the United States, several current and former members of the Board of Governors of the Federal Reserve System use the notion to explain and rationalize policy decisions.

Johnson and Keleher (1996), for example, offer a textbook treatment on how market prices, and most notably bond prices, can be used to formulate policy. Their monograph identifies three useful indicators and explains how those indicators can be jointly used in policy-making. In particular, Johnson and Keleher advocate prices from central auction markets as pivotal policy indicators, namely commodity prices, exchange rates, and bond rates. These can be jointly used to locate the real rate of interest consistent with price stability (Wicksell's natural rate). Johnson and Keleher (p. 6) recognize that

> (e)ach of these indicators serves as a proxy measure of the value of money from a somewhat unique perspective. Commodity prices measure the value of money in terms of traded goods. Foreign exchange measures the price of money in terms of other monies. The price of bonds represents the exchange rate between money and bonds, or to put it another way, the price of money in terms of future money.

Monetary policy can affect all these variables, but the authors warn that so, too, can non-policy factors. Thus, "it is the careful and joint assessment of these proxies for the value of money that enables perceptive policymakers to contribute significantly to price stability" (p. 6).

Blinder (1998), a former Fed vice-chairman, also advocates the use of an equilibrium rate when formulating policy. His rough estimate of the real equilibrium rate is about 2.75 per cent, which represents a historical average of the real interest rate. Blinder believes that such a rate should be formally estimated by equating the steady-state IS curve to potential output each period, which would yield an interest rate for which the rate of inflation would remain constant. Bomfim (1997), however, estimating such a rate, finds that the resulting series is highly volatile.

Meyer (2000), a member of the Fed's Board of Governors, uses an equilibrium rate to rationalize the challenges that policy-makers face. He explains that the level of the real equilibrium rate should be tied to the level of productivity in the economy, with higher productivity resulting in higher real equilibrium rates: "The equilibrium real interest rate at full employment has to balance saving (driven by the thrift motive) and investment (responding to the productivity and, hence, profitability of capital)" (p. 8). The goal of monetary policy is to keep market real interest rates in line with the equilibrium rate to achieve the objective of price stability.

2.2 United Kingdom

Equilibrium interest rates form the basis of a *Financial Times* article by Brittan (2001), who notes that the estimate of the real equilibrium rate would "almost certainly come in the range of 3-5 per cent." Part of the motivation for the article stemmed from the views of DeAnn

Julius, member of the Bank of England's Monetary Policy Committee (MPC); she justifies her positions on interest rate movements by drawing upon her beliefs about the level of the equilibrium rate. Consider her testimony (Julius 1998) at a recent House of Commons Select Committee on Treasury:

> *Question*: In the latest minutes that we have available, you recommended a cut in interest rates (question #302), whereas your colleagues either recommended to hold interest rates steady or recommended an increase. What do you think was the major factor determining your resolve that the interest rates should be cut (#303)?
>
> *Julius*: The stance of monetary policy needs to take into account of where we are in the cycle and what is to come over the next year or year and a half because . . . the effect is a year and a half or two years away, so it seemed to me that the stance of policy was too tight for that phase of the cycle. When I say that, I am referring particularly to the height of real short-term interest rates which . . . is probably somewhere around 4 per cent, and also the shape of the yield curve with short rates being higher than long rates, both of those are indicators that monetary policy is pretty tight and at this stage of the cycle it is not clear to me that is the appropriate stance for monetary policy as the economy is slowing, so I voted for a reduction in interest rates as a means of moving towards a more neutral position of monetary policy which I felt would be more appropriate for this phase of the cycle.

3. The Model

This section describes a sticky-price DSGE model with capital adjustment costs. The model includes households, firms, and a monetary authority, and is very similar to the one used in Neiss and Nelson (2003). There are two types of firms: final- and intermediate-goods producers. Final-goods producers behave competitively and produce output using intermediate goods. On the other hand, intermediate-goods firms are monopolistic competitors that produce a differentiated product using capital and labour based on existing technology. These firms set nominal prices on a staggered basis.

3.1 Firms

We assume that there are a large number of final-goods producers that behave competitively and produce a homogeneous good, Y_t, using intermediate goods, $Y_t(z)$. We also assume that there is a continuum of intermediate-goods producers owned by consumers, indexed by the letter *z*, that operate in a Dixit-Stiglitz style imperfectly competitive economy.

3.2 Final-goods producers

Final-goods producers use the following production function to transform intermediate goods into final output:

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where $\theta > 1$. This production function exhibits constant returns to scale, diminishing marginal product, and constant elasticity of substitution. In each period, the final-goods firms choose inputs $Y_t(z)$ for all $z \in [0,1]$, and output to maximize profits subject to equation (1). This maximization problem is given by:

$$\max\ P_t Y_t - \int P_t(z) Y_t(z) dz \quad \text{s.t. equation (1)}, \tag{2}$$

where $P_t(z)$ is the price of the intermediate good $Y_t(z)$ and P_t refers to the aggregate price level.

The solution to this problem yields the constant price elasticity demand function, *z*, that is homogeneous of degree one in total final output:

$$Y_t(z) = \left[\frac{P_t(z)}{P_t} \right]^{-\theta} Y_t. \tag{3}$$

Combining this demand function with the production function (1), we obtain the following price index for intermediate goods:

$$P_t = \left[\int_0^1 P_t(z)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{4}$$

3.3 Intermediate-goods producers

Each intermediate-goods producer is indexed by $z \in [0,1]$, operates in a Dixit-Stiglitz style imperfectly competitive economy, and faces a downward-sloping demand function for its product (equation (3)). In addition, each firm produces intermediate goods subject to the following technology constraint:

$$Y_t(z) = A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha}, \tag{5}$$

where $N_t(z)$ and $K_t(z)$ are, respectively, the amount of labour and capital hired by the firm to produce output, and A_t is a technology shock that follows this process:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A \text{ and } \varepsilon_t^A \sim iid(0, \sigma_A). \tag{6}$$

The intermediate-goods producer is assumed to choose the optimal amount of physical capital and labour to maximize profits taking the productivity of the firm as given, subject to equation (5). Thus the firm solves:

$$\min_{N_t(z), K_t(z)} [w_t(z) + r_t K_t(z)] \text{ s.t. equation (5)}, \tag{7}$$

where w_t is the real wage rate and r_t is the rental rate of capital.

The first-order conditions to this minimization problem are given by:

$$w_t = \alpha mc_t \frac{Y_t(z)}{K_t(z)}, \text{ and} \tag{8}$$

$$r_t = (1-\alpha) mc_t \frac{Y_t(z)}{K_t(z)}, \tag{9}$$

where mc_t is the real marginal cost.

Using the above equations, we can see the link between marginal cost and the gross markup. Defining the markup as the inverse of real marginal costs and rearranging (8) and (9), we obtain a link between marginal cost and the gross markup.

Rearranging (8) and (9), we have,

$$\alpha \frac{Y_t(z)}{N_t(z)} = \mu_t w_t \text{, and} \tag{10}$$

$$(1-\alpha)\frac{Y_t(z)}{N_t(z)} = \mu_t r_t , \tag{11}$$

where $\mu_t = \frac{1}{mc_t}$.

Equations (10) and (11) imply that firms adjust their inputs to equate the marginal product of each input with the markup times the price of the input.

3.4 Inflexible prices

Intermediate-goods producers firms set nominal prices on a staggered basis. As with Roberts (1995), Yun (1996), and Clarida, Galí, and Gertler (1999), we use the model of Calvo (1983) and assume that firms are allowed to reset the price of their good in any given period with probability (1-s).[4] Hence, the firms keep their price fixed with probability s. The parameter s governs the degree of nominal price rigidity in the model. As s approaches zero (one), prices become more flexible (rigid).[5]

4. An alternative is to follow Christiano, Eichenbaum, and Evans (2001), who modify Calvo's framework by assuming a dynamic price-updating scheme instead of a static one. They assume that the proportion of producers who can adjust their prices do so by indexing their prices to last period's inflation.

5. If firms face a probability (1-s) of changing their price, the price will remain fixed on average for $\frac{1}{1-s}$.

Firms that are able to adjust their price will do so optimally and will maximize expected profits, taking the aggregate output (Y_t), aggregate price level (P_t), nominal marginal cost (MC_t), and constraint on the frequency of price adjustment as given. Firms that are not able to change their price instead adjust output to meet demand. However, all firms minimize their costs given demand.

The problem of a firm changing prices at time t consists of choosing to maximize the following:

$$E_t \sum_{i=0}^{\infty} (s\beta)^i \left[\Lambda_{t,t+i} \left(\frac{P_t(z) - MC_{t+1}}{P_{t+i}} \right) Y_{t,t+i}(z) \right], \tag{12}$$

subject to the demand for its good, $Y_{t,t+i}(z) = \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\theta} Y_{t+i}$, where $\Lambda_{t,t+1}$ is the ratio of the marginal utility of consumption at time $t+i$ and t and $\beta\Lambda_{t,t+1}$ is the rate at which the firm discounts earnings at time $t+i$. Substituting the above equation into equation (12), the objective function can then be written as:

$$E_t \sum_{i=0}^{\infty} (s\beta)^i \left[\Lambda_{t,t+i} \left(\left[\frac{P_t(z)}{P_{t+i}} \right]^{1-\theta} - \left[\frac{MC_{t+i}}{P_{t+i}} \right] \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\theta} \right) Y_{t+i} \right]. \tag{13}$$

The first-order condition to this maximization problem can be written as

$$P_t(z) = \mu E_t \sum_{i=0}^{\infty} \omega_{t,t+i} MC_{t+i}, \tag{14}$$

where $\omega_{t,t+i} = \dfrac{(s\beta)^i \Lambda_{t,t+i} D_{t,t+i}}{E_t \sum_{i=0}^{\infty} (s\beta)^i \Lambda_{t,t+i} D_{t,t+i}}$, $\mu = \dfrac{\theta}{\theta - 1}$ is the steady-state markup or the inverse of the steady-state real marginal cost, and $D_{t,t+i} = \left[\frac{P_t(z)}{P_{t+i}} \right]^{1-\theta} Y_{t+i}$ denotes the firm's revenues at $t+i$ conditional on $P_t(z)$, as in Galí and Gertler (1999).

Because firms know that they cannot change their price in each period, their optimal price will be a function of past and expected future demand. As a result, the aggregate price index will be a weighted average of the optimal price and prices set in period t-1 to reflect the proportion of firms that are not able to change their price. The aggregate price index can thus be expressed as

$$P_t^{1-\theta} = (1-s)\widetilde{P}_t^{1-\theta} + sP_{t-1}^{1-\theta} . \tag{15}$$

By log-linearizing the first-order condition (FOC) of the firm and equation (15) above, we obtain a "New Keynesian" Phillips curve. This supply function is given by

$$\pi_t = \beta E_t(\pi_{t+1}) + \varphi mc_t \text{ where } \varphi_t = \left[\frac{s}{1-s}\right]\left[1-(1-s)\beta\right]. \tag{16}$$

As Clarida, Galí, and Gertler (1999) point out, unless a serially correlated cost-push shock is added to this equation, the New Keynesian Phillips curve does not generate realistic inflation dynamics. For that reason, we use a hybrid supply function as a sensitivity test in our analysis. We follow Galí and Gertler (1999) and assume that a proportion of firms use a simple rule of thumb that is based on the recent history of aggregate price behaviour to set prices.[6] Galí and Gertler show that, when this is combined with the model of Calvo, the following hybrid supply function results:

$$\pi_t = \phi\pi_t + (1-\phi)E_t(\pi_{t+1}) + \varphi mc_t . \tag{17}$$

In equations (16) and (17), the parameter governs the degree to which prices are sticky. The larger (smaller) □ is, the more flexible (rigid) prices are. The equilibrium real rate is obtained by calibrating □ to a very large number, whereas the observed (actual) rate is obtained by assuming that prices are rigid. Once the two measures are obtained, we can construct the interest rate gap; i.e., the difference between the equilibrium rate (flexible-price value) and the observed rate (sticky-price value). Although the specifications of the supply function and of the rule for setting interest rates do not affect our measure of the equilibrium rate, however, the observed or actual interest rate will depend on how the model is specified.

6. See Christiano, Eichenbaum, and Evans (2001) for a derivation of (17) that does not rely on a rule-of-thumb assumption.

3.5 Households

The economy is composed of a continuum of infinitely lived agents, each of whom consumes a final good (C_t), holds real money balances $\left(\frac{M_t}{P_t}\right)$, and supplies labour (N_t). The agents are assumed to maximize the sum of discounted expected utility by choosing the optimal quantity of goods to consume and the amount of hours to work and invest in physical capital (K_{t+1}) each period, given prices (P_t), wages (W_t), and interest rates (r_t). Assuming that □ is the discount rate of the consumer, the representative household chooses a sequence of consumption, nominal money balances, one-period bond holdings (B_{t+i}), capital (K_{t+i}), and employment to maximize the following lifetime utility function:

$$Max\ E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\xi_t\frac{\sigma}{\sigma-1}\left(\frac{C_t}{C_{t-1}^h}\right)^{\frac{\sigma-1}{\sigma}}+\frac{\gamma_m}{1-\gamma_m}\left(\frac{M_t}{P_t}\right)^{1-\gamma_m}+a_nL_t\right]\right\}, \tag{18}$$

subject to a series of period budget constraints:

$$C_{t+i}+I_{t+i}+\frac{M_{t+i}}{P_{t+i}}+\frac{B_{t+i+1}}{P_{t+i+1}}=w_{t+i}N_{t+i}+r_{t+i}K_{t+i}+\Pi_{t+i}+\frac{M_{t-1+i}}{P_{t+i}}+\frac{(1+R_{t-1+i})B_{t+i}}{P_{t+i}}-H(I_{t+i}) \tag{19}$$

$$L_t+N_t\leq 1, \tag{20}$$

where

$$I_{t+i}=K_{t+1+i}-(1-\delta)K_{t+i},\quad \delta\in[0,1], \tag{21}$$

$$H(I_{t+i})=\psi I_{t+i}^{\eta},\ \text{and} \tag{22}$$

$$\xi_t=\rho_\xi\xi_{t-1}+\varepsilon_t^\xi. \tag{23}$$

Consumption in the model displays habit formation, since households derive utility not only from current but from past consumption. The parameter h measures the strength of habit persistence. In the limiting case, where $h=0$, the function reduces to a standard time-separable utility function. We follow Fuhrer (2000), Neiss and Nelson (2003), and Christiano, Eichenbaum, and Evans (2001) and set h to a fraction. In that case, households derive utility from current and also from past consumption. The parameter □ measures the curvature of the consumption function and corresponds to the intertemporal elasticity of substitution. ξ_t represents a preference shock that will be specified below.

In equation (19), r_t represents the rental rate of capital, R_t the gross nominal interest rate, and Π_t lump-sum firm profits. I_t denotes investment and it is related to the capital stock, as in equation (21), where □ is the depreciation rate. We assume that households cannot adjust their capital stock instantaneously (equation (22)). We follow Abel (1983), Casares and McCallum (2000), and Neiss and Nelson (2003) and assume that capital accumulation in the model is subject to some adjustment cost, as shown. With □=2, equation (22) amounts to quadratic costs of adjustment. We allow, however, for a more general specification and let □ take on different values.

Letting λ_t denote the Lagrangian multiplier in (18), we have the following first-order conditions for the representative household:

Consumption

$$\left[\xi_t C_{t-1}^{-h\left(\frac{\sigma-1}{\sigma}\right)} C_t^{-\frac{1}{\sigma}}\right] - \beta h E_t\left[\xi_{t+1} C_{t+1}^{\frac{\sigma-1}{\sigma}} C_t^{-h\left(\frac{\sigma-1}{\sigma}\right)-1}\right] - \lambda_t = 0, \tag{24}$$

Employment

$$-a_n + \lambda_t w_t = 0, \tag{25}$$

Money demand

$$a_m\left(\frac{M_t}{P_t}\right)^{-\gamma_m} - \lambda_t + \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0, \tag{26}$$

Capital

$$\lambda_t(1+H'(I_t)) - \beta E_t \lambda_{t+1}\left[(1-\delta)(1+H'(I_{t+1})) + r_{t+1}\right], \text{and} \tag{27}$$

Bonds

$$-\lambda_t + \beta(1+R_t)E_t\lambda_{t+1}\frac{P_t}{P_{t+1}} = 0. \qquad (28)$$

3.6 Monetary authority

Monetary policy in the model is characterized by an interest rate rule. We estimate a forward-looking rule for Canada for the period 1990Q1 to 2000Q4 using generalized method of moments. This rule is given by:

$$R_t = 0.816R_{t-1} + (1-0.816)[2.075E_t\pi_{t+1} + 1.08(y_t - \bar{y}_t)] + \varepsilon_t^R, \qquad (29)$$

where ε_t^R is a policy shock and $y_t - \bar{y}_t$ is the output gap.

As a sensitivity test, we also consider an alternative rule that Côté et al. (2004) find to be robust in some models of the Canadian economy. It is a Taylor-type rule but with a weight of 2 on the deviations of inflation from its target.

3.7 Market clearing and equilibrium

We define equilibrium as a collection of allocations for final-goods producers, Y_t, $Y_t^D(z)$ for all $z \in [0,1]$; allocations for intermediate-goods producers, $N_t(z)$, $K_t(z)$ for all $z \in [0,1]$; allocations for consumers, $C_t, N_t, I_t, M_t, B_{t+1}, K_{t+1}$; together with a price vector, $W_t, Z_t, P_t, P_t(z), R_{t+1}^n, MC_t, H(I_t)$, for all $z \in [0,1]$, such that all agents are maximizing subject to the constraints they face, supply equals demand in each market, and all resource constraints are satisfied, given the values of the predetermined and exogenous variables. To examine the dynamics of the model, we log-linearize the optimality conditions around their steady state. Appendix A shows the log-linearization.

4. Calibration

In this section, we describe the parameter values for our benchmark model, which are shown in Table 1. In calibrating the model, we follow Dib (2002), who estimates several of these parameters for Canada. We set the discount factor, □, to its usual value of 0.99; the depreciation parameter, □, to 0.025; and the labour share, □, to 0.7. The probability that a firm is able to change its price, *s*, is set to 0.25, which implies that prices are fixed on average for a year. ϕ, which measures the degree of backward-lookingness in the hybrid supply function, is set at 0.5, an estimate similar to that by Fuhrer and Moore (1995). The steady-state markup, μ, is set at 1.25; the adjustment cost parameters, ψ, η, to 0.2 and 2, respectively (we allow for a quadratic cost adjustment process—see Casares and McCallum 2000); and the habit persistence parameter, *h,* to 0.6 (see Fuhrer 2000). The standard deviations of the preference and technology shock are from Dib (2002) and they are set, respectively, at 0.015 and 0.01. We assume that both shocks are very persistent and set the serial correlation parameter to 0.875 and 0.9, respectively.

The intertemporal elasticity of substitution, σ, is set to 0.6. This parameter is typically calibrated to a lower value in closed-economy models (for example, Dib 2002 has an estimate of 0.45 for this parameter). Because we want the model to capture some of the dynamics of a small open-economy model (at least on the demand side), we follow Neiss and Nelson (2003) and calibrate this parameter to a higher value.[7] In an open-economy framework, net exports are an important component of aggregate demand. Their influence on aggregate demand can be partially captured in this model by increasing the value that σ takes.[8] We assume a value of 0.45 and 0.15, respectively, for the interest responsiveness of consumption and net exports (hence, a value of 0.6 for σ). This three-to-one ratio captures a common rule of thumb about IS coefficients (see, for example, Duguay 1994). In addition to our baseline model, we present three alternative models that reflect some of the sensitivity tests we have performed. These sensitivity tests are shown in Table 2.

---

7. Although we calibrate the intertemporal elasticity of substitution to a higher value to capture some of the dynamics of a small open-economy model on the demand side, our model does not allow for such effects on the supply side, since the exchange rate is not included in the Phillips curve. Many studies, however, have shown that, for Canada, there is no empirical justification to include the exchange rate in the Phillips curve. As a result, our baseline Calvo and hybrid supply functions are reasonable approximations of open-economy Phillips curves.
8. See Neiss and Nelson (2003) for more details.

5. Equilibrium Interest Rates

5.1 The model's impulse-response function

As stated in the introduction, the equilibrium real rate is obtained by eliminating all nominal rigidities in the model.[9] In a model where capital is exogenous and habit formation is absent, the equilibrium real rate can be easily obtained, because it is simply a function of the state vector of the model, which contains only exogenous variables. On the other hand, in a model with habit formation and where the capital stock is time-varying and endogenous, calculating the equilibrium rate is not as straightforward. In such a model, the state vector is no longer just a function of exogenous variables, but also contains endogenous variables: the capital stock at period t and consumption at period t-1. Consequently, we cannot simply express the equilibrium rate as a function of the state vector, since the latter does not contain only exogenous variables.

To circumvent this problem, we employ the same methodology as Neiss and Nelson (2003). They argue that the equilibrium rate can be expressed as a function of current and past dated exogenous variables if the two endogenous variables (capital stock and consumption) are substituted out of the state vector. Since the exogenous elements of the state vector are assumed to follow an autoregressive process, the capital stock and consumption can also be expressed as a function of the exogenous variables of the state vector. As a result, they can be substituted out of the state vector. We can then write the equilibrium rate as a linear function of current and past dated shocks of the model (a distributed lag of the innovations of the model).

To obtain the impulse-response functions and empirical estimates of the equilibrium real rate, we proceed as follows: we first solve the model under flexible prices and then, using stochastic simulations, generate artificial data for several variables of the model, including the innovations. Since the equilibrium interest rate and potential output are linear combinations of current and past dated shocks, using the artificial data generated from each stochastic simulation, we run the following regressions for an i that is high enough for a good fit[10]:

$$r_t^* = \Phi_1\xi_t + \Phi_2\xi_{t-1} + \ldots.. + \Phi_i\xi_{t-i} + \Theta_1 a_t + \Theta_2 a_{t-1} + \ldots.. + \Theta_i a_{t-i}, \text{ and} \qquad (30)$$

9. The equilibrium level of output is obtained using a similar methodology.

10. The fit of these equation is extremely good, implying that our regression approximates well the true measure of the equilibrium rates.

$$y_t^* = \widetilde{\Phi}_1 \xi_t + \widetilde{\Phi}_2 \xi_{t-1} + \ldots.. + \widetilde{\Phi}_i \xi_{t-i} + \widetilde{\Theta}_1 a_t + \widetilde{\Theta}_2 a_{t-1} + \ldots.. + \widetilde{\Theta}_i a_{t-i} . \tag{31}$$

We then calculate the average of each of the above coefficients across the simulations. The average of these coefficients will be used subsequently to generate our model-based and empirical series for the equilibrium real rate.

Second, the model is solved under the assumption of sticky prices using one of the supply functions and policy rules we have specified.[11] Third, the real interest rate and output gaps are calculated using the series derived with and without price rigidities. The impulse-response functions for the output gap, real interest rate gap, and the equilibrium rate are shown in Figures 1 to 6 for four different versions of our model.

Figures 1 to 3 show the response of the economy to a preference shock. Since output increases more than potential, this shock leads to a positive output gap. The equilibrium real rate as well as the actual real rate also increases. Because the latter increases by more than the former, however, a negative interest rate gap opens, signalling that policy is too loose.

A positive technology shock (Figures 4 to 6), on the other hand, leads to a negative output gap as potential output increases by more than actual output. In this model, a positive technology shock leads to a countercyclical response of the equilibrium rate: the equilibrium interest rates fall rather than rise. The countercyclical response of the natural rate to a positive technology shock is explained below.

A positive technology shock (Figures 4 to 6) leads to a rise in investment and this, in turn, puts upward pressure on the equilibrium real interest rate. Since it is costly to adjust capital in this model, however, investment does not increase by as much as it would if capital were allowed to adjust freely. As a result, the increase in the equilibrium rate is muted.

The positive technology shock also increases income and hence consumption. Because households smooth consumption in the model, however, they shift resources from current to future periods and do not consume all of their higher income immediately. In other words, they increase their savings. This puts downward pressure on the equilibrium real rate. In the model, since the fall in the equilibrium rate due to higher savings more than offsets the increase due to

11. The choice of the policy rule and supply function does not matter when prices are fully flexible.

higher investment demand, the equilibrium real rate falls following a technology shock (Figure 6).

Before presenting our empirical estimates of the equilibrium real rate, we present (Table 3) some basic correlations between the real interest rate, output gap, and inflation. In all four models, the real interest rate gap has a strong negative relationship with inflation. Moreover, the correlation between the observed real interest rate and the interest rate gap is very high in all four models, which indicates that the variations in the interest rate gap can be mostly explained by changes in the observed real rate of interest.

5.2 Empirical estimate of the equilibrium real rate

We have already shown that the equilibrium real rate in our model can be expressed as a linear function of the technology and preference shocks. Using data on preference and technology shocks and equation (30), we construct an empirical series for the equilibrium rate. Whereas technology shocks are typically obtained from Solow residuals, preference shocks, on the other hand, are much harder to measure. Using the Euler equations of the model, however, we can derive an empirical series for this shock.[12] Our methodology is explained below.

In most models, technology shocks are measured by Solow residuals, which are usually obtained by subtracting the log values of labour and capital inputs, weighted accordingly, from the log of total output. Although we have estimated and derived Solow residuals using the productions function, we use the Solow residuals series from the Bank of Canada's Quarterly Projection Model (QPM).[13] In fact, we find that our results are very robust and do not depend on how the Solow residuals are derived.

To construct our preference shocks, we proceed as follows. Using the log-linearized equations for consumption (equation (A1)), bonds (equation (A4)), and equation (23), we have the

12. We are aware that the identification of preference shocks using first-order conditions is by no means perfect and has been criticized by many. In this paper, we make the strong assumption that the term ξ_t captures preference shocks.

13. Because our model does not allow for sectoral growth, the series for the Solow residuals correspond to detrended total factor productivity.

following equation, which relates preference shocks to consumption, prices, and interest rates (more details are provided in Appendix B):

$$\xi_t = \frac{1}{\upsilon_6}\left[\upsilon_2 \Delta c_t + \upsilon_3 E_t \Delta c_{t+2} - \upsilon_4 E_t \Delta c_{t+1} - \upsilon_5 R_t + \upsilon_5 E_t \pi_{t+1}\right]. \tag{32}$$

In equation (32), c_t is the log of consumption excluding durables, π_t is annual core inflation, R_t is the 90-day commercial paper rate, and ξ_t is a preference shock at time t. To generate expected values for consumption, we use a 3-variable VAR estimated over the period 1980Q1 to 2001Q4.[14] Using our calibrated values, the coefficients estimated in equation (30), our series on Solow residuals, and the preference shocks series that we constructed, we can obtain an empirical series for the equilibrium real rate. Our series spans from 1984Q1 to 2001Q4.

Using Model 4, the equilibrium real interest rate is given by

$$\begin{aligned} r_t^* = & -0.0163\xi_t + 0.053\xi_{t-1} + 0.024\xi_{t-2} + 0.011\xi_{t-3} + 0.006\xi_{t-4} + 0.003\xi_{t-5} + 0.002\xi_{t-6} + 0.001\xi_{t-7} \\ & + 0.001\xi_{t-8} - 0.09a_t + 0.045a_{t-1} + 0.0153a_{t-2} + 0.003a_{t-3} - 0.001a_{t-4} - 0.003a_{t-5} \\ & - 0.003a_{t-6} - 0.002a_{t-7} - 0.002a_{t-8} - 0.002a_{t-9} - 0.001a_{t-10}. \end{aligned} \tag{33}$$

The next section evaluates the quality of our real interest rate gap series; in particular, it evaluates the forecasting ability of this series.

6. Estimates of Equilibrium Rates

In Figure 7 we plot four different measures of policy stance, each defined as the difference between an estimate of the (time-varying) real equilibrium rate and the real 90-day commercial paper rate. In addition, we plot the familiar yield spread, measured as the difference between the long-term (10-year and over) interest rate and the 90-day rate. As discussed, the leading-indicator properties of the yield spread are thought to originate from the fact that, through

14. The VAR contain eight lags of the variables $\Delta c_t, \pi_t$, and ΔR_t, with a dummy at 1992Q1 to capture changes in the monetary policy regime.

the expectations hypothesis, the long-term rate can be viewed as a proxy for the equilibrium short-term rate, and so the yield spread could be viewed as a measure of the policy stance.

Figure 7 shows that our four model-based measures of policy stance move in a similar fashion over time, differing only in the degree of policy stimulus in effect. With zero representing, by definition, a neutral policy stance, we note that, according to our model estimates, policy has been tight for most of the time since 1984. This is further evidenced by the fact that the average measures of policy stance (Table 4) have been negative since 1984. Estimates from our model reveal that the policy rate in 2002Q4 was between 104 (Model 4) and 179 (Model 2) basis points below neutral, which is estimated to be 1.24 (Model 4) and 1.98 (Model 2) per cent, in real terms. Assuming that the equilibrium rates remain constant, our model indicates that short-term real interest rates in 2002Q4 were between 100 and 175 basis points below the estimated neutral value.

6.1 Output-indicator models

A key indicator of success of any measure of policy stance is its ability to lead output growth and inflation. To this end, we estimate simple indicator models of the year-over-year growth rates of both real output and the core price level. Table 5 provides in-sample estimates of output-growth models, using as regressors only the yield spread and our four stance measures. The yield spread has long been shown to be a good indicator of output growth at this horizon, and so provides a strong benchmark against which to compare other measures of policy stance. Based purely on in-sample fit, Model 4 is the only model-based measure of policy stance that can improve upon the fit of the yield spread.

Examining the regression diagnostics, we find that our use of overlapping data results in serial correlation problems; we adjust our estimated standard errors using the Newey-West variance-covariance matrix to account for this issue. Parameter instability, however, does not appear to be an issue with these models, as all estimated constant and slope parameters appear to be stable over the sample. Our final diagnostic, the RESET test to uncover possible non-linearities in the residuals, reveals that there are indeed significant non-linearities present in these simple models. To partly account for such non-linear effects, we augment each model with the square of each stance measure. The effect of adding such a variable results in a notable jump in the fit of the yield-spread model, with the adjusted-R^2 climbing from 0.30 to 0.39. The improved fit can also be seen in Figure 8. The positive value of 1.06 on the spread indicates that the relationship between the lagged spread and output growth is positive, but the negative value on the parameter of the squared term indicates that the relationship is concave. Performing the RESET test on this augmented model reveals that the residuals have been cleansed of all non-linearities, which indicates that a simple quadratic specification is a simple way of explaining the relationship between the yield spread and output growth within a parametric framework. This finding of significant non-linearities supports the nonparametric neural network models estimated in Tkacz (2001), but within a more tractable parametric model.

The trick of adding squared values of the stance measures to the other models, however, does not prove to be effective in improving the modelled relationships, because the in-sample fits do not improve as much, and the models continue to fail the RESET tests, which indicates that a more complex method must be used to capture the non-linearities.

It is well known that a strong in-sample fit does not always lead to a strong forecasting performance. For that reason, we also perform an out-of-sample forecasting exercise in which we estimate each model using data from 1985Q1 to 1992Q4, produce an output-growth forecast for 1993Q4, re-estimate each model using an additional data point, and produce another forecast 4 quarters ahead. This procedure is repeated until a forecast is obtained for 2001Q4. Our forecasting sample is chosen to be sufficiently long to draw meaningful conclusions regarding relative forecast performance.

The results (Table 6) reveal that the model-based stance measures outperform the simple yield spread in forecasting output growth, even when the yield spread is augmented with a quadratic term. The best forecasting model is Model 4, which produces a root mean squared error of 1.53. Figure 9 shows that both the yield spread and Model 4 forecasted output growth very well until 1998, but that the forecast performance has since deteriorated, which indicates that non-policy-related factors (e.g., productivity growth) have been superior indicators of output growth.

6.2 Inflation-indicator models

Because the Bank of Canada has a formal inflation target range of 1 to 3 per cent, another useful feature of a stance measure is an ability to predict the rate of inflation. In this section, we compare the different stance measures that explain the year-over-year growth in the core price level.

Table 7 shows the in-sample estimates of the inflation models. The initial results indicate that the stance measures and yield spread are poor indicators of future inflation, because the adjusted R^2s are very low, and the slope parameters are insignificant and/or have the wrong sign. Upon examining the diagnostics, however, we find that a severe break occurred in 1991Q3 for all models. That period is near the beginning of formal inflation targets (which were announced in February 1991) and a drop in overall inflation that resulted from the 1990–91 recession. When we re-estimate the models with a dummy variable to capture the break in the relationship, we find that the model fits improve dramatically, and that the slope parameters are of the correct sign and in some cases significant. Furthermore, all parameters of the dummy-augmented models are found to be stable. Also, the inflation rates that emerge in these models when policy is neutral (i.e., when the model-based stance variables equal zero, which occurs only when the policy rate equals the equilibrium rate) are consistent with the Bank's inflation target. For Models 1 through 4, the steady-state inflation rates are found to be in the range of 1.89 (Model 2) to 1.95 (Model 4) per cent.

The effects of the structural break in the inflation models are most apparent in Figures 10 and 11. The structural break is clearly discernible in Figure 10, while Figure 11 demonstrates the requirement for a dummy variable in the forecasting model.

7. Conclusion

This paper has attempted to provide an estimate of the equilibrium interest rate that relies on a DSGE equilibrium model. Using a dynamic general-equilibrium model with nominal rigidities, we have defined the equilibrium rate as that which would prevail each period were the rigidities removed; that is, the rate that would prevail were all markets in equilibrium. To verify the robustness of our findings, we estimated four different equilibrium rates under different calibration assumptions.

Our findings have revealed that the derived stance measures are fairly robust to the calibration assumptions, in the sense that all stance measures move in similar patterns over time. Comparing the model-based stance measures with the yield spread, we detected one stance measure in particular (Model 4) that is as good as or better than the yield spread at explaining future output and inflation.

Bibliography

Abel, A.B. 1983. "Optimal Investment under Uncertainty." *American Economic Review* 73: 228–33.

Andrews, D.W.K. 1993. "Tests for Parameter Instability and Structural Change with Unknown Change Point." *Econometrica* 61: 821–56.

Blinder, A.S. 1998. *Central Banking in Theory and Practice*. Cambridge: MIT Press.

Bomfim, A.N. 1997. "The Equilibrium Fed Funds Rate and the Indicator Properties of Term-Structure Spreads." *Economic Inquiry* 35: 830–46.

———. 2001. "Measuring Equilibrium Real Interest Rates: What Can We Learn from Yields on Indexed Bonds?" Federal Reserve Board, Finance and Economics Discussion Series No. 53.

Brittan, S. 2001. "Alternatives to Inflation Targets." *Financial Times* (1 March).

Calvo, G.A. 1983. "Staggered Contracts in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12: 383–98.

Casares, M. and B. McCallum. 2000. "An Optimising IS-LM Framework with Endogenous Investment." NBER Working Paper Series, No. 7908.

Christiano, L., M. Eichenbaum, and C. Evans. 2001. "Nominal Price Rigidities and the Dynamic Effects of a Shock to Monetary Policy." Working Paper, Northwestern University.

Clarida, R., J. Galí, and M. Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37: 1661–1707.

Côté, D., J. Kuszczak, J-P. Lam, Y. Liu, and P. St-Amant. 2004. "The Performance and Robustness of Simple Monetary Policy Rules in Models of the Canadian Economy." *Canadian Journal of Economics*, forthcoming.

Dib, A. 2002. "Nominal Rigidities and Monetary Policy Rule in Canada Since 1981." Bank of Canada Working Paper No. 2002-25.

Duguay, P. 1994. "Empirical Evidence on the Strength of the Monetary Transmission Mechanism in Canada: An Aggregate Approach." *Journal of Monetary Economics* 33(1): 39–61.

Fuhrer, J. 2000. "Optimal Monetary Policy in a Model with Habit Formation." Federal Reserve Bank of Boston Working Paper No. 00-5.

Fuhrer, J.C. and G.R. Moore. 1995. "Inflation Persistence." *Quarterly Journal of Economics* 109: 127–59.

Gagnon, E. and H. Khan. 2001. "New Phillips Curve with Alternative Marginal Cost Measures for Canada, the United States, and the Euro Area." Bank of Canada Working Paper No. 2001-25.

Galí, J. and M. Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics* Vol. 44: 1237–70.

Johnson, M.H. and R.E. Keleher. 1996. *Monetary Policy, A Market Price Approach.* Westport, CT: Quorom Books.

Julius, D. 1998. House of Commons (U.K.) Select Committee on Treasury, Minutes of Evidence, Questions 300-319, Thursday 18 June.

Laubach, T. and J.C. Williams. 2003. "Measuring the Natural Rate of Interest." *The Review of Economics and Statistics* 85(4): 1063–70.

Meade, J. 1933. *The Rate of Interest in a Progressive State.* London: Macmillan.

Meyer, L. 2000. "The Economic Outlook and the Challenges Facing Monetary Policy." Remarks before the Toronto Association for Business and Economics, 12 April.

Neiss, K.S. and E. Nelson. 2003. "The Real Interest Rate Gap as an Inflation Indicator." *Macroeconomic Dynamics* 7(2): 239–62.

Ramsey, J.B. 1969. "Tests for Specification Errors in Classical Linear Least-Squares Analysis." *Journal of the Royal Statistical Society B* 31: 350–71.

Roberts, J. 1995. "New Keynesian Economics and the Phillips Curve." *Journal of Money, Credit and Banking* 27: 975–84.

Thornton, H. 1802. *An Enquiry Into the Nature and Effects of the Paper Credit of Great Britain.* Reprinted 1978. Fairfield, NJ: A.M. Kelley.

Tkacz, G. 2001. "Neural Network Forecasting of Canadian GDP Growth." *International Journal of Forecasting* 17: 57–69.

White, H. 1980. "A Heteroskedasticity-Consistent Covariance Matrix Estimator and Direct Test for Heteroskedasticity." *Econometrica* 48: 817–38.

Wicksell, K. 1898. *Interest and Prices: A Study of the Causes Regulating the Value of Money*. Translated by R.F. Kahn 1936, reprinted 1965. New York: A.M. Kelley.

Woodford, M. 2000. "A Neo-Wicksellian Framework for Analyzing Monetary Policy." Working Paper, Princeton University.

Yun, T. 1996. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics* 37: 345–70.

Table 1: Parameter Values for Benchmark Model

Parameters	Description	Value
β	Discount factor	0.99
δ	Depreciation rate	0.025
α	Share of labour	0.7
h	Habit persistence	0.6
μ	Steady-state markup	1.25
ψ	Adjustment cost parameter	0.2
η	Adjustment cost parameter	2
σ	Intertemporal elasticity of substitution	0.6
ρ_{ξ}	Autocorrelation of preference shock	0.875
$\sigma_{\varepsilon^{\xi}}$	Std deviation of preference shock	0.015
ρ_{A}	Autocorrelation of technology shock	0.9
$\sigma_{\varepsilon^{A}}$	Std deviation of technology shock	0.01

Table 2: Model Specification

Models	Parameters	Aggregate supply	Policy rule
Model 1	Baseline	Calvo	Forward looking
Model 2	Baseline except $\sigma = \frac{2}{3}$	Hybrid	Forward looking
Model 3	Baseline except $h = 0.8$	Hybrid	Simple rule
Model 4	Baseline except $h = 0.8$ and $\rho_{\xi} = 0.85$	Hybrid	Forward looking

Table 3: Basic Correlations using Artificial Data

Variables	Model 1	Model 2	Model 3	Model 4
$Corr(\pi_t, rgap_t)$	-0.985	-0.906	-0.546	-0.906
$Corr(\pi_t, rgap_{t-1})$	-0.322	-0.724	-0.148	-0.731
$Corr(\pi_t, rgap_{t-2})$	-0.087	-0.379	-0.030	-0.393
$Corr(\pi_t, rgap_{t-3})$	-0.030	-0.119	0.014	-0.139
$Corr(\pi_t, ygap_t)$	0.989	0.580	0.471	0.560
$Corr(\pi_t, ygap_{t-1})$	0.327	0.590	0.216	0.584
$Corr(\pi_t, ygap_{t-2})$	0.090	0.361	0.049	0.369
$Corr(\pi_t, ygap_{t-3})$	0.031	0.145	0.045	0.171
$Corr(rgap_t, ygap_t)$	-0.989	-0.861	-0.894	-0.842
$Corr(r_t^*, r_t)$	0.240	0.139	0.305	0.144
$Corr(r_t^*, rgap_t)$	0.955	0.979	0.938	0.983
$Corr(y_t^*, y_t)$	0.776	0.804	0.986	0.869
$Corr(y_t, ygap_t)$	0.615	0.580	0.159	0.482
$Corr(r_t, y_t)$	-0.638	-0.513	-0.165	-0.429

Table 4: Basic Statistics, 1984Q1 to 2002Q1

Variable	Mean	Std dev	Minimum	Maximum	Latest value (2002Q1)
Real 90-day rate (R90)	4.71	2.16	0.11	10.36	0.20
Real 10-year rate (R10)	5.64	1.43	3.17	9.03	4.01
R_1^*	3.16	1.34	1.00	6.51	1.70
R_2^*	3.61	1.52	1.18	7.49	1.98
R_3^*	2.95	1.24	0.90	6.29	1.59
R_4^*	2.31	0.97	0.70	4.95	1.24
$R10 - R90$	0.93	1.47	-2.55	3.82	3.81
$R_1^* - R90$	-1.55	1.39	-4.82	1.88	1.50
$R_2^* - R90$	-1.10	1.44	-4.13	2.29	1.79
$R_3^* - R90$	-1.76	1.57	-5.37	2.02	1.39
$R_4^* - R90$	-2.39	1.64	-6.44	1.56	1.05
Annual output growth $(\dot{Y})$	2.95	2.14	-3.47	6.34	0.30
Annual inflation (π)	2.66	1.15	1.06	5.15	1.99

Note: Sample for output growth ends in 2001Q4.

Table 5: In-Sample Estimates, Output Growth, 1985Q1 to 2001Q4

Explanatory variables (lagged 4 quarters)	Estimated parameters (*t*-stat)	R^2	Breusch-Godfrey serial correlation	White (1980) Heterosced-asticity	Andrews (1993) parameter stability	Ramsey (1969) non-linearity RESET (2, 3, and 4)
Constant $R10-R90$	2.09 (4.30) 0.81 (3.17)	0.304	44.8*	1.06	8.27 (95Q1)	10.5* 5.26* 3.53*
Constant R_1^*-R90	4.13 (8.01) 0.79 (2.41)	0.232	47.9*	0.56	9.45 (98Q3)	3.07 5.75* 3.84*
Constant R_2^*-R90	3.59 (7.76) 0.65 (1.88)	0.165	51.0*	0.46	9.15 (98Q2)	4.20* 4.74* 3.13*
Constant R_3^*-R90	4.23 (8.38) 0.76 (2.72)	0.277	47.5*	0.49	9.53 (98Q3)	5.10* 9.21* 6.04*
Constant R_4^*-R90	4.85 (8.59) 0.80 (3.27)	0.339	44.8*	0.44	8.56 (98Q3)	3.98 6.69* 4.39*
Constant $R10-R90$ $(R10-R90)^2$	2.53 (5.39) 1.06 (5.25) -0.23 (-2.64)	0.392	39.8*	0.17	3.00 (95Q4)	0.21 0.42 1.29
Constant R_1^*-R90 $(R_1^*-R90)^2$	3.69 (6.04) -0.02 (-0.03) -0.21 (-1.36)	0.255	45.3	4.08*	5.61 (98Q3)	7.23* 4.44* 2.92*
Constant R_2^*-R90 $(R_2^*-R90)^2$	3.56 (7.53) -0.04 (-0.07) -0.25 (-1.47)	0.203	47.3*	2.50	6.06 (98Q3)	4.89* 2.42 1.59
Constant R_3^*-R90 $(R_3^*-R90)^2$	3.71 (6.31) -0.11 (-0.21) -0.193 (-1.74)	0.319	45.5*	6.11*	5.66 (88Q4)	11.68* 6.52* 4.34*
Constant R_4^*-R90 $(R_4^*-R90)^2$	3.94 (6.39) -0.04 (-0.09) -0.14 (-1.68)	0.368	43.5*	4.39*	7.26 (88Q4)	8.70* 4.95* 3.75*

Table 6: Out-of-Sample Forecasts, Output Growth, 1993Q4 to 2001Q4

Explanatory variables (lagged 4 quarters)	Root mean squared error	Mean absolute error	Confusion rate
Constant $R10 - R90$	1.91	1.43	0.41
Constant $R_1^* - R90$	1.67	1.24	0.34
Constant $R_2^* - R90$	1.77	1.31	0.34
Constant $R_3^* - R90$	1.62	1.19	0.38
Constant $R_4^* - R90$	1.53	1.18	0.38
Constant $R10 - R90$ $(R10 - R90)^2$	1.74	1.44	0.69
Constant $R_1^* - R90$ $(R_1^* - R90)^2$	1.68	1.32	0.50
Constant $R_2^* - R90$ $(R_2^* - R90)^2$	1.80	1.43	0.56
Constant $R_3^* - R90$ $(R_3^* - R90)^2$	1.65	1.29	0.56
Constant $R_4^* - R90$ $(R_4^* - R90)^2$	1.53	1.19	0.44

Table 7: In-Sample Estimates, Inflation, 1985Q1 to 2001Q4

Explanatory variables (lagged 4 quarters)	Estimated parameters (*t*-stat)	R^2	Breusch-Godfrey serial correlation	White (1980) heteroscedasticity	Andrews (1993) parameter stability	Ramsey (1969) non-linearity RESET(2, 3, and 4)
Constant $R10-R90$	2.77 (9.89) -0.21 (-2.56)	0.053	59.0*	0.27	169.2* (91Q3)	0.27 1.09 0.73
Constant R_1^*-R90	2.39 (6.17) -0.12 (-0.92)	0.004	60.2*	0.06	222.1* (91Q3)	0.06 0.05 0.04
Constant R_2^*-R90	2.51 (7.24) -0.07 (-0.49)	-0.008	60.4*	0.00	229.0* (91Q3)	0.00 0.11 0.19
Constant R_3^*-R90	2.35 (6.19) -0.13 (-1.18)	0.013	60.2*	0.06	229.2* (91Q3)	0.06 0.24 0.17
Constant R_4^*-R90	2.14 (5.27) -0.18 (-1.86)	0.043	59.7*	0.25	223.9* (91Q3)	0.25 0.13 0.10
Constant $R10-R90$ *Dummy*	3.91 (22.4) 0.07 (1.28) -2.22 (-10.5)	0.839	29.4*	1.42	6.66 (89Q3)	2.88 7.18* 5.45*
Constant R_1^*-R90 *Dummy*	4.16 (22.4) 0.11 (2.26) -2.23 (-11.57)	0.847	28.3*	6.47*	5.50 (91Q3)	10.9* 7.41* 5.37*
Constant R_2^*-R90 *Dummy*	4.11 (24.2) 0.12 (2.47) -2.22 (-11.8)	0.850	28.2*	6.85*	5.61 (91Q3)	10.3* 6.66* 5.49*
Constant R_3^*-R90 *Dummy*	4.16 (21.0) 0.09 (1.99) -2.23 (-11.3)	0.846	28.3*	8.43*	6.60 (91Q3)	14.3* 9.85* 7.33*
Constant R_4^*-R90 *Dummy*	4.19 (18.5) 0.08 (1.69) -2.24 (-11.0)	0.843	28.9*	9.58*	7.22 (91Q3)	17.4* 11.5* 8.38*

Table 8: Out-of-Sample Forecasts, Inflation, 1993Q4 to 2001Q4

Explanatory variables (lagged 4 quarters)	Root mean squared error	Mean absolute error	Confusion rate
Constant $R10-R90$	1.36	1.28	0.47
Constant R_1^*-R90	1.38	1.32	0.53
Constant R_2^*-R90	1.40	1.34	0.53
Constant R_3^*-R90	1.38	1.31	0.50
Constant R_4^*-R90	1.36	1.29	0.53
Constant $R10-R90$ *Dummy*	0.44	0.36	0.53
Constant R_1^*-R90 *Dummy*	0.45	0.37	0.53
Constant R_2^*-R90 *Dummy*	0.44	0.36	0.56
Constant R_3^*-R90 *Dummy*	0.46	0.38	0.56
Constant R_4^*-R90 *Dummy*	0.48	0.39	0.53

Figure 1:output gap – preference shock

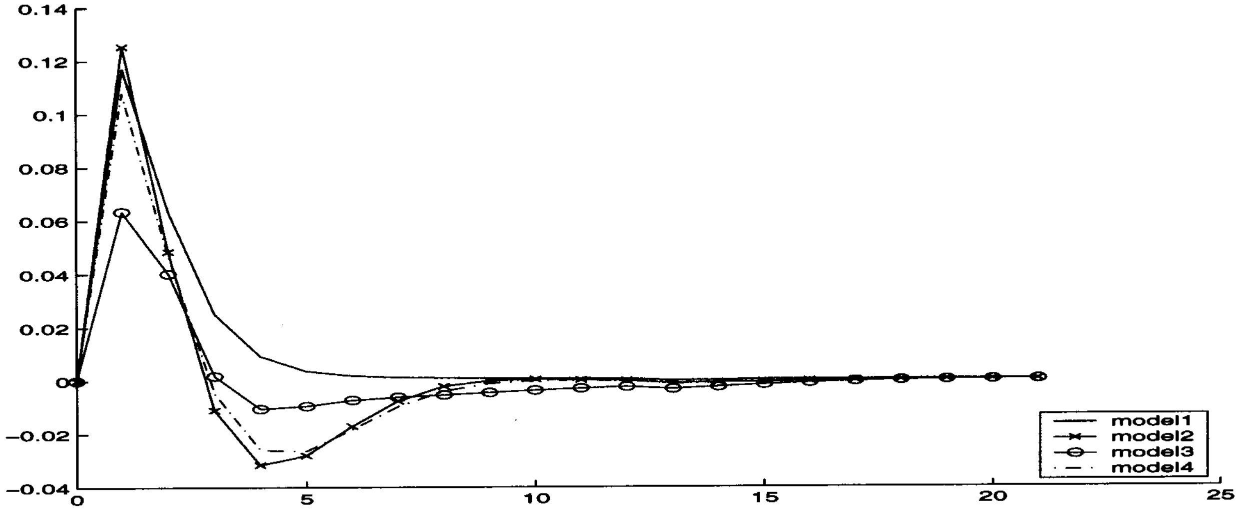


Figure 2:rgap – preference shock

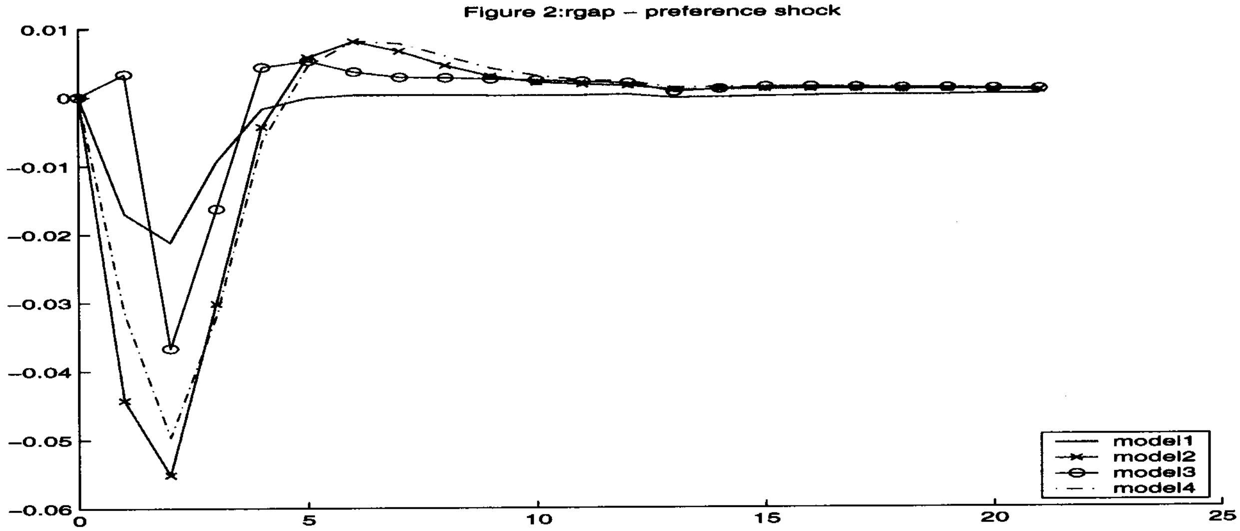
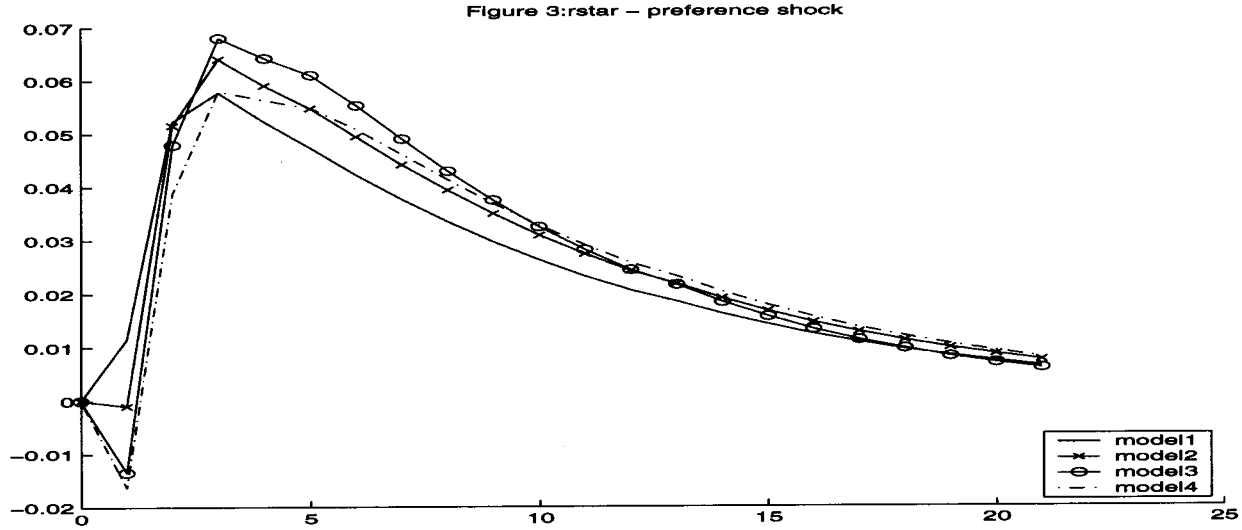


Figure 3:rstar – preference shock



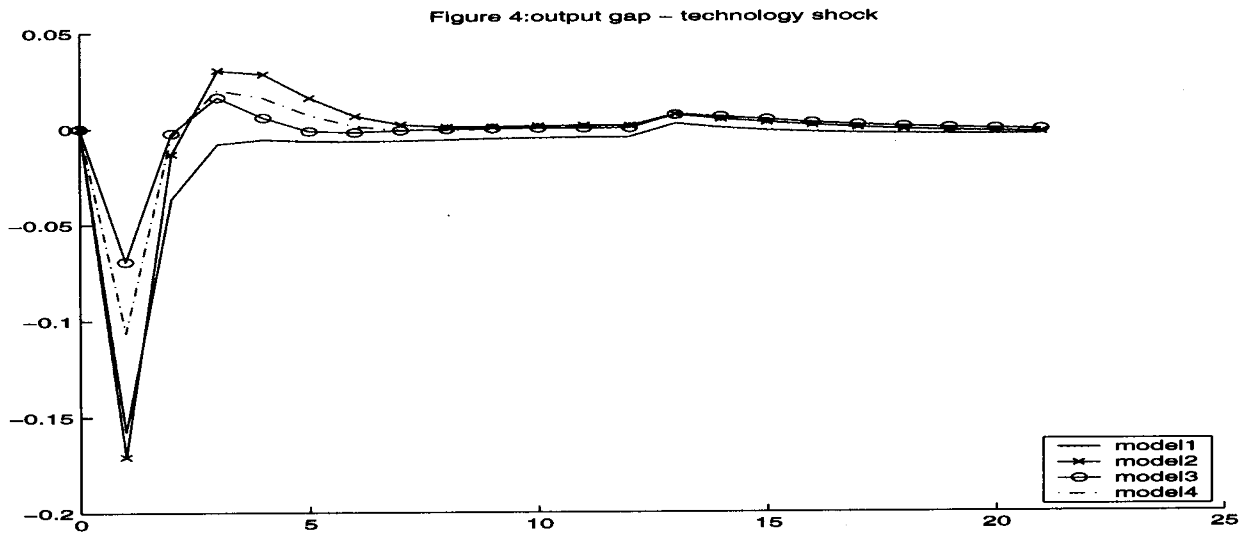


Figure 4:output gap – technology shock
0.05
0
−0.05
−0.1
−0.15
−0.2
0
5
10
15
20
25
model1
model2
model3
model4

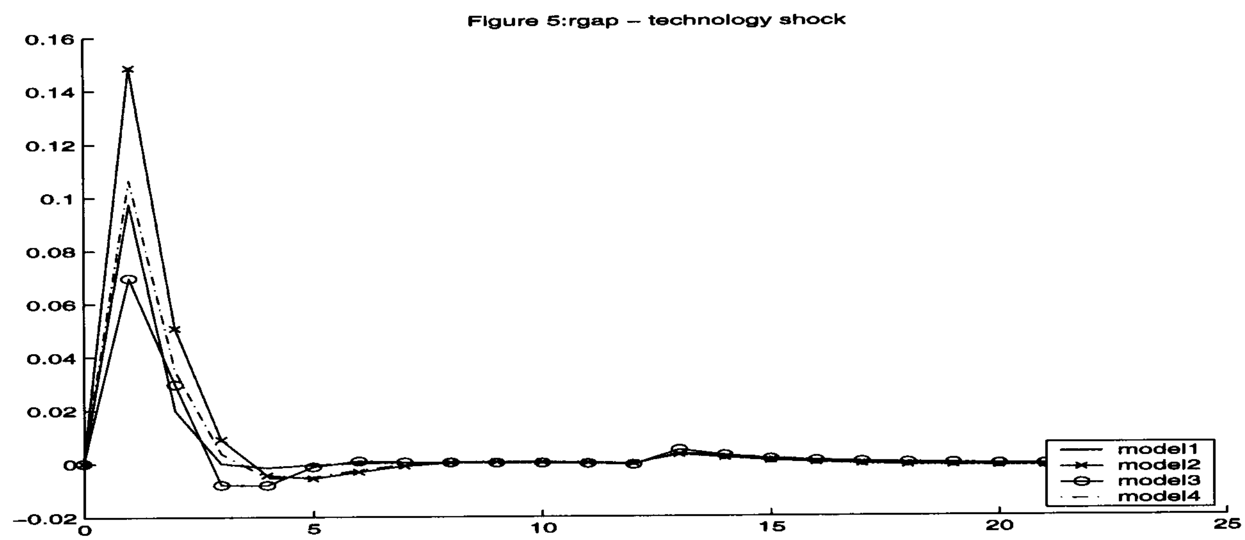


Figure 5:rgap – technology shock
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
−0.02
0
5
10
15
20
25
model1
model2
model3
model4

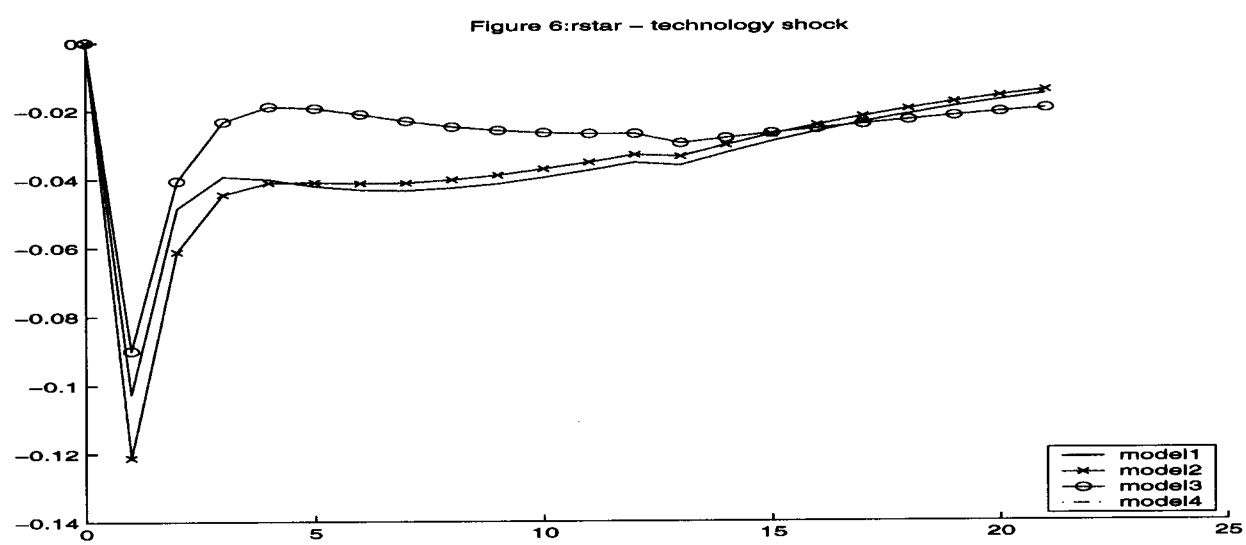


Figure 6:rstar – technology shock
0
−0.02
−0.04
−0.06
−0.08
−0.1
−0.12
−0.14
0
5
10
15
20
25
model1
model2
model3
model4

Figure 7: Yield Spread and Policy Stance Measures, 1984Q1 to 2002Q1

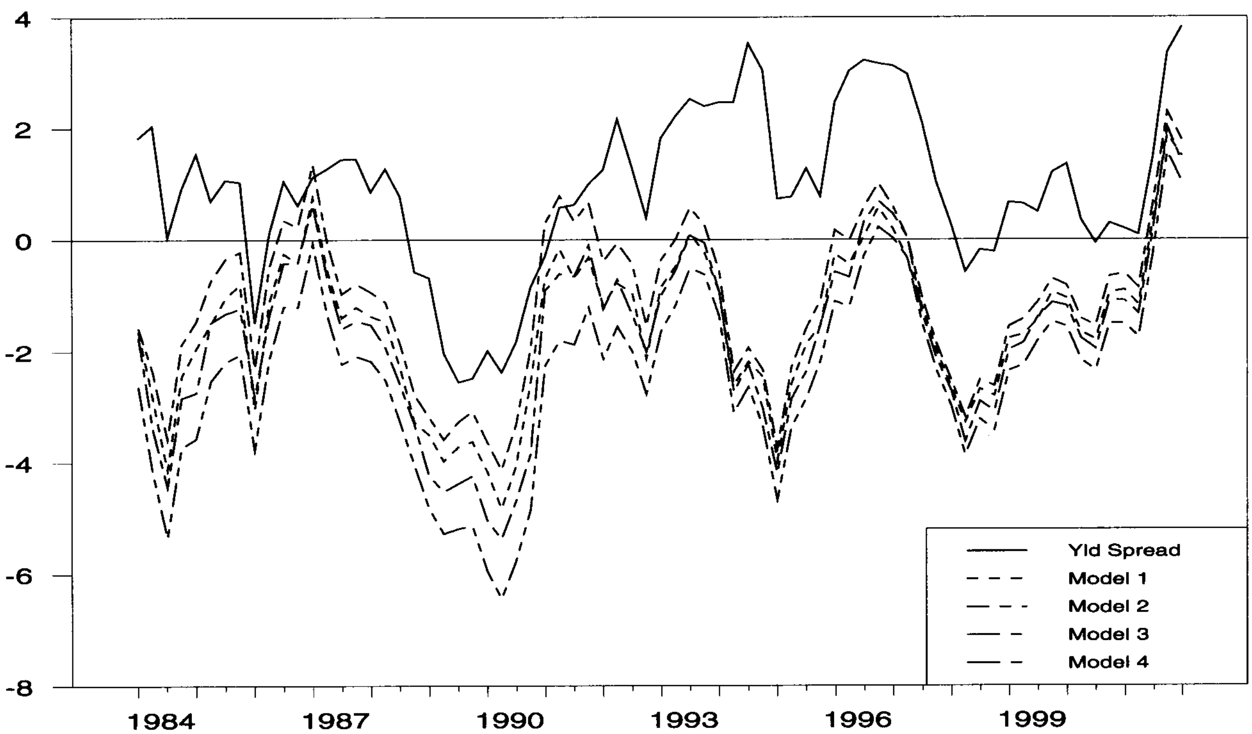


Figure 8: Actual and Fitted Output, 1985Q1 to 2001Q4

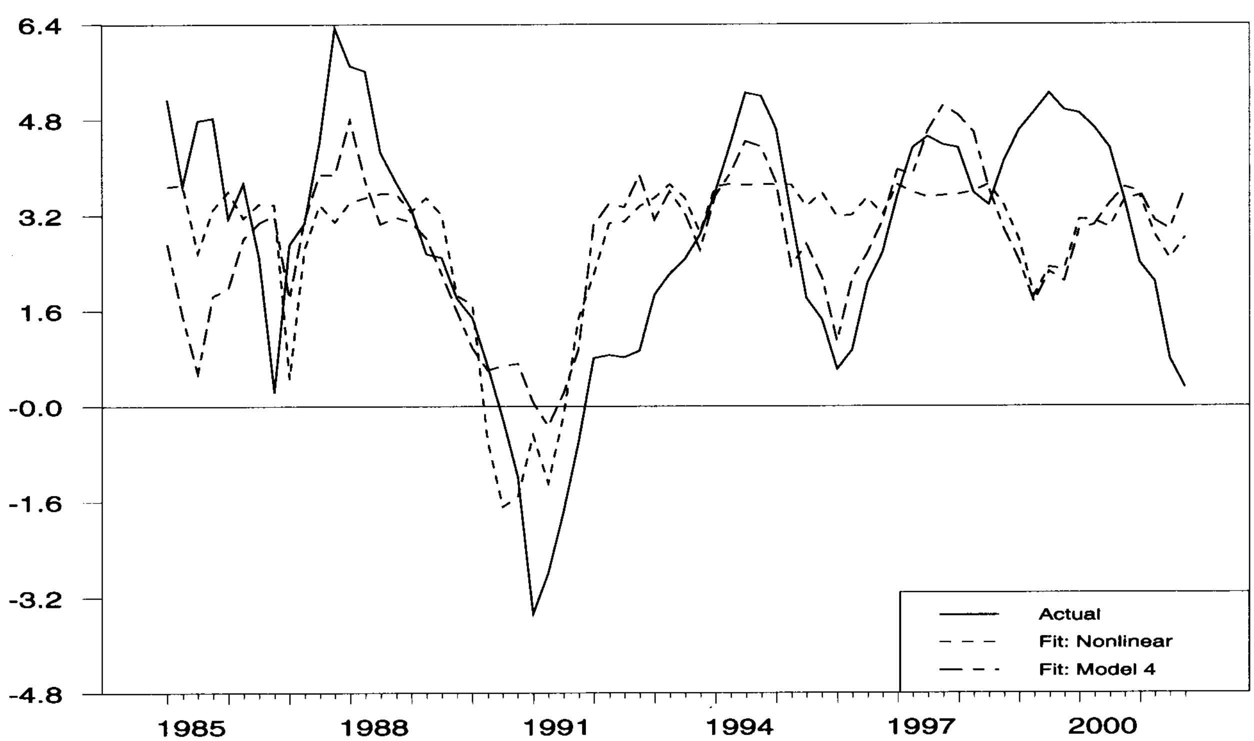


Figure 9: Actual and Forecasted Output (Model 4), 1993Q4 to 2002Q4

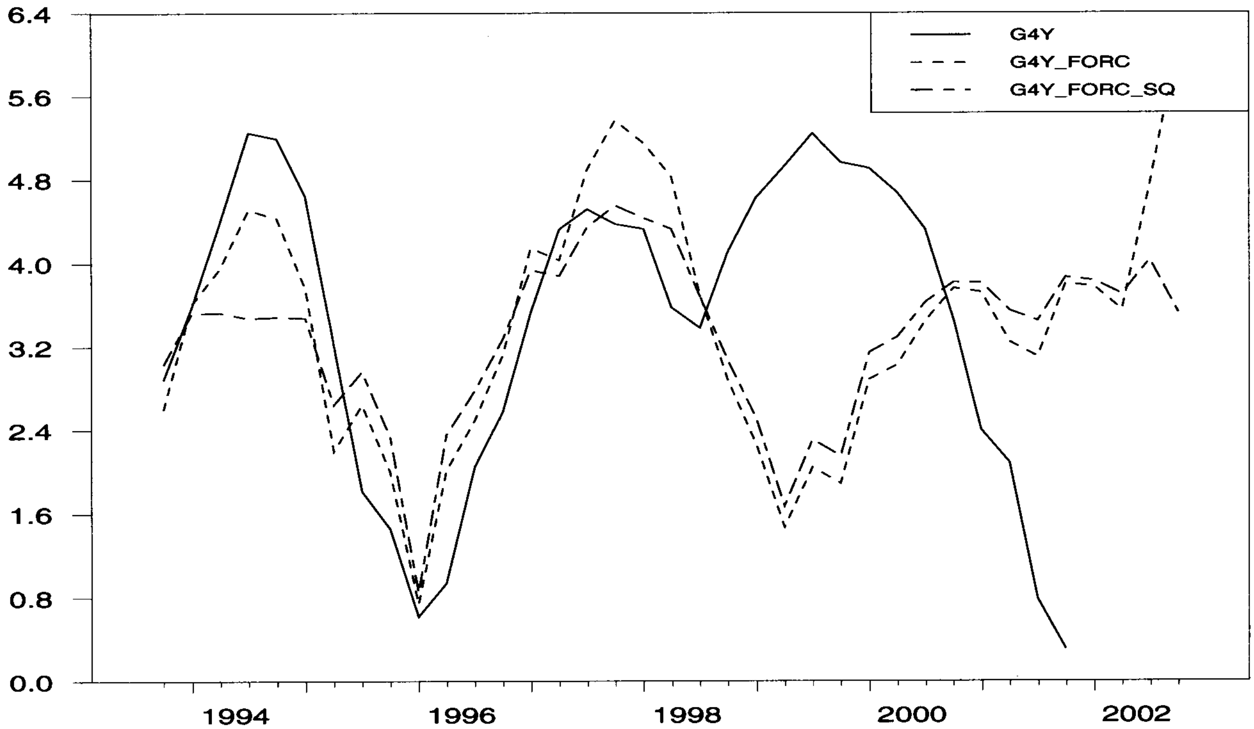


Figure 10: Actual and Fitted Inflation, 1985Q1 to 2001Q4

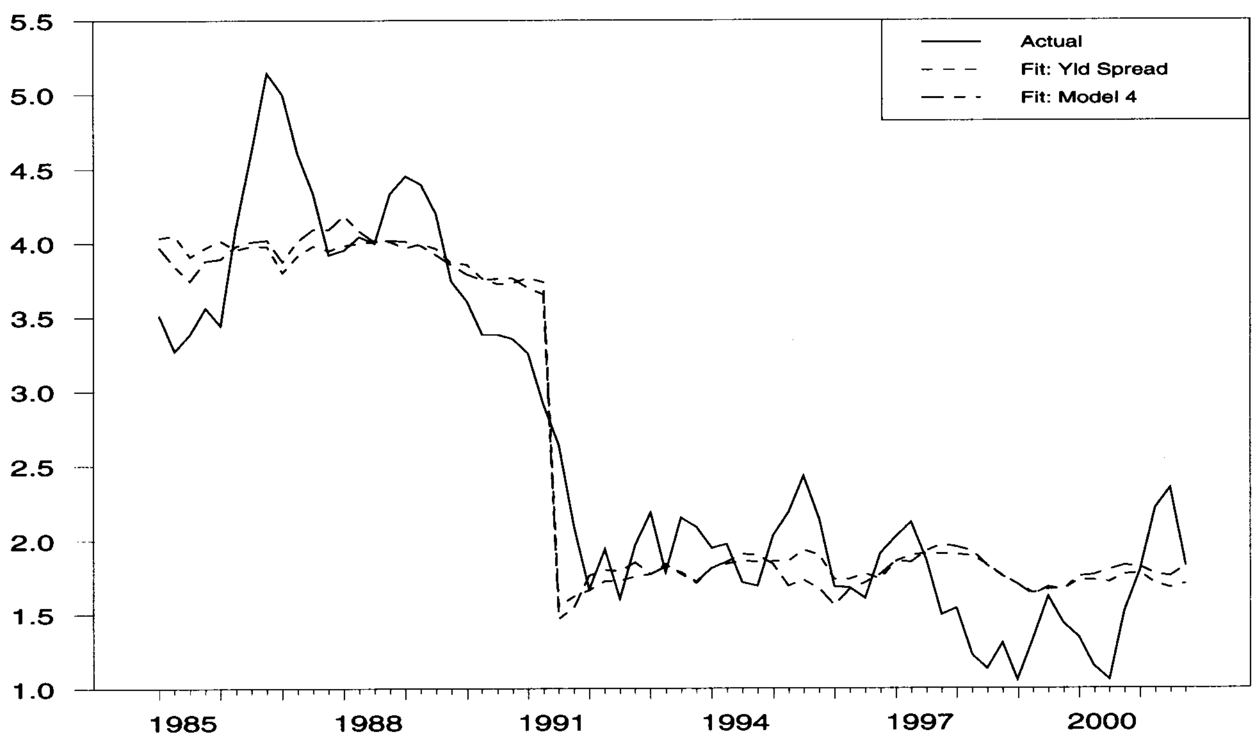
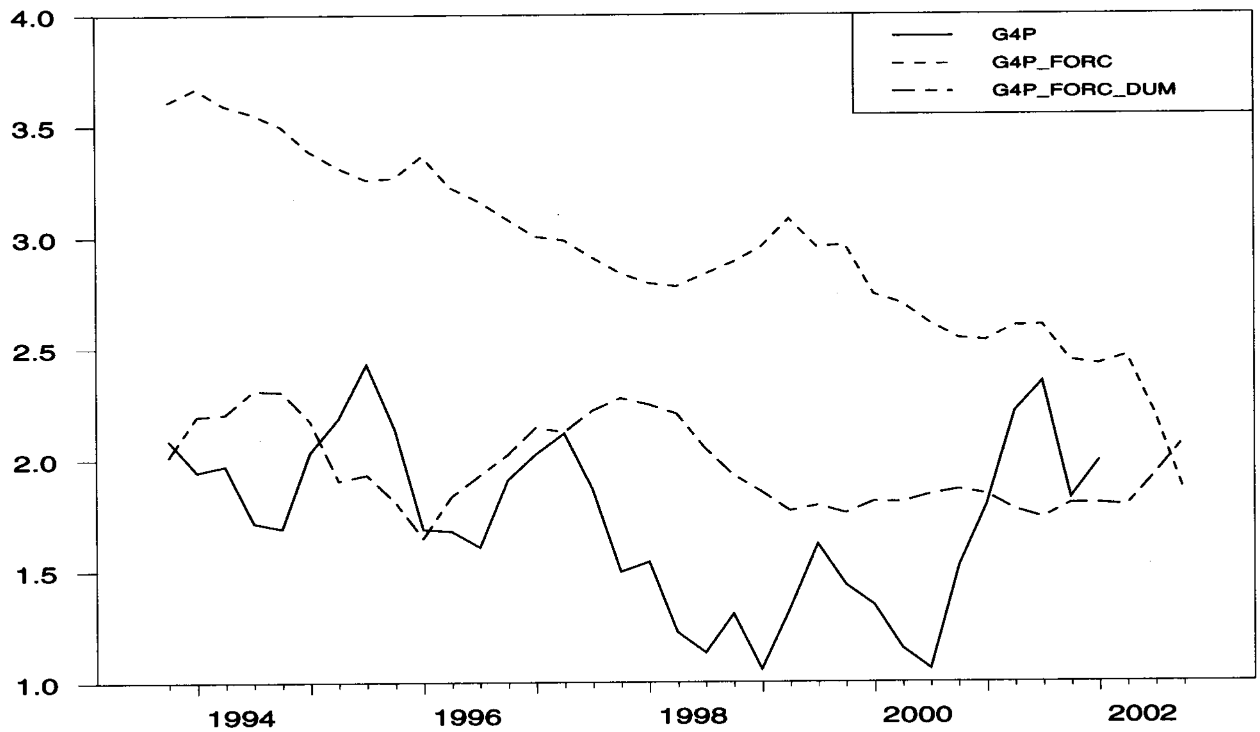


Figure 11: Actual and Forecasted Inflation (Model 4), 1993Q4 to 2002Q4



Appendix A: Log-Linearized Conditions

The log-linear model is given by the equations below. The lower case letters are the log deviations from their steady-state values.

Consumption

$$\left[\frac{1-\beta h\rho_{\xi}}{1-\beta h}\right]\xi_t - \left[\frac{1}{\sigma(1-\beta h)}\right]\left[h(\sigma-1)c_{t-1} - (\beta h(\sigma-1))E_t c_{t+1} + \left(\beta h^2(\sigma-1)+\beta h\sigma - 1\right)c_t\right] - \lambda_t = 0 \quad \text{(A1)}$$

Employment

$$y_t - n_t = \mu_t - \lambda_t \quad \text{(A2)}$$

Money demand

$$-\frac{1}{\gamma_m} - \tilde{m}_t - \left[\frac{1}{\gamma_m}\frac{R_t}{R^{ss}}\right] = 0 \quad \text{(A3)}$$

Euler equation

$$E_t\lambda_{t+1} - \lambda_t - r_t = 0 \quad \text{(A4)}$$

Fisher equation

$$r_t = R_t - \mathrm{E}_t(p_{t+1} - p_t) \quad \text{(A5)}$$

Production function

$$y_t = a_t + \alpha n_t + (1-\alpha)k_t \quad \text{(A6)}$$

Resource constraint

$$y_t = \frac{\overline{C}}{\overline{Y}}c_t + \left(\frac{\bar{I} + \psi\eta\bar{I}^{\eta}}{Y}\right)i_t \quad \text{(A7)}$$

Capital law of motion

$$k_{t+1} = i_t + (1-\delta)k_t \tag{A8}$$

Calvo pricing

$$\pi_t = \beta E_t(\pi_{t+1}) + \varphi mc_t \tag{A9}$$

Capital

$$i_t = (1-\delta)E_t(I_{t+1}) + E_t\left[\left(\frac{1}{(\eta-1)\psi\eta\bar{I}^{\eta-1}}\right)\left(\left(\frac{(1-\alpha)\bar{Y}}{\bar{K}}\right)(y_{t+1} - k_{t+1} - mc_{t+1}) + r_t\right)\right] \tag{A10}$$

Appendix B: Derivation of Preference Shocks

From (A1), we have

$$\upsilon_1 \xi_t = \upsilon_2 c_{t-1} + \upsilon_3 E_t c_{t+1} - \upsilon_4 c_t + \upsilon_5 \lambda_t, \tag{B1}$$

where $\upsilon_1 = \sigma(1-\beta h \rho_\xi), \upsilon_2 = h(\sigma - 1), \upsilon_3 = \beta h(\sigma - 1), \upsilon_4 = \beta h^2(\sigma - 1) + \beta h \sigma - 1$, and

$\upsilon_5 = \sigma(1 - \beta h)$.

Writing the above equation forward one period and taking expectations at time *t*, we have

$$\upsilon_1 E_t \xi_{t+1} = \upsilon_2 c_t + \upsilon_3 E_t c_{t+2} - \upsilon_4 E_t c_{t+1} + \upsilon_5 E_t \lambda_{t+1}. \tag{B2}$$

Subtracting (B1) from (B2), and using equations (23) and (28) in the text, one obtains

$$\upsilon_1 (\rho_\xi - 1)\xi_t = \upsilon_2 \Delta c_t + \upsilon_3 E_t \Delta c_{t+2} - \upsilon_4 E_t \Delta c_t + \upsilon_5 (E_t \pi_{t+1} - R_t), \tag{B3}$$

which is equation (32) in the text with $\upsilon_6 = \upsilon_1(\rho_\xi - 1)$.

Bank of Canada Working Papers
Documents de travail de la Banque du Canada

Working papers are generally published in the language of the author, with an abstract in both official languages. ***Les documents de travail sont publiés généralement dans la langue utilisée par les auteurs; ils sont cependant précédés d'un résumé bilingue*.**

2004

2004-8	The Economic Theory of Retail Pricing: A Survey	O. Secrieru
2004-7	The Demand for Money in a Stochastic Environment	J. Atta-Mensah
2004-6	Bank Capital, Agency Costs, and Monetary Policy	C. Meh and K. Moran
2004-5	Structural Change and Forecasting Long-Run Energy Prices	J.-T. Bernard, L. Khalaf, and M. Kichian
2004-4	A Structural Small Open-Economy Model for Canada	S. Murchison, A. Rennison, and Z. Zhu
2004-3	Modélisation << PAC >> du secteur extérieur de l'économie américaine	M.-A. Gosselin and R. Lalonde
2004-2	Exact Tests of Equal Forecast Accuracy with an Application to the Term Structure of Interest Rates	R. Luger
2004-1	The Effect of Adjustment Costs and Organizational Change on Productivity in Canada: Evidence from Aggregate Data	D. Leung

2003

2003-44	Common Trends and Common Cycles in Canadian Sectoral Output	F. Barillas and C. Schleicher
2003-43	Why Does Private Consumption Rise After a Government Spending Shock?	H. Bouakez and N. Rebei
2003-42	A Structural VAR Approach to the Intertemporal Model of the Current Account	T. Kano
2003-41	Anatomy of a Twin Crisis	R.H. Solomon
2003-40	Poignée de main invisible et persistance des cycles économiques : une revue de la littérature	C. Calmès
2003-39	Alternative Targeting Regimes, Transmission Lags, and the Exchange Rate Channel	J.-P. Lam
2003-38	Simple Monetary Policy Rules in an Open-Economy, Limited-Participation Model	S. Hendry, W-M. Ho, and K. Moran

Copies and a complete list of working papers are available from:
Pour obtenir des exemplaires et une liste complète des documents de travail, prière de s'adresser à :

Publications Distribution, Bank of Canada
234 Wellington Street, Ottawa, Ontario K1A 0G9
E-mail: publications@bankofcanada.ca
Web site: http://www.bankofcanada.ca

Diffusion des publications, Banque du Canada
234, rue Wellington, Ottawa (Ontario) K1A 0G9
Adresse électronique : publications@banqueducanada.ca
Site Web : http://www.banqueducanada.ca