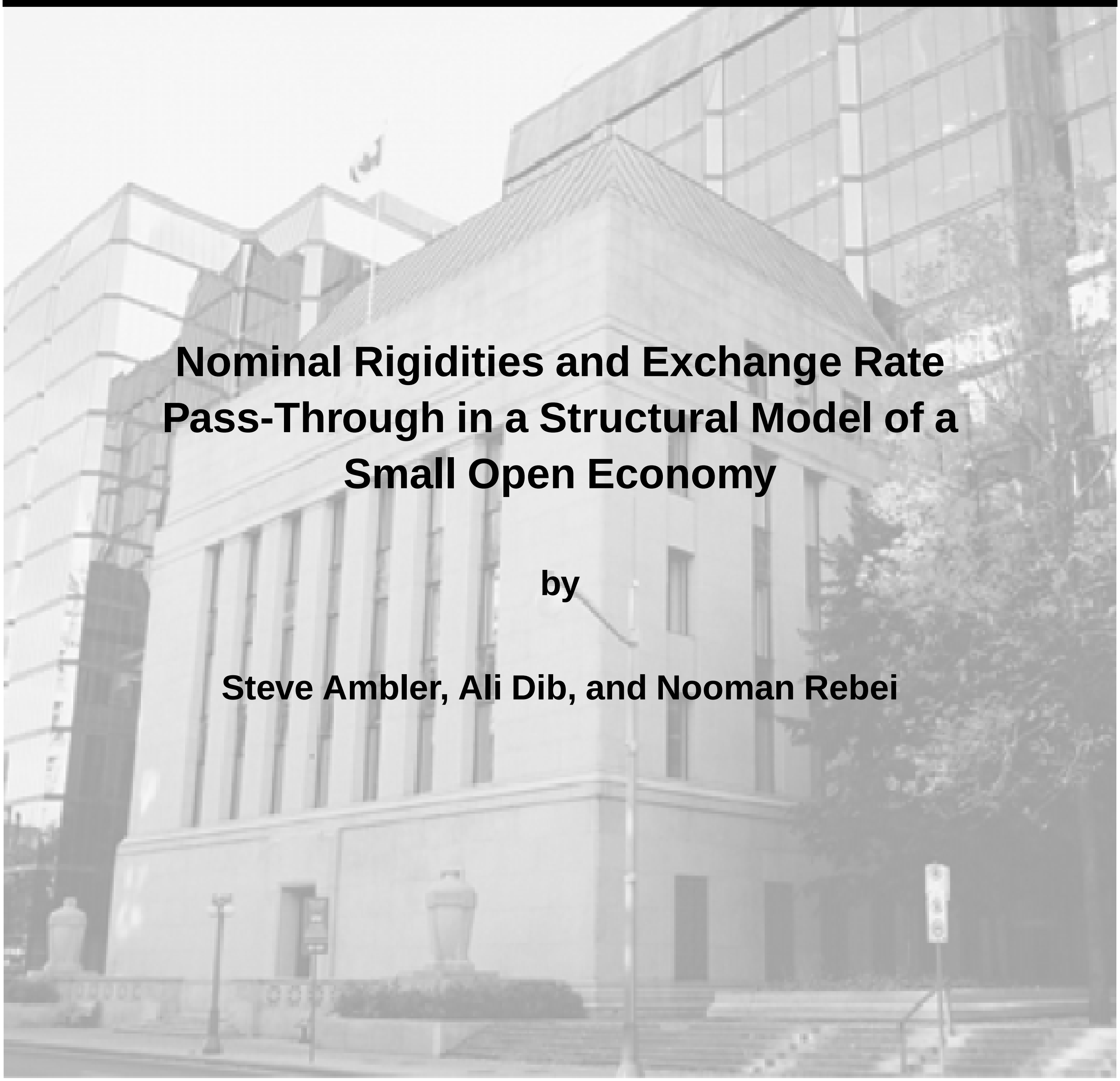
Bank of Canada Banque du Canada

Working Paper 2003-29/ Document de travail 2003-29



Nominal Rigidities and Exchange Rate Pass-Through in a Structural Model of a Small Open Economy

by

Steve Ambler, Ali Dib, and Nooman Rebei

ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2003-29

September 2003

Nominal Rigidities and Exchange Rate Pass-Through in a Structural Model of a Small Open Economy

by

Steve Ambler,[1] Ali Dib,[2] and Nooman Rebei[3]

[1]CIRPEE
Université du Québec à Montréal
C.P. 8888 Succ. Centre-Ville
Montréal, QC, Canada H3C 3P8
ambler.steven@uqam.ca

[2]Monetary and Financial Analysis Department
[3]Research Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
adib@bankofcanada.ca
nrebei@bankofcanada.ca

Contents

Acknowledgements iv
Abstract/Résumé v

1. Introduction 1
2. The Model 3
   2.1 Households 3
   2.2 Goods production 6
   2.3 Monetary authority 10
   2.4 The government 11
3. Equilibrium 11
   3.1 New Keynesian Phillips Curves 12
4. Parameter Estimation and Simulation Methodology 13
   4.1 Parameter values 15
5. Simulation Results 18
   5.1 Unconditional moments 18
   5.2 Impulse-response functions 19
   5.3 Exchange rate pass-through 19
   5.4 Sensitivity analysis 20
6. Conclusions 22

Bibliography 23

Tables 26

Figures 30

Appendix A: Data and Data Sources 44

Appendix B: Moments Used to Estimate the Model 46

Appendix C: Equilibrium Conditions 47

Acknowledgements

We thank Brian Doyle, Hafedh Bouakez, and seminar participants at the Bank of Canada and the Canadian Economics Association annual meetings for helpful comments.

Abstract

The authors analyze exchange rate pass-through in an estimated structural model of a small open economy that incorporates three types of nominal rigidity (wages and the prices of domestically produced and imported goods) and eight different structural shocks. The model is estimated using quarterly data from Canada and the United States. It predicts a remarkably similar dynamic relationship between the nominal exchange rate and prices in response to the different structural shocks: the nominal exchange rate overshoots its long-run level, and changes in the nominal exchange rate are passed through slowly to the domestic price level. The authors show that, although pricing to market (the slow adjustment of the domestic-currency prices of imported goods) is necessary to generate slow pass-through to the prices of imported goods, it is not necessary to generate slow pass-through to the overall price level. Sticky domestic wages also generate slow exchange rate pass-through, even when the prices of imported goods adjust instantaneously to changes in the exchange rate.

JEL classification: F2, F31, F33
Bank classification: Business fluctuations and cycles; Economic models; Exchange rates; Inflation and prices; International topics

Résumé

Les auteurs analysent l'incidence des mouvements du taux de change sur les prix dans le cadre d'un modèle structurel de petite économie ouverte qui intègre trois types de rigidité nominale (des salaires, des prix des biens produits au pays et des prix des importations) ainsi que huit chocs structurels. Estimé à l'aide de données trimestrielles se rapportant au Canada et aux États-Unis, le modèle fait ressortir une relation dynamique remarquablement semblable entre le taux de change nominal et les prix intérieurs dans leur profil de réaction aux divers chocs : le premier surréagit systématiquement à court terme et les seconds s'ajustent lentement aux fluctuations du taux de change nominal. Les auteurs montrent que, si l'établissement des prix en fonction du marché (soit l'ajustement lent des prix en monnaie nationale des biens importés) est une condition nécessaire à la lenteur de la transmission des mouvements du taux de change aux prix des importations, cela n'est pas vrai en ce qui concerne l'ensemble des prix. La rigidité des salaires intérieurs contribue elle aussi à ralentir la transmission des variations du taux de change, même lorsque les prix des importations s'ajustent instantanément aux mouvements de ce taux.

Classification JEL : F2, F31, F33
Classification de la Banque : Cycles et fluctuations économiques; Modèles économiques; Taux de change; Inflation et prix; Questions internationales

1. Introduction

The pass-through of exchange rate changes to import prices and to domestic inflation is of obvious interest to central banks that have strict inflation targets. Imperfect exchange rate pass-through also has important consequences in theoretical models of optimal monetary policy. A large literature has developed that analyzes optimal monetary policy in the context of the New Open-Economy Macroeconomics (NOEM), a class of open-economy dynamic general-equilibrium models with explicit microfoundations, nominal rigidities, and imperfect competition.[1] For example, Smets and Wouters (2002) show that optimal monetary policy with sticky domestic- and imported-goods prices involves the minimizing of a weighted average of domestic and import price inflation.[2] This result provides a rationale for exchange rate stabilization, and qualifies the results of Galí and Monacelli (1999). The latter study, based on a model with instantaneous pass-through of exchange rate changes to imported goods prices, concluded that optimal monetary policy is identical in open and closed economies and involves stabilizing the overall price level, without regard to exchange rate fluctuations.

In this paper, we analyze exchange rate pass-through in the context of a NOEM model of a small open economy with three types of nominal rigidity: wages, domestic output prices, and imported goods prices are all set in advance by monopolistically competitive agents. We attempt to answer three related questions raised by the recent theoretical and empirical literature.

First, it is clear that exchange rate pass-through is a conditional phenomenon: since the exchange rate and prices are endogenous in general equilibrium, the joint dynamics of the exchange rate and prices may differ in response to different structural shocks. In contrast, empirical studies of exchange rate pass-through typically focus on reduced-form equations[3] or vector autoregressions with a limited number of variables.[4] Implicitly, these studies estimate the joint dynamics of the exchange rate and prices in response to "average" shocks and ignore the possibility that the dynamics may be heterogeneous across different types of shock. This raises the question of whether this simplification is justified. We try to answer this question by incorporating a large number of different shocks: to technology, domestic money demand, domestic interest rates, government spending and tax rates, and foreign inflation, foreign interest rates, and foreign output.

Second, there are several competing definitions of pass-through in the literature. Some authors (for example, Campa and Goldberg 2001) use a narrow definition and take pass-through to mean how changes in the exchange rate are transmitted to the prices of imported

[1]The NOEM literature, spawned by the pioneering work of Obstfeld and Rogoff (1995), has been successful in explaining phenomena such as high real exchange rate volatility and persistence, and the strong impact of monetary policy shocks on real exchange rates. See Sarno (2001), Lane (2001), and Bowman and Doyle (2003) for recent surveys.

[2]Similarly, Corsetti and Pesenti (2001) show in a similar model that it is optimal for the central bank to minimize a CPI-weighted average of markups charged in the domestic market by domestic and foreign producers.

[3]See, for example, Campa and Goldberg (2001) and Bailliu and Fujii (2003).

[4]For example, Kim (1998) and McCarthy (2000).

goods. Other studies (for example, Devereux 2001; Devereux and Yetman 2003) use a broader definition, taking pass-through to mean the transmission of exchange rate changes to the overall price level, which can be either the producer price index (PPI) or the consumer price index (CPI). This raises the question of the extent to which the dynamic responses of different price indexes are similar. We simulate the response of the import price index, the PPI and the CPI, along with the nominal exchange rate, to compare the joint dynamics of all four variables.

Third, the recent theoretical literature on imperfect pass-through has stressed the role of pricing to market (PTM), which assumes that the prices of imported goods are set in advance in the local currency.[5] It has been almost axiomatic that PTM is both necessary and sufficient to generate imperfect pass-through in the dynamic general-equilibrium model. This raises the question of whether other mechanisms can also generate slow pass-through. We investigate the roles of the three different types of nominal rigidity in generating slow pass-through, and examine the response of the exchange rate and prices to shocks when all three nominal rigidities are shut down.

Our main findings can be summarized as follows. The response of the exchange rate and prices to the model's different structural shocks is remarkably similar. The nominal exchange rate overshoots, with its maximum response occurring during the period in which the shock hits. In contrast, import prices and the domestic price level adjust gradually. In most cases, the path followed by the price level is hump-shaped. The adjustment of the price level is more gradual than that of the price of imports. While PTM is necessary for slow pass-through to imported goods prices and sufficient for slow pass-through to the overall price level, it is not necessary for the latter. Sticky wages and domestic prices also generate slow exchange rate pass-through to the CPI, even when the prices of imported goods adjust instantaneously to changes in the exchange rate.

We estimate most of the model's structural parameters with Canadian and U.S. data using a methodology that combines the generalized method of moments (GMM) and the simulated method of moments (SMM).[6] We are able to estimate most of the model's structural parameters precisely, and the model passes a J-test of its overidentifying restrictions. The model is also capable of reproducing other features of the data. It generates a large amount of persistence of the real exchange rate, of real variables such as output, and of inflation. The nominal and real exchange rates generated by the model are highly correlated, as in the data.

This paper is organized as follows. In section 2, we describe the key ingredients of the structural model. In section 3, we summarize the equations that define the economy's stationary equilibrium. In section 4, we discuss the estimation strategy that we use to attribute values to the model's structural parameters. Simulation results and sensitivity analyses are described in section 5. Section 6 offers some conclusions.

[5] Implicitly, this assumes that goods arbitrage is not feasible at least in the short run, so that the law of one price does not hold.

[6] Ambler, Guay, and Phaneuf (2003) use a similar methodology to estimate a closed-economy business cycle model.

2. The Model

We model an economy that is small in the sense that it faces fixed prices on world markets for imported intermediate goods. Its domestic output is an imperfect substitute for foreign goods, and it faces a downward-sloping demand curve for its output on world markets. It also faces an upward-sloping supply curve for funds on international capital markets.

The production structure of the model is summarized in Figure 1.[7] There is one primary factor, labour, which is used with final output in the production of differentiated intermediate inputs. There are different labour types, which are imperfect substitutes for each other in the production process. Labour inputs are denoted by $h_t(j,i)$: the j argument refers to the type of labour, which is associated with a particular household (that acts as a monopolistic competitor in the labour market), and the i argument refers to the intermediate-goods firm using the labour input. Domestic intermediate goods, $Y_t(i)$, are produced by monopolistically competitive firms using labour and the final composite good as inputs, and combined together by a representative competitive firm to form a composite-domestic good, Y_t. Some of this good, Y_t^x, is exported, and some is combined with the composite-imported good, Y_t^m, to form the final composite good, Z_t. As in McCallum and Nelson (1999, 2001), imports enter the production process for the domestic final good, rather than being consumed directly.[8] The final composite good is used for consumption, government consumption, and as an input into the production of domestic intermediate goods. The composite imported good is produced by aggregating individual imported intermediate goods, $Y_t^m(j)$. These intermediate imports are imported by monopolistically competitive importers.

There are therefore three sources of monopoly distortion in the economy, and three sources of nominal rigidity. Households set their nominal wages in advance, and importers and the firms that produce domestic intermediate goods set their prices in advance. Following Calvo (1983), we suppose that price and wage setters maintain constant prices and wages unless they receive a signal to revise them, which arrives at the beginning of each period with a constant probability. This assumption makes aggregation simple, allows us to vary the average duration of the nominal rigidity without varying the number of state variables in the model, and allows us to estimate the length of the nominal rigidities along with other structural parameters of the model.

2.1 Households

Households offer differentiated labour skills. There is a continuum of different households on the unit interval, indexed by j. The j^{th} household's preferences are described by the following expected utility function:

$$U_0(j) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t(j), \frac{M_t(j)}{P_t}, h_t(j,\cdot)\right), \tag{1}$$

[7] The prices associated with the different goods and factors in the model are given in parentheses beside the goods and factors to which they correspond.

[8] Bergin (2003) and Kollmann (2002) develop similar models.

where $\beta \in (0,1)$ is the discount factor, E_0 is the conditional expectations operator, $C_t(j)$ is consumption, $M_t(j)$ denotes nominal money balances held at the end of the period, P_t is the price level, and $h_t(j,\cdot)$ denotes hours worked by the household. The single-period utility function is given by:

$$u(\cdot) = \frac{\gamma}{\gamma-1}\log\left(C_t(j)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}}\left(\frac{M_t(j)}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right) + \eta\log\left(1-h_t(j,\cdot)\right), \tag{2}$$

where γ and η are positive structural parameters. Total time available to the household in the period is normalized to equal one. The shock, b_t, is interpreted as a shock to money demand. It follows the first-order autoregressive process given by:

$$\log(b_t) = (1-\rho_b)\log(b) + \rho_b\log(b_{t-1}) + \varepsilon_{bt}, \tag{3}$$

where $\rho_b \in (-1,1)$ and the serially uncorrelated shock, ε_{bt}, is normally distributed with zero mean and standard deviation σ_b.

The household's budget constraint is given by:

$$\begin{aligned} P_tC_t(j) + M_t(j) + \frac{D_t^g(j)}{R_t} + \frac{e_tB_t^*(j)}{\kappa_tR_t^*} &= (1-\tau_t)W_t(j)h_t(j,\cdot) \\ &+M_{t-1}(j) + D_{t-1}^g(j) + e_tB_{t-1}^*(j) + T_t + D_t. \end{aligned} \tag{4}$$

Labour income is taxed at an average marginal tax rate, τ_t. B_t^* and D_t^g are foreign-currency and domestic-currency bonds purchased in t, and e_t is the nominal exchange rate. Domestic-currency bonds are used by the government to finance its deficit. R_t and R_t^* denote, respectively, the gross nominal domestic and foreign interest rates between t and $t+1$; κ_t is a measure of a risk premium that reflects departures from uncovered interest parity. The household also receives $D_t = D_t^d + D_t^m$, nominal profits, from monopolistically competitive firms that produce domestic intermediates and that import intermediates, and T_t in lump-sum nominal transfers from the government. The risk premium, κ_t, is a positive convex function that depends on the ratio of net foreign assets to domestic output:

$$\log(\kappa_t) = \varphi\left[\exp\left(\frac{e_tB_t^*}{P_t^dY_t}\right) - 1\right], \tag{5}$$

where P_t^d is the GDP deflator, which corresponds to the domestic output price index. The introduction of a risk premium that depends on the level of indebtedness of the economy ensures that the model has a unique steady state. If the domestic real interest rate is equal to the foreign real interest rate, the time paths of domestic consumption and wealth follow random walks.[9]

[9]For an early discussion of this problem, see Giavazzi and Wyplosz (1984). Our risk premium equation is similar to the one used by Senhadji (1997). For alternative ways of ensuring that stationary paths exist for consumption in small open-economy models, see Schmitt-Grohé and Uribe (2003).

The foreign nominal interest rate, R_t^*, evolves according to the following stochastic process:

$$\log(R_t^*) = (1-\rho_{R^*})\log(R^*) + \rho_{R^*}\log(R_{t-1}^*) + \varepsilon_{R^*t}, \tag{6}$$

where $\rho_{R^*} \in (-1,1)$ and the serially uncorrelated shock, ε_{R^*t}, is normally distributed with zero mean and standard deviation σ_{R^*}. Households also face a no-Ponzi-game restriction:

$$\lim_{T\to\infty}\left(\prod_{t=0}^{T}\frac{1}{\kappa_t R_t^*}\right)B_T(j) = 0.$$

Each household chooses $C_t(j)$, $M_t(j)$, $D_t^g(j)$, and $B_t^*(j)$, (and $W_t(j)$ if it is allowed to change its wage) to maximize the expectation of the discounted sum of its utility flows subject to the budget constraint, equation (4), and to intermediate firms' demand for their labour type, j. Aggregate labour input is a composite of the labour of different skill types, and is given by:

$$h_t = \left(\int_0^1 h_t(j,\cdot)^{\frac{\sigma-1}{\sigma}}dj\right)^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

so that σ is the elasticity of substitution between different labour skills. This implies the following conditional demand for labour of type j:

$$h_t(j,\cdot) = \left(\frac{W_t(j)}{W_t}\right)^{-\sigma}h_t, \tag{8}$$

where h_t is aggregate employment and W_t is an exact index of average nominal wages, given by:

$$W_t = \left(\int_0^1 W_t(j)^{1-\sigma}dj\right)^{\frac{1}{1-\sigma}}. \tag{9}$$

The household's first-order conditions are as follows:

$$\frac{C_t(j)^{\frac{-1}{\gamma}}}{C_t(j)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}}\left(\frac{M_t(j)}{P_t}\right)^{\frac{\gamma-1}{\gamma}}} = \Lambda_t(j)\frac{P_t}{P_t^d}; \tag{10}$$

$$\frac{b_t^{\frac{1}{\gamma}}\left(\frac{M_t(j)}{P_t}\right)^{\frac{-1}{\gamma}}\left(\frac{P_t^d}{P_t}\right)}{C_t(j)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}}\left(\frac{M_t(j)}{P_t}\right)^{\frac{\gamma-1}{\gamma}}} = \Lambda_t(j) - \beta E_t\left[\frac{P_t^d}{P_{t+1}^d}\Lambda_{t+1}(j)\right]; \tag{11}$$

$$\frac{\Lambda_t(j)}{R_t} = \beta E_t\left[\frac{P_t^d}{P_{t+1}^d}\Lambda_{t+1}(j)\right]; \tag{12}$$

$$\frac{\Lambda_t(j)}{\kappa_t R_t^*} = \beta E_t\left[\frac{P_t^d}{P_{t+1}^d}\frac{e_{t+1}}{e_t}\Lambda_{t+1}(j)\right], \tag{13}$$

where $\Lambda_t(j)$ is the Lagrangian multiplier associated with the time t budget constraint. In addition, there is a first-order condition for setting the nominal wage when the household is allowed to do so. This happens with probability d_w at the beginning of each period. Maximizing utility with respect to the nominal wage $\tilde{W}_t(j)$ yields:

$$\tilde{W}_t(j) = \left(\frac{\sigma}{\sigma - 1}\right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_w)^l \frac{\eta h_{t+l}(j)}{1-h_{t+l}(j)}}{E_t \sum_{l=0}^{\infty} (\beta d_w)^l (1-\tau_{t+l}) h_{t+l}(j) \Lambda_{t+l}(j)/P_{t+l}^d}. \tag{14}$$

This first-order condition can be manipulated to yield a New Keynesian Phillips curve for wage inflation (see section 3.1). The wage index evolves over time according to the recursive equation given by:

$$W_t = \left[d_w (W_{t-1})^{1-\sigma} + (1-d_w)(\tilde{W}_t)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{15}$$

where $\tilde{W}_t$ is the average wage of those workers who revise their wage at time t.

2.2 Goods production

2.2.1 Domestic intermediate goods

The firm producing intermediate good i has a production function given by:[10]

$$Y_t(i) = X_t(i)^{\phi} \left(A_t h_t(\cdot, i)\right)^{1-\phi}, \quad \phi \in (0,1), \tag{16}$$

where $h_t(\cdot, i)$ is the quantity of the aggregate labour input employed by firm i; $X_t(i)$ is the quantity of the final composite good, Z_t, used by firm i; and A_t is an aggregate technology shock that follows the stochastic process given by:

$$\log A_t = A + \log(A_{t-1}) + \varepsilon_{At}, \tag{17}$$

where ε_{At} is a normally distributed, serially uncorrelated shock with zero mean and standard deviation σ_A. The firm chooses $X_t(i)$ and $h_t(\cdot, i)$ to maximize its stock market value. When allowed to do so (with probability d_p each period), it also chooses the price of its output, $\tilde{P}_t^d(i)$. Its problem, including the output price among its choice variables, can be written as follows:

$$\max_{\{X_t(i), h_t(\cdot,i), \tilde{P}_t^d(i)\}} E_t \left[\sum_{l=0}^{\infty} (\beta d_p)^l \left(\frac{\Lambda_{t+l}}{\Lambda_t}\right) \frac{D_{t+l}^d(i)}{P_{t+l}^d}\right], \tag{18}$$

where Λ_t is the marginal utility of wealth for a representative household, and

$$D_{t+l}^d(i) \equiv \tilde{P}_t^d(i) Y_{t+l}(i) - W_{t+l} h_{t+l}(\cdot, i) - P_{t+l} X_{t+l}(i),$$

[10]We include $X_t(i)$ in the production of domestic intermediates for two reasons. First, without $X_t(i)$, the response of the real wage to demand shocks is too highly countercyclical. Second, as shown in similar models by McCallum and Nelson (1999, 2001), the presence of intermediates in the production function for domestic goods affects the correlation between the nominal exchange rate and domestic inflation.

where P_t is the price of the final output good, Z_t. The maximization is subject to the firm's production function and to the derived demand for the firm's output (discussed in section 2.2.3), and is given by:

$$Y_{t+l}(i) = \left(\frac{\tilde{P}_t^d(i)}{P_{t+l}^d} \right)^{-\theta} Y_{t+l}, \tag{19}$$

where P_t^d is the exact price index of the composite domestic good. The elasticity of the derived demand for the firm's output is equal to $-\theta$, assumed to be greater than one in absolute value. The first-order conditions are:

$$\frac{W_t}{P_t^d} = \xi_t(i)(1-\phi)\frac{Y_t(i)}{h_t(\cdot, i)}; \tag{20}$$

$$\frac{P_t}{P_t^d} = \xi_t(i)\phi\frac{Y_t(i)}{X_t(i)}; \tag{21}$$

$$\tilde{P}_t^d(i) = \left(\frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_p)^l \Lambda_{t+l} \xi_{t+l}(i) Y_{t+l}(i)}{E_t \sum_{l=0}^{\infty} (\beta d_p)^l \Lambda_{t+l} Y_{t+l}(i) / P_{t+l}^d}, \tag{22}$$

where $\xi_t(i)$ is the Lagrange multiplier associated with the production function constraint. It measures the real marginal cost of the firm in units of final output (the Z_t good). The first-order conditions with respect to the choice of each of the two inputs state that the marginal cost of the input should be equal to its marginal product weighted by the real marginal cost of output. The first-order condition with respect to the firm's price relates the price to the expected future price of final output and to expected future real marginal costs. This first-order condition will be instrumental in deriving a New Keynesian Phillips curve relationship for the rate of change of domestic output prices (see section 3.1).

2.2.2 Imported intermediate goods

The economy imports a continuum of foreign intermediate goods on the unit interval. There is monopolistic competition in the market for imported intermediates, which are imperfect substitutes for each other in the production of the composite imported good, Y_t^m, produced by a representative competitive firm. When allowed to do so (with probability d_m each period), the monopolistically competitive importer of good i sets the price, $\tilde{P}_t^m(i)$, that maximizes its weighted expected profits. The import firm that is allowed to set a new price maximizes:

$$\max_{\{\tilde{P}_t^m(i)\}} E_t \left[\sum_{l=0}^{\infty} (\beta d_m)^l \left(\frac{\Lambda_{t+l}}{\Lambda_t} \right) \frac{D_{t+l}^m(i)}{P_{t+l}^d} \right], \tag{23}$$

where:

$$D_{t+l}^m(i) = \left(\tilde{P}_t^m(i) - e_{t+l} P_{t+l}^* \right) \left(\frac{\tilde{P}_t^m(i)}{P_{t+l}^m} \right)^{-\vartheta} Y_{t+l}^m. \tag{24}$$

For convenience, we assume that the price in foreign currency of all imported intermediates is P_t^*, which is also equal to the foreign price level. The elasticity of the derived demand for the imported good, i, is $-\vartheta$, assumed to be greater than one in absolute value. The first-order condition of this optimization problem is:

$$\tilde{P}_t^m(i) = \left(\frac{\vartheta}{\vartheta - 1}\right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_m)^l \Lambda_{t+l} Y_{t+l}^m(i) e_{t+l} P_{t+l}^* / P_{t+l}^d}{E_t \sum_{l=0}^{\infty} (\beta d_m)^l \Lambda_{t+l} Y_{t+l}^m(i) / P_{t+l}^d}. \tag{25}$$

This equation can be used to derive a New Keynesian Phillips curve relationship for the rate of change of intermediate input prices (see section 3.1).

2.2.3 Composite domestic good

The composite domestic good, Y_t, is produced using a constant elasticity of substitution (CES) technology with a continuum of domestic intermediate goods, $Y_t(i)$, as inputs:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}. \tag{26}$$

The composite good is produced by a representative competitive firm, which maximizes its profits by choosing optimal levels of the domestic intermediate goods. Its profit-maximization problem is:

$$\max_{\{Y_t(i)\}} P_t^d Y_t - \int_0^1 P_t^d(i) Y_t(i) di, \tag{27}$$

subject to the production function (26). The first-order conditions yield the derived demand functions for the domestic intermediate goods given by equation (19). The exact price index for the composite domestic good is given by:

$$P_t^d = \left(\int_0^1 P_t^d(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}. \tag{28}$$

Given the Calvo contract on prices, and the price index for the composite domestic good, the price level obeys the following law of motion:

$$P_t^d = \left[d_p (P_{t-1}^d)^{1-\theta} + (1-d_p)(\tilde{P}_t^d)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{29}$$

where $\tilde{P}_t^d$ is the price index derived by aggregating over all firms that are allowed to change their price at time t.

Composite domestic output, Y_t, is divided between domestic use, Y_t^d, and exports, Y_t^x, so that $Y_t = Y_t^d + Y_t^x$. The foreign demand function for domestic exports is given by:[11]

$$Y_t^x = \alpha_x \left(\frac{P_t^d}{e_t P_t^*}\right)^{-\varsigma} Y_t^*, \tag{30}$$

[11]This condition can be derived from a foreign importing firm that combines non-perfectly substitutable imported goods.

where Y_t^* is foreign output.[12] The elasticity of demand for domestic output is given by $-\varsigma$, and $\alpha_x > 0$ is a parameter determining the fraction of domestic exports in foreign spending. Domestic exports form an insignificant fraction of foreign expenditures, and have a negligible weight in the foreign price index. This is another sense in which the domestic economy is small.

The foreign variables P_t^* and Y_t^* are both exogenous and, when stationarized, evolve according to

$$\log(P_t^*/P_{t-1}^*) = (1-\rho_{\pi^*})\log(\pi^*) + \rho_{\pi^*}\log(P_{t-1}^*/P_{t-2}^*) + \varepsilon_{\pi^* t}, \tag{31}$$

and

$$\log Y_t^* = (1-\rho_{y^*})\log(Y^*) + \rho_{y^*}\log(Y_{t-1}^*) + \varepsilon_{y^* t}, \tag{32}$$

where π^* is the steady-state rate of foreign inflation, ρ_{π^*} and ρ_{y^*} are autocorrelation coefficients, and $\varepsilon_{\pi^* t}$ and $\varepsilon_{y^* t}$ are zero-mean, serially uncorrelated shocks with standard errors σ_{π^*} and σ_{y^*}, respectively.

2.2.4 Composite imported good

The composite imported good, Y_t^m, is produced using a CES technology with a continuum of imported-intermediate goods, $Y_t^m(i)$, as inputs:

$$Y_t^m \leq \left(\int_0^1 Y_t^m(i)^{\frac{\vartheta-1}{\vartheta}} di \right)^{\frac{\vartheta}{\vartheta-1}}. \tag{33}$$

The composite imported good is produced by a representative competitive firm. Its profit maximization problem implies the derived demand function for intermediate imported good $Y_t^m(i)$ given by:

$$Y_t^m(i) = \left(\frac{P_t^m(i)}{P_t^m} \right)^{-\vartheta} Y_t^m. \tag{34}$$

The exact price index for the composite imported goods is given by:

$$P_t^m = \left(\int_0^1 P_{mt}(i)^{1-\vartheta} di \right)^{\frac{1}{1-\vartheta}}. \tag{35}$$

Given this price index and the price-fixing behaviour of importers, the price index obeys the following law of motion:

$$P_t^m = \left[d_m (P_{t-1}^m)^{1-\vartheta} + (1-d_m)(\tilde{P}_t^m)^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}, \tag{36}$$

where $\tilde{P}_t^m$ is a price index derived by aggregating over all importers that change their price in time t.

[12] To ensure the existence of a balanced growth path for the economy, we assume that foreign output grows at the same trend rate as domestic output.

2.2.5 Final-goods production

The final good, Z_t, is produced by a competitive firm that uses Y_t^d and Y_t^m as inputs subject to the following CES technology:

$$Z_t = \left[\alpha_d^{\frac{1}{\nu}} (Y_t^d)^{\frac{\nu-1}{\nu}} + \alpha_m^{\frac{1}{\nu}} (Y_t^m)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \tag{37}$$

where $\alpha_d > 0$, $\alpha_m > 0$, $\nu > 0$, and $\alpha_d + \alpha_m = 1$. The final good, Z_t, is used for domestic consumption, C_t; inputs used in the production of domestic intermediate goods, X_t; and government purchases, G_t. The final good is produced by a competitive firm whose maximization problem can be expressed as follows:

$$\max_{\{Y_t^d, Y_t^m\}} P_t Z_t - P_t^d Y_t^d - P_t^m Y_t^m, \tag{38}$$

subject to the production function (37). Profit maximization entails:

$$Y_t^d = \alpha_d \left(\frac{P_t^d}{P_t} \right)^{-\nu} Z_t, \tag{39}$$

and

$$Y_t^m = \alpha_m \left(\frac{P_t^m}{P_t} \right)^{-\nu} Z_t. \tag{40}$$

Furthermore, the final-good price, P_t, is given by:

$$P_t = \left[\alpha_d (P_t^d)^{1-\nu} + \alpha_m (P_t^m)^{1-\nu} \right]^{1/(1-\nu)}. \tag{41}$$

2.3 Monetary authority

Following Taylor (1993), Dib (2003), and Ireland (2003), the central bank manages the short-term nominal interest rate, R_t, in response to deviations of detrended output, $y_t = Y_t/A_t$; inflation, $\pi_t = P_t/P_{t-1}$; money growth, $\mu_t = M_t/M_{t-1}$; and the real exchange rate, $s_t = e_t P_t^*/P_t$. Its interest rate reaction function is given by:

$$\begin{aligned} \log(R_t/R) &= \varrho_y \log(y_t/y) + \varrho_\pi \log(\pi_t/\pi) \\ &+ \varrho_\mu \log(\mu_t/\mu) + \varrho_s \log(s_t/s) + \varepsilon_{Rt}, \end{aligned} \tag{42}$$

where y, π, μ, and s are the steady-state values of y_t, π_t, μ_t, and s_t, where R is the steady-state value of the gross nominal interest rate, and where ε_{Rt} is a zero-mean, serially uncorrelated monetary policy shock with standard deviation σ_R.

2.4 The government

The government budget constraint is given by:

$$P_tG_t + T_t + D_{t-1}^g = \tau_t W_t h_t + M_t - M_{t-1} + \frac{D_t^g}{R_t}. \tag{43}$$

The left side of (43) represents uses of government revenue: goods purchases, transfers, and debt repayments. The right side includes tax revenues, money creation, and newly issued debt.

The government also faces a no-Ponzi constraint:

$$\lim_{T\to\infty}\left(\prod_{t=0}^{T}\frac{1}{R_t}\right)D_T^g = 0,$$

which, jointly with (43), implies that the present value of government expenditures equals the present value of tax revenue plus the initial stock of public debt, D_0^g.

Public debt in this model is Ricardian in that, given D_0^g and policy choices on government purchases and tax rates, the competitive equilibrium can be represented with the government debt path dictated by (43), or represented with adjustments in lump-sum transfers to households, T_t, by the amount required to balance the budget constraint each period. Therefore, without loss of generality, the budget rule can be rewritten as:

$$P_tG_t + T_t = \tau_t W_t h_t + M_t - M_{t-1}. \tag{44}$$

This assumption implies a value of domestic bonds, D_t^g, equal to zero at each period. Government spending and the tax rate are determined by the following exogenous stochastic processes:

$$\log(G_t/A_t) = (1-\rho_g)\log(g) + \rho_g\log(G_{t-1}/A_{t-1}) + \varepsilon_{gt}, \tag{45}$$

and

$$\log(\tau_t) = (1-\rho_\tau)\log(\tau) + \rho_\tau\log(\tau_{t-1}) + \varepsilon_{\tau t}. \tag{46}$$

Given these stochastic processes, and given that the nominal money stock is determined by money demand once the monetary authority fixes the nominal interest rate according to equation (42), lump-sum taxes are determined residually to balance the government's budget.

3. Equilibrium

Some of the model's shocks give rise to permanent changes in its endogenous variables, which therefore contain a unit root. There are three different stochastic trends in the model. The first is a stochastic trend in the foreign price level, which arises from the specification of the stochastic process for P_t^* in terms of rates of change in equation (31). The second is a stochastic trend in the price of domestic output (that is common to all other domestic nominal variables), which arises from the fact that the monetary authority adjusts the domestic nominal interest rate as a function of inflation, rather than the price level according to equation (42). The third is a stochastic trend in domestic output (that is common to other domestic real variables, such as consumption and real balances), which comes from the unit root in the technology process given in equation (17).

Solving the model requires that we work with stationary transformations of the variables that contain unit roots. We use the following transformations: $p_t \equiv P_t/P_t^d$, $m_t \equiv M_t/(A_t P_t)$, $w_t \equiv W_t/(A_t P_t^d)$, $p_t^m \equiv P_t^m/P_t^d$, $\tilde{p}_t^d \equiv \tilde{P}_t^d/P_t^d$, $\pi_t \equiv P_t/P_{t-1}$, $\pi_t^d \equiv P_t^d/P_{t-1}^d$, $\pi_t^* \equiv P_t^*/P_{t-1}^*$, $s_t \equiv e_t P_t^*/P_t^d$, and $\lambda_t \equiv A_t \Lambda_t$. All the other variables are transformed according to the following formula: $x_t \equiv X_t/A_t$.[13] The complete system of equations in stationary variables that characterize the model's equilibrium are given in Appendix C.

3.1 New Keynesian Phillips Curves

The price- and wage-setting equations are not implementable for either empirical estimation or numerical simulation, since they involve infinite summations. By linearizing these equations around the steady-state values of the variables, and assuming zero inflation in the steady state, we obtain three Phillips curves relationships that determine the rates of inflation of locally produced goods intermediates, imported intermediates, and the nominal-wage index. Defining $\pi_t^m \equiv P_t^m/P_{t-1}^m$, and $\pi_t^w \equiv W_t/W_{t-1}$, we get:

$$\hat{\pi}_t^d = \beta \hat{\pi}_{t+1}^d + \frac{(1-\beta d_p)(1-d_p)}{d_p}\hat{\xi}_t; \tag{47}$$

$$\hat{\pi}_t^m = \beta \hat{\pi}_{t+1}^m + \frac{(1-\beta d_m)(1-d_m)}{d_m}\hat{s}_t; \tag{48}$$

and

$$\hat{\pi}_t^w = \beta \hat{\pi}_{t+1}^w + \frac{(1-\beta d_w)(1-d_w)}{d_w}\left[\left(\frac{h}{1-h}\right)\hat{h}_t - \hat{\lambda}_t + \left(\frac{\tau}{1-\tau}\right)\hat{\tau}_t - \hat{w}_t\right], \tag{49}$$

where hats over variables denote deviations from steady-state values. The New Keynesian Phillips curve for domestic output inflation is the same as in Galí and Gertler (1999). It relates inflation to expected future inflation and to the real marginal cost of output. The equation for import price inflation is analogous, with real marginal cost captured by the real exchange rate. The wage inflation equation is also analogous. The term in square brackets measures the marginal rate of substitution (the real marginal cost to workers of their work effort) minus the real wage. The household's first-order condition for the nominal wage can be interpreted as a markup over the average marginal cost of work effort over the life of the wage contract.

[13]This includes foreign output, which must grow at the same trend rate as domestic output for the economy to be on a balanced growth path in the long run.

4. Parameter Estimation and Simulation Methodology

The model is too complex to permit an analytical solution. Our results are derived by numerical simulations, which entail assigning numerical values to the model's structural parameters. For this, we use a combination of estimation and calibration. A small number of parameters prove impossible to identify econometrically. For these parameters, we resort to calibrated values based on previous results in the literature. Conditional on these calibrated values, we manage to estimate a large subset of the structural parameters of the model using a combination of GMM and SMM techniques, using quarterly Canadian and U.S. data.[14] We first describe our empirical methodology, and then discuss the parameter values themselves, which are summarized in Tables 1 through 3.

We use a two-step estimation procedure. In the first step, we estimate the parameters of the exogenous stochastic processes involving observable variables, using standard GMM techniques. In the second step, we use SMM to estimate the remaining structural parameters of the model.

The model has five exogenous stochastic processes involving observable variables: R_t^*, π_t^*, y_t^*, g_t, and τ_t. The stochastic processes for the logs of these variables all take the following form:

$$\mathbf{z}_t = (1-\rho_z)\mathbf{z} + \rho_z \mathbf{z}_{t-1} + \epsilon_{zt}, \tag{50}$$

where $\mathbf{z}_t$ is the unconditional sample mean of the (log of) variable $\mathbf{z}_t$. To obtain precise estimates of the persistence parameters, ρ_z, we use moment conditions of the following form:

$$E\left[\mathbf{z}_{t-1}\left(\mathbf{z}_t - (1-\rho_z)\mathbf{z} - \rho_z \mathbf{z}_{t-1}\right)\right] = 0. \tag{51}$$

We use the first lag of $\mathbf{z}_t$ as the only instrument to achieve exact identification. We use moment conditions of the following form to estimate the unconditional means of the $\mathbf{z}$ variables:

$$E\left[\underline{1}\left(\mathbf{z}_t - \mathbf{z}\right)\right] = 0, \tag{52}$$

where we specify a vector of ones ($\underline{1}$) as the instrument. To obtain precise estimates of the variances of the innovations to the stochastic processes, we impose the following moment conditions:

$$E\left[\left(\mathbf{z}_t - (1-\rho_z)\mathbf{z} - \rho_z \mathbf{z}_{t-1}\right)^2 - \sigma_z^2\right] = 0. \tag{53}$$

Conditional on the calibrated parameter values, and on the GMM estimates of parameters of the stochastic shock processes, we use SMM to estimate the remaining structural parameters. The estimator of the parameter vector (Θ) is the solution to the following problem:

$$\hat{\Theta}_T = \arg\min_{\Theta}\left(\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{y}_t,\Theta)\right)' W_T \left(\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{y}_t,\Theta)\right), \tag{54}$$

where W_T is a random non-negative symmetric matrix, $\mathbf{y}_t$ is a vector containing a subset of the model's variables, and $f(\mathbf{y}_t,\Theta)$ is a q-vector of unconditional moment restrictions.

[14] Data sources are described in Appendix A.

As discussed below, most of the moment restrictions involve the difference between an unconditional moment predicted by the model and the corresponding moment in the data, where the predicted moments are calculated for given parameter values using the linearized version of the model without simulating. An optimal weighting matrix, W_T, is obtained as the inverse of the variance-covariance matrix of the moment conditions evaluated at a set of first-step estimates, in which W_T is set equal to the identity matrix. This matrix is consistently estimated using the estimator proposed by Newey and West (1994). Heuristically, it gives more weight to moments that are precisely estimated in the data.

This methodology has several attractive features. First, it allows flexibility in selecting the moments used in the estimation. These can include unconditional means, variances, covariances, and autocovariances of any of the variables in the model. Information from the data can be used in the estimation that cannot be used with some alternative methods. For example, matching the impulse-response functions of variables in the model with those of an estimated vector autoregression (VAR) reproduces as closely as possible the entire joint distribution of variables in the VAR. However, only as many variables can be included as there are structural shocks in the model: moments involving excluded variables cannot be used at all. Second, the method can rely on variables that are measured accurately. For instance, net foreign indebtedness is poorly measured in the data: moments involving this variable can be excluded from the estimation. Methods that use GMM directly to estimate the optimality or orthogonality conditions of structural models are forced to use data on such poorly measured variables. Finally, when the dimension of the vector of moments (q) is greater than the dimension of the vector of structural parameters, the overidentifying restrictions implied by the model can be tested formally using a J-test.[15]

The parameters of the stochastic shock processes for A_t and b_t are estimated as part of our SMM procedure. Both processes involve unobserved variables, but can be expressed in terms of observable variables and one or more of the model's structural parameters. Because they involve other structural parameters of the model, we cannot estimate them in the first round by GMM along with the parameters of the other exogenous stochastic processes. In the case of the technology shock, we use the factor demand equations of the domestic intermediate-goods-producing firm, along with its production function, to solve for A_t in terms of observable variables. Using equations (16), (20), and (21) gives, after some manipulation (and after aggregating):

$$\log(A_t) = \frac{1}{1-\phi}\left[\log(Y_t/h_t) - \phi\log(W_t/P_t) - \phi\log(\phi/(1-\phi))\right]. \tag{55}$$

With the assumption that technology follows a random walk with drift, this variable is non-stationary, but in first differences we have:

$$\log(A_t) - \log(A_{t-1}) = A + \epsilon_{At}. \tag{56}$$

[15]Bergin (2003) estimates the parameters of a similar model using maximum-likelihood techniques. He tests its overidentifying restrictions by re-estimating an exactly identified version of the model and implementing a likelihood ratio test.

We estimate A by adding the following moment condition:

$$E\left[\underline{1}\left(\Delta\log(A_t)-A\right)\right]=0, \tag{57}$$

and we estimate the variance of the innovation to the technology process by adding the following moment condition:

$$E\left[\left(\Delta\log(A_t)\right)^2-\sigma_A^2\right]=0. \tag{58}$$

In the case of the money-demand shock b_t, we use equations (10), (11), and (12) to derive the following money-demand equation:

$$\log(m_t/c_t)=\gamma\log(R_t/(R_t-1))+\log(b_t). \tag{59}$$

Adding and subtracting the mean of the stochastic process generating $\log(b_t)$, we get:

$$\log(m_t/c_t)=\log(b)+\gamma\log(R_t/(R_t-1))+\log(b_t/b). \tag{60}$$

We treat $\log(b_t/b)$ as a mean-zero AR(1) error term. We estimate ρ_b using the following moment condition:

$$\begin{aligned}&E\big[R_{t-2}\big(\log(m_t/c_t)-\rho_b\log(m_{t-1}/c_{t-1})-(1-\rho_b)\log(b)\\&+\gamma\log(R_t/(R_t-1))-\rho_b\gamma\log(R_{t-1}/(R_{t-1}-1))\big)\big]=0.\end{aligned} \tag{61}$$

We use the second lag of the nominal interest rate to achieve exact identification of ρ_b. We estimate the variance of the money-demand innovation using the following second-moment condition:

$$\begin{aligned}&E\big[\big(\log(m_t/c_t)-\rho_b\log(m_{t-1}/c_{t-1})-(1-\rho_b)\log(b)\\&+\gamma\log(R_t/(R_t-1))-\rho_b\gamma\log(R_{t-1}/(R_{t-1}-1))\big)^2-\sigma_b^2\big]=0.\end{aligned} \tag{62}$$

Finally, we estimate the constant term $\log(b)$ using the following moment condition:

$$E\left[\underline{1}\left(\log(m_t/c_t)-\gamma\log(R_t/(R_t-1))-\log(b).\right)\right] \tag{63}$$

4.1 Parameter values

Table 1 summarizes the parameter values fixed by calibration. The subjective discount rate, β, is given a standard value, which implies an annual real interest rate of 4 per cent in the steady state. The weight on leisure in the utility function, η, is calibrated so that the representative household spends about one third of its total time working in the steady state. The foreign supply and demand parameters come from equation (30), which gives foreign demand for domestic exports, and from the risk premium equation (5), which relates to the elasticity of supply of foreign funds to the domestic economy: it also affects the economy's net indebtedness in the long run. The α_x parameter is a normalization that ensures that the current account is balanced in the long run. The parameter φ is set equal to -0.06, which gives an average risk premium of 93 basis points as in Clinton (1998).

In the case of the individual demand elasticities, σ, θ, and ϑ, their influence on the stochastic properties of the model is very indirect. When we use equations (14) and (15) to derive the linearized Phillips curve for wage inflation, the σ parameter drops out. Similarly, θ and ϑ no longer appear in the Phillips curves for domestic- and imported-goods inflation. By influencing the size of the markups over marginal cost, however, these parameters influence the steady-state levels of the domestic production of intermediate goods, imported intermediate goods, and employment. Because certain coefficients in the linearized model depend on the steady-state levels of endogenous variables, the moments predicted by the model are related to these parameters. Unfortunately, the influence of variations in these parameters on the moments predicted by the model is so weak that it is impossible to estimate them precisely. The θ and ϑ parameters give the elasticity of substitution across different types of intermediate goods in the production of the composite domestic good and the composite imported good. Setting $\theta = \vartheta = 8.0$ gives a steady-state markup of 14 per cent, which agrees well with estimates in the empirical literature of between 10 per cent and 20 per cent (see, for example, Basu 1995). The σ parameter gives the elasticity of substitution across different labour types in the production of individual domestic intermediate goods. The value of 6.0 corresponds to estimates from microdata in Griffin (1992).[16]

Table 2 summarizes the parameters of the model's exogenous stochastic processes. Apart from the parameters of the stochastic process for public spending, the individual parameter estimates are all significant at the 1 per cent level.[17] Except for foreign inflation, the processes are all fairly persistent, with AR(1) parameters above 0.65. The standard deviations of the innovations to the processes vary widely in magnitude, ranging from 0.0016 in the case of foreign interest rate shocks to 0.0402 in the case of tax rate shocks. The volatility of foreign shocks is smaller than that of domestic shocks, which suggests the relative importance of domestic shocks for business cycle fluctuations in the Canadian economy.

Table 3 summarizes the parameters estimated by SMM. Appendix B summarizes the moments used to obtain these estimates. The model passes a test of its overidentifying restrictions at the 10 per cent level: the marginal significance level for the J-test is 13.18 per cent.[18] For this subset of parameters, our estimates are very precise. In particular, the nominal rigidity parameters are highly significant (at the 0.1 per cent level). They are also of plausible magnitude and well within the range of values in previous empirical studies and in calibrated general-equilibrium models. The estimate of d_p implies that the prices of domestic intermediate goods remain fixed for, on average, slightly more than three quarters.

[16] It also agrees with the estimated value in Ambler, Guay, and Phaneuf (2003). They succeed in estimating the value of the equivalent parameter in their model by calibrating the equivalent of the d_w parameter.

[17] The constants of the stochastic processes for foreign output and foreign inflation are normalized to equal zero, so estimates for these parameters do not appear in the table.

[18] As described in the preceding section, the parameters of the interest rate rule, money-demand shock, and technology process are estimated using exactly identified GMM. We estimate them simultaneously with the other parameters in the table, but for the eighteen parameters unconditional moments are used to estimate the other eight parameters.

The other prices are slightly more sluggish, but still well within the range of plausibility. Import prices remain fixed for, on average, about 3.4 quarters. Nominal wages remain fixed for, on average, slightly more than four quarters.

The ϱ_i parameters are from the interest rate reaction function. They are intended to capture a fairly standard Taylor-type rule. Since the sum of ϱ_π and ϱ_μ is greater than unity, the long-run level of the inflation rate is determinate. For our base-case simulation results, we set $\varrho_s = 0$ to avoid some anomalous responses of some variables to shocks,[19] but the coefficient is built into the structural model in order to allow for sensitivity analysis. We estimated ϱ_y and found it to be close to zero and insignificant, so we set it to zero in our simulations, as in Dib (2003).

The ν and ς parameters, which capture the elasticities of demand for composite intermediate imports and for exports, would enter symmetrically into the domestic and foreign aggregate production functions for final goods, which suggests that their values should be similar. Their estimated values are close to the calibrated value of 0.6 set by McCallum and Nelson (2001) for the equivalents of both parameters.[20] With our estimated values, the sum of import and export elasticities exceeds one, so the static Marshall-Lerner condition is satisfied by the model. The parameter α_d gives the relative weight on the composite domestic intermediate good in the production of the final good. The estimated value of 0.7413 is estimated precisely and reflects a degree of home bias in the demand for composite intermediate goods. ϕ gives the weight on the composite good used as an input in the production of domestic intermediate goods. The estimated value implies that labour accounts for just over 70 per cent of the value added in the intermediate-goods sector.

The money-demand process is highly persistent, with an autoregressive coefficient that is indistinguishable from a random walk, even though our theoretical model implies that the observable variables used to estimate the process should be stationary. This could reflect the fact that many empirical studies find it difficult to reject the null hypothesis of a unit root in short-term nominal interest rates. The standard deviation of the innovation to the money-demand shock process, 0.0610, is larger than the standard deviation of any other forcing process in the model. This, combined with the degree of persistence of the money-demand shock, means that the unconditional variance of money demand is very large compared with that of the other shocks in the model.

The drift term in the technology process is highly significant and entails an annual per-capita growth rate of 1.4 per cent, which is close to the per-capita growth rate of output in our sample period. The standard deviation of the innovation to the technology process is 0.0026, which is smaller than the volatility of any other domestic innovation. This suggests that technology shocks have not been the primary driving force of the Canadian business cycle.

[19] With ϱ_s set equal to its estimated value, the response of the exchange rate to several types of shocks is highly non-monotonic, with the long-run change in the exchange rate being of a different sign than the initial change.

[20] Kollmann also uses values of 0.6 for the equivalents of both parameters in a similar model. He justifies these values as being approximately equal to the median values of the import and export elasticities estimated by Hooper and Marquez (1995).

The model is simulated by linearizing its equilibrium conditions around their deterministic steady-state values. This yields a set of saddlepoint-stable linear difference equations, which we simulate using standard techniques described by Blanchard and Kahn (1980) and King, Plosser, and Rebelo (1987).

5. Simulation Results

5.1 Unconditional moments

Table 4 summarizes some of the unconditional second moments predicted by the model. The second column gives the model's prediction concerning the moment defined in the first column.[21] The third column gives the estimated value (using exactly identified GMM) of the moment in our data set, and the fourth column gives the standard error of this estimate. To compute the model's predictions, we use stochastic simulations with shocks generated using the standard deviations from Tables 2 and 3, and imposing independence across the different types of shocks. The calculated statistics, both for the model and the data, are for series measured in growth rates.

The volatility of output in the model exceeds that in the data. Consumption volatility in the model is also too high. This reflects the lack of capital in the model, which reduces the consumption-smoothing opportunities that are available to agents. This also leads to a relative volatility of hours that is too high when hours are measured in levels. The volatility of employment growth is, however, comparable with that in the data. Diminishing marginal returns to labour in the production functions for intermediate goods mean that, in response to any type of shock except a technology shock, hours must be more volatile than output. The real exchange rate is 2.15 times as volatile as output. This relative volatility is somewhat lower, but of the same order of magnitude, as in the data. The nominal and real exchange rates are highly correlated, which also corresponds well with what we observe in the data.

Figure 2 shows in graphical form some of the unconditional autocorrelations predicted by the model. The series are filtered using the Hodrick-Prescott filter, for comparability with results in the literature. Output and the nominal and real exchange rates are quite persistent, although somewhat less than in the data: the first-order autocorrelations in the Canadian data are, respectively, 0.77, 0.82, and 0.79. The model's prediction concerning the persistence of inflation is quite striking. Generating inflation persistence has been a strong challenge to recent business cycle models, both closed and open economy, as Fuhrer and Moore (1995) note. Although the normalized level of output is strongly autocorrelated, the first-order autocorrelation of output growth (not shown) is only mildly positive (0.12), which means that the autocorrelation of output growth predicted by the model does not match that in the data. It is clear that reproducing this feature of the data would require

[21] The predictions of the model concerning unconditional moments are calculated using the asymptotic variance-covariance matrices of the state variables and endogenous variables.

adding features to the model such as the slow adjustment of employment.[22]

5.2 Impulse-response functions

The impulse responses of various endogenous variables to 1 per cent shocks to the different exogenous stochastic shocks in the model are shown in Figures 3 through 10. The model exhibits prolonged output responses to most types of shocks, including nominal shocks. The maximum response of output to most types of shocks, however, occurs during the same period that the shock hits. Output has a hump-shaped response only in the case of a tax rate shock. In response to a foreign output shock, domestic output initially falls, but its response changes sign after the first period and then peaks after several periods. Only technology shocks have permanent effects on real variables in the model, because of the unit root in the technology process. On the other hand, because the interest rate rule responds to the rate of change of prices and money-supply growth, most shocks have permanent effects on the model's nominal variables.

5.3 Exchange rate pass-through

Figure 11 summarizes the model's predictions concerning the effects of different shocks on the nominal exchange rate and on three different price indexes: P_t, which is the equivalent in our model of the CPI; P_t^d, the equivalent of the PPI; and P_t^m, the price index of imported goods. The results are striking. In response to all types of shocks, except technology shocks, the maximum response of the nominal exchange rate to the shock occurs immediately upon impact. As stated earlier, the model's nominal variables have a common stochastic trend due to the interest rate rule that responds to inflation rather than to the price level. This implies that the exchange rate overshoots (its short-run response is greater than its long-run response) in response to each of the eight structural shocks in the model. The responses of import prices, the PPI, and the CPI are invariably gradual. Even in the case of technology shocks, the immediate response of the exchange rate is much more pronounced than that of the price level, and exceeds the response of the CPI at horizons up to 40 periods. Pass-through is also incomplete (to the PPI and the CPI) after 40 periods in response to each of the different structural shocks in the model, in spite of the fact that pass-through to the import price index is essentially complete after 10 periods in response to all shocks. The CPI has a hump-shaped response to all types of shocks except shocks to the domestic interest rate, where its response can be characterized as a gradual descent to a plateau.

In the case of tax rate shocks, the impact effect on the nominal exchange rate is negative, whereas the impact effect on the overall price level is positive. This means that pass-through to the CPI is actually negative in the very short run in response to tax shocks. It takes approximately 10 periods for the price level to return to its initial level, so the degree of pass-through after 10 periods is approximately zero.

[22] Ambler, Guay, and Phaneuf (2003) find that employment adjustment costs were crucial in allowing a closed-economy business cycle model to reproduce the positive autocorrelations at low orders of output growth.

The impulse-response functions of the exchange rate and prices make it clear that exchange rate pass-through is a conditional phenomenon. The dynamics of pass-through depend on the type of shock that hits the economy. In response to all of the structural shocks in the model, however, pass-through is less than complete in the short run (negative in the case of tax rate shocks). The degree of pass-through in the medium run is quite consistent across different types of shocks. Pass-through to imported goods prices is close to complete after 10 quarters in response to all types of shocks. Omitting tax rate shocks, which are discussed in the preceding paragraph, after 10 quarters pass-through is lowest in the case of foreign output shocks: about 30 per cent of the change in the exchange rate has been passed through to the CPI. Pass-through is most complete after 10 quarters in response to foreign nominal interest rate shocks: more than 50 per cent of the change in the nominal exchange rate has been passed through to the CPI.

After 40 quarters, exchange rate pass-through to the CPI is close to complete in response to all shocks except foreign output shocks, in that the absolute differences between the values of the exchange rate and the price level are small. However, because the nominal exchange rate initially overshoots in response to the different structural shocks, the relative responses of the price level compared with that of the exchange rate remains quite small, even after 40 quarters. Figure 11 shows that exchange rate pass-through is complete in the long run in response to all of the types of shocks in the model. This reflects the fact that slow pass-through in the model is the result of short-term frictions (nominal rigidities). There is no strategic pricing in our model that would lead to long-run deviations from purchasing-power parity: models of international product differentiation with strategic pricing are popular micro-based explanations for slow and incomplete exchange rate pass-through. Ghosh and Wolf (2001) conclude that the empirical evidence supports the case for complete long-run pass-through and hence for macro-based models of sticky prices, rather than micro-based models of strategic pricing.

A VAR analysis of the bivariate dynamics of the nominal exchange rate and the price level generated by the model would be incapable of distinguishing between the large number of different structural shocks in our model. In fact, to identify different types of structural shocks would be difficult, given that most of the model's shocks lead to permanent long-run effects on the exchange rate and the price level, and have qualitatively similar effects on the exchange rate and the price level in the very short run (except for money-demand shocks). However, the uniformity across shocks of the responses of the exchange rate and domestic prices should be picked up by VAR estimates applied to artificial data generated by the model.

5.4 Sensitivity analysis

In this section, we show how our results pertaining to pass-through are sensitive to the degree of nominal rigidity of both domestic goods prices and import prices. Figure 12 shows the impulse-response functions of the exchange rate and the overall domestic price level (CPI) to a domestic interest rate shock when domestic output prices are no longer sticky. For these simulations, we set $d_p = 0$, with all other parameters set equal to their values in the base-case scenario discussed in section 5.3. The exchange rate still overshoots in response to each of the eight structural shocks. There is still slow pass-through, even to the price of domestic output, despite the fact that, by assumption, the prices of domestic intermediate goods can adjust instantaneously to shocks, except in the case of technology shocks. In the latter case, a positive technology shock lowers the marginal cost of production by so much that the price of domestic output drops initially by more than the exchange rate. In the very short run, pass-through to the PPI is more than 100 per cent. For all of the other structural shocks, the degree of pass-through both in the short run and the medium run (after 10 quarters) is almost the same as in the base-case scenario.

The results with $dp = 0$ suggest that nominal wage rigidity may be an important part of the explanation of slow pass-through. Figures 13 and 14 confirm this hypothesis. In Figure 13, we shut down both types of nominal price rigidity, so that the only nominal rigidity left in the model is wage rigidity. Since the prices of imported goods now adjust instantaneously, the import price index, P^m, tracks the nominal exchange rate quite closely, including in response to technology shocks (in this case, import prices respond slightly more strongly in the short run than the exchange rate). The exchange rate, however, still overshoots in response to each of the structural shocks and the CPI adjusts slowly. The quantitave measure of pass-through both in the very short run and after 10 quarters is again little changed from the base-case scenario.

Figure 14 shows the response of the nominal exchange rate and of three different price indexes when all three of the nominal rigidities are removed from the model. The exchange rate continues to overshoot in response to most types of structural shocks, although in response to interest rate shocks its response is essentially flat after the first period. In response to domestic nominal shocks (interest rate shocks and money-demand shocks), exchange rate pass-through to the CPI is immediate. Even with no nominal rigidities, however, most real shocks (tax rate shocks, government spending shocks, and foreign shocks) lead the CPI to respond less than the nominal exchange rate does in the very short run. In response to tax rate shocks, the exchange rate and the CPI initially move in opposite directions. In contrast to the scenarios with at least one type of nominal rigidity, the response of the CPI is no longer hump-shaped. It responds either monotonically (foreign output shocks), is flat after the first period (interest rate shocks and technology shocks), or overshoots its long-run response (all other shocks).

The main conclusion that can be drawn from our sensitivity analyses is that, although pricing to market is sufficient to generate slow exchange rate pass-through, it is not necessary. Wage rigidity is an important structural feature of our model that leads to slow pass-through in response to structural shocks. Even with no nominal rigidities, real shocks can also lead to incomplete pass-through in the short run. These results qualify the conclusions of earlier theoretical models, such as those by Betts and Devereux (2000) and Smets and Wouters (2002), that pricing to market is crucial in generating slow exchange rate pass-through.

6. Conclusions

Our structural small open-economy model is capable of distinguishing between the effects of a large number of different types of shocks. Because the exchange rate and prices are endogenous variables, exchange rate pass-through in the model is always conditional on the type of shock. However, there is a remarkable degree of uniformity in the dynamics of exchange rate pass-through across the different types of structural shocks in the model. The effect of shocks on the price level is much smaller than on the exchange rate. After 10 quarters, the degree of exchange rate pass-through varies between 50 and 75 per cent. After 40 quarters, the degree of exchange rate pass-through is greater than 90 per cent in response to all shocks except foreign output shocks. Our sensitivity analysis leads us to conclude that sticky imported-goods prices (pricing to market) are sufficient to generate slow exchange rate pass-through, but that they are not necessary. Other structural features, such as nominal-wage rigidities, can also by themselves result in sluggish exchange rate pass-through. Even a model with no nominal rigidities can generate slow pass-through in response to real shocks.

Bibliography

Ambler, S., A. Guay, and L. Phaneuf. 2003. "Labor Market Imperfections and the Dynamics of Postwar Business Cycles." CIRPÉE, Université du Québec à Montréal. Draft.

Bailliu, J. and E. Fujii. 2003. "Exchange Rate Pass-Through in Industrialized Countries: Has it Really Declined?" Bank of Canada. Draft.

Bansal, R., A.R. Gallant, R. Hussey, and G. Tauchen. 1995. "Nonparametric Estimation of Structural Models for High-Frequency Currency Market Data." *Journal of Econometrics* 66: 251-87.

Basu, S. 1995. "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare." *American Economic Review* 85: 512-31.

Bergin, P. 2003. "Putting the 'New Open Economy Macroeconomics' to a Test." *Journal of International Economics.* Forthcoming.

Betts, C. and M. Devereux. 2000. "Exchange Rate Dynamics in a Model of Pricing to Market." *Journal of International Economics* 50: 215-44.

Blanchard, O. and C. Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations." *Econometrica* 48: 1305-11.

Bowman, D. and B. Doyle. 2003. "New Keynesian Open Economy Models and their Implications for Monetary Policy." In *Price Adjustment and Monetary Policy.* Proceedings of a conference held by the Bank of Canada, November 2002. Forthcoming.

Calvo, G. 1983. "Staggered Contracts and Exchange Rate Policy." In *Exchange and International Economics*, edited by J. Frenkel. Chicago: University of Chicago Press.

Campa, J. and L. Goldberg. 2001. "Exchange Rate Pass-Through into Import Prices: A Macro and Micro Phenomenon." Federal Reserve Bank of New York. Draft.

Chari, V.V., P. Kehoe, and E.R. McGrattan. 2002. "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" Federal Reserve Bank of Minneapolis. Staff Report No. 277.

Clinton, K. 1998. "Canada-U.S. Long-Term Interest Differentials in the 1990s." *Bank of Canada Review* (Spring): 17–38.

Cogley, T. and J. Nason. 1995. "Output Dynamics in Real Business Cycle Models." *American Economic Review* 85: 492-511.

Corsetti, G. and P. Pesenti. 2001. "International Dimensions of Optimal Monetary Policy." National Bureau of Economic Research, Working Paper No. 8230.

Devereux, M. 2001. "Monetary Policy, Exchange Rate Flexibility and Exchange Rate Pass-Through." In *Revisiting the Case for Flexible Exchange Rates*, 47-82. Proceedings of a conference held by the Bank of Canada, November 2000. Ottawa: Bank of Canada.

Devereux, M. and J. Yetman. 2003. "Price Setting and Exchange Rate Pass-Through: Theory and Evidence." In *Price Adjustment and Monetary Policy.* Proceedings of a conference held by the Bank of Canada, November 2002. Forthcoming.

Dib, A. 2003. "Monetary Policy in Estimated Models of Small Open and Closed Economies." Bank of Canada Working Paper No. 2003-27.

Fuhrer, J. and G. Moore. 1995. "Inflation Persistence." *Quarterly Journal of Economics* 110: 127-59.

Galí, J. and M. Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics* 44: 195-222.

Galí, J. and T. Monacelli. 1999. "Optimal Monetary Policy and Exchange Rate Variability in a Small Open Economy." Boston College. Draft.

Ghosh, A. and H. Wolf. 2001. "Imperfect Exchange Rate Pass-Through: Strategic Pricing and Menu Costs." CESifo Working Paper No. 436.

Giavazzi, F. and C. Wyplosz. 1984. "The Real Exchange Rate, the Current Account, and the Speed of Adjustment." In *Exchange Rate Theory and Practice*, edited by J. Bilson and R. C. Marston. Chicago: University of Chicago Press.

Goldberg, P. and M. Knetter. 1997. "Goods Prices and Exchange Rates: What Have We Learned?" *Journal of Economic Literature* 35: 1243-72.

Griffin, P. 1992. "The Impact of Affirmative Action on Labor Demand: A Test of Some Implications of the Le Chatelier Principle." *Review of Economics and Statistics* 74: 251-60.

Hooper, P. and J. Marquez. 1995. "Exchange Rates, Prices and External Adjustment in the United States and Japan." In *Understanding Interdependence* Princeton: Princeton University Press.

Ireland, P.N. 2003. "Endogenous Money or Sticky Prices." *Journal of Monetary Economics.* Forthcoming.

Jones, J. 2002. "Has Fiscal Policy Helped Stabilize the Postwar U.S. Economy?" *Journal of Monetary Economics* 49: 709-46.

Kim, K. 1998. "U.S. Inflation and the Dollar Exchange Rate: A Vector Error Correction Model." *Applied Economics* 30: 613-19.

King, R.G., C.I. Plosser, and S. Rebelo. 1987. "Production, Growth, and Business Cycles: Technical Appendix." University of Rochester. Mimeo.

Kollmann, R. 2002. "Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles." *Journal of Monetary Economics* 49: 989-1015.

Lane, P. 2001. "The New Open Economy Macroeconomics: A Survey." *Journal of International Economics* 54: 235-66.

McCallum, B. and E. Nelson. 1999. "Nominal Income Targeting in an Open-Economy Optimizing Model." *Journal of Monetary Economics* 43: 553-78.

McCallum, B. and E. Nelson. 2001. "Monetary Policy for an Open Economy: An Alternative Framework with Optimizing Agents and Sticky Prices." *Oxford Review of Economic Policy* 16: 74-91.

McCarthy, J. 2000. "Pass-Through of Exchange Rates and Import Prices to Domestic Inflation in Some Industrialized Economies." Federal Reserve Bank of New York. Staff Report No. 111.

Mendoza, E., A. Razin, and L. Tezar. 1994. "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption." *Journal of Monetary Economics* 34: 297-323.

Newey, W. and K. West. 1994. "Automatic Lag Selection in Covariance Matrix Estimation." *Review of Economic Studies* 61: 631-53.

Obstfeld, M. and K. Rogoff. 1995. "Exchange Rate Dynamics Redux." *Journal of Political Economy* 103: 624-60.

Sarno, L. 2001. "Toward a New Paradigm in Open Economy Modeling: Where do we Stand?" *Federal Reserve Bank of St. Louis Review* May-June: 21-36.

Schmitt-Grohé, S. and M. Uribe. 2003. "Closing Small Open Economy Models." *Journal of International Economics*. Forthcoming.

Senhadji, A. 1997. "Sources of Debt Accumulation in a Small Open Economy." International Monetary Fund. Working Paper No. 97/146.

Smets, F. and R. Wouters. 2002. "Openness, Imperfect Exchange Rate Pass-Through and Monetary Policy." *Journal of Monetary Economics* 49: 947-81.

Taylor, J. B. 1993. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39: 195-214.

Table 1: Calibrated Parameters

Parameter	Value
Preferences	
β	0.99
η	1.35
Production	
σ	6.00
θ	8.00
ϑ	8.00
Foreign supply/demand	
α_x	0.074
φ	-0.06

Table 2: First-Step Estimation (exactly identified GMM)

Parameter	Value	Standard deviation	t-stat	p-value
ρ_g	0.8548	0.7590	1.1262	0.2101
$\bar{g}$	6.2232	0.0729	85.3962	0.0000
σ_g	0.0098	0.0131	0.7488	0.2994
τ	0.2908	0.0096	30.3564	0.0000
ρ_τ	0.6729	0.1363	4.9386	0.0000
σ_τ	0.0402	0.0104	3.8509	0.0004
ρ_{R^*}	0.8102	0.0933	8.6845	0.0000
R^*	0.0150	0.0014	10.4156	0.0000
σ_{R^*}	0.0016	0.0004	4.3298	0.0001
ρ_{y^*}	0.8835	0.0519	17.0074	0.0000
σ_{y^*}	0.0059	0.0010	5.9069	0.0000
ρ_{π^*}	0.4273	0.0907	4.7134	0.0000
σ_{π^*}	0.0035	0.0005	7.0488	0.0000

Table 3: Parameter Estimation (SMM)

Parameter	Value	Standard deviation	t-stat	p-value
Nominal rigidity				
d_p	0.6763	0.1200	5.6348	0.0000
d_m	0.7045	0.1078	6.5326	0.0000
d_w	0.7572	0.0652	11.6156	0.0000
Interest rate rule				
r	0.0193	0.0013	14.3899	0.0000
ϱ_π	0.7391	0.0538	13.7352	0.0000
ϱ_μ	0.5059	0.1421	3.5599	0.0011
ϱ_s	0.0525	0.0241	2.1803	0.0388
σ_R	0.0023	0.0054	0.4354	0.3609
Foreign supply/demand				
ς	0.5251	0.1166	4.5053	0.0001
Production				
ν	0.5521	0.0738	7.4789	0.0000
α_d	0.7413	0.0839	8.8358	0.0000
ϕ	0.2966	0.1068	2.7757	0.0099
Money-demand shock				
b	0.3820	0.1521	2.5111	0.0188
ρ_b	0.9999	0.0014	698.8693	0.0000
σ_b	0.0610	0.0279	2.1857	0.0383
Preferences				
γ	0.2485	0.1187	2.0937	0.0462
Technology process				
A	0.0035	0.0014	2.4952	0.0195
σ_A	0.0098	0.0026	3.7067	0.0007

J-stat=12.4619, p-value=0.1318

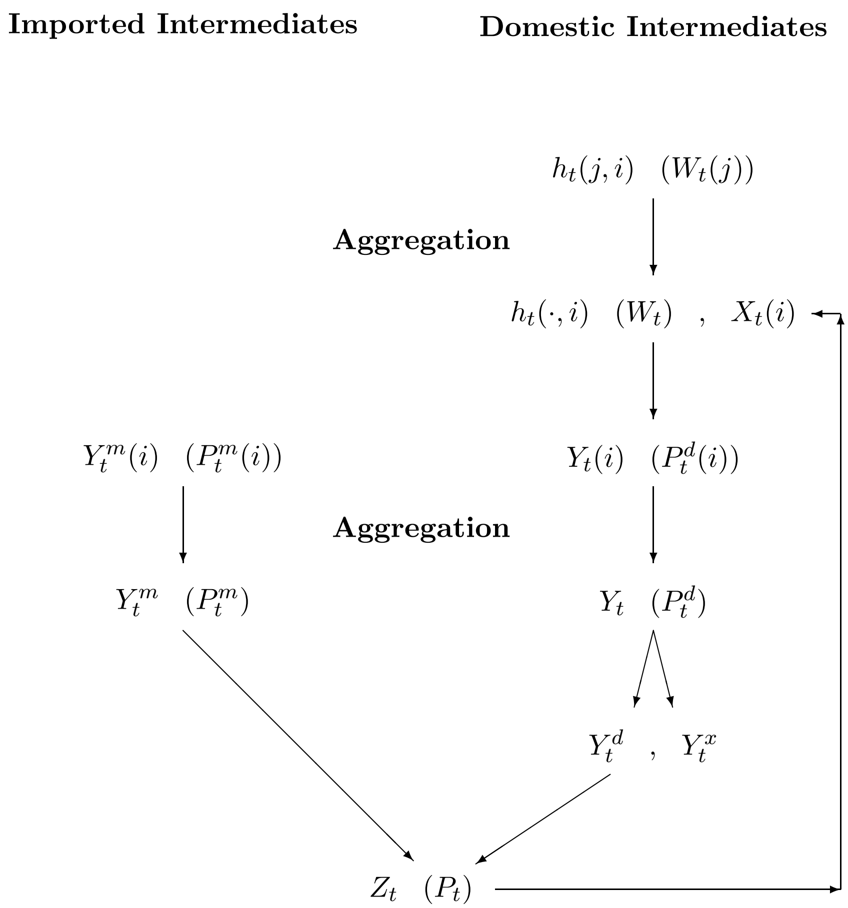
Table 4: Standard Deviations and Correlations

Moment	Model	Data	S.E.†
$\sigma_{\Delta y}$	0.0110	0.0074	0.0012
$\sigma_{\Delta c}$	0.0126	0.0077	0.0012
$\sigma_{\Delta h}$	0.0083	0.0081	0.0016
$\sigma_{\Delta e}$	0.0279	0.0190	0.0017
$\sigma_{\Delta s}{}^{*}$	0.0240	0.0177	0.0015
σ_{π}	0.0094	0.0192	0.0053
$\sigma(\Delta e_t, \Delta s_t)$	0.9345	0.9567	0.1548

†: standard error of value estimated from data

*: with $s_t \equiv e_t p_t^{*}/p_t$

Figure 1: Production Structure of the Model



$$Z_t = C_t + G_t + X_t$$

Figure 2: Autocorrelation Functions

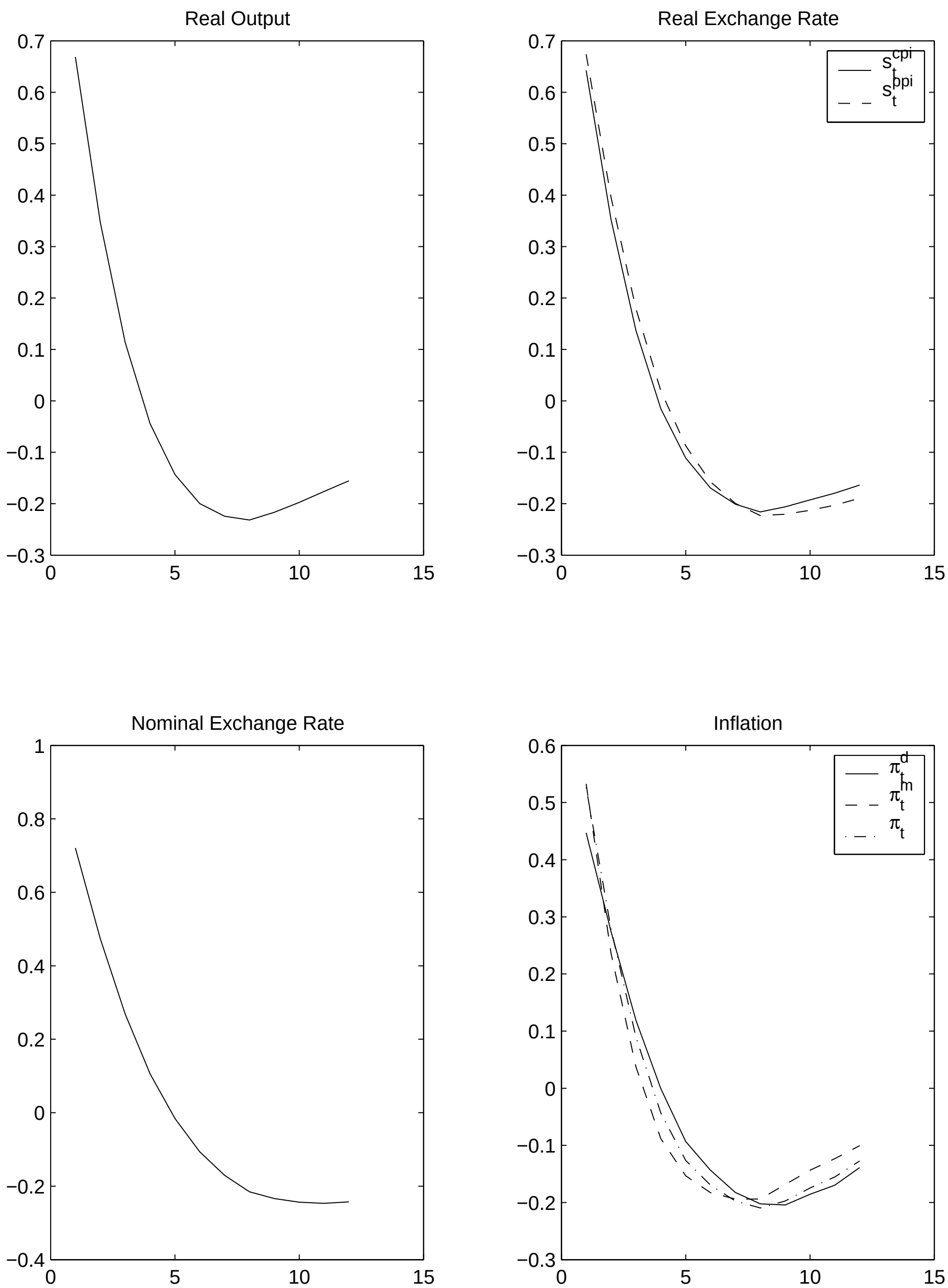


Figure 3: Effects of 1 per cent Technology Shock

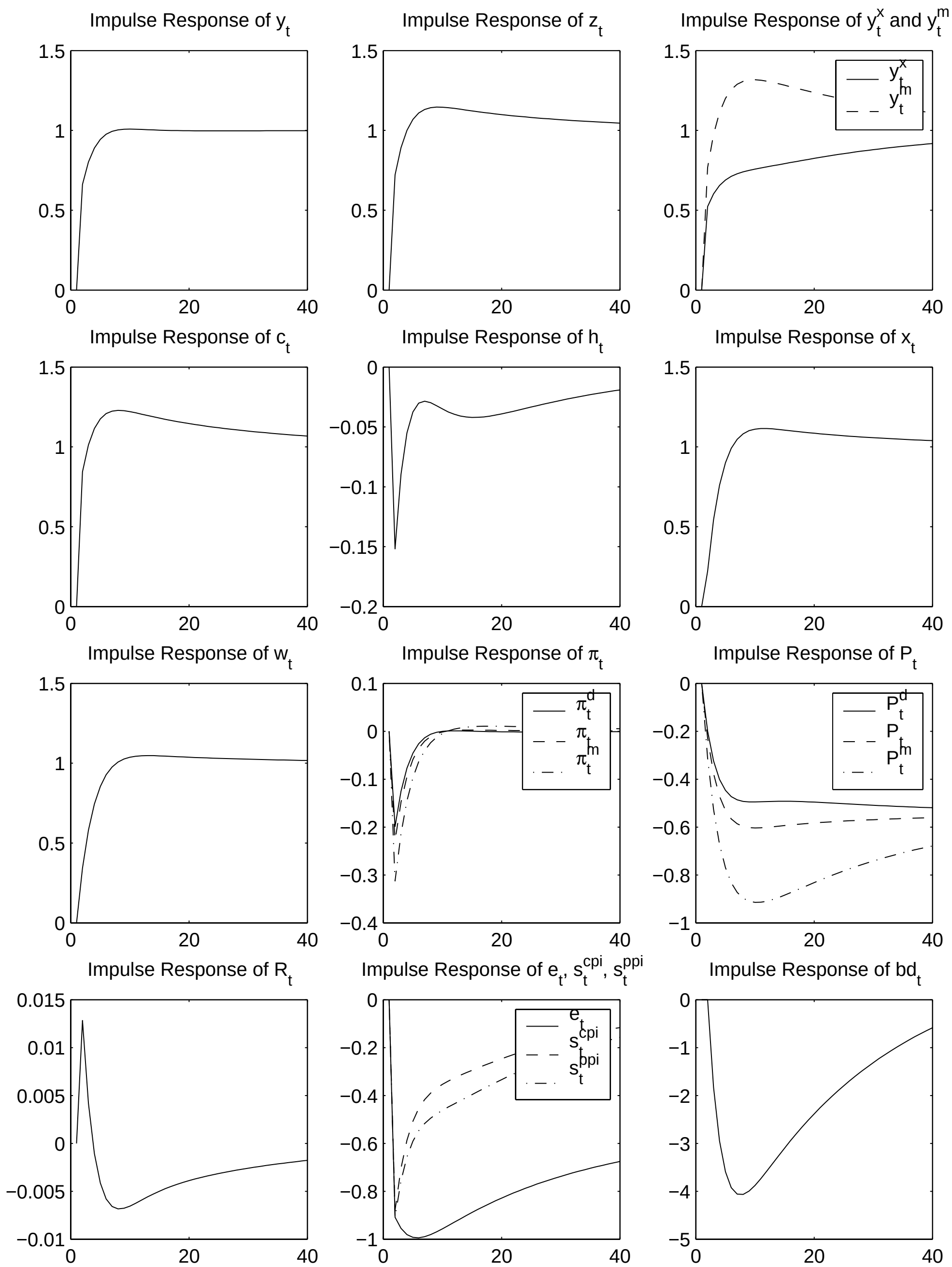


Figure 4: Effects of 1 per cent Local Nominal Interest Rate Shock

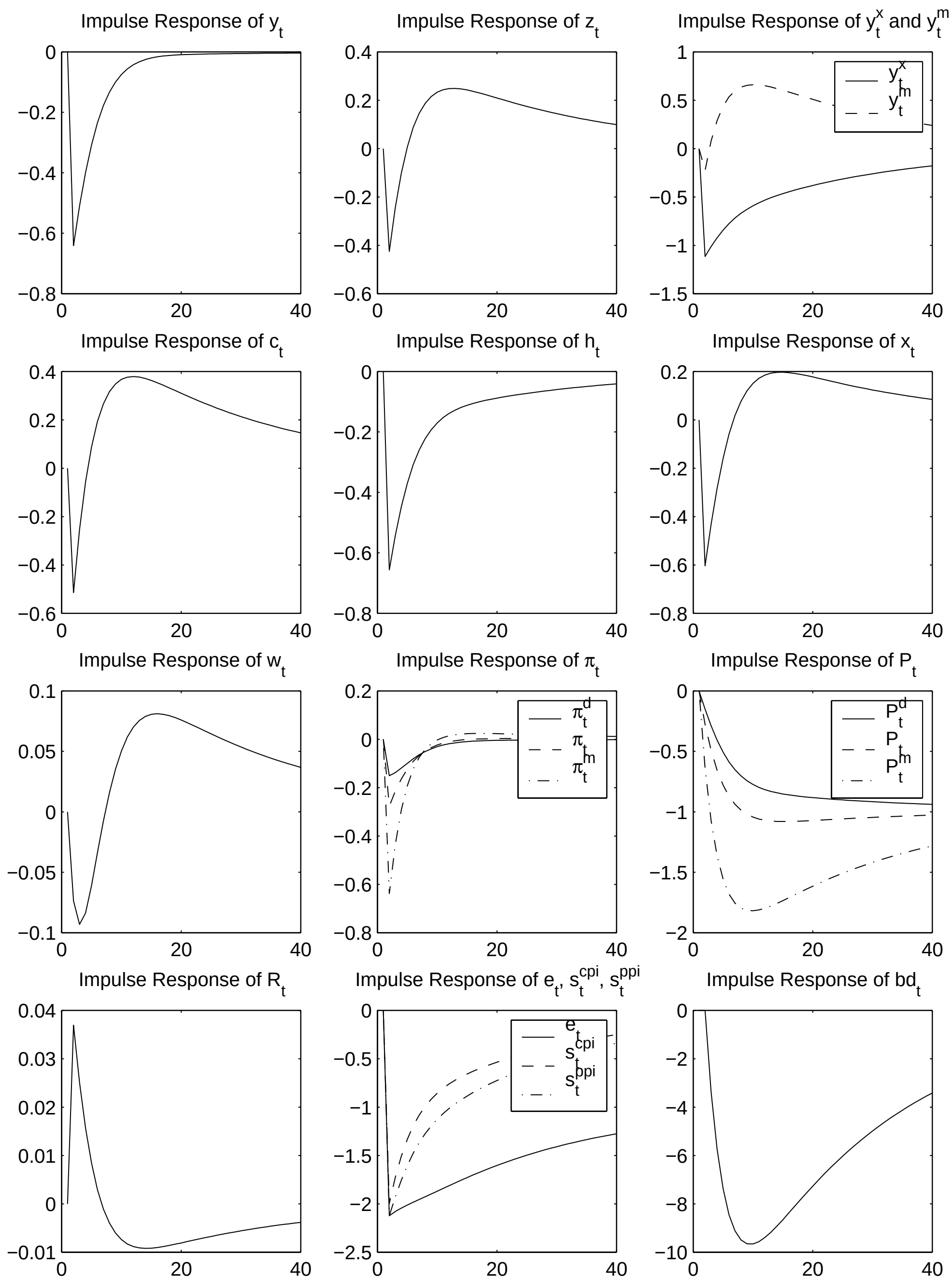


Figure 5: Effects of 1 per cent Money-Demand Shock

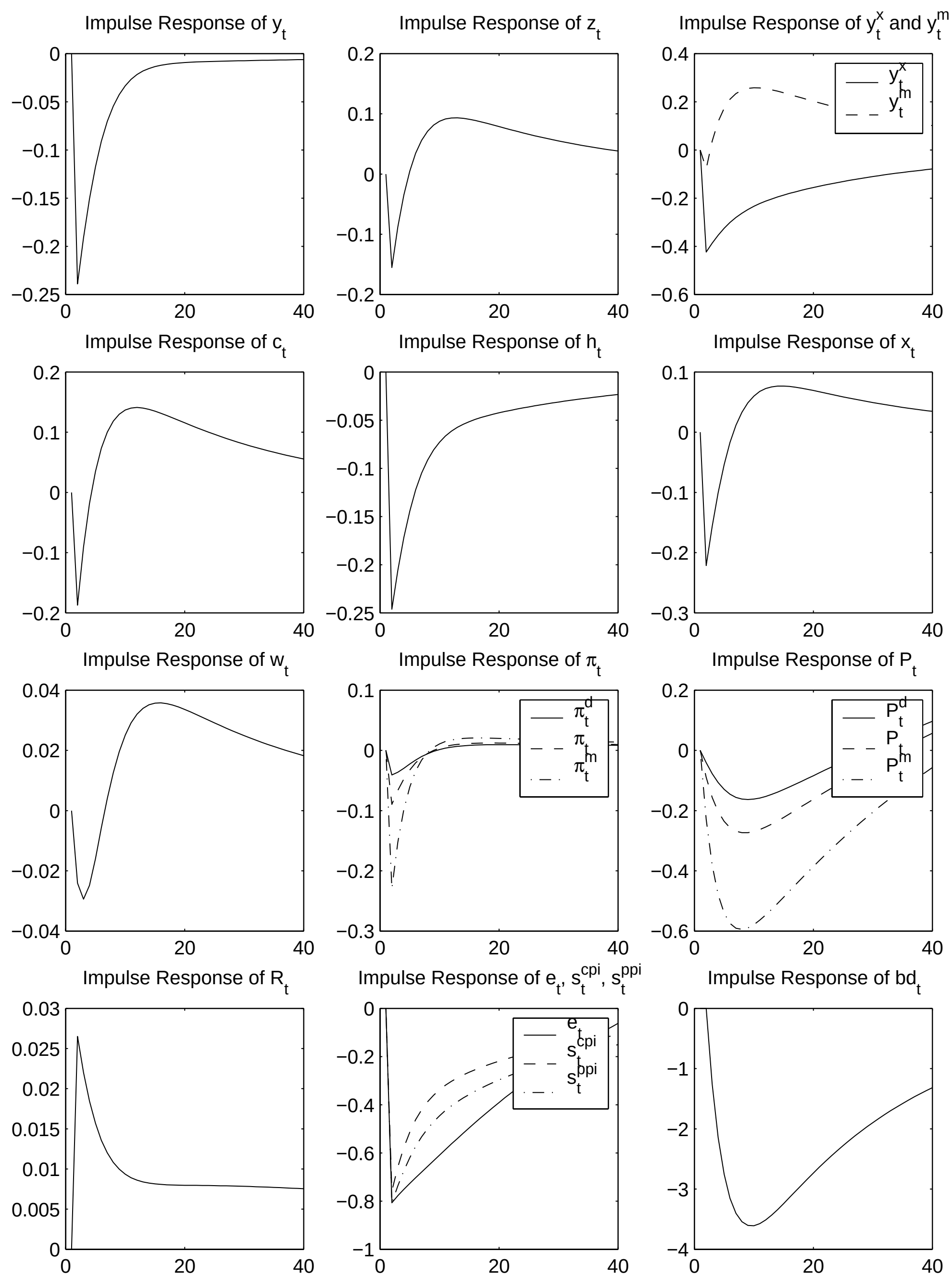


Figure 6: Effects of 1 per cent Government Spending Shock

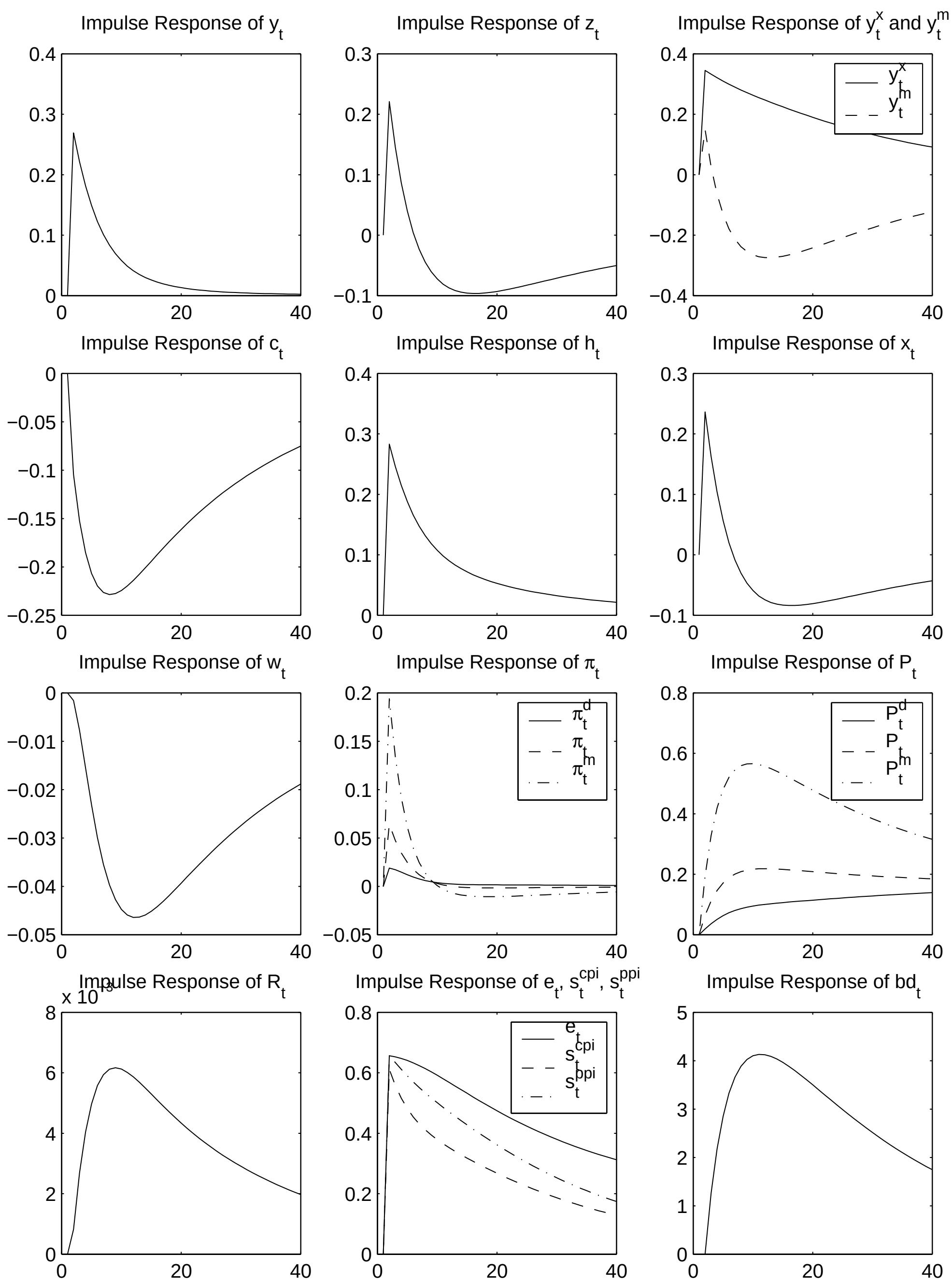


Figure 7: Effects of 1 per cent Tax Rate Shock

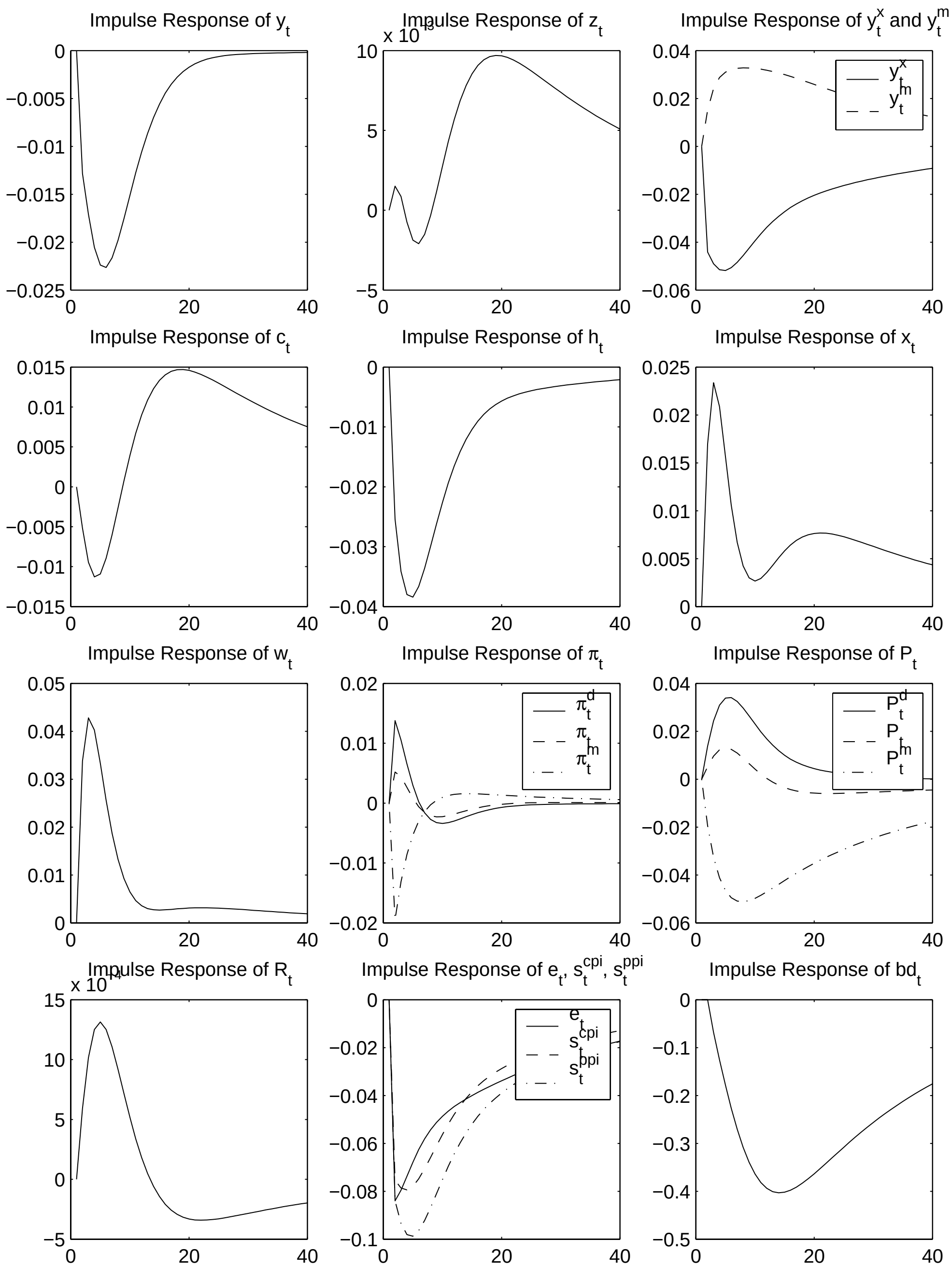


Figure 8: Effects of 1 per cent Foreign Nominal Interest Rate Shock

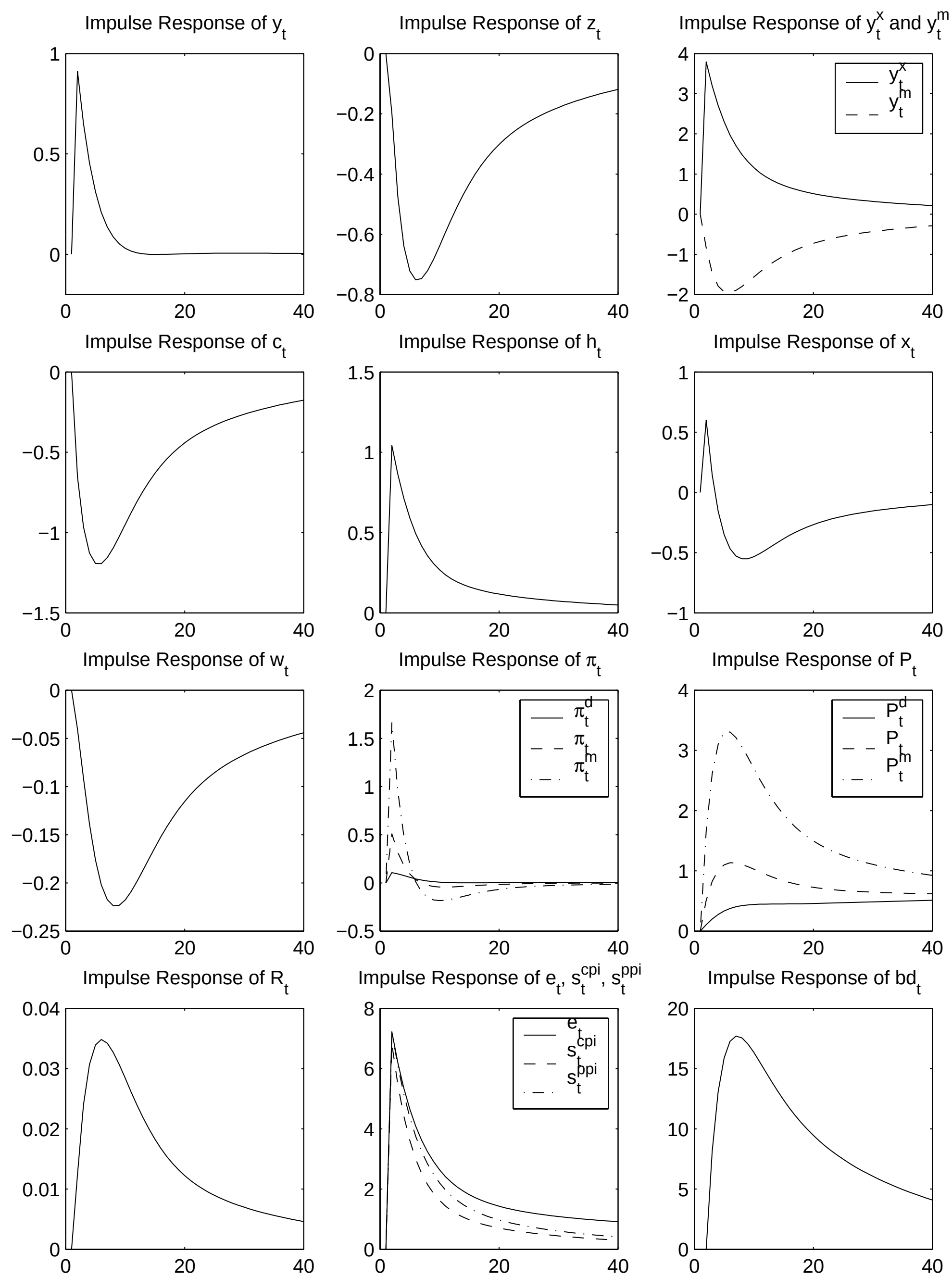


Figure 9: Effects of 1 per cent Foreign Output Shock

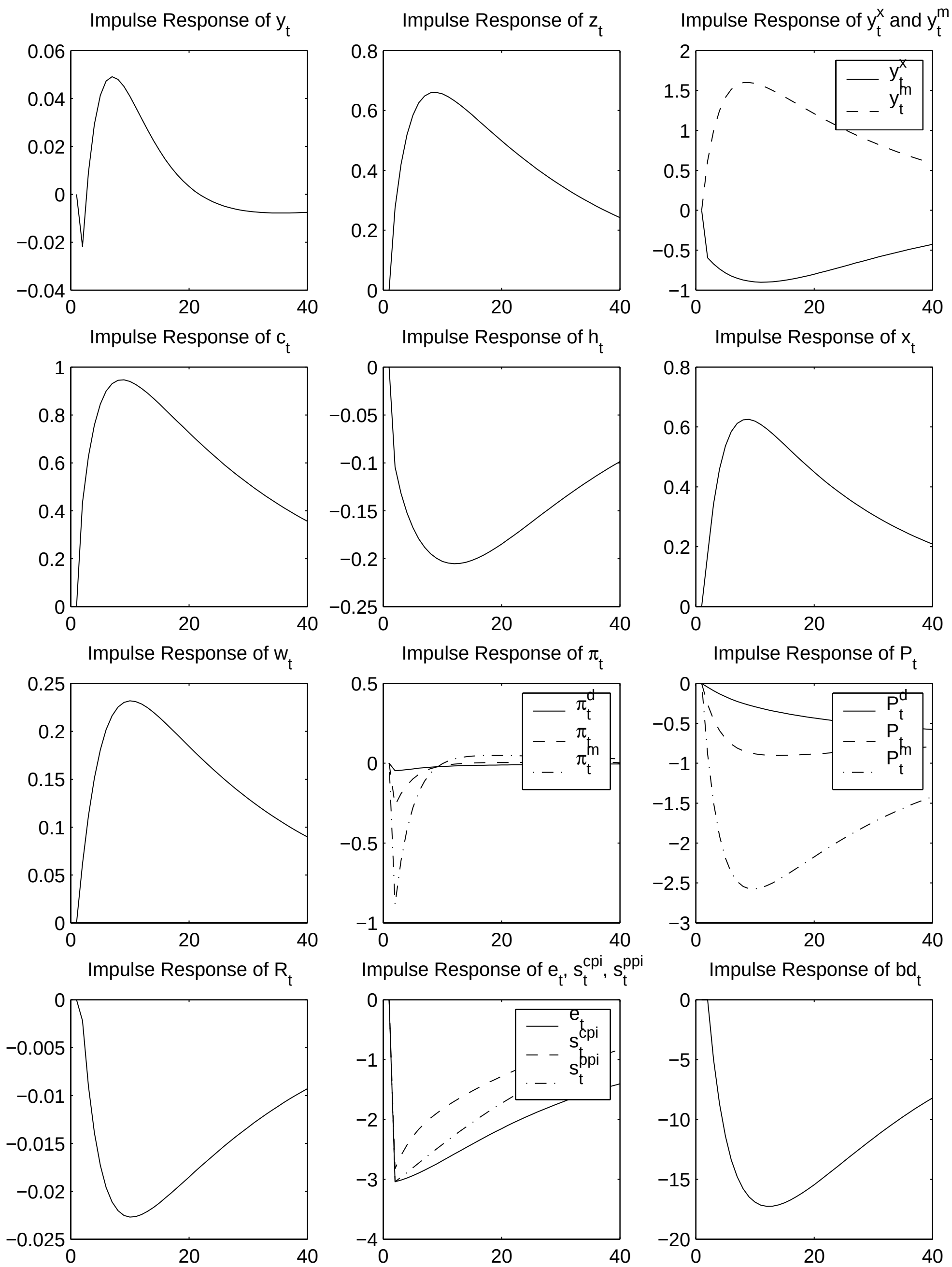


Figure 10: Effects of 1 per cent Foreign Inflation Shock

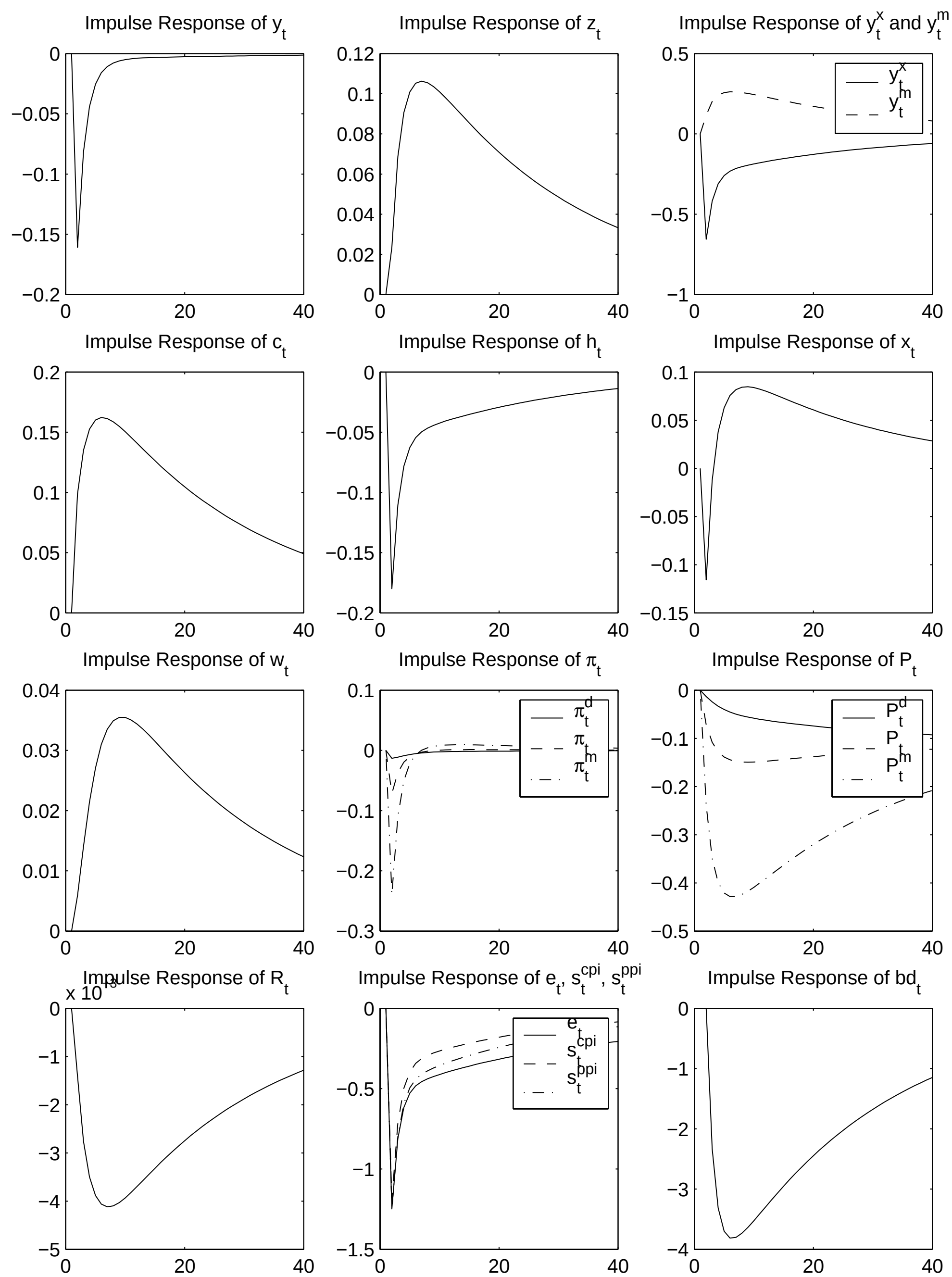


Figure 11: Pass-Through

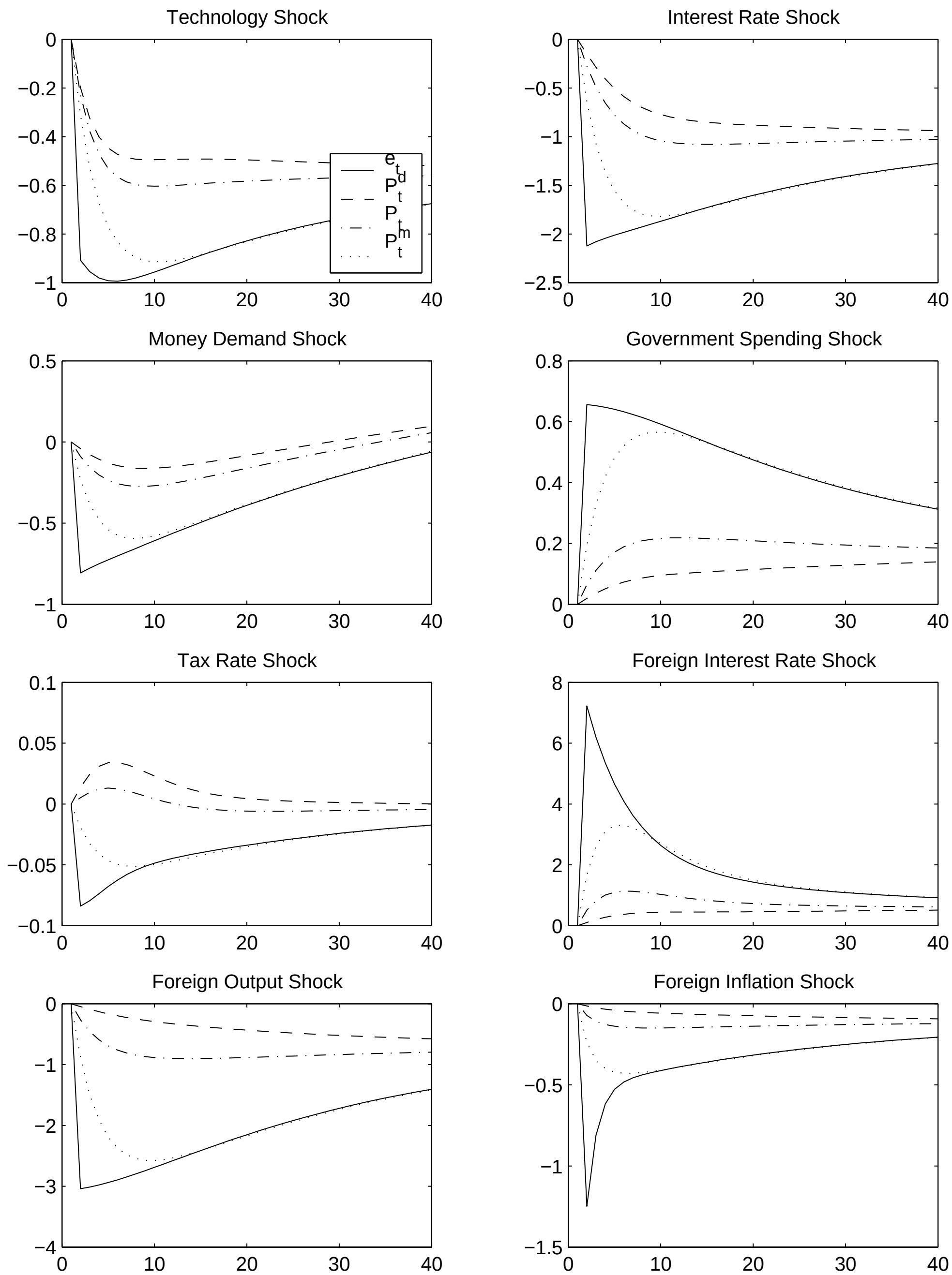


Figure 12: Pass-Through with $d_p = 0$

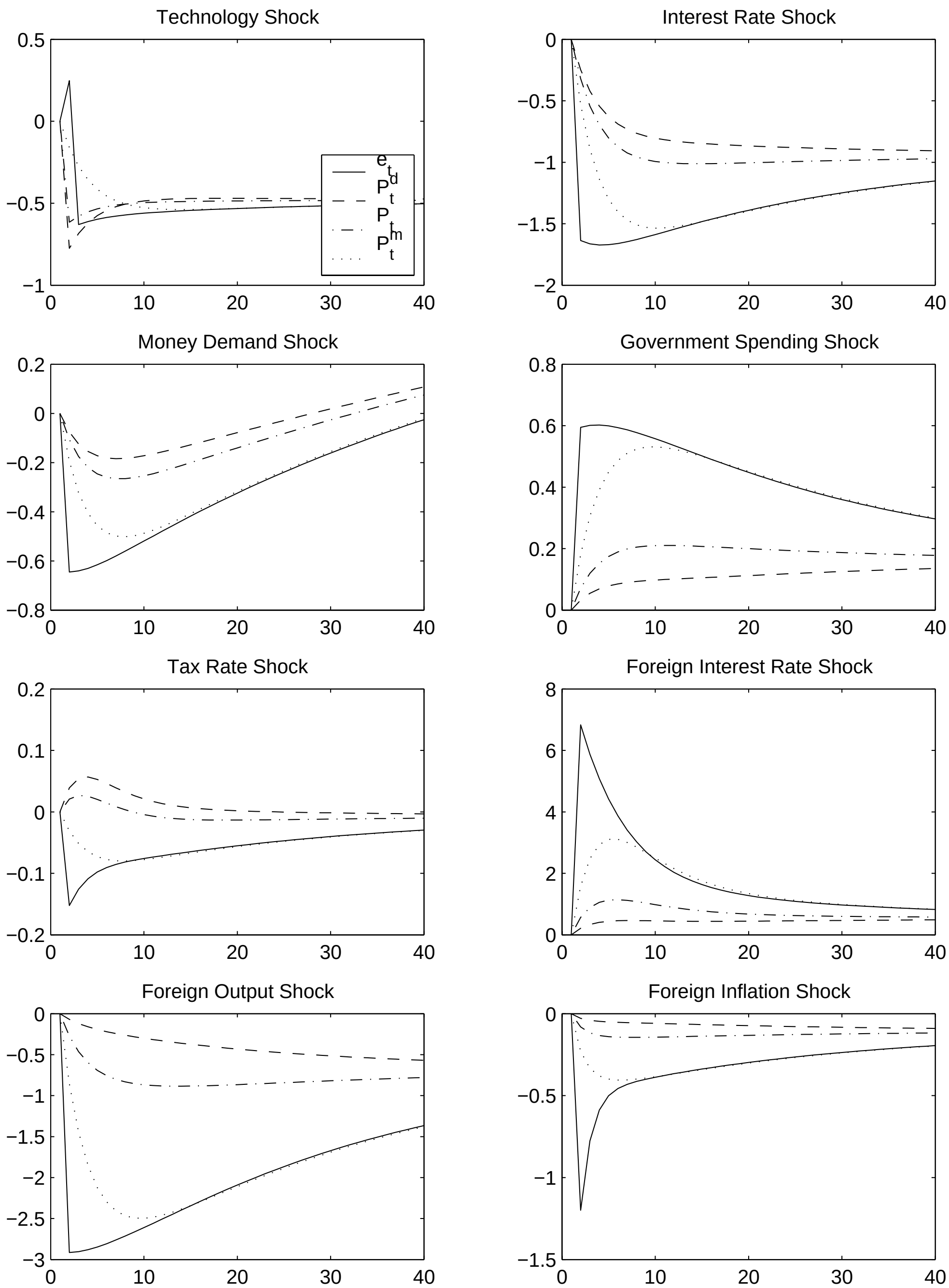


Figure 13: Pass-Through with $d_m = d_p = 0$

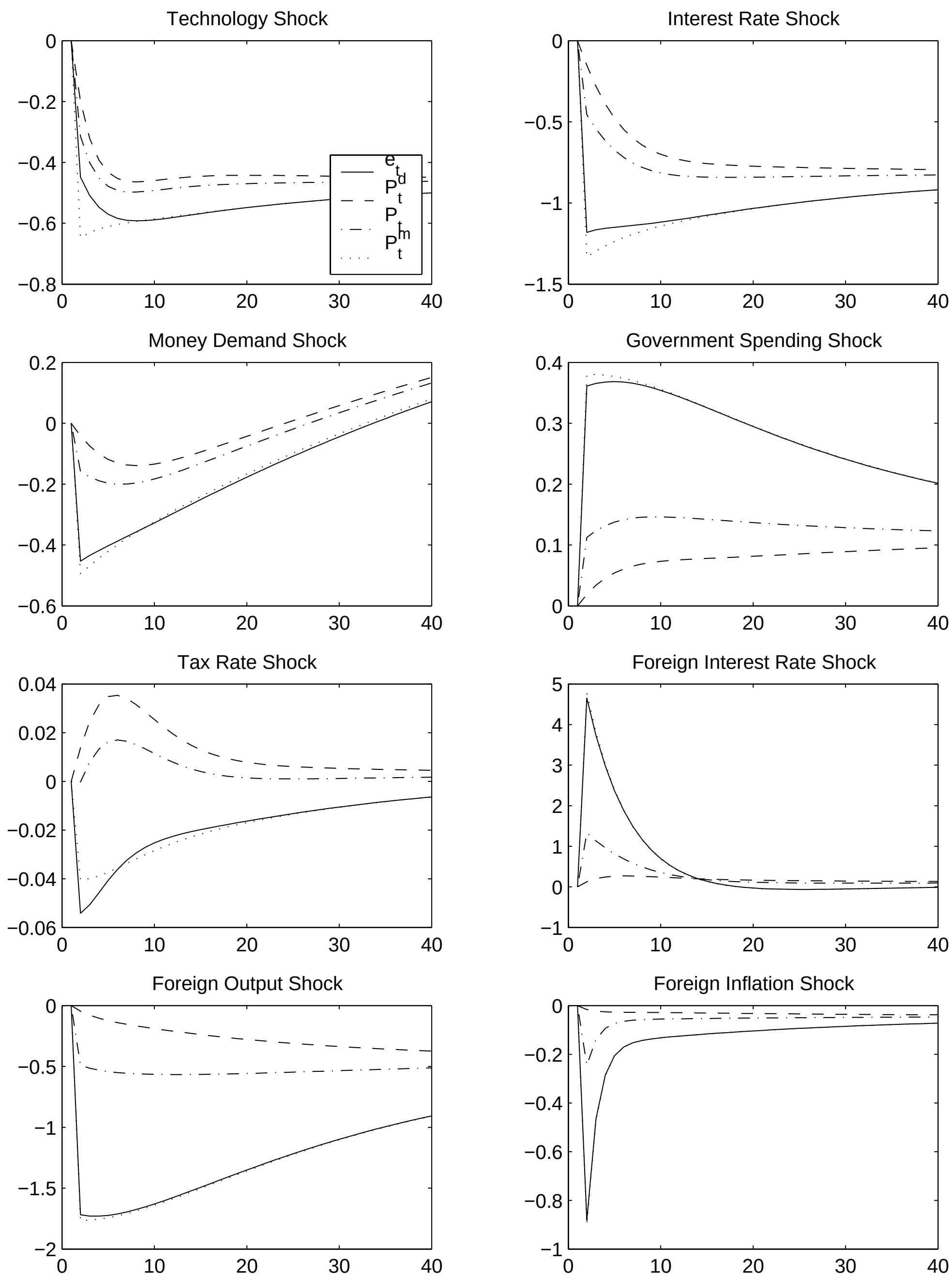
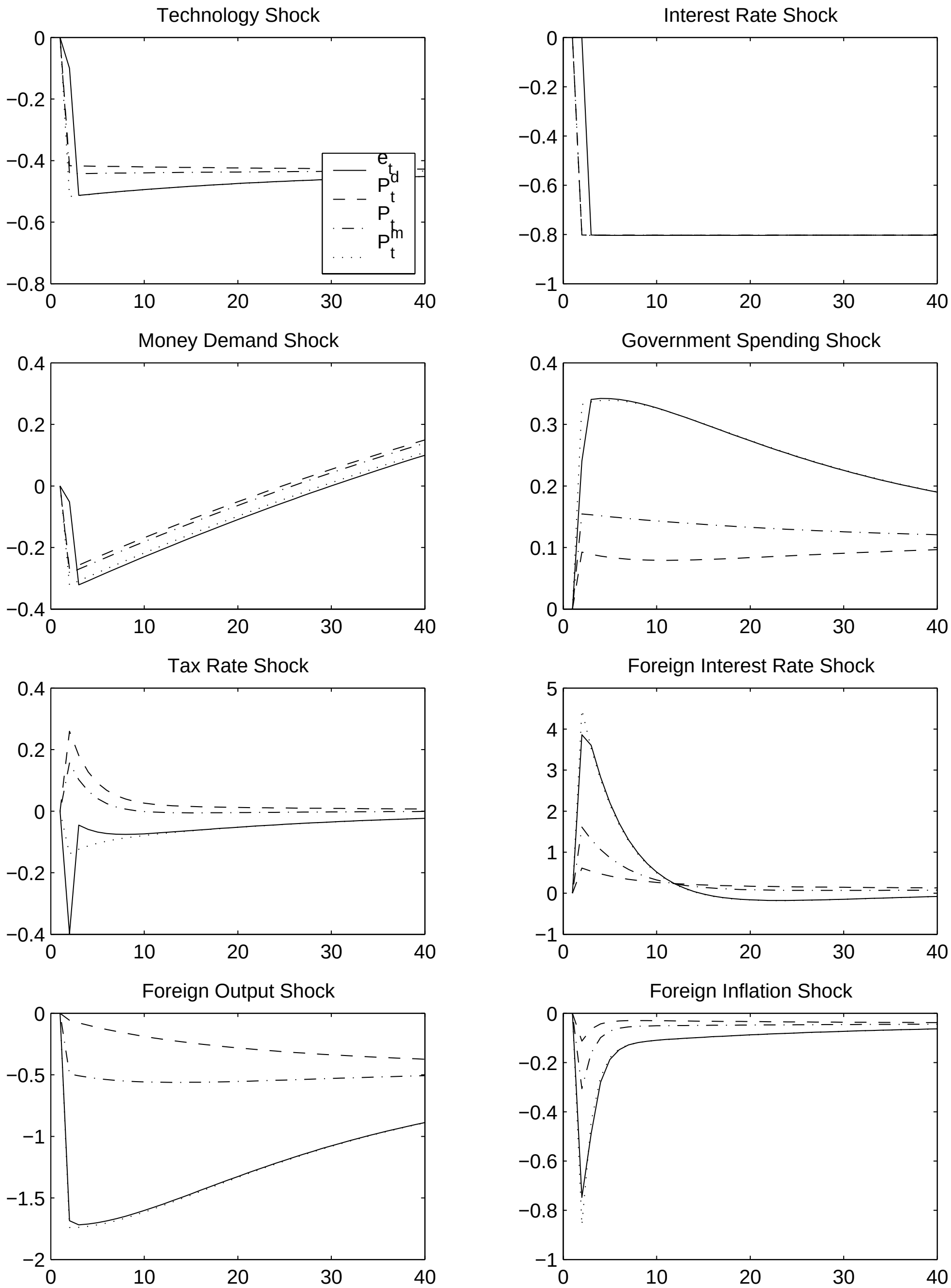


Figure 14: Pass-Through with $d_m = d_p = d_w = 0$



Appendix A: Data and Data Sources

Our data set is available on request. The data are from Canada and the United States and are quarterly from 1981Q3 to 2001Q4. The Canadian data are from *Bank of Canada Banking and Financial Statistics*, a monthly publication by the Bank of Canada. Series numbers are indicated in brackets and correspond to Cansim databank numbers.

- Consumption, C_t, is measured by real personal spending on non-durable goods and services in 1997 dollars (non-durables [v1992047] + services [v1992119]).
- The CPI inflation rate, π_t, is measured by changes in the consumer price index, P_t [v18702611].
- The PPI inflation rate, π_t^d, is measured by changes in the GDP implicit deflator, P_t^d [v1997756].
- The short-term nominal interest rate, R_t, is measured by the rate on Canadian three-month treasury bills [v122531].
- The output-growth rate, ΔY_t, is measured by changes in real per-capita GDP [v1992067].
- The money growth rate, μ_t, is measured by changes in nominal per-capita M2 stock [v37124].
- Exports, Y_t^x, are measured by real per-capita exports of goods and services [v1997750].
- Imports, Y_t^m, are measured by real per-capita imports of goods and services [v1997753].
- The average nominal wage, W_t, is measured by average hourly labour earnings (wages and salaries [v498076] / total hours worked [v4391505]).
- Employment, h_t, is measured by average weekly hours worked (total hours worked [v4391505] / all employees [v2062811]).
- The nominal exchange rate, e_t, is average Canadian dollars per unit of U.S. dollars [v37426].
- Government spending, G_t, is measured by government expenditures on goods and services (total domestic demand [v1992068] − total personal expenditures [v1992115] − construction [v1992053 + v1992055] − machinery and equipment [v1992056]).
- The labour tax rate, τ_t, is measured by the effective labour tax rate (calculated following the methodology of Jones 2002; and Mendoza, Razin, and Tezar 1994).
- The series in per-capita terms are obtained by dividing each series by the Canadian civilian population aged 15 and over (civilian labour force [v2062810] / labour force participation [v2062816]).

The U.S. data are from the Federal Reserve Bank of St. Louis, with the series numbers in brackets. The world series are approximated by some of the U.S. series.

- World output, Y_t^*, is real U.S. GDP per capita in 1996 dollars [GDPC96] divided by the U.S. civilian non-institutional population [CNP16OV].
- The world nominal interest rate, R_t^*, is measured by the rate on U.S. three-month Treasury Bills [TB3MS].
- The world inflation rate, π_t^*, is measured by changes in the U.S. GDP implicit price deflator, P_t^* [GDPDEF].

Appendix B: Moments Used to Estimate the Model

The set of unconditional moments used to estimate the structural parameters of the model are:

- $\mathrm{Corr}(\pi_t, \pi_{t-i})$ for $i = 1, 2, 3$;
- $\mathrm{Corr}(\pi_t^d, \pi_{t-i}^d)$ for $i = 1, 2, 3$;
- $\mathrm{Corr}(\pi_t^w, \pi_{t-i}^w)$ for $i = 1, 2, 3$;
- $\mathrm{Corr}(\Delta y_t, \Delta c_t)$;
- $\mathrm{Corr}(\Delta y_t, \Delta y_t^x)$;
- $\mathrm{Corr}(\Delta y_t, \Delta y_t^m)$;
- $\mathrm{Corr}(\Delta y_t, \Delta h_t)$;
- $\mathrm{Corr}(\Delta e_t, \pi_t^d)$;
- $\sigma_{\Delta c_t}/\sigma_{\Delta y_t}$;
- $\sigma_{\Delta e_t}/\sigma_{\Delta y_t}$;
- $\sigma_{\Delta s_t}/\sigma_{\Delta y_t}$;
- $\sigma_{\Delta h_t}/\sigma_{\Delta y_t}$.

Appendix C: Equilibrium Conditions

The following system of equations defines the economy's equilibrium:

$$\frac{c_t^{\frac{-1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t p_t; \tag{C.1}$$

$$\frac{b_t^{\frac{1}{\gamma}} m_t^{\frac{-1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t P_t \left(1 - \frac{1}{R_t}\right); \tag{C.2}$$

$$\frac{R_t}{\kappa_t R_t^*} = E_t \left[\frac{s_{t+1}\pi_{t+1}}{s_t \pi_{t+1}^*}\right]; \tag{C.3}$$

$$\frac{\lambda_t}{R_t} = \beta E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}^d} \exp(-A - \varepsilon_{At+1})\right]; \tag{C.4}$$

$$\tilde{w}_t = \left(\frac{\sigma}{\sigma-1}\right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_w)^l \ \eta h_{t+l}/(1-h_{t+l})}{E_t \sum_{l=0}^{\infty} (\beta d_w)^l (1-\tau_{t+l}) \lambda_{t+l} h_{t+l} \prod_{k=1}^{l} (\pi_{t+k}^d)^{-1} \exp(-A - \varepsilon_{At+k})}; \tag{C.5}$$

$$w_t^{1-\sigma} = d_w \left(\exp(-A - \varepsilon_{At})\right)^{1-\sigma} \left(\frac{w_{t-1}}{\pi_t^d}\right)^{1-\sigma} + (1-d_w)\tilde{w}_t^{1-\sigma}; \tag{C.6}$$

$$y_t = x_t^{\phi} h_t^{1-\phi}; \tag{C.7}$$

$$w_t = (1-\phi)\xi_t \frac{y_t}{h_t}; \tag{C.8}$$

$$p_t = \phi \xi_t \frac{y_t}{x_t}; \tag{C.9}$$

$$\tilde{p}_t^d = \left(\frac{\theta}{\theta-1}\right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_p)^l \lambda_{t+l} y_{t+l} \xi_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta d_p)^l \lambda_{t+l} y_{t+l} \prod_{k=1}^{l} (\pi_{t+k}^d)^{-1}}; \tag{C.10}$$

$$1 = d_p \left(\frac{1}{\pi_t^d}\right)^{(1-\theta)} + (1-d_p)(\tilde{p}_t^d)^{(1-\theta)}; \tag{C.11}$$

$$\tilde{p}_t^m = \left(\frac{\vartheta}{\vartheta-1}\right) \frac{E_t \sum_{l=0}^{\infty} (\beta d_m)^l \lambda_{t+l} y_{t+l}^m s_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta d_m)^l \lambda_{t+l} y_{t+l}^m \prod_{k=1}^{l} (\pi_{t+k}^m)^{-1}}; \tag{C.12}$$

$$(p_t^m)^{(1-\vartheta)} = d_m \left(\frac{p_{t-1}^m}{\pi_t^d}\right)^{(1-\vartheta)} + (1-d_m)\left(\tilde{p}_{mt}\right)^{(1-\vartheta)}; \tag{C.13}$$

$$(p_t)^{(1-\nu)} = \alpha_d + \alpha_m \left(p_t^m\right)^{(1-\nu)}; \tag{C.14}$$

$$z_t = c_t + x_t + g_t; \tag{C.15}$$

$$y_t = y_t^x + y_t^d; \tag{C.16}$$

$$y_t^x = \alpha_x s_t^{\varsigma} y_t^*; \tag{C.17}$$

$$y_t^d = \alpha_d \left(\frac{1}{p_t}\right)^{-\nu} z_t; \tag{C.18}$$

$$y_t^m = \alpha_m \left(\frac{p_{mt}}{p_t}\right)^{-\nu} z_t; \tag{C.19}$$

$$\frac{b_t^*}{\kappa_t R_t^*} - \frac{b_{t-1}^*}{\pi_t^*} \exp(-A - \varepsilon_{At}) = y_t^x - s_t y_t^m; \tag{C.20}$$

$$\log(\kappa_t) = \varphi \left[\exp\left(\frac{s_t b_t^*}{y_t}\right) - 1\right]; \tag{C.21}$$

$$\begin{aligned} \log(R_t/R) = {} & \varrho_y \log(y_t/y) + \varrho_\pi \log(\pi_t/\pi) \\ & + \varrho_\mu \log(\mu_t/\mu) + \varrho_s \log(s_t/s) + \varepsilon_{Rt}; \end{aligned} \tag{C.22}$$

$$\pi_t = \frac{m_{t-1}}{m_t} \exp(A + \varepsilon_{At}) \mu_t; \tag{C.23}$$

$$\log A_t = (A) + \log(A_{t-1}) + \varepsilon_{At}; \tag{C.24}$$

$$\log(b_t) = (1-\rho_b)\log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}; \tag{C.25}$$

$$\log(g_t) = (1-\rho_g)\log(g) + \rho_g \log(g_{t-1}) + \varepsilon_{gt}; \tag{C.26}$$

$$\log(\tau_t) = (1-\rho_\tau)\log(\tau) + \rho_\tau \log(\tau_{t-1}) + \varepsilon_{\tau t}; \tag{C.27}$$

$$\log(R_t^*) = (1-\rho_{R^*})\log(R^*) + \rho_{R^*} \log(R_{t-1}^*) + \varepsilon_{R^* t}; \tag{C.28}$$

$$\log(\pi_t^*) = (1-\rho_{\pi^*})\log(\pi^*) + \rho_{\pi^*} \log(\pi_{t-1}^*) + \varepsilon_{\pi^* t}; \tag{C.29}$$

$$\log y_t^* = (1-\rho_{y^*})\log(y^*) + \rho_{y^*} \log(y_{t-1}^*) + \varepsilon_{y^* t}, \tag{C.30}$$

where equation (C.20) gives the trade balance of the economy.

Bank of Canada Working Papers
Documents de travail de la Banque du Canada

Working papers are generally published in the language of the author, with an abstract in both official languages. ***Les documents de travail sont publiés généralement dans la langue utilisée par les auteurs; ils sont cependant précédés d'un résumé bilingue*.**

2003

2003-28	An Empirical Analysis of Liquidity and Order Flow in the Brokered Interdealer Market for Government of Canada Bonds	C. D'Souza, C. Gaa, and J. Yang
2003-27	Monetary Policy in Estimated Models of Small Open and Closed Economies	A. Dib
2003-26	Measuring Interest Rate Expectations in Canada	G. Johnson
2003-25	Income Trusts—Understanding the Issues	M.R. King
2003-24	Forecasting and Analyzing World Commodity Prices	R. Lalonde, Z. Zhu, and F. Demers
2003-23	What Does the Risk-Appetite Index Measure?	M. Misina
2003-22	The Construction of Continuity-Adjusted Monetary Aggregate Components	J. Kottaras
2003-21	Dynamic Factor Analysis for Measuring Money	P.D. Gilbert and L. Pichette
2003-20	The U.S. Stock Market and Fundamentals: A Historical Decomposition	D. Dupuis and D. Tessier
2003-19	A Small Dynamic Hybrid Model for the Euro Area	R. Djoudad and C. Gauthier
2003-18	Technological Change and the Education Premium in Canada: Sectoral Evidence	J. Farès and T. Yuen
2003-17	Explaining and Forecasting Inflation in Emerging Markets: The Case of Mexico	J. Bailliu, D. Garcés, M. Kruger, and M. Messmacher
2003-16	Some Notes on Monetary Policy Rules with Uncertainty	G. Srour
2003-15	The Syndicated Loan Market: Developments in the North American Context	J. Armstrong
2003-14	An Index of Financial Stress for Canada	M. Illing and Y. Liu
2003-13	Un modèle << PAC >> d'analyse et de prévision des dépenses des ménages américains	M.-A. Gosselin and R. Lalonde
2003-12	The Macroeconomic Effects of Military Buildups in a New Neoclassical Synthesis Framework	A. Paquet, L. Phaneuf, and N. Rebei

Copies and a complete list of working papers are available from:
Pour obtenir des exemplaires et une liste complète des documents de travail, prière de s'adresser à :

Publications Distribution, Bank of Canada
234 Wellington Street, Ottawa, Ontario K1A 0G9
E-mail: publications@bankofcanada.ca
Web site: http://www.bankofcanada.ca

Diffusion des publications, Banque du Canada
234, rue Wellington, Ottawa (Ontario) K1A 0G9
Adresse électronique : publications@banqueducanada.ca
Site Web : http://www.banqueducanada.ca