MUSE: The Bank of Canada’s New Projection Model of the U.S. Economy

by

Marc-André Gosselin and René Lalonde
MUSE: The Bank of Canada’s New Projection Model of the U.S. Economy

Marc-André Gosselin and René Lalonde

International Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9

The views expressed in this report are solely those of the authors.
No responsibility for them should be attributed to the Bank of Canada.
Acknowledgements ................................................................. iv
Abstract .................................................................................. v
Résumé ...................................................................................... vi
1. Introduction ........................................................................... 1
2. Motivation and goals ............................................................. 2
3. Basic Structure ....................................................................... 3
4. Polynominal Adjustment Cost (PAC) ......................................... 5
5. GDP ...................................................................................... 7
  5.1 Household spending ............................................................. 7
  5.2 Business investment ........................................................... 10
  5.3 Government spending ......................................................... 14
  5.4 International trade ............................................................. 17
6. Inflation .................................................................................. 21
7. Interest Rates ......................................................................... 22
8. Real Exchange Rate ............................................................... 24
9. Shock Analysis ....................................................................... 26
  9.1 A shock to private domestic demand .................................... 26
  9.2 A shock to the federal funds rate ......................................... 28
  9.3 A shock to the exchange rate ................................................. 29
  9.4 A shock to government spending .......................................... 29
  9.5 A shock to the interest rate for business ............................... 30
  9.6 An inflation shock ............................................................... 31
  9.7 An oil-price shock ............................................................... 31
  9.8 A shock to total factor productivity ...................................... 32
  9.9 A shock to the inflation target .............................................. 33
  9.10 A permanent reduction in the size of government ................. 34
  9.11 A permanent reduction in the ratio of government debt to GDP .......................................................... 34
10. Conclusion ............................................................................ 35
References ................................................................................ 36
Appendix A: Estimation Protocol for PAC Equations ......................... 39
Appendix B: The Term Structure of Interest Rates ............................ 41
Appendix C: Dynamic Response of MUSE .................................... 43
Acknowledgements

The authors would like to thank Robert Lafrance, Tiff Macklem, James Powell, Larry Schembri, and participants at seminars held by the Bank of Canada, Department of Finance Canada, Federal Reserve Board, Bank of England, OECD, and European Central Bank for valuable comments and suggestions. Special thanks to Jean-François Perrault for his support at all stages of this project. Thanks also to Glen Keenleyside for editorial review.
Abstract

The analysis and forecasting of developments in the U.S. economy have always played a critical role in the formulation of Canadian economic and financial policy. Thus, the Bank places considerable importance on generating internal forecasts of U.S. economic activity as an input to the Canadian projection. Over the past year, Bank staff have been using a new macroeconometric model, MUSE (Model of the U.S. Economy). The model is a system of estimated equations that describe, in a stock-flow framework, the interactions among the principal macroeconomic variables, such as gross domestic product (GDP), inflation, interest rates, and the exchange rate. The stock-flow equilibrium is fully described in MUSE. In steady state, the model defines specific values for all stocks, including capital stock, government debt, financial wealth, and net foreign assets.

In MUSE, most behavioural equations are governed by a polynomial adjustment cost (PAC) structure. This approach is widely used in the U.S. Federal Reserve Board’s FRB/US model. By allowing for lags in the dynamic equations in the context of forward-looking rational expectations, the PAC approach strikes a balance between theoretical structure and forecasting accuracy. MUSE, therefore, makes an explicit distinction between dynamic movements caused by changes in expectations and those caused by adjustment costs. Moreover, GDP is decomposed into household expenditures, business investment, government spending, exports, and imports. Hence, MUSE can be used to predict the consequences of a wide variety of shocks to the U.S. economy.

*JEL classification:* E37, C53, E17, E27, F17

*Bank classification:* Economic models; Business fluctuations and cycles
Résumé

L’analyse et la prévision de l’évolution économique aux États-Unis ont toujours joué un rôle déterminant dans l’élaboration des politiques économiques et financières canadiennes. C’est pourquoi la Banque du Canada accorde beaucoup d’importance à la réalisation de prévisions internes de l’activité économique américaine dans la formulation des projections canadiennes. Au cours de la dernière année, le personnel de la Banque a commencé à utiliser un nouveau modèle macroéconométrique, du nom de MUSE pour Model of the U.S. Economy, aux fins d’analyse et de prévision. Ce modèle est formé d’un système d’équations estimées décrivant les liens entre les principales variables macroéconomiques, telles que le produit intérieur brut (PIB), l’inflation, les taux d’intérêt et le taux de change. L’équilibre entre les stocks et les flux est pleinement explicité dans MUSE. En régime permanent, le modèle définit des valeurs pour tous les stocks, dont le stock de capital des entreprises, la dette publique, la richesse financière et le niveau des actifs nets à l’étranger.

La plupart des équations de comportement de MUSE sont fondées sur une structure de coûts d’ajustement polynomiaux. Cette approche est employée dans plusieurs composantes du modèle FRB/US, conçu à la Réserve fédérale américaine. En permettant d’introduire des retards au sein des équations dynamiques dans un contexte d’anticipations rationnelles prospectives, l’approche des coûts d’ajustement polynomiaux réussit à concilier contenu théorique et qualité des prévisions. MUSE établit en effet une distinction explicite entre les mouvements qu’induisent les modifications des anticipations et ceux qui se produisent avec un retard en raison de la présence de coûts d’ajustement. En outre, le PIB est décomposé en cinq agrégats : dépenses des ménages, investissement des entreprises, dépenses publiques, exportations et importations. MUSE peut donc servir à prédire les conséquences d’un large éventail de chocs susceptibles de toucher l’économie américaine.

Classification JEL : E37, C53, E17, E27, F17
Classification de la Banque : Modèles économiques; Cycles et fluctuations économiques
1. Introduction

The Bank of Canada has a long history of forecasting Canadian economic activity. Its pioneering work began in the 1960s with the development of RDX1. In the 1980s, the Bank developed RDXF (the Research Department Experimental Forecasting model), which built on the earlier RDX1 and RDX2 models. RDXF was a highly disaggregated macroeconometric model that did not include explicitly forward-looking expectations or the long-run budget constraints needed to describe the complete stock-flow equilibrium. With advances in economic theory and increased computing power, the Bank developed the Quarterly Projection Model (QPM) in the mid-1990s. This model reflected the state of the art in terms of theoretical structure and dynamic adjustment, and has been the Bank’s main model for Canadian economic projections and policy advice for the past decade. Bank staff will soon complete work on a new Canadian projection model called TOTEM (Terms Of Trade Economic Model), which is at the cutting edge of technology and embodies most of the latest developments in dynamic general-equilibrium modelling.

Although different generations of Canadian models have undergone significant changes in terms of theoretical underpinnings or macroeconomic structure, they have always relied on other models or sources of information for estimates of external economic activity. The analysis and forecasting of developments in the U.S. economy have always played a critical role in the formulation of Canadian economic and financial policy. Thus, the Bank places considerable importance on generating internal forecasts of U.S. economic activity as an input to the Canadian projection (Macklem 2002).

Over the past year, Bank staff have been using a new macroeconometric model, MUSE (Model of the U.S. Economy), to analyze and forecast the U.S. economy. The model
is a system of estimated equations that describe the interactions among the principal macroeconomic variables, such as gross domestic product (GDP), inflation, interest rates, and the exchange rate. MUSE replaces USM (United States Model), a small reduced-form model used since 2000.

In MUSE, most behavioural equations are governed by a polynomial adjustment cost (PAC) structure (Tinsley 1993). This approach is widely used in the U.S. Federal Reserve Board's FRB/US model. By allowing for lags in the dynamic equations in the context of forward-looking rational expectations, the PAC approach strikes a balance between theoretical structure and forecasting accuracy. Moreover, in MUSE, GDP is decomposed into household expenditures, business investment, government spending, exports, and imports. Hence, MUSE can be used to predict the consequences of a wide variety of shocks to the U.S. economy.

This report is organized as follows. Section 2 reviews the motivation and goals behind the construction and use of MUSE at the Bank, and section 3 reviews the model’s basic structure. Section 4 describes the PAC approach, and section 5 reviews the modelling of the GDP components. Sections 6, 7, and 8 describe the inflation, interest rate, and real exchange rate equations, respectively. Section 9 reports the results of several shock simulations, and section 10 offers some conclusions.

2. **Motivation and Goals**

There is an important conceptual difference between the Bank staff’s projections for Canada and the United States. The Canadian projection is the staff’s assessment of the most likely path for the economy and includes a recommendation to the Bank’s Governing Council on the optimal profile for the overnight interest rate to bring inflation to the 2 per
cent midpoint of the inflation-control target range. In contrast, the primary role of the projection for the United States is not to provide specific monetary policy recommendations, but to forecast U.S. economic activity and inflation, together with the Federal Reserve’s most likely interest rate response. The U.S. model is therefore not as oriented towards policy analysis as the Canadian model.

Even though the main goal of the U.S. model is to provide a forecast input to the Canadian projection, a developed model can be helpful in understanding various influences on the U.S. economy, such as oil-price shocks, productivity shocks, and fiscal policy shocks. It can also be useful for comparisons with the Canadian economy.

Before the introduction of MUSE, the U.S. economic projection relied on USM: a small model consisting of an IS curve, a Phillips curve, and a monetary policy reaction function (Lalonde 2000). USM has a number of limitations: it is based on backward-looking expectations and therefore is not well suited to shock analyses; given its small size, it can answer only a limited number of questions; and both potential output and exchange rate are exogenous. With the PAC specification and the disaggregation of GDP, MUSE overcomes these deficiencies of USM.1

3. Basic Structure

MUSE is medium-sized. It contains 35 behavioural equations, of which 33 are estimated. The largest component is the output block, where GDP is decomposed into household expenditures (consumption plus housing), business investment, government spending, exports, imports, and inventories. The household block is inspired by the

---

1 Using the FRB/US model directly would be too resource intensive, given the relatively limited amount of staff devoted to the U.S. projection and the Bank’s needs. Building MUSE had the benefit of developing human capital while taking advantage of the core insights and advancements of the FRB/US model.
permanent-income hypothesis, in which spending is a function of interest rates, personal disposable income, and various wealth stocks. We use a Cobb-Douglas production function with three types of capital goods to model business investment, so that potential output is partly endogenous. In MUSE, the fiscal sector adjusts to reach targets for government size and the ratio of debt to GDP. Exports and imports are modelled similarly: both react to relative prices, and they react to foreign and domestic income, respectively.

The stock-flow equilibrium is fully described in MUSE. In steady state, the model defines specific values for all stocks. The equilibrium capital-output ratio is determined by businesses’ financing costs and asset-specific depreciation rates and relative prices. The tax rate adjusts to meet a target level of debt and size of government in the steady state. The model also converges to a constant ratio of net foreign assets (NFAs) to GDP. This convergence is facilitated by the exchange rate. Human wealth at equilibrium is influenced by personal income, taxes, transfers, and households’ rate of time preference.

In MUSE, expectations are rational. As in the FRB/US model, there is an explicit distinction between dynamic movements caused by changes in expectations and those caused by adjustment costs. By using a very general description of frictions (i.e., PAC models), MUSE is able to closely match the persistence in the historical time-series data. As a result, it stands halfway between general-equilibrium models, in which the dynamics are entirely explained by theory, and reduced-form models, which are based solely on data.

As is the case in USM, MUSE builds on the output-gap paradigm. The difference between actual and potential output is a key driver of inflation in the model. Still, inflation is determined in the context of forward-looking rational expectations, and the persistence in the inflation process is explained by adjustment costs. The monetary and fiscal authorities are modelled according to simple forward-looking rules. The central bank reacts with the
objective of closing both the current output gap and the inflation gap 4 quarters ahead, and
transfer spending is counter-cyclical. The other key adjustment mechanism in the model is
the real exchange rate, which reacts to interest rate differentials and the current account
balance so as to restore the NFA target position. The exchange rate equations are partly
calibrated.

4. Polynomial Adjustment Cost (PAC)

MUSE relies extensively on the PAC approach for its dynamic properties. As Brayton
et al. (1997) note, traditional structural macroeconomic models do not allow researchers to
determine whether current movements in a variable are the result of changing expectations
or the lagged response to a previous decision. PAC models eliminate this ambiguity by
decomposing the dynamic behaviour of a time series into changes that are induced by
expectations and those that are delayed responses to previous decisions. In such models,
agents make decisions on the basis of forecasts of the target level of the variable of interest.
This target level is attained gradually, since moving to it from the current level entails
adjustment costs. PACs are characterized by disequilibrium: the target outcome is not
achieved in the short run, despite the fact that agents are rational. It is precisely because of
these adjustment costs that agents are forward looking. Decisions subject to higher
adjustment costs require longer planning horizons.

In the case of investment, for instance, these costs can be as diverse as those related
to information gathering, plant or product design, testing, and regulatory approval.
Deviations from target levels are often the result of unanticipated shocks, such as a
difference between expected and actual income.
In PAC models, agents minimize the joint expected costs of diverging from a target level and the costs associated with modifying spending patterns to return to it. Expected future costs are discounted such that adjustment costs in the distant future have less bearing on current decisions than those in the near future. This results in the minimization of a cost function specified in terms of the discounted current and future costs of a decision variable. For instance, this cost function would have the following specification in the case of consumption:

\[
E_{t-1} \left\{ \sum_{i=0}^{\infty} \beta^i \left[ \kappa_0 (C_{t+i} - C^*_t)^2 + \kappa_1 (\Delta C_{t+i})^2 + \kappa_2 (\Delta^2 C_{t+i})^2 + \ldots \right] \right\},
\]

(1)

where \( E_{t-1} \{ \} \) is a forecast of the costs of diverging from the target level of consumption, \( C^* \). The first squared expression is the cost of diverging from \( C^* \) in period \( t+i \), and \( \kappa_0 \) is the unit cost associated with it. The other expressions are a general characterization of the order of the frictional costs related to changes in \( C \) in future periods, \( t+i \).

Most macroeconomic models assume that the principal source of friction is captured by the quadratic term \( \kappa_1 (\Delta C_{t+i})^2 \). PAC models permit a more general specification of adjustment costs, since \( \kappa_2 \) is the unit cost of changing the rate of change in \( C \), \( \kappa_3 \) the cost of changing the rate of acceleration in \( C \), and so on. By minimizing equation (1), Tinsley (1993) derives the following decision rule:

\[
\Delta C_t = -a_0 (C_{t-1} - C^*_t) + \sum_{j=1}^{m-1} a_j \Delta C_{t-j} + E_{t-1} \left\{ \sum_{i=0}^{\infty} e^i \Delta C^*_t \right\}.
\]

(2)

The decision rule stipulates that the cost-minimizing adjustment at time \( t, \Delta C_t \), depends on: the per cent difference between the previous period of consumption and its

\[ C^* \] is the target level of consumption in that it is the optimal level in the absence of adjustment costs.
target level, \((C_{t-1} - C^*_{t-1})\); previous changes in the level of consumption, \(\Delta C_{t-1}\); and a weighted sum of expected changes in the target level of consumption, \(\Delta C^*_{t+i}\). With the exception of the last term, equation (2) is a basic error-correction model. The inclusion of the final term imparts a forward-looking, or rational, element to the equation. Appendix A describes the estimation protocol for PAC equations.

5. GDP

Instead of using a simple IS curve to forecast aggregate GDP, we focus on the main components of the national accounts identity separately. GDP is decomposed into four main blocks: household spending, business investment, government spending, and international trade. The GDP forecast is given by the sum of these components. For the sake of brevity, we report results for only the parameter estimates. \(R^2\) and \(t\)-statistics are available from the authors. Parameter stability tests and out-of-sample forecasting exercises are also conducted (but not reported here).

5.1 Household spending

According to the permanent-income hypothesis (PIH), a household’s consumption in any given period is equal to its permanent income, defined as the current value of household wealth (human and non-human). In this context, consumption changes when

---

3 The \(\beta\) coefficients are functions of the discount rate, \(\beta\), and the cost parameters \((\kappa_0, \kappa_1, \kappa_2, ... )\). This implies that the relative importance of expected changes in the target level depends on the nature of the frictions that are characteristic of that variable. In a PAC model, there are an infinite number of leads. When running MUSE, we use a truncated number of leads (such that a significant portion of the asymptotic value of the infinite leads is taken into account). This number varies across the different components of the model, depending on the relative importance of the adjustment costs.

4 In equation (2), \(m\) is the order of adjustment costs. It is determined empirically.

5 Inventories also enter the aggregation of GDP in MUSE. These are modelled in proportion to potential GDP, and are a function of a lag and the expected output gap.
agents change their expectations of future income, or when agents make expectational errors. If rational agents are assumed, then these forecast errors or changes in expectations follow a random walk, implying that consumption also follows a random walk (Hall 1978). Campbell and Mankiw (1990) and Shea (1995), however, find that consumption does not follow a random walk, thereby rejecting the pure PIH. Flavin (1985) and Carroll (2001) argue that liquidity-constrained households limit the applicability of the PIH.\(^6\) Carroll (1994) also argues, however, that households may choose to save for precautionary reasons.\(^7\) In both cases, consumption may be more closely related to current rather than permanent income.

In addition, the PIH is conceptually problematic when applied to the consumption of durable goods. Households derive utility from durable goods that extends well beyond the purchase date. For this reason, Blinder and Deaton (1985), Campbell (1987), and Galí (1990) estimate consumption equations that exclude durable goods. Stock-adjustment models are typically used in the literature to determine expenditures on durable goods. These are simplified error-correction models in which frictions cause a delay in the adjustment of purchases to their target level. This approach is used by McCarthy and Peach (2002), among others, to model housing investment.

In an effort to strike the appropriate balance between the costs and benefits of disaggregation, MUSE models household spending (defined as the sum of total consumption and residential investment) in a single equation. Based on the aforementioned deviations from the PIH, we model household purchases using the PIH in the long run, and we allow for deviations from permanent income along the dynamic path (Gosselin and Lalonde 2003).

---

\(^6\) Restricted access to credit may preclude some households from borrowing when permanent income rises. Consumption is then determined by current rather than permanent income.

\(^7\) Uncertainty about future income may compel households to increase current savings in case of a shortfall in future revenues.
**Target level:**

The target level of real per-capita household spending is given by:

\[
\log C_t^* = -2.03 + 0.59 \log W_{t, \text{human}} + 0.18 \log W_{t, \text{fin}} + 0.23 \log W_{t, \text{house}} - 0.56 r_{t, \text{fed}},
\]

where \( W_{t, \text{human}} \) is human wealth, defined as the expected discounted sum of future real disposable income flows over the next 40 quarters; \( W_{t, \text{fin}} \) is real financial wealth; \( W_{t, \text{house}} \) is real housing wealth; and \( r_{t, \text{fed}} \) is the real federal funds rate.\(^8,9\) Although this is an issue of some theoretical debate, we separate financial wealth from housing wealth to allow for different elasticities.\(^10\) Financial wealth is given by:

\[
W_{t, \text{fin}} = K_t + DEBT_t + NFA_t + W_{t, \text{fin, real}},
\]

where \( K \) is the total capital stock, \( DEBT \) is government debt, \( NFA \) is the net foreign asset position, and \( W_{t, \text{fin, real}} \) is an exogenous revaluation term. Thus, internal as well as external imbalances influence the profile for consumption in MUSE.

**Dynamic specification:**

Ljung-Box tests imply that the adjustment costs for household expenditures are of order three. Spending is determined by the rational error-correction term, two lags \((m = 3)\), real disposable income in the current period, expected changes in the target level of household expenditures, expected changes in the output gap, and the change in the nominal mortgage rate in the current period.\(^11\) The discounted expected future path of the output gap enters the equation as a proxy for cyclical uncertainty: the lower the value, the less certain

---

8 In line with the PIH, we impose the condition that the coefficients on the three types of wealth sum to unity.
9 Personal income is equal to GDP times the labour share of income. Although labour input is exogenous in MUSE, the labour share of income is a function of a labour input gap. Disposable income is given by personal income minus taxes plus transfers.
10 There is limited debate about this issue in the empirical literature. See, for example, Case, Quigley, and Shiller (2005) and Ludwig and Slok (2002).
11 The change in the nominal mortgage rate is included to approximately capture the temporary effects of down-payment requirements and other borrowing constraints on housing investment when nominal interest rates change.
agents are about future economic outcomes.\textsuperscript{12} Owing to liquidity-constrained consumers, a negative effect that results from higher oil prices is included to proxy the impact of this variable on disposable income. Although these variables do not fit with the pure PAC specification, they are added to improve forecasting capacity. To ensure convergence to a stable ratio of household spending to GDP in the steady state, the sum of the coefficients on lagged spending, real disposable income, and future changes in target consumption are constrained to unity in estimation:

\[
\Delta \log C_t = -0.13(\log C_{t-1} - \log C^*_t) + 0.03 \Delta \log C_{t-1} + 0.13 \Delta \log C_{t-2} + 0.27 \Delta \log YD_t \\
+ 0.57 E_t \left\{ \sum_{i=0}^{30} \Delta \log C_{t+i}^* \right\} + 0.18 E_t \left\{ \sum_{i=0}^{40} \log Y_{t+i} \right\} - 0.41 \Delta Y_{t+1}^{mortgage} - 0.01 \Delta \log OIL_{t+1}. \tag{5} \]

The percentage of households that are liquidity constrained or rule-of-thumb spenders is 27 per cent; 73 per cent are forward looking. Consequently, the spending patterns of households adjust relatively sluggishly to differences between expenditures in the previous period and their target level. This 27 per cent share is relatively small. It could be explained by the fact that household spending includes housing.

\subsection*{5.2 Business investment}

Firms are forward looking in MUSE. In the long run, business output is governed by a Cobb-Douglas production function for labour ($L$) and capital ($K$).\textsuperscript{13} The dynamic path of investment depends on the target level of the capital stock, the gap between the current capital stock and the target capital stock, and the costs of closing this difference. Thus, output is determined by:

\textsuperscript{12} A variable defined in terms of the second moment of the output gap (i.e., variance) would have been preferable. Nevertheless, the addition of this variable improves the forecasting performance and has little impact on the model's properties.

\textsuperscript{13} This section draws upon the work of Kiley (2001).
\[ Y = 1 L^\alpha K^\beta, \] (6)

where \( L \) is the labour input; \( K \) corresponds to the capital services generated by three capital inputs: structures \((K^{\text{str}})\), high-tech equipment \((K^{\text{ht}})\), and equipment and software excluding high-tech equipment \((K^{\text{es}})\); \( \alpha \) is the labour share of income; and \( \Theta \) is total factor productivity (TFP).

Given the upward trend in the share of income from high-tech capital, we cannot use the Cobb-Douglas function to identify components of the target capital stock. Rather, we rely on the dual approach and use translog cost functions to estimate capital-share equations. This functional form allows technology—defined as the relative share of each type of capital—to change over time, so that substitution and complementarity effects are taken into account.

For \( K \in \{K^{\text{str}}, K^{\text{ht}}, K^{\text{es}}\} \), the factor-share equations are determined by:

\[
\frac{u^k_t K^k_t}{(1-\alpha)Y_t} = s^k_t = b^k + \sum_k B_k \log(u^k),
\] (7)

where \( k = \text{str, ht, es, alternately,} \) and \( u^k_t \) is a traditionally defined user cost of capital:

\[
u^k_{t,t} = [f^\text{st}_t + \delta^k_{t,t} - (1 - \delta^k_{t,t})\Delta p^\text{rel}_{t,t}^k] p^k_{t,t},
\] (8)

where \( f^\text{st} \) is an interest rate for business-financing costs, \( \delta^k \) is the depreciation rate for each type of capital asset, \( \Delta p^\text{rel}_{t,t}^k \) is a moving average of the growth of the relative price of capital, and \( p^k \) is the price of capital.\(^{14}\) The equation implies that the return on capital is sufficient to earn the rate of interest, recover depreciation, and recoup any capital losses caused by movements in the price of the asset. The firm’s financing costs are measured as a weighted

\(^{14}\) The asset-specific depreciation rates are computed by inverting the capital accumulation rule for each asset. The relative price of each type of investment is modelled with simple error-correction equations or in deviations relative to their Hodrick-Prescott (HP)-filtered trend.
average of borrowing costs in debt and equity markets. (See Appendix B for a definition.)

Cointegration estimates for the share equations (7) are:

\[ s_{it}^{str} = 0.67 + 0.13 \log u_{it}^{str} + 0.04 \log u_{it}^{ht}, \]  \hspace{1cm} \text{(9)}

\[ s_{it}^{ht} = 0.19 - 0.05 \log u_{it}^{str} - 0.07 \log u_{it}^{ht}, \]  \hspace{1cm} \text{(10)}

\[ s_{it}^{es} = 0.14 - 0.08 \log u_{it}^{str} + 0.03 \log u_{it}^{ht}. \]  \hspace{1cm} \text{(11)}

The homogeneity restriction ensures that trending shares are offset by trends in other shares, thereby maintaining a constant capital income share \((1-\alpha)\).

The semi-elasticity of the high-tech share of capital income with respect to the user cost of high-tech equipment is negative. Thus, the growing share of high-tech capital reflects the rapid pace of decline in the price of these investment goods. In turn, declining prices for high-tech capital have led to a decline in the share of structures and non-high-tech equipment and software. (Note the positive coefficient on the user cost of high-tech capital in share equations for structures and for equipment and software.) The positive coefficient on the user cost of structures in equation (9) reflects the fact that investment in structures is less sensitive to changes in the cost of capital than the other types of investment goods.

**Target level:**

Based on the rule of perpetual inventory capital accumulation, the target level of capital stock is used to determine targeted investment:

\[ I_{t}^{*} = (g_{t+1} + \delta)K_{t}^{*}, \]  \hspace{1cm} \text{(12)}

where \(g\) is the growth rate of each capital asset. This is proxied by potential output growth.

\hspace{1cm} \text{\(15 b_{str} + b_{ht} + b_{es} = 1\) across equations (9), (10), and (11). This restriction imposes that the capital income share equals the share implied by the production function.}
Dynamic specification:

We find evidence of time-to-build effects; i.e., the disequilibrium term lagged twice, for all types of investment except high-tech equipment. Business expenditures are determined by the rational error-correction term, two lags \((m = 3)\), expected changes in the target level of business spending, and, in some cases, output growth in the current or previous period. This last term is included to capture cash-flow effects for some subset of financially constrained firms:

\[
\Delta \log I_i^{str} = -0.03 \left( \log I_{t-2}^{str} - \log I_{t-2}^{str} \right) + 0.19 \Delta \log I_{t-1}^{str} + 0.13 \Delta \log I_{t-2}^{str}
+ 0.20 E_{t-1} \left\{ \sum_{i=0}^{20} f_i \Delta \log I_{t+i}^{str*} \right\} + 0.48 \Delta \log Y_{t-1},
\]

\[
\Delta \log I_i^{ht} = -0.06 \left( \log I_{t-1}^{ht} - \log I_{t-1}^{ht} \right) + 0.16 \Delta \log I_{t-1}^{ht} + 0.25 \Delta \log I_{t-2}^{ht}
+ 0.59 E_{t-1} \left\{ \sum_{i=0}^{20} f_i \Delta \log I_{t+i}^{ht*} \right\},
\]

\[
\Delta \log I_i^{es} = -0.05 \left( \log I_{t-2}^{es} - \log I_{t-2}^{es} \right) + 0.10 \Delta \log I_{t-1}^{es} + 0.22 \Delta \log I_{t-2}^{es}
+ 0.35 E_{t-1} \left\{ \sum_{i=0}^{20} f_i \Delta \log I_{t+i}^{es*} \right\} + 0.33 \Delta \log Y_t.
\]

In all cases, investment exhibits substantial inertia to movements in output or user costs. Although still highly persistent, high-tech spending is the least persistent of the capital assets. The percentage of firms that are financially constrained (or rule-of-thumb investors) is 48 per cent for investment in structures and 33 per cent for investment in non-high-tech equipment and software. Cash-flow effects are not significant in the case of high-tech spending.

Using the capital-accumulation rule with the investment-flow forecasts, we compute a forecast of the capital stock. In combination with exogenous assumptions for trend labour input and TFP, this forecast feeds into the production function to generate a forecast for potential output over the projection period. Instead of using a production function over
history to estimate potential output, we use a combination of extended multivariate HP filters and structural vector autoregressions (VARs) based on the labour market (Gosselin and Lalonde 2002). This choice is motivated by the fact that data on U.S. capital stock are available on only an annual basis and are subject to frequent historical revision.

5.3 Government spending

The government sector plays an important role in MUSE. Aside from government consumption and investment, which feed directly into the national accounting identity, taxes and transfers partly determine personal disposable income. Government debt influences consumption through its effect on household wealth. It also influences the risk premium on Treasury bonds, which affects the cost of capital and mortgage rates. The Jorgensen-style user cost of capital utilized in MUSE is invariant to changes in corporate tax rates.

In MUSE, the government targets a debt level in the steady state. Thus, the government sector’s dynamic response to shocks is best characterized as a fiscal policy rule. Governments adjust revenues to ensure achievement of their debt target in the long run. In keeping with other components of MUSE, this adjustment is subject to frictional costs, which slow the response of revenues to unanticipated shocks. Note that all fiscal variables are deflated by potential, and not actual, output.

Expenditures

In the post-Vietnam War era, total primary government expenditures as a proportion of potential output have been stationary around a constant. This fact is used to anchor the model’s fiscal assumptions, and argues against the use of a PAC model for this component

---

16 Total primary government expenditures are defined as the sum of government spending on capital and other goods and services plus transfers. In MUSE, no distinction is made between federal, state, and local expenditures or revenues.
of MUSE. Thus, the ratio of aggregate primary government spending to potential GDP is simply determined by:

\[
\left( \frac{G}{Y_{pot}} \right)_t = 0.032 + 0.89 \left( \frac{G}{Y_{pot}} \right)_{t-1} - 0.06 \log Y_{\text{gap}},
\]

(14)

where the ratio of total government spending to output in the steady state is:

\[
\alpha = \frac{0.032}{1 - 0.89} = 0.2909.
\]

In equation (14), the output gap captures the cyclical variations in \( G \) that result from the operation of automatic stabilizers. The greater the recession or degree of excess supply, the higher are government expenditures. In the long run, government size converges to its steady-state ratio of 29 per cent.

Because \( G \) is the sum of government consumption and investment plus transfer payments, we use a share equation to derive separate estimates for government spending on a national accounts basis and for the transfer payments that affect labour income. Since the share of transfers in total primary government spending is stationary around its average and is highly cyclical, we use a simple equation that links the share of transfers in government spending to the output gap to parse out the expenditure component of government spending from the transfer element.\(^{17}\)

**Revenues**

A PAC equation is used for tax rate dynamics. Given the preferences for steady-state government size \( \alpha \) and the government’s steady-state debt target, its target aggregate tax

---

\(^{17}\) Estimation suggests that there are three breaks in the transfer\(/G\) ratio in the post-Vietnam War era. These are identified using the Bai and Perron (1998) tests for structural breaks. The resulting subperiods are estimated separately. In MUSE, we use the most recent period of stability as the basis for the dynamics of the transfer share in the projection. This period corresponds to the post-Cold War period. So far, we have not found statistical evidence of a structural break in the post-September 11 period.
rate, $T^*$, eliminates the difference between the current level of debt and its long-run value over time:

$$T^*_t = \alpha_1 + 0.15\text{debt}_t^\text{gap} + r_t^\text{gov} (\text{debt}_t^*)^\gamma,$$  \hspace{1cm} (15)

where $\text{debt}_t^\text{gap} = (\text{debt}_t^*/Y_t^*) - (\text{debt}_{t-1}^*/Y_{t-1}^*)$ and $r_t^\text{gov}$ is the steady-state real government interest rate.

Institutional and political constraints prevent the government from adjusting the tax rate to its target level in the near term. An example of institutional constraints could be the lengthy budget deliberations in the U.S. Congress. For obvious reasons, political considerations can also delay required tax adjustments. Thus, the aggregate tax rate adjusts slowly to its target level. With $m = 2$, its dynamic path is governed by:

$$\Delta T_t = -0.128(T_{t-1} - T_{t-1}^*) - 0.235\Delta T_{t-1} + 0.59E_t \left\{ \sum_{i=0}^{12} f_i \Delta T_{t+i}^* \right\} + 0.08\log Y_t^\text{pop}. \hspace{1cm} (16)$$

Cyclical conditions affect $T$; all else equal, tax revenues relative to potential GDP are less in a recession than in an expansion. Alternative specifications of equation (16) in MUSE include financial wealth, to account for the temporary effect of changes in capital gains on taxes.

MUSE relies on an estimated share equation to separate personal income taxes from the other sources of government revenues embedded in $T$. This is also a PAC model, in which the target share of personal income taxes in total revenue is a function of financial wealth, and the dynamic profile is a function of the output gap. Results of this equation are available from the authors.
5.4 International trade

Estimation of the model’s trade equations is subject to two principal constraints. The first is that MUSE is a one-good model: it does not differentiate between traded and non-traded goods. Given that labour productivity in the traded-goods sector has increased much more rapidly than in the non-traded-goods sector, the relative price of imports and exports has fallen over the past few decades.\textsuperscript{18} To account for the trend in the relative price of tradable goods, we include a deterministic trend in the long-run profile for import prices.

Second, we assume unit income elasticity of both imports and exports to ensure model convergence in the steady state. Most empirical studies find a marked difference between the income elasticity of U.S. imports and that of U.S. exports, generally in the order of two to one; see Hooper, Johnson, and Marquez (2000) for a summary. Gosselin and Lalonde (2004) find that this elasticity puzzle largely disappears if proxies for globalization are included in both the import and export equations. MUSE relies on these results for the specification of both target imports and exports by incorporating proxies for openness to global trade and imposing unit income elasticity in the model’s estimation.

Adjustment costs can be important in the tradable goods sector. Gagnon (1989) finds that both U.S. importing and exporting firms face substantial adjustment costs. These costs may include shipping delays, linguistic barriers, lack of familiarity with domestic commercial practices, and commercial policies (such as taxes and tariffs). Given these costs, profit-maximizing firms must be forward looking in their behaviour, anticipating domestic and foreign demand in order to reduce the costs of sudden shifts in demand. MUSE therefore uses PAC models to represent import and export volumes.

\textsuperscript{18} For a complete review of the international decline in the prices of tradable goods, see Gagnon, Sabourin, and Lavoie (2004).
5.4.1 Import volumes

Target level:

The target level of real imports is determined by the standard paradigm of income and relative prices, augmented with a proxy for globalization 19:

\[ M_t^* = -1.08 + 1.00 \log DEM_t + 0.50 OPEN_t - 0.90 \log PM_t. \]  

(17)

Real imports are an increasing function of private domestic demand, \( DEM \), and openness to trade, \( OPEN \), and a decreasing function of the relative price of imports, \( PM \). 20

Dynamic specification:

The dynamic path for imports is governed by the following PAC model, with \( m = 2 \):

\[
\begin{align*}
\Delta \log M_t &= -0.20(\log M_{t-1} - \log M_{t-1}^*) + 0.12 \Delta \log M_{t-1} + 0.88 E_t \left\{ \sum_{i=0}^{12} f_i \Delta \log M_{t+i}^* \right\} \\
&+ 1.55 \Delta \log y_{t, \text{gap}} + 0.04 \Delta \log OIL_{t-1}.
\end{align*}
\]

(18)

Note the rapid adjustment of the disequilibrium term. Each quarter, 20 per cent of the lagged difference between the actual and target levels is closed, which suggests that adjustment costs are low for import volumes. The change in the output gap is added to account for the fact that the short-run income elasticity of imports is much higher than its long-run value (Hooper, Johnson, and Marquez 2000). The model includes a positive effect from higher oil prices to account for the fact that the demand for oil imports is inelastic in the short run.

---

19 The trend towards openness to trade is proxied by the volume of trade between OECD countries.

20 \( DEM \) is the sum of household spending and business investment. The relative price of imports is the price of imports in relation to the GDP deflator. At estimation, the coefficient on \( DEM \) is set to unity. Given the inclusion of the \( OPEN \) variable, this restriction is not rejected by the data.
5.4.2 Import prices

Import prices play an important role in MUSE, since they help determine import volumes and feed into the inflation process. As noted earlier, the relative price of imports exhibits a marked downward trend. This poses an empirical problem, given MUSE’s one-good specification. We use a time trend to account for the decline in import prices.

**Target level:**

The target level of the relative price of imports (deflated by the GDP deflator) is determined by the real effective exchange rate, $FX_t$, a time trend, and the relative price of oil (West Texas Intermediate crude oil):

$$\log PM_t^* = 2.16 - 0.45 \log FX_t - 0.0049 \text{TREND} + 0.14 \log OIL_t. \quad (19)$$

**Dynamic specification:**

In contrast to the typical error-correction specification of import prices, we use a PAC equation to set the dynamic path for relative import prices. With $m = 2$, we have:

$$\Delta \log PM_t = -0.25(\log PM_{t-1} - \log PM_{t-1}) + 0.09 \Delta \log PM_{t-1} + 0.91E_t\left\{\sum_{i=0}^{12} f_i \Delta \log PM_{t+i}\right\}$$

$$+ 0.09 \Delta \log OIL_t. \quad (20)$$

As with import volumes, relative import prices converge quickly to optimal levels, since adjustment costs are small. The change in oil prices also affects import prices in the very short run.
5.4.3 Export volumes

Target level:

Ideally, the target level of real exports should be a function of foreign output, the relative price of exports, and globalization. We are unable to estimate a stable and well-performing export equation using the relative price of exports, perhaps because of measurement problems. As a substitute, we specify a long-run export equation as a function of the real effective exchange rate, foreign output, $Y_{t foreign}$, openness to trade, and a time trend:

$$\log X_t^* = 10.15 + 1.00 \log Y_{t foreign} + 0.50 \log OPEN_t - 0.69 \log FX_t + 0.0050TREND.$$ (21)

As with real imports, we impose unit income elasticity of foreign demand to ensure convergence of the trade balance in the steady state.

Dynamic specification:

With $m = 2$, the path of exports is set by:

$$
\Delta \log X_t = -0.09(\log X_{t-1} - \log X_{t-1}^*) + 0.28 \Delta \log X_{t-1}^* \\
+ 0.72 E_t \left\{ \sum_{i=0}^{30} f_i \Delta \log X_{t+i}^* \right\} + 1.7 \Delta \log Y_{t-1 foreign}^*.
$$ (22)

Although exports adjust quickly to divergences from the long-run level, their speed of adjustment is slower than that of real imports.

---

21 Although export prices are specified in MUSE, they have little dynamic impact on the model. For this reason, we do not elaborate on their specification. They play a role in determining the nominal trade balance, and thus indirectly influence current account dynamics, which in turn influence the real effective exchange rate.

22 Foreign output is the weighted sum of output in Mexico, Canada, the euro area, the United Kingdom, Japan, and emerging Asia (which includes China and India). Output is aggregated using U.S. export weights. The time trend and the exchange rate are substitutes for the relative price of exports.
6. **Inflation**\(^{23}\)

The ability to match the historical persistence of inflation is of critical importance in a macroeconometric forecasting model, since it improves out-of-sample forecasting. Furthermore, persistence in the inflation process increases the importance of the monetary authority’s role. Inflation models that exhibit low persistence can generate forecasts in which inflation returns to target with little action being taken by the monetary authority.

There is a wide range of ways to motivate and specify sticky prices, from menu costs to Calvo- (1983) and Taylor- (1980) type price-setting behaviour. More recent research focuses on New Keynesian Phillips curves or their variants. Hybrid specifications, such as that of Galí and Gertler (1999), can identify significant inflation persistence with the use of lagged values of inflation.

Instead of choosing one of these approaches, we do not take a rigid stance on the theory of inflation determination. As in Koziicki and Tinsley (2002), we use a more general PAC approach and let the data determine the persistence of inflation, rather than impose it by specification. This approach assumes rational economic agents that balance the costs of price adjustments against the costs of diverging from the optimal price level. The costs associated with changing prices lead firms to smooth the inflation profile, generating persistence in the inflation process. While non-specific about the nature of adjustment costs in the inflation process, the PAC approach is consistent with a number of price-setting frictions. Leads and lags enter the specification, but some structure is imposed in the spirit of New Keynesian models.

---

\(^{23}\) Inflation is defined as the annual change in the deflator of personal consumption expenditures excluding food and energy.
Kozicki and Tinsley (2002) show that the distance of actual inflation from its equilibrium path can be approximated by a factor that is proportional to the output gap. Thus, the inflation rate is expressed as:

\[
\pi_t = c_1 + \frac{(G_1 \psi E_t \pi_{t+1} + G_2 \psi^2 E_t \pi_{t+2} + G_3 \psi^3 E_t \pi_{t+3} + G_1 \pi_{t-1} + G_2 \pi_{t-2} + G_3 \pi_{t-3})}{(1 + G_1 \beta + G_2 \beta^2 + G_3 \beta^3)} \]  
\[+ \alpha \log Y_{\text{gap}}^{\text{gap}} + \theta \Delta \log PM_{t-1}, \tag{23} \]

where \(Y_{\text{gap}}^{\text{gap}}\) is a 4-quarter moving average of the output gap; \(PM_t\) is the relative price of imports; \(G_1 = 1 - G_2 - G_3\); there are third-order adjustment costs in the inflation process \((m = 4)\); \(\psi\) is the discount factor; and \(E_t \pi_{t+1}, E_t \pi_{t+2}, E_t \pi_{t+3}\) are inflation expectations.\(^{24}\)

Equation (23) can be rewritten in reduced form as follows:

\[
\pi_t = 0.11 \pi_{t-3} + 0.11 \pi_{t-2} + 0.28 \pi_{t-1} + 0.28 \pi_{t+1} + 0.11 \pi_{t+2} + 0.11 \pi_{t+3} 
+ 0.046 \log Y_{\text{gap}}^{\text{gap}} + 0.04 \Delta \log PM_{t-1} + 0.03 \Delta \log PM_{t-2}. \tag{24} \]

Having examined the estimated adjustment costs from the structural equation, we do not reject the hypothesis that \(\psi = 1\). This implies that our model is not statistically different from Taylor price-setting behaviour, where the sum of the coefficients on the leads and lags of inflation is 0.5. The coefficients on changes in the relative prices of imports are partly calibrated.

7. Interest Rates

A number of interest rates are used in MUSE which, in turn, influence various elements of the model. They are all anchored, in one way or another, to the federal funds rate. Appendix B gives details on the term structure of interest rates.

\(^{24}\) At the estimation stage, inflation expectations are from a Federal Reserve Bank of Philadelphia survey.
In its current version, the model’s policy-setting rule is derived from English, Nelson, and Sack (2002). In their work, the policy-setting rule is determined by the following three-equation model:

\[
\begin{align*}
\hat{i}_t &= i^* + b_x \pi_t^{pop} + b_y y_t^{pop}, \\
i_t &= (1 - \lambda) \hat{i}_t + \lambda i_{t-1} + \nu_t, \\
\nu_t &= \rho \nu_{t-1} + \epsilon_t,
\end{align*}
\]  

(25)

where \( \hat{i}_t \) is the nominal federal funds rate prescribed by a forward-looking Taylor (1993) rule, \( i^* \) is the neutral rate, \( \pi_t^{pop} \) is the difference between the 4-quarter-ahead forecast of inflation and its target, and \( y_t^{pop} \) is the current output gap. The neutral rate is fixed at its steady-state value. The steady-state value is endogenous and is equal to the unique value of the real interest rate that makes aggregate demand equal to aggregate supply in the steady state. In MUSE, \( b_\pi \) and \( b_y \) are equal to 4.0 and 0.7, respectively. Inflation is defined as the annual change in the deflator of personal consumption expenditures excluding food and energy, and the target is 2.0 per cent. In this policy-setting rule, the nominal federal funds rate gradually converges to the target Taylor rule prescription at \((1 - \lambda)\) per quarter, where \( \lambda \) represents the Federal Reserve’s preference for interest rate smoothing \((\lambda = 0.66 \text{ in MUSE})\).

The model’s policy-setting rule includes \( \nu_t \), which permits a temporary deviation from the Taylor rule target for reasons other than interest rate smoothing. With \( \rho = 0.67 \), its autoregressive specification generates some persistence in the policy reaction to these considerations. From the equations in (25), the dynamic equation for the federal funds rate can be expressed as:

---

25 These weights are inspired by English, Nelson, and Sack (2002). This is not the optimal monetary policy rule, but a good representation of Federal Reserve actions. This is a key difference from the Canadian model. (See section 2.)

26 Recent examples of such factors are geopolitical turmoil and turbulence in financial markets.
\[ \Delta i_t = (1 - \lambda)\Delta \hat{i}_t + (1 - \lambda)(1 - \rho)(\hat{i}_{t-1} - i_{t-1}) + \lambda \rho \Delta i_{t-1} + \varepsilon_t. \] (26)

The first term of equation (26) captures the partial adjustment to the federal funds rate given by the Taylor rule; the other terms reflect the gradual reduction in the difference between the observed federal funds rate and that given by the Taylor rule plus inertia in the response to past interest rate changes.

8. Real Exchange Rate

Stock-flow dynamics and the adjustment towards stock equilibrium play a key role in MUSE. As explained in section 5.1, NFAs are a component of household financial wealth. Successive current account deficits reduce NFAs and have a negative impact on the target consumption profile. In MUSE, the significant decline in NFAs over the past few years has depressed consumption.

**Target level:**

In the steady state, the model converges to a target ratio of NFAs to GDP. This convergence is facilitated by the real effective exchange rate. In MUSE, the real exchange rate, measured by the Federal Reserve’s real broad effective exchange rate, generates an improvement in the trade balance that is sufficient to attain the target NFA ratio. The long-run value of the exchange rate is governed by:

\[ \log FX_t^* = \log FX_{-SS} + 0.7 \left[ \frac{NFA_t}{Y_t} - \left( \frac{NFA_t}{Y_t} \right)^s \right]. \] (27)

The slope of the equation (i.e., 0.7) is calibrated to generate a reasonable adjustment persistence of the ratio of NFAs to GDP.\textsuperscript{27} \( FX_{-SS} \) is endogenous and corresponds to the value of the real effective exchange rate that is compatible with the target ratio of NFAs to GDP.

\textsuperscript{27} The slope is also partly determined by estimated error-correction models.
GDP. Given the steady-state version of the model, there is a unique value of the exchange rate (i.e., $FX_{SS}$) such that the ratio of NFAs to GDP converges to its target level.

**Dynamic specification:**

The dynamic exchange rate equation is a modified version of the Lalonde and Sabourin (2003) specification. In contrast to most of MUSE’s other equations, the dynamic exchange rate equation is an error-correction model, since it is conceptually difficult to rationalize adjustment costs of financial variables.\(^{28}\):

\[
\Delta \log FX_t = -0.17(\log FX_{t-1} - \log FX^*_{t-1}) + 0.25\Delta \log FX_{t-1} \\
+ 0.04rdiff_t - 0.13\Delta \left( \frac{NFA}{Y_{pot}} \right)_t.
\]

(28)

This dynamic equation links percentage changes in the real effective exchange rate to a disequilibrium term, one lag, the interest rate differential between the United States and its major trading partners, and short-run movements in the ratio of NFAs to potential GDP. This last term captures the effect of short-term capital inflows/outflows. Consequently, there is a dichotomy between the short-run and the long-run response of the exchange rate.

Following a demand shock, the exchange rate appreciates in the short run because of capital inflows and positive interest rate differentials, but then depreciates in order to generate a trade surplus consistent with a restoration of the NFA target. There are no revaluation effects in the current version of the model: changes in exchange rates or asset prices have no influence on the outstanding stock of foreign assets. As Tille (2003) reports, such revaluation effects can have a large impact on NFAs. In the context of MUSE, abstracting from these

\(^{28}\) One rationalization is provided by the literature on monetary policy that adopts the limited-participation approach. These models assume that monetary policy has real effects because of portfolio-adjustment costs.
revaluation effects implies that a greater improvement in the trade balance is required to attain the NFA target.\textsuperscript{29}

9. Shock Analysis

To summarize the empirical properties of MUSE, in this section we report the model’s response to various shocks. The complete listing of the model (in Troll) is available from the authors. Several relevant shocks help to illustrate the dynamic behaviour of MUSE. We focus on seven transitory shocks: a shock to demand, a shock to the federal funds rate, a shock to the exchange rate, a shock to government spending, a shock to the interest rate for business, an inflation shock, and an oil-price shock. We also focus on four permanent shocks: a shock to TFP, a shock to the inflation target, a permanent reduction in government size, and a permanent reduction in the ratio of government debt to GDP. Figures C3 to C13 in Appendix C show the response of MUSE to these shocks. Figures C1 and C2 summarize the linkages across the various components of MUSE following a demand and a supply shock, respectively. The dark arrows indicate the direct impacts, and the light arrows indicate the different adjustment mechanisms in the model; i.e., monetary policy, fiscal policy, and the exchange rate. In all simulations, we assume that foreign output and interest rates do not respond to shocks in the United States; agents have perfect foresight, since expectations are fully model-consistent.

9.1 A shock to private domestic demand

In this scenario, a demand shock stems from household spending and business investment, which both temporarily increase by 1 per cent relative to a base case.

\textsuperscript{29} By increasing the dollar value of foreign investment by U.S. citizens, this depreciation should increase NFAs. Thus, NFAs would improve more rapidly than they would by simply summing current account outcomes over the forecasting period.
(Appendix C, Figure C3). The shock translates into an excess demand peaking at about 0.8 per cent and lasting around 8 quarters. The opening of the output gap yields a small but persistent increase in inflation of 0.25 per cent. Reacting to both the current output gap and the inflation gap 4 quarters ahead, the monetary authority raises the policy rate by 100 basis points. The Fed engineers a small degree of excess supply to bring inflation back to the target. This rate increase feeds through the term structure and the cost of capital, thereby pushing consumption and investment back to control. Since fiscal policy is counter-cyclical, government transfers decrease following the shock. Note that counter-cyclical fiscal policy operates only through transfers. Lower transfers reduce personal income flows and human wealth, and depress household spending. The increase in private domestic demand has a positive effect on imports in the short run. Since this scenario assumes no response in foreign variables, exports fall in response to a short-term appreciation in the real exchange rate that results from higher domestic interest rates. The deterioration in the trade balance leads to a temporary worsening of the NFA position, thereby requiring a depreciation of the real exchange rate in the longer run in order to return NFA to the target position. This depreciation leads to a lower profile for imports in the longer run. Note the differing speeds of adjustment for the various components of GDP. Adjustment costs are highest for investment, which explains why this component is the slowest to return to equilibrium. This shock also highlights one of the model’s strengths: there is very little secondary cycling. In response to the 0.8 per cent shock to the output gap, the monetary and fiscal rules generate very little excess supply in subsequent years.

---

30 There is also an accelerating effect of output on investment.
9.2 A shock to the federal funds rate

The response to a shock of 100 basis points is shown in Appendix C, Figure C4. This is a peculiar shock in MUSE, since the monetary authority responds aggressively to its own actions. Thus, despite imposing a shock of 100 basis points on the nominal federal funds rate, these rates rise by slightly less than one in the base period, because the forward-looking monetary authority immediately responds to the disinflationary pressures created by the sudden rise in interest rates. In fact, the target rate falls immediately. Still, owing to smoothing, policy rates remain above control for 6 quarters. Through the term structure of interest rates, the change in short-term interest rates affects all interest rates in the model. Business-financing costs, for instance, rise by about 40 basis points. Higher interest rates reduce both consumption and investment in the early years of the simulation. At its peak, consumption is about 0.3 per cent lower than in the base case after one year, and investment is almost 0.5 per cent lower after about three years. This reflects the greater sensitivity of investment to interest rates. The impact on consumption would be greater but for the fiscal response, which generates an increase in government transfers, lending support to disposable income. Positive interest rate differentials generate an appreciation of the dollar, leading to lower exports in the short run. Imports fall as well, as the short-run effect that results from the reduction in private income dominates. Higher interest rates raise the interest costs of servicing NFA. To compensate, MUSE must generate a depreciation of the dollar in the longer run to improve the trade balance and return NFA to its target. On balance, an excess supply of about 0.25 per cent is generated by year two of the simulation. The resulting pressure on inflation forces a rapid response by the monetary authority, which

---

31 Higher adjustment costs also delay the response of investment over that of consumption. The target level for investment is significantly more variable than its dynamic path. Because the target level is a function of the user cost of capital, it varies with changes in business-financing costs.
cuts interest rates aggressively. In the simulation, inflation is reduced by almost 0.20 per cent by year two. The Fed must generate a small degree of excess demand in years four and five to ensure that inflation returns to target.

### 9.3 A shock to the exchange rate

In this scenario, there is a 1 per cent temporary appreciation of the exchange rate (Appendix C, Figure C5). The direct channels through which this variable affects the economy are relative import prices, exports, and relative investment prices. This positive terms-of-trade shock leads to a 0.15 per cent increase in imports and a 0.2 per cent reduction in exports around year two of the simulation. The deterioration in the trade balance is offset by a persistent increase in business investment, peaking at 0.1 per cent by year five. This positive response is attributed mainly to a temporary reduction in the relative price of imported equipment and software, in reaction to the exchange rate appreciation. The shock has virtually no impact on the other components of domestic demand: consumption and government expenditures rise only marginally. On balance, the output gap is reduced by only 0.03 per cent at its peak. Inflation is reduced slightly, in line with the output-gap response and the reduction in import prices. Interest rates show little reaction (-10 basis points). A more persistent exchange rate shock would generate larger impacts.

### 9.4 A shock to government spending

In this scenario, the level of government expenditures on goods and services rises temporarily by 1 per cent (Appendix C, Figure C6). The fiscal block is adjusted entirely through the tax rate. Consequently, a very persistent increase in the tax rate, peaking at 4 basis points, is required in order to bring the ratio of debt to GDP back to its steady-state target. As a result, consumption is directly affected via both the spending and tax channels.
Household spending rises very temporarily because of rule-of-thumb spending, but then falls in response to higher taxes in year two. An excess demand of 0.2 per cent is created, which leads the Fed to tighten policy by 20 basis points. Real long-term interest rates rise by 8 basis points, leading to a temporary negative effect of 0.1 per cent on business investment, peaking in year four. Owing to an increase in import volumes, the trade balance deteriorates in the short run. Again, the exchange rate appreciates over this horizon, in line with higher domestic interest rates and a weaker NFA position, but depreciates in the long run in order to restore external equilibrium (not shown).

9.5 A shock to the interest rate for business

In this scenario, there is a temporary 1 per cent shock to the real long-run interest rate for business (Appendix C, Figure C7). The shock lasts about two years and feeds directly into businesses’ borrowing costs. Business investment is the only variable that is directly affected by the shock. Higher user cost of capital reduces target investment by close to 6 per cent in the first year of the simulation. Adjustment costs, however, greatly limit the reaction of actual business investment, which falls by 0.5 per cent. The shock is offset by a 12-basis-point reduction in the federal funds rate and a slight rise in transfers, both of which contribute to a rise in consumption. Negative interest rate differentials and a stronger short-run NFA position reduce the exchange rate in the early years of the simulation. An opposite reaction of the dollar is, however, required in the longer run, to restore external balance. Although relatively persistent, the shock has a limited impact on the economy as a whole, since the reduction in the output gap and inflation is only about 0.03 per cent, because of the rapid fiscal and monetary responses.
9.6 An inflation shock

In this scenario, the inflation rate is temporarily increased by 1 per cent (Appendix C, Figure C8). The shock lasts about two years, which reflects the adjustment costs inherent in the inflation process. The Fed reacts quickly, tightening policy by 65 basis points, which seems rather small, but more than half of the inflation shock dissipates within the first year of the simulation. The transmission channels of the rate increase are as described in section 9.2. Private domestic demand and imports fall by about 0.3 per cent. Transfers rise by 0.7 per cent, offsetting part of the shock to household spending. Exports show little reaction, while the dollar appreciates owing to the creation of a positive interest rate differential. Overall, an excess supply of -0.25 per cent is created by year two, which is sufficient to return inflation to its target. Potential output falls temporarily, owing to weaker investment. The response is, however, very persistent, as a result of significant adjustment costs in structures investment.

9.7 An oil-price shock

In this scenario, there is a 20 per cent temporary increase in the relative price of West Texas Intermediate oil (Appendix C, Figure C9). The shock lasts one year. The direct channels through which oil prices affect the economy in MUSE are consumption, imports, and import prices. Higher oil prices affect rule-of-thumb spending, reducing consumption by 0.25 per cent one quarter after the shock. The shock to consumption is mitigated by the fiscal response. Import prices rise, so that the target level of imports is significantly reduced. Nevertheless, import volumes fall by only 0.1 per cent by the end of year one, since the
demand for oil imports is considered to be inelastic in the very short run. The other components of demand are largely unaffected. The reaction of inflation reflects that of the change in import prices: it is benign, since the oil-price shock is not persistent. On balance, an excess supply of 0.18 per cent is created and the Fed reacts by lowering the federal funds rate by 13 basis points. The persistence of the oil-price shock is key to determining the interest rate reaction. For instance, following an oil-price increase that is persistent enough to feed into expectations for, say, five years, the impact on inflation is such that the Fed raises, not lowers, rates. The persistence threshold at which the Fed reverses its reaction to the oil-price shock is around two years.

9.8 A shock to total factor productivity

In this scenario, there is an unexpected 1 per cent permanent increase in the level of TFP (Appendix C, Figure C10). This productivity shock is specific to the United States, since foreign output is exogenous in the simulation. Although demand reacts quickly, adjustment costs are such that the shock initially creates an excess supply of about 0.8 per cent. The output gap closes rapidly as demand adjusts to its new equilibrium. This positive supply shock has a disinflationary impact of -0.14 per cent. The Fed responds by decreasing interest rates by about 60 basis points. Household spending is positively affected by the permanent increase in human wealth, while investment flows rise in order to return to the equilibrium capital-output ratio. Owing to the negative output gap, government transfers increase significantly in the short run. Government expenditures rise by the same amount as output in the long run, to restore the steady-state size of government. Imports rise permanently, in line with the permanent increase in private domestic demand. The increase

---

32 The effect on import volumes of a shock on import prices is partly mitigated when oil prices are the source of the shock. This is true only in the case of a transitory shock. See equation (18).
in imports in the longer run deteriorates NFA such that, in order to bring NFA back to its target, a permanent depreciation of the exchange rate is needed. This depreciation raises exports by 1.1 per cent in the steady state.\(^{33}\) Had this shock been shared with the rest of the world, it would have had a neutral effect on the exchange rate in the steady state.

### 9.9 A shock to the inflation target

In this scenario, the monetary authority decides to permanently reduce the inflation target by 1 per cent (Appendix C, Figure C11). To convince agents that the reduction in inflation is permanent, the Fed needs to tighten policy by 65 basis points in real terms. This dampens consumption and business investment by 0.3 per cent and 1.0 per cent, respectively. Government transfers increase by 0.8 per cent, offsetting part of the shock to consumption. Higher domestic interest rates lead to a temporary appreciation of the exchange rate. Although the trade balance improves, higher interest-servicing costs dominate the NFA dynamics, so that MUSE must generate a depreciation of the dollar around year four to preserve the external balance. Overall, an excess supply lasting five years and peaking at 0.3 per cent is created.\(^{34}\) The sacrifice ratio (i.e., the cumulative loss of output needed to reduce the inflation target by 1 per cent) is 1.0 in MUSE. This is somewhat low relative to the existing literature. It could reflect either a reduction in the persistence of the inflation process or increased central bank credibility.\(^{35}\)

\(^{33}\) MUSE generates a completely different response in the context of an expected productivity shock. For instance, an increase in TFP that is expected to occur two years from now is inflationary, not deflationary, since it initially creates an excess demand: agents anticipate the shock to future income and increase demand immediately. In this case, the Fed raises rates and creates an excess supply, which eventually brings inflation back to target.

\(^{34}\) The fact that potential output falls very persistently is not very appealing. It occurs because of the very high adjustment cost required to explain the persistence in structures investment. In an alternative specification, we could explore the possibility of calibrating this equation.

\(^{35}\) This explanation is plausible, given that we account for structural breaks in the estimation of the Phillips curve. We obtain a sacrifice ratio of 1.9 if we do not account for breaks in the mean of inflation.
9.10 A permanent reduction in the size of government

In this scenario, the ratio of total government spending (including transfers) to GDP is lowered by 1 per cent (Appendix C, Figure C12). A permanent reduction of 3.5 per cent in the level of spending is required to decrease the size of government by 1 per cent. Fiscal adjustment in MUSE operates entirely through the tax rate, implying that a 1 per cent permanent reduction in the tax rate is needed to restore the steady-state ratio of debt to GDP. This change raises human wealth permanently and consumption is increased by 0.7 per cent in the steady state. Consequently, import volumes rise. A permanent 1 per cent depreciation of the dollar is required to generate the 0.7 per cent increase in exports necessary to restore the NFA target. Although very persistent, the negative effect on business investment is marginal. Interest rates rise by 10 basis points in reaction to expected inflation pressures and a small excess demand in year two to three of the simulation.

9.11 A permanent reduction in the ratio of government debt to GDP

In this scenario, we lower the steady-state target ratio of government debt to GDP by 10 per cent (Appendix C, Figure C13).\(^{36}\) The graphs in Figure C13 show 25 years of simulation, since adjustment to this shock is particularly slow. To reach this new equilibrium ratio, MUSE must generate a persistent increase in the tax rate, peaking at 0.8 per cent. Consumption is negatively affected by this change (-0.2 per cent at its peak). A 10 per cent permanent reduction in the debt ratio lowers the steady-state risk premium on long-term rates by about 30 basis points. This has a substantial impact on businesses’ financing costs, yielding an increase in business investment of 2 per cent in the long run. Higher investment

\(^{36}\) To be more realistic, this shock would require a recalibration of the steady-state target of the ratio of NFA to GDP. Typically, a 50 per cent rule of thumb is used in this case; i.e., a 10 per cent reduction in the ratio of government debt to GDP is accompanied by a 5 per cent rise in the ratio of NFA to GDP.
flows lead to a permanent increase in potential output of 0.35 per cent. Government expenditures and transfers rise equivalently to restore the equilibrium target for government size. Imports rise in the long run owing to the increase in investment. The dollar depreciates in order to generate the trade surplus necessary to attain the NFA target (not shown). Overall, this shock has virtually no impact on the output gap, inflation, and short-term interest rates.

10. Conclusion

Through an extensive application of PAC models, we have developed in MUSE what we believe to be a good balance between theoretical structure and forecasting accuracy (Appendix A) and between size and level of detail. As a consequence, it is hoped that this model will result in a more enlightened perspective on current and future economic developments in the United States. This is of critical importance to the conduct of Canadian monetary policy.

Importantly, MUSE can also be used for policy simulations. It can, for example, be used to examine issues ranging from how the U.S. economy might respond to an unsustainable current account deficit to the implications of rising government debt, and even analyze the impact of financial turbulence.

The work described in this report should be viewed as a first step in an approach to more accurately modelling economic developments in the United States. Much work remains to be done. For instance, some elements of the model that are currently exogenous, such as the short-term neutral interest rate or housing, will be endogenized in a future version. The option to run shocks under VAR expectations or using gradual recognition rules might also be useful. Nevertheless, based on the simulations reported here, as well as other simulations, we are hopeful that MUSE will prove to be a useful tool in the formulation of Canadian monetary policy.
References


Appendix A: Estimation Protocol for PAC Equations

The PAC models used in MUSE are estimated in three steps. In the first, the target level of a variable is estimated. This is the level of the variable that would be attained in the absence of adjustment costs. This level is best viewed as a cointegrating relationship between the variable of interest \( C \) and its long-run determinants \( X \). In the case of consumption, for instance, \( X \) could include permanent income or some measure of wealth:

\[
C_t = \delta X_t .
\]

Equation (A1) is estimated using Stock and Watson’s (1993) leads and lags methodology. In the second step, the expected future level of \( C \) is forecast using the \( \hat{\delta} \) estimated in (A1) and forecasts of the variables in \( X \). Typically, this is done with a VAR containing all information relevant to \( X \).\(^1\) \( C^* \) is obtained from the VAR forecasts and \( \hat{\delta} \).

The third step is to estimate the associated dynamic equation using ordinary least squares or generalized method of moments. This is an iterative process that uses the \( \hat{\delta} \), the VAR parameters, a calibrated discount rate (\( \beta \) from equation (1)), and starting-point values for \( a_j \).

Since MUSE is both a forecasting and a projection model, much weight is placed on forecasting ability in its empirical design. To this end, the results of the PAC models chosen in MUSE are compared with those of traditional error-correction specifications and naïve forecast rules. Although not included in this report, in most cases the PAC models have better in- and out-of-sample forecasting properties than either alternative error-correction

---

\(^1\) As a result, estimation does not assume rational expectations. MUSE, on the other hand, does assume fully model-consistent expectations. Thus, there exists some inconsistency between the estimated equations and the expectations-formulation process in simulation.
models (in both the in- and out-of-sample exercises) or autoregressive models (for out-of-sample forecasts).\(^2\) The fact that the forecasting performance of our PAC equations is always as good as, or better than, standard error-correction models implies that theoretical structure can be introduced without deteriorating the empirical properties of a model. In addition, the PAC models selected for MUSE exhibit greater parameter stability than alternative approaches. This characteristic is of primary importance for the model’s longer-term viability.

\(^2\) Results are available from the authors.
Appendix B: The Term Structure of Interest Rates

B.1 Government Bond Yields

In MUSE, government bond yields have a direct impact on government finances, and an indirect impact on household spending (through mortgage rates) and business investment (through the cost of capital).

The yield on government debt is determined by an error-correction mechanism, in which the long-run yield on Treasuries is determined by the expectations hypothesis. Thus, the long-run yield on Treasuries is a geometric average of future short-term interest rates, multiplied by a term premium:

\[
i_t^L = \left( 1 + \frac{\sum_k \mu_i^k i_{t+k}}{\sum_k \mu^k} \right)^* (1 + 0.03 \left( \frac{\text{debtfund}/Y_t}{\mu} \right)) - 1, \tag{B1}
\]

where \( i \) is the nominal federal funds rate, the discount factor (\( \mu \)) is set at 0.70, \( k \) corresponds to the bond maturity (40 quarters), and the term premium is a function of the ratio of government debt to GDP. The dynamic path for Treasuries is based on the following specification:

\[
\Delta i_t^L = -0.19(i_{t-1}^L - i_{t-1}^L) + 0.23\Delta i_{t-1}^L. \tag{B2}
\]

B.2 Mortgage Rates

In the model, mortgage rates are based on the yield on Treasuries and a multiplicative risk premium:

\[
i_t^M = ((1 + i_t^L)(1 + rpr^M)) - 1. \tag{B3}
\]

The risk premium is equal to the historical average spread (200 basis points).
B.3 Corporate Bond Rate

In the long run, the yield on high-grade corporate debt is governed by the long-run yield on Treasuries and a risk premium ($pr$) of 156 basis points:

$$i^B_t = ((1 + i^L_t)(1 + pr^B)) - 1,$$  \hspace{1cm} (B4)

with a dynamic profile set by the following error-correction mechanism:

$$\Delta i^B_t = -0.18(i^B_{t-1} - i^B_t) + 0.18\Delta i^B_{t-1}.$$ \hspace{1cm} (B5)

B.4 Cost of Equity Capital

Over history, the return on capital is given by the current and expected return on Standard and Poor’s 500 dividends:

$$req_t = rdiv_t + g_t,$$ \hspace{1cm} (B6)

where $rdiv_t$ are current dividend returns and $g_t$ are expected dividends per share given by a VAR. The equity-risk premium is then determined as the difference between $req_t$ and the real costs of high-grade corporate debt, $r_t^B$. Over the forecast horizon, the equity-risk premium is variable and characterized by an AR(1) process with a 0.69 root. The cost of equity capital is the sum of the equity-risk premium and the yield on corporate debt.

B.5 Business Financing Costs

Business financing costs enter the calculation of the user cost of capital in the business investment block. These costs are a weighted average of both the equity cost of capital and the yield on high-grade corporate debt:

$$fcst_t = 0.50r_t^B + (1 - 0.50)req_t.$$ \hspace{1cm} (B7)
Appendix C: Dynamic Response of MUSE

Figure C1: Demand Shock

Dark arrows: direct impact
Light arrows: adjustment mechanisms
Figure C2: Supply Shock

Dark arrows: direct impact
Light arrows: adjustment mechanisms
Figure C3: A 1% Shock to Demand (response in %)\(^1\)

\(^1\) For each chart in this appendix, the dashed line represents the item identified in brackets.
Figure C3 (concluded): A 1% Shock to Demand (response in %)
Figure C4: A 1% Shock to the Federal Funds Rate (response in %)
Figure C4 (concluded): A 1% Shock to the Federal Funds Rate (response in %)
Figure C5: A 1% Shock to the Exchange Rate (response in %)

Output (and Potential Output) $(1=1\%)$

Output Gap $(1=1\%)$

Reference Inflation Rate $(1=1\%)$

Real Federal Funds Rate $(1=1\%)$

Nominal Federal Funds Rate (and Target Rate) $(1=1\%)$

Real Long-Run Interest Rate for Business $(1=1\%)$
Figure C5 (concluded): A 1% Shock to the Exchange Rate (response in %)
Figure C6: A 1% Shock to Government Spending (response in %)
Figure C6 (concluded): A 1% Shock to Government Spending (response in %)

- **Consumption (and Target Consumption)**
  - $(1=1\%)$

- **Investment (and Target Investment)**
  - $(1=1\%)$

- **Government Spending (and Transfers)**
  - $(1=1\%)$

- **Exports (and Target Exports)**
  - $(1=1\%)$

- **Imports (and Target Imports)**
  - $(1=1\%)$

- **Tax Rate (and Target Tax Rate)**
  - $(1=1\%)$
Figure C7: A 1% Shock to the Interest Rate for Business (response in %)

Output (and Potential Output) (1=1%)

Output Gap (1=1%)

Reference Inflation Rate (1=1%)

Real Federal Funds Rate (1=1%)

Nominal Federal Funds Rate (and Target Rate) (1=1%)

Real Long-Run Interest Rate for Business (1=1%)

(continued)
Figure C7 (concluded): A 1% Shock to the Interest Rate for Business (response in %)
Figure C8: A 1% Inflation Shock (response in %)

Output (and Potential Output) (1=1%)

Reference Inflation Rate (1=1%)

Real Federal Funds Rate (1=1%)

Nominal Federal Funds Rate (and Target Rate) (1=1%)

Real Long-Run Interest Rate for Business (1=1%)

(continued)
Figure C8 (concluded): A 1% Inflation Shock (response in %)

- Consumption (and Target Consumption) (1=1%)
- Investment (and Target Investment) (1=1%)
- Government Spending (and Transfers) (1=1%)
- Exports (and Target Exports) (1=1%)
- Imports (and Target Imports) (1=1%)
- Exchange Rate (and Target Exchange Rate) (1=1%)
Figure C9: A 20% Shock to the Relative Price of West Texas Intermediate Oil (response in %)
Figure C9 (concluded): A 20% Shock to the Relative Price of West Texas Intermediate Oil (response in %)
Figure C10: A 1% Permanent Shock to TFP (response in %)
Figure C10 (concluded): A 1% Permanent Shock to TFP (response in %)
Figure C11: A 1% Permanent Reduction in the Inflation Target (response in %)

(continued)
Figure C11 (concluded): A 1% Permanent Reduction in the Inflation Target (response in %)
Figure C12: A 1% Permanent Reduction in Government Size (response in %)

(continued)
Figure C12 (concluded): A 1% Permanent Reduction in Government Size (response in %)
Figure C13: A 10% Permanent Reduction in the Ratio of Debt to GDP (response in %)
Figure C13 (concluded): A 10% Permanent Reduction in the Ratio of Debt to GDP (response in %)
2003
95 Essays on Financial Stability  J. Chant, A. Lai, M. Illing, and F. Daniel
94 A Comparison of Twelve Macroeconomic Models of the Canadian Economy  D. Côté, J. Kuszczak, J.-P. Lam, Y. Liu, and P. St-Amant
93 Money in the Bank (of Canada)  D. Longworth

2002
92 The Performance and Robustness of Simple Monetary Policy Rules in Models of the Canadian Economy  D. Côté, J. Kuszczak, J.-P. Lam, Y. Liu, and P. St-Amant
91 The Financial Services Sector: An Update on Recent Developments  C. Freedman and J. Powell
90 Dollarization in Canada: The Bucks Stops There  J. Murray and J. Powell

2001
89 Core Inflation  S. Hogan, M. Johnson, and T. Laflèche

2000

1999
87 The Regulation of Central Securities Depositories and the Linkages between CSDs and Large-Value Payment Systems  C. Freedman
86 Greater Transparency in Monetary Policy: Impact on Financial Markets  P. Muller and M. Zelmer
85 Inflation Targeting under Uncertainty  G. Srour
84 Yield Curve Modelling at the Bank of Canada  D. Bolder and D. Stréliski

1998
83 The Benefits of Low Inflation: Taking Stock  B. O’Reilly
82 The Financial Services Sector: Past Changes and Future Prospects  C. Freedman and C. Goodlet

Copies and a complete list of technical reports are available from:

Publications Distribution, Bank of Canada  Diffusion des publications, Banque du Canada
234 Wellington Street, Ottawa, Ontario K1A 0G9  234, rue Wellington, Ottawa (Ontario) K1A 0G9
Email: publications@bankofcanada.ca  Adresse électronique : publications@banqueducanada.ca
Website: http://www.bankofcanada.ca  Site Web : http://www.banqueducanada.ca