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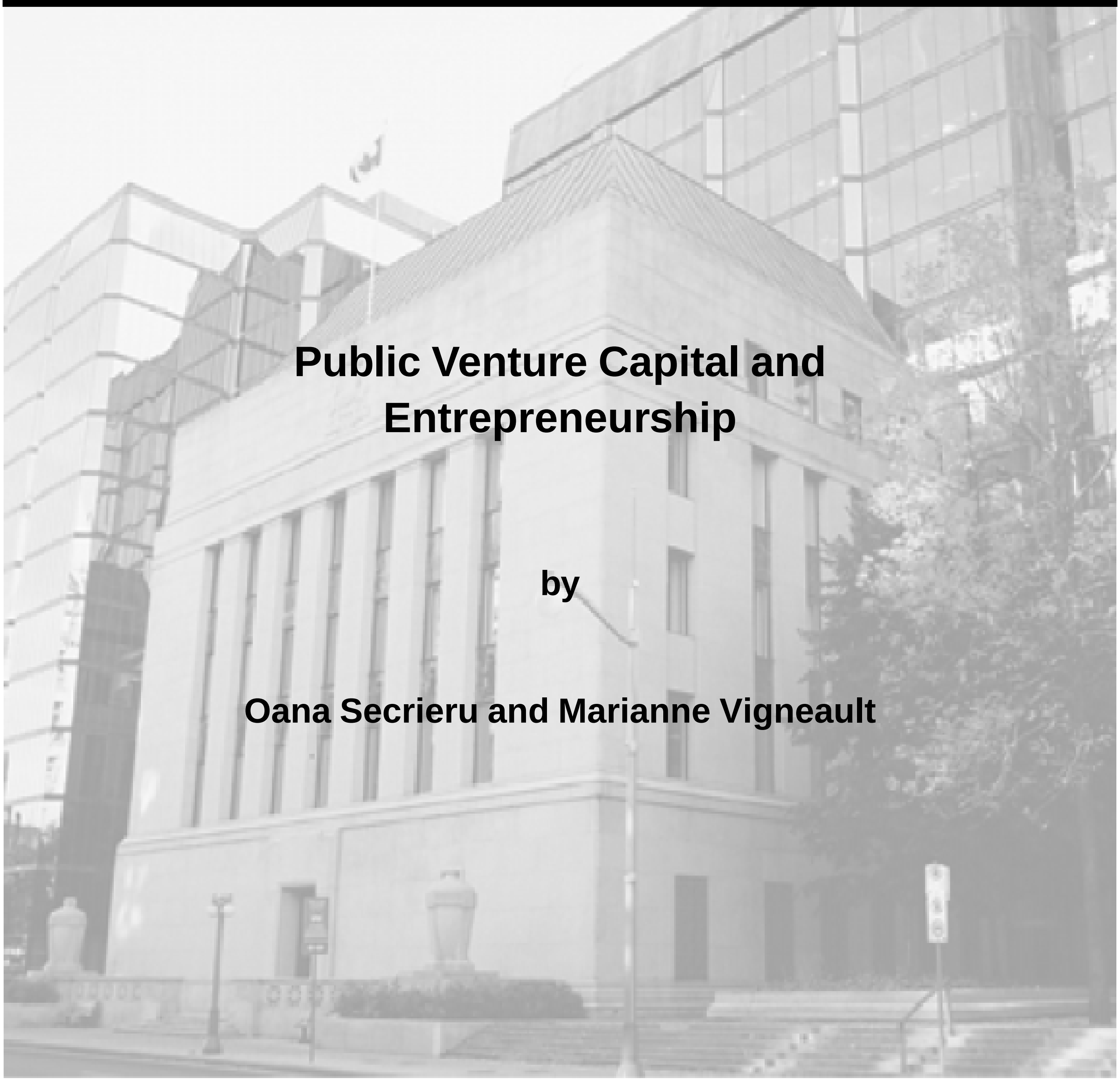

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Abstract

Entrepreneurship is a key factor in promoting growth in output and employment. Consequently, to encourage new start-ups, most governments in developed countries have public venture capital programs. The authors develop a model that endogenously determines the number of entrepreneurs and the optimal quantity of financing and managerial advice provided by a public venture capital program. Their analysis is based on a model of occupational choice that has informational asymmetries regarding the ability of entrepreneurs. The authors identify circumstances under which over- or underinvestment can occur. They also show that the equilibrium is characterized by an inefficient number (too many or too few) of less-able entrepreneurs. Furthermore, the authors find that the government faces disincentives in providing small amounts of managerial advice; larger amounts of such advice may be optimal.

Note: This study is based on a hypothetical model. The authors’ theoretical findings relate to public venture capital programs in general and not to the Business Development Bank of Canada.

JEL classification: D28, G24, G28, J24, M13
Bank classification: Financial markets; Fiscal policy; Labour markets

Résumé

L’entrepreneuriat est un ingrédient clé de la croissance de la production et de l’emploi. C’est pourquoi la plupart des États industrialisés possèdent des programmes conçus pour faciliter l’accès des jeunes entreprises au capital de risque. Les auteures ont mis au point un modèle qui détermine de manière endogène le nombre d’entrepreneurs ainsi que la quantité de financement et de conseils managériaux fournie optimalement par un programme public d’accès au capital de risque. Leur analyse repose sur un modèle de choix professionnel où la compétence des entrepreneurs pose un problème d’asymétrie d’information. Les auteures établissent quelles conditions peuvent donner lieu à un surinvestissement ou, au contraire, à un sous-investissement. Elles montrent par ailleurs que l’équilibre se caractérise par la présence en nombre inefficient (trop grand ou trop faible) d’entrepreneurs de moindre compétence. Elles constatent enfin que l’intervention des autorités publiques a pour effet de réduire le bien-être si elle ne s’accompagne que d’une quantité limitée de conseils managériaux; il pourrait donc être optimal d’offrir davantage de conseils aux entreprises.

Note : L’étude est fondée sur un modèle hypothétique. Les résultats théoriques obtenus par les auteurs se rapportent aux programmes publics de capital de risque en général, et non à ceux de la Banque de développement du Canada.

Classification JEL : D28, G24, G28, J24, M13
Classification de la Banque : Marchés financiers; Politique budgétaire; Marchés du travail

1. Introduction

Most governments recognize the fact that entrepreneurs play an important role in creating employment and promoting growth in output and productivity (Audretsch and Thurik 2001). As a result, a large number of government programs are aimed at encouraging the development of new businesses.

In this paper, we investigate the effects of a government venture capital program on entrepreneurship, the allocation of capital, and managerial advice. The government venture capital program consists of loans to new entrepreneurs along with business advice. The government runs the program through a government bank, which we call the Business Development Bank (BDB). Our main result shows that the government program cannot induce the optimal number of entrepreneurs, because when individuals are free to choose their occupation, they do so without taking into account the effect of their choice on the welfare of other individuals—entrepreneurs and workers. This makes the occupational-choice equilibrium suboptimal compared with the case where the government can control access to occupations: the number of entrepreneurs is too low or too high. We also find that an increase in the amount of managerial advice, starting from a situation where no advice is provided, reduces welfare. This is somewhat surprising, because we would expect that a bit of advice is always beneficial. Providing a small amount of advice, however, has a very small impact on the probability of the success of a business, while it has a huge cost (providing advice is costly). Thus, a little bit of advice can be more detrimental for welfare than no advice at all. We also show that larger amounts of managerial advice are optimal, which implies that governments should not interfere in business decisions beyond the provision of capital, unless they have enough expertise to provide the right level of advice. With respect to capital allocation, we find that the optimal allocation involves either over- or underinvestment relative to the profit-maximizing allocation, as a result of the redistributive motive of the government.

Economic theory provides three rationales for government intervention in the supply of entrepreneurship. The first rationale is the positive externality created through research and development (R&D), which serves to make the social rate of return on R&D

expenditures exceed the private rate of return by a considerable amount (see, for example, Griliches 1992). The second rationale stems from the empirical evidence of firm formation, which shows that entrepreneurs are liquidity-constrained. For example, Evans and Jovanovic (1989) find that most individuals who enter self-employment face a binding liquidity constraint and therefore use a suboptimal level of capital to start up their business. Other studies find that the probability of becoming an entrepreneur increases with the size of assets held by the individual (Evans and Leighton 1989), and depends positively on whether the individual ever received an inheritance or gift (Blanchflower and Oswald 1998). Furthermore, Holtz-Eakin, Joulfain, and Rosen (1994) find that liquidity constraints are not only important for entry into entrepreneurship, but are also important in determining the likelihood of entrepreneurial failure: the probability of enterprise survival increases with the size of an inheritance. Recent studies have shown that liquidity constraints limit many forms of business investment and investment in R&D (Hall 1992, Hao and Jaffe 1993, Himmelberg and Petersen 1994, and Hubbard 1998).

Liquidity constraints are caused by informational asymmetries that result from adverse selection and moral hazard problems. Adverse selection problems arise when an entrepreneur is better informed about their ability and, thus, their probability of success, than are outside investors. Moral hazard problems arise when entrepreneurial effort is unobservable by outside investors. Informational asymmetries can thus make external capital more expensive than internally generated capital.[1] Furthermore, because the wealth of potential entrepreneurs is limited, they require substantial outside financing. The lack of collateral and a track record make it difficult for new entrepreneurs to obtain bank financing. The dominant form of external financing is thus venture capital. Venture capitalists provide both financial assistance and managerial expertise, and attempt to address informational asymmetries by extensively scrutinizing and monitoring entrepreneurial projects. Informational asymmetries are not completely eliminated, however.

The third rationale for government intervention in the supply of entrepreneurship is

[1] See Jensen and Meckling (1976), Myers and Majluf (1984), Greenwald, Stiglitz, and Weiss (1984), and Stiglitz and Weiss (1981) for in-depth analyses of the types of problems that arise from asymmetric information between entrepreneurs and outside investors.

the fact that public venture capital programs can play a role in certifying new firms to outside investors (Lerner 2002). This is one way to overcome the informational asymmetries described above. The idea is that government programs can identify and support the creation of new firms in industries that do not attract private venture capital (for example, technology-intensive industries). According to financial theory, this failure to attract capital might be due to a type of "herding" behaviour; i.e., private venture capitalists herding themselves into particular industries. Government certification of promising firms might shift some private venture capital into these neglected areas. This rationale is not based on the assumption that the government has some advantage over the private sector in certifying new firms; rather, it recognizes the fact that the private sector is not willing to assume the certification role, owing to possible free-riding problems. This view is consistent with evidence that private venture capital focuses more on the later stages of a firm's growth and development than on the early stages of a start-up (Amit, Brander, and Zott 1997).

Although most of the literature focuses on private venture capital, a considerable proportion of capital is publicly financed (see, for example, OECD 1996). Vaillancourt (1997) shows that 44 per cent of the stock of venture capital in Canada in 1994 was in the form of public funds (funds financed by the government, or funds that benefited from tax incentives).[2] The U.S. Small Business Administration provides financing to start-up businesses.[3] The governments of Great Britain, France, Belgium, and the Netherlands also have financial programs, to assist unemployed workers who start businesses (Bendick and Egan 1987, OECD 1996 and 1997).

Despite the importance of public venture capital programs, economic analysis of them has largely been ignored.[4] In this paper, we consider liquidity constraints to be exogenous and develop a model that endogenously determines the optimal quantity of financing and managerial advice provided by the public venture capital program. Optimality is achieved

[2] This was up from 15 per cent in 1989, 23 per cent in 1991, and 43 per cent in 1993.

[3] Lerner (1996) briefly describes the U.S. government programs towards entrepreneurship between 1958 and 1996.

[4] There is an extensive literature on private venture capital. See, for example, Kanniainen and Keuschnigg (2000 and 2001), and Kanniainen and Leppämäki (2002), Keuschnigg and Nielsen (2000).

by maximizing a utilitarian social welfare function, and thus involves redistribution from the more-able to the less-able entrepreneurs. The model assumes that the government faces the following two informational asymmetries: it is unable to observe the ability of entrepreneurs, and it is unable to observe entrepreneurial effort. Given these two constraints, the first-best allocation cannot be achieved, because the optimal contract involving financing and managerial advice must provide the right incentives for self-selection. Our model thus combines the literature on self-selection[5] with that on occupational choice.[6]

The rest of this paper is organized as follows. In section 2 we describe our model. Section 3 considers the case where the government has full information regarding entrepreneurs' abilities, and section 4 examines the asymmetric case. Section 5 provides concluding remarks.

2. The Model

2.1 Preferences

Individuals in this economy can become either entrepreneurs or workers. Workers are endowed with one unit of labour, which they supply inelastically. All workers are assumed to be equally productive. Entrepreneurs, however, differ in ability according to an ability parameter θ, with $\theta^2 > \theta^1$.[7] There are N^i individuals of ability type $i = 1, 2$, and $N = \Sigma_i N^i$ is the total population. The preferences of entrepreneurs and workers for income are identical and are given by the concave utility function $u(\cdot)$, $u''(\cdot) < 0 < u'(\cdot)$, $u(0) = 0$, which reflects the fact that individuals are risk-averse.

2.2 Production

Entrepreneurs of type i supply effort, e^i, hire labour, ℓ^i, and borrow capital, K^i, to produce a homogeneous good according to the production technology $F^i(\ell^i, K^i)$. The production function obeys the standard properties: $F^i_j > 0$, $F^i_{jj} < 0$, $F^i_{jj'} > 0$, $j, j' \in \{\ell, K\}$, $j \neq j'$.

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5 See, for example, Stiglitz (1982).

6 See, for example, Kihlstrom and Laffont (1979), Kanbur (1981), Boadway, Marchand, and Pestieau (1991), and Boadway et al. (1998).

7 Alternatively, we could have written the model with n discrete types of individuals. This would not change our results, but would make the notation more complicated.

Production is risky; the probability of success depends on the entrepreneurial input, defined as $\varepsilon^i = e^i\theta^i$, and the managerial advice from the government, a. We will come back to this shortly. Neither e^i nor θ^i is directly observable by the government, but capital, labour, and output are, and so, as a result, is entrepreneurial input, ε^i. Entrepreneurial effort is costly, with the cost function, $h(e^i)$, convex in effort, $h'(\cdot) > 0$, $h''(\cdot) > 0$.

2.3 The government's objective

As stated in the introduction, entrepreneurs who lack managerial experience (i.e., have no track record) and their own resources find it difficult to raise external financing. This provides the government with a rationale to intervene and finance entrepreneurial projects that would otherwise never be completed. The government sets up a public venture capital program, which is implemented by the BDB. (Throughout this paper, we refer to the BDB and the government interchangeably as the same entity.) In addition to financial assistance, the government provides managerial advice, which increases the entrepreneur's probability of success. The role of the BDB is thus threefold: to provide financing of entrepreneurial projects that are subject to market failure in capital markets, to provide managerial advice, and to redistribute from more-able to less-able entrepreneurs. This redistribution involves the maximization of a utilitarian social welfare function, as defined below.

We use a to denote the managerial input provided by the BDB, and we assume that the probability of success of a type i entrepreneur is a function of the entrepreneurial input $\varepsilon^i = e^i\theta^i$ and the managerial input a. For simplicity, we assume the following functional form for the probability of success: $p(\varepsilon^i, a) = \varepsilon^i p(a)$, where $\varepsilon^i \in [0,1]$, $p : I\!R_+ \to [0,1]$, $p(0) = 0$.

2.4 Sequence of decisions

Stage 1: The government offers financial contracts, (r^1, K^1) and (r^2, K^2), and managerial advice, a, to maximize a utilitarian social welfare function.

Stage 2: Individuals decide whether to become workers or entrepreneurs. Individuals who become entrepreneurs choose the financial contract designed for their type.

Stage 3: Entrepreneurs select the amount of labour to hire, and the entrepreneurial effort required to maximize their expected utility.

The equilibrium concept we employ is that of subgame perfect equilibrium. To solve for the equilibrium, we start, as usual, at the end of the game.

2.5 The entrepreneur's production decision

Assuming that the entrepreneur earns zero profits in the event of a failure, we can write the expected utility of a type i entrepreneur as

$$\varepsilon^i p(a) u\Big[F^i(\ell^i, K^i) - w\ell^i - (1+r^i)K^i - h\Big(\frac{\varepsilon^i}{\theta^i}\Big)\Big], \tag{1}$$

where we have written entrepreneurial income in terms of the variables that are directly observable by the government, with the exception of the entrepreneur's type, θ^i. We assume that, if entrepreneurs are not successful, they earn zero revenue. In this case, workers are laid off and receive no pay. Capital is assumed to be specific to the firm; if the project is not successful, entrepreneurs repay nothing to the bank. The entrepreneur chooses effort, labour, and capital to maximize the expected utility of income before the uncertainty is resolved. To examine this, we separate the entrepreneurs' problem into two stages, with capital chosen first, and effort and labour second. Solving backwards, an entrepreneur of type i chooses effort and labour so as to maximize (1). The first-order conditions

$$u^i = \varepsilon^i u^{i\prime} \frac{h'}{\theta^i}, \tag{2}$$

$$F^i_\ell(\ell^i, K^i) = w, \tag{3}$$

allow us to obtain the effort function $\varepsilon^i(w, r^i, K^i)$ and the labour-demand function $\ell^i(w, K^i)$, with $\varepsilon^i_w < 0$, $\varepsilon^i_K \gtreqless 0$, $\varepsilon^i_r < 0$, $\ell^i_w < 0$, and $\ell^i_K > 0$.[8]

Substituting $\varepsilon^i(w, r^i, K^i)$ and $\ell^i(w, K^i)$ into the entrepreneurs' objective function defines the indirect utility function $\Omega^i(a, w, r^i, K^i)$.

[8] Comparative statics on the first-order conditions (2) and (3) yield $\partial\varepsilon^i/\partial w = -(1/\Delta)(u^{i\prime} - \varepsilon^i u^{i\prime\prime}(h'/\theta^i))\ell^i < 0$, $\partial\ell^i/\partial w = 1/F^i_{\ell\ell} < 0$, $\partial\varepsilon^i/\partial K^i = (1/\Delta)(u^{i\prime} - \varepsilon^i u^{i\prime\prime}(h'/\theta^i))[F^i_K - (1+r^i)] \gtreqless 0$, $\partial\ell^i/\partial K^i = -F^i_{\ell K}/F^i_{\ell\ell} > 0$, $\partial\varepsilon^i/\partial r^i = -(1/\Delta)(u^{i\prime} - \varepsilon^i u^{i\prime\prime}(h'/\theta^i))K^i < 0$, where $\Delta \equiv 2u^{i\prime}(h'/\theta^i) - \varepsilon^i u^{i\prime\prime}((h'/\theta^i))^2 + \varepsilon^i u^{i\prime}(h''/(\theta^i)^2) > 0$.

Lemma 1 The indirect utility function, $\Omega^i(a, w, r^i, K^i)$, satisfies:

$$\frac{\partial \Omega^i}{\partial a} = \varepsilon^i p'(a) \cdot u^i > 0, \tag{4}$$

$$\frac{\partial \Omega^i}{\partial w} = -\varepsilon^i p(a) \cdot u^{i'} \cdot \ell^i < 0, \tag{5}$$

$$\frac{\partial \Omega^i}{\partial r^i} = -\varepsilon^i p(a) \cdot u^{i'} \cdot K^i < 0, \tag{6}$$

$$\frac{\partial \Omega^i}{\partial K^i} = \varepsilon^i p(a) \cdot u^{i'} \cdot [F_K^i - (1 + r^i)] \gtreqless 0. \tag{7}$$

For the analysis of the government's problem, it is helpful to construct entrepreneurs' indifference curves in $(K, r)-$space. The marginal rate of substitution between K and r can easily be shown to be:

$$-\frac{\partial \Omega^i}{\partial K^i} \Big/ \frac{\partial \Omega^i}{\partial r^i} = \left.\frac{dr^i}{dK^i}\right|_{\Omega^i} = \frac{[F_K^i - (1 + r^i)]}{K^i} \gtreqless 0 \quad \text{as} \quad F_K^i \gtreqless (1 + r^i). \tag{8}$$

Given (8), it follows that the indifference curves have the shape indicated in Figure 1. We assume throughout this paper that the indifference curves satisfy a single-crossing property. There are two scenarios for which a single-crossing property holds:

(i) $F_K^1 < F_K^2 \Leftrightarrow \left.\frac{dr^1}{dK^1}\right|_{\Omega^1} < \left.\frac{dr^2}{dK^2}\right|_{\Omega^2}, \forall (K, r)$;

(ii) $F_K^1 > F_K^2 \Leftrightarrow \left.\frac{dr^1}{dK^1}\right|_{\Omega^1} > \left.\frac{dr^2}{dK^2}\right|_{\Omega^2}, \forall (K, r)$.

The two possibilities show that indifference curves of different types can intersect on either (i) their increasing region, or (ii) their decreasing region. Figure 1(a) illustrates the case where single-crossing property (i) holds and Figure 1(b) illustrates the case where single-crossing property (ii) holds. As the figure indicates, property (i) implies that the more-able entrepreneurs have higher marginal rates of substitution between r and K for any given (K, r). This, in turn, implies that, for a given interest rate, more-able entrepreneurs prefer a higher level of capital. The opposite holds for single-crossing property (ii). Thus, differences in marginal rates of substitution provide a basis for self-selection.

Before discussing the government's problem in detail, we wish to emphasize that the government offers entrepreneurs (r^i, K^i) contracts that differ for the two types. If an entrepreneur were able to borrow as much capital as they would like at the interest rates offered by the government, the entrepreneur would select the level of capital that maximizes utility, or equivalently profits, taking as given the interest rate r^i, $i = 1, 2$. If the government had full information about entrepreneurs' types, it would charge entrepreneurs of different types different interest rates, so that $r^1 \neq r^2$. At the given interest rate r^i, a type i entrepreneur chooses K^i to solve the familiar first-order condition, $F_K^i = 1 + r^i$. From (8), this implies that $MRS^i_{r,K} = 0$ when profits are maximized. Such an outcome is depicted in Figures 1(a) and 1(b). From this point on, we will refer to the allocation of capital that would be chosen by the entrepreneurs if they could freely choose capital as the profit-maximizing allocation.

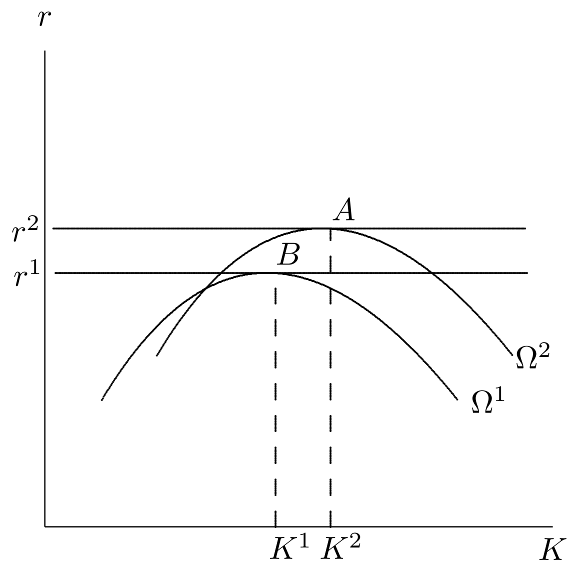

Figure 1(a)

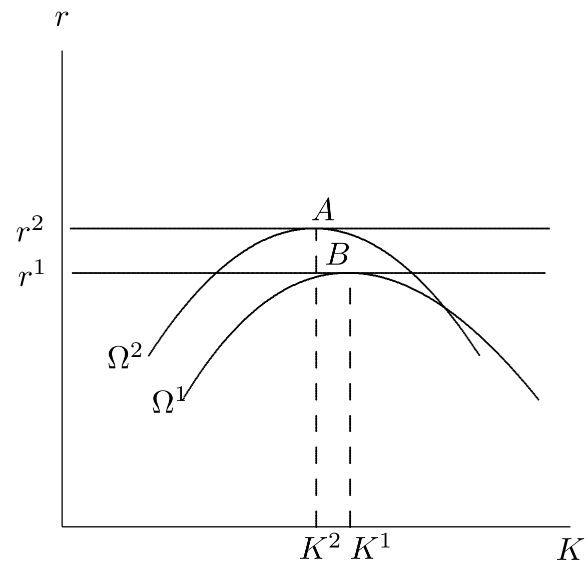

Figure 1(b)

As we will show shortly, the profit-maximizing allocation is, in general, suboptimal; that is, social welfare can be increased if the government chooses both the interest rate and the amount of capital employed by the entrepreneur. Thus, the government's redistributive motive can result in a failure to obtain allocations A and/or B in Figures 1(a) and 1(b), even when the government has full information about entrepreneurs' types. A fortiori, this will also be true in the asymmetric information case, as allocations A and B are not incentive-compatible. That is, more-able entrepreneurs have an incentive to mimic those who are less able, to obtain a lower interest rate. The government is aware of this incentive when selecting its optimal policy.

2.6 The occupational choice

Before we examine the government's problem, we need to determine the division of the population between entrepreneurs and workers. Recall that individuals are free to select their occupation, and they will do so based on a comparison of the utility obtained from becoming a worker or an entrepreneur. The marginal individual is indifferent between becoming an entrepreneur and becoming a worker. To fix ideas, let us assume that the marginal individual is of type 1.

Assumption 1: The occupational-choice equilibrium is interior and the marginal entrepreneur is type 1.

Since the utility function is increasing in θ^i, it follows that all type 2 individuals become entrepreneurs. Assumption 1 thus implies that, in equilibrium, we have two types of entrepreneurs, which ensures that the government is able to redistribute from the more-able to the less-able entrepreneurs.

At this stage, individuals take the optimal contracts (r^i, K^i) as given and anticipate the utility of being an entrepreneur $\Omega^i(a, w, r^i, K^i)$. For the marginal individual, the following condition must hold:

$$\Omega^1(a, w, r^1, K^1) = u(w). \tag{9}$$

Occupational-choice condition (9) determines the wage rate $w(a, r^1, K^1)$, with $w_a > 0$, $w_r < 0$, $\mathrm{d}w_K \gtreqless 0$.[9]

Let E^1 denote the number of entrepreneurs of type 1. The equilibrium value of E^1 must satisfy the following labour-market clearing condition:

$$N^2(1 + \ell^2) + E^1(1 + \ell^1) = N. \tag{10}$$

Equation (10) determines the number of entrepreneurs of type 1 $E^1(a, r^1, K^1, K^2)$, the properties of which are given in Lemma 2.

[9] Comparative statics on (9) give: $\partial w/\partial a = -(\partial\Omega^1/\partial a)/((\partial\Omega^1/\partial w) - u^{1\prime}) > 0$, $\partial w/\partial r^1 = -(\partial\Omega^1/\partial r^1)/((\partial\Omega^1/\partial w) - u^{1\prime}) < 0$, $\partial w/\partial K^1 = -(\partial\Omega^1/\partial K^1)/(\partial\Omega^1/\partial w) - u^{1\prime}) \gtreqless 0$.

Lemma 2 $E^1(a, r^1, K^1, K^2)$ has the following properties:

$$\frac{\partial E^1}{\partial a} = -\frac{1}{1+\ell^1}\left[N^2\frac{\partial \ell^2}{\partial w} + E^1\frac{\partial \ell^1}{\partial w}\right]\frac{\partial w}{\partial a} > 0, \tag{11}$$

$$\frac{\partial E^1}{\partial r^1} = -\frac{1}{1+\ell^1}\left[N^2\frac{\partial \ell^2}{\partial w} + E^1\frac{\partial \ell^1}{\partial w}\right]\frac{\partial w}{\partial r^1} < 0, \tag{12}$$

$$\frac{\partial E^1}{\partial K^1} = -\frac{1}{1+\ell^1}\left[\left(N^2\frac{\partial \ell^2}{\partial w} + E^1\frac{\partial \ell^1}{\partial w}\right)\frac{\partial w}{\partial K^1} + E^1\frac{\partial \ell^1}{\partial K^1}\right] \gtreqless 0, \tag{13}$$

$$\frac{\partial E^1}{\partial K^2} = -\frac{1}{1+\ell^1}N^2\frac{\partial \ell^2}{\partial K^2} < 0. \tag{14}$$

The effect of a change in a on the number of entrepreneurs is shown in Equation (11): an increase in a increases the number of entrepreneurs by raising the wage rate and, hence, reducing the demand for labour. Equation (12) shows that an increase in r^1 has the opposite effect. Equation (13) shows that an increase in K^1 has an ambiguous effect on the number of entrepreneurs, and depends on the sign of $(\partial w/\partial K^1)$. That is, an increase in K^1 directly increases the demand for labour and decreases the number of entrepreneurs. If $(\partial w/\partial K^1) > 0$, however, the general-equilibrium effect of a change in K^1 on the wage may offset this direct effect on labour demand by increasing the wage and decreasing the demand for labour. Equation (14) shows that an increase in K^2 directly increases the demand for labour and reduces the number of entrepreneurs.

2.7 The government's problem

We are ready to consider the BDB's problem. Assume that the BDB does not discriminate with respect to the amount of advice it provides to different types of entrepreneurs.[10] The government chooses the managerial input a and the bundles (r^i, K^i) to maximize the following utilitarian social welfare function:

$$\max_{\{a,r^1,r^2,K^1,K^2\}} W \equiv E^1\Omega^1(a, w, r^1, K^1) + N^2\Omega^2(a, w, r^2, K^2) + (N^1 - E^1)u(w). \tag{15}$$

The BDB faces three constraints. The first is a zero-profit constraint, given by

$$\varepsilon^1 p(a)(1 + r^1)E^1K^1 + \varepsilon^2 p(a)(1 + r^2)N^2K^2 - a = (1 + r)(E^1K^1 + N^2K^2), \tag{16}$$

[10] Although interesting, the analysis becomes very complicated if we assume that the BDB offers different amounts of advice to different types of entrepreneurs.

where r is the exogenous risk-free interest rate. The zero-profit condition implies that either $r^2 < r < r^1$ or $r^2 > r > r^1$. Given that one of the BDB's goals is to redistribute from type 2 entrepreneurs to type 1, we focus on equilibrium allocations for which $r^2 > r > r^1$. The zero-profit constraint implies that there is no cross-subsidization from the rest of the economy. We choose to work with this assumption for simplicity.[11]

The zero-profit constraint can be used to solve for r^2 as a function of a, r^1, K^1, and K^2. It can be shown that $r^2(a, r^1, K^1, K^2)$ is not monotonic in any of its arguments.[12] The second and third constraints faced by the BDB ensure that type 2 entrepreneurs have no incentive to mimic type 1, and vice versa. These are the self-selection constraints. Using a "hat" to denote the variables that apply to the mimicking entrepreneur, the self-selection constraints are given by

$$\Omega^2(a, w, r^2, K^2) \geq \widehat{\Omega}^2(a, w, r^1, K^1), \tag{17}$$

$$\Omega^1(a, w, r^1, K^1) \geq \widehat{\Omega}^1(a, w, r^2, K^2). \tag{18}$$

It is straightforward that only the first self-selection constraint can be binding when the single-crossing property is satisfied.

The Lagrangian for the BDB is thus

$$\begin{aligned} \max_{\{a, r^1, K^1, K^2\}} \mathcal{L} = & E^1\Omega^1(a, w, r^1, K^1) + N^2\Omega^2(a, w, r^2, K^2) + (N^1 - E^1)u(w) \\ & + \lambda[\Omega^2(a, w, r^2, K^2) - \widehat{\Omega}^2(a, w, r^1, K^1)], \end{aligned} \tag{19}$$

[11] If we allow that the BDB could be subsidized, the zero-profit constraint becomes

$$\varepsilon^1 p(a)(1 + r^1)E^1K^1 + \varepsilon^2 p(a)(1 + r^2)N^2K^2 - a + s = (1 + r)(E^1K^1 + N^2K^2),$$

where s is the subsidy. In this case, with a positive subsidy we can have $r > r^2 > r^1$. This possibility would not change our results.

[12] Totally differentiating the zero-profit condition (16) yields the following properties: $\partial r^2/\partial a = (1 - \varepsilon^1 p'(a)(1 + r^1)E^1K^1 - \varepsilon^2 p'(a)(1 + r^2)N^2K^2 + \delta^1(\partial E^1/\partial a))/\varepsilon^2 p(a)N^2K^2 \gtreqless 0$, $\partial r^2/\partial r^1 = (-(\partial\varepsilon^1/\partial r^1)p(a)(1 + r^1)E^1K^1 - \varepsilon^1 p(a)E^1K^1 + \delta^1(\partial E^1/\partial r^1))/\varepsilon^2 p(a)N^2K^2 \gtreqless 0$, $\partial r^2/\partial K^1 = (-(\partial\varepsilon^1/\partial K^1)p(a)(1 + r^1)E^1K^1 - \varepsilon^1 p(a)E^1K^1 + \delta^1(\partial E^1/\partial K^1))/\varepsilon^2 p(a)N^2K^2 \gtreqless 0$, $\partial r^2/\partial K^2 = (\delta^2 N^2 + \delta^1 K^1(\partial E^1/\partial K^2) - (\partial\varepsilon^2/\partial K^2)p(a)(1 + r^2)N^2K^2)/\varepsilon^2 p(a)N^2K^2 \gtreqless 0$, where $\delta^i \equiv [(1 + r) - \varepsilon^i p(a)(1 + r^i)] \gtreqless 0$, $i = 1, 2$.

where we take into account that r^2 is a function of (a, r^1, K^1, K^2). The government chooses (a, r^i, K^i), $i = 1, 2$ to maximize (19), anticipating the effect of its choice on wages $w(a, r^1, K^1)$, labour $\ell^i(w, K^i)$, entrepreneurial input $\varepsilon^i(w, r^i, K^i)$, and the number of type 1 entrepreneurs $E^1(a, r^1, K^1, K^2)$.

3. The Full-Information Case

We first consider the case where the BDB has full information about entrepreneurs' abilities. Thus, the self-selection constraint (17) is not binding. The optimum allocation in the full-information case is obtained by setting λ to zero in the Lagrangian function described in section 2.7. We are interested in comparing the optimum allocation with the profit-maximizing allocation that solves $F^i_K = 1 + r^i$, $i = 1, 2$, represented by A and B in Figures 1(a) and 1(b). To avoid rendering the notation too cumbersome, we use the same notation for the full-information allocation as we did for the profit-maximizing allocation.

Proposition 1 *With full information about entrepreneurs' types, the optimal allocation involves entrepreneurs employing either too much, too little, or just enough capital relative to the profit-maximizing allocation:*

(i) $MRS^1_{rK} = \frac{\partial r^1}{\partial K^1} \gtreqless 0,$

(ii) $MRS^2_{rK} = \frac{\partial r^2}{\partial K^2} \gtreqless 0.$

The intuition behind Proposition 1 is straightforward. The government, when selecting (r^i, K^i), sets the marginal rate of substitution equal to the slope of the budget constraint. Given that $(\partial r^1/\partial K^1)$ and $(\partial r^2/\partial K^2)$ are ambiguous in sign, the sign of the marginal rate of substitution between r and K is ambiguous for both types of entrepreneurs. Thus, compared with the profit-maximizing allocation for the same interest rates, both types of entrepreneurs employ either too little, too much, or just enough capital at the optimal allocation with full information. Four possible cases for the full-information allocation are illustrated in Figure 2 for the case where single-crossing property (i) holds. Essentially, Figure 2 compares the optimal allocation (r^i, K^i) chosen by the government with the profit-maximizing allocation for the same interest rates, r^i.[13] The optimum al-

[13] Alternatively, we could compare the optimal allocation chosen by the government with the profit-

location for a more-able entrepreneur is at A or A', whereas the optimum allocation for a less-able entrepreneur can be at B or B'. At A' and B', both types of entrepreneurs are credit-rationed compared with the profit-maximizing allocation, whereas at A and B they overinvest compared with the profit-maximizing outcome. A similar figure can be drawn for the case where single-crossing property (ii) holds, with the same results applying relative to the profit-maximizing allocation.

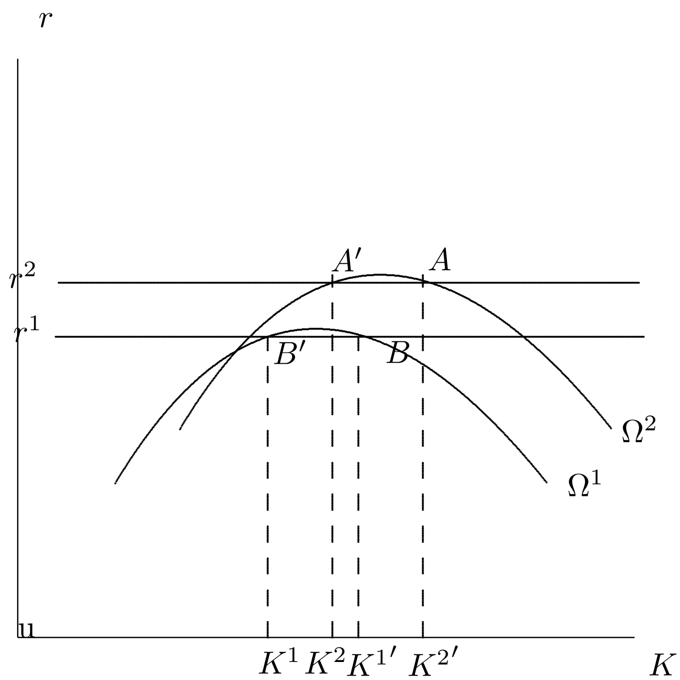

Figure 2

The figure shows that entrepreneurs, if allowed to select their levels of capital freely, have no incentive to take into account the effect their capital choice will have on the BDB's budget constraint. The BDB does take this into account when maximizing welfare by setting the marginal rates of substitution equal to the slope of the budget constraint.

4. Asymmetric Information

When the BDB is unable to observe entrepreneurial ability, type 2 entrepreneurs have an incentive to mimic type 1 entrepreneurs, to obtain a more favourable rate of interest. The

maximizing allocation of capital when the government chooses the interest rates that maximize the social welfare function W. It is easy to see that the result of Proposition 1 holds for this alternative comparison, as well. When the government chooses r^i to maximize its objective, this can shift the indifference curves of the two types up or down. Given the position of the new equilibrium allocations relative to the profit-maximizing ones, we end up with one of the four cases depicted in Figure 2.

BDB takes this behaviour into account when deciding upon the optimal bundles (a, r^i, K^i).

Proposition 2 *With asymmetric information on entrepreneurs' types, type 1 entrepreneurs are constrained to employ either too little or too much capital relative to the full-information allocation, whereas type 2 entrepreneurs' employment of capital is non-distorted relative to the full-information allocation:*

(i) $MRS^1_{rK} > \frac{\partial r^1}{\partial K^1}$,

(ii) $MRS^2_{rK} = \frac{\partial r^2}{\partial K^2}$.

Proposition 2 indicates the effect of asymmetric information on the optimal employment of capital; it is illustrated in Figure 3(a) for the case where single-crossing property (i) applies, and in Figure 3(b) for the case where single-crossing property (ii) applies. For the former case, preventing type 2 entrepreneurs from mimicking type 1 entrepreneurs implies that those who are type 1 are constrained to employ too little capital relative to the full-information and profit-maximizing outcomes. Thus, asymmetric information results in credit-rationing of type 1 entrepreneurs for this case. Because the sign of $(\partial r^2/\partial K^2)$ is ambiguous, type 2 entrepreneurs may employ either too little or too much capital relative to the profit-maximizing outcome. Figure 3(a) depicts the latter case. This result is similar to one obtained by Boadway et al. (1998), where over- or underinvesting results because of adverse selection in the private credit market. In their model, however, government intervention via a subsidy on capital income is used to remedy this inefficiency.

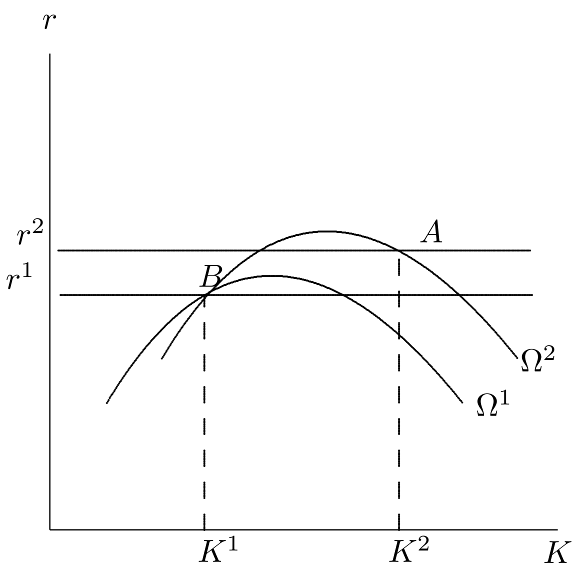

Figure 3(a)

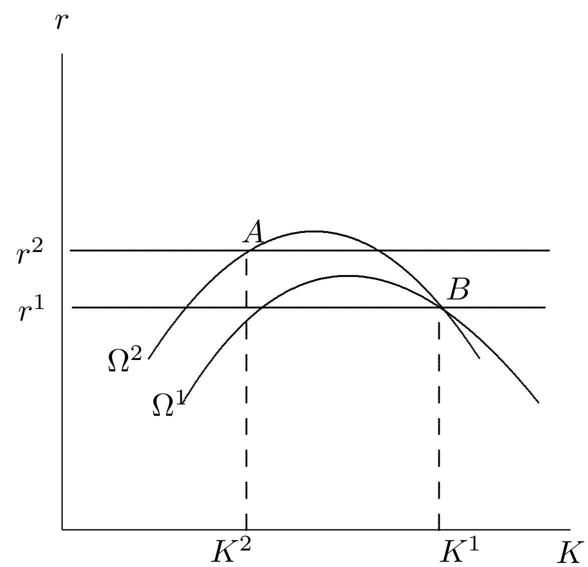

Figure 3(b)

The implications of Proposition 2 for the supply of entrepreneurship when the single-crossing property (i) holds is summarized in Corollary 2.1. A similar implication can be obtained for the case of single-crossing property (ii).

Corollary 2.1 *In the presence of asymmetric information, credit-rationing of less-able entrepreneurs has an ambiguous effect on the number of entrepreneurs.*

Equation (13) shows that the effect of a reduction in K^1 on E^1 is ambiguous. It depends on (i) how labour demand is directly affected by a change in K^1, and (ii) the general-equilibrium effect that a change in K^1 has on the wage rate. If the latter effect is small, then a reduction in K^1 relative to the full-information case increases the number of entrepreneurs by reducing the proportion of individuals who are employed as workers. The result in Proposition 3 compares the number of type 1 entrepreneurs in the equilibrium with occupational choice with the number chosen by the government if it had direct control over access to occupations. We refer to the latter as the efficient E^1.

Proposition 3 *Irrespective of the informational assumption, the occupational-choice equilibrium is characterized by an inefficient number of type 1 entrepreneurs.*

A formal proof of this result is given in Appendix C. If the government can control access to occupations, it chooses E^1 type 1 individuals to enter the entrepreneurial occupation to maximize welfare subject to the zero-profit condition and the self-selection constraints. In this case, the wage rate is determined by the labour-market clearing condition, $w(E^1, K^1, K^2)$. We can see the intuition behind this result by analyzing the indirect effects of an additional type 1 individual entering the pool of entrepreneurs. When one more individual decides to become an entrepreneur, the demand for labour increases and, as a result, so do wages. An increase in wages, in turn, raises the welfare of workers and reduces that of entrepreneurs (both type 1 and type 2). At the same time, the higher number of type 1 entrepreneurs increases the cost of capital for type 2 entrepreneurs, thus reducing their welfare. Furthermore, a higher E^1 may tighten or relax the self-selection constraint. This constraint requires that a type 2 entrepreneur has no incentive to mimic a type 1 entrepreneur. An increase in the number of type 1 entrepreneurs increases the wage rate, which in turn lowers the utility of a type 2 entrepreneur. At the same time, a higher wage rate lowers the utility of the mimicker. The effect on the difference between the two utilities is ambiguous; it may tighten or relax the self-selection constraint. Since a type 1 individual who decides to become an entrepreneur ignores these effects, the equilibrium number of type 1 entrepreneurs is inefficient—too high or too low. This result is a consequence of the BDB's redistributive motive.

The important insight of Proposition 3 is that government venture capital alone cannot induce the optimal number of entrepreneurs: when individuals choose their occupation freely, they ignore the effect of their choices on the welfare of other entrepreneurs and workers. This suggests that the government might need additional instruments to achieve the optimal supply of entrepreneurship.

The result summarized in Proposition 3 holds whether there is full or asymmetric information. It is difficult, however, to quantify the effect of asymmetric information on the number of type 1 entrepreneurs, because changes in their number have an ambiguous effect on the self-selection constraint.

We next consider the effect of changes in managerial input a on welfare. To do so, it is useful to rewrite the derivative of the Lagrangian as

$$\frac{\partial \mathcal{L}}{\partial a} = E^1 \frac{\partial \Omega^1}{\partial a} + N^2 \frac{\partial \Omega^2}{\partial a} + \left[E^1 \frac{\partial \Omega^1}{\partial w} + N^2 \frac{\partial \Omega^2}{\partial w} + (N^1 - E^1) u^{1\prime} \right] \frac{\partial w}{\partial a} + N^2 \frac{\partial \Omega^2}{\partial r^2} \frac{\partial r^2}{\partial a}$$

$$+\lambda\Big[\frac{\partial\Omega^1}{\partial a}-\frac{\partial\widehat{\Omega}^1}{\partial a}+\Big(\frac{\partial\Omega^1}{\partial w}-\frac{\partial\widehat{\Omega}^1}{\partial w}\frac{\partial w}{\partial a}+\frac{\partial\Omega^2}{\partial r^2}\frac{\partial r^2}{\partial a}\Big)\Big]=0. \tag{20}$$

The first two terms in (20) show the direct effect of an increase in managerial input a on welfare. This effect is positive because a higher managerial input increases the utility of both types of entrepreneurs. The second term shows the indirect effect of an increase in a on welfare operating through a change in the wage rate. A higher a increases the wage rate, which in turn has an ambiguous effect on welfare: it increases workers' utility while reducing the utility of both types of entrepreneurs. The third term in (20) shows the indirect effect of a change in a on welfare operating through a change in the rate of interest faced by type 2 entrepreneurs. We also know that a higher a has an ambiguous effect on r^2. The last term in the first-order condition (20) shows the direct and indirect effects of an increase in a on the self-selection constraint. It is easy to see that the sign of this term is ambiguous. The last term disappears in the full-information case. The government weighs these various direct and indirect effects when determining the optimal amount of managerial advice.

At this point, we are interested in the effect that a change in the level of managerial advice—specifically, an increase in managerial advice—has on welfare. Given the difficulty of characterizing the welfare change in general, we focus on the area around zero managerial advice, $a = 0$. Around $a = 0$, the slope of the zero-profit constraint with respect to a becomes infinity:

$$\lim_{a\to 0}\frac{\partial r^2}{\partial a}=\infty. \tag{21}$$

Using this, we have:

Proposition 4 *Starting from a situation where the BDB does not provide managerial advice, an increase in managerial input reduces welfare.*[14]

The result in Proposition 4 is counterintuitive. Starting from a situation where no managerial input is provided, there is a cost of increasing a that is reflected in an increase in the interest rate r^2, which directly reduces the welfare of type 2 entrepreneurs. Around

[14] Evaluating (20) around $a = 0$, we obtain $(\partial\mathcal{L}/\partial a)|_{a=0} = (N^2+\lambda)(\partial\Omega^2/\partial r^2)(\partial r^2/\partial a)|_{a=0} = -(N^2+\lambda)/N^2u^{2\prime} < 0$.

$a = 0$, this cost is not matched by an increase in entrepreneurs' probability of success. In essence, Proposition 4 says that the government has no incentive to provide a small amount of managerial advice starting from a position of zero advice. According to (20), however, larger amounts of managerial advice may be optimal.[15] Proposition 4 suggests that governments should not interfere in business decisions beyond the provision of capital, unless they have enough expertise to provide the right level of advice. Thus, a little bit of advice can be more detrimental for welfare than no advice at all.

5. Conclusion

The literature on entrepreneurship has long recognized the fact that venture capital is a major form of external financing for new firms. Despite evidence that most governments in developed countries have public venture capital programs in place, no economic analyses of these programs exist in the literature. The main objective of this paper has been to build a model that endogenously determines the optimal amount of venture capital and managerial advice provided by the Business Development Bank. Our analysis is based on an occupational-choice model with informational constraints.

Our results for the supply of capital, supply of entrepreneurs, and managerial input depend on the environment and on the informational assumptions we consider. We find that, with full information, entrepreneurs who are more or less able may be credit-rationed or overinvest compared with the profit-maximizing outcome. Furthermore, the introduction of asymmetric information has no effect on the use of capital by more-able entrepreneurs. Less-able entrepreneurs, however, may be credit-rationed compared with the full-information case.

When we examine the effect of public venture capital on the supply of entrepreneurs, we find that the occupational-choice equilibrium is characterized by an inefficient number of less-able entrepreneurs compared with the number chosen by a government that controls access to occupations. This result holds irrespective of the informational assumption—full

[15] In a different setting, Keuschnigg and Nielsen (2000) show that a tax on entrepreneurship and/or a tax on start-up investment increases managerial advice and has a positive first-order effect on welfare.

or asymmetric information. The occupational-choice equilibrium has too many or too few less-able entrepreneurs, depending on the relative effect of an increase in their number—versus that of workers—on the welfare of all entrepreneurs. If entry into entrepreneurship increases workers' welfare by more than it reduces the welfare of entrepreneurs, then we can conclude that the equilibrium has too few entrepreneurs. The opposite holds if entry into entrepreneurship reduces entrepreneurs' welfare by more than it increases workers' welfare.

One of the main roles of venture capitalists, besides supplying capital to start-ups, is to provide managerial expertise. In this paper, the effect of managerial advice on welfare has been, in general, ambiguous. We find, however, that, starting with a situation in which the BDB does not provide managerial advice, a small increase in the level of advice has a negative effect on welfare; it results in an increase in the interest rate for more-able entrepreneurs that is not offset by an increase in their probability of success.

One limitation of our model is that only public venture capital is available. The simplest way to introduce private venture capital into the model is to have individuals first shop for it, and have those who are rejected apply for public venture capital financing. In this case, individuals who are more able obtain private venture capital; less-able individuals apply for public venture capital. It is clear that the equilibrium in the market for public venture capital is still characterized in terms of our results. The problem with this simple framework is that only less-able entrepreneurs are financed with public venture capital. An alternative would be to assume that individuals first apply for private venture capital, as before, and that they get matched to a venture capitalist according to a matching function. Those who do not get matched with a private venture capitalist can apply for public venture capital. This scenario would allow us to avoid the problem of having only less-able types apply for public venture capital. Another possibility is to have individuals differ with respect to two characteristics; for example, ability and risk aversion. Unfortunately, the analysis becomes too complicated to allow us to obtain simple results. Our model is, however, consistent with evidence that most of the private venture capital finances the growth and development stages of a firm, rather than the early stages of a start-up (Amit, Brander, and Zott 1997).

In this paper, we have analyzed the equilibrium with public venture capital in the form of government loans. As stated in the introduction, governments also directly supply capital in the form of equity investments in start-ups. An interesting extension of our model would consider a scenario where the government offers both debt and equity financing. It would, however, be technically very difficult to solve for an interior solution where both debt and equity are offered in equilibrium. The literature has recognized this fact and has thus far considered debt and equity financing separately.[16] Overcoming these technical difficulties is an area for future work.

[16] See Hellman and Stiglitz (2000) for a discussion of this point.

Bibliography

Amit, R., J. Brander, and C. Zott (1997): "Venture Capital Financing of Entrepreneurship in Canada," in P. Halpern (ed), *Financing Growth in Canada*, Industry Canada Research Series: 237-77.

Audretsch, D. and R. Thurik (2001): "Linking Entrepreneurship to Growth," OECD STI Working Paper No. 2001/2.

Banerjee, A. V. and A. F. Newman (1993): "Occupational Choice and the Process of Development," *Journal of Political Economy* **101**: 274-98.

Banerji, S. and N. van Long (2000): "Wealth Distribution, Moral Hazard, and Entrepreneurship," manuscript.

Bendick, M. Jr. and M. L. Egan (1987): "Transfer Payment Diversion for Small Business Development: British and French Experience," *Industrial and Labour Relations Review* **40**: 528-42.

Bhattacharya, U. and B. Ravikumar (1999): "Capital Markets and the Evolution of Family Business," manuscript.

Black, J. and D. de Meza (1997): "Everyone May Benefit from Subsidising Entry into Risky Occupations," *Journal of Public Economics* **66**: 409-24.

Blanchflower, D. G. and A. J. Oswald (1998): "What Makes an Entrepreneur?," *Journal of Labour Economics* **16(1)**: 26-60.

Boadway, R., M. Marchand, and P. Pestieau (1991): "Optimal Linear Income Taxation in Models with Occupational Choice," *Journal of Public Economics* **46**: 133-62.

Boadway, R., N. Marceau, M. Marchand, and M. Vigneault (1998): "Entrepreneurship, Asymmetric Information, and Unemployment," *International Tax and Public Finance* **5**: 307-27.

Chamley, C. (1983): "Entrepreneurial Abilities and Liabilities in a Model of Self-Selection," *Bell Journal of Economics* **14**: 70-80.

Christiansen, V. (1988): "Choice of Occupation, Tax Incidence and Piecemeal Tax Revision," *Scandinavian Journal of Economics* **90**: 141-59.

Cremer, H. and P. Pestieau (1996): "Redistributive Taxation and Social Insurance," *International Tax and Public Finance* **3**: 281-95.

De Meza, D. and D. Webb (1987): "Too Much Investment: A Problem of Asymmetric Information," *Quarterly Journal of Economics* **102**: 281-92.

Evans, D. S. and B. Jovanovic (1989): "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints," *Journal of Political Economy* **97**: 808-27.

Evans, D. S. and L. S. Leighton (1989): "Some Empirical Aspects of Entrepreneurship," *American Economic Review* **79**: 519-35.

Gordon, R. H. (1998): "Can High Personal Tax Rates Encourage Entrepreneurial Activity?," IMF *Staff Papers* **45**: 49-80.

Greenwald, B.C., J.E. Stiglitz, and A. Weiss (1984): "Information Imperfections in the Capital Market and Macroeconomic Fluctuations," *American Economic Review Papers and Proceedings* **74(1)**: 94-199.

Griliches, Z. (1992): "The Search for R&D Spillovers," *Scandinavian Journal of Economics* 94, supple.: S29-S47.

Hall, B. H. (1992): "Investment and Research and Development: Does the Source of Financing Matter?," Working Paper No. 92-194, Department of Economics, University of California at Berkeley.

Hao, K. and A.B. Jaffe (1993): "Effect of Liquidity on Firms's R&D Spending," *Economics of Innovation and New Technology* **2**: 275-82.

Hellman, T. and J. Stiglitz (2000): "Credit Rationing and Equity Rationing in Markets with Adverse Selection," *European Economic Review* **44**: 281-304.

Himmelberg, C. P. and B. P. Petersen (1994): "R&D and Internal Finance: A Panel Study of Small Firms in High-Tech Industries," *Review of Economics and Statistics* **76**: 38-51.

Holtz-Eakin, D., D. Joulfain, and H. S. Rosen (1994): "Sticking It Out: Entrepreneurial Survival and Liquidity Constraints," *Journal of Political Economy* **102**: 53-75.

Hubbard, G. (1998): "Capital-Market Imperfections and Investment," *Journal of Economic Literature* **36**: 193-225.

Ilmakunnas, P. and V. Kanniainen (2000): "Entrepreneurship, Economic Risk, and Risk Insurance in the Welfare State: Results with OECD Data 1978-93," manuscript.

Jensen, M.C. and W.H. Meckling (1976): "Theory of the Firm: Managerial Behaviour, Agency Costs and Ownership Structure," *Journal of Financial Economics*, **3**: 305-60.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica* **50**: 649-70.

Kanbur, S. M. (1979): "Of Risk Taking and the Personal Distribution of Income," *Journal of Political Economy* **87**: 769-97.

——— (1981): "Risk Taking and Taxation," *Journal of Public Economics* **15**: 163-84.

Kanniainen, V. and C. Keuschnigg (2000): "The Optimal Portfolio of Start-up Firms in Venture Capital Support," CESifo Discussion Paper No. 381.

——— (2001): "Start-up Investment with Scarce Venture Capital Support," CESifo Discussion Paper No. 439.

Kanniainen, V. and M. Leppämäki (2002): "Financial Institutions and the Allocation of Talent," Bank of Finland Discussion Paper No. 5/2002.

Keuschnigg, C. and S.B. Nielsen (2000): "Tax Policy, Venture Capital and Entrepreneurship," CEPR Discussion Paper No. 2626.

Kihlstrom, R. E. and J.-J. Laffont (1979): "A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion," *Journal of Political Economy* **87(4)**: 719-48.

Lerner, J. (1996): "The Government as Venture Capitalist: The Long-Run Impact of the SBIR Program," NBER Working Paper No. 5753.

——— (2002): "When Bureaucrats Meet Entrepreneurs: The Design of Effective 'Public Venture Capital' Programs," *Economic Journal* **112** (*February*): F73-F84.

Myers, S.C. and N. Majluf (1984): "Corporate Financing and Investment Decisions When Firms Have Information That Investors do not Have," *Journal of Financial Economics* **13**: 187-221.

Organisation for Economic Co-operation and Development (OECD).(1996): *Government Programmes for Venture Capital.* Paris: Organisation for Economic Co-operation and Development.

——— (1997): *Government Venture Capital for Technology-Based Firms.* Paris: Organisation for Economic Co-operation and Development.

Peck, R. M. (1989): "Taxation, Risk, and Returns to Scale," *Journal of Public Economics* **40**: 319-30.

Poterba, J.M. (1989a): "Capital Gains Tax Policy Toward Entrepreneurship," *National Tax Journal* **42**: 375-89.

——— (1989b): "Venture Capital and Capital Gains Taxation," in *Tax Policy and the Economy*, vol 3: 47-67.

Stiglitz, J.E. (1982): "Self-Selection and Pareto Efficient Taxation," *Journal of Public Economics* **17**: 213-40.

Stiglitz, J.E. and A. Weiss (1981): "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, **71**: 393-410.

Vaillancourt, F. (1997): “Labour-Sponsored Venture Capital Funds in Canada: Institutional Aspects, Tax Expenditures, and Employment Creation,” in *Financing Growth in Canada* by P. Halpern (ed), Industry Canada Research Series, Calgary, University of Calgary.

Zhang, W. (2000): “Why are Entrepreneurs Liquidity-Constrained?,” *Annals of Economics and Finance*, **1**: 165-88.

Appendix A: Proof of Proposition 1

The first-order conditions for the BDB's problem are the following:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial r^1} =& E^1\Big[\frac{\partial \Omega^1}{\partial r^1}+\frac{\partial \Omega^1}{\partial w}\frac{\partial w}{\partial r^1}\Big]+N^2\Big[\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial r^1}+\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial r^1}\Big]+(N^1-E^1){u^1}'\frac{\partial w}{\partial r^1}\\
&+\lambda\Big[\Big(\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial r^1}+\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial r^1}\Big)-\Big(\frac{\partial \widehat{\Omega}^2}{\partial w}\frac{\partial w}{\partial r^1}+\frac{\partial \widehat{\Omega}^2}{\partial r^1}\Big)\Big]=0, &\text{(A1)}\\
\frac{\partial \mathcal{L}}{\partial K^1} =& E^1\Big[\frac{\partial \Omega^1}{\partial K^1}+\frac{\partial \Omega^1}{\partial w}\frac{\partial w}{\partial K^1}\Big]+N^2\Big[\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial K^1}+\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial K^1}\Big]+(N^1-E^1){u^1}'\frac{\partial w}{\partial K^1}\\
&+\lambda\Big[\Big(\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial K^1}+\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial K^1}\Big)-\Big(\frac{\partial \widehat{\Omega}^2}{\partial w}\frac{\partial w}{\partial K^1}+\frac{\partial \widehat{\Omega}^2}{\partial K^1}\Big)\Big]=0, &\text{(A2)}\\
\frac{\partial \mathcal{L}}{\partial K^2} =& N^2\Big[\frac{\partial \Omega^2}{\partial K^2}+\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial K^2}\Big]+\lambda\Big[\frac{\partial \Omega^2}{\partial K^2}+\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial K^2}\Big]=0, &\text{(A3)}\\
\frac{\partial \mathcal{L}}{\partial a} =& E^1\Big[\frac{\partial \Omega^1}{\partial a}+\frac{\partial \Omega^1}{\partial w}\frac{\partial w}{\partial a}\Big]+N^2\Big[\frac{\partial \Omega^2}{\partial a}+\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial a}+\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial a}\Big]+(N^1-E^1){u^1}'\frac{\partial w}{\partial a}\\
&+\lambda\Big[\Big(\frac{\partial \Omega^2}{\partial a}+\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial a}+\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial a}\Big)-\Big(\frac{\partial \widehat{\Omega}^2}{\partial w}\frac{\partial w}{\partial a}+\frac{\partial \widehat{\Omega}^2}{\partial a}\Big)\Big]=0. &\text{(A4)}
\end{aligned}$$

The proof is straightforward. Recall that, when maximizing utility or, equivalently, profits, the entrepreneur selects capital so that $F^i_K = 1 + r^i$, and so $MRS^i_{r,K} = 0$. For the government's problem, the first-order conditions (A1) and (A2) with $\lambda = 0$ give

$$\begin{aligned}
MRS^1_{rK} =& -\frac{\partial \Omega^1}{\partial K^1}\Big/\frac{\partial \Omega^1}{\partial r^1}\\
=& -\frac{N^2\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial K^1}+\Big[E^1\frac{\partial \Omega^1}{\partial w}+N^2\frac{\partial \Omega^2}{\partial w}+(N^1-E^1){u^1}'\Big]\frac{\partial w}{\partial K^1}}{N^2\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial r^1}+\Big[E^1\frac{\partial \Omega^1}{\partial w}+N^2\frac{\partial \Omega^2}{\partial w}+(N^1-E^1){u^1}'\Big]\frac{\partial w}{\partial r^1}}.
\end{aligned}$$

Let

$$\begin{aligned}
a \equiv& N^2\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial K^1},\\
b \equiv& \Big[E^1\frac{\partial \Omega^1}{\partial w}+N^2\frac{\partial \Omega^2}{\partial w}+(N^1-E^1){u^1}'\Big]\frac{\partial w}{\partial K^1},\\
c \equiv& N^2\frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial r^1},\\
d \equiv& \Big[E^1\frac{\partial \Omega^1}{\partial w}+N^2\frac{\partial \Omega^2}{\partial w}+(N^1-E^1){u^1}'\Big]\frac{\partial w}{\partial r^1}.
\end{aligned}$$

With this notation, it follows that

$$MRS^1_{rK} = -\frac{a+b}{c+d} = -\frac{a}{c} - d\frac{b/d - a/c}{c+d}$$

$$= -\frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1} - d \frac{\frac{\partial w}{\partial K^1} / \frac{\partial w}{\partial r^1} - \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1}}{c+d}.$$

Since

$$\frac{\partial w}{\partial K^1} \Big/ \frac{\partial w}{\partial r^1} = \frac{\partial \Omega^1}{\partial K^1} \Big/ \frac{\partial \Omega^1}{\partial r^1} = -MRS^1_{rK},$$

we obtain

$$MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1} = d \frac{MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1}}{c+d}.$$

Factoring out $MRS^1_{rK} + (\partial r^2/\partial K^1)/(\partial r^2/\partial r^1)$, we get

$$\Big[MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1} \Big] \frac{c}{c+d} = 0.$$

Since $c > 0$, it follows that we must necessarily have

$$MRS^1_{rK} = -\frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1} = \frac{\partial r^1}{\partial K^1} \gtreqless 0. \quad Q.E.D.$$

Appendix B: Proof of Proposition 2

We first specify the following notation:

$$\begin{aligned}
\alpha \equiv & E^1 \frac{\partial \Omega^1}{\partial w} + (N^2 + \lambda_2) \frac{\partial \Omega^2}{\partial w} + (N^1 - E^1) u^{1\prime} - \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial w}, \\
a \equiv & (N^2 + \lambda_2) \frac{\partial \Omega^2}{\partial r^2} \frac{\partial r^1}{\partial K^1}, \\
b \equiv & \alpha \frac{\partial w}{\partial K^1} - \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial K^1}, \\
c \equiv & (N^2 + \lambda_2) \frac{\partial \Omega^2}{\partial r^2} \frac{\partial r^1}{\partial r^1}, \\
d \equiv & \frac{\partial w}{\partial r^1} - \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial r^1}.
\end{aligned}$$

Next, the first-order conditions (A1) and (A2) yield

$$MRS^1_{rK} = -\frac{a+b}{c+d} = \frac{a}{c} + d\frac{b/d - a/c}{c+d}.$$

Then,

$$\begin{aligned}
\frac{b}{d} - \frac{a}{c} &= \frac{\alpha \frac{\partial w}{\partial r^1} [\frac{\partial w}{\partial K^1} / \frac{\partial w}{\partial r^1} - \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1}] + \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial r^1} [-\frac{\partial \widehat{\Omega}^2}{\partial K^1} / \frac{\partial \widehat{\Omega}^2}{\partial r^1} + \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1}]}{\alpha \frac{\partial w}{\partial r^1} - \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial r^1}} \\
&= \frac{-\alpha \frac{\partial w}{\partial r^1} [MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1}] + \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial r^1} [\widehat{MRS}^2_{rK} + \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1}]}{\alpha \frac{\partial w}{\partial r^1} - \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial r^1}}, \\
&= \frac{1}{d} \Big\{ - \alpha \frac{\partial w}{\partial r^1} \Big[MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1} \Big] + \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial r^1} \Big[\widehat{MRS}^2_{rK} + \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1} \Big] \Big\},
\end{aligned}$$

where we made use of the fact that, from (8) and (9), we can write

$$\frac{\partial w}{\partial K^1} / \frac{\partial w}{\partial r^1} = \frac{\partial \Omega^1}{\partial r^1} / \frac{\partial \Omega^1}{\partial r^1} = -MRS^1_{rK}.$$

Using this result, we can write the MRS^1_{rK} as

$$\begin{aligned}
MRS^1_{rK} &= -\frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1} \\
&= -\frac{1}{c+d} \Big\{ - \alpha \frac{\partial w}{\partial r^1} \Big[MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1} \Big] + \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial r^1} \Big[\widehat{MRS}^2_{rK} + \frac{\partial r^2}{\partial K^1} / \frac{\partial r^2}{\partial r^1} \Big] \Big\}, \quad \text{(B1)}
\end{aligned}$$

or

$$
\begin{aligned}
&MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1} \\
&= -\frac{1}{c+d}\Big\{ -\alpha \frac{\partial w}{\partial r^1}\Big[MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1}\Big] + \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial r^1}\Big[\widehat{MRS}^2_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1}\Big]\Big\}. \qquad \text{(B2)}
\end{aligned}
$$

From the single-crossing property, we know that $MRS^1_{rK} < \widehat{MRS}^2_{rK}$, $\forall(r, K)$.

Suppose that $c + d > 0$. We can then say that

$$
\begin{aligned}
&MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1} \\
&> -\frac{1}{c+d}\Big\{ -\alpha \frac{\partial w}{\partial r^1}\Big[MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1}\Big] + \lambda_2 \frac{\partial \widehat{\Omega}^2}{\partial r^1}\Big[MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1}\Big]\Big\} \\
&= \frac{d}{c+d}\Big[MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1}\Big]. \qquad \text{(B3)}
\end{aligned}
$$

This implies that

$$\frac{c}{c+d}\Big[MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1}\Big] > 0. \qquad \text{(B4)}$$

Since $c > 0$ and $c + d > 0$, it follows that

$$MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1} > 0,$$

and hence

$$MRS^1_{rK} > -\frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1}. \qquad \text{(B5)}$$

If, on the other hand, $c + d < 0$, then

$$\frac{c}{c+d}\Big[MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1}\Big] < 0.$$

Since $c > 0$ and $c + d < 0$, we have once again that

$$MRS^1_{rK} + \frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1} > 0,$$

or

$$MRS^1_{rK} > -\frac{\partial r^2}{\partial K^1} \Big/ \frac{\partial r^2}{\partial r^1}. \quad Q.E.D.$$

For part (ii) of the proposition, gathering like terms in (A3) yields

$$\frac{\partial \Omega^2}{\partial K^2} + \frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial K^2} = 0 \quad \text{and hence} \quad MRS^2_{rK} = \frac{\partial r^2}{\partial K^2}. \quad Q.E.D.$$

Appendix C: Proof of Proposition 3

If the government can directly control access to occupations, it chooses $\{E^1, r^1, r^2, K^1, K^2\}$ to maximize the utilitarian social welfare function

$$W \equiv E^1\Omega^1(w, r^1, K^1) + N^2\Omega^2(w, r^2, K^2) + (N^1 - E^1)u(w), \tag{C1}$$

subject to the zero-profit condition

$$(1 + r^1)E^1K^1 + (1 + r^2)N^2K^2 = (1 + r)(E^1K^1 + N^2K^2), \tag{C2}$$

the labour-market clearing condition

$$N^2(1 + \ell^2) + E^1(1 + \ell^1) = N, \tag{C3}$$

and the self-selection constraint

$$\Omega^2(w, r^2, K^2) \geq \widehat{\Omega}^1(w, r^2, K^2). \tag{C4}$$

The zero-profit condition can be solved for r^2 as a function of (E^1, r^1, K^1, K^2), with properties

$$\begin{aligned}
\frac{\partial r^2}{\partial E^1} &= \frac{(r - r^1)K^1}{N^2K^2} > 0,\\
\frac{\partial r^2}{\partial r^1} &= -\frac{E^1K^1}{N^2K^2} < 0,\\
\frac{\partial r^2}{\partial K^1} &= \frac{(r - r^1)E^1}{N^2K^2} > 0,\\
\frac{\partial r^2}{\partial K^2} &= \frac{(r - r^2)N^2}{N^2K^2} < 0.
\end{aligned}$$

The labour-market clearing condition allows us to determine the wage rate as a function of (E^1, K^1, K^2), with

$$\begin{aligned}
\frac{\partial w}{\partial E^1} &= -\,(1 + \ell^1)\Big/\left[N^2\frac{\partial \ell^2}{\partial w} + E^1\frac{\partial \ell^1}{\partial w}\right] > 0,\\
\frac{\partial w}{\partial K^1} &= -\,E^1\frac{\partial \ell^1}{\partial K^1}\Big/\left[N^2\frac{\partial \ell^2}{\partial w} + E^1\frac{\partial \ell^1}{\partial w}\right] > 0,\\
\frac{\partial w}{\partial K^2} &= -\,N^2\frac{\partial \ell^2}{\partial K^2}\Big/\left[N^2\frac{\partial \ell^2}{\partial w} + E^1\frac{\partial \ell^1}{\partial w}\right] > 0.
\end{aligned}$$

The Lagrangian for the BDB problem is

$$\max_{\{E^1,r^1,K^1,K^2\}} \mathcal{L} = E^1\Omega^1(w,r^1,K^1) + N^2\Omega^2(w,r^2,K^2) + (N^1-E^1)u(w) \\ + \lambda[\Omega^2(w,r^2,K^2) - \widehat{\Omega}^2(w,r^1,K^1)].$$

The first-order condition for E^1 is

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial E^1} =& \Omega^1(w,r^1,K^1) - u(w) \\ &+ E^1\frac{\partial \Omega^1}{\partial w}\frac{\partial w}{\partial E^1} - N^2\Big(\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial E^1} + \frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial E^1}\Big) + (N^1-E^1)u^{1\prime}\frac{\partial w}{\partial E^1} \\ &+ \lambda\Big[\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial E^1} + \frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial E^1} - \frac{\partial \widehat{\Omega}^2}{\partial w}\frac{\partial w}{\partial E^1}\Big] = 0. \end{aligned} \tag{C5}$$

Evaluating at the free occupational-choice equilibrium $E^1 = (E^1)^{OC}$, characterized by $\Omega^1(w,r^1,K^1) = u(w)$, we get

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial E^1}\Big|_{E^1=(E^1)^{OC}} =& E^1\frac{\partial \Omega^1}{\partial w}\frac{\partial w}{\partial E^1} + N^2\Big(\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial E^1} + \frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial E^1}\Big) + (N^1-E^1)u^{1\prime}\frac{\partial w}{\partial E^1} \\ &+ \lambda\Big[\frac{\partial \Omega^2}{\partial w}\frac{\partial w}{\partial E^1} + \frac{\partial \Omega^2}{\partial r^2}\frac{\partial r^2}{\partial E^1} - \frac{\partial \widehat{\Omega}^2}{\partial w}\frac{\partial w}{\partial E^1}\Big]. \end{aligned} \tag{C6}$$

Equation (C6) shows that, when individuals freely choose their occupation, they ignore the effects of their choice on welfare operating through changes in the wage rate, w, and the rate of interest faced by type 2 entrepreneurs, r^2. An increase in E^1 increases w, which increases the utility of workers and decreases the utility of both types of entrepreneurs. A higher E^1 also increases r^2 and thus lowers the utility of type 2 entrepreneurs.

With perfect information about types $\lambda = 0$, the overall effect of an increase in E^1 is ambiguous. Thus, the equilibrium with occupational choice is characterized by too few or too many entrepreneurs.

This result also holds under the assumption of asymmetric information about types. In this case, $\lambda \neq 0$ and, besides the effects of an increase in E^1 identified above, there is an additional effect on the self-selection constraint (C4): it requires that a type 2 entrepreneur has no incentive to mimic a type 1 entrepreneur. An increase in the number of type 1 entrepreneurs increases the wage rate, which, in turn, lowers the utility of a type 2 entrepreneur. At the same time, a higher wage rate also lowers the utility of the mimicker. The effect on the difference between the two utilities is ambiguous. Thus, a higher E^1 can tighten or relax the self-selection constraint. Again, we conclude that, in the equilibrium with occupational choice, there are too few, too many, or just the right number of entrepreneurs, depending on whether

$$\left.\frac{\partial \mathcal{L}}{\partial E^1}\right|_{E^1=(E^1)^{OC}} \gtreqless 0. \quad Q.E.D.$$

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