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Abstract

This paper studies the persistent effects of monetary shocks on output. Previous empirical literature documents this persistence, but standard general-equilibrium models with sticky prices fail to generate output responses beyond the duration of nominal contracts. The paper constructs and estimates a general-equilibrium model with price rigidities, habit formation, and costly capital adjustment. The model is estimated by the maximum-likelihood method using U.S. data on output, the real money stock, and the nominal interest rate. Econometric results indicate that habit formation and adjustment costs to capital play an important role in explaining the output effects of monetary policy. In particular, impulse-response analysis indicates that the model generates persistent, hump-shaped output responses to monetary shocks.

JEL classification: E3, E4, E5
Bank classification: Business fluctuations and cycles; Transmission of monetary policy

Résumé

Dans cet article, les auteurs étudient les effets persistants des chocs monétaires sur la production. Cette persistance est bien établie dans la littérature empirique, mais les modèles standard d'équilibre général avec rigidité des prix ne réussissent pas à générer des réponses de la production qui persistent au-delà de la durée des contrats nominaux. Dans cette étude, les auteurs construisent un modèle d'équilibre général à prix rigides où les consommateurs forment des habitudes et où l'ajustement du capital est coûteux. Le modèle est estimé par la méthode du maximum de vraisemblance en utilisant des données américaines sur la monnaie, la production et le taux d'intérêt. Les résultats économétriques indiquent que la formation d'habitudes et les coûts d'ajustement du capital sont importants dans l'explication des effets de la politique monétaire sur la production. En particulier, l'analyse des fonctions de réponse montre qu'après un choc monétaire, le modèle génère des réactions de la production qui sont persistantes et en forme de « bosse ».

Classification JEL : E3, E4, E5
Classification de la Banque : Cycles et fluctuations économiques; Transmission de la politique monétaire

1. Introduction

In a recent paper, Chari, Kehoe, and McGrattan (2000) show that standard dynamic stochastic general-equilibrium (DSGE) models with sticky prices fail to generate persistent output effects to monetary shocks. More precisely, the response of output to a money-growth shock does not last beyond the duration of price contracts, even if contracts are staggered. Hence, unless one assumes an implausibly large degree of price rigidity, this type of model cannot replicate the persistent output response obtained using, for example, a benchmark vector autoregression (VAR). Previous empirical studies based on VARs document a persistent, hump-shaped response of output to a monetary shock with a peak at around four to six quarters after the shock (see Bernanke and Mihov 1998 and Christiano, Eichenbaum, and Evans 1999). The failure of DSGE models to replicate this feature of the data is called "the persistence problem."

This paper studies the effects of monetary policy on output using a DSGE model with sticky prices, habit formation, and adjustment costs to capital. Price rigidity is modelled as in Calvo (1983), where each firm has a constant exogenous probability of changing its price in every period. Habit formation has been employed previously by (among others) Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) to study the equity premium puzzle; by Carrol, Overland, and Weil (2000) to explain the growth-to-savings causality; and by Fuhrer (2000) to explain the excess smoothness of consumption and inflation inertia. Because habit-forming agents dislike large changes in consumption, the consumption response to shocks is smoother and more persistent than predicted by the permanent income hypothesis (PIH) with a time-separable utility. Since consumption is the largest component in GDP, habit formation is a plausible candidate to explain the persistent and hump-shaped output response to monetary policy shocks.

The model is estimated by the maximum-likelihood (ML) method using U.S. data on output, the real money stock, and the nominal interest rate. The ML procedure yields plausible estimates of the structural parameters. Impulse-response analysis indicates that monetary shocks lead to a persistent and hump-shaped output response. Up to 95 per cent of the initial effect of a money-growth shock on output persists beyond the average duration of price contracts. A comparison of impulse responses and persistence measures for different values of the habit formation and capital-adjustment cost parameters indicates that habit formation, by itself, does not solve the persistence problem. Instead, habit formation interacts non-linearly with costly capital adjustment to increase the propagation of monetary shocks in the model. When the fit of the estimated DSGE model is compared with that of an unrestricted VAR, the mean squared error (MSE) of the DSGE model is smaller than that of the VAR for output and the real money stock and only slightly larger for the nominal interest rate. Variance decomposition indicates that money growth explains more than 50 per cent of the (conditional) output variability at horizons of less than one year. In the long run, money growth explains only 27.1 per cent of the unconditional output variability, while 71.4 per cent is explained by technology shocks.

Related papers include those by Bergin and Feenstra (2000), Dotsey and King (2001), and Dib and Phaneuf (2001). Bergin and Feenstra construct a model where the interaction of materials inputs and translog preferences leads to endogenous output persistence. Translog preferences dissuade firms from charging higher prices by making the elasticity of demand that a given firm faces depend on the firm's relative price. Dotsey and King construct a model that incorporates variable capital utilization, and materials input and labour flexibility. Results indicate that these three features are mutually reinforcing and magnify output persistence. Dib and Phaneuf construct a DSGE model with sticky prices and costly adjustment to labour. Their results show that adding adjustment costs to the labour input generates endogenous output persistence to monetary shocks. After our research was completed, we found a closely related paper by Christiano, Eichenbaum, and Evans (2001). These authors examine both output and inflation persistence using a limited-participation model that incorporates price and wage rigidities, optimizing and non-optimizing price- and wage-setting, habit formation, adjustment costs in investment, and variable capital utilization. Their results suggest that wage rigidity and variable capital utilization are also important for explaining output persistence in response to monetary shocks. Although their modelling strategy is similar to ours, Christiano, Eichenbaum, and Evans obtain empirical estimates by minimizing the distance between the impulse responses in a VAR and the ones predicted by the model, whereas we estimate the model by full-information ML using the Kalman filter. The Kalman filter allows us to deal with poorly measured or unobserved variables (like the stock of capital), and yields the optimal solution to the problem of predicting and updating state-space models. Furthermore, we propose a different propagation mechanism than the one emphasized in the earlier models. In fact, although apparently distinct, the crucial features of these models work through the same channel to increase output persistence. They prevent a rapid change in the real marginal cost after a monetary shock, and lead to stronger nominal rigidity.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 describes the estimation procedure, reports empirical results, and discusses the impulse-response functions and variance decompositions implied by the estimated model. Section 4 concludes.

2. The Model

The economy consists of (i) an infinitely lived representative household, (ii) a representative final-good producer, (iii) a continuum of intermediate-good producers indexed by $i \in [0,1]$, and (iv) a government. Intermediate goods are used in the production of the final good. The final good is perishable and can be used for either consumption or investment. There is no population growth. The population size is normalized to one.

2.1 Households

The representative household maximizes lifetime utility, defined by

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} u_s(c_s, c_{s-1}, m_s, \ell_s),$$

where $\beta \in (0,1)$ is the subjective discount factor and $u(\cdot)$ is the instantaneous utility function. Households derive utility from the consumption of the final good (c_t), real money balances (m_t), and leisure (ℓ_t). The household's preferences exhibit internal habit formation. That is, utility depends on current consumption relative to a habit stock determined by the household's own past consumption. Thus, consumption levels in adjacent periods are complements. In particular, the instantaneous utility function is assumed to be

$$u_t(c_t, c_{t-1}, m_t, \ell_t) = \frac{(c_t/c_{t-1}^{\gamma})^{1-\eta_1}}{1-\eta_1} + \frac{b_t(m_t)^{1-\eta_2}}{1-\eta_2} + \frac{\psi(\ell_t)^{1-\eta_3}}{1-\eta_3}, \tag{1}$$

where $m_t = M_t/P_t$, M_t is the nominal money stock, P_t is the aggregate price index, b_t is a preference shock, $\psi > 0$ measures the weight of leisure in the utility function, and η_1, η_2, and η_3 are positive preference parameters different from one. In the special case where $\eta_j \to 1$ for all j, the logarithmic utility function is obtained. In the special case where $\gamma = 0$, there is no habit formation and households care only about the absolute level of current consumption. In principle, the habit stock could include consumption levels prior to time $t-1$. Fuhrer (2000) estimates a model where the habit stock is a weighted average of past consumption and finds that the habit-formation reference level is essentially the previous period's consumption level.

In addition to money, households can hold interest-bearing, one-period nominal bonds. The gross nominal interest rate on bonds due at time $t+1$ is denoted by R_t. The household's resources in period t consist of the principal and the return on bonds purchased at time $t-1$, money holdings set aside in period $t-1$, wages and rents received from selling labour and renting capital to firms, dividends, and lump-sum transfers from the government.

The household's income in period t is allocated to consumption, investment, money holdings, and the purchase of nominal bonds. Investment increases the household's stock of capital according to

$$k_{t+1} = (1-\delta)k_t + x_t, \tag{2}$$

where $\delta \in (0,1)$ is the depreciation rate of capital. The capital stock is costly to adjust. The adjustment-cost function is assumed to be quadratic in investment and strictly convex:

$$\Gamma(x_t, k_t) = (\chi/2)(x_t/k_t - \delta)^2 k_t, \tag{3}$$

where $\chi \geq 0$. Investment beyond that required to replace depreciated capital entails a positive quadratic cost that is proportional to the current capital stock.

The representative household's budget constraint (expressed in real terms) is

$$c_t + a_t + m_t + x_t \leq (R_{t-1}/\pi_t)a_{t-1} + (m_{t-1}/\pi_t) + w_t n_t + q_t k_t + d_t + \tau_t - (\chi/2)(x_t/k_t - \delta)^2 k_t, \tag{4}$$

where $a_t = A_t/P_t$ is the real value of nominal bond holdings, A_t are nominal bond holdings, π_t is the gross rate of inflation between $t-1$ and t, w_t is the real wage, n_t is the number of hours worked, q_t is the real rental rate of capital, d_t are dividends, and τ_t are lump-sum transfers or taxes. The household's total endowment of time is normalized to one. Thus

$$\ell_t + n_t = 1. \tag{5}$$

The representative household maximizes its lifetime utility subject to constraints (2), (4), (5), and the no-Ponzi-game condition. The first-order necessary conditions associated with the optimal choice of $c_t, M_t, \ell_t, k_{t+1}$, and A_t for this problem are

$$\begin{aligned}
\lambda_t &= (1/c_{t-1}^{\gamma})(c_t/c_{t-1}^{\gamma})^{-\eta_1} - \beta\gamma E_t[(c_{t+1}/c_t^{1+\gamma})(c_{t+1}/c_t)^{-\eta_1}], && (6)\\
b_t m_t^{-\eta_2} &= \lambda_t[(R_t-1)/R_t], && (7)\\
(1-n_t)^{-\eta_3} &= \lambda_t w_t/\psi, && (8)\\
\lambda_t &= \frac{\beta E_t\{\lambda_{t+1}[1+q_{t+1}-\delta+\chi(x_{t+1}/k_{t+1}-\delta)+(\chi/2)(x_{t+1}/k_{t+1}-\delta)^2]\}}{1+\chi(x_{t+1}/k_{t+1}-\delta)}, && (9)\\
\lambda_t &= \beta R_t E_t\left(\lambda_{t+1}/\pi_{t+1}\right), && (10)
\end{aligned}$$

where λ_t is the Lagrange multiplier associated with the household's budget constraint at time t and equals the marginal utility of consumption at time t. Condition (7) determines money demand by equating the marginal rate of substitution of money and consumption to $(R_t-1)/R_t$, where R_t is the gross return of the nominal bond. The interest elasticity of money is equal to $-1/\eta_2$.[1] The preference shock, b_t, can be interpreted as a money-demand shock. Condition (8) determines the labour supply by equating the marginal rate of substitution between labour and consumption to the real wage. Condition (9) prices the (marginal unit of) capital. Condition (10) prices the nominal bond. Conditions (9) and (10) imply that the ex-ante real interest rate should be equal to the ex-ante real return on capital.

2.2 The final-good producer

Final-good producers are perfectly competitive and aggregate the intermediate goods into a single perishable commodity. Their technology is constant elasticity of substitution (CES):

$$y_t = \left[\int_0^1 y_t(i)^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}, \tag{11}$$

where $y(i)$ is the input of intermediate good i, and $\theta > 1$ is the elasticity of substitution between different goods. As $\theta \to \infty$, goods become perfect substitutes in production. The final-good producer solves the static problem

$$\underset{\{y_t(i)\}}{Max} \quad P_t y_t - \textstyle\int_0^1 P_t(i) y_t(i) di,$$

subject to (11). $P_t(i)$ is the price of the intermediate good i and P_t is the aggregate price index. The solution of this problem yields the input demand of good i:

$$y_t(i) = (P_t(i)/P_t)^{-\theta} y_t, \tag{12}$$

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[1] Strictly speaking, $-1/\eta_2$ is the elasticity with respect to $(R_t-1)/R_t$, rather than $R_t - 1$.

where the elasticity of demand is θ. The zero-profit condition implies that the aggregate price index is given by

$$P_t = \left[\int_0^1 P_t(i)^{(1-\theta)} di \right]^{1/(1-\theta)}. \tag{13}$$

2.3 The intermediate-good producer

The representative firm i produces its differentiated good using the Cobb-Douglas technology:

$$y_t(i) = z_t k_t(i)^{\alpha} n_t(i)^{1-\alpha}, \tag{14}$$

where $0 < \alpha < 1$ and z_t is a serially correlated technology shock. The technology shock is common to all intermediate-good producers. Unit-cost minimization determines the demands for labour and capital inputs. Formally,

$$\underset{\{n_t(i), k_t(i)\}}{Min} \qquad w_t n_t(i) + q_t k_t(i),$$

subject to $z_t k_t(i)^{\alpha} n_t(i)^{1-\alpha} = y_t(i) \geq 1$. First-order conditions are

$$w_t = (1-\alpha)\,\phi_t[y_t(i)/n_t(i)], \tag{15}$$

and

$$q_t = \alpha\phi_t[y_t(i)/k_t(i)], \tag{16}$$

where the real marginal cost (ϕ_t) is the Lagrange multiplier associated with the constraint. Since technology is common, and labour and capital are perfectly mobile across industries, conditions (15) and (16) imply that all firms must have the same capital/labour ratio.

Intermediate-good producers are monopolistically competitive. Each firm faces the downward-sloping demand curve (12) for its differentiated good. Firm i chooses its (nominal) price $P(i)$ taking as given the aggregate demand and the price level. Nominal prices are assumed to be sticky. Price stickiness is modelled *à la* Calvo (Calvo 1983): a firm changes its price with constant and exogenous probability $1-\varphi$ in every period.[2] Alternatively, one could assume explicit costs of changing prices or Taylor's staggered price-setting. Quadratic costs of price adjustments, as in Rotemberg (1982), can be shown to lead to an aggregate pricing equation similar to the one obtained using Calvo's model. Moreover, aggregation is somewhat easier using Calvo-type than Taylor-type price rigidity, because it is not necessary

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[2] Hence, the average duration of price contracts is given by $1/(1-\varphi)$.

to keep track of heterogeneous price cohorts. From the viewpoint of estimating the average length of price contracts using ML, Calvo's model is also easier to implement because the log-likelihood function is continuous on φ. This follows from the fact that the probability of price changes is continuous in the interval $[0, 1]$. On the other hand, the contract length in Taylor's model is an integer number and, consequently, the log-likelihood function is discontinuous on this parameter.

Let us denote by P_t^* the optimal price set by a typical firm at period t. It is not necessary to index P_t^* by firm, because all the firms that change their prices at a given time choose the same price (see Woodford 1996). The total demand facing this firm at time s for $s \geq t$ is $y_s^* = (P_t^*/P_s)^{-\theta} y_s$. The probability that P_t^* "survives" at least until period s, for $s \geq t$, is φ^{s-t}. Then, the intermediate-good producer chooses P_t^* to maximize

$$E_t \sum_{s=t}^{\infty} (\beta\varphi)^{s-t} \Lambda_{t,s} \left(P_t^* - \Phi_s\right) y_s^*,$$

where $\Lambda_{t,s} = (\lambda_s/P_s)/(\lambda_t/P_t)$ and Φ_s is the nominal marginal cost at time s. Differentiating with respect to P_t^* and equating to 0 yields

$$P_t^* = \left(\frac{\theta}{\theta - 1}\right) \left(\frac{E_t \sum_{s=t}^{\infty} (\beta\varphi)^{s-t} \Lambda_{t,s} y_s^* \Phi_s}{E_t \sum_{s=t}^{\infty} (\beta\varphi)^{s-t} \Lambda_{t,s} y_s^*}\right). \tag{17}$$

Equation (17) shows that the optimal price depends on current and expected future demands and nominal marginal costs. Owing to price stickiness, the equilibrium markup is not constant, as it would be if prices were flexible.

Assuming that price changes are independent across firms, the law of large numbers implies that $1 - \varphi$ is also the proportion of firms that set a new price each period. The proportion of firms that set a new price at time s and have not changed it as of time t (for $s \leq t$) is given by the probability that a time-s price is still in effect in period t. It is easy to show that this probability is $\varphi^{t-s}(1 - \varphi)$. It follows that the aggregate price level can be written as

$$P_t = \left((1 - \varphi) \sum_{s=-\infty}^{t} \varphi^{t-s} (P_t^*)^{1-\theta}\right)^{\frac{1}{1-\theta}}.$$

This expression can be written in recursive form as

$$P_t^{1-\theta} = \varphi P_{t-1}^{1-\theta} + (1 - \varphi)(P_t^*)^{1-\theta}. \tag{18}$$

2.4 The government

The government comprises both fiscal and monetary authorities. There is no government spending or investment. The government makes lump-sum transfers to households each period. Transfers are financed by printing additional money in each period. Thus, the government budget constraint is

$$\tau_t = m_t - m_{t-1}/\pi_t, \tag{19}$$

where the term on the right-hand side is seigniorage revenue at time t. Money is supplied exogenously by the government according to $M_t = \mu_t M_{t-1}$, where μ_t is the (stochastic) gross rate of money growth.[3] In real terms, this process implies

$$m_t \pi_t = \mu_t m_{t-1}. \tag{20}$$

2.5 Stochastic shocks

The economy is subject to shocks to technology (z_t), money-supply growth (μ_t), and money demand (b_t). These shocks follow the exogenous stochastic processes

$$\begin{aligned} \ln z_{t+1} &= (1-\rho^z)\ln z + \rho^z \ln z_t + \epsilon_{z,t}, & (21)\\ \ln \mu_{t+1} &= (1-\rho^\mu)\ln \mu + \rho^\mu \ln \mu_t + \epsilon_{\mu,t}, & (22)\\ \ln b_{t+1} &= \left(1-\rho^b\right)\ln b + \rho^b \ln b_t + \epsilon_{b,t}, & (23) \end{aligned}$$

where ρ^z, ρ^μ, and ρ^b are strictly bounded between -1 and 1, and the innovations $\epsilon_t = (\epsilon_{z,t}, \epsilon_{\mu,t}, \epsilon_{b,t})'$ are assumed to be normally distributed with a zero mean and variance-covariance matrix:

$$\mathbf{V} = Var(\epsilon_t \epsilon_t') = \begin{bmatrix} \sigma^2_{\epsilon_z} & 0 & 0 \\ 0 & \sigma^2_{\epsilon_\mu} & 0 \\ 0 & 0 & \sigma^2_{\epsilon_b} \end{bmatrix}. \tag{24}$$

Since households are identical, the net supply of (private) bonds is zero. Goods-market clearing requires that aggregate output be equal to aggregate demand:

$$y_t = c_t + x_t. \tag{25}$$

A symmetric equilibrium for this economy is a collection of 13 sequences (c_t, m_t, n_t, x_t, k_{t+1}, y_t, λ_t, ϕ_t, P_t, P_t^*, q_t, w_t, and $R_t)_{t=0}^{\infty}$ satisfying (i) the accumulation equation (2),

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[3] It is easy to extend the model to allow an endogenous process for money supply whereby money growth (or the nominal interest rate) follows, for example, a Taylor-type rule. In such an extension of the model, the endogenous reaction of the government might also increase the persistence of monetary shocks.

(ii) the household's maximization conditions (equations (6) to (10)), (iii) the production function (14), (iv) the cost-minimization conditions (equations (15) and (16)), (v) the pricing conditions (equations (17) and (18)), (vi) the market-clearing condition (25), and (vii) the money-supply process (20), given the initial stocks of habit, real money, and capital, and the exogenous stochastic processes (z_t, μ_t, b_t).

Since the model cannot be solved analytically, the equilibrium conditions are log-linearized around the deterministic steady state to obtain a system of linear difference equations. (Appendix A gives the log-linearized version of the model.) After some manipulations, the log-linearized version of the model can be written as

$$\begin{aligned} \begin{bmatrix} \mathbf{X}_{t+1} \\ E_t \mathbf{Y}_{t+1} \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{X}_t \\ \mathbf{Y}_t \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \mathbf{Z}_{t+1}, \quad (26) \\ \mathbf{Z}_{t+1} &= \rho \mathbf{Z}_t + \epsilon_{t+1}, \quad (27) \end{aligned}$$

where $\mathbf{X}_t = (\hat{k}_t,\ \hat{m}_{t-1},\ \hat{c}_{t-1})'$ is a 3×1 vector that contains the predetermined variables of the system (the circumflex denotes percentage deviations from the deterministic steady state); $\mathbf{Y}_t = (\hat{c}_t,\ \hat{\pi}_t,\ \hat{\lambda}_t,\ \hat{q}_t)'$ is a 4×1 vector that contains the forward-looking variables; $\mathbf{Z}_t = (\hat{z}_t,\ \hat{\mu}_t,\ \hat{b}_t)'$ is a 3×1 vector that contains the exogenous shocks; $\epsilon_t = (\epsilon_{z,t},\ \epsilon_{\mu,t},\ \epsilon_{b,t})'$ is a 3×1 vector with the innovations of z_t, μ_t, and b_t, respectively; ρ is a 3×3 diagonal matrix with elements ρ^z, ρ^μ, and ρ^b; and $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$, $\mathbf{B}_1$, and $\mathbf{B}_2$ are submatrices of appropriate size that contain combinations of structural parameters. The Blanchard-Kahn (1980) forward-backward solution method can be applied to (26) to obtain

$$\begin{aligned} \mathbf{X}_{t+1} &= \mathbf{A}_{11}\, \mathbf{X}_t + \mathbf{A}_{12} \mathbf{Y}_t + \mathbf{B}_1 \mathbf{Z}_t, \quad (28) \\ \mathbf{Y}_t &= \mathbf{F}_1 \mathbf{X}_t + \mathbf{F}_2 \mathbf{Z}_t, \end{aligned}$$

where $\mathbf{F}_1$ and $\mathbf{F}_2$ are both 4×3 matrices that include non-linear combinations of the structural parameters contained in $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$, $\mathbf{B}_1$, and $\mathbf{B}_2$. For the precise form of these matrices and the conditions for a unique solution, see Blanchard and Kahn (1980). The remaining (static) variables of the model can be collected in the 6×1 vector $\mathbf{S}_t = (\hat{x}_t,\ \hat{n}_t,\ \hat{w}_t,\ \hat{\phi}_t,\ \hat{R}_t,\ \hat{y}_t)'$ that follows:

$$\mathbf{S}_t = \mathbf{C}\mathbf{X}_t + \mathbf{D}\mathbf{Z}_t, \quad (29)$$

where $\mathbf{C}$ and $\mathbf{D}$ are matrices of size 6×3 whose elements are also non-linear combinations of structural parameters.

3. Econometric Analysis

3.1 Estimation method and data

The model is estimated by ML using the Kalman filter. Earlier studies that use ML procedures to estimate DSGE models include Christiano (1988), Altug (1989), Bencivenga (1992), McGrattan (1994), Hall (1996), McGrattan, Rogerson, and Wright (1997), Kim (2000), and Ireland (2001). Our estimation strategy is closest to that used by Ireland (2001). The Kalman filter allows us to deal with unobserved or poorly measured predetermined variables (like the stock of capital) and yields the optimal solution to the problem of predicting and updating state-space models. Hansen and Sargent (1998) show that the ML estimator obtained by applying the Kalman filter to the state-space representation of DSGE models is consistent and asymptotically efficient.

For the Kalman-filter estimation procedure, the transition (or state) equation is constructed using equations (27) and (28) to collect the predetermined and exogenous variables of the system into the 6×1 vector $\mathbf{H}_t = (\mathbf{X}_t \ \mathbf{Z}_t)' = (k_t,\ m_{t-1}, c_{t-1},\ z_t,\ \mu_t,\ b_t)'$ that follows the process

$$\mathbf{H}_{t+1} = \mathbf{Q}\mathbf{H}_t + \mathbf{e}_{t+1}, \tag{30}$$

where

$$\mathbf{Q} = \begin{bmatrix} \mathbf{A}_{11}+\mathbf{A}_{12}\mathbf{F}_1 & \mathbf{A}_{12}\mathbf{F}_2+\mathbf{B}_1 \\ \mathbf{0} & \rho \end{bmatrix}$$

is a 6×6 matrix and $\mathbf{e}_t = (0,0,0,\epsilon_t)' = (0,0,0,\epsilon_{zt},\epsilon_{\mu t},\epsilon_{bt},)'$ is a 6×1 vector.

The measurement equation consists of the processes for output, the real money stock, and the nominal interest rate. After some fairly straightforward transformations, these variables are written as functions of $\mathbf{H}_t$:

$$\xi_t = \mathbf{W}\mathbf{H}_t, \tag{31}$$

where $\xi_t = (m_t, y_t, R_t)'$ is a 3×1 vector and $\mathbf{W}$ is a 3×6 matrix.[4] The elements of $\mathbf{Q}$ and $\mathbf{W}$ are non-linear functions of the structural parameters of the model. These elements are

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[4] As is well known, estimating DSGE models using more observable variables than structural shocks leads to a singular variance-covariance matrix of the residuals. See Ingram, Kocherlakota, and Savin (1994) for a discussion in the special case where the only shock is a technology shock. One way to address this issue is to add measurement errors to the observable variables (as in McGrattan 1994). A possible drawback to this approach is that measurement errors lack structural interpretation and essentially capture specification errors. Still, in preliminary work, we considered this approach. When we added measurement errors to all observable variables, we found that not all variances were identified, or that some of them converged to zero. When we added only as many errors as needed to make the system non-singular, we found that the results were very sensitive to the variable that was assumed to be measured with noise.

computed from the Blanchard-Kahn solution of the DSGE model in each iteration of the optimization procedure. Note that the estimation procedure imposes all restrictions implied by the theoretical model. Standard errors are computed as the square root of the diagonal elements of the inverted Hessian of the (negative) log-likelihood function evaluated at the maximum. To assess the robustness of the results to deviations from the assumption of normality, robust quasi-maximum likelihood (QML) standard errors (White 1982) are also computed. At the estimated ML parameters, the condition for the existence of a unique model solution is satisfied. That is, the number of explosive characteristic roots of the system of linear difference equations equals the number of non-predetermined variables.

The model is estimated using quarterly U.S. data on output, real money, and the rate of nominal interest. The series are taken from the database of the Federal Reserve Bank of St. Louis. The sample period is from 1960Q1 to 2001Q2. Output is measured by real GDP per capita. The stock of nominal money is measured by M2 per capita. By measuring these two series in per-capita terms, we aim to make the data compatible with our model, where there is no population growth. Population is measured by the civilian, non-institutional population, 16 years old and over. The gross nominal interest rate is measured by the three-month U.S. Treasury bill rate. Because the variables in the model are expressed in percentage deviations from the steady state, the output and real money series were logged and detrended linearly. The nominal interest rate series was logged and demeaned. We also estimated the model using Hodrick-Prescott (H-P) filtered data, obtaining very similar results to the ones reported below.[5]

3.2 Estimates of structural parameters

The structural parameters estimated are the preference parameters η_2 and η_3, the habit-persistence parameter (γ), the probability of a price change by an intermediate-good producer (φ), the parameter of the capital-adjustment-cost function (χ), and parameters of the shock processes (ρ^z, ρ^μ, ρ^b, σ^z, σ^μ, and σ^b). Remaining parameters were either poorly identified or additional evidence about their magnitude is available. Data on national income accounts suggest that a plausible value for the share of capital in production is 0.36. The subjective discount factor is fixed to 0.99, meaning that the steady-state quarterly gross real interest rate is approximately 1.01. The rate of depreciation is fixed to 0.025. The gross rate of money growth (and inflation) at the steady state is fixed to 1.017. This value

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[5] Results using H-P filtered data are available upon request.

corresponds to the average gross rate of money growth during the sample period. Two important structural parameters that are poorly identified are the curvature parameter of the consumption component in the utility function (η_1) and the elasticity of demand (θ). Markup estimates reported by Basu and Fernald (1994) for U.S. data indicate that θ is approximately 10. Estimates of the curvature of the utility function with respect to consumption range from 0.5 to 5. We assume that $\eta_1 = 2$, but sensitivity analysis indicates that the results do not depend crucially on the magnitudes of θ and η_1.[6] Finally, fixing the proportion of time worked in steady state (n) amounts to fixing either the mean of the technology shock (z) or the weight of leisure in the utility function (ψ). We do not assign particular values to these parameters during the estimation procedure. Instead, we adjust them so that, along with the ML estimate of η_3, $n = 0.31$. This means that the proportion of time worked in steady state is approximately one third.

ML estimates of the parameters and their asymptotic and QML standard errors are reported in Table 1. Since asymptotic and QML standard errors have very similar magnitudes, conclusions regarding the statistical significance of the parameters do not depend on the estimate of the standard error employed to construct the t-statistic. The ML estimate of the habit-formation parameter (γ) is 0.98 (0.016). The term in parenthesis is the asymptotic standard error of the estimate. This estimate is significantly different from zero, but is not significantly different from one, at standard levels. This estimate is larger than, but still consistent with, the values of 0.80 (0.19) and 0.90 (1.83), reported by Fuhrer (2000); 0.63 (0.14), reported by Christiano, Eichenbaum, and Evans (2001); 0.73, reported by Boldrin, Christiano, and Fisher (2001); and 0.938 (1.775), reported by Heaton (1995).

The estimated value of the adjustment-cost parameter (χ) is 85.19 (18.94). To give meaning to this estimate and to allow its comparison with estimates based on other functional forms, it is useful to compute the elasticity of investment with respect to the price of installed capital. The elasticity implied by the estimate of χ is 0.47. This value is higher than the point estimates of 0.34 and 0.28 reported by, respectively, Kim (2000) and Christiano, Eichenbaum, and Evans (2001), but it is considerably lower than the typical value used to calibrate standard real business cycle (RBC) models (see, for example, Baxter and Crucini 1993).

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[6] We also performed single and joint Lagrange multiplier (LM) tests of the null hypothesis that the true values of β, δ, η_1, α, and θ are the ones assumed during estimation. In all cases, one cannot reject the null hypothesis. These results, however, should be interpreted with caution because they might also reflect low test power.

The estimated probability of not changing price in a given quarter or, equivalently, the proportion of firms that do not change prices in a given quarter, is 0.847 (0.034). This implies that the average length of price contracts is $1/(1-0.847) = 6.56$ (1.44) quarters. Previous estimates of the average time between price adjustments vary substantially. Galí and Gertler (1999) find that θ is approximately 0.83. Their estimate implies that prices are fixed between five and six quarters. Cecchetti (1986) reports that the average number of years since the last price adjustment for U.S. magazines ranges from 1.8 to 14. Kashyap (1995) finds that the average time between price changes in 12 mail-order catalogue goods is approximately 4.9 quarters. Taylor (1999) surveys empirical studies on price-setting and finds that the average duration of price contracts is about four quarters in the United States. Bils and Klenow (2001) document substantial heterogeneity in the frequency of price adjustments among the goods surveyed by the U.S. Bureau of Labor Statistics and report a median price duration of only 1.66 quarters. Christiano, Eichenbaum, and Evans (2001) find that the average length of price contracts is about two quarters and that of wage contracts is roughly 3.3 quarters.

The parameter estimates imply that the interest elasticity of money is 0.32 and the consumption elasticity of money is 0.65. The former estimate is very close to that of 0.39 reported by Chari, Kehoe, and McGrattan (2000), but larger than the estimates of 0.10 and 0.11 found by, respectively, Christiano, Eichenbaum, and Evans (2001) and Dib and Phaneuf (2001).

From the estimate of the curvature parameter of the leisure component in the utility function (η_3), we can compute an estimate of the elasticity of labour supply with respect to the real wage (for a given marginal utility of consumption) as $(1-n)/(\eta_3 n) = (1-0.31)/(1.591 \cdot 0.31) = 1.4$ (2.99). (See Appendix A.) This estimate is too imprecise, however, to allow reliable conclusions.

Estimates of the shock processes' autoregressive coefficients indicate that all shocks are very persistent. Very persistent technology and money-demand shocks are also reported by Kim (2000), Ireland (2001), and Dib and Phaneuf (2001). The estimate of ρ^{μ} is higher than values found when money growth is estimated using a univariate process (as in Chari, Kehoe, and McGrattan 2000).

3.3 Fit and specification tests

This section evaluates the model's goodness of fit, compares it with that of an unrestricted VAR, and performs specification tests on the model's residuals. Figure 1 plots the actual and predicted series of U.S. real money stock, output, and the nominal interest rate. This figure indicates that the model tracks the dynamics of these variables very well. A standard measure of the goodness of fit is the R^2, which measures the proportion of the total variation in the dependent variable that is explained by the model. The R^2s for the real money stock, output, and the nominal interest rate are 0.945, 0.948, and 0.893, respectively. Thus, roughly 95 per cent of the total variation of the real money stock and output can be explained by the DSGE model with sticky prices, habit formation, and costly capital adjustment. The model does not explain as well the behaviour of the nominal interest rate, but it still can account for more than 89 per cent of the total variation of this series.

It is instructive to compare the fit of the model with the one of an unrestricted VAR. The VAR is of order one and contains the following U.S. variables: real money stock, output, and the nominal interest rate. The comparison is made in terms of the MSE, defined as[7]

$$MSE = \left(\sum_{t=2}^{T} (X_t - X_t^p)^2 \right) / (T-1),$$

where $T = 166$ is the number of observations, X_t is either output, real money stock, or the rate of nominal interest, and X_t^p is the value predicted by the model. Since the VAR uses the first observation in the sample to construct the lag, the number of observations used to construct the MSE is $T - 1 = 165$. Table 2 reports the MSE from the estimated DSGE model and the VAR. The DSGE model outperforms the VAR when explaining the behaviour of U.S. output and real money stock in that its MSE is smaller. For the nominal interest rate, however, the MSE of the VAR is slightly less than that of the DSGE model.

Table 3 reports test results for serial correlation of the residuals (Panel A) and neglected autoregressive conditional heteroscedasticity (ARCH) (Panel B). Consider first the Durbin-Watson test for first-order autocorrelation. Comparing the test statistic with the 5 per cent critical value of its tabulated distribution indicates that (i) one cannot reject the null hypothesis of no autocorrelation for the real money stock and output residuals, but (ii) one can reject it for the nominal interest rate residuals. Similarly, results of Pormanteau tests for the first- to third-order autocorrelations of the residuals yield statistics that are below

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[7] Note that the state-space and VAR models are non-nested, and that therefore standard likelihood ratio, LM, and Wald tests would not be appropriate.

(above) their 5 per cent critical value for real money stock and output (nominal interest rate).[8]

The LM tests for neglected ARCH were computed as the product of the number of observations and the uncentred R^2 of the OLS regression of the squared residual on a constant and one to three of its lags. Under the null hypothesis of no conditional heteroscedasticity, the statistic is distributed chi-square with as many degrees of freedom as the number of lagged squared residuals included in the regression. Results in Panel B indicate that the null hypothesis of no conditional heteroscedasticity cannot be rejected at the 5 per cent level for output and the real money stock, but that it can be rejected for the nominal interest rate in some cases. All these results indicate that the DSGE model tracks well the behaviour of U.S. output and real money stock, but that it is somewhat less successful in explaining the nominal interest rate.

The DSGE model also generates predictions regarding series whose actual data were not used in the estimation procedure; for example, consumption, investment, the rate of inflation, and the real marginal cost. The real marginal cost is not directly observable, but under certain conditions it can be proxied by the labour share in national income (see Galí and Gertler 1999 for a detailed discussion). Figure 2 plots the actual and predicted series of U.S. consumption, investment, inflation, and real marginal cost. The figure shows that the model generates consumption and investment dynamics that are similar to the ones of their detrended U.S. counterparts. Predicted investment, however, is much smoother than the data.

The DSGE model does poorly in explaining the behaviour of the real marginal cost and inflation. This result reflects a drawback of inflation models based on forward-looking pricing rules. It is possible to show that, under Calvo-type pricing, the inflation deviation from steady state equals the present discounted value of current and future expected real marginal cost deviations from steady state.[9] This means that inflation inherits the dynamic properties of the real marginal cost and that current inflation is not helpful in predicting future inflation. Because lagged inflation is absent from the inflation equation, forward-looking pricing rules imply that inflation is less persistent than usually found in the data. To address this shortcoming of the model, some authors (for example, Galí and Gertler

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[8] Under the null hypothesis of no autocorrelation, the Portmanteau test statistic is distributed chi-square with as many degrees of freedom as autocorrelations are tested for.

[9] This result can be easily derived by rewriting equation (A.1) in Appendix A with current inflation in the left-hand side and iterating forward.

1999) assume the existence of rule-of-thumb firms that fix their prices as a function of past inflation. Another problem with our model is that the real marginal cost is more volatile than the labour share in national income would suggest. One possibility is that the labour share in national income is a poor empirical proxy for the real marginal cost. More likely, the real marginal cost in our model is excessively volatile because it abstracts from supply-side features like variable capital utilization and adjustment costs to labour input.

3.4 Impulse-response analysis

This section examines the response of the economy to a shock to the money-supply growth rate, hereafter called a money-supply shock. Our intention is to (i) assess the ability of the model to match the persistent output effect of monetary policy shocks documented in the VAR literature, and (ii) investigate the role of habit formation and costly capital adjustment in solving the persistence problem. We compare the impulse-response functions calculated using the estimated parameters with those obtained using two polar, counterfactual versions of the model. The first version assumes adjustment costs of capital but no habit formation. The second version assumes habit formation but no adjustment costs of capital.

Figure 3 plots the impulse responses of output, investment, consumption, labour, inflation, and the nominal interest rate to a 1 per cent money-supply shock. Following the shock, there is an increase in aggregate demand that causes output and consumption to increase. The consumption response is hump-shaped because, under habit formation, agents smooth both the level and the change of consumption. The output response is also hump-shaped, as in previous VAR literature. The peak of the output (consumption) response takes place after two (four) quarters, rather than the four to six quarters usually found in VAR models.

Figure 3 shows that the dynamic path of output is quite persistent. As a measure of the endogenous persistence of output generated by the model, we compute the proportion of the impact effect that persists beyond the average length of price contracts. Recall that the estimated probability of price changes implies an average duration of price contracts of 6.56 quarters. Thus, the measure of endogenous persistence is

$$\zeta \equiv \kappa(7)/\kappa(0),$$

where $\kappa(j)$ is the impulse-response coefficient at lag j.[10] In this case, $\zeta = 0.95$, meaning that 95 per cent of the initial effect of the monetary shock on output persists beyond

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[10] Of course, this measure of persistence applies only if $\kappa(0)$ is different from 0. This condition is satisfied in our case since output is a non-predetermined variable in our model.

seven quarters. This indicates that the estimated model produces a substantial amount of endogenous persistence.

Figure 3 also shows that investment and labour increase following a (positive) monetary shock. This result is due to the fact that aggregate demand is expected to increase in subsequent periods because prices adjust slowly. The nominal interest rate also rises after a positive monetary shock. Thus, the model does not generate a liquidity effect. A more detailed explanation of this result is presented below.

Figure 4 plots the impulse responses generated from a model with price stickiness and adjustment costs to capital but no habit formation. The parameter γ is set to zero and the remaining parameters are set to their ML estimates. In contrast to the previous model, output and consumption responses are not hump-shaped. Both variables jump immediately after the monetary shock and return gradually to their steady-state levels. The output response is less persistent than the one in Figure 3. Since $\zeta = 0.30$, this version of the model with no habit formation delivers only 30 per cent of endogenous persistence. This suggests that habit formation might be important in explaining the persistence of output in response to monetary shocks.

Figure 5 shows the impulse responses corresponding to a model with price stickiness and habit formation but no adjustment costs to capital. The parameter χ is set to zero and the remaining parameters are set to their ML estimates. A positive monetary shock triggers a large initial increase in output, investment, hours worked, and inflation, but the variables drop sharply in the following period and return close to their steady-state levels. The output response is caused by the fact that investment must increase to accommodate the upward shift in future demand. Because capital is free to adjust, however, all the required increase in investment takes place immediately after the shock. This version of the model does not generate any significant amount of endogenous persistence: $\zeta = 0.03$, meaning that only 3 per cent of the initial effect of the monetary shock persists after seven quarters. Kim (2000) and Dib and Phaneuf (2001) report a similar dynamic path of output using models with price stickiness only. This suggests that habit formation alone does not solve the persistence problem. Instead, habit formation plays the role of a catalyst that, combined with additional features, helps to spread out the effects of monetary shocks. In this model, the additional feature is the adjustment costs of capital.

The increase in the nominal interest rate following a positive monetary shock is larger in Figure 4 than in Figure 5. This result is consistent with Kim's (2000) finding that real rigidities help to generate a liquidity effect in DSGE models. As Figure 3 shows, however, adjustment costs to the capital stock are not enough to generate a liquidity effect in this model. The reason is that the estimated money-growth process is highly autocorrelated. Thus, after a positive money-supply shock, expected inflation increases by a magnitude that is larger in absolute value than the decrease in the real interest rate. As a result, the net effect of the money shock on the nominal interest rate is positive.

In summary, impulse-response analysis indicates that both habit formation and adjustment costs to capital are likely to be important features in a model that seeks to explain the persistent output response to monetary policy shocks. To further understand the relationship between endogenous output persistence and the parameters that control habit formation and capital adjustment costs, the persistence measure, ζ, is computed for different combinations of the parameters γ and χ. Figure 6 plots the resulting three-dimensional graph. In this figure, γ varies from 0 to 1, and χ varies between 0 and 100. The figure shows that habit formation increases the output persistence of monetary shocks only to the extent that capital adjustment costs are not in the neighbourhood of zero. The increase in persistence is bounded at fairly low levels unless γ is sufficiently large. Hence, habit formation and adjustment costs of capital interact in a non-linear way to increase the output persistence of monetary policy shocks. This finding parallels the one in Bergin and Feenstra (2000), where the non-linear interaction of materials inputs and translog preferences increases endogenous output persistence.

Although we are primarily concerned with the effects of monetary policy shocks, the estimated model generates predictions regarding the effect of technology and money-demand shocks. Figure 7 plots the response of output, investment, consumption, labour, inflation, and the nominal interest rate to a 1 per cent technology shock. Because prices are rigid, the aggregate supply curve is upward-sloping. A positive technology shock shifts the aggregate supply curve to the right. Consequently, output increases and prices decrease. The response of output and consumption is persistent and hump-shaped. Hours worked decrease in a persistent manner following a technology shock. The intuition of this result is as follows. After a positive technology shock, the firm is able to satisfy current demand with a lower level of inputs, so labour input will decrease on impact. Eventually, as demand increases and capital is adjusted, labour demand increases. A similar decline of labour in response to a technology shock is reported by Galí (1999) using a structural VAR, and by Dib and Phaneuf (2001) and Vigfusson (2002) using DSGE models.

Figure 8 plots the impulse response functions generated by a 1 per cent money-demand shock. Because money supply is unchanged and prices are rigid, this shock produces a downward shift of aggregate demand in current and subsequent periods. Consequently, output, consumption, labour, and investment decrease. As a result of habit formation, the response of consumption has an inverted hump shape, with a trough around three periods after the shock.

3.5 Variance decomposition

In this section, we study the relative importance of monetary shocks for the fluctuations of output, investment, consumption, labour, inflation, and the nominal interest rate. To that effect, we compute the fraction of the conditional variance of the forecasts at different horizons that is attributed to each of the model's shocks. This variance decomposition is plotted in Figure 9. As the horizon increases, the conditional variance of the forecast error of a given variable converges to the unconditional variance of that variable. Table 4 reports the decomposition of the unconditional variances. Recall that a money-supply shock is a shock to the growth rate of the money supply, while a money-demand shock is a shock to the preference parameter of money in the utility function. Several results are apparent from Figure 9 and Table 4. First, money-demand shocks play an important role in explaining the fluctuations of the nominal interest rate. At horizons of less than six quarters, money-demand shocks explain more than 50 per cent of the conditional variance of the nominal interest rate. In the long run, money-demand shocks explain roughly 45 per cent of the conditional variance of the nominal interest rate. Second, money-supply shocks explain most of the fluctuations of the rate of inflation at all horizons. Third, technology shocks explain most of the variation in hours worked at all horizons. Fourth, money-supply shocks account for the largest part of the conditional variance in forecasting investment in the short run. As the horizon increases, the contribution of technology shocks increases and that of money-supply shocks decreases, but, even in the long run, money-supply shocks explain half of the variance of investment. Fifth, money-supply shocks and technology shocks are equally important in explaining the conditional variance of consumption in the very short run. As the horizon increases, however, the contribution of technology shocks increases and that of money-supply shocks decreases. In the long run, 77.3 per cent of the variance of consumption is explained by technology shocks and only 21.6 per cent by money-supply shocks. Finally, money-supply shocks account for the largest part of the conditional variance in forecasting output in the short run (i.e., less than a year). At higher horizons, most of the conditional variance is due to technology shocks. In the long run, 27 per cent of the unconditional variance of output is attributed to money-supply shocks, 2 per cent to money-demand shocks, and 71 per cent to technology shocks.

4. Conclusion

This paper has constructed a DSGE model with sticky prices, habit formation, and costly capital adjustment that accounts for the persistent and hump-shaped response of output to monetary shocks. Although habit formation, by itself, does not solve the persistence problem, it interacts non-linearly with costly capital adjustment to increase the internal propagation mechanism of the model.

The model was estimated by the ML method using U.S. data on output, the real money stock, and the nominal interest rate. Econometric results indicate that U.S. prices are fixed, on average, for six-and-a-half quarters. Although the peak of the output response takes place after two quarters (that is, less than the four to six quarters usually found in VAR models), up to 95 per cent of the initial effect of a monetary shock on output persists beyond the average duration of price contracts. Variance decomposition indicates that money growth explains more than 50 per cent of the (conditional) output variability at horizons of less than one year. In the long run, money growth explains only 27.1 per cent of the unconditional output variability, while 71.4 per cent is explained by technology shocks.

The DSGE fits U.S. output and real money stock better than an unrestricted VAR and does only slightly worse for the nominal interest rate. The model also tracks well the behaviour of consumption and investment, but it does poorly in explaining the U.S. inflation rate. This is partly the result of the forward-looking pricing rule and the prediction that the real marginal cost is much more volatile than the data. The inclusion of additional features would allow DSGE models to capture other stylized facts, such as inflation persistence and, perhaps, the liquidity effect. In future work, we intend to extend this model to allow for a backward-looking component in the price rule that arises directly from first principles.

Appendix A: The Log-Linearized Model

In this appendix, variables without time subscripts denote steady-state values, and the circumflex denotes percentage deviation from steady state. For example, $\hat{x}_t = (x_t - x)/x$ is the percentage deviation of investment from its steady state at time t. Linearizing (2) and the first-order conditions (6)-(10) yields

$$\begin{aligned}
\hat{k}_{t+1} &= (1-\delta)\hat{k}_t + \delta\hat{x}_t, \\
E_t\hat{c}_{t+1} &= \left(\frac{\beta\gamma\left(\gamma\left(1-\eta_1\right)+1\right)-\eta_1}{\beta\gamma\left(1-\eta_1\right)}\right)\hat{c}_t - \left(\frac{1}{\beta}\right)\hat{c}_{t-1} + \left(\frac{\beta\gamma-1}{\beta\gamma\left(1-\eta_1\right)}\right)\hat{\lambda}_t, \\
\hat{R}_t &= \eta_2\left(\frac{\pi-\beta}{\beta}\right)\left(\hat{\pi}_t - \hat{m}_{t-1} - \hat{\mu}_t\right) - \left(\frac{\pi-\beta}{\beta}\right)\hat{\lambda}_t + \left(\frac{\pi-\beta}{\beta}\right)\hat{b}_t, \\
\hat{n}_t &= \left(\frac{1-n}{n\eta_3}\right)\hat{\lambda}_t + \left(\frac{1-n}{n\eta_3}\right)\hat{w}_t, \\
E_t\hat{q}_{t+1} &= \left(\frac{1}{\beta q}\right)(\hat{\lambda}_t - E_t\hat{\lambda}_{t+1}) + \left(\frac{\chi(1+\beta\delta)}{\beta q}\right)\hat{k}_{t+1} - \left(\frac{\chi}{\beta q}\right)\hat{k}_t - \left(\frac{\chi\delta}{q}\right)\hat{x}_{t+1}, \\
E_t\hat{\lambda}_{t+1} &= \hat{\lambda}_t + E_t\hat{\pi}_{t+1} - \hat{R}_t.
\end{aligned}$$

The production function (14) and first-order conditions for cost minimization by the intermediate-good producer (equations (15) and (16)) become

$$\begin{aligned}
\hat{y}_t &= \alpha\hat{k}_t + (1-\alpha)\,\hat{n}_t + \hat{z}_t, \\
\hat{w}_t &= \hat{\phi}_t + \hat{y}_t - \hat{n}_t, \\
\hat{\phi}_t &= \hat{q}_t - \hat{y}_t + \hat{k}_t.
\end{aligned}$$

The linearized versions of equations (17) and (18), together, imply

$$E_t\hat{\pi}_{t+1} = \frac{1}{\beta}\hat{\pi}_t - \left(\frac{(1-\varphi)\,(1-\varphi\beta)}{\varphi\beta}\right)\hat{\phi}_t. \tag{A.1}$$

Linearizing the equation that defines money growth (20) and the market-clearing condition (25) yields

$$\begin{aligned}
\hat{m}_t &= \hat{m}_{t-1} - \hat{\pi}_t + \hat{\mu}_t, \\
\hat{x}_t &= \left(\frac{y}{\delta k}\right)\hat{y}_t - \left(\frac{c}{\delta k}\right)\hat{c}_t.
\end{aligned}$$

Finally, the stochastic processes of the shocks (equations (21)-(23)) are linearized as

$$\begin{aligned}
\hat{z}_{t+1} &= \rho^z\hat{z}_t + \epsilon_{z,t}, \\
\hat{\mu}_{t+1} &= \rho^\mu\hat{\mu}_t + \epsilon_{\mu,t}, \\
\hat{b}_{t+1} &= \rho^b\hat{b}_t + \epsilon_{b,t}.
\end{aligned}$$

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Table 1. Maximum-Likelihood Estimates

Description	Parameter	Estimate	Asymptotic S.E.	Robust S.E.
Habit-formation parameter	γ	0.982	0.016	0.021
Probability of no price change	φ	0.847	0.037	0.023
Adjustment cost	χ	85.188	20.728	29.402
Preference parameter	η_2	3.089	0.827	1.462
Preference parameter	η_3	1.591	3.530	3.732
AR coefficient of technology shock	ρ^z	0.867	0.055	0.055
AR coefficient of money-supply shock	ρ^μ	0.879	0.035	0.053
AR coefficient of money-demand shock	ρ^b	0.924	0.019	0.029
S.D. of technology shock	σ_{ϵ_z}	0.040	0.027	0.032
S.D. of money-supply shock	σ_{ϵ_μ}	0.007	0.002	0.003
S.D. of money-demand shock	σ_{ϵ_b}	0.077	0.005	0.008

Notes: S.D. and S.E. are standard deviation and standard error, respectively. The restrictions imposed on the parameters are γ, $\varphi \in (0,1)$, ρ^z, ρ^μ, $\rho^b \in (-1,1)$, and η_2, η_3, χ, σ_{ϵ_z}, σ_{ϵ_μ}, $\sigma_{\epsilon_b} \in (0,\infty)$.

Table 2. Goodness of Fit

Variable	MSE ($\times 10^{-5}$)	
	DSGE model	Unrestricted VAR(1)
Output	6.073	6.562
Real money stock	4.599	7.841
Nominal interest rate	0.416	0.381

Notes: MSE is mean squared error.

Table 3. Test for Serial Correlation and Neglected ARCH

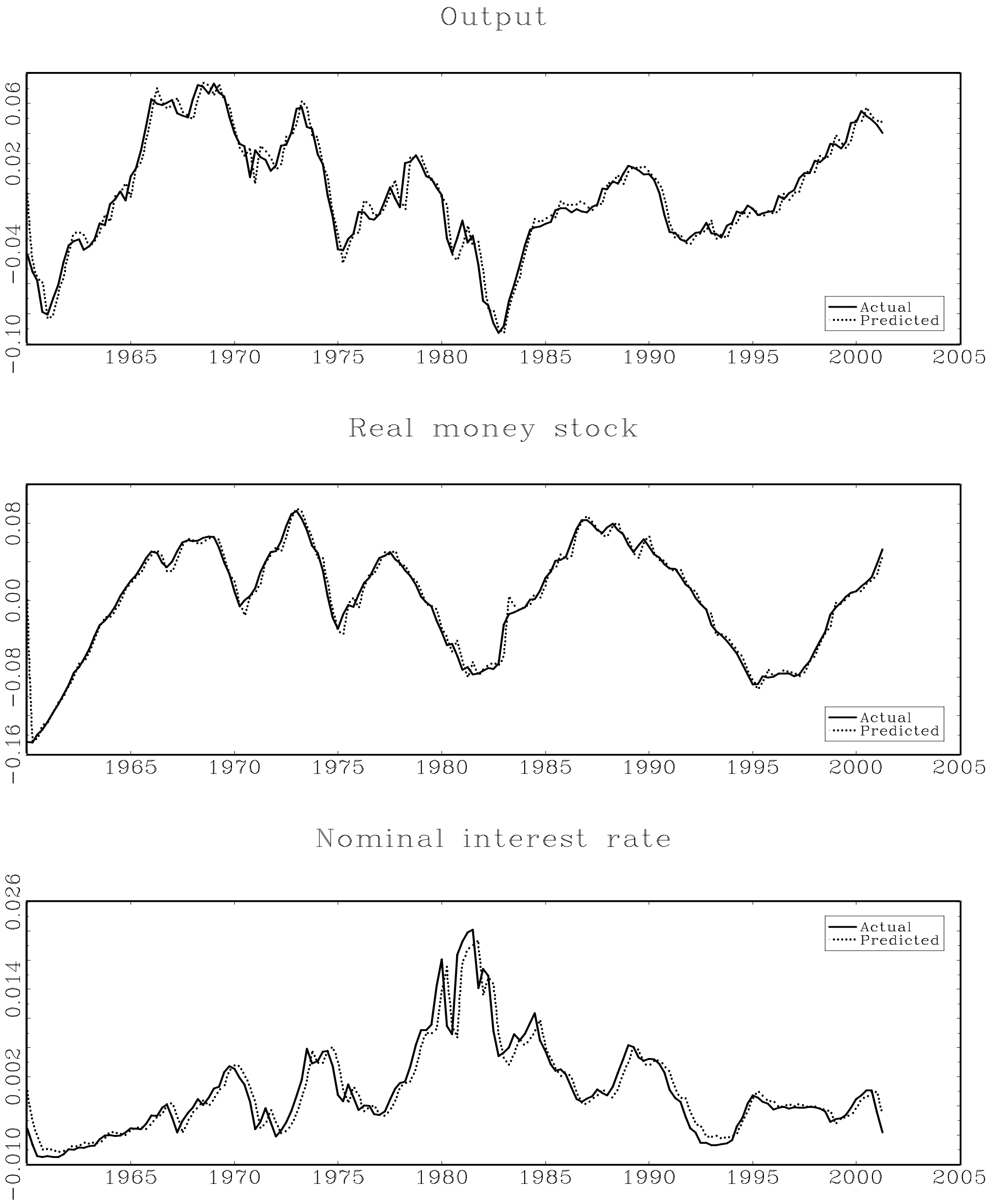
	Output	Real money stock	Nominal interest rate
Panel A. Test for Serial Correlation			
Durbin-Watson	2.15	1.98	1.50
Portmanteau			
One autocorrelation	1.11	0.002	9.38*
Up to two autocorrelations	1.11	1.86	14.27*
Up to three autocorrelations	2.21	4.18	17.32*
Panel B. LM Test for Neglected ARCH			
Number of squared lags			
One	0.90	3.62	1.82
Two	1.48	3.63	26.77*
Three	1.63	3.74	28.88*

Notes: The superscript * denotes the rejection of the null hypothesis that the parameter is zero at the 5 per cent significance level.

Table 4. Unconditional Variance Decomposition

	Fraction of the unconditional variance due to		
Variable	Technology shocks	Money-supply shocks	Money-demand shocks
Output	0.714	0.271	0.015
Investment	0.469	0.493	0.038
Consumption	0.773	0.216	0.011
Labour	0.872	0.120	0.008
Inflation rate	0.221	0.756	0.023
Nominal interest rate	0.163	0.389	0.448

Notes: The money-supply shock is a shock to the growth rate of the money supply. The money-demand shock is a shock to the preference parameter of money in the utility function.


Output

Real money stock

Nominal interest rate

Figure 1: Actual and predicted values of variables in the measurement equation

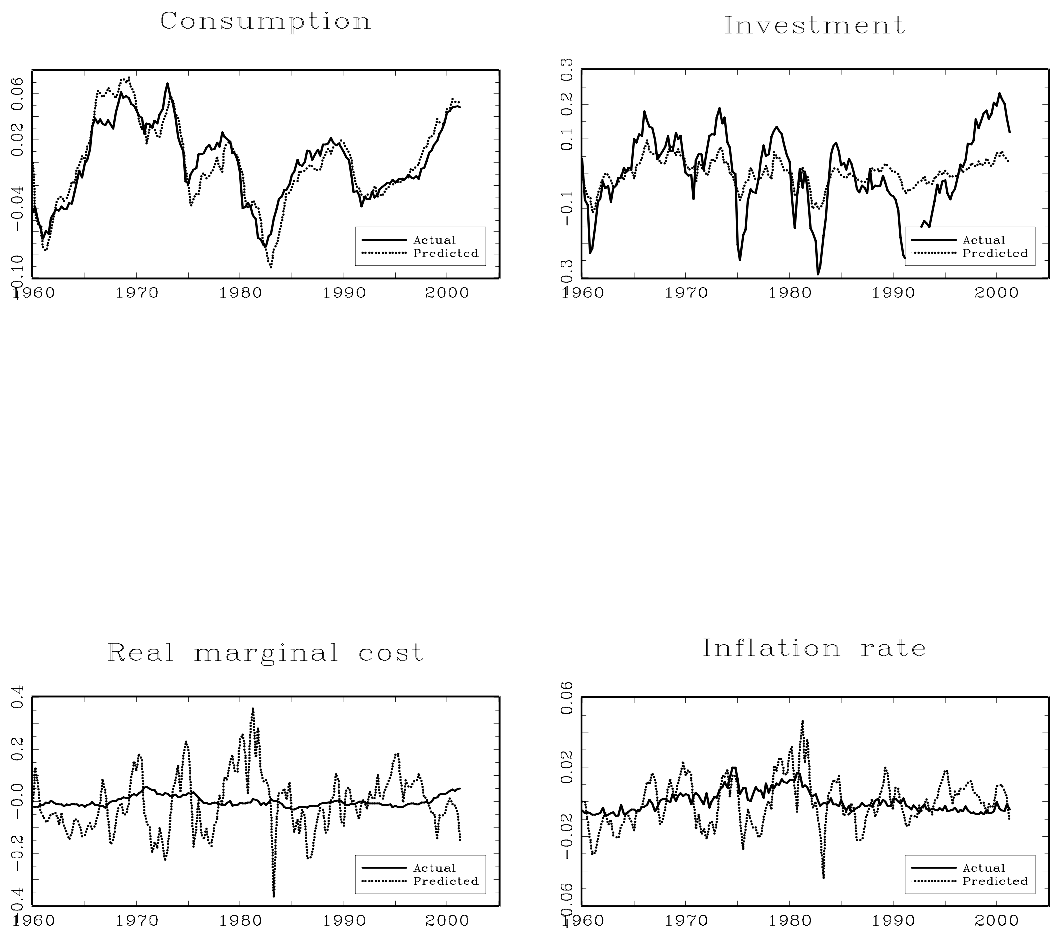


Figure 2: Actual and predicted values of other model variables

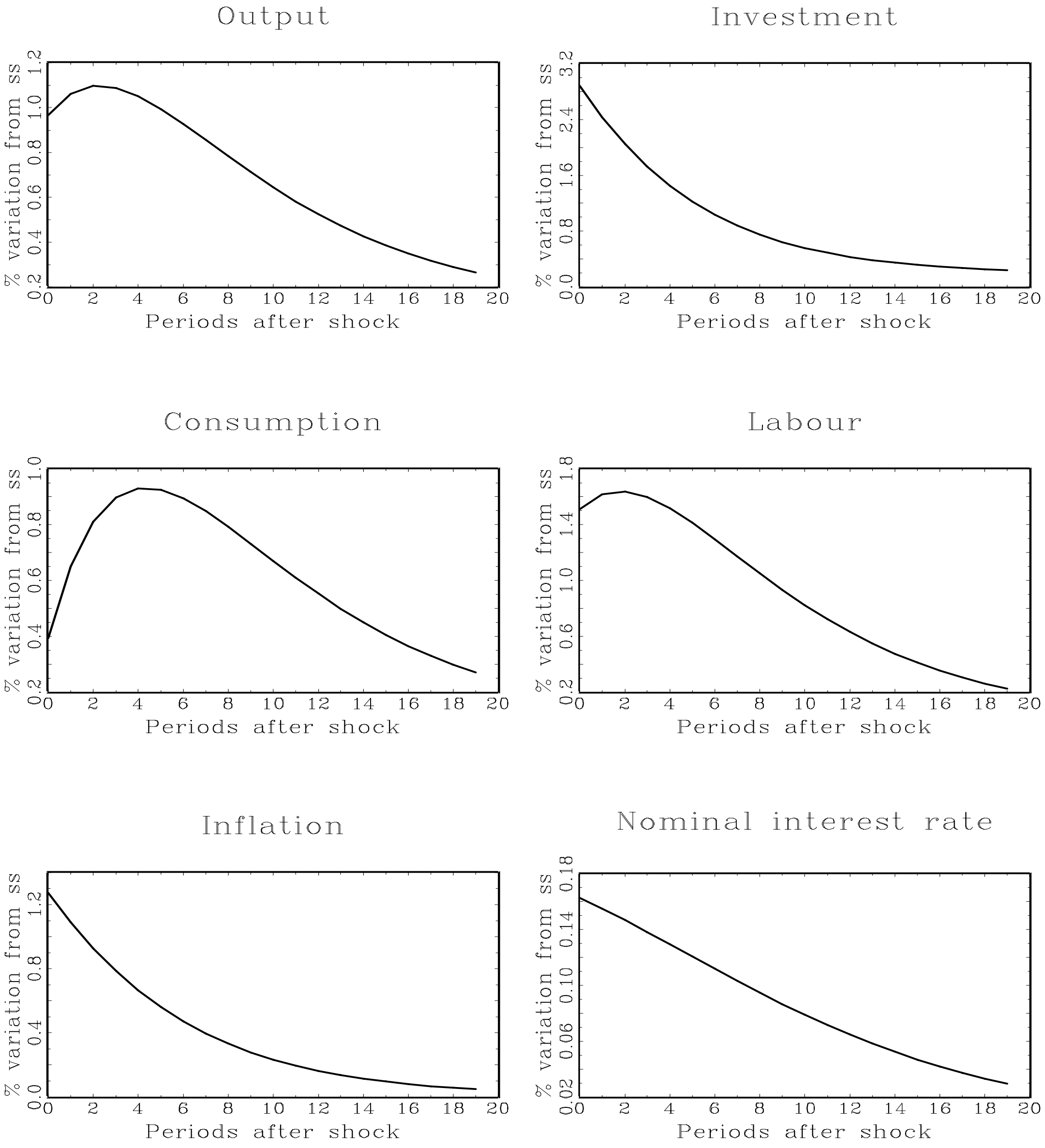


Figure 3: Impulse responses to a 1 per cent money-supply shock

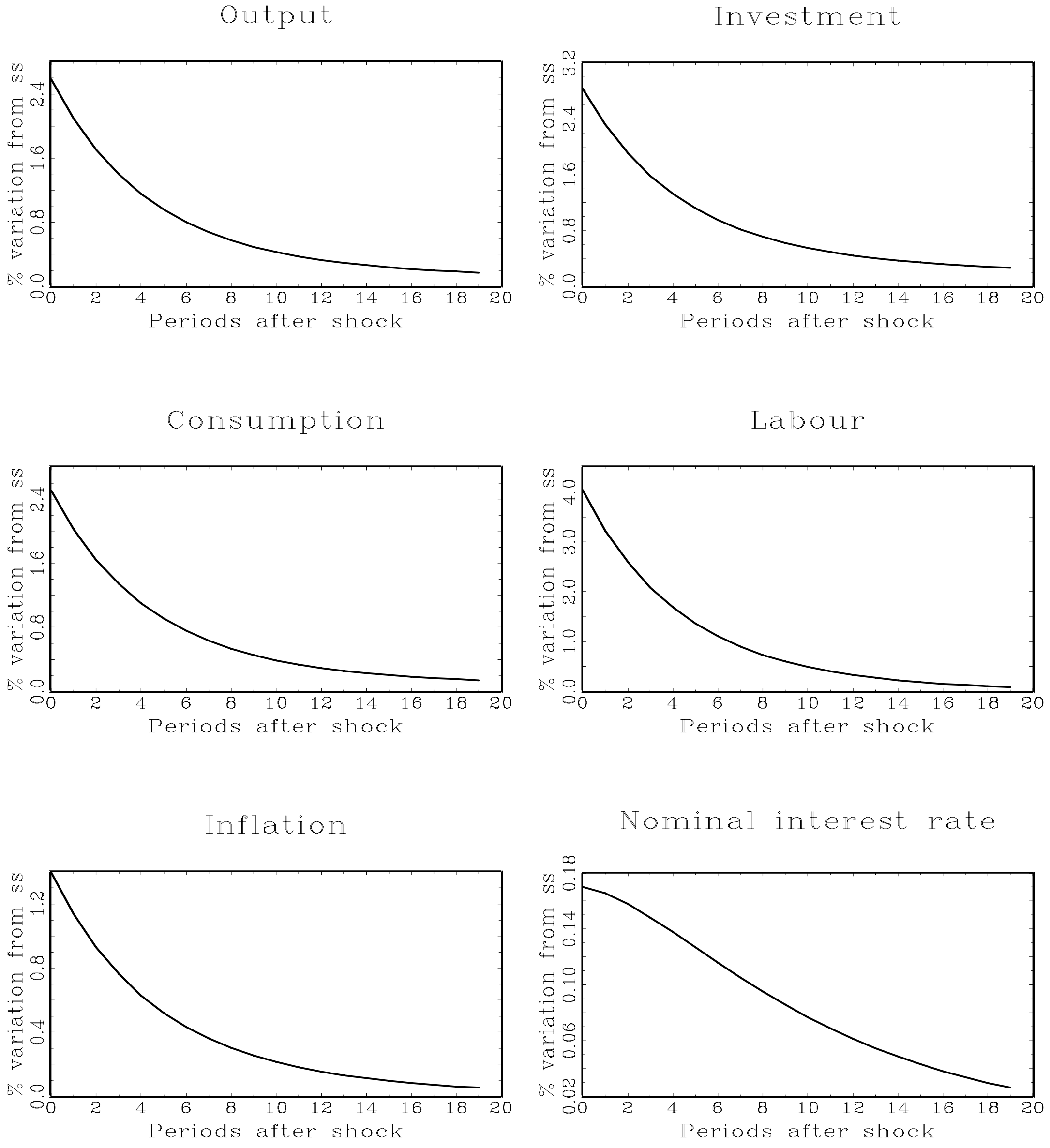


Figure 4: Impulse responses to a 1 per cent money-supply shock ($\gamma = 0$)

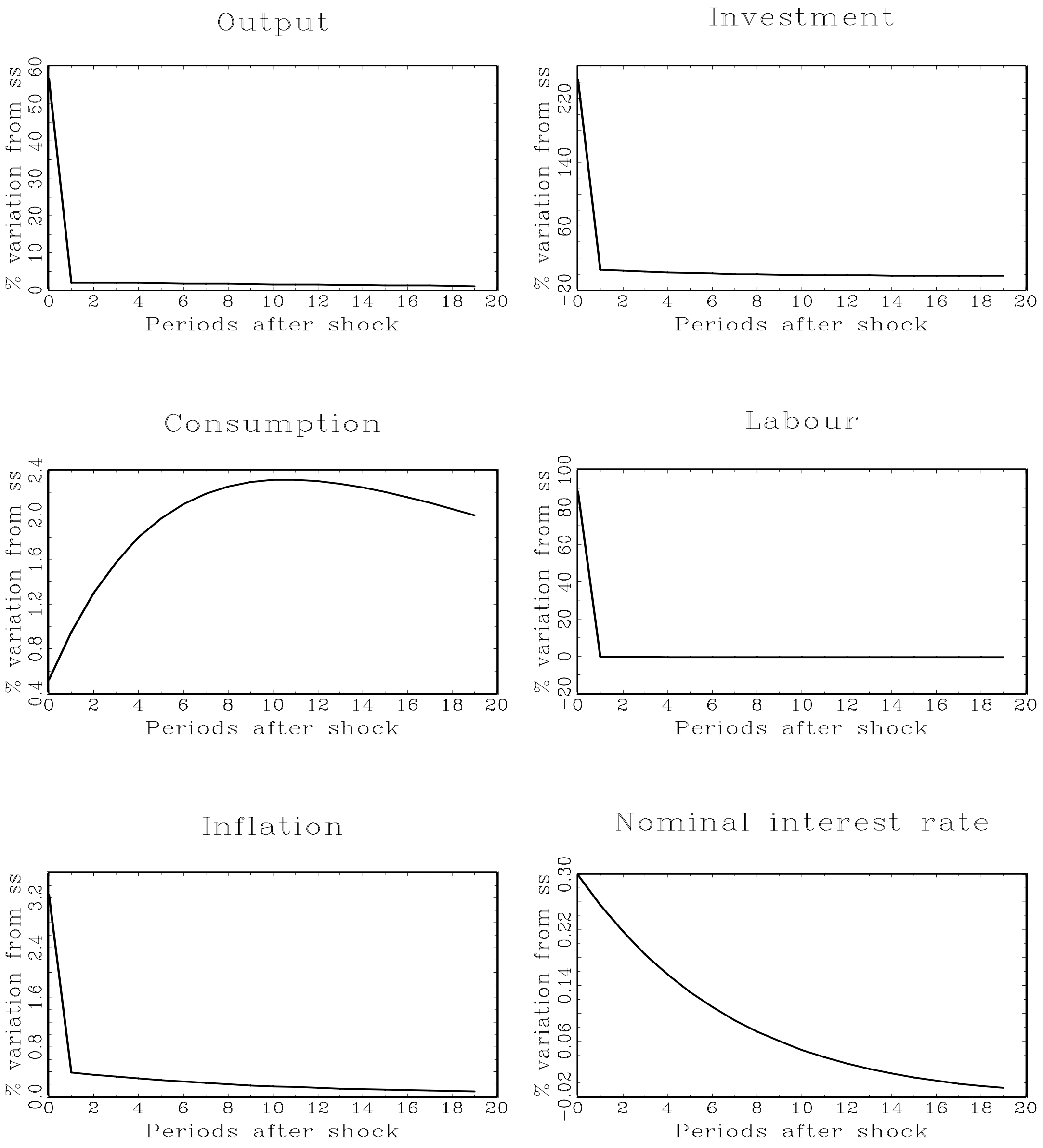


Figure 5: Impulse responses to a 1 per cent money-supply shock ($\chi = 0$)

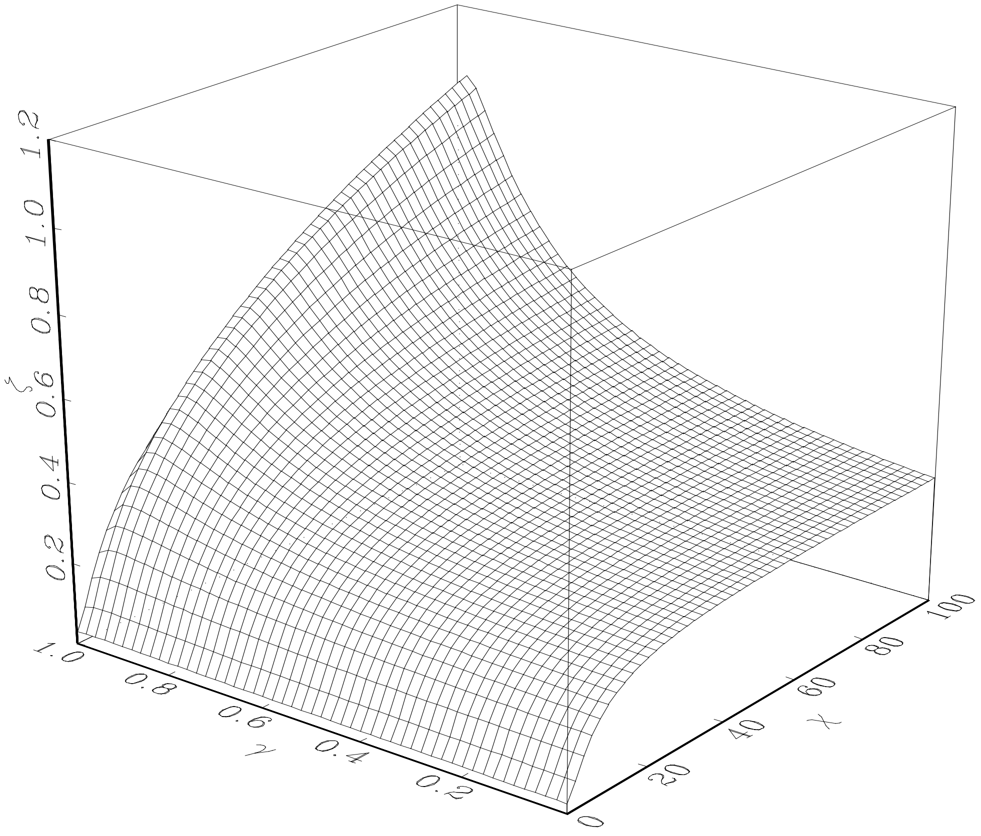


Figure 6: Endogenous persistence as a function of γ and χ

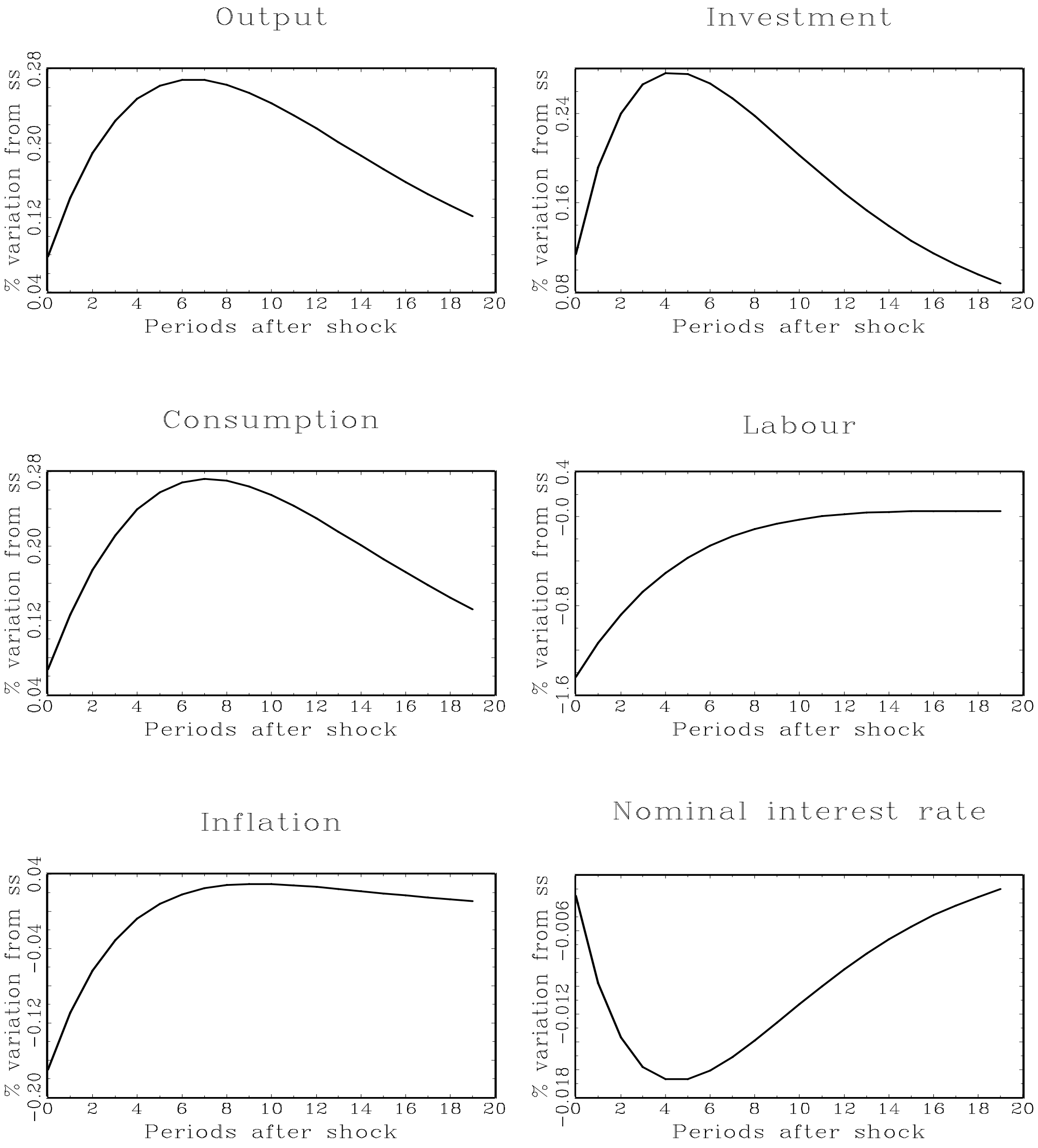


Figure 7: Impulse responses to a 1 per cent technology shock

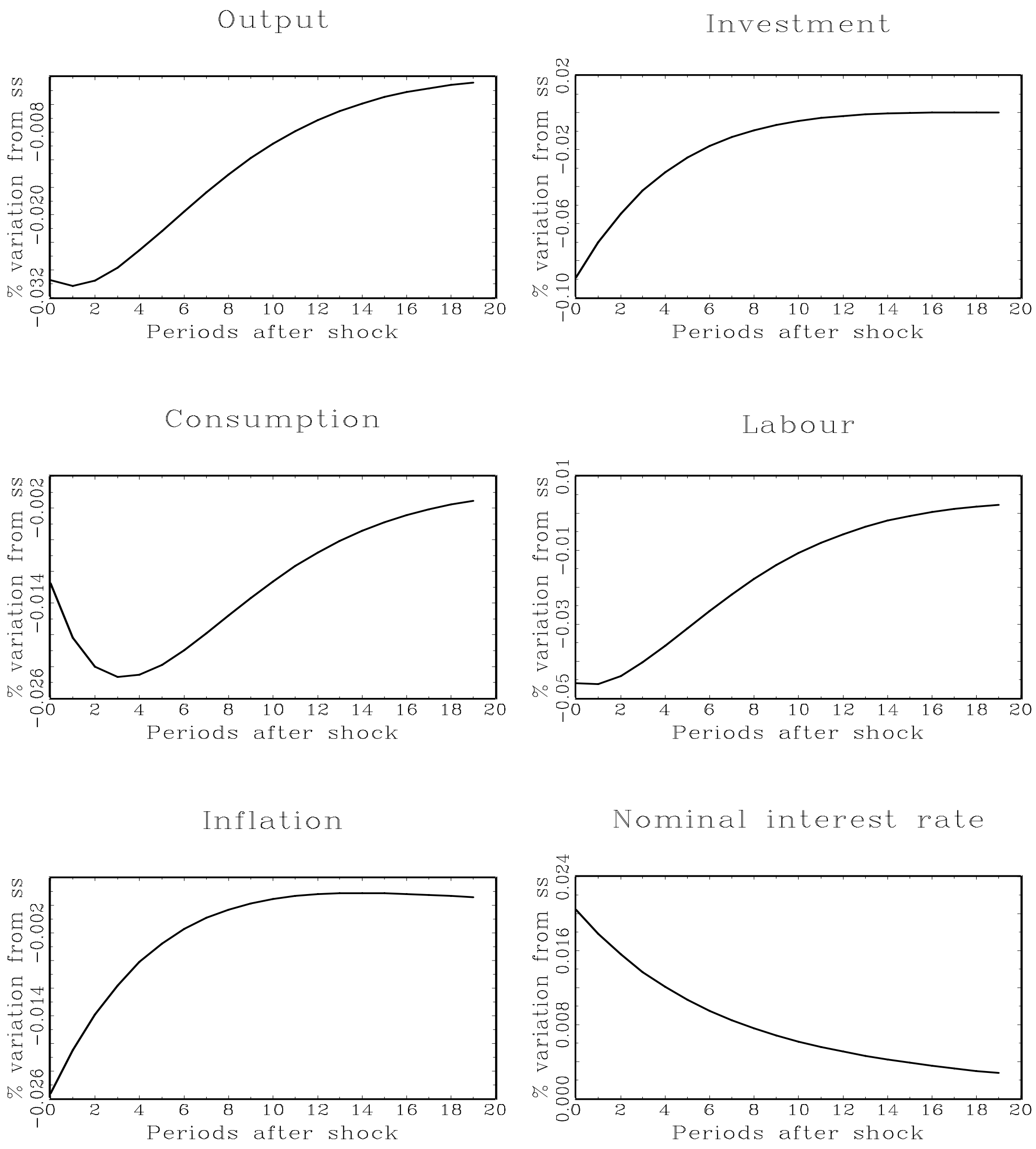


Figure 8: Impulse responses to a 1 per cent money-demand shock

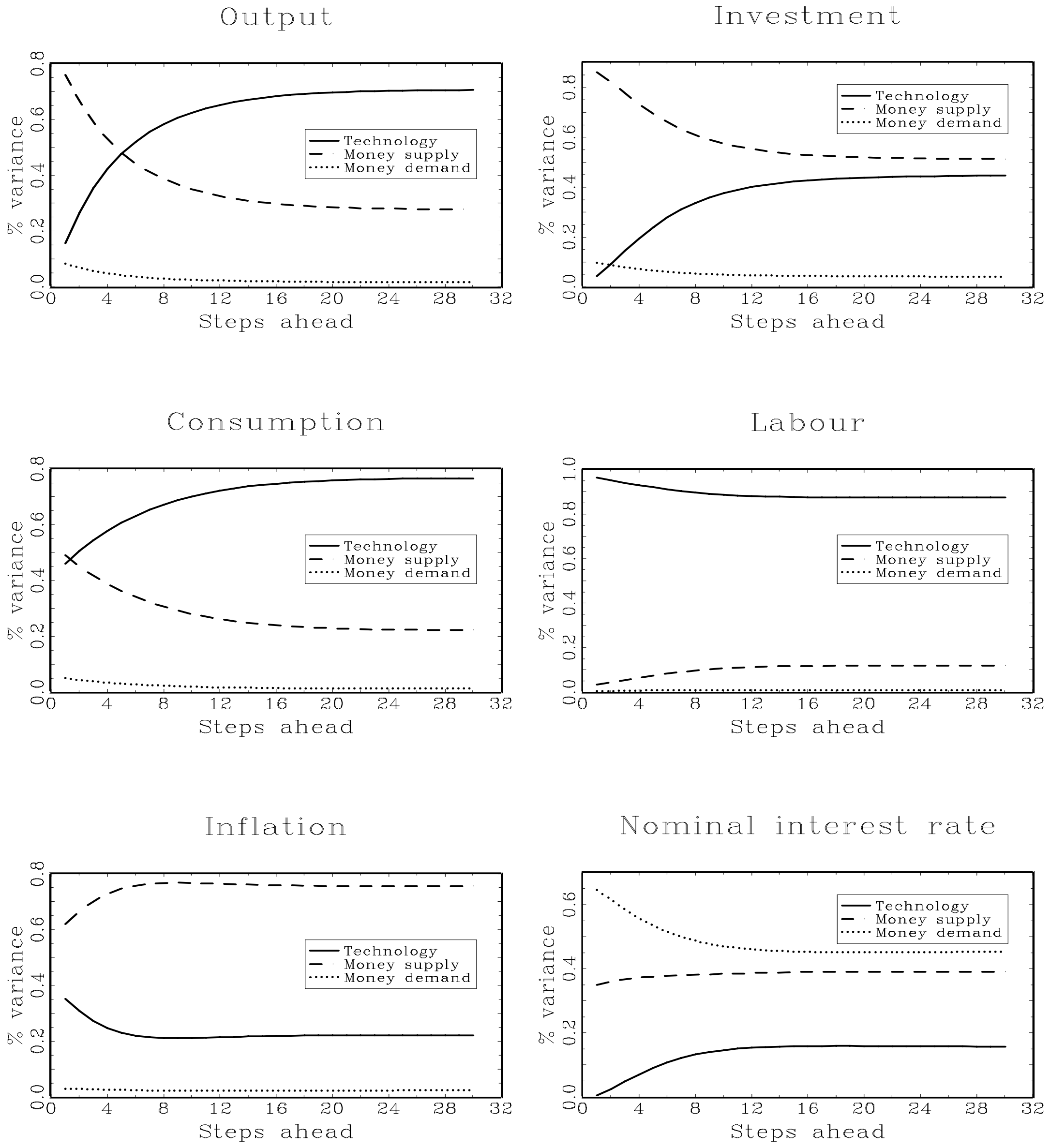


Figure 9: Variance decomposition

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