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A Practical Guide to Swap Curve Construction

by

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Abstract

The swap market has enjoyed tremendous growth in the last decade. With government issues shrinking in supply and increased price volatilities, the swap term structure has emerged as an alternative pricing, benchmark, and hedging mechanism to the government term structure. This paper outlines the advantages of using the swap curve, and provides a detailed methodology for deriving the swap term structure for marking to market fixed-income products. The paper concludes with a discussion of the proposed swap term structure derivation technique.

JEL classification: G12, G15
Bank classification: Asset pricing; International financial markets

Résumé

Le marché des swaps a connu un essor prodigieux durant les années 1990. Avec la réduction de l'offre des titres d'État et la volatilité accrue de leurs prix, la structure à terme des taux de swaps offre une solution de rechange à celle des emprunts d'État, établissant ainsi une nouvelle courbe de référence pour l'évaluation des titres à revenu fixe et la couverture des risques. L'auteur décrit les avantages que présente l'utilisation de la courbe des taux de swaps et expose en détail une méthode permettant de déterminer celle-ci pour fins d'évaluation des prix au marché des titres à revenu fixe. L'auteur conclut sur une discussion de la méthode qu'il propose.

Classification JEL : G12, G15
Classification de la Banque : Évaluation des actifs; Marchés financiers internationaux

1 Introduction

The swap market has enjoyed substantial growth in size and turnover in the past few years.[1] Swaps are increasingly used by governments, financial intermediaries, corporations, and investors for hedging, arbitrage, and to a lesser extent speculation. Swaps are also used as benchmarks for evaluating the performance of other fixed-income markets, and as reference rates for forecasting.

Swaps offer an operationally efficient and flexible means of transforming cash-flow streams. The swap market has little or no government regulation, and provides a high degree of privacy. The swap market's liquidity, depth, and high correlation with fixed-income products, other than plain vanilla government bonds, render its derived term structure a fundamental pricing mechanism for these products and a relevant benchmark for measuring the relative value of different fixed-income products.[2]

The role of the swap term structure as a relevant benchmark for pricing and hedging purposes is expected to increase as government fiscal situations improve. An improved fiscal situation reduces the size of government debt programs, in effect decreasing the liquidity and efficiency of government debt markets.[3] Furthermore, the financial markets crisis in the fall of 1998 reinforced the "flight to quality" phenomenon, where spreads between governments' issues and other fixed-income securities widened substantially under adverse market conditions, thereby calling into question the role of the government market as a relevant benchmark for non-government issues. The swap term structure again emerges as a potential substitute.

With the increased importance of the swap market, practitioners recognize the importance of a consistent and computationally efficient swap term structure for marking to market financial transactions; marking to market is

[1] According to the International Swaps and Derivatives Association (ISDA) 1999 year-end market survey, the combined total of outstanding interest rate swaps, currency swaps, and interest rate options stood at US$58.265 trillion in notional principal at 31 December 1999 compared with US$50.997 trillion one year earlier. The year-to-year increase was 14.3 per cent compared with 1998's record 76 per cent.

[2] See Fleming (2000) for correlations of swap rates and other fixed-income rates for the U.S. market.

[3] The U.S. Congressional Budget Office (1999) projects budget surpluses for the next ten years, rising from US$161 billion in fiscal year 2000 to US$413 billion in fiscal year 2009. In addition, public debt is expected to fall from US$3.6 trillion on 30 September 1999 to US$0.9 trillion at the end of fiscal year 2009.

the practice of valuing an instrument to reflect current market conditions. While the general framework for the construction of the swap term structure is widely known, the derivation details are vague and not well documented. This paper attempts to bridge this gap by carefully covering all angles of the swap term structure derivation procedure while leaving enough flexibility to adjust the constructed term structure to the specific micro requirements and constraints of each primary swap market.

Marking to market fixed-income portfolios is instrumental for trading, accounting, performance valuation, and satisfying interinstitution collaterization requirements. The current methodology in capital markets for marking to market fixed-income securities is to estimate and discount future cash flows using rates derived from the appropriate term structure. The swap term structure is increasingly used as the foundation for deriving relative term structures and as a benchmark for pricing and hedging.

Section 2 describes the motivation for using the swap term structure as a benchmark for pricing and hedging fixed-income securities. Section 3 gives a detailed description of a swap term structure derivation technique designed to mark to market fixed-income products. Section 4 discusses different aspects of the derived swap term structure.

2 The Swap Curve Advantage

The swap market offers a variety of advantages. It has almost no government regulations, making it more comparable across different markets; some sovereign issues offer a variety of tax benefits to domestic and/or foreign investors, making government curve comparative analysis across countries latently inconsistent. The swap market is an increasingly liquid market, with narrow bid-ask spreads and a wide spectrum of maturities. The supply of swaps is solely dependent on the number of counterparties wishing to transact at any given time. No position in an underlying asset is required, avoiding any potential repo "specials" effects.[4] Given the liquidity and large size of the swap market, new swaps with standard maturities are issued daily,

[4]A repo transaction is the borrowing of money by selling securities to a counterparty and buying them back at a later date at a pre-agreed price. The repo rate is the interest rate embedded in a repurchase agreement. Repo "specials" carry different rates, thereby introducing inconsistencies to the derived term structure, such as the government term structure. See Jordan and Jordan (1997) and Duffie (1996a).

keeping a constant forecast horizon, mitigating any potential coupon effects; bonds with high coupons tend to have lower yields to maturity than bonds with low coupons (see Malz 1998). The fungibility of swaps also prevents swaps with similar cash flows from trading at substantially different rates, contributing to market efficiency.

Swaps have similar credit-risk properties across countries, making them more comparable than the government term structure. Government debt is considered risk-free; however, governments entail different credit-risk qualities across countries. Credit risk is embedded in the swap curve as swaps are based on the balance sheet of the banking sector (see Appendix 1 for inputs). In addition, swap rates are highly correlated with yields on other fixed-income securities, even under adverse market conditions, making swaps latently a better hedging vehicle than government issues. Other fixed-income securities include agency debt, corporate debt, and mortgage-backed securities.

Swap prices are frequently quoted as a spread over government issues, therefore serving as a rough indicator of credit risk of the banking sector. A swap spread is the difference between the fixed rate on an interest rate swap contract and the yield on a government bond with an equivalent tenor. The fixed swap rate is the rate that equates the present value of the swap to zero. Quoting the swap curve as a spread over the government curve can be unreliable, as there is a maturity mismatch and coupon effect between the different quoted government notes and their corresponding swap issues. Swap rates should be quoted directly off the swap market. Quoting the swap rate as a spread over government issues is common mainly in Anglo-Saxon swap markets.

The most prominent impediment to swap market liquidity is swap counterparty credit exposure, which is balance-sheet intensive, in that it is a bilateral contract. The risk is the potential loss to a counterparty of the present value of a swap position if a swap party defaults. Therefore, parties to a swap transaction must be confident in the credit quality of their swap counterparty. A variety of credit-enhancement mechanisms have been developed to somewhat reduce this potential credit exposure. Some of the mechanisms include the use of credit-enhanced subsidiaries, credit derivatives, and an automatic swap unwind clause triggered by a credit event.

In summary, the swap term structure offers several advantages over government curves, and is a robust tool for pricing and hedging fixed-income products. Correlations among governments and other fixed-income products have declined, making the swap term structure a more efficient hedging and pricing vehicle (Theobald and Singh 2000). With the supply of government issues declining and high correlations of credit spreads to swap spreads, the swap term structure is a potential alternative to the government term structure as a benchmark for measuring the relative value of different debt classes. Section 3 presents a methodology for deriving the swap term structure.

3 Swap Curve Construction

The swap curve depicts the relationship between the term structure and swap rates. The swap curve consists of observed market interest rates, derived from market instruments that represent the most liquid and dominant instruments for their respective time horizons, bootstrapped and combined using an interpolation algorithm. This section describes a complete methodology for the construction of the swap term structure.

3.1 Curve inputs

In deriving the swap curve, the inputs should cover the complete term structure (i.e., short-, middle-, and long-term parts). The inputs should be observable, liquid, and with similar credit properties. Using an interpolation methodology, the inputs should form a complete, consistent, and smooth yield curve that closely tracks observed market data. Once the complete swap term structure is derived, an instrument is marked to market by extracting the appropriate rates off the derived curve.

The technique for constructing the swap term structure, as constructed by market participants for marking to market purposes, divides the curve into three term buckets. The short end of the swap term structure is derived using interbank deposit rates. The middle area of the swap curve is derived from either forward rate agreements (FRAs) or interest rate futures contracts. The latter requires a convexity adjustment[5] to render it equivalent to FRAs. The long end of the term structure is constructed using swap par rates derived from the swap market.

A combination of the different interest rates forms the basis for the swap curve term structure. For currencies where the future or forward market is

[5]The adjustment required to convert a futures interest rate to a forward interest rate.

illiquid, inefficient, or non-existent for certain tenors,[6] it is customary to use longer-term interbank deposit rates and rely more heavily on interpolation. On the other hand, for currencies such as the U.S. dollar, where an efficient liquid futures or forward market exists, for longer-term maturities it is customary to use futures contracts or FRAs with longer maturities (i.e., beyond two years out to five years).

The inputs used to construct the term structure are currency-dependent. Some currencies offer more liquid and deeper markets than others; see Appendix 1. A swap term structure should be constructed given these micro constraints.

3.2 Deriving the swap curve

To derive the swap term structure, observed market interest rates combined with interpolation techniques are used; also, dates are constructed using the applicable business-day convention. Swaps are frequently constructed using the modified following business-day convention, where the cash flow occurs on the next business day unless that day falls in a different month. In that case, the cash flow occurs on the immediately preceding business day to keep payment dates in the same month (ISDA 1999). The swap curve yield calculation convention frequently differs by currency. Table 1 lists the different payment frequencies, compounding frequencies, and day count conventions, as applicable to each currency-specific interest rate type.

[6] Time to maturity of financial instrument.

Table 1

Yield calculation conventions by currency

Currency/rate	Payment freq	Compounding freq	Day count convention
CAD cash rates			ACT/365
CAD swap rates	S/A	S/A	ACT/365
EUR cash rates			ACT/360
EUR swap rates	A	A	30/360
JPY cash rates			ACT/360
JPY swap rates	S/A	S/A	ACT/365
GBP cash rates			ACT/365
GBP swap rates	S/A	S/A	ACT/365
USD cash rates			ACT/360
USD swap rates	S/A	S/A	30/360

3.2.1 *The short end of the swap curve*

The short end of the swap curve, out to three months, is based on the overnight, one-month, two-month, and three-month deposit rates. The short-end deposit rates are inherently zero-coupon rates and need only be converted to the base currency swap rate compounding frequency and day count convention. The following equation is solved to compute the continuously compounded zero swap rate (r_c):

$$r_c = \frac{t_y}{t_m} \times \ln[1 + \frac{r_d}{\frac{t_y}{t_m}}] \tag{1}$$

where n represents the number of years, r_d represents the observed market deposit rate, t_m represents the number of days to maturity, and t_y represents the number of days in a year as specified according to the day count convention used. Continuously compounded interest rates are used for consistency with other parts of this paper.

3.2.2 *The middle area of the swap curve*

The middle area of the swap curve up to two years is derived from either FRA rates or interest rate futures contracts. FRAs are preferable, as they carry a fixed time horizon to settlement and settle at maturity, whereas futures contracts have a fixed settlement date and are marked to market daily. FRAs for most currencies, however, are not observable or suffer from lack of liquidity. On the other hand, futures contracts are exchange traded, rendering them more uniform, liquid, and transparent. Extracting forward rates from futures rates requires a convexity adjustment. It is an adjustment for the difference in convexity characteristics of futures contracts and forward rates. Most interest rate futures have zero convexity, a fixed payoff per basis point change, regardless of the level of underlying interest rates, whereas FRAs are convex instruments. The convexity bias is positively correlated to the futures contract maturity, and is of the magnitude of one to two basis points for maturities around one year, gradually increasing with term to maturity.

A long position in FRAs or swaps and a short position in futures has net positive convexity. The short futures position has a positive payoff when interest rates rise and lower losses when interest rates fall, as they can be refinanced at a lower rate. This mark to market positive effect of futures contracts creates a bias in favour of short sellers of futures contracts. This bias must be removed from futures contracts prices to derive an unbiased estimator of the equivalent forward rates.

Convexity adjustment estimation Estimating the convexity adjustment requires an estimation of the future path of interest rates up to the future contract maturity. Convexity adjustments for several futures markets are provided by brokers or from market data vendors. An alternative methodology is to use the Hull-White (1990) term structure model to estimate the convexity bias. In the Hull-White model, the continuously compounded forward rate, lasting between times t_1 and t_2 (denominated in years from current date), equals the continuously compounded future rate less the following convexity adjustment:

$$\left(\frac{\frac{1-e^{-a(t_2-t_1)}}{a}}{t_2-t_1}\right)\left[\left(\frac{\frac{1-e^{-a(t_2-t_1)}}{a}}{t_2-t_1}\right)\left(1-e^{-2at_1}\right)+2a\frac{\frac{1-e^{-a(t_1)}}{a}}{t_1}\right]\frac{\sigma^2}{4a} \tag{2}$$

where σ is the standard deviation of the change in short-term interest rates expressed annually, and a is the mean reversion rate.

Mean reversion rate estimation Convexity bias estimation requires an estimate of the mean reversion rate (a), and the standard deviation (σ) of the change in short-term interest rates expressed annually. Historical data can be used to estimate the mean reversion rate. A typical range of values for the mean reversion rate is 0.001 for negligible effects to 0.1, which could have material effects. For simplicity, a constant default value for mean reversion speed could be assumed. For example, Bloomberg assumes a constant mean reversion rate of 0.03.

We assume that the short-term interest rates follow the following Vasicek (1977) discount bond prices stochastic process:

$$dr_t = a\left[\theta - r_t\right]dt + \sigma dz_t \tag{3}$$

where r_t is the short-term interest rate at time t, and dz_t is the increment of a standard Wiener process. Parameters α and β specify the drift and mean reversion of the process.

To estimate the Vasicek continuous stochastic time model, the model must be discretized. We discretized and estimated the continuous time model as follows:

$$\Delta r_t = \varphi + \delta r_{t-1} + \varepsilon_t \tag{4}$$

where,

$$\varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2) \tag{5}$$

The parameter δ is used to estimate the negative of the mean reversion rate, $-a$, where I_{t-1} is the information set at time $t-1$.

Interest rates volatility estimation There are several alternative methodologies for estimating the standard deviation (σ) of the change in short-term interest rates. Two derivation methodologies are explored next.

The first methodology flows from the mean reversion estimation process. It estimates the conditional standard deviation of short-term interest rates using the $GARCH(1,1)$ model:

$$\sigma_t^2 = \alpha + \beta\varepsilon_{t-1}^2 + \gamma\sigma_{t-1}^2 \tag{6}$$

The conditional density of Δr_t is:

$$f(r_t|I_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(\frac{-\varepsilon_t^2}{2\sigma_t^2}\right) \tag{7}$$

The log-likelihood function, where N represents the total number of observations,

$$L = \sum_{t=1}^{N} \log f(r_t|I_{t-1}) \tag{8}$$

is then maximized numerically with respect to the population parameters. Maximizing the log-likelikood function gives estimates of α, β, and γ. The annualized standard deviation equals $\sigma_t\sqrt{252}$, assuming there are 252 trading days in a year.

The second methodology uses the implied volatility from interest rate caps that correspond to the appropriate time horizon. An interest rate cap comprises q caplets, where q is the number of reset dates. Each caplet corresponds to the rate at time t_k and provides payoff at time t_{k+1}. An interest rate cap provides insurance against adverse upward movements in floating rate obligations during a future period. An interest rate caplet provides the cap holder with the following payoff:

$$n\delta_k \max(R_k - R_x, 0) \tag{9}$$

where n denotes caplet notional, R_x denotes the cap rate, R_k is the reset rate at time t, and $\delta_k = t_{k+1} - t_k$. As an interest rate caplet market value is observable, assuming R_k is lognormal, the implied interest rate caplet volatility σ_k can be computed using the following extension to the Black-Scholes model (Hull 1999):

$$n\delta_k P(0, t_{k+1}) \left[F_k N(d_1) - R_x N(d_2)\right] \tag{10}$$

where

$$d_1 = \frac{\ln(F_k/R_x) + \sigma_k^2 t_k/2}{\sigma_k\sqrt{t_k}}$$

$$d_2 = d_1 - \sigma_k \sqrt{t_k}$$

$P(0,t)$ is the spot price of a zero-coupon bond paying \$1 at time T. F_k denotes the forward rate for the period between t_k and t_{k+1}. $N(x)$ is the cumulative probability distribution function, where $x \sim N(0,1)$. The volatility σ_k is solved for the period between t_k and t_{k+1}.

The estimated conditional standard deviation or the implied volatility, for the period between t_k and t_{k+1}, and the mean reversion rate are used in combination with the Hull-White model to adjust for the interest rates futures convexity bias. Futures rates with maturities from the six-month to the two-year time horizon are frequently used. For currencies with highly liquid interest rates futures markets, interest rate futures could be used out to five years.

Futures prices Futures prices are quoted as ($100-$future interest rate$\times 100$). The quarterly compounded future interest rates adjusted for convexity are converted to continuously compounded zero rates as follows.

Convert the quarterly compounded future rate to the continuously compounded future rate using equation (1), where t_m equals the future's accrual period (difference in days between two consecutive futures contracts).

The continuously compounded future rate is then converted to a continuously compounded zero rate using the following transformation:

$$r_2 = \frac{r_f(t_2 - t_1) + r_1 t_1}{t_2} \tag{11}$$

where r_f is the continuously compounded future rate for the period between t_1 and t_2, and r_1 and r_2 are the continuously compounded zero rates for maturities t_1 and t_2, respectively.

3.2.3 *The long end of the swap curve*

The long end of the swap curve out to ten years is derived directly from observable coupon swap rates. These are generic plain vanilla interest rate swaps with fixed rates exchanged for floating interest rates. The fixed swap rates are quoted as par rates and are usually compounded semi-annually (see Table 1). The bootstrap method is used to derive zero-coupon interest rates from the swap par rates. Starting from the first swap rate, given all the continuously compounded zero rates for the coupon cash flows prior to maturity, the continuously compounded zero rate for the term of the swap is bootstrapped as follows:

$$r_T = -\frac{\ln\left[\frac{100-\sum_{i=m}^{T-m}\left(\frac{c}{m}e^{-r_i \times t_i}\right)}{100+\frac{c}{m}}\right]}{T} \tag{12}$$

where m is the swap payment frequency per annum, c is the coupon per annum, which is equal to the observed swap rate times the swap notional, and r_i represents the continuously compounded zero rate for time t_i. The bootstrapped interest rate, r_T, is the continuously compounded zero rate for time T.

Progressing recursively along the observed swap rates interpolating between market observations as required forms the complete long end of the swap curve.

3.3 Interpolation algorithm

There is no single correct way to link deposit, futures, and swap interest rates to construct the complete swap term structure; however, several fundamental characteristics and conventions should be followed, to ensure yield curve validity. The derived yield curve should be consistent and smooth, and should closely track observed market data points. However, over-smoothing the yield curve might cause the elimination of valuable market pricing information. This is the main criticism against the use of more advanced interpolation yield curve modeling techniques for pricing derivatives, such as the Nelson and Siegel (1987) and Svensson (1994) functions. These functions fit the market data very loosely, which is appropriate for extracting expectations or comparative analysis across countries, but is not appropriate for market pricing. The market convention has been to use several interpolation techniques to generate a complete term structure that closely mimics the observed market data for marking to market purposes. The most prevalent algorithms of interpolation used in practice to create a swap term structure include linear interpolation and cubic splines. For other non-linear curve modelling techniques see Satyajit (1998).

3.3.1 *Piecewise linear interpolation*

All observed market data points are connected by a straight line to form a complete term structure. The value of a new data point is assigned according to its position along a straight line between observed market data points. Linear interpolation is simple to implement and closely tracks observed market interest rates. However, it tends to produce kinks around transition areas where the yield curve is changing slope. Therefore, linear interpolation is inappropriate for modeling yield curves that change slope frequently and exhibit significant term structure curvature. As illustrated in Appendix 2, Figures 2.1 to 2.5, the swap term structure is not characterized by a continuously changing slope nor does it exhibit significant curvature.

Constructing piecewise linear interpolation Piecewise linear interpolation can be presented in a closed form, which simplifies the interpolation process.

$$R(t) = R(t_i) + [\frac{(t - t_i)}{(t_{i+1} - t_i)}] \times [R(t_{i+1}) - R(t_i)] \tag{13}$$

Here, i is the market observation index with time to maturity of t_i, and $R(t)$ represents the interest rate corresponding to maturity t, where $t_i \leq t \leq t_{i+1}$.

3.3.2 *Piecewise cubic spline interpolation*

Use of polynomial functions that pass through the observed market data points create a fitted smooth yield curve that does not oscillate wildly between observations. It is possible to either use a single high-order polynomial of degree $n-1$ (n is the number of observations), or piece together low-order polynomials (e.g., quadratic, cubic). The advantage of using a number of lower-order polynomials (splines) is that the extra degrees of freedom can be used to impose additional constraints to ensure smoothness and prevent wild oscillatory patterns between observations. The piecewise cubic spline technique goes through all observed data points and creates by definition the smoothest curve that fits the observations and avoids kinks.

Constructing a piecewise cubic spline To construct a set of cubic splines, let the function $R_i(t)$ denote the cubic polynomial associated with the t segment $[t_i, t_{i+1}]$:

$$R_i(t) = a_i(t - t_i)^3 + b_i(t - t_i)^2 + c_i(t - t_i) + r_i \tag{14}$$

where n is the number of market observations, r_i represents market observation (knot point) i, and t_i represents time to maturity of market observation i.

There are n market observations, $n - 1$ splines, and three coefficients per spline. Overall, there are $3n - 3$ unknown coefficients. The coefficients of the cubic spline function defined over the interval $[t, T]$ can be obtained by imposing the following constraints:

$$a_i(t_{i+1} - t_i)^3 + b_i(t_{i+1} - t_i)^2 + c_i(t_{i+1} - t_i) = r_{i+1} - r_i$$

$$3a_{i-1}(t_i - t_{i-1})^2 + 2b_{i-1}(t_i - t_{i-1}) + c_{i-1} - c_i = 0$$

$$6a_{i-1}(t_i - t_{i-1}) + 2b_{i-1} - 2b_i = 0$$

$$b_1 = 0$$

$$6a_{n-1}(t_n - t_{n-1}) + 2b_{n-1} = 0$$

The first set of $n - 1$ constraints require that the spline function join perfectly at the knot points. The second and third set of $2n - 2$ constraints require that first and second derivative constraints match adjacent splines. Finally, the last 2 constraints are end point constraints that set the derivative equal to zero at both ends.

The linear algebraic system consists of $3n - 3$ equations and $3n - 3$ unknowns that can be solved to produce the optimal piecewise cubic spline. Press et al. (1998) describe a routine for cubic spline interpolation.

3.4 Consolidation

The complete term structure is formed by joining the different parts of the swap term structure together using the chosen interpolation methodology. The end result is a complete swap term structure that is a fundamental tool in marking to market fixed-income securities.

The construction of the swap term structure is not a uniform practice. The substitutable inputs, overlapping instrument maturity dates, inconsistencies between different inputs, different alternatives for transition points between different sections of the term structure, and variety of instruments and derivation techniques all combine to form a variety of plausible swap term structures. The most prominent problems arise around the transition areas between inputs as especially exhibited in Appendix 2, Figure 2.4. The transition areas especially around the two-year mark lack smoothness and an oscillatory pattern is observable. Several possible solutions include using different term structures for different applications and adjustments to the set of rates utilized. In general, institutions tend to adopt their own approaches to these issues. However, over-adjustment and over-smoothing of the term structure can be counterproductive. By eliminating variation, valuable pricing information embedded in the term structure might be "smoothed" away.

The swap term structures for major currencies are presented in Appendix 2. In general, both linear interpolation and piecewise cubic spline derivation techniques generate similar zero and forward swap term structures. However, after zooming in on relatively unstable areas of the term structure, one can detect the better fit of piecewise cubic spline over linear interpolation in preserving a term structure curvature and smoothness. Nevertheless, cubic splines may produce inconsistent or implausible forward term structures such as exhibited at the long end of Appendix 2, Figure 2.4. As these are estimates of the swap term structure, it is impossible to determine precisely which estimate serves as a better benchmark. The swap zero and forward term structures for major currencies are much smoother and consistent than those for the less-prevalent currencies. This attribute characterizes more liquid developed and deeper markets.

4 Conclusions

The swap term structure is a pivotal element in pricing fixed-income products, measuring the relative value of debt classes, and measuring interest rate expectations. The swap term structure also offers many advantages over the government term structure. This paper has outlined a methodology for deriving the swap term structure. The derived zero term structure is used to price to market financial instruments by estimating and discounting their future cash flows to derive their present value. The different time buckets of the swap term structure are extracted from different market rates and instruments. The variety of plausible extraction and interpolation techniques and data availability problems prevent the derivation of a completely uniform efficient yield curve. The outlined model carefully preserves variations in market observations, thereby maintaining important pricing information. However, linear interpolation can introduce inaccuracies when there is significant curvature in the term structure, or sparse or noisy data. Cubic spline interpolation, on the other hand, may produce inconsistent or implausible forward term structures. The most problematic area of the term structure tends to be the transition area between time buckets. Nevertheless, linear interpolation and cubic splines are the most prevalent yield curve generation techniques used in the marketplace for marking to market purposes. To get mark-to-market prices that are consistent with the marketplace, institutions use the specified inputs and derivation techniques. However, an institution may develop more robust term structure derivation techniques for identifying mispriced securities, such as a multiple factor model. Duffie (1996b) summarizes term structure models.

With the size of federal debt shrinking, the weight of government issues in bond indices decreasing, spread volatilities and valuation relative to government term structures becoming less important, and federal agencies status as government-sponsored enterprises under review (DeStefano 2000), the swap term structure is becoming more influential. Given the above trends in financial markets, the importance of the swap term structure as a benchmark for pricing fixed-income products and for comparative equity valuation[7] is expected to increase.

[7] Equities are valued against bonds through the reverse price to earnings ratio to government yield. With the decreasing role of government bonds as a benchmark for fixed-income debt and their increased price volatility and scarcity, the swap term structure, which shows greater stability, is an ideal substitute.

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Appendix 1

The following are the most commonly used inputs to derive a complete swap curve term structure for different currencies:

Canadian Dollar (CAD)

- Interbank overnight financing rate.
- Banker's acceptance out to three months.
- BAX futures out to two years.
- Swap rates out to ten years.

European Dollar (EUR)

- Interbank overnight financing rate.
- Interbank deposit rates out to three months.
- LIFFE three-month EURIBOR futures or Euro LIBOR futures out to two years.
- Swap rates out to ten years.

Japanese Yen (JPY)

- Interbank overnight financing rate.
- Interbank deposit rates out to three months.
- CME three-month Yen LIBOR futures out to two years.
- Swap rates out to ten years.

United Kingdom Sterling (GBP)

- Interbank overnight financing rate.
- Interbank deposit rates out to three months.
- LIFFE three-month Sterling LIBOR futues out two years.
- Swap rates out to ten years.

US Dollar (USD)

- Interbank overnight financing rate.
- LIBOR fixings out to three months.
- Eurodollar futures or FRAs out to five years.
- Swap rates out to ten years (frequently quoted as government bond yield for chosen benchmark adjusted for swap spreads).

Appendix 2

Figure 2.1

USD swap zero curve (continuously compounded) as of 14 April 2000

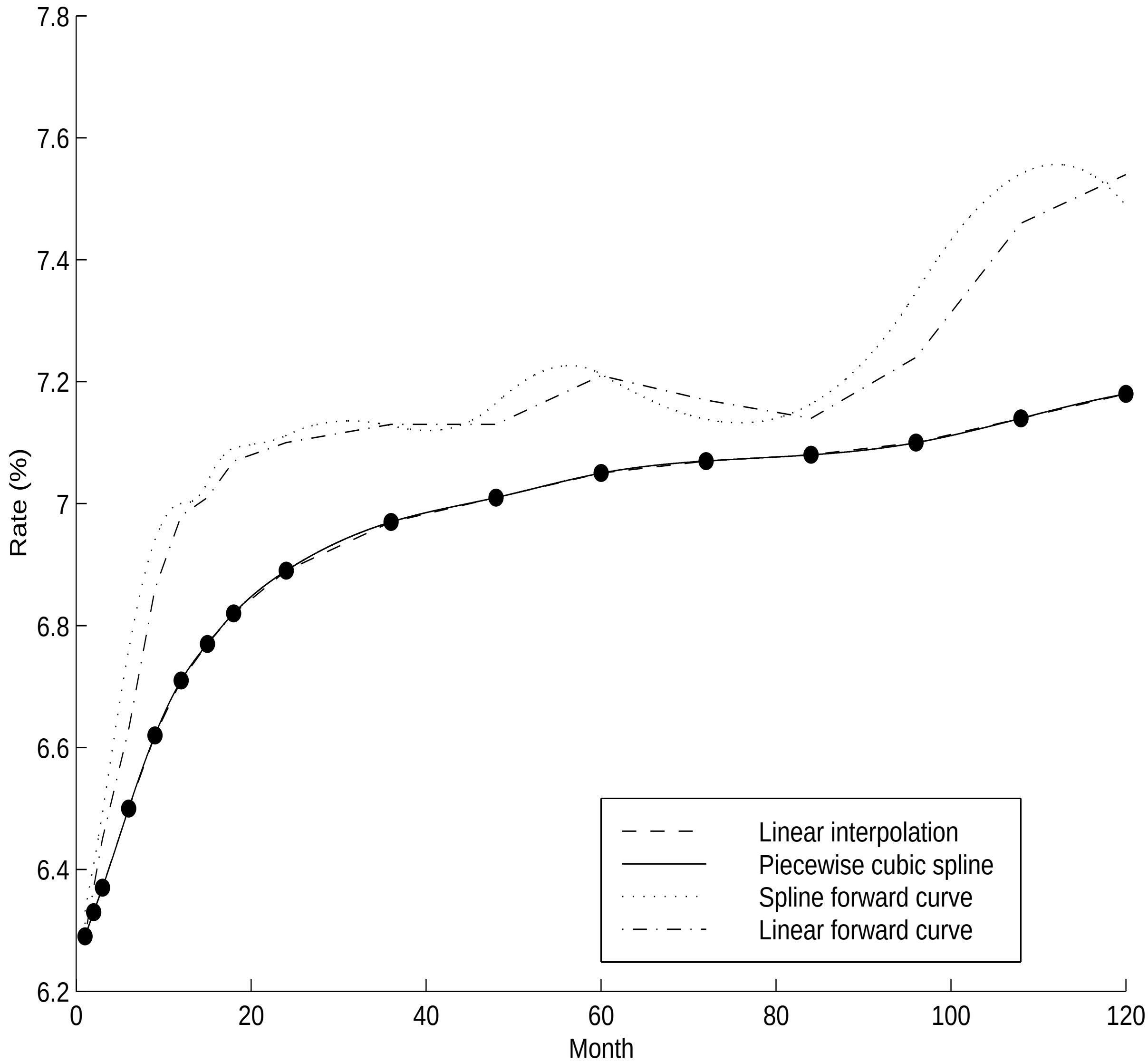


Figure 2.2

JPY swap zero curve (continuously compounded) as of 14 April 2000

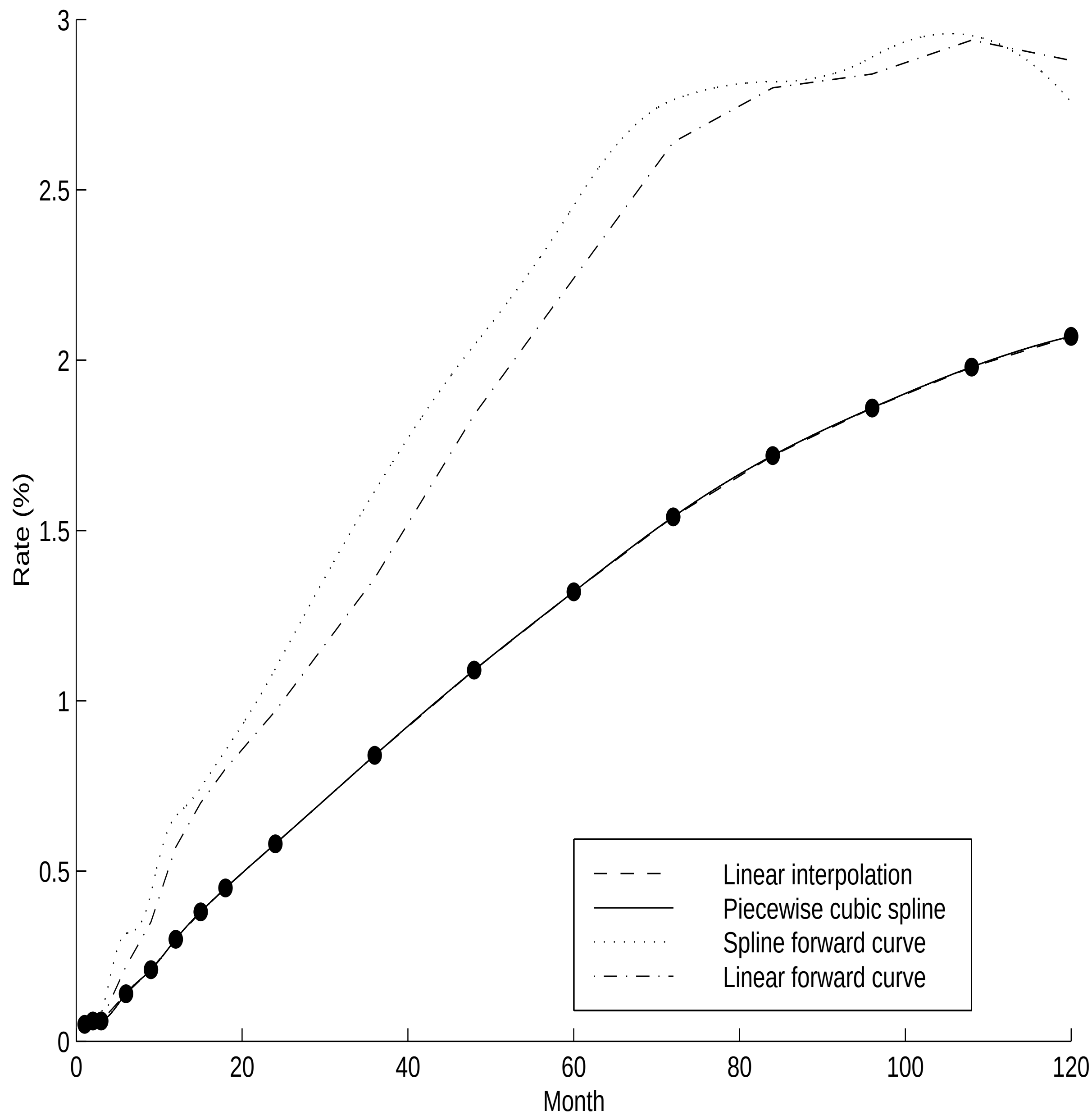


Figure 2.3

EUR swap zero curve (continuously compounded) as of 14 April 2000

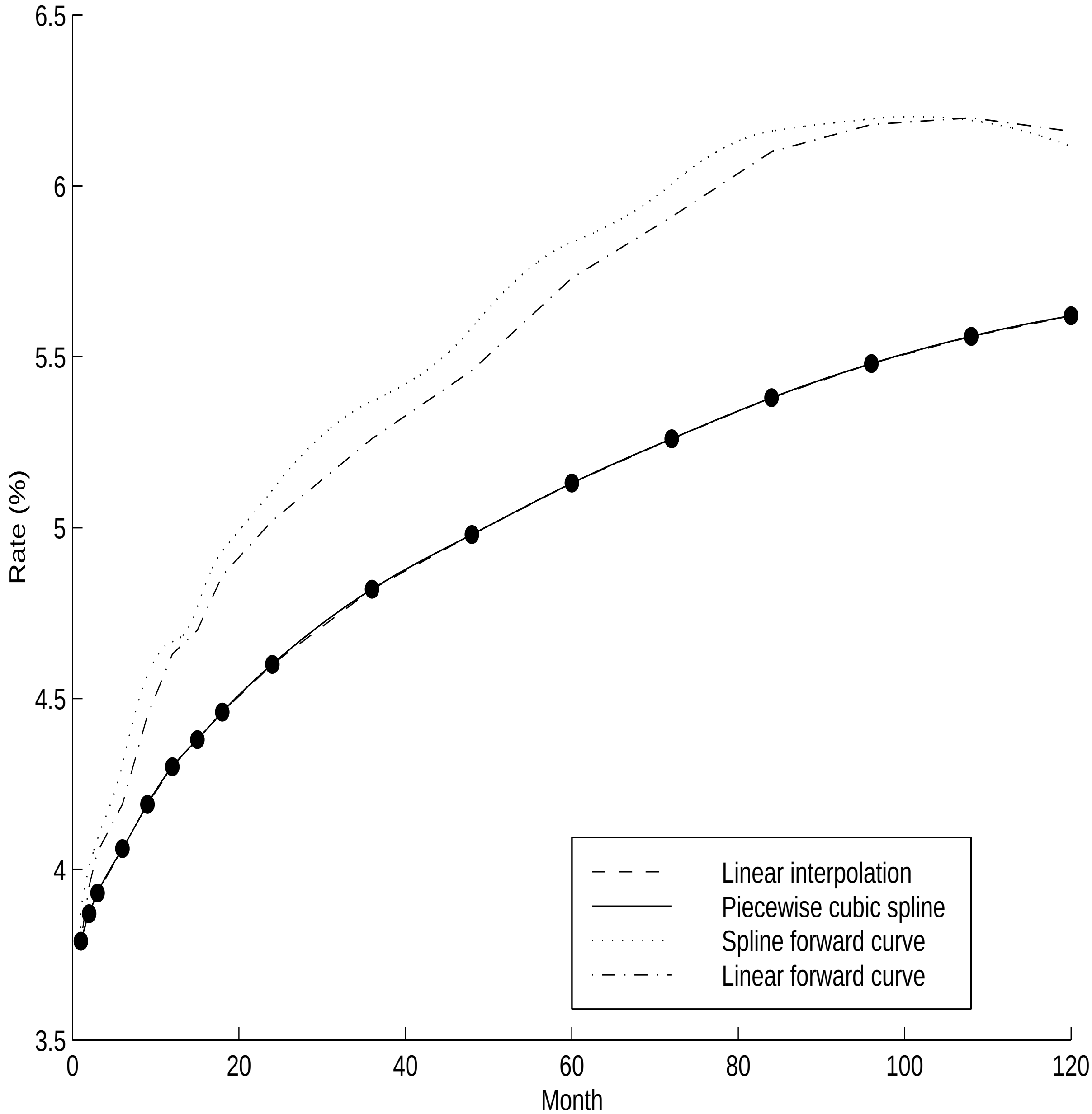


Figure 2.4

CAD swap zero curve (continuously compounded) as of 14 April 2000

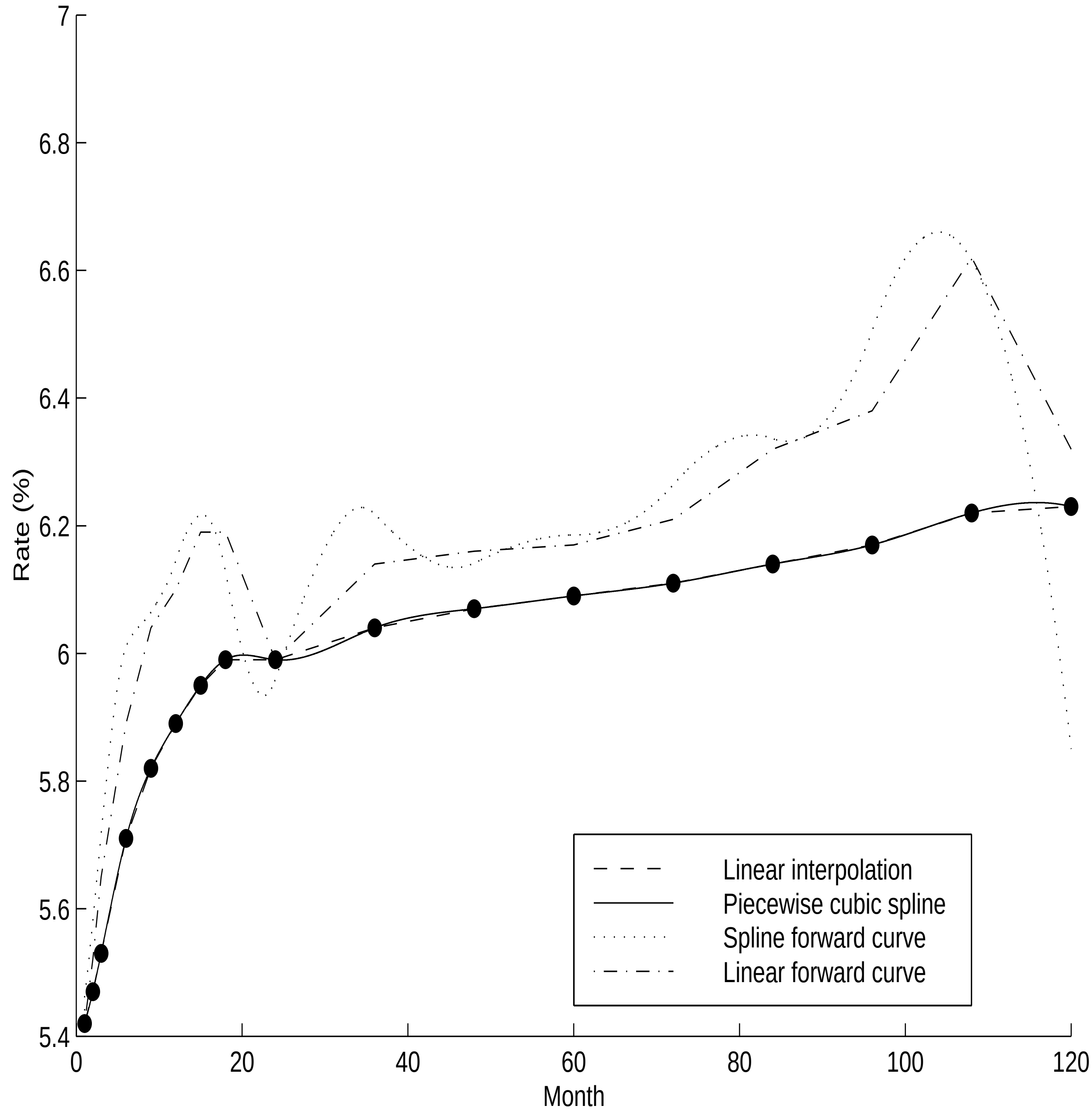


Figure 2.5a

Linear interpolation: swap zero curve by currency (continuously compounded)

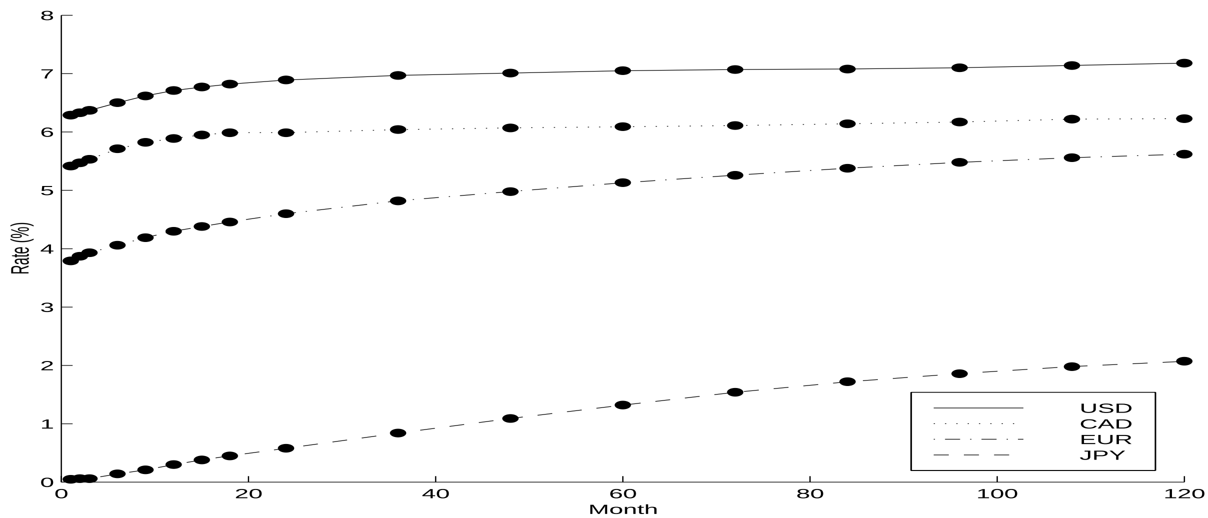
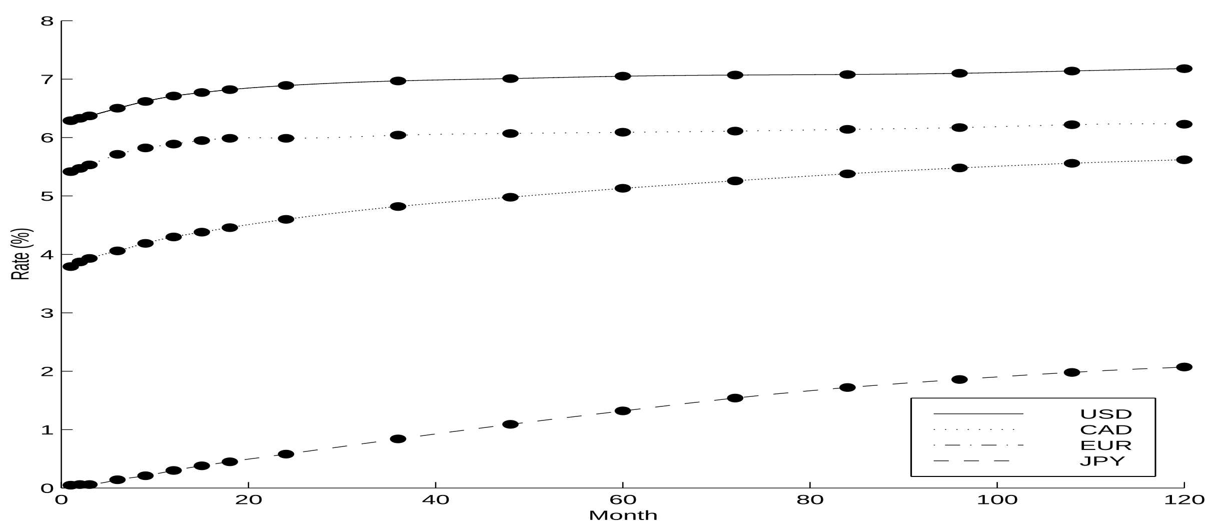


Figure 2.5b

Piecewise cubic spline: swap zero curve by currency (continuously compounded)



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