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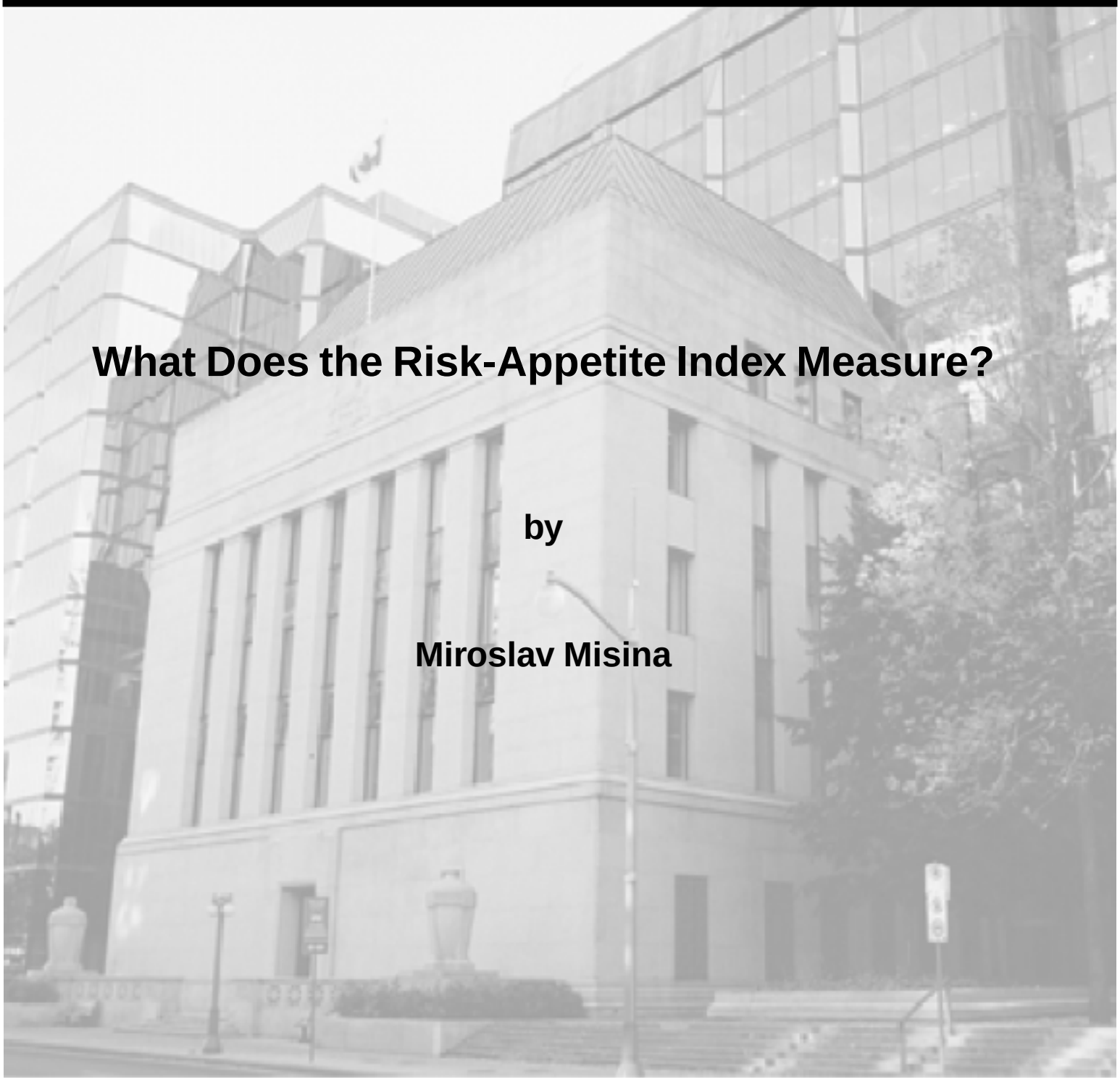

What Does the Risk-Appetite Index Measure?

by

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The views expressed in this paper are those of the author.
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Abstract

Explanations of changes in asset prices as being due to exogenous changes in risk appetite, although arguably controversial, have been popular in the financial community and have also received some attention in attempts to account for recent financial crises. Operational versions of these explanations are based on the assumption that changes in asset prices can be decomposed into a part that can be attributed to changes in riskiness and a part attributable to changes in risk aversion, and that some quantitative measure can capture these effects in isolation. One such measure, the risk-appetite index (RAI)—used in the financial community as well as in assessments of financial stability in emerging markets—is based on the rank correlation between assets' riskiness and excess returns. The author seeks to provide a theoretical foundation for this measure. He summarizes the arguments behind the index in two propositions and attempts to derive these propositions within a class of well-specified asset-pricing models. His results indicate that, whereas the exclusive attribution of the rank effect to changes in risk aversion is problematic in general, a specific set of circumstances can be identified in which this attribution is permissible. The key assumption is identified, and its empirical implications are examined. In cases where this assumption is shown to be empirically valid, the model provides a theoretical foundation for the RAI.

JEL classification: G12
Bank classification: Economic models; Financial markets

Résumé

Bien que d'aucuns considèrent qu'elle prête à controverse, la thèse selon laquelle les variations des prix des actifs s'expliquent par des modifications exogènes de la propension à prendre des risques connaît une certaine popularité au sein des milieux financiers et soulève l'intérêt des universitaires qui se penchent sur les causes des crises financières récentes. Les versions opérationnelles de cette thèse se fondent sur l'hypothèse voulant que les variations des prix des actifs soient imputables en partie aux modifications du niveau de risque des actifs et en partie à celles du degré d'aversion pour le risque, et que ces effets puissent être quantifiés isolément. L'une des mesures utilisées à cette fin est l'indice de propension à prendre des risques (indice RAI), qui est basé sur la corrélation de rang entre le niveau de risque et les excédents de rendement des actifs. L'auteur cherche à donner un fondement théorique à cet indice, qui est employé dans le monde de la finance et sert également à évaluer la stabilité financière des économies de marché émergentes. Il résume les arguments intuitifs à l'appui de cet indice en deux

propositions, qu'il tente de formaliser à l'aide d'une catégorie précise de modèles correctement spécifiés d'évaluation des actifs. Les résultats obtenus indiquent que, si l'on ne peut généralement pas attribuer l'effet de rang aux seules modifications du degré d'aversion pour le risque, il existe des circonstances particulières où une telle explication est admissible. L'auteur cerne la principale condition nécessaire à cet égard et en tire les conséquences empiriques. Dans les cas où cette condition se vérifie sur le plan empirique, le modèle proposé par l'auteur fournit un fondement théorique à l'indice de propension à prendre des risques.

Classification JEL : G12
Classification de la Banque : Modèles économiques; Marchés financiers

1. Introduction

The explanations of asset price changes based on changes in investors' risk appetite[1] figure prominently in the financial community. Trading strategies such as momentum and contrarian trading are based on the idea that it is possible to quantify the movements in prices that are due to changes in risk appetite and exploit them either by "riding the wave" or "trading against the crowd." Various indices have been constructed that attempt to capture changes in prices due to changes in risk appetite.

In marked contrast to the financial community, modelling price changes as being due to exogenous changes in risk aversion has not been a popular approach in academic research.[2] Two types of arguments have been made against this approach. The first is methodological: allowing for changes in risk aversion relaxes an essential constraint—constant preferences—that safeguards rigour in economic research.[3] The second argument is based on the observational equivalence of changes in prices due to changes in asset riskiness and changes due to changing risk aversion. Recently, however, exogenous changes in risk aversion have been used in the academic literature to explain the financial crises of the late 1990s and to elucidate the mechanisms that lead to financial contagion.[4]

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[1] The terms "risk appetite" and "risk aversion" are used interchangeably, the former being more common in the financial community, and the latter in academic research. Note that increasing risk appetite means declining risk aversion; decreasing risk appetite indicates increasing risk aversion.

[2] Some evidence suggests that the approach based on state-dependent preferences, supported by experimental evidence in favour of changing risk aversion, is gaining recognition. See Campbell, Lo, and MacKinlay (1997, Chapter 8.4), for a discussion and references, and Danthine et al. (2003) for a recent example.

[3] Without this or a similar constraint, the concern is that one will be able to generate any kind of result and thus explain anything. Misina (2003) discusses these problems and offers an example that illustrates the pitfalls of using models with time-varying beliefs in which no a priori constraints on beliefs are imposed.

[4] See Kumar and Persaud (2002) and the references cited therein.

Setting the methodological argument aside, this paper focuses on the second argument. It is clear that, to make these explanations operational, one must argue that the observational equivalence can be broken. The argument consists of two parts:

(i) **demonstrate** that different sources of price changes will result in qualitatively different effects (a separability issue), and

(ii) construct a quantitative measure that would capture these different effects.

The first part should be based on theoretical arguments. The objective is to develop a model within which the observational distinctness can be established. The quantitative measure follows from this step. If the first step is ignored, one cannot ascertain that a proposed quantitative measure captures what its proponents claim.

This paper focuses on a measure of changes in risk aversion—the risk-appetite index (RAI)—based on the rank correlation between assets' riskiness and their excess returns.[5] This measure, which originated within the financial community, seems to be the first to try to support informal appeals to changing risk appetite as explanations of price movements. It has also received wider attention in attempts to assess the financial stability of emerging markets.[6] Whereas the case in favour of the RAI can be made on intuitively plausible grounds, the question is to what extent one can provide a theoretical foundation for this approach.

The objective of this paper is to give a theoretical foundation for this measure. To establish the validity of the claim that the RAI measures changes in risk aversion, one

[5] This measure is discussed in Kumar and Persaud (2002), where it is named the "global risk-appetite index." They describe the main idea behind the index as well as its application to financial contagion. The idea of measuring the risk appetite in the way suggested seems to have a somewhat longer tradition. Kumar and Persaud provide additional references.

[6] See, for example, IMF (2002, 2003).

must provide a theoretical model that would imply this measure. We propose to examine a wide class of asset-pricing models to try to establish the validity of this index. The intuition behind the RAI is summarized by two propositions, after which a theoretical model, which represents the starting point in the attempt to justify the intuition, is given. Various versions of the basic model are examined to determine whether the justification can be achieved. Although the presence of the rank effect, which is the basis for using the RAI, can be established theoretically, difficulties arise in attempting to attribute this effect exclusively to changes in risk aversion.

This work demonstrates that, although the exclusive attribution of the rank effect to changes in risk aversion is problematic in general, it is possible to give a theoretical foundation to the interpretation that the RAI captures changes in risk aversion in specific circumstances. The key condition of the linear independence of asset returns is identified and its empirical implications drawn. This assumption can be easily tested in any given data set. If the condition is shown to hold, the model provides a theoretical foundation for the index.

The results also imply that, in circumstances in which the key condition does not hold, great caution is necessary in interpreting the results obtained using this measure, or in construing them as evidence supportive of explanations that market phenomena such as financial contagion are due to changes in risk aversion.

The paper is organized as follows. In section 2, the arguments in favour of observational equivalence and separability are given; these arguments form the basis of the RAI. The formal model is given in section 3. In section 4, links are established between intuitive claims and the model, and some resulting problems are discussed. The key assumption necessary for the model to support the RAI is identified and its empirical

implications are discussed. Section 5 offers some conclusions about the nature of the results obtained and their bearing on the use of the RAI.

2. Arguments

2.1 Observational equivalence

The argument against separability is based on the observational equivalence of changes in risk appetite and changes in asset riskiness. The argument can be illustrated by means of an example.

Suppose we have a portfolio that consists of two assets: a riskless asset with a rate of return R^f and a risky asset with a rate of return R. Suppose that the riskiness of the risky asset increases due to an exogenous shock. Assuming unchanging risk aversion, investors will want to rebalance their portfolio in the direction of the riskless asset. This will lead to an increase in the expected rate of return on the risky asset and a decrease in the expected return on the riskless asset.

Consider now the same portfolio, but suppose that instead of a change in risk, the risk aversion of investors increases. The rebalancing will be in the direction of the riskless asset, with qualitatively identical consequences for expected returns on both assets.

Although the generality of this argument can be questioned, those who claim it is possible to establish separability have to break this observational equivalence.

2.2 Separability

The RAI, discussed in Kumar and Persaud (2002), is based on the rank correlation of excess returns on assets in a portfolio and their riskiness.[7] The use of this index is based on the assumption that it can distinguish price changes due to changes in asset riskiness from those due to changes in risk aversion. Changes in this rank correlation over time are interpreted as evidence of changes in risk aversion.

Why would one expect the rank correlation of assets' riskiness and excess returns to provide a measure of changes in risk aversion? The intuition can be summarized in the following two propositions.

Part 1 (H_1)

Assumption 1 *The riskiness of assets is exogenously given and constant.*

Let $k = 1, ..., K$ index the risk classes of assets, with $k = 1$ denoting the riskless assets and $k = K$ denoting the riskiest class of assets.

Proposition 1 *A change in investors' risk aversion will have monotonic effects on assets in different risk classes: the impact on returns will depend on the riskiness of a particular asset.*

If investors become more risk-averse, they will rebalance their portfolio away from the riskiest assets, bidding down their price and increasing the excess returns on these classes of assets. The opposite holds in the case of a decrease in risk aversion.

More formally, let R_k^{ex} denote the excess return on a risky asset, ρ the coefficient of investors' risk aversion, and μ_k a measure of the riskiness of an asset in class k.[8] The

[7] See footnote 5 for additional details.

[8] The nature of the relationship implied by the proposition does not depend on the measure of risk used.

above proposition states that, when there is a change in risk aversion, there will be a *rank effect*,

$$\mu_j > \mu_l \Rightarrow \triangle\ R_j^{ex} > \triangle\ R_l^{ex},\ \ \forall j > l,$$

when the risk aversion increases, and the opposite effect when it decreases. Quantitatively, this effect can be captured by the rank correlation. To establish the existence of the rank effect in the data *and* draw the inference that the presence of this effect is due to changes in risk aversion, one has to address two issues:

(i) Proposition 1 is not directly testable, since it relates changes in excess returns to changes in the unobservable risk-aversion parameter.

(ii) The presence of the rank effect might emerge for reasons other than changes in risk aversion.

Proposition 2 addresses both of these issues.

Part 2 (H_0)

Assumption 2 *Investors' risk aversion is exogenously given and constant.*

Proposition 2 *A change in the riskiness of an asset will **not** have monotonic effects on excess returns across different asset classes. The impact on returns will not depend on the riskiness of a particular asset.*

The first issue is dealt with by relating two observable variables: asset riskiness and excess returns. The second issue is addressed by a claim that the rank effect will not occur if risk aversion is held constant. Proposition 2 is of key importance in the argument: if this proposition is valid, it permits inferences about the unobserved risk-aversion parameter by computing a statistic relating two observables. The absence of the rank effect would indicate that the observed change in prices is due to a change in

asset riskiness. On the other hand, the presence of the rank effect, captured by rank correlation, would be attributed to changes in the unobserved risk-aversion parameter, since proposition 2 precludes other possibilities.

To illustrate, let $R_1^{ex}, ..., R_K^{ex}$ denote the returns on asset classes $1, ..., K$ and let $\mu_1, ..., \mu_K$ represent some measure of their riskiness. For each asset $i = 1, ..., K$, replace (R_i, μ_i) by its ranking in terms of its riskiness, $r_i(\mu_i)$, and its ranking in terms of its excess return, $r_i(R_i)$. The sequence of rankings, $(r_i(R_i), r_i(\mu_i))$, is used to compute the rank correlation, $corr(r_i(R_i), r_i(\mu_i))$. The result, $corr(r_i(R_i), r_i(\mu_i)) = 0$, would indicate that there is no relationship between the ranking of assets in terms of their riskiness and their ranking in terms of their excess returns.

Propositions 1 and 2 can be used to formulate a statistical test:

$$H_0 : corr(r_i(R_i), r_i(\mu_i)) = 0,$$

$$H_1 : corr(r_i(R_i), r_i(\mu_i)) \neq 0.$$

The rejection of H_0 would indicate that the change in observed riskiness can be attributed to a change in underlying risk aversion.

Whereas the argument outlined above seems intuitively plausible, the validity of the proposed test, and of the rank correlation as a measure of changes in risk aversion, depends on the validity of the argument offered. Note that one cannot simply compute rank correlations and then use the test to make inferences about risk aversion. The validity of the test depends on the validity of the argument. One is always free to compute the rank correlation between asset riskiness and excess returns. However, to establish the interpretation that it captures changes in risk aversion, one must first demonstrate the validity of the arguments underlying this interpretation. Consequently, the question

is whether these arguments can be given a theoretical foundation. In section 3, a model is given that can be used as a first step in deriving the propositions stated above.

3. A Model

To formalize the argument outlined in section 2, a candidate model should have the following characteristics:

– explicit links between excess returns, asset riskiness, and risk aversion

– multiple assets with different levels of risk

– a representative agent ("common but changing appetite for risk")[9]

– exogenous changes in risk aversion

Multiple assets with different levels of risk are needed, since with only two classes the observational equivalence cannot be broken. Modelling changes in risk aversion as exogenous necessitates the use of the constant risk-aversion (CRA) class of utility functions.[10] General representatives of this class are exponential utility (constant absolute risk aversion, CARA) and power utility (constant relative risk aversion, CRRA).

The starting point of the exercise is the basic asset-pricing relationship

$$p_t = E_t \left[m_{t+1} x_{t+1} \right], \tag{1}$$

[9] Cf. Kumar and Persaud (2002, 404).

[10] To model the changes in risk aversion as endogenous is not satisfactory, given the nature of the problem. Non-constant risk-aversion utility functions relate risk aversion to variables such as consumption (as in the habit-persistence case). It is difficult in this setting to accommodate the explanations of sudden price changes and phenomena such as financial contagion as being due to psychological factors, which seem to be behind most of the appeals to changes in risk appetite as an explanatory device. See Campbell, Lo, and MacKinlay (1997, Chapter 8.4) and the references therein for a detailed discussion and comparison of these two types of explanations.

where $m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the stochastic discount rate, with c denoting consumption, and x_{t+1} is the asset's payoff. This relationship can be derived from a whole class of models. In a representative-agent model, this relationship is the individual's optimality condition as well as the equilibrium condition.

The following restrictions are imposed:

(i) exponential utility (CARA class)

$$u(c) = -e^{-\rho c};$$

(ii) a set of assets, i, grouped into risk classes, $k = 1, .., K$. The return on an asset in class k is R_k;

(iii) the riskless asset has a return R^f;

(iv) asset returns are normally distributed.

This model is a standard CAPM with linear m_{t+1}. Solving the model yields[11]

$$E(R_k) = R^f + \rho \sigma_{R_k, R^W},$$

or

$$R_k^{ex} = \rho \sigma_{k,W}. \tag{2}$$

Here, $R_k^{ex} = E(R_k) - R^f$ is the excess return on asset class k, and $\sigma_{k,W} \equiv \sigma_{R_k, R^W}$ is the covariance of the returns of the class-k assets with the return of the market portfolio, R^W. In this model, $\sigma_{k,W} \equiv \mu_k$: the covariance measures the contribution of the class-k

[11] Cf. Cochrane (2001, 154).

assets to the variance of the overall portfolio, and thus represents the riskiness of this class of assets as part of the portfolio.

The effects of changes in risk aversion and riskiness can be obtained from (2) in a straightforward way. The objective is to relate these results to propositions 1 and 2 in section 2.

Effect of changes in risk aversion:

$$\frac{\partial R_k^{ex}}{\partial \rho} = \sigma_{k,W},\ \forall k. \tag{3}$$

Since $\sigma_{j,W} > \sigma_{l,W},\ \forall j > l$,

$$\triangle\ R_j^{ex} > \triangle\ R_l^{ex},\ \ \forall j > l.$$

This establishes proposition 1 (H_1).

Although this, without the proof of proposition 2, does not validate the approach, the result is of interest because it establishes the existence of an effect that did not occur in the observational equivalence arguments. The rank effect is characteristic of asset-price changes due to changes in risk aversion. This does not break the observational equivalence, since one needs to ensure that the effect established here does not occur in circumstances other than changes in risk aversion. The key part of the argument is contained in proposition 2, since that proposition enables us to establish the link to the observables and conduct the test.

Effect of changes in riskiness:

$$\frac{\partial R_k^{ex}}{\partial \sigma_{k,W}} = \rho, \forall k. \tag{4}$$

The effect of changes in riskiness on excess returns is invariant to the asset class. The question is whether (4) establishes proposition 2.

4. Problems

To determine whether (4) establishes proposition 2, take a closer look at expression (2). Let α_i represent the share of asset i in the market portfolio, with $\sum_{i=1}^{K} \alpha_i = 1$. The return to the market portfolio is

$$R^W = \sum_{i=1}^{K} \alpha_i R_i.$$

Using this, the covariance term in (2) can be rewritten as

$$cov\left(R_k, R^W\right) \equiv \sigma_{k,W} = \sum_{i=1}^{K} \alpha_i \sigma_{k,i} = \alpha_k \sigma_k^2 + \sum_{i \neq k} \alpha_i \sigma_{i,k}. \tag{5}$$

Expressions (2) and (5) imply that a change in excess returns can be written as

$$dR_k^{ex} = \frac{\partial R_k^{ex}}{\partial \rho} d\rho + \frac{\partial R_k^{ex}}{\partial \sigma_k^2} d\sigma_k^2 + \sum_{i \neq k} \frac{\partial R_k^{ex}}{\partial \sigma_{i,k}} d\sigma_{i,k}, \ k = 1, ..., K. \tag{6}$$

The first term represents changes in R_k^{ex} due to changes in risk aversion, the second term captures changes due to a change in the variance of the asset, and the last term denotes changes due to changes in the covariance of asset k with other assets in the portfolio.

Focusing on changes in returns due to a change in riskiness, by setting $d\rho = 0$, gives

$$dR_k^{ex} = \frac{\partial R_k^{ex}}{\partial \sigma_k^2} d\sigma_k^2 + \sum_{i \neq k} \frac{\partial R_k^{ex}}{\partial \sigma_{i,k}} d\sigma_{i,k}, \ k = 1, ..., K. \tag{7}$$

Changes in the riskiness of the class-k assets will, in general, affect expected returns in classes other than k, and will, in turn, be affected by changes in the riskiness of class j assets. While this type of dependence, in itself, does not directly imply the occurrence of the rank effect, such a possibility cannot be excluded.

An obvious way to preclude the possibility of these patterns occurring is to postulate that the cross-effects are zero:

$$\frac{\partial R_k^{ex}}{\partial \sigma_{i,k}} = 0, \ \forall i \neq k.$$

Within the model studied here, there are two possible ways to proceed:

(i) postulate that assets are independent goods, or

(ii) postulate independent returns.

Each of these options is explored below.

4.1 Independent goods

The assumption of independent goods implies that a change in the price of one asset will not have any effect on the quantity demanded of other assets. This assumption is, however, logically inconsistent with the above model, since the exponential utility does not yield independent goods. Within the current model, the possibility of correlations occurring under H_0 cannot be eliminated.

The next step is to change the utility function. The other class of utility functions to be considered is the CRRA. The general representative of this class is the power utility, but this type of utility function will not result in independent goods either. A special

case of the power utility is the log utility,

$$u\left(c\right)=\ln\left(c\right),$$

which does imply independent goods, but this utility does not allow for exogenous changes in risk aversion, since the coefficient of relative risk aversion is always equal to -1. This implies that, with log utility, (6) is reduced to

$$dR_k^{ex}=\frac{\partial R_k^{ex}}{\partial\sigma_k^2}d\sigma_k^2,$$

since $d\rho=0$ in all cases. Log utility disables the basic mechanism that relates changes in asset prices to changes in risk aversion. Hence, the theoretical basis for the RAI cannot be established within this class of models by using the assumption of independent goods.

4.2 Assumptions on returns

4.2.1 Consequences of dependence in returns

One might argue that even with the type of dependence referred to above, the emergence of the rank effect under H_1 is an unlikely event and, as such, of no great concern. To argue that independent returns are necessary, one needs to demonstrate that the rank effect will occur under H_0, at least under some circumstances, when returns are dependent. The problem at this point is to identify the conditions under which the rank effect will occur.

The analysis starts from the expression for covariance in (5). Substituting this expression into (2) yields

$$R_k^{ex} = \rho\sigma_{k,W} \equiv \rho\alpha_k\sigma_k^2 + \rho\sum_{i\neq k}\alpha_i\sigma_{i,k}.$$

Consider two assets, indexed by i and k, with $\sigma_{k,W} > \sigma_{i,W}$. Suppose that there is a change in the riskiness of asset k due to a change in its covariance with asset i. Then,

$$\frac{dR_k^{ex}}{d\sigma_{k,W}} = \rho\alpha_i > 0.$$

Since $\sigma_{k,i} = \sigma_{i,k}$, it follows that

$$\frac{dR_i^{ex}}{d\sigma_{i,W}} = \rho\alpha_k > 0.$$

Thus, if $\alpha_i > \alpha_k$,

$$\sigma_{k,W} > \sigma_{i,W} \Rightarrow \frac{dR_k^{ex}}{d\sigma_{k,W}} > \frac{dR_i^{ex}}{d\sigma_{i,W}},$$

and, if $\alpha_i < \alpha_k$,

$$\sigma_{k,W} > \sigma_{i,W} \Rightarrow \frac{dR_k^{ex}}{d\sigma_{k,W}} < \frac{dR_i^{ex}}{d\sigma_{i,W}}.$$

In both cases, the rank effect is present: in the former case the effect is equivalent to increasing risk aversion, and in the latter it is equivalent to decreasing risk aversion. Hence, the rank effect will occur whenever $\alpha_i \neq \alpha_k$.

4.2.2 Independent returns

The assumption of independent returns means that $\sigma_{i,k} = 0,\ \forall i, k$. In other words, the variance-covariance matrix associated with the market portfolio should be diagonal:

$$\begin{bmatrix} \sigma_1^2 & 0 & . & . & 0 \\ 0 & . & . & 0 & . \\ . & . & \sigma_k^2 & . & . \\ . & 0 & . & . & 0 \\ 0 & . & . & 0 & \sigma_K^2 \end{bmatrix}.$$

To investigate the conditions under which this assumption will preclude the occurrence of the rank effect, two cases should be considered.

Case 1: Independent returns, idiosyncratic shock

Under these assumptions, it follows that a change in the riskiness of asset k will have the effect

$$\frac{dR_k^{ex}}{d\sigma_{k,W}} = \rho\alpha_k.$$

Since the shock is idiosyncratic and asset returns are independent, $\frac{dR_i^{ex}}{d\sigma_{k,W}} = 0,\ \forall i \neq k$; the rank effect will not occur.

Case 2: Independent returns, common shock

Under the assumption of independent returns, a common shock means that $d\sigma_k^2 > 0$, or $d\sigma_k^2 < 0, \forall k$. Then,

$$\frac{dR_k^{ex}}{d\sigma_{k,W}} = \rho\alpha_k,\ \forall k.$$

If, for example, $\alpha_k > \alpha_i$, and $\sigma_{k,W} > \sigma_{i,W}$, one gets

$$\sigma_{k,W} > \sigma_{i,W} \Rightarrow \frac{dR_k^{ex}}{d\sigma_{k,W}} > \frac{dR_i^{ex}}{d\sigma_{i,W}},$$

and the rank effect occurs.

The results obtained can be summarized as follows:

- with dependent returns, the rank effect will emerge under H_0 when the riskiness of an asset changes in all portfolios except those in which assets are equally weighted.
- with independent returns,
  - the rank effect will *not* occur under H_0 if the shock to riskiness is idiosyncratic,
  - the rank effect will occur under H_0 if the shock is common in all but equally weighted portfolios.

These findings have practical implications. To the extent that the identification of the common shock is apparent and changes in rank correlation can be traced to it, the assumption of independent returns is the key to the validity of the interpretation that the RAI captures changes in risk aversion. Under the model given here, the RAI can be interpreted as a measure of changes in risk aversion only if the assumption of independent returns is demonstrated to hold. This is the key requirement, the validity of which has to be checked in any sample where the RAI is computed, to interpret it as capturing changes in risk aversion. This assumption is easily verified by computing the covariance matrix of the data used to compute the RAI. In practical applications, the assumption of exact zero covariance will be translated into a test of whether the covariances are statistically different from zero.

4.3 Summary

The starting point in an attempt to provide a theoretical foundation for the RAI is the present-value relationship, (1), which is common to a variety of asset-pricing models. The success of the model depends on the ability to demonstrate the presence of a rank

effect when risk aversion changes, and its absence when risk aversion is assumed constant. Two possibilities were explored: independent goods and independent returns.

Working with constant risk-aversion utility functions, the above analysis shows that one cannot guarantee the absence of the rank effect under H_0 without assuming independence of goods. But this assumption precludes the occurrence of the rank effect under H_1 by disallowing changes in risk aversion.

The theoretical foundation for the RAI can be provided if returns are linearly independent. This requirement, which is equivalent to zero cross-correlations, can be empirically verified. For samples in which returns are independent, the RAI will indeed capture changes in risk aversion in isolation. Evidence of dependence in returns, on the other hand, implies that the claim that the RAI captures this effect in isolation cannot be validated by appealing to the class of asset-pricing models investigated here.

5. Conclusions

The results of this study suggest that the form of skepticism based on the observational equivalence argument (described in section 2) is partially justified: the observational equivalence result is quite robust. The study did show, however, that the reasoning based on two classes of assets prevents one from seeing some interesting relationships. In multiple asset settings, the rank effect has been shown to occur when there is a change in risk aversion. The problem is that this effect can occur for other reasons as well.

The contribution of this work is to offer a model and demonstrate that the observational equivalence can be broken and that, under certain conditions, the RAI can be interpreted as capturing changes in risk aversion. Moreover, the key condition of independent returns can be easily checked in any data set.

Of course, the condition of independent returns is relevant only in the context of the class of asset-pricing models studied in this paper. While this is a rather large class, there may be a model that could be used to provide a theoretical foundation for the RAI that relies on weaker assumptions. The existence of such a model is an open question.

It is important to note that the violation of the assumption of independent returns does not necessarily negate the use of the RAI, for example as a predictive device. The finding does imply that great caution is necessary in interpreting the results obtained using this measure, or in construing them as evidence to support explanations that market phenomena such as financial contagion are due to changes in risk aversion.

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