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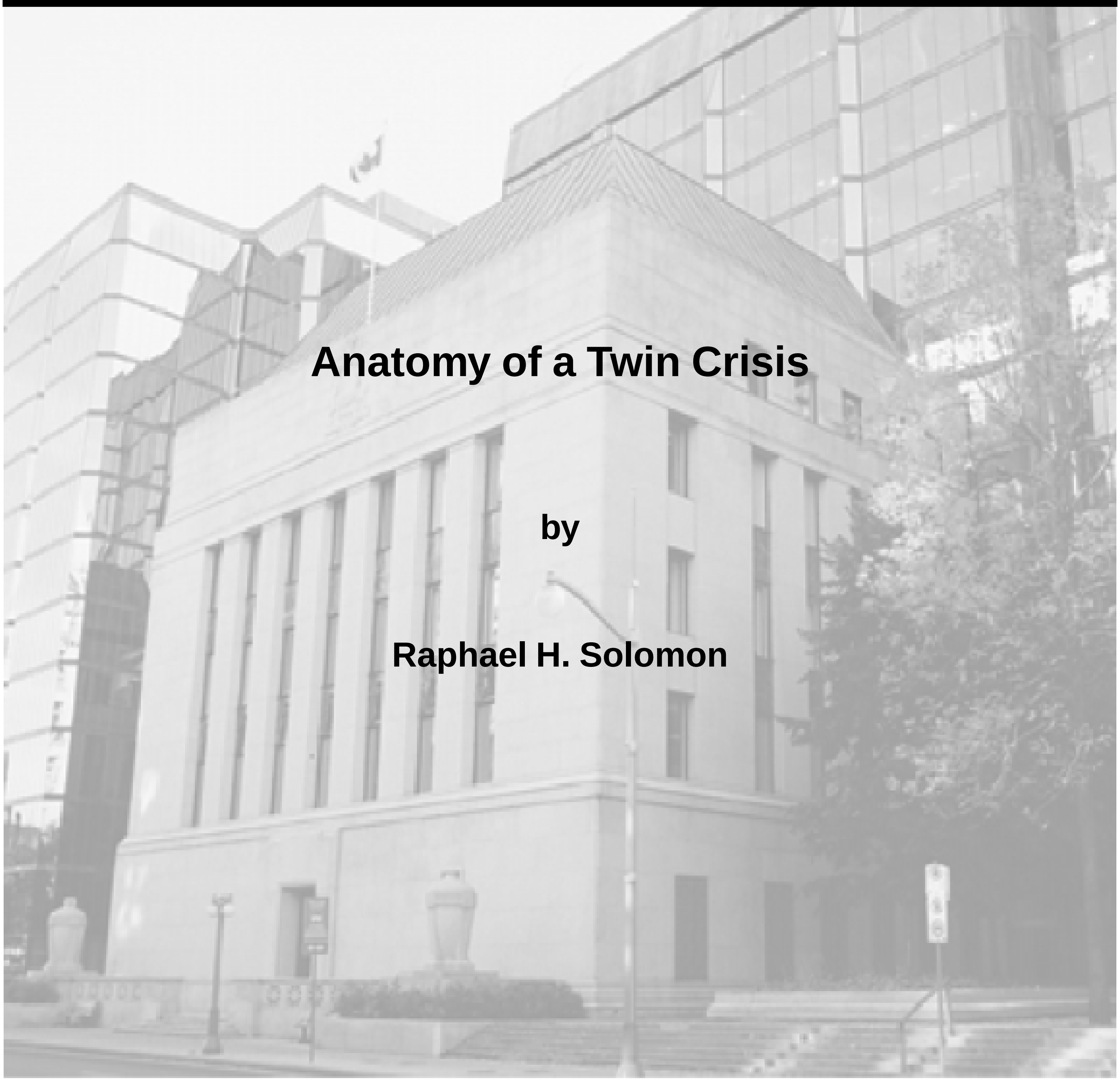

Anatomy of a Twin Crisis

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Abstract

The author presents a model of a twin crisis, in which foreign and domestic residents play a banking game. Both “honest” and run equilibria of the post-deposit subgame exist; some run equilibria lead to a currency crisis, as agents convert domestic currency to foreign currency. In the subgame, sunspot variables can affect the equilibrium. The author calculates the unique equilibrium of the game numerically, taking into account the possible realizations of the sunspot variables. He also calibrates the model to the Turkish economy, providing insight into the Turkish twin crisis of 2001.

JEL classification: E58, F30, G21
Bank classification: Exchange rates; Financial institutions

Résumé

L’auteur propose un modèle des crises jumelles (crise de liquidité bancaire suivie d’une crise de change) dans lequel des agents nationaux et étrangers interagissent dans le cadre d’un jeu bancaire. Dans le sous-jeu auquel se livrent les agents après leurs dépôts à la seule banque du modèle, deux équilibres existent : un équilibre « honnête » et un équilibre de panique bancaire. Le second type d’équilibre peut causer une crise de change, dès lors que les agents convertissent en la devise étrangère leurs fonds exprimés en monnaie locale. Dans le sous-jeu, des aléas de type « tache solaire » peuvent déterminer l’équilibre. L’auteur calcule numériquement l’équilibre unique du jeu en tenant compte des aléas possibles. Il étalonne son modèle en fonction de l’économie de la Turquie afin d’analyser la crise qui a frappé ce pays en 2001.

Classification JEL : E58, F30, G21
Classification de la Banque : Taux de change; Institutions financières

1. Introduction

Between 1976 and 2002, 38 countries experienced at least one period of twin crisis;[1] that is, a period when a currency crisis followed a banking crisis.[2] A twin crisis often leads to a severe recession. For example, the gross domestic product (GDP) of Korea, Indonesia, and Thailand each fell by more than 13 per cent after the Asian crisis of 1997. Turkey's GDP fell by 33 per cent in the six months following its crisis in 2001.

The reasons for studying twin crises are therefore several. First, twin crises are a worldwide phenomenon, affecting many countries directly and almost all countries through trade and financial linkages. Understanding how and why these crises occur aids both governments and firms. Second, twin crises are pernicious economic events; by studying them we may perceive ways to prevent them. Third, "[m]uch as the study of disease is one of the most effective ways to learn about human biology, the study of financial crises provides one of the most revealing perspectives on the functioning of monetary economies" (Eichengreen and Portes 1987, 10).

The twin crisis literature has grown out of the currency crisis literature. Currency crisis theorists belong to one of three schools. First-generation modellers believe that "bad" macroeconomic fundamentals cause currency crises. Second-generation modellers believe that random shifts in expectations cause currency crises.[3] Krugman (1999) and others present models that include the financial sector, referred to collectively as third-generation models.[4] These models typically draw on Diamond and Dybvig's (1983) model of bank runs.[5] Chang and Velasco (2000a, b, 2001) model a currency crisis in a Diamond and Dybvig framework. A bank-caused currency crisis is not the only way fragilities in the financial system can spill over into the currency markets; other examples include debt and equity markets. This paper, like many in the literature, models only the bank-runs channel, without prejudice to other channels in the financial sector that typically operate in real-world crises.

[1] These figures are based on Glick and Hutchison (2001).

[2] Kaminsky and Reinhart (1999) coined the term "twin crisis."

[3] The first papers in the first- and second-generation literatures are Krugman (1979) and Obstfeld (1986), respectively.

[4] For a review of the first- and second-generation models, see Jeanne (2000). For a review of early third-generation models, see Marion (1999).

[5] An early example is Velasco (1987).

Earlier twin crisis models have advanced our understanding of the twin crisis phenomenon by focusing on poor policy choices and multiple equilibria, as well as some of the links between components of the financial system. Indeed, many of these models explain particular twin crises well. There are three areas in which these models need improvement. First, because previous models effectively assume that every bank run leads to a currency crisis, they cannot explain the fact that some banking crises leave the currency regime in place.[6] Second, since these models do not explain equilibrium selection, they cannot measure the riskiness of the currency peg. Third, since they do not allow domestic and foreign residents to have different expectations between domestic residents and foreigners, they cannot account for their differing behaviour in times of crisis.[7]

The model in this paper improves on all three areas by striking new ground in the sunspots literature.[8] Sunspots are not new to models of financial crises. In the bank-runs literature, however, only one sunspot variable is used (Peck and Shell 2003). In the currency crisis literature, where two sunspot variables are used, they are uncorrelated (Cole and Kehoe 1996). I innovate by introducing two imperfectly correlated sunspot variables into a bank-run game: one is observed only by domestic agents and the other is observed only by foreign agents. The correlation of the sunspots allows for a rich array of equilibria of the bank-run game.[9]

The addition of two imperfectly correlated sunspots to a model of twin crisis alters the interpretation of excess bank withdrawals—that is, more withdrawals than the bank anticipated. Without any sunspots in the model, excess withdrawals represent a coordination failure among depositors. In a model with sunspots, excess withdrawals represent equilibrium behaviour. With only one sunspot in the model, excess withdrawals signal that a twin

[6] See Kaminsky and Reinhart (1999) and Glick and Hutchison (2001) for evidence.

[7] In many financial crises, domestic residents begin withdrawing from banks and the domestic currency before foreigners do. The locals set the stage for the crisis, although foreign participation in the run may be required for the crisis to become full-blown.

[8] Shell (1977) and Cass and Shell (1983) pioneer the mathematical treatment of sunspot variables. I define the term sunspot in section 2.

[9] If the two sunspots were perfectly correlated, every bank run would be by both domestic depositors and foreigners. In this case, every bank run leads to a currency crisis, a feature this paper strives to avoid. If the two sunspots were orthogonal, foreign agents could not make inferences about the behaviour of domestic agents, or vice-versa. This lack of information causes foreigners optimally to ignore their sunspot signal, another undesirable property.

crisis will occur with certainty. In my model, with two imperfectly correlated sunspots, excess withdrawals signal that a twin crisis is more likely to occur than if there had been no excess withdrawals, but it does not guarantee that it will occur.

The model developed in this paper is rich enough to answer six questions, most of which are unanswerable in earlier models. How does the interplay between foreign investors and domestic residents change the character, likelihood, or timing of a twin crisis? Can foreign investors, acting on their own, cause a twin crisis even when domestic investors do not withdraw their bank deposits? Can domestic investors, acting on their own, precipitate a twin crisis, when foreign investors maintain a high level of confidence in the economy and do not liquidate their investments? If foreign investors and domestic investors withdrew their funds from the banking system simultaneously, would the combined actions of both groups of investors exacerbate the effects of a twin crisis or make the occurrence of a twin crisis more likely? Should theorists focus on shifts in investor expectations to explain how the crisis occurs? How much overlap is there between the expectations of domestic and foreign investors?

The rest of this paper has the following structure. Section 2 explains the basics of the model and section 3 details the timing of the game. In section 4, I define and characterize some equilibria. Section 5 explains the parameterization of the model. In section 6, I present a numerical solution of the model. Section 7 provides concluding thoughts and ideas for extension. Proofs are reserved for the appendix.

2. The Model

The model follows the Diamond-Dybvig tradition of bank-run models and borrows notation from Chang and Velasco (2000b).

2.1 People and institutions

A small open endowment economy lasts for three periods: 0, 1, and 2. There are three types of households—domestic impatient, domestic patient, and foreign—and two institutions—a bank and a government. This section describes these actors; section 2.2 enumerates the choices they face.

A unit-measure continuum of domestic agents consists of patient and impatient agents. Impatient agents receive utility only from consuming in period 1, whereas patient agents receive utility whenever they consume. All domestic agents are risk-averse, but their utility depends on when they consume, not on who they are. Agents consuming c_I in period 1 receive utility $g(c_I)$.[10] Patient agents holding deposits in period 1 and consuming c_P in period 2 receive utility $g(A[c_P, m])$, where m represents the lira value of the stream of utility services obtained by holding deposits and the option to withdraw them.[11] A known fraction, λ, of domestic agents is impatient; domestic agents costlessly learn if they are patient in period 1. The patience or impatience of each agent is private information. There is also a unit-measure continuum of foreign agents. I assume that foreign agents deposit a small fraction of their investment portfolio in this bank and are thus risk-neutral over this small investment. Foreign agents earn ρ_1 per dollar deposited and held until period 1; deposits held until period 2 earn ρ_2. I assume that the bank can distinguish foreign and domestic agents (for tax purposes) and that it is not convenient for a foreign agent to contract with a domestic agent to receive the latter's return. If these assumptions were not present, in equilibrium, domestic and foreign agents would receive the same return and there would be little point in introducing foreign agents into the Diamond-Dybvig framework.

A single bank exists for risk-sharing purposes. Agents deposit at the bank, hoping to earn a high return. The bank's objective function is expected domestic utility.[12] The bank invests depositors' resources in two assets described below.

The government is both the fiscal and monetary authority. To accumulate foreign exchange reserves, the government sets a tax rate for domestic deposits, $(1-\eta)$, and for domestic withdrawals, τ. The deposit tax generates foreign exchange reserves in this model, because domestic deposits come from agents' dollar-denominated endowment. Since domestic agents also withdraw dollars, a withdrawal tax acts in a similar manner. As a sim-

[10] A list of all symbols used in the paper appears in the appendix.

[11] I assume that g is smooth, concave, positive, and bounded below but not bounded above. The boundedness assumptions ensure that expected utility is defined for all feasible contracts and that a solution to the optimization problem is computable. I also assume that A is smooth, concave, and positive. The function A converts consumption in period 2 and utility from deposits held in period 1 to a consumption equivalent.

[12] Were the two functions not identical, another bank could emerge, pay higher returns, and capture all of the first bank's business.

plifying assumption, I do not model the government's choice of η and τ; their values are exogenous.[13] The government also explicitly guarantees deposits of its citizens.

2.2 Assets, currencies, and goods

This model has two assets: world and productive. A unit invested in the world asset yields one unit whenever the investment is liquidated. A unit invested in the productive asset yields R_2 units if liquidated in period 2, but only R_1 units if liquidated in period 1. The values of R_1 and R_2 are fixed and known; they obey the relation $0 < R_1 < 1 < R_2$.[14] Let γ_b be the share of deposits the bank invests in the productive asset. I detail below how the bank chooses γ_b, as well as how agents would allocate their resources between the two assets in the event that they choose not to deposit at the bank.

There are two currencies in this model; the home currency is the lira and the other currency is the dollar. Initially, the government fixes the exchange rate at unity. There is a single good, usable both as an input to production and for consumption. For simplicity, I fix the price of this good at one dollar. One way to interpret this restriction is that the economy under study is so open that its composite consumption good is best modelled as an import. Since I do not model post-crisis welfare losses, it is not necessary to specify the price of consumption, except trivially. The bank invests in the two assets in period 0 and liquidates these investments in periods 1 or 2 for dollars. Similarly, the products or services underlying the productive asset are exported, which explains why liquidating the productive asset yields dollars.

The currency in which agents are paid is exogenous but requires some explanation (Table 1).

Domestic agents receive payments in dollars because they consume the imported composite good. Foreign agents receive payments in liras for two reasons. First, it allows them to be exposed to exchange rate risk, which is an important component in foreign investment decisions. Second, it separates the interests of the government and the bank. As will be shown later,

[13] Sensitivity analysis suggests that the optimal contract does not vary much for small changes in η and τ.

[14] The condition $R_1 < 1$ is necessary so that the productive asset does not dominate the world asset. Cooper and Ross (1998) note that, if R_1 is not sufficiently small, the bank can meet its liquidity needs by liquidating the productive asset.

Table 1: Currency of Payment

Agent	Currency of Payment
Domestic	Dollars
Foreign	Liras

this specification implies that the bank does not care about a devaluation whereas the government potentially does. It is useful to think of the currency specification as a reduced form; it allows the model to mimic certain aspects of foreign and domestic investment without fully specifying the currency investment functions of foreign and domestic agents. One drawback of the currency specification is that domestic agents do not mind if their currency devalues; since I do not perform a post-crisis welfare analysis in this paper, this consideration diminishes in importance.

2.3 A sunspot vector and its distribution

Cass and Shell (1983, 194) define a sunspot variable as "extrinsic uncertainty, that is, random phenomena that do not affect tastes, endowments or production possibilities." In some models with multiple equilibria, a sunspot variable serves as the equilibrium selection mechanism. Since Chang and Velasco (2000a, b, 2001) do not model equilibrium selection, it is impossible to quantify the risk of a twin crisis in their model. Although Cole and Kehoe (1996) model equilibrium selection in the Mexican financial crisis with a foreign and a domestic sunspot variable, their two sunspot variables are statistically independent. This is equivalent to assuming that there is no overlap in the extrinsic information observed by domestic and foreign agents. The model presented here relaxes these assumptions by having a vector of two correlated sunspot variables.[15]

A simple story explains why a dependent specification is relevant. Suppose foreign agents and domestic agents read a newspaper article to learn the state of the economy and its banks. Domestic agents and foreign agents

[15] Brennan and Cao (1997) show that domestic investors have an informational advantage when investing in the stock market. But they also show that some information is common to domestic and foreign investors. In this paper, foreign and domestic investors learn different information, but the information of domestic agents is not, per se, superior to that of foreign agents.

both read "between the lines" of the article, but they do so differently. The "common" aspect of the sunspot vector, which requires that the specification be dependent, is the newspaper article itself. The "semi-private" (i.e., common to one group of agents but not to the other) aspect of the sunspot vector is each group's interpretation of the article. Semi-private information requires that the correlation between the sunspots be imperfect.

Denote the domestic and foreign sunspot variables by s_d and s_f, respectively. Domestic agents observe only s_d, whereas foreign agents observe only s_f. Sunspot signals are either zeroes or ones. Without loss of generality, I interpret a one as a signal to run and a zero as a signal to wait. The joint distribution of the sunspot vector (s_d, s_f) is as follows[16]:

$$\begin{aligned}
&Pr(s_d = 0, s_f = 0) = (1 - \pi_1)(1 - \pi_3), \\
&Pr(s_d = 0, s_f = 1) = (1 - \pi_1)\pi_3, \\
&Pr(s_d = 1, s_f = 0) = \pi_1(1 - \pi_2), \\
&Pr(s_d = 1, s_f = 1) = \pi_1\pi_2,
\end{aligned}$$

where π_1, π_2, π_3 are in the open-unit interval and $\pi_2 > \pi_3$. The triple (π_1, π_2, π_3) describes the distribution of (s_d,s_f) completely.

3. The Game

3.1 Timing of the model

The model unfolds over three periods, as described below.

Period 0:

1. Domestic agents receive an endowment, e_d units of the consumption good. Foreigners arrive with investment funds, e_f dollars.

2. The government announces the tax rates, τ and $1 - \eta$.

[16] The sunspot vector (s_d, s_f) may be generated using the following process. Nature draws s_d first, taking on the value 1 with probability π_1. Nature draws s_f after s_d (but recall that foreigners do not observe s_d). The variable s_f takes on the value 1 with probability π_2 if s_d has taken on the value 1. On the other hand, if s_d has taken on the value 0, s_f takes on the value 1 with probability π_3. Both variables have conditional Bernoulli distributions, where the Bernoulli parameter for the unconditional distribution of s_f depends on the realization of s_d.

3. The bank announces the contract, $C(c_I, c_P, \gamma_b, \rho_1, \rho_2)$.

4. Foreign and domestic agents deposit at the bank.

5. The bank sends the deposit tax to the central bank. The bank invests the rest in the two assets according to the contract.

Period 1:

1. Domestic agents learn their type—patient or impatient.

2. Nature draws s_d and s_f, revealing them to domestic and foreign agents, respectively.

3. The bank opens for business. Agents of various types arrive in random order. Agents claiming to be impatient receive $(1-\tau)\, c_I$ dollars, if available. Agents claiming to be patient receive utility services valued at m liras. If foreign agents arrive, they receive $\rho_1 e_f$ liras. The bank borrows these liras from the central bank in exchange for a promise to repay $\rho_2 e_f$ dollars in period 2, if possible. The bank withholds taxes from domestic withdrawals.

4. If the bank serves all customers in queue, go to item 5. The bank liquidates assets to serve domestic agents. Deposit guarantees may be triggered.

5. Foreigners holding liras trade them for dollars at the central bank.

Period 2:

1. Any remaining investment in the productive asset matures.

2. The bank pays $(1-\tau)\, c_P$ dollars to domestic agents who claimed to be patient in period 1, if available. The bank remits τc_P dollars to the government as taxes. The bank also pays $\rho_2 e_f$ liras to any remaining foreign agents. If the bank can and needs to do so, it repays the loan of $\rho_2 e_f$ dollars to the government.

3. Foreign agents holding liras trade them for dollars at the central bank.

4. The economy ends.

3.2 The contract and related variables

The bank's decisions in period 0 are critical. In period 0, the bank offers a contract $C(c_I, c_P, \gamma_b,\rho_1, \rho_2) \in \Re^5_+$ to which agents respond. Determining which is the optimal contract is the subject of the equilibrium section. Notice that I preclude suspension of convertibility, since the payment to domestic agents is not contingent on the history of withdrawals. That is, the bank must continue paying c_I to domestic agents until it runs out of dollars and has liquidated all assets. This restriction is effectively a "strong form" of sequential service.

Let the value of holding deposits be proportional to expected domestic payouts conditional on there being no bank runs—$m = \kappa\left[\lambda c_I + (1-\lambda)\, c_P\right]$. Determinacy of m frequently poses a problem; Chang and Velasco (2000a) determine m by imposing a satiation level of money demand.

3.3 Rules of the game

In period 0, if agents deposit at the bank, the bank splits the deposits between the two assets. If agents do not deposit, domestic agents can divide their endowment between the two assets. Foreigners may not invest directly in the productive asset.

In period 1, agents arrive at the bank. The bank can always accommodate the demands of foreigners, since foreigners receive liras when withdrawing from the bank. The central bank prints liras costlessly, equating supply and demand. Domestic agents claiming to be patient present no problems for the bank either, since the bank also pays them liras. The bank can obtain dollars only by liquidating assets. As domestic agents claiming to be impatient arrive, the bank liquidates some assets and pays each agent $(1-\tau)\, c_I$, remitting τc_I to the central bank as taxes.[17]

The rules of the game include the bank's liquidation policy. Let a_d be the measure of domestic agents that claim to be impatient and let a_f be the measure of foreign agents that arrive in period 1. First, note that the bank is indifferent to the value of a_f. The bank pays foreign agents in liras; it need not liquidate any assets to pay foreigners.[18] (A corollary of this fact is

[17] Because of the return structure of the two assets, the bank will not liquidate any of the productive asset until it is forced to do so by the presence of domestic agents claiming to be impatient.

[18] That the bank does not need to liquidate assets to serve foreign agents comes from

the bank's indifference to a currency crisis.) Let $L(a_d)$ be the amount of the productive asset the bank will liquidate in period 1 if a_d agents claim to be impatient. Then,

$$L(a_d) = \max\left[\min\left(\gamma_b(\eta e_d + e_f), \frac{a_d c_I - (1-\gamma_b)(\eta e_d + e_f)}{R_1}\right), 0\right]. \quad (1)$$

Liquidation is bounded between 0 and total investment, $\gamma_b(\eta e_d + e_f)$. The fractional term in $L(a_d)$ represents the amount liquidated to pay a_d agents c_I. The bank may liquidate all its assets while additional domestic agents remain in queue claiming to be impatient.

After the bank has liquidated assets, the government deposit guarantee can be triggered. The deposit guarantee is subject to the following limits and restrictions[19]:

- If any foreign agents demand liras in period 1, the guarantee is not triggered.[20]

- The deposit guarantee is limited by the lesser of the taxes collected and the amount required to pay every domestic depositor in line c_I.[21] Unlike many real-world guarantees, this guarantee is based on the amount that depositors receive, c_I, not on the amount they originally deposited, e_d.

the central bank's lending liras inelastically to the bank.

[19] This deposit guarantee does not increase moral hazard, in that it does not cause the bank to invest more in the productive asset than it would in the absence of the guarantee. The guarantee is present to increase the realism of the model. If bank runs occur in equilibrium with a deposit guarantee, they would also occur in equilibrium without one.

[20] This rule highlights the idea that a foreign presence entails higher scrutiny of government actions. Foreign agents know that the use of the deposit guarantee reduces the dollar value of the liras they have just received from the bank by lowering government reserves. The government's policy of not paying its citizens the guarantee when foreigners are present encourages foreign depositors to have confidence in the bank. By arriving in period 1, foreign agents "pay a monitoring cost," since their expected return is lower in period 1 than in period 2. The main reason for this rule is to create the possibility of two distinct occurrences, each of which has different consequences for the financial system: one in which all domestic agents claim to be impatient and all foreigners arrive in period 1, and one in which all domestic agents claim to be impatient and all foreigners arrive in period 2. I show later that both of these occurrences can be equilibrium outcomes. If this rule were not in place, either one or the other of these possibilities might not occur in equilibrium, or they might both occur but have identical consequences for the financial system.

[21] The fact that the payout is bounded by the taxes collected guarantees that the dollar

- A domestic agent not in line when the guarantee is paid in period 1 receives nothing from the government in period 2.[22]

After a government payout occurs, the bank distributes $(1-\tau)\, c_I$ to each agent until its resources are again exhausted.

Foreigners proceed to the central bank after collecting liras at the bank. If dollars in the reserve vault exceed the liras in circulation, each lira can be exchanged for one dollar. If not, the central bank pays foreigners proportionally, effectively devaluing the lira.

In period 2, domestic agents that claimed to be patient return to the bank. If the bank has unliquidated assets, these assets mature; the bank uses the dollars from maturing investments to pay domestic agents claiming to be patient $(1-\tau)\, c_P$, remitting τc_P to the government as taxes.[23]

After all assets have matured and the bank has paid all domestic agents claiming to be patient, the bank repays any outstanding loans to the central bank, if possible. This guarantees that bank profits cannot exceed zero. Finally, if foreign agents have liras, they convert them to dollars at the central bank. The payment procedure is the same as the one used in period 1.

4. Equilibrium

4.1 Classes of contract

Diamond and Dybvig (1983, 409) show that the contract the bank has offered determines whether the post-deposit subgame has a run equilibrium. The bank can offer a contract that prevents runs or a contract that allows runs.[24]

value of liras received by foreigners is strictly positive in all states of the world. If this rule were not present, it would affect foreign equilibrium behaviour. The fact that the payout is bounded by the amount required to restore domestic depositors to whole guarantees that the bank does not make positive profits in a run.

[22] This rule guarantees that domestic run equilibria exist. If it were not present, domestic agents would always arrive in period 2 and be guaranteed c_I.

[23] If the bank cannot serve every domestic agent claiming to be patient, it serves as many agents as possible, but does not change the amount each agent receives. Even if the bank cannot serve everyone in period 2, the government does not pay depositors in period 2. These considerations are irrelevant in equilibrium.

[24] A contract is said to *prevent runs* if the set of equilibria of the subgame played after the announcement of that contract does not include a run equilibrium. A contract that does not prevent runs is said to *allow runs*. I define a bank run more precisely later.

A non-naive bank should understand the possibility of a crisis; it should act to prevent it in some circumstances and not do so in others. Peck and Shell (2003) demonstrate that the optimal contract may come from either the No-Run Class or the Run Class.[25] Since these classes are disjointed, the bank can compute the optimal contract from each one separately and compare the optima in terms of expected utility. The bank then chooses the "best of the best."

4.2 Equilibrium of the post-deposit subgame

The subgame consists of a move by Nature, "choosing" the realization of the sunspot vector, followed by choices by the three types of agents. Two simplifications can sharpen the focus. First, I consider only type-symmetric equilibria; that is, equilibria in which all agents of the same type act identically. Second, I note that impatient agents always rationally claim to be impatient. I can express an equilibrium as a strategy for patient agents and a strategy for foreign agents such that no one wishes to deviate. More formally, let

$\sigma_d : s_d \longrightarrow \{CI, CP\}$, and let $\sigma_f : s_f \longrightarrow \{A1, A2\}$, where
CI means claim to be impatient; CP means claim to be patient;
$A1$ means arrive in period 1; $A2$ means arrive in period 2.

Each type of agent has four possible strategy functions (Table 2).

Table 2: Agents' Strategies by Type

Strategy	$s_d = 0$	$s_d = 1$	Strategy	$s_f = 0$	$s_f = 1$
$\sigma_{d,1}$	CP	CI	$\sigma_{f,1}$	$A2$	$A1$
$\sigma_{d,2}$	CI	CI	$\sigma_{f,2}$	$A2$	$A2$
$\sigma_{d,3}$	CP	CP	$\sigma_{f,3}$	$A1$	$A1$
$\sigma_{d,4}$	CI	CP	$\sigma_{f,4}$	$A2$	$A1$

Let $\Sigma_d = \{\sigma_{d,1}, \sigma_{d,2}, \sigma_{d,3}, \sigma_{d,4}\}$ and $\Sigma_f = \{\sigma_{f,1}, \sigma_{f,2}, \sigma_{f,3}, \sigma_{f,4}\}$ be the strategy sets for domestic and foreign agents with typical elements σ_d and σ_f, respectively. Each individual agent has an expected payoff function—$\mathrm{E}_s U_d$ for domestic agents and $\mathrm{E}_s U_f$ for foreign agents—which depends on the strategy

[25] Peck and Shell's (2003) paper contains a bank-run model; I apply their methodology to a twin crisis model.

played by all other agents of the same nationality, the strategy played by all agents of the opposite nationality, the strategy played by that individual agent, and the contract offered by the bank.

Definition 1 *A pair of strategies* (σ_d, σ_f) *is a type-symmetric Nash equilibrium of the post-deposit subgame if two conditions hold:*

1. For all patient agents in $[0, 1-\lambda]$ *and for all* $\tilde{\sigma}_d \in \Sigma_d$,
$E_s U_d(\sigma_d, \sigma_f, \sigma_d, \cdot) \geq E_s U_d(\sigma_d, \sigma_f, \tilde{\sigma}_d, \cdot)$.[26]

2. For all foreign agents in $[0, 1]$ *and for all* $\tilde{\sigma}_f \in \Sigma_f$,
$E_s U_f(\sigma_d, \sigma_f, \sigma_f, \cdot) \geq E_s U_f(\sigma_d, \sigma_f, \tilde{\sigma}_f, \cdot)$.

Note that the Nash equilibrium of the subgame takes Nature's move into account; agents may cooordinate on the realization of the sunspot vector. Aumann (1974, 90–92) discusses the equivalence of sunspot games and correlated equilibrium games. In particular, Aumann notes that there is a 1-1 correspondence between the Nash equilibria of the game where Nature is included as a player and the correlated equilibria of the game that Nature does not play. The equilibria found here differ from those of the bank-run literature, because of the informational structure: information is neither all public nor all private.

4.3 Equilibrium of the subgame: No-Run Class

For contracts in this class, domestic bank runs are suboptimal by construction. The bank does not consider foreign utility when designing the contract, except insofar as is necessary to ensure foreign deposits. Despite this fact, foreigners will not run here, either.

Theorem 1 *If domestic agents are known not to be running on the bank, it is always optimal for foreigners to arrive in period 2.*[27]

Proof. See the appendix. ■

[26] The first three arguments in $E_s U_d$ and $E_s U_f$ are the action played by all patient agents, the action played by all foreign agents, and the action played by a deviating agent, respectively. The suppressed arguments in $E_s U_d$ and $E_s U_f$ stand for the contract vector $(c_I, c_P, \gamma_b, \rho_1, \rho_2)$.

[27] All theorems in this paper implicitly assume the model environment presented in section 2 and the rules of the game enumerated in section 3.

The unique equilibrium of the subgame is $(\sigma_{d,3}, \sigma_{f,2})$. Note that sunspots do not matter for this equilibrium. Players make the same choices, regardless of what Nature does beforehand. Since domestic agents ignore sunspots when faced with a contract in this class, foreign agents are also better off selecting a pure strategy.

4.4 Equilibrium of the subgame: Run Class

Theorem 2 *One Nash equilibrium of this subgame is* $(\sigma_{d,1}, \sigma_{f,1})$.

Proof. See the appendix. ∎

Note that, for this equilibrium, sunspots matter.[28] In some situations, it is better to run on the bank; in other situations, it is better not to run. This is the classical case of Diamond and Dybvig (1983).

4.5 Game equilibrium: assumptions and conditions

Following the announcement of a contract of the No-Run Class, the subgame has a unique equilibrium. Following the announcement of a contract of the Run Class, there are at least two type-symmetric equilibria: one for which sunspots matter and one for which sunspots do not matter. I assume that agents choose the equilibrium for which sunspots matter.[29] Relaxing this assumption complicates the bank's objective function greatly.

I assume that agents have the choice of depositing all of their endowment in the bank or of not depositing at all. Depositing at the bank must be individually rational. Agents must receive a higher expected return by depositing than in autarchy. Consider the deposit decision for foreigners. Since foreigners may not invest in the productive asset except through the bank, their net autarchic return is 0. They receive e_f when their investment of e_f matures. Since foreigners are risk-neutral, they deposit so long as the gross expected return equals e_f. Domestic agents can split their endowment between the two assets. Let γ_{aut} be the fraction of a typical domestic agent's endowment invested in the productive asset. The autarchic return

[28] The equilibrium derived in section 4.3 is also an equilibrium for subgames played after announcing a contract of the Run Class.

[29] Peck and Shell (2003) also make this assumption. If one wanted to complicate the game greatly, one could remove this assumption and replace it with a contract-dependent equilibrium selection mechanism.

is $r_{aut}(\gamma_{aut}) = \lambda g(e_d[R_1\gamma_{aut} + 1 - \gamma_{aut}]) + (1-\lambda) g(e_d[R_2\gamma_{aut} + 1 - \gamma_{aut}])$. There is a unique $\gamma^*_{aut} \in [0,1]$ that maximizes this expression.[30] Thus, the autarchy return is $r_{aut}(\gamma^*_{aut})$. Any contract must satisfy two conditions: expected domestic utility equals or exceeds $r_{aut}(\gamma^*_{aut})$, and expected foreign utility equals e_f.[31]

4.6 Aggregation of the equilibrium

The bank is concerned less with the action of its depositors individually than with their actions collectively. We can aggregate domestic and foreign actions as the pair (a_d, a_f).

Definition 2 *A domestic bank run occurs when* $a_d = 1$.

Definition 3 *A bank run by foreigners occurs when* $a_f = 1$.

Definition 4 *A currency crisis occurs when the demand for dollars exceeds the supply of dollars in the central bank's reserves.*

In some equilibria, the pair (a_d, a_f) depend on the state (the realization of the sunspot). There are four possible states:
$(s_d, s_f) = (1,0)$ [state 1]; $(s_d, s_f) = (1,1)$ [state 2];
$(s_d, s_f) = (0,0)$ [state 3]; $(s_d, s_f) = (0,1)$ [state 4].

4.7 Equilibrium of the game: No-Run Class

Suppose the bank chooses the best contract from the No-Run Class. The bank's problem can be written as follows:

[30] I prove that γ^*_{aut} is unique in the appendix. The existence of γ^*_{aut} follows from the compactness of $[0,1]$ and the continuity of r_{aut} in γ_{aut}. Note that the value of γ^*_{aut} depends on λ as well as the curvature parameters of the utility function.

[31] That the foreign individual rationality condition holds with equality follows from the fact that the bank maximizes expected domestic utility but is indifferent to foreign utility.

$$\max_{C\in\Re_+^5} \lambda g((1-\tau)\,c_I)+(1-\lambda)\,g(A\,[(1-\tau)\,c_P,m]), \text{ s.t.}, \tag{2}$$

$$\lambda c_I=(1-\gamma_b)\,(\eta e_d+e_f)\,, \tag{3}$$

$$[1-\gamma_b+R_1\gamma_b]\,(\eta e_d+e_f)>c_I, \tag{4}$$

$$A\,(c_P,m)>c_I, \tag{5}$$

$$\lambda g\left((1-\tau)\,c_I\right)+(1-\lambda)\,g\left(A\,[(1-\tau)\,c_P,m]\right)\geq r_{aut}\left(\gamma^*_{aut}\right), \tag{6}$$

$$\lambda c_I+(1-\lambda)\,c_P+e_f=[1-\gamma_b+R_2\gamma_b]\,(\eta e_d+e_f)\,. \tag{7}$$

Some explanation of the above equations is necessary. The maximand, (2), represents expected domestic utility. The bank calculates expected domestic utility differently for contracts of the No-Run Class than it does for contracts of the Run Class. If the contract is of the No-Run Class, the only uncertainty remaining in the game is idiosyncratic. The bank knows that exactly λ domestic agents claim to be impatient and receive $g((1-\tau)\,c_I)$, and that exactly $(1-\lambda)$ domestic agents claim to be patient and receive $g\left(A\,[(1-\tau)\,c_P,m]\right)$.

Since the bank is certain there are no runs, it invests exactly λc_I in the world asset. Investing less requires costly early liquidation of the productive asset. Investing more is inefficient, because the bank can earn higher returns by using the information that runs will not occur. Equation (3) specifies this restriction on investment. Inequalities (4) and (5) define the No-Run Class. Inequality (4) guarantees that patient agents receive at least c_I regardless of what they claim. Inequality (5) ensures that patient agents receive payment strictly greater than c_I if they claim to be patient. Inequality (6) is the individual rationality constraint for domestic agents; if (6) is satisfied, domestic agents will deposit their resources with the bank. There is no individual rationality constraint for foreigners, since foreign individual rationality is satisfied with any pair $(\rho_1,\rho_2)=(\rho_1,1)$, where $\rho_1<1$. In this contract, $\rho_2=1$, because the bank wants to satisfy the (implicit) foreign individual rationality constraint with equality. Since this contract induces no runs and the best alternative opportunity for foreigners is storage (the "world" asset), $\rho_2=1$ solves the foreign constraint exactly. The maximum profit the bank can make is zero. Equation (7) is the zero profit constraint.

Theorem 3 *Inequalities (4) and (5) are necessary and sufficient for a contract to be of the No-Run Class.*

Proof. See the appendix. ■

4.8 Equilibrium of the game: Run Class

Suppose the bank selects the best contract from the Run Class. The problem can be written as follows:

$$\max_{C\in\Re^5_+} \sum_{s=1}^{4} \Pr_s U_{d,s}(\sigma_{d,1},\sigma_{f,1}), \text{ s.t.,} \tag{8}$$

$$\lambda c_I \le (1-\gamma_b)(\eta e_d + e_f), \tag{9}$$

$$[1-\gamma_b + R_1\gamma_b](\eta e_d + e_f) \le c_I, \tag{10}$$

$$A(c_P, m) \ge c_I, \tag{11}$$

$$\sum_{s=1}^{4} \Pr_s U_{d,s}(\sigma_{d,1},\sigma_{f,1}) \ge r_{aut}(\gamma^*_{aut}), \tag{12}$$

$$\sum_{s=1}^{4} \Pr_s U_{f,s}(\sigma_{d,1},\sigma_{f,1}) = e_f, \tag{13}$$

$$\lambda c_I + (1-\lambda)c_P + \rho_2 e_f = [1-\gamma_b + R_2\gamma_b](\eta e_d + e_f). \tag{14}$$

In the above maximization problem, $U_{d,s}$ is the expected domestic utility of state s, $U_{f,s}$ is the expected foreign utility of state s, and $\Pr_s$ is the probability of state s. These probabilities come from the distribution of the sunspot vector, as noted by Aumann (1987). The expected utility depends upon the Nash equilibrium. The details of $U_{d,s}$ and $U_{f,s}$ are given in the appendix.

Inequality (9) guarantees that, if there is no bank run, the bank need not liquidate the productive asset. It is the analogue of (3). This constraint does not necessarily bind, since the bank may choose to have additional resources on hand. Inequalities (10) and (11) are the conditions for a contract to be of the Run Class. Inequality (12) and equation (13) are the individual rationality constraints; as long as they are met, foreign and domestic agents deposit, eschewing autarchy. Equation (14) is the zero profit constraint.

Theorem 4 *Inequality (10) is sufficient for contracts to be of the Run Class.*

Proof. See the appendix. ■

The results in this section are comparable to those in Peck and Shell (2003). In their paper, there is a Run and a No-Run Class; sunspots matter only for contracts in the Run Class. In this paper, when a contract is of the Run Class, all four states listed in the aggregation section can occur with positive probability. Since some of these states imply a twin crisis and some of them do not, my model shows how some banking crises leave the exchange rate regime in place. In addition, in some of these states foreign and domestic agents behave differently. Finally, if a contract is of the Run Class, it is possible to calculate the riskiness of the currency peg from the sunspot distribution. I thus achieve my three modelling goals; the numerical results follow.

5. Calibration

The complexity of the model does not admit closed-form solutions. Calibrating the model thus has two purposes. First, it allows me to describe the solution and to show its existence. Second, it allows me to analyze policy.

Many countries have experienced twin crises. The Turkish case is particularly useful for analyzing this model, because of the role of foreign depositors. The crisis unfolded as follows (*The Economist* 2000a, b, 2001). In January 2000, Turkey adopted a fixed exchange rate path, in an effort to control inflation.[32] In September, the Turkish government created new banking regulations. By December, ten Turkish banks had failed and fallen under government supervision. As the year 2001 began, investors noted the diminished capacity of the Turkish government to maintain the fixed exchange rate; confidence in the peg dropped precipitously. On 19 February, foreigners withdrew $5 billion from Turkish investments, an amount that exceeded one-fourth of the foreign exchange reserves of the central bank. Three days later, the government floated the Turkish Lira (TL).

In this paper, I calibrate the model loosely to Turkish data; I evaluate the Turkish fixed-exchange rate policy explicitly in the following sections.

[32] The Governor of the Central Bank of the Republic of Turkey, Gazi Erçel, explained the fixed exchange rate to foreign investors in London: "It will be noted that while we are making [a] strong commitment to a pre-announced exchange rate path, we are simultaneously announcing our exit strategy, which should allay concerns about the difficulty of making a smooth exit from such systems" (Erçel 2000, 6).

5.1 Utility functions

Based on Holman's (1998) empirical results, I use a Cobb-Douglas form for the money-in-the-utility (MIU) function.[33] In particular, let

$$A\left[c_P, m\right] = c_P^{\beta} m^{1-\beta},\ 0 < \beta < 1. \tag{15}$$

I set $\beta = 0.98$, following Holman. A high value of β implies that liquidity services from holding deposits are not valuable, something common to countries with weak banking systems. For the overall utility function, I choose the following "hybrid"[34]:

$$g\left(c_I\right) = -\frac{1}{\alpha}\exp\left[-\alpha c_I\right] + \frac{1}{\alpha} + \zeta c_I,\ \alpha > 0,\ \zeta > 0. \tag{16}$$

From the estimates of Antle (1987) and Wolf and Pohlman (1983), I calibrate α to values ranging from 3.5 to 5. I thus check the solution for sensitivity to the parameter α.[35] The parameter ζ is atheoretical; computational considerations dictated that I set it to 10^{-10}.

5.2 Other parameters

I take estimates of π_1, π_2, and π_3 from Kaminsky and Reinhart (1999). They estimate the unconditional probability of a bank run ($\pi_1 = 0.1$), the probability of a currency crisis conditional on a bank run ($\pi_2 = 0.46$), and the probability of a currency crisis conditional on no bank run ($\pi_3 = 0.29$). I set τ to 0.145 to match the average tax/GDP ratio for Turkey from 1987–2000. A value for R_2 of 1.7 accords with the annualized dollar-based return

[33] Holman estimates a variety of functional forms for the MIU function using American annual data from 1889 to 1991. Using generalized method of moments (GMM) and tests for overidentifying restrictions, she cannot reject the Cobb-Douglas functional form. For more on MIU functions, see Feenstra (1986) and Brock (1974).

[34] This function has the same properties as a constant absolute risk aversion (CARA) utility function for small values of c_I; it also possesses the properties of a linear utility function. The coefficient of absolute risk aversion for the function g is $\frac{-\alpha e^{-\alpha c_I}}{e^{-\alpha c_I}+\zeta}$. It ranges from $\frac{-\alpha}{1+\zeta}$ when $c_I = 0$ to 0 as c_I approaches ∞. This function is positive, increasing, concave, and bounded below, but not bounded below for all non-negative values of c_I.

[35] The papers by Antle (1987) and Wolf and Pohlman (1983) estimate the parameter α in a CARA function.

from the Istanbul Stock Exchange Composite Index from 1986–1999. Allen and Gale (2000) suggest that R_1 should be fairly small, so that banks do not liquidate the productive asset to meet predictable liquidity needs; on this basis, I set $R_1 = 0.3$. Because there is evidence that inverse monetary velocity is non-stationary,[36] I fix $\kappa = 0.16$ to match the value of inverse M1 velocity in Turkey in the last quarter before the fixed exchange rate. The ratio of foreign deposits to total deposits in Turkey exceeded 50 per cent in 1995; to capture this feature, I set $e_d = e_f = 10$.[37] Finally, since the equilibrium should be sensitive to my choice of λ, I choose several values ranging from 0.25 to 0.85. Table 3 summarizes all the parameterizations used in the empirical model.[38]

Table 3: Parameters Used in the Calibration

Parameter	Value(s)	Parameter	Value(s)
e_d	10	β	0.98
e_f	10	λ	0.25 to 0.85, step 0.05
R_1	0.3	η	0.94
R_2	1.7	τ	0.145
		π_1	0.1
α	3.5 to 5, step 0.5	π_2	0.46
ζ	10^{-10}	π_3	0.29

6. Results and Discussion

It is convenient to partition the set on which λ is defined into three values: low, medium, and high. Optimal contracts for low and high values of λ have no bank runs in equilibrium, and thus no twin crises. Optimal contracts for medium values of λ give rise to bank runs in equilibrium; as a result, twin crises occur with positive probability.

[36]For a non-stationary time series, no long-run expected value exists. Accordingly, the average over a particular sample is a meaningless statistic.

[37]The foreign deposit ratio is $\frac{e_f}{e_f+e_d}$.

[38]In Table 3, "step" denotes the size of the increment in an arithmetic sequence with a given start and finish.

Table 4: Optimal Contracts – Low Values of λ

α	λ	c_I	c_P	γ_b
3.5	0.25	6.7	26.84	0.91
4	0.25	5.9	27.29	0.92
4.5	0.25	5.2	27.69	0.93
5	0.25	4.7	27.97	0.94
3.5	0.3	6.7	27.95	0.90
4	0.3	5.9	28.53	0.91
4.5	0.3	5.2	29.04	0.92
5	0.3	4.7	29.4	0.92
3.5	0.35	6.7	29.22	0.88
4	0.35	5.9	29.95	0.89
4.5	0.35	5.2	30.59	0.91
5	0.35	4.7	31.05	0.92

Table 5: Optimal Contracts – Medium Values of λ

α	λ	c_I	c_P	ρ_1	ρ_2	γ_b
4	0.40	10.3	15.1	0.06	1.53	0.67
4	0.43	10.3	15.1	0.03	1.54	0.67
4	0.50	10.3	16.1	0.07	1.53	0.67
4	0.52	10.3	16.1	0.04	1.54	0.67
4	0.55	10.3	17.1	0.10	1.51	0.67

6.1 Contracts of the No-Run Class

There are three notable patterns in Tables 4 to 6.[39] First, consumption given to agents claiming to be impatient depends negatively on the curvature of the utility function; that is, on the degree of risk aversion.[40] This result can

[39]Foreign consumption is defined by ρ_1 and ρ_2. For contracts of the No-Run Class, $\rho_2 \equiv 1$ and ρ_1 is an arbitrary value between 0 and 1.

[40]The coefficient of absolute risk aversion is $ARA(\alpha) = \frac{-\alpha e^{-\alpha c_I}}{e^{-\alpha c_I}+\zeta}$. After some rearranging, $ARA\prime(\alpha) = \frac{\zeta e^{-\alpha c_I}\left(\alpha^2-1\right)-e^{-2\alpha c_I}}{\left[e^{-\alpha c_I}+\zeta\right]^2}$. If $\alpha < 1$, $ARA\prime(\alpha) < 0$ for any value of ζ. Given that $\zeta = 10^{-10}$, the negative term dominates the positive term for any reasonable value of c_I. With relative risk aversion defined as a negative number, a fall in $ARA(\alpha)$ implies

Table 6: Optimal Contracts – High Values of λ

α	λ	c_I	c_P	γ_b
3.5	0.75	6.7	57.75	0.74
4	0.75	5.9	61.83	0.77
4.5	0.75	5.2	65.4	0.8
5	0.75	4.7	67.95	0.82
3.5	0.8	6.7	69.34	0.72
4	0.8	5.9	74.78	0.76
4.5	0.8	5.2	79.54	0.79
5	0.8	4.7	82.94	0.81
3.5	0.85	6.7	88.66	0.71
4	0.85	5.9	96.36	0.74
4.5	0.85	5.2	103.1	0.77
5	0.85	4.7	107.9	0.79

be explained as follows. As domestic agents become more risk-averse, the bank must pay more to agents claiming to be patient, to induce them to report their types accurately. Although the amount available to the bank to pay domestic agents is not fixed (it depends on γ, a choice variable), it is bounded by the bank's total resources. More consumption to patient agents entails less consumption to impatient agents.

Second, investment in the productive asset increases with risk aversion, because patient agents demand more consumption the more risk-averse they are, and because the bank pays agents claiming to be patient from the maturing productive asset.

Third, investment in the productive asset falls as the proportion of impatient agents rises, because dollars available for investment are fixed. As the proportion of impatient agents increases, more investment dollars have to be diverted to the world asset to prevent unnecessary liquidations of the productive asset.

increasing risk aversion. Thus, an increase in α also increases absolute risk aversion.

6.2 Contracts of the Run Class

The pattern of payments to domestic agents in contracts of the Run Class differs from that of the previous section. Payment to agents claiming to be impatient (c_I) does not vary with either the curvature parameter, α, or the impatience parameter, λ. Payments to agents who claim to be patient increase with λ but do not vary with α, because runs occur with positive probability. The bank keeps a precautionary reserve, in that it optimally[41] invests more in the world asset than it needs to meet the demands of impatient agents. This reserve allows the bank to increase c_P without decreasing c_I. The payment results for contracts of the Run Class differ from those in Peck and Shell (2003). The difference emerges from the strong sequential service constraint.

6.3 Currency crises

Several other results bear mention. First, for any contract that leads to a twin crisis with positive probability, there is a positive probability of a domestic bank run alone (although these probabilities are not equal). Furthermore, if a twin crisis occurs, a domestic bank run must have occurred. These two results replicate the literature by Chang and Velasco (2000a, b, 2001). One new result is that a bank run by foreigners need not lead to a currency crisis. This emerges from two assumptions: first, that foreign agents are paid in liras but domestic agents in dollars, and second, that foreign and domestic agents choose their strategies simultaneously.

The probability of a currency crisis depends on several parameters. First, it depends on λ being in the range where there are contracts of the Run Class. Second, it depends on the other parameters being such that a currency crisis occurs in at least one of the four states. In particular, a currency crisis occurs when the sunspot vector is in state 1 if $(1-\eta)\,e_d < \rho_2 e_f$. A currency crisis occurs when the sunspot vector is in state 2 or 4 if $(1-\eta)\,e_d + \tau\lambda c_I < \rho_1 e_f$. A currency crisis never occurs in state 3, by construction. For the values used in the calibration above, a currency crisis occurs only in state 1. That is, for the calibrated values, the probability of a currency crisis is the probability

[41] The fraction of deposits invested in the productive asset γ_b is optimal in that it maximizes expected domestic utility. The bank chooses to maintain this reserve despite the fact that the reserve is insufficient either to stop a bank run from occurring or to pay every agent in a run in the absence of the government deposit guarantee.

of the sunspot vector being in state 1: $\pi_1 (1 - \pi_2)$. While this probability is increasing in the probability of a domestic bank run, it is decreasing in the probability of a foreign bank run. This result comes from the currency specification, as well.

Table 7: Variance of Prcc with the Pi Vector

π_1	π_2	π_3	Prcc
0.05	0.45	0.29	0.0275
0.1	0.4	0.29	0.06
0.1	0.45	0.29	0.055
0.1	0.46	0.29	0.054
0.15	0.3	0.29	0.105

It is interesting to explore the sensitivity of the currency crisis probability (Prcc) to the triple (π_1, π_2, π_3). In Table 7, I do not vary π_3, since Prcc does not depend on π_3, given the other parameters of the model. The selected values of π_1 and π_2 are chosen to be representative of a set for which the quantity $PRB = \dfrac{\pi_1 \pi_2}{\pi_1 \pi_2 + (1 - \pi_1) \pi_3}$ has a value less than or equal to its value in the calibrations. As I explain in more detail in the appendix, when PRB is small enough, it is equilibrium behaviour for foreigners to run and locals not to run when the sunspot vector's realization is $(0, 1)$. As shown in Table 7, the riskiness of the currency peg depends critically on the sunspot distribution. A government willing to peg its currency if a domestic bank run is unlikely might be unwilling to do so if a domestic bank run were slightly more likely.

6.4 General discussion

The interplay between foreign and domestic agents affects the timing of the twin crisis. Without foreign agents, all financial crises occur in period 1, as in Diamond and Dybvig (1983). The presence of foreign depositors also affects the character of twin crises. In models with only one group of depositors, the occurrence of a crisis depends on the actions of that group. In this model, both the foreign and domestic depositors have the ability to run on the bank; does this cause a crisis if they act alone? Foreign investors alone cannot precipitate a twin crisis. A foreign bank run does not cause the bank to collapse, because it does not have to supply precious foreign currency to do so. The foreign bank run causes a currency crisis, but since there is no bank collapse—that is, since the bank remains solvent—it is not a twin crisis. Domestic investors alone can cause a twin crisis. Their withdrawal of funds from the bank causes the bank to collapse. Foreign investors who arrive in period 2 receive worthless scrip (that is, domestic currency). But the foreign currency backing up the domestic currency has been depleted in bailing out the bank's depositors. Thus, the fixed exchange rate fails and there is a currency crisis.

Another possibility not present in earlier models is a simultaneous run on the bank by both foreign and domestic agents. This combined assault has several repercussions for the financial system. The fact that foreign agents run means there is no payout from the deposit guarantee. This means that domestic depositors lose and foreign depositors gain. In the range of parameter values in which such a "double run" is possible, the losses by domestic depositors outweigh the gains by foreign depositors in dollar terms. However, since domestic depositors are risk-averse and foreign depositors are risk-neutral, it may be more appropriate to compare utility losses with utility gains. Domestic utility losses are smaller than foreign utility gains. It is not possible to say that a twin crisis with foreign participants is worse than one without them, unless one decides what weight to give to the utility functions of each.

The model suggests that theorists should continue to focus on shifts in investor expectations, since these affect the selection from among the four type-symmetric equilibria.

7. Conclusion

This paper has presented a model of a twin crisis in which fragilities in the banking sector can spill over into a currency crisis. It is unique in the banking crisis literature, because it models foreign and domestic depositors as having different preferences and different expectations. It is unique in the sunspots literature, because it contains two imperfectly correlated sunspots. Finally, it is unique in the twin crisis literature because (for many parameter values), there can be either a banking crisis but no currency crisis, a currency crisis but no banking crisis, a twin crisis, or no crisis at all.

One logical extension of the model is in the area of modelling expectations. The present model does not tie expectations to the fundamentals. This is unrealistic, according to Demirgüç-Kunt and Detragiache (1997, 5), who argue that "crises do not appear to be solely driven by self-fulfilling expectations as in Diamond and Dybvig (1983)." Within the framework of this model, one can account for fundamentals by allowing the probabilities π_1, π_2, and π_3 to evolve over time due to "real" events in the model. Extensions in this direction require more periods, where each collection of three periods is a realization like the present model.[42] One could derive a definition of economic growth and let the probabilities depend on economic growth. This may be a useful direction for future research.

[42] This idea borrows from Temzelides (1997).

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Appendix

A.1 Details of Expected Utility

The expected utilities earned by domestic and foreign agents in each of the four states are the following:

$$
\begin{aligned}
U_{d,1} &= \left(\min\left[1, \frac{[1-\gamma_b+R_1\gamma_b]\left(\eta e_d+e_f\right)}{(1-\tau)c_I}\right]\right) g\left(\left(1-\tau\right)c_I\right), \\
U_{d,2} &= \left(\frac{[1-\gamma_b+R_1\gamma_b]\left(\eta e_d+e_f\right)}{c_I}\right) g\left(\left(1-\tau\right)c_I\right), \\
U_{d,3} &= \lambda g\left(\left(1-\tau\right)c_I\right)+\left(1-\lambda\right)g\left(A\left[\left(1-\tau\right)c_P, \kappa\left(1-\tau\right)m\right]\right), \\
U_{d,4} &= U_{d,3}, \\
U_{f,1} &= \min\left[\rho_2 I, \left(1-\eta\right)e_d\right], \\
U_{f,2} &= \min\left[\rho_1 I, \left(1-\eta\right)e_d+\tau\left[1-\gamma_b+R_1\gamma_b\right]\left(\eta e_d+e_f\right)\right], \\
U_{f,3} &= \rho_2 I, \\
U_{f,4} &= \min\left[\rho_1 I, \left(1-\eta\right)e_d+\tau\lambda c_I\right].
\end{aligned}
$$

A.2 Lemmata

Lemma 5 *$c_P > c_I$ is a necessary condition for $A[c_P, m] > c_I$.*

Proof. Let $c_P = \theta c_I$. Either $\theta < 1$ or $\theta = 1$ imply that $c_I > A[c_P, m]$. Recall that $A[c_P, m] = c_P^{\beta} m^{1-\beta}$, $0 < \beta < 1$ and $m = \kappa\left[\lambda c_I + \left(1-\lambda\right)c_P\right]$, $0 < \lambda < 1$ and $0 < \kappa < 1$.[43]
By substitution, $A[c_P, m] = c_I \theta^{\beta} \kappa^{1-\beta}\left[\lambda + \left(1-\lambda\right)\theta\right]^{1-\beta}$.
Substitution of $\theta = 1$ yields the result. ■

Lemma 6 *$c_P > c_I$ is not sufficient for $A[c_P, m] > c_I$.*

Proof. It is easy to find a counter-example. Write $c_P = c_I\left(1+\varepsilon\right)$. By the same logic and similar substitutions as those in the previous lemma, one can write $A[c_P, m] = c_I\left(1+\varepsilon\right)^{\beta}\left(\kappa + \left(1-\lambda\right)\kappa\varepsilon\right)^{1-\beta}$. In the simulations, $\beta = 0.98$, $\kappa = 0.16$. Let $\varepsilon = 0.001$. Then, for $\lambda = 0.2$, the counter-example condition is satisfied. ■

[43] The restriction $0 < \kappa < 1$ comes not from theory but from data. $\kappa = 0$ is a cashless economy, whereas $\kappa = 1$ is a cash-in-advance economy. It stands to reason that most economies are somewhere in between.

Lemma 7 γ^*_{aut} *is unique.*

Proof. $r_{aut}(\gamma_{aut}) = \lambda g(e_d[R_1\gamma_{aut} + 1 - \gamma_{aut}])$ $+ (1-\lambda) g(e_d[R_2\gamma_{aut} + 1 - \gamma_{aut}])$. Since g is smooth, so is r_{aut}. The first and second derivatives are:

$$\begin{aligned} r'_{aut}(\gamma_{aut}) &= e_d\lambda(R_1 - 1) g'(e_d[R_1\gamma_{aut} + 1 - \gamma_{aut}]) \\ &+ e_d(1-\lambda)(R_2 - 1) g'(e_d[R_2\gamma_{aut} + 1 - \gamma_{aut}]). \\ r''_{aut}(\gamma_{aut}) &= e_d^2\lambda(R_1 - 1)^2 g''(e_d[R_1\gamma_{aut} + 1 - \gamma_{aut}]) \\ &+ e_d^2(1-\lambda)(R_2 - 1)^2 g''(e[R_2\gamma_{aut} + 1 - \gamma_{aut}]). \end{aligned}$$

Since g is strictly concave, $r''_{aut} < 0$. There are two possibilities:

$$r'_{aut}(0) > 0 \text{ and}$$
$$r'_{aut}(0) \leq 0.$$

If $r'_{aut}(0) > 0$, $\gamma^*_{aut} > 0$. If not, $\gamma^*_{aut} = 0$. Thus, γ^*_{aut} is unique. ■

A.3 Proofs of Theorems in the Main Text

Proof of Theorem 1. Recall that foreigners are risk-neutral in dollar terms. If there are no domestic bank runs, and foreigners arrive in period 2 demanding liras, they will receive $\rho_2 e_f$ liras, whose real value is $\rho_2 e_f$ dollars. If foreigners come in period 1, they receive $\rho_1 e_f$ liras, the real value of which is: min $[\rho_1 e_f, (1-\eta) e_d + \tau\lambda c_I]$. Since $\rho_1 \leq \rho_2$, min $[\rho_1 e_f, (1-\eta) e_d + \tau\lambda c_I] \leq \rho_2 e_f$. Thus, foreigners always prefer to arrive in period 2 if they know that there will be no domestic bank runs. ■

Proof of Theorem 2 (Part One). In this part, I prove the following assertion. Suppose that all agents except for a small group of domestic agents act in accordance with $\sigma_{d,1}$, and $s_d = 1$. It is optimal for that small group of agents to use $\sigma_{d,1}$ as their strategy function. Let ε be the measure of that group of patient agents. Suppose that $a_d = \dfrac{(1-\gamma_b + R_1\gamma_b)(\eta e_d + e_f)}{c_I} \equiv \psi$. Note that $L(\psi) = \gamma_b(\eta e_d + e_f)$ and, for all $a_d > \psi$, $L(a_d) = \gamma_b(\eta e_d + e_f)$. Consider any number ε, where $0 < \varepsilon < 1 - \psi$. If a group of size ε does not run on the bank, it will receive zero utility with certainty. If the members of the group were to run on the bank, they would receive an expected utility of $\psi g\left(c_I\right) + \left(1-\psi\right) g\left(0\right) > 0$. ■

Proof of Theorem 2 (Part Two). The previous proof depended on the fact that $\psi < 1$; i.e., that $\left(1-\gamma_b + R_1\gamma_b\right)\left(\eta e_d + e_f\right) < c_I$. For brevity, I omit the proof where $\psi = 1$. It is available upon request. ■

Proof of Theorem 2 (Part Three). In this part, I consider the analogue to parts one and two, but here, $s_d = 0$. Let ε be the size of a group of patient agents. Let $\chi = \dfrac{\left(1-\gamma_b\right)\left(\eta e_d + e_f\right)}{c_I}$. For all $\varepsilon < \min\left[\chi, \psi\right] - \lambda$, the bank need not liquidate productive assets to accommodate the defecting group of measure ε. Members of the defecting group receive $g\left(c_I\right)$, whereas they receive $g\left[A[c_P, m]\right]$ if they did not defect. Since $A[c_P, m] > c_I$, no domestic agent will run on the bank when $s_d = 0$. ■

Proof of Theorem 2 (Part Four). In this part of the proof, I examine deviations from the equilibrium strategy $\left(\sigma_{d,1}, \sigma_{f,1}\right)$ by foreign agents when $s_f = 0$. Let

$$PRA = \frac{\pi_1\left(1-\pi_2\right)}{\pi_1\left(1-\pi_2\right) + \left(1-\pi_1\right)\left(1-\pi_3\right)}.$$

If there is no deviation, foreigners receive expected utility,

$$\left(1-PRA\right)\rho_2 e_f + PRA\min\left(\rho_2 e_f, \left(1-\eta\right)e_d\right).$$

If some foreigners deviate, they receive expected utility,
$\left(1-PRA\right)\min(\rho_1 e_f, \left(1-\eta\right)e_d + \tau\lambda c_I) + PRA\min\left(\rho_2 e_f, \left(1-\eta\right)e_d\right)$. It is easy to see that foreigners who deviate receive less expected utility than those who play $\sigma_{f,1}$ when $s_f = 0$. ■

Proof of Theorem 2 (Part Five). The final portion of the theorem cannot be verified analytically. But here is the condition to be verified numerically. Let $PRB = \dfrac{\pi_1\pi_2}{\pi_1\pi_2 + (1-\pi_1)\pi_3}$. Let ε be the size of the deviating group. The return to deviation is:

$$\begin{aligned} &PRB\left(\max\left[0,(1-\eta)\,e_d-(1-\varepsilon)\,\rho_1 e_f\right]\right) \\ &+(1-PRB)\left(\min(\max\left[0,T\right]+\tau\,(1-\lambda)\,c_P,\varepsilon\rho_2 e_f)\right), \end{aligned}$$

where $T=(1-\eta)\,e_d-(1-\varepsilon)\,\rho_1 e_f+\tau\lambda c_I$.

The return to following $\sigma_{f,1}$ is:

$\varepsilon\left[(1-\eta)\,e_d+(PRB)\,\tau\left((1-\gamma_b+R_1\gamma_b)\,(\eta e_d+e_f)\right)+(1-PRB)\,\tau\lambda c_I\right]$.

For small values of ε, the difference between these two returns is zero. ■

Proof of Theorem 3. If $(1-\gamma_b+R_2\gamma_b)\,(\eta e_d+e_f)<c_I$, bank runs are possible, since not all domestic agents can be served by the bank. See Chang and Velasco (2000a) for more discussion. From Theorem 2 (Part Two), $c_I = (1-\gamma_b+R_1\gamma_b)\,(\eta e_d+e_f)$ is insufficient to guarantee the absence of bank runs. Therefore, $(1-\gamma_b+R_1\gamma_b)\,(\eta e_d+e_f)>c_I$ is necessary to prevent runs. If $A[c_P,m]<c_I$, domestic agents prefer to consume in period 1, even though they could be served in period 2. $A[c_P,m]\geq c_I$ is also necessary to prevent runs. Note that, if both conditions hold, no patient agent has an incentive to represent himself as impatient; thus, a run will never occur. These conditions are also sufficient. ■

Proof of Theorem 4. If $(1-\gamma_b+R_1\gamma_b)\,(\eta e_d+e_f)\leq c_I$, there is a non-zero probability that patient agents will not be served in period 2. This probability is a function of how many domestic agents represent themselves as impatient in period 1. This is the case discussed by Diamond and Dybvig (1983). ■

Table A1: Latin Letters and Their Meanings

Letter	Meaning
A	Part of the utility function of the patient
a_d	Measure of domestic agents claiming to be impatient
a_f	Measure of foreign agents arriving in period 1
C	The contract
c_I	Payment promised to those claiming to be impatient
c_P	Payment promised to those claiming to be patient
e_d	Domestic endowment
e_f	Foreign investment
g	Part of the domestic utility function
L	Liquidation function
m	Real money holdings
Prcc	The probability of a currency crisis
R_1	Return on the productive asset by period 1
R_2	Return on the productive asset by period 2
r_{aut}	Autarchic return function
s_d	Domestic sunspot
s_f	Foreign sunspot
T	Temporary variable used in appendix proofs
PRA	Temporary variable used in appendix proofs
PRB	Temporary variable used in appendix proofs

Table A2: Greek Letters and Their Meanings

Letter	Meaning
α	Curvature parameter from the utility function
β	Elasticity of consumption parameter from the utility function
γ_{aut}	Fraction invested in the productive asset in autarchy
γ^*_{aut}	Fraction optimally invested in the productive asset in autarchy
γ_b	Fraction invested in the productive asset by the bank
ε	Temporary variable used in appendix proofs
η	Reserve requirement
θ	Temporary variable used in appendix proofs
κ	Cambridge k
λ	Fraction of impatient agents
π_1	Probability of $\mathrm{s}_d = 1$
π_2	Probability of $\mathrm{s}_f = 1$ conditional on $\mathrm{s}_d = 1$
π_3	Probability of $\mathrm{s}_f = 1$ conditional on $\mathrm{s}_d = 0$
ρ_1	Return to foreign deposits, 1 period
ρ_2	Return to foreign deposits, 2 period
Σ_d	Domestic strategy set
Σ_f	Foreign strategy set
σ_d	Typical domestic strategy
σ_f	Typical foreign strategy
$\sigma_{d,i}$	Domestic strategy "i"
$\sigma_{f,i}$	Foreign strategy "i"
τ	Taxation rate for domestic deposits
χ	Temporary variable used in appendix proofs
ψ	Temporary variable used in appendix proofs

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