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Do Mechanical Filters Provide a Good Approximation of Business Cycles?

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ABSTRACT

In this paper, the authors examine how well the Hodrick-Prescott filter (HP) and the band-pass filter recently proposed by Baxter and King (BK) extract the business-cycle component of macroeconomic time series. The authors assess these filters using two different definitions of the business-cycle component. First, they define that component to be fluctuations lasting no fewer than six and no more than thirty-two quarters; this is the definition of business-cycle frequencies used by Baxter and King. Second, they define the business-cycle component on the basis of a decomposition of the series into permanent and transitory components. In both cases the conclusions are the same. The filters perform adequately when the spectrum of the original series has a peak at business-cycle frequencies. When the spectrum is dominated by low frequencies, the filters provide a distorted business cycle. Since most macroeconomic series have the typical Granger shape, the HP and BK filters perform poorly in terms of identifying the business cycles of these series.

RÉSUMÉ

Dans la présente étude, les auteurs cherchent à évaluer l'efficacité avec laquelle le filtre de Hodrick-Prescott (HP) et le filtre passe-bande récemment proposé par Baxter et King (BK) permettent d'isoler la composante cyclique des séries macroéconomiques. Ils utilisent deux définitions du cycle économique pour comparer la performance de ces filtres. Selon la première définition (celle que retiennent Baxter et King), la composante cyclique correspond à des fluctuations d'une durée minimale de six trimestres et maximale de trente-deux trimestres. L'autre définition du cycle consiste dans la décomposition de la série en deux composantes, l'une permanente et l'autre transitoire. Les auteurs parviennent aux mêmes conclusions peu importe la définition utilisée. Les filtres donnent des résultats satisfaisants lorsque le spectre de la série initiale atteint un sommet au voisinage des fréquences comprises entre six et trente-deux trimestres. Lorsque le spectre est dominé par les basses fréquences, le cycle économique obtenu donne une image faussée de la réalité. Comme la forme spectrale de la plupart des séries macroéconomiques ressemble à celle que Granger a mise en lumière, les filtres HP et BK réussissent mal à isoler la composante cyclique de ces séries.

CONTENTS

1 INTRODUCTION

Identifying the business-cycle component of macroeconomic time series is essential for applied business-cycle researchers. Since the influential paper of Nelson and Plosser (1982), which suggested that macroeconomic time series could be better characterized by stochastic trends than by linear trends, methods for stochastic detrending have been developed. In particular, this has led to the increasing use of mechanical filters to identify permanent and cyclical components of a time series. The most popular filter-based method is probably that proposed by Hodrick and Prescott (1981), known as the HP filter. More recently, Baxter and King (1995) have proposed a band-pass filter, the BK filter, whose purpose is to isolate certain frequencies in the data. This filter has already been used in empirical studies.[1]

The use of the HP filter has already been criticized. King and Rebelo (1993) provide examples of how measures of persistence, variability, and comovement are altered when the HP filter is applied to observed time series and to series simulated with real business-cycle models. Harvey and Jaeger (1993) and Cogley and Nason (1995a) show that spurious cyclicality is induced when the HP filter is applied to the level of a random-walk process. Osborn (1995) reports a similar result for a simple moving-average detrending filter. The above results were obtained by comparing the cyclical component obtained by applying the filters to the level of the series with the component corresponding to the business-cycle frequencies of time series in difference.

The objective of this paper is to examine how well the Hodrick-Prescott (HP) and Baxter-King (BK) filters extract the business-cycle component of macroeconomic series. In particular, we seek to characterize the conditions necessary to obtain a good approximation of the cyclical component with the HP and BK filters. Previous papers aimed at evaluating the performance of the HP filter have focussed on specific processes and used

1. See Baxter (1994), King, Stock and Watson (1995), and Cecchetti and Kashyap (1995). Other types of band-pass filters have also been proposed. For example, see Hasler et al. (1994).

unclear definitions of the business-cycle component. For example, one might ask how well filters perform if macroeconomic time series are not simply random walks but do, in fact, possess a business cycle. Our aim is to obtain general results that can be applied to a large class of time-series processes and to provide clear indications on the appropriateness of the HP and BK filters in applied macroeconomic work. We also hope that our findings will shed some light on the results obtained by previous studies.

To do this, we need to define the business-cycle component of macroeconomic series. In the first part of this paper, we retain the definition of business cycles proposed by researchers at the National Bureau for Economic Research and adopted by Baxter and King, which is based on the method put forward by Burns and Mitchell (1946). These authors define business cycles as fluctuations lasting no fewer than six and no more than thirty-two quarters. An ideal filter should extract this specific range of periodicities without altering the properties of the extracted component. To assess the performance of the HP and BK filters on this basis, we compare the spectra of unfiltered series at these frequencies with those of their filtered counterparts for several processes.

Our main conclusion is as follows. The HP and BK filters do well in terms of extracting the business-cycle frequencies of time series whose spectra peak at those frequencies. Unfortunately, the peak of spectral density in most macroeconomic series is at lower frequencies. Indeed, it is well known that macroeconomic series have the typical spectral shape identified by Granger (1966), with most of their power at low frequencies and spectra that decrease sharply and monotonically at higher frequencies. For such series, the HP and BK filters perform poorly in terms of extracting business-cycle frequencies. The intuition behind this result is simple: much of the power of typical macroeconomic time series at business-cycle frequencies is concentrated in the band where the squared gain of both the HP and BK filters differs from that of an ideal filter. Moreover, the shape of the squared gain of those filters, when applied to typical macroeconomic time series, induces a peak in the spectrum of the cyclical component that is absent from the original series. When the HP and BK filters are applied, they induce spurious dynamic properties and they extract a cyclical component that fails to capture a significant part of the variance contained in business-cycle frequencies.

However, macroeconomic time series are often represented as an unobserved permanent component containing a unit root and an unobserved cyclical component. While the HP and BK filters do not provide a good approximation of business-cycle frequencies for the series in level, they might still provide a good approximation of an unobserved cyclical component if this component were characterized by a peak in its spectrum at business-cycle frequencies. We explore this possibility through a simulation study. The data-generating process is a structural time-series model composed of a random walk plus a cyclical component. Both components are uncorrelated, and the cyclical component can have a peak in its spectrum at business-cycle frequencies. The filters perform adequately when the spectrum of the original series (including the permanent and cyclical components) has a peak at business-cycle frequencies. However, when the series is dominated by low frequencies, the HP and BK filters provide a distorted cyclical component. The series is dominated by low frequencies when the permanent component is large relative to the cyclical component or the cyclical component has its peak at zero frequencies. Since most macroeconomic series have the typical Granger shape, the application of these mechanical filters is likely to provide a distorted cyclical component. Our result also holds for more general specifications of the permanent component and for a specification containing a cyclical component correlated with the permanent component.

These results help us understand the findings of King and Rebelo (1993) for simulated series obtained with a real business-cycle model. It is now well known that this model has few internal propagation mechanisms.[2] Indeed, the dynamic of output for this model corresponds almost exactly to the dynamic of exogenous shocks. King and Rebelo report persistence, volatilities and comovement of simulated series for cases where the exogenous process is a first-order autoregressive process with coeffi-

2. See King, Plosser and Rebelo (1988), Cogley and Nason (1995b), and Rotemberg and Woodford (1996) for a discussion of this point.

cients of 0.9 and 1. For these processes, the spectral densities of output, consumption and investment in level are dominated by low frequencies. Applying the HP filter to these simulated series provides distorted cyclical properties. The same argument explains the findings of Harvey and Jaeger (1993) and Cogley and Nason (1995a) for a random-walk process.

The paper is organized as follows. In Section 2, we present the HP and BK filters and briefly discuss the existing literature on the HP filter. In Section 3, we examine how well the HP and BK filters extract frequencies corresponding to fluctuations of between six and thirty-two quarters. In Section 4, we present a simulation study to assess how well these filters retrieve the cyclical component of a time series composed of a random walk and a transitory component. In Section 5 we compare the cyclical component resulting from the application of the HP and BK filters with those obtained with the detrending methods proposed by Watson (1986) and Cochrane (1994) for U.S. output. We then present our conclusions and propose alternative methods to identify the business-cycle component.

2 THE HP AND BK FILTERS

2.1 The HP filter

The HP filter decomposes a time series y_t into an additive cyclical component (y_t^c) and a growth component (y_t^g):

$$y_t = y_t^g + y_t^c.$$

Applying the HP filter involves minimizing the variance of the cyclical component y_t^c subject to a penalty for the variation in the second difference of the growth component y_t^g,

$$\{y_t^g\}_{t=0}^{T+1} = argmin \sum_{t=1}^{T} [(y_t - y_t^g)^2 + \lambda[(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2],$$

where λ, the smoothness parameter, penalizes the variability in the growth component. The larger the value of λ, the smoother the growth component. As λ approaches infinity, the growth component corresponds to a linear time trend. For quarterly data, Hodrick and Prescott propose to set $\lambda = 1600$. King and Rebelo (1993) show that the HP filter can render stationary any integrated process of up to the fourth order.

A number of authors have studied the HP filter's basic properties. As shown by Harvey and Jaeger (1993) and King and Rebelo (1993), the infinite-sample version of the HP filter can be rationalized as the optimal linear filter of the trend component for the following process:[3]

$$y_t = \mu_t + \varepsilon_t,$$

where ε_t is an $NID(0, \sigma_\varepsilon^2)$ irregular component and the trend component, μ_t, is defined by

3. That is, the filter that minimizes the mean square error $MSE = (1/T)\sum_{t=1}^{T}(\hat{y}_t^c - y_t^c)^2$, where y_t^c is the true cyclical component and $\hat{y}_t^c$ is its estimate.

$$\mu_t = \mu_{t-1} + \beta_{t-1},$$

$$\beta_t = \beta_{t-1} + \zeta_t,$$

with $\zeta_t \sim NID(0,\sigma^2)$. β_t is the slope of the process and ζ_t is independent of the irregular component. Note that this trend component is integrated of order two, i.e., stationary in second differences.

The use of the HP filter to identify the cyclical component of most macroeconomic time series cannot be justified on the basis of optimal filtering arguments because of likely problems with the following associated assumptions:

1. *No correlation between transitory and trend components.* This implies that the growth and cyclical components of a time series are generated by distinct economic forces; this is often incompatible with business-cycle models (see Singleton 1988 for a discussion).

2. *The process y_t is integrated of order two.* This is often incompatible with priors on macroeconomic time series. For example, it is usually assumed that real GDP is integrated of order one or stationary around a breaking trend.

3. *The transitory component is white noise.* This is also questionable. For example, it is unlikely that the stationary component of output is strictly white noise. King and Rebelo (1993) show that this condition can be replaced by the following assumption: *an identical dynamic mechanism propagates changes in the trend component and innovations to the cyclical component.* However, this condition is also very restrictive.

4. *The parameter controlling the smoothness of the trend component, λ, is appropriate.* Note that λ corresponds to the ratio of the variance of the irregular component to that of the trend component. Economic theory provides little or no guidance as to what this ratio should be. While attempts have been made to estimate this parameter using maximum-likelihood methods (see Harvey and Jaeger 1993 or Côté

and Hostland 1994), it appears difficult to estimate λ with reasonable precision.

Moreover, for the finite-sample version of the HP filter, the user should not be interested in data points near the beginning or the end of the sample. This is simply a consequence of the fact that the HP filter, a two-sided filter, changes its nature and becomes more like a one-sided filter as it approaches the beginning or the end of a time series. Indeed, after studying the properties of the HP filter at those extremities, Baxter and King (1995) recommend that three years of data be dropped at both ends of a time series when the HP filter is applied to quarterly or annual data.[4]

Despite these shortcomings, Singleton (1988) shows that the HP filter can still be a good approximation of a high-pass filter when it is applied to stationary time series. Here we need to introduce some elements of spectral analysis. A zero-mean stationary process has a Cramer representation as follows:

$$y_t = \int_{-\pi}^{\pi} \varepsilon^{i\omega t} dz(\omega),$$

where $dz(\omega)$ is a complex value of orthogonal increments, i is the imaginary number ($\sqrt{(-1)}$) and ω is frequency measured in radians, i.e., $-\pi \le \omega \le \pi$ (see Priestley 1981, Chapter 4). In turn, filtered time series can be expressed as

$$y_t^f = \int_{-\pi}^{\pi} \alpha(\omega) e^{i\omega t} dz(\omega),$$

with

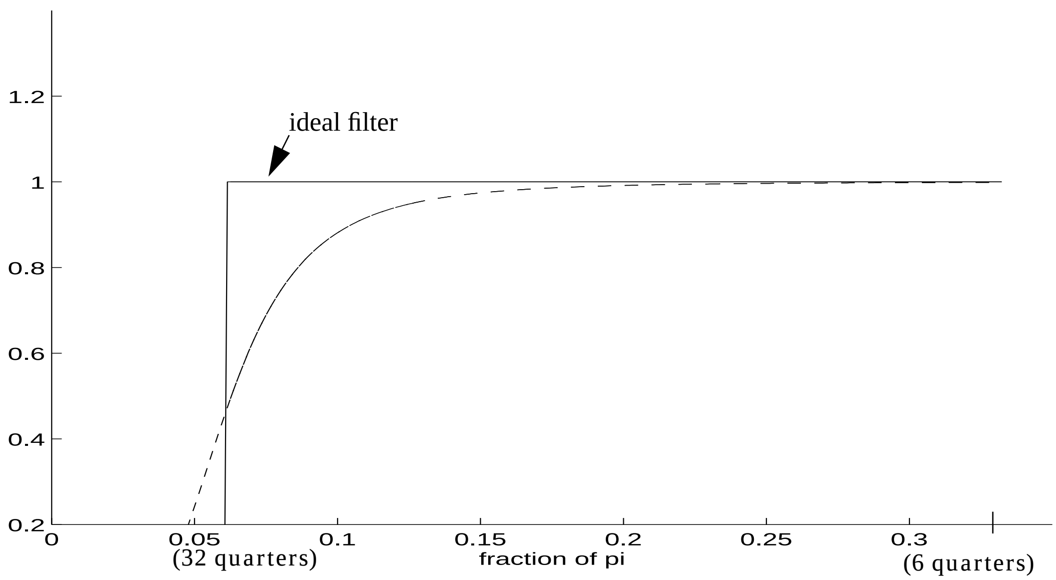
$$\alpha(\omega) = \sum_{h=-k}^{k} a_h e^{-i\omega h}. \tag{1}$$

4. This is clearly a problem for policy makers hoping to use the HP filter to estimate current potential output. This is discussed in Laxton and Tetlow (1992) and van Norden (1995).

Equation 1 is the frequency response (Fourier transform) of the filter. That is, $\alpha(\omega)$ indicates the extent to which y_t^f responds to y_t at frequency ω and can be regarded as the weight attached to the periodic component $e^{i\omega t}dz(\omega)$. In the case of symmetric filters, the Fourier transform is also called the gain of the filter.

An ideal high-pass filter would remove low frequencies or long-cycle components and allow high frequencies or short-cycle components to pass through, so that $\alpha(\omega) = 0$ for $|\omega| \leq \omega^p$, where ω^p has some predetermined value, and $\alpha(\omega) = 1$ for $|\omega| > \omega^p$. Chart 1 shows the squared gain of the HP filter. We see that the squared gain is 0 at zero frequency and is close to 1 from around frequency $\pi/10$ and higher. Thus, the HP filter appears to be a good approximation of a high-pass filter, in that it removes low frequencies and passes through higher frequencies.

Chart 1: Squared gain of the HP filter



A major problem is that most macroeconomic time series are either integrated or highly persistent processes, so that they are better characterized in small samples as non-stationary processes rather than stationary. In their study of the implications of applying the HP filter to integrated or highly persistent time series, Cogley and Nason (1995a) argue that the HP filter is equivalent to a two-step linear filter that initially first-differences the data to make them stationary and then smooths the differenced data with the resulting asymmetric filter. The filter tends to amplify cycles at business-cycle frequencies in the detrended data and to dampen long-run and short-run fluctuations. Cogley and Nason conclude that the filter can generate business-cycle periodicity even if none is present in the data. Harvey and Jaeger (1993) make the same point.[5] To better understand this result, consider the following I(1) process

$$(1-L)y_t = \varepsilon_t, \tag{2}$$

where ε_t is zero-mean and stationary. King and Rebelo (1993) show that the HP cyclical filter can be rewritten as $(1-L)^4 H(L)$. We define $|HP(\omega)|^2$ as the squared gain corresponding to the HP cyclical filter, where $HP(\omega)$ is the Fourier transform of $(1-L)^4 H(L)$ at frequency ω. When the HP filter is applied to the level of the series y_t, the spectrum of the cyclical component is defined as

$$f_{y^c}(\omega) = |HP(\omega)|^2 |1-\exp(-i\omega)|^{-2} f_{\varepsilon}(\omega),$$

where $(1-\exp(-i\omega))$ is the Fourier transform of $(1-L)$ and $f_{\varepsilon}(\omega)$ is the spectrum of ε_t, which is well defined since ε_t is a stationary process. Obviously, $|1-\exp(-i\omega)|^{-2}$ is not defined for $\omega = 0$. The expression $|1-\exp(-i\omega)|^{-2} f_{\varepsilon}(\omega)$ is often called the pseudo-spectrum of y_t (see Gouriéroux and Monfort 1995).

Cogley and Nason (1995a) and Harvey and Jaeger (1993) calculate the squared gain of the HP cyclical component for $(1-L)y_t$. In this case, the squared gain is equal to $(1-L)^3 H(L)$, given that

5. Classic examples of filter-induced cyclicality in the context of stationary time series are Slustsky (1937) and Howrey (1968). These examples are discussed in Chapter 11 of Sargent (1987).

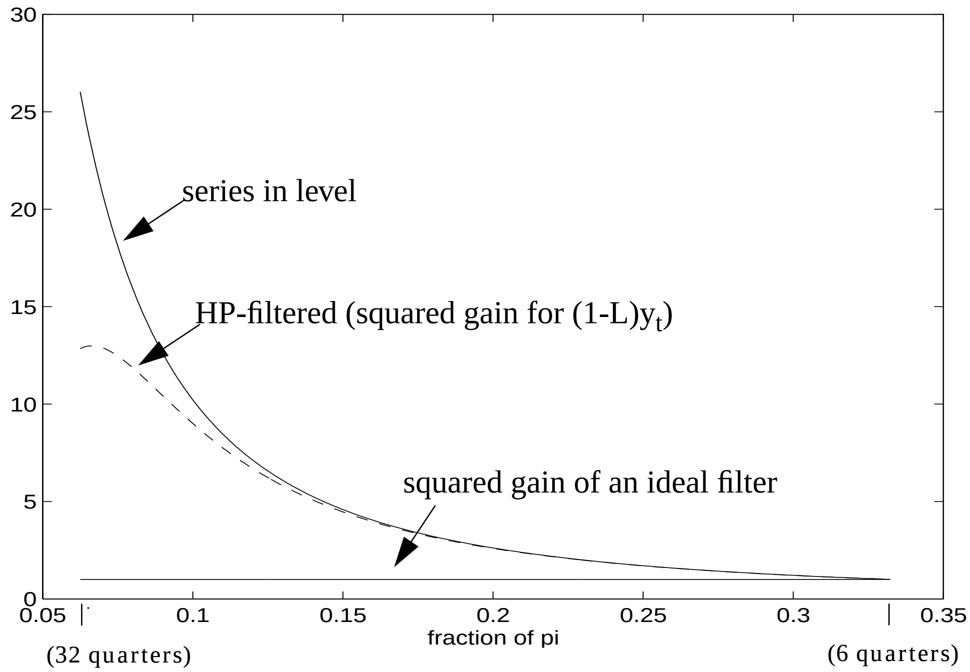
$(1-L)^4 H(L) y_t = (1-L)^3 H(L)(1-L) y_t$. By the Fourier transform, the squared gain corresponding to the filter applied to $(1-L)y_t$ is $|HP(\omega)|^2 |1-\exp(-i\omega)|^{-2}$. The dashed line in Chart 2 represents this squared gain. These authors conclude that applying the HP filter to the level of a random walk produces detrended series that have the characteristics of a business cycle. When this squared gain is compared with the ideal squared gain for the series in difference, we can see that the filter amplifies business-cycle frequencies and produces spurious dynamics.

Now suppose that ε_t in equation 2 is a white-noise process with variance equal to 2π, so that the spectrum of ε_t is equal to 1 at each frequency. We choose this example because the squared gain calculated by Cogley and Nason corresponds to the cyclical component extracted by the HP filter in this specific case. Chart 2 presents the pseudo-spectrum of y_t and the spectrum of the cyclical component identified by the HP filter for business-cycle frequencies. We can see that the effect of the HP filter is quite different depending on whether we are interested in retrieving the component corresponding to business-cycle frequencies for the level of the series y_t or for the series in difference $(1-L)y_t$.[6] Indeed, if the performance of the HP filter is to be judged by how well it extracts a specified range of periodicities, which is the first of the six objectives set by Baxter and King (1995) for their band-pass filter, the spectrum of the extracted component should be compared with the spectrum (or pseudo-spectrum) of the series in level. The conclusion then differs from that of Cogley and Nason (1995a) and Harvey and Jaeger (1993). We still find that the spectrum of the cyclical component identified by the HP filter has a peak corresponding to a period of 30 quarters that is absent from the spectrum of the original series. However, we also find that the filter actually dampens business-cycle fluctuations so that business-cycle frequencies become relatively less important. Thus, the conclusion depends on the definition of the business-cycle component. Moreover, the conclusions of Cogley and Nason (1995a) and of Harvey and Jaeger (1993) may not hold for the cyclical com-

6. The fact that we are interested in extracting business-cycle frequencies from the level of integrated series may appear problematic. Note that we could also consider an AR(1) process with a coefficient of 0.95 and obtain the same result.

ponent of processes other than random walks. We consider these points in Sections 3 and 4 respectively.

CHART 2: Spectrum of y_t with and without HP-filtering
(at frequencies between six and thirty-two quarters)



2.2 The BK filter

While an ideal high-pass filter removes low frequencies from the data, an ideal band-pass filter removes both low and high frequencies. Baxter and King (1995) propose a finite moving-average approximation of an ideal band-pass filter based on Burns and Mitchell's (1946) definition of a business cycle. The BK filter is designed to pass through components of time series with fluctuations between six and thirty-two quarters while removing higher and lower frequencies.

When applied to quarterly data, the band-pass filter proposed by Baxter and King takes the form of a 24-quarter moving average,

$$y_t^f = \sum_{h=-12}^{12} a_h y_{t-h} = a(L) y_t,$$

where L is the lag operator. The weights a_h can be derived from the inverse Fourier transform of the frequency-response function (see Priestley 1981, 274). Baxter and King adjust the band-pass filter by imposing a constraint that the gain is 0 at zero frequency. This constraint implies that the sum of the moving-average coefficients must be 0. When using the BK filter, 12 quarters are sacrificed at the beginning and the end of the time series, seriously limiting its usefulness for analysing contemporaneous data.

To study some time and frequency domain properties of the BK filter, assume the following data-generating process for y_t:

$$y_t = (1-L)^{-r} \varepsilon_t, \tag{3}$$

where r determines the order of integration of y_t and ε_t is a zero-mean stationary process. Baxter and King show that their filter can be factorized as

$$a(L) = (1-L)^2 a^*(L),$$

so that it is able to render stationary any time series containing up to two unit roots.

CHART 3a: Autocorrelations corresponding to the BK filter

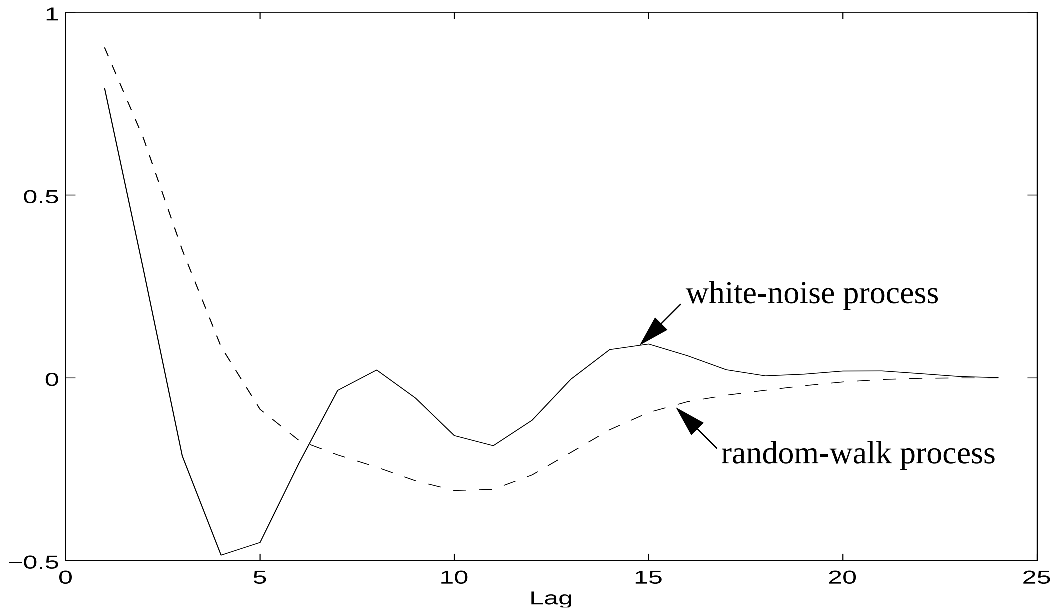


Chart 3b: Autocorrelations corresponding to the HP filter

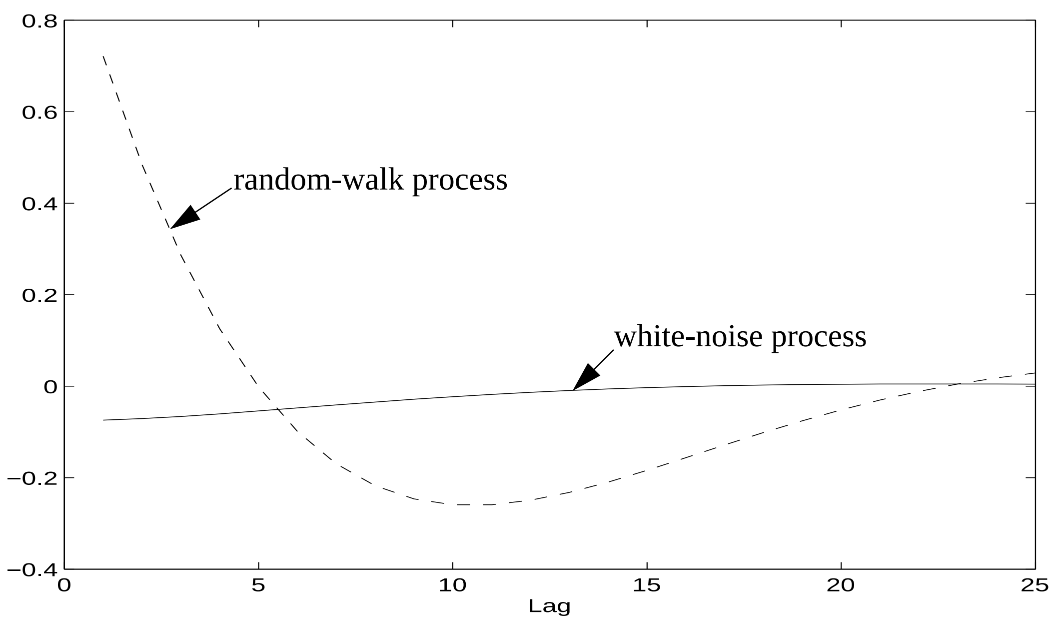


Chart 3a shows the autocorrelation functions for the BK-filtered version of a white-noise process and a random-walk process. In both cases, the cyclical component identified by the BK filter possesses strong positive autocorrelations at shorter horizons. The result for the random walk is similar to what Cogley and Nason (1995a) find for the HP filter (shown in Chart 3b). However, in contrast to the HP filter, the cyclical component identified by the BK filter displays strong dynamics for a white-noise process. This result precludes using the autocorrelation functions resulting from this band-pass filter to evaluate the internal dynamic propagation mechanism of business-cycle models.

The spectrum of the cyclical component obtained by applying the BK filter is

$$f_{y^c}(\omega) = |BK(\omega)|^2 f_y(\omega),$$

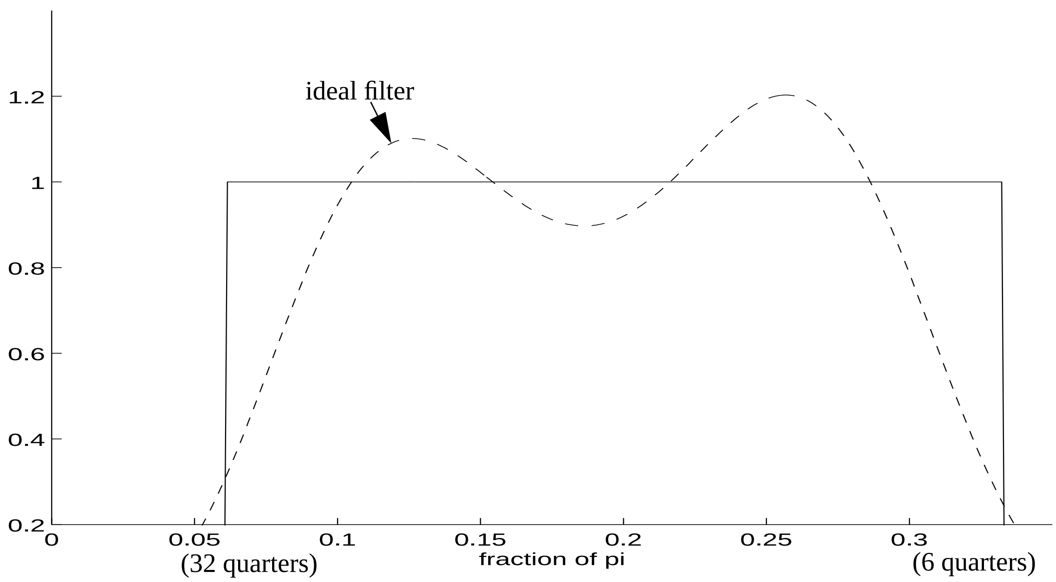
where $|BK(\omega)|^2$ is the squared gain of the BK filter and $f_y(\omega)$ is the spectrum of y_t. The squared gain $|BK(\omega)|^2$ is equal to $|a(\omega)|^2$, where $a(\omega)$ denotes the Fourier transform of $a(L)$ at frequency ω. The pseudo-spectrum of y_t is equal to

$$f_y(\omega) = |1 - \exp(-i\omega)|^{-2r} f_\varepsilon(\omega) = 2^{-2r} (sin^2(\omega/2))^{-r} f_\varepsilon(\omega)$$

for $\omega \neq 0$ (see Priestley 1981, 597), where $f_\varepsilon(\omega)$ is the spectrum of the process ε_t, which is well defined since ε_t is stationary.

Chart 4a presents the squared gain of the BK filter and compares it with the squared gain of the ideal filter. The BK filter is designed to remove low and high frequencies from the data. This is basically what is obtained. The filter passes through most components with fluctuations of between six and thirty-two quarters (respectively $\pi/3$ and $\pi/16$), while removing components at higher and lower frequencies. However, the BK filter does not correspond exactly to the ideal band-pass filter (also shown on the graph) because it is a finite approximation of an infinite moving-average filter. In particular, at lower and higher frequencies we observe a compression effect, so that the squared gain is less than 1.

CHART 4a: Squared gain of the BK filter



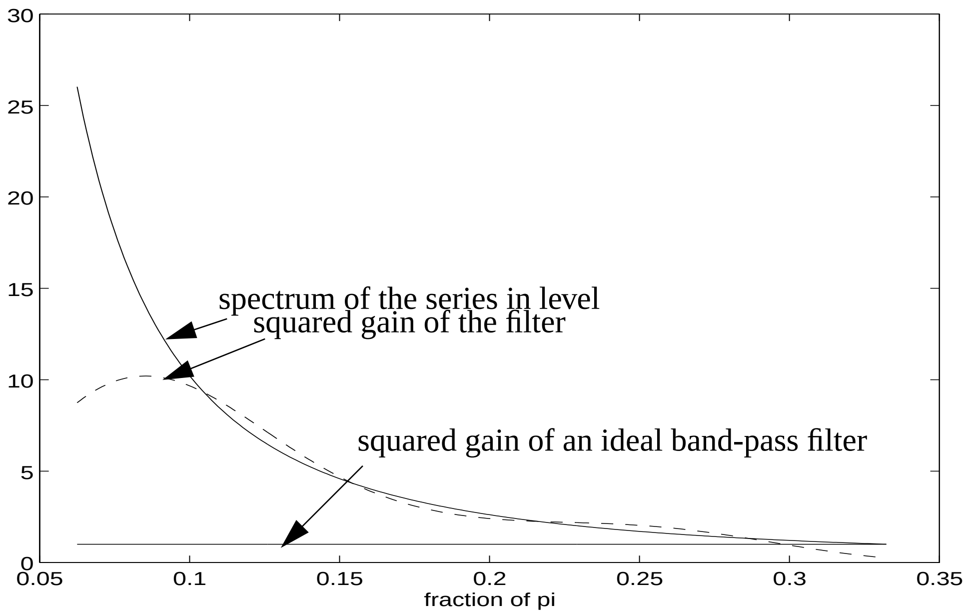
As in Section 2.1, we now assume that r=1 and that ε_t is white noise with variance equal to 2π in equation 3. The spectrum of ε_t is then equal to 1 at all frequencies, and the cyclical component obtained with the BK filter corresponds exactly to the squared gain for the BK filter calculated by Cogley and Nason (1995a) and by Harvey and Jaeger (1993) for the HP filter:

$$|BK(\omega)|^2|1-\exp(-i\omega)|^{-2} = |BK(\omega)|^2 2^{-2} \ (\sin^2(\omega/2))^{-1}.$$

Chart 4b presents the pseudo-spectrum of y_t and the spectrum of the cyclical component identified by the BK filter at business-cycle frequencies. The conclusion once again depends on whether we are interested in retrieving the component corresponding to business-cycle frequencies for the level of series y_t or for the series in difference $(1-L)y_t$. In the latter case, as noted by Cogley and Nason and by Harvey and Jaeger for the HP filter, the BK filter greatly amplifies business-cycle frequencies and creates spurious cycles compared with the ideal squared gain for the series in difference. For example, it amplifies by a factor of ten the variance of cycles with a periodicity of around 20 quarters ($\pi/10$). Also, as in the case of the HP filter, business-cycle frequencies of the BK-filtered series are less important than those of the original series in level, and the cyclical component identified by the BK filter has a peak corresponding to a period of 20 quarters (compared with 30 quarters in the case of the HP filter) that is absent from the spectrum of the level of the series y_t.

CHART 4b: Squared gain of the BK filter

(Case of a random-walk process)



3 ABILITY OF THE FILTERS TO EXTRACT CYCLICAL PERIODICITIES

In this section, we examine how well the BK and HP filters capture the cyclical component of macroeconomic time series. Baxter and King's (1995) first objective is to adequately extract a specified range of periodicities without altering the properties of this extracted component. We use the same criteria to assess the performance of the HP and BK filters. We show that when the peak of the spectral-density function of these series lies within business-cycle frequencies, these filters provide a good approximation of the corresponding cyclical component. If the peak is located at zero frequency, so that the bulk of the variance is located in low frequencies, those filters cannot identify the cyclical component adequately.

To show this, we consider the following data-generating process (DGP),

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t, \tag{4}$$

where $\phi_1 + \phi_2 < 1$. A second-order autoregressive process is useful for our purpose because its spectrum may have a peak at business-cycle frequencies or at zero frequency. The spectrum of this process is equal to

$$f_y(\omega) = \frac{\sigma_\varepsilon^2}{1 + \phi_1^2 + \phi_2^2 - 2\phi_1(1-\phi_2)\cos\omega - 2\phi_2\cos 2\omega}$$

and the location of its peak is given by

$$-\sigma_\varepsilon^{-2} f_y(\omega)^2((2\sin\omega)[\phi_1(1-\phi_2) + 4\phi_2\cos\omega]).$$

Thus, $f_y(\omega)$ has a peak at frequencies other than zero for

$$\phi_2 < 0 \text{ and } \left|\frac{-\phi_1(1-\phi_2)}{4\phi_2}\right| < 1. \tag{5}$$

Therefore, $f_y(\omega)$ has its peak at $\omega = cos^{-1}(-\phi_1(1-\phi_2)/4\phi_2)$ (see Priestley 1981). For other parameter values, the spectrum has a trough at non-zero frequencies if $\phi_2 > 0$ and $|\phi_1(1-\phi_2)/4\phi_2| < 1$.

Charts 5 and 6 show the spectra of autoregressive processes and the spectrum of the cyclical component identified with the HP and BK filters. When the peak is located at zero frequency (i.e., most of the power of the series is located at low frequencies), the spectrum of the cyclical component resulting from the application of both filters is very different from that of the original series, especially at lower frequencies (Chart 5). In particular, the HP and BK filters induce a peak at business-cycle frequencies that is absent from the original series, and they fail to capture a significant part of the variance contained in the business-cycle frequencies. On the other hand, when the peak is located at business-cycle frequencies, the spectrum of the cyclical component identified by HP- and BK-filtering matches fairly well the true spectrum at these frequencies (Chart 6). This result is robust for different sets of parameters ϕ_1 and ϕ_2. Note that the BK filter does not perform as well as the HP filter at frequencies corresponding to cycles of around six to eight quarters. Indeed, the BK filter amplifies cycles of around eight quarters but compresses those of around six quarters. This results from the shape of the squared gain of the BK filter at those frequencies (see Chart 4a). The absence of a peak at business-cycle frequencies does not imply that macroeconomic series lack business cycles (see Sargent 1987 for a discussion). In fact, while most macroeconomic series feature the typical Granger shape, the growth rate of these series is often characterized by a peak at business-cycle frequencies. King and Watson (1996) call this “the typical spectral shape of growth rates.”

CHART 5: Series with the typical Granger shape

(AR(2) coefficients: 1.26 -0.31)

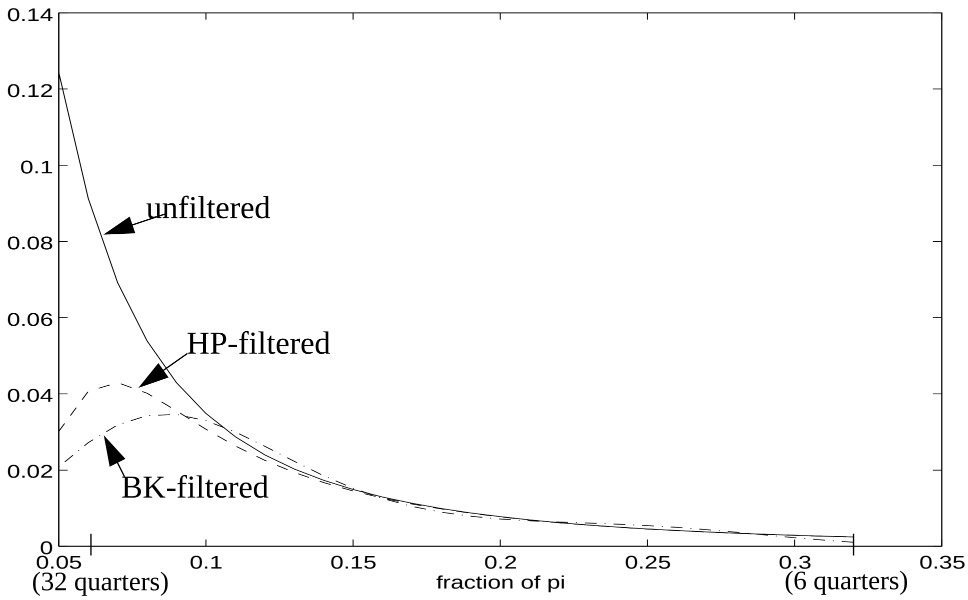
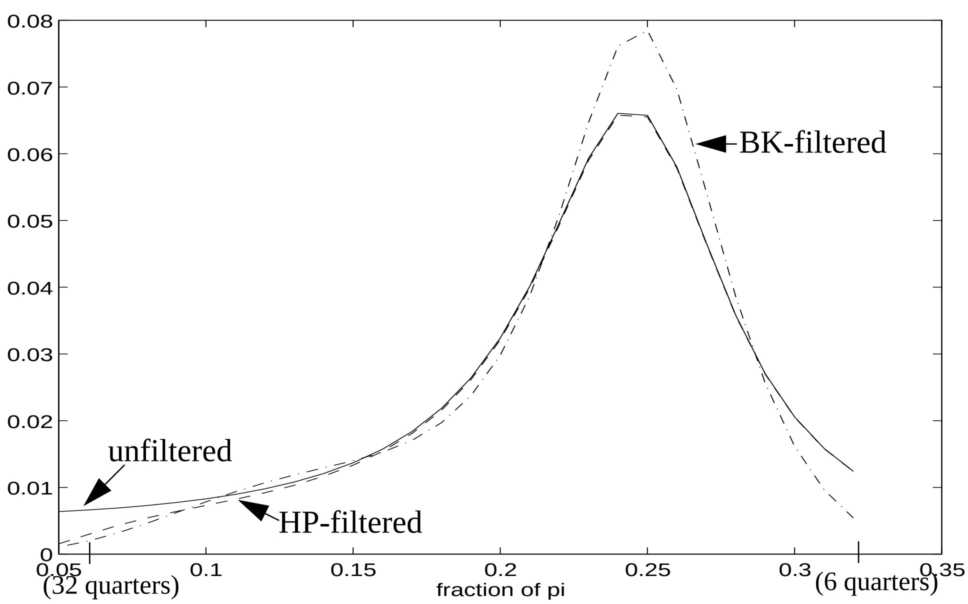


CHART 6: Series with a peak at business-cycle frequencies

(AR(2) coefficients: 1.26 -0.78)



To examine this question in more detail, we perform the following exercise. First, we establish a DGP by choosing $\theta = (\phi_1, \phi_2)$ for the second-order autoregressive process of equation 4. Second, we extract the corresponding cyclical component with the HP or BK filter. Third, we search among second-order autoregressive processes for the parameters ϕ_1 and ϕ_2 that minimize the distance, at business-cycle frequencies, between the spectrum of this process and the spectrum of the HP- or BK-filtered true second-order autoregressive processes. The problem is the following:

$$\tilde{\theta} = \arg min \int_{\omega 1}^{\omega 2} (S_{yf}(\omega;\theta_0) - S_{\tilde{y}}(\omega;\theta))^2 d\omega ,$$

where $\omega 1 = \pi/16$, $\omega 2 = \pi/3$, $S_{yf}(\omega;\theta_0)$ is the spectrum of the filtered DGP (where θ_0 is the vector of true values for the parameters ϕ_1 and ϕ_2), and $S_{\tilde{y}}(\omega;\theta)$ is the spectrum of the evaluated autoregressive process. Thus, in the case where the HP and BK filters adequately extract the range of periodicities corresponding to fluctuations of between six and thirty-two quarters (respectively, $\omega = \pi/3$ and $\omega = \pi/16$), $\tilde{\theta}$ will be equal to the true vector θ_0. Otherwise, the filter will extract a cyclical component corresponding to a second-order autoregressive process differing from the true one.

Table 1 presents our results for a DGP where the autoregressive parameter of order one is set at 1.20 while the parameter of order two is allowed to vary. Using the restrictions implied by equation 5, the peak of the spectrum lies within business-cycle frequencies when $\phi_2 < -0.43$. We report results for the HP filter only, but the results with the BK filter are almost identical.[7] The results of this exercise corroborate those obtained from visual inspection. The second-order autoregressive process that minimizes the distance between its spectrum at business-cycle frequencies and that of the business-cycle component identified by the HP and BK filters for the true process is very different from the true second-order autoregressive process when the peak of the DGP is located at zero frequency. When

7. These results are robust to the use of alternative values for θ_0, so that the restrictions are respected.

the peak is located at business-cycle frequencies, the resulting second-order autoregressive process is close to the true second-order autoregressive process.

TABLE 1: Fitted values for the HP filter

DGP (θ_0)		**HP ($\tilde{\theta}$)**	
ϕ_1	ϕ_2	ϕ_1	ϕ_2
1.20	-0.25	-0.09	0.72
1.20	-0.30	0.12	0.40
1.20	-0.35	0.48	-0.15
1.20	-0.40	0.87	-0.20
1.20	-0.45	1.09	-0.41
1.20	-0.50	1.16	-0.50
1.20	-0.55	1.19	-0.56
1.20	-0.60	1.20	-0.61
1.20	-0.65	1.20	-0.66
1.20	-0.70	1.20	-0.70
1.20	-0.75	1.20	-0.75
1.20	-0.80	1.20	-0.80

The spectrum of the level of macroeconomic time series typically resembles that of the unfiltered series shown on Chart 5. The spectrum's peak is located at zero frequency and the bulk of its variance is located in the low frequencies. This is known as the typical Granger shape. Charts 7, 8, 9 and 10 display the estimated spectra of U.S. real GDP, real consumption, consumer price inflation and the unemployment rate, as well as the spectra of the filtered counterparts to these series.[8] It is clear that the filters perform badly in terms of capturing business-cycle frequencies in these cases.

8. We use a parametric estimator of the spectrum. An autoregressive process was fitted and the order of that process was determined on the basis of the Akaike criteria.

The intuition behind this result is simple. Charts 1 and 4a (Section 2) show that the gains of the HP and BK filters at low business-cycle frequencies are significantly smaller than that of the ideal filter. Indeed, the squared gain of the BK filter is 0.34 at frequencies corresponding to 32-quarter cycles, while that of the HP filter is 0.49. In the case of the HP filter, the squared gain does not reach 0.95 before frequency $\pi/8$ (cycles of 16 quarters). The problem is that much of the power of typical macroeconomic time series at business-cycle frequencies is concentrated in the band where the squared gains of the HP and BK filters differ from that of an ideal filter. Also, the shape of the squared gain of those filters, when applied to typical macroeconomic time series, induces a peak in the spectrum of the cyclical component that is absent from the original series. In short, applying the HP and BK filters to series dominated by low frequencies results in the extraction of a cyclical component that fails to capture a significant part of the variance contained in business-cycle frequencies of the original series and that induces spurious dynamic properties.

One might argue that macroeconomic time series are actually composed of a permanent component and a cyclical component, so that the peak of the spectrum of the series would be at zero frequency while the peak of the spectrum of the cyclical component would be at business-cycle frequencies. For example, the permanent component could be driven by a random-walk technological process with drift, while transitory monetary or fiscal policy shocks, among others, could generate the cyclical component with a peak in its spectrum at business-cycle frequencies. If this were true, then the HP and BK filters might be able to adequately capture the cyclical component. We examine this issue in the next section.

CHART 7: Spectrum of the logarithm of U.S. real GDP

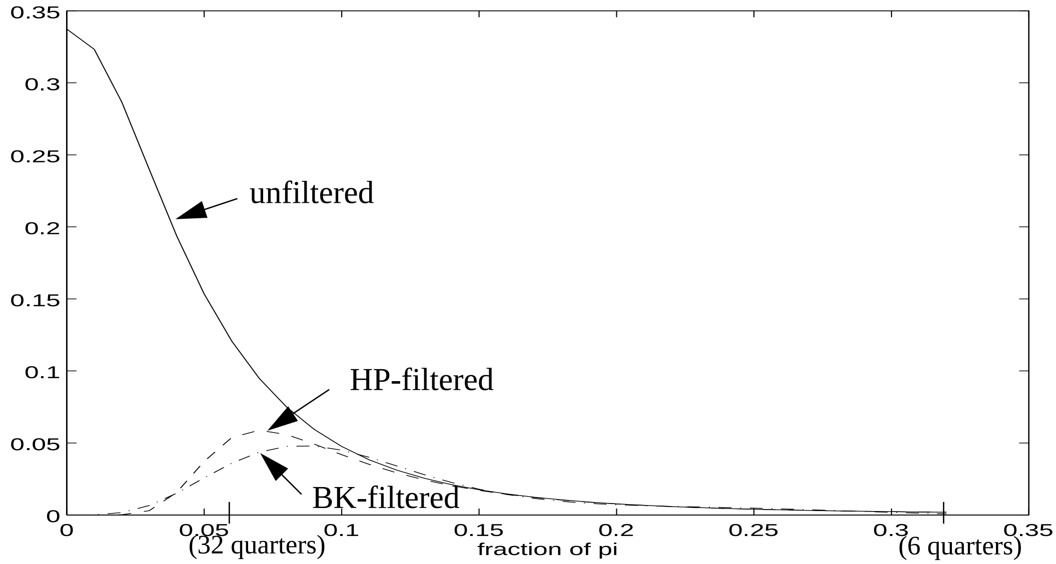


CHART 8: Spectrum of the logarithm of U.S. real consumption

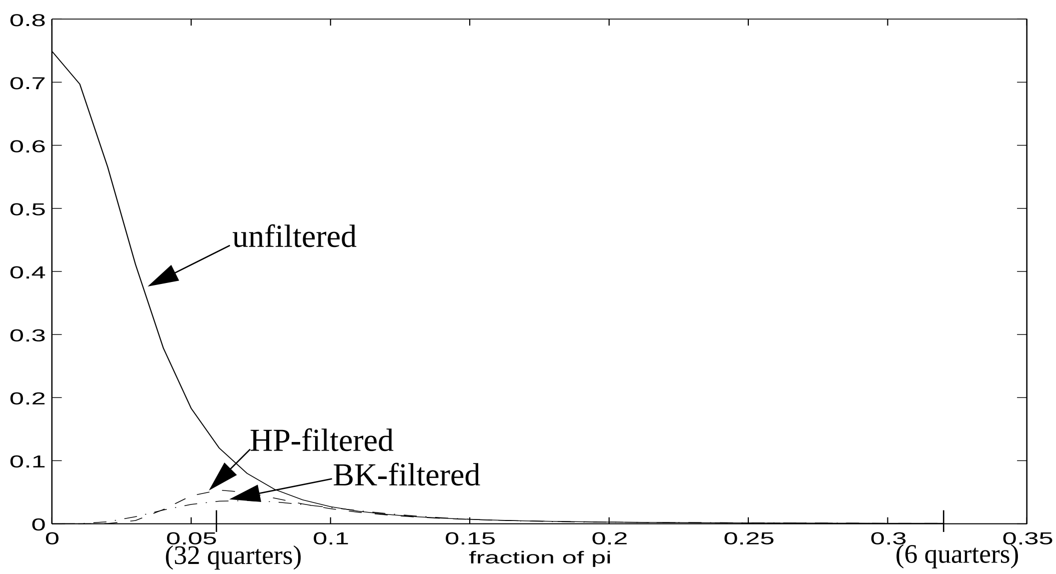


CHART 9: Spectrum of U.S. consumer price inflation

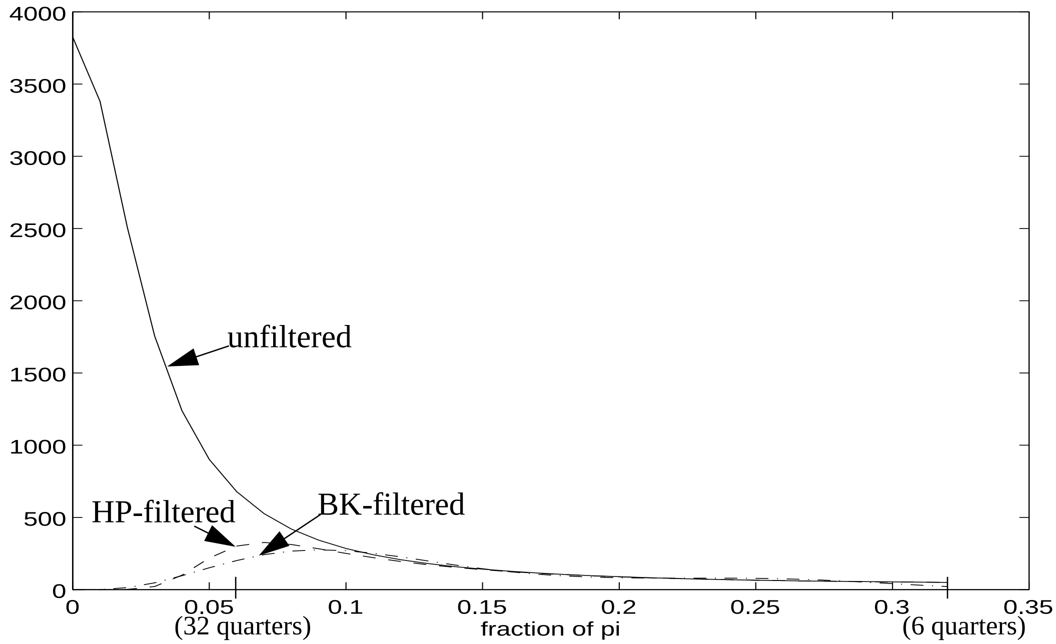
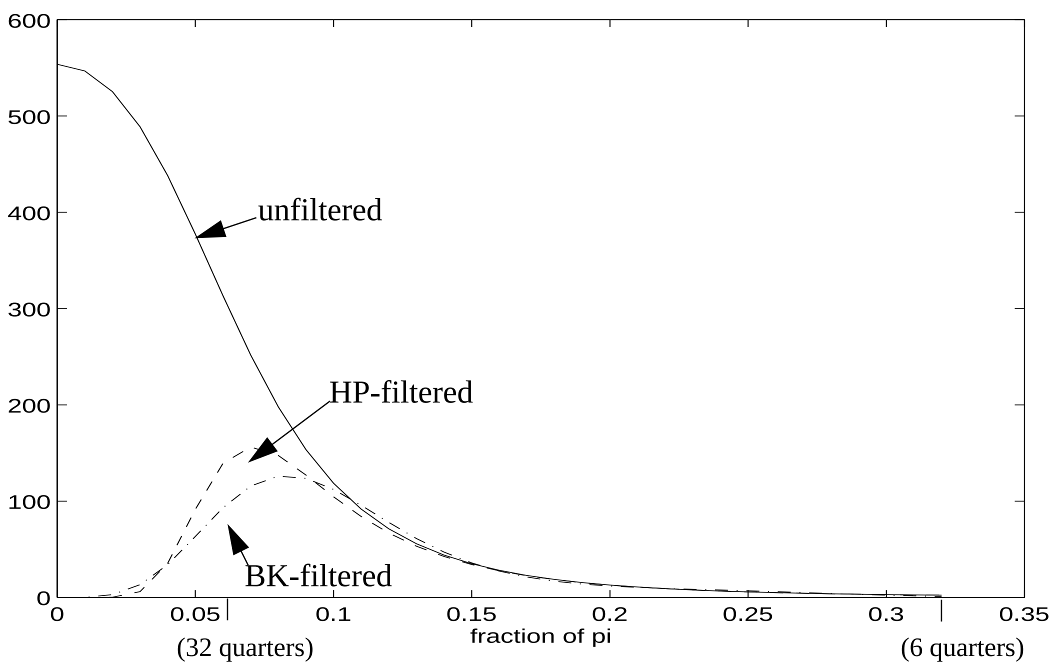


CHART 10: Spectrum of U.S. unemployment rate



4 A SIMULATION STUDY

Consider the following DGP:

$$y_t = \mu_t + c_t, \tag{6}$$

where

$$\mu_t = \mu_{t-1} + \varepsilon_t$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \eta_t$$

and

$$\varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \ \eta_t \sim NID(0, \sigma_\eta^2).$$

Equation 6 defines y_t as the sum of a permanent component, μ_t, which in this case corresponds to a random walk, and a cyclical component, c_t.[9] The dynamics of the cyclical component are specified as a second-order autoregressive process, so that the peak of the spectrum could be at zero frequency or at business-cycle frequencies. We assume that ε_t and u_t are uncorrelated.

Data are generated from equation 6 with ϕ_1 set at 1.2 and different values assigned to ϕ_2 to control the location of the peak in the spectrum of the cyclical component. We also vary the standard-error ratio for the disturbances $\sigma_\varepsilon / \sigma_\eta$ to change the relative importance of each component. We follow the standard practice of assigning the value 1600 to λ, the HP filter smoothness parameter. We also follow Baxter and King's suggestion of dropping 12 observations at the beginning and at the end of the sample. The resulting series contains 150 observations, a standard size for quarterly macroeconomic data. The number of replications is 500.

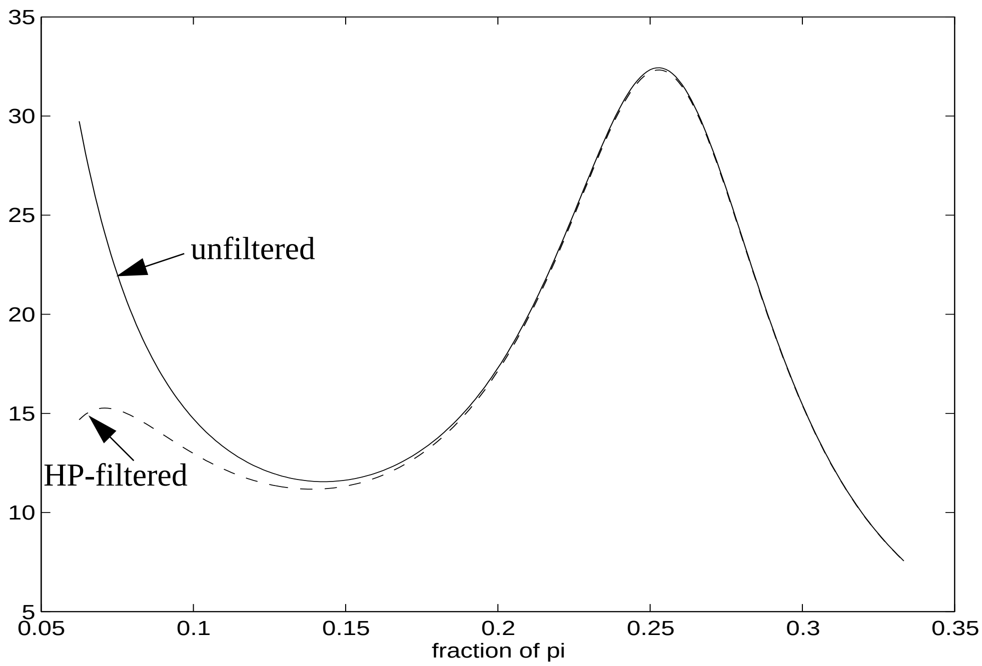
9. This is Watson's (1986) specification for U.S. real GDP.

The performance of the HP and BK filters is assessed by comparing the autocorrelation function of the cyclical component of the true process with that obtained from the filtered data. We also calculate the correlation between the true cyclical component and the filtered cyclical component and report their relative standard deviations ($\hat{\sigma}_c / \sigma_c$). Table 2 presents the results for the HP filter and Table 3 those for the BK filter.

Table 2 shows that the HP filter performs particularly poorly when there is an important permanent component. Indeed, for high $\sigma_\varepsilon / \sigma_\eta$ ratios, in most cases the correlation between the true and the filtered components is not significantly different from zero. The estimated autocorrelation function is invariant to the change in the cyclical component in these cases (the values of the true autocorrelation functions are given in parentheses in the tables). When the ratio $\sigma_\varepsilon / \sigma_\eta$ is equal to 0.5 or 1 and the peak of the cyclical component is located at zero frequency ($\phi_2 < -0.43$), the dynamic properties of the true and the filtered cyclical components are significantly different, as indicated by the estimated parameter values. In general, the HP filter adequately characterizes the series dynamics when the peak of the spectrum is at business-cycle frequencies and the ratio $\sigma_\varepsilon / \sigma_\eta$ is small. However, even when the ratio of standard deviations is equal to 0.01 (i.e., the permanent component is almost absent), the filter performs poorly when the peak of the spectrum of the cyclical component is at zero frequency. Indeed, for $\phi_2 = -0.25$, the dynamic properties of the filtered component differ significantly from those of the true cyclical component; moreover, the correlation is only equal to 0.66, and the standard deviation of the filtered cyclical component is half that of the true cyclical component.

It is interesting to note that the HP filter does relatively well when the ratio $\sigma_\varepsilon / \sigma_\eta$ is equal to 1, 0.5, or 0.01 and the spectrum of the original series has a peak at zero frequency and at business-cycle frequencies (i.e., the latter frequencies contain a significant part of the variance of the series). This is reflected in Chart 11, which shows the spectrum for the case where $\sigma_\varepsilon / \sigma_\eta = 1$ and $\phi_2 = -0.75$. Consequently, the following conditions are required to adequately identify the cyclical component with the HP filter: the spectrum of the original series must have a peak located at business-cycle frequencies, which must account for an important part of the variance of the series. If the variance of the series is dominated by low frequencies, which is the case for most macroeconomic series in levels, the HP filter does a poor job of extracting the cyclical component.

CHART 11: Spectrum for $\sigma_\varepsilon/\sigma_\eta = 1$ and $\phi_2 = -0.75$



The results for the BK filter are similar to those for the HP filter, although the dynamic properties of the filtered cyclical component seem to be invariant (or almost invariant) to the true process. For example, when $\sigma_\varepsilon/\sigma_u = 0.01$, $\phi_1 = 0$, and $\phi_2 = 0$, which corresponds to the case where the cyclical component is white noise and dominates the permanent component, the filtered cyclical component is a highly autocorrelated process. Thus, the BK filter would appear to be of limited value as a way to identify with any confidence the cyclical dynamics of a macroeconomic time series. As noted previously, this result precludes the use of the BK filter to assess the internal dynamic properties of a business-cycle model, since the filter produces a series with dynamic properties that are almost invariant to the true process.

TABLE 2: Simulation results for the HP filter

DGP			Estimated values				
$\sigma_\varepsilon/\sigma_\eta$	ϕ_1	ϕ_2	Autocorrelations 1	2	3	Correlation	$\hat{\sigma}_c/\sigma_c$
10	0	0	.71[0] (.59,.80)	.46[0] (.30,.60)	.26[0] (.08,.43)	.08 (-.07,.21)	12.96 (10.57,15.90)
10	1.2	-.25	.71[.96] (.61,.80)	.47[.90] (.31,.61)	.27[.84] (.08,.44)	.08 (-.11,.28)	4.19 (2.77,6.01)
10	1.2	-.40	.71[.86] (.60,.80)	.46[.63] (.30,.60)	.26[.41] (.08,.44)	.13 (-.12,.36)	6.34 (4.82,8.07)
10	1.2	-.55	.71[.77] (.60,.80)	.46[.38] (.29,.60)	.26[.03] (.06,.43)	.14 (-.08,.33)	6.93 (5.36,8.70)
10	1.2	-.75	.71[.69] (.60,.78)	.46[.27] (.30,.59)	.25[-.19] (.07,.41)	.15 (-.01,.31)	6.37 (4.79,7.95)
5	0	0	.69[0] (.58,.78)	.45[0] (.30,.58)	.26[0] (.09,.41)	.15 (.02,.27)	6.50 (5.28,7.85)
5	1.2	-.25	.71[.96] (.61,.80)	.46[.90] (.32,.61)	.26[.84] (.08,.43)	.16 (-.01,.36)	2.11 (1.43,3.04)
5	1.2	-.40	.72[.86] (.61,.80)	.46[.63] (.31,.60)	.25[.41] (.08,.42)	.23 (-.01,.45)	3.26 (2.47,4.15)
5	1.2	-.55	.71[.77] (.61,.80)	.46[.38] (.30,.59)	.24[.03] (.06,.41)	.24 (.01,.44)	3.60 (2.83,4.52)
5	1.2	-.75	.70[.69] (.61,.79)	.43[.27] (.26,.57)	.20[-.19] (.00,.38)	.29 (.11,.44)	3.30 (2.53,4.17)
1	0	0	.43[0] (.27,.57)	.28[0] (.11,.42)	.20[0] (-.02,.31)	.59 (.49,.70)	1.61 (1.41,1.85)
1	1.2	-.25	.76[.96] (.67,.83)	.51[.90] (.37,.62)	.29[.84] (.11,.44)	.51 (.33,.68)	.66 (.44,.91)
1	1.2	-.40	.75[.86] (.67,.81)	.44[.63] (.28,.55)	.16[.41] (-.03,.33)	.71 (.56,.82)	1.02 (.83,1.22)
1	1.2	-.55	.72[.77] (.66,.78)	.34[.38] (.21,.47)	.01[.03] (-.17,.19)	.76 (.56,.82)	1.15 (.83,1.22)
1	1.2	-.75	.68[.69] (.63,.72)	.15[.27] (.04,.27)	-.27[-.19] (-.44,.10)	.83 (.75,.89)	1.16 (1.04,1.29)
.5	0	0	.16[0] (.01,.32)	.10[0] (-.04,0.24)	.04[0] (-.10,.18)	.82 (.75,.88)	1.16 (1.07,1.27)
.5	1.2	-.25	.79[.96] (.71,.85)	.53[.90] (.38,.65)	.30[.84] (.11,.46)	.61 (.41,.79)	.55 (.37,.76)
.5	1.2	-.40	.77[.86] (.69,.81)	.43[.63] (.29,.54)	.13[.41] (-.05,.29)	.84 (.73,.92)	.87 (.74,.99)
.5	1.2	-.55	.72[.77] (.67,.78)	.28[.38] (.17,.39)	-.10[.03] (-.25,.06)	.89 (.83,.94)	.98 (.89,1.07)
.5	1.2	-.75	.67[.69] (.63,.71)	.07[.27] (-.03,.18)	-.42[-.19] (-.57,-.27)	.94 (.90,.96)	1.02 (.97,1.08)
.01	0	0	-.08[0] (-.21,.06)	-.06[0] (-.21,.06)	-.06[0] (-.19,.06)	.98 (.96,.99)	.97 (.94,.99)

TABLE 2: (Continued)

DGP			Estimated values				
$\sigma_\varepsilon/\sigma_\eta$	ϕ_1	ϕ_2	Autocorrelations			Correlation	$\hat{\sigma}_c/\sigma_c$
			1	2	3		
.01	1.2	-.25	.80[.96] (.72,.86)	.54[.90] (.38,.67)	.30[.84] (.11,.48)	.66 (.45,.83)	.51 (.34,.69)
.01	1.2	-.40	.78[.86] (.72,.83)	.43[.63] (.30,.55)	12[.41] (-.05,.28)	.90 (.82,.96)	.81 (.71,.90)
.01	1.2	-.55	.73[.77] (.67,.77)	.26[.38] (.15,.37)	-.14[.03] (-.30,.01)	.96 (.91,.99)	.92 (.86,.96)
.01	1.2	-.75	.67[.69] (.62,.71)	.02[.27] (-.08,.13)	-.50[-.19] (-.61,-.35)	.99 (.97,1.0)	.97 (.95,.99)

TABLE 3: Simulation results for the BK filter

DGP			Estimated values				
$\sigma_\varepsilon/\sigma_\eta$	ϕ_1	ϕ_2	Autocorrelations			correlation	$\hat{\sigma}_c/\sigma_c$
			1	2	3		
10	0	0	.90[0] (.87,.93)	.65[0] (.52,.75)	.33[0] (.13,.51)	.03 (-.11,.16)	11.55 (9.05,14.38)
10	1.2	-.25	.90[.96] (.87,.93)	.65[.90] (.55,.74)	.34[.84] (.17,.49)	.08 (-.13,.32)	3.71 (2.34,5.45)
10	1.2	-.40	.90[.86] (.87,.93)	.64[.63] (.54,.73)	.33[.41] (.16,.48)	.11 (-.16,.36)	5.67 (4.19,7.18)
10	1.2	-.55	.90[.77] (.87,.93)	.64[.38] (.53,.73)	.33[.03] (.14,.48)	.12 (-.12,.33)	6.23 (4.71,7.93)
10	1.2	-.75	.90[.69] (.86,.92)	.63[.27] (.52,.73)	.31[-.19] (.13,.48)	.16 (-.04,.36)	5.69 (4,37,7.16)
5	0	0	.90[0] (.87,.90)	.64[0] (.53,.73)	.33[0] (.14,.49)	.05 (-.09,.20)	5.80 (4.54,7.16)
5	1.2	-.25	.90[.96] (.87,.93)	.65[.90] (.54,.73)	.34[.84] (.16,.49)	.17 (-.05,.38)	1.94 (1.25,2.74)
5	1.2	-.40	.90[.86] (.87,.93)	.64[.63] (.53,.74)	.32[.41] (.14,.49)	.23 (-.03,.47)	2.93 (2.15,3.76)
5	1.2	-.55	.89[.77] (.87,.92)	.62[.38] (.52,.72)	.30[.03] (.12,.46)	.26 (.03,.46)	3.19 (2.45,3.98)
5	1.2	-.75	.88[.69] (.85,.92)	.60[.27] (.47,.70)	.26[-.19] (.06,.44)	.28 (.09,.45)	2.97 (2.24,3.77)
1	0	0	.89[0] (.85,.92)	.61[0] (.48,.71)	.27[0] (.06,.45)	.19 (.05,.32)	1.21 (.96,1.43)
1	1.2	-.25	.90[.96] (.87,.93)	.65[.90] (.53,.74)	.34[.84] (.15,.50)	.53 (.36,.71)	.60 (.39,.84)

TABLE 3: (Continued)

DGP			**Estimated values**				
$\sigma_\varepsilon/\sigma_\eta$	ϕ_1	ϕ_2	Autocorrelations 1	2	3	correlation	$\hat{\sigma}_c/\sigma_c$
1	1.2	-.40	.88[.86] (.85,.91)	.58[.63] (.47,.68)	.22[.41] (.03,.39)	.70 (.55,.81)	.95 (.78,1.12)
1	1.2	-.55	.85[.77] (.81,.89)	.48[.38] (.36,.60)	.05[.03] (-.15,.24)	.73 (.61,.83)	1.06 (.89,1.23)
1	1.2	-.75	.79[.69] (.75,.83)	.27[.27] (.14,.40)	-.26[-.19] (-.45,-.06)	.79 (.69,.87)	1.08 (.96,1.20)
.5	0	0	.86[0] (.81,.89)	.50[0] (.35,.63)	.10[0] (-.12,.30)	.36 (.25,.47)	.77 (.63,.91)
.5	1.2	-.25	.90[.96] (.87,.93)	.65[.90] (.55,.74)	.34[.84] (.16,.50)	.63 (.45,.78)	.51 (.34,.71)
.5	1.2	-.40	.88[.86] (.84,.91)	.56[.63] (.43,.66)	.17[.41] (-.02,.34)	.81 (.71,.88)	.81 (.67,.93)
.5	1.2	-.55	.83[.77] (.80,.87)	.41[.38] (.30,.53)	-.06[.03] (-.24,.12)	.85 (.78,.91)	.91 (.81,1.01)
.5	1.2	-.75	.76[.69] (.72,.79)	.17[.27] (.06,.29)	-.42[-.19] (-.57,-.25)	.89 (83,.93)	.96 (.89,1.03)
.01	0	0	.79[0] (.75,.83)	.29[0] (.16,.42)	-.22[0] (-.40,-.03)	.55 (.48,.63)	.51 (.43,.58)
.01	1.2	-.25	.91[.96] (.88,.93)	.66[.90] (.57,.74)	.35[.84] (.20,.50)	.68 (.52,.82)	.48 (.32,.64)
.01	1.2	-.40	.87[.86] (.84,.90)	.54[.63] (.44,.64)	15[.41] (-.03,.31)	.86 (.79,.92)	.76 (.65,.86)
.01	1.2	-.55	.83[.77] (.79,.86)	.39[.38] (.27,.49)	-.11[.03] (-.29,.06)	.90 (.85,.94)	.86 (.78,.92)
.01	1.2	-.75	.74[.69] (.71,.78)	.13[.27] (.03,.23)	-.48[-.19] (-.61,-.34)	.93 (.89,.96)	.92 (.86,.97)

The results of our simulation study are clear regarding the performance of the HP and BK filters when they are applied to decompositions between permanent and cyclical components that are more general than equation 6. For instance, the trend component can be an I(1) process with transient dynamic (e.g., $\varepsilon_t = d(L)\zeta_t$).[10] Also, the cyclical component can be correlated with the permanent component. For example, the decompo-

10. Lippi and Reichlin (1994) argue that modelling the trend component in real GNP as a random walk is inconsistent with the standard view concerning the diffusion process of technological shocks. Blanchard and Quah (1989) and King et al. (1991) used a multivariate representation to obtain a trend component with an impulse function whose short-run impact was smaller than its long-run impact. Accordingly, the effect of the permanent shock gradually increased to its long-run impact.

sition proposed by Beveridge and Nelson (1981) implies permanent and transitory components that are perfectly correlated. However, to reproduce the typical Granger shape, any decomposition must have an important permanent component relative to the cyclical component or else a cyclical component dominated by low frequencies. In both cases, the HP and BK filters provide a distorted cyclical component.[11]

11. The results of complementary simulations with different processes are available on request. For brevity these are not discussed here.

5 COMPARISON WITH OTHER APPROACHES

In this section, we compare the cyclical component obtained using the HP and BK filters with those of other approaches. Watson (1986) proposes an unobserved stochastic trend decomposition into permanent and cyclical components. His model for U.S. real GDP corresponds to equation 6 in the previous section.

We investigated whether the HP or BK filter is able to capture the cyclical component of the above DGP. Using Kuttner's (1994) estimates ($\phi_1 = 1.44$, $\phi_2 = -0.47$, $\sigma_\varepsilon^2 = 0.0052$, and $\sigma_\eta^2 = 0.0069$),[12] we simulated data on the basis of this DGP, filtered it, and compared the dynamic properties and the correlation of the true and the filtered components. The results are shown in Table 4. Both the HP and BK filters produce cyclical components with dynamic properties significantly different from the true one. Notably, the cyclical components identified by both filters are much less persistent than the true one. Also, the correlation is rather small. These results are not surprising, given that the spectrum of the cyclical component has its peak at zero frequency and the bulk of the variance is located at low frequencies.

TABLE 4: Simulation results with and without HP- and BK-filtering

	Autocorrelations			Correlation
	1	2	3	
Theoretical values	0.98	0.94	0.89	—
BK filter	0.92 (0.89-0.94)	0.70 (0.60-0.78)	0.42 (0.25-0.56)	0.47 (0.30-0.67)
HP filter	0.84 (0.78-0.89)	0.61 (0.48-0.73)	0.38 (0.20-0.54)	0.56 (0.35-0.76)

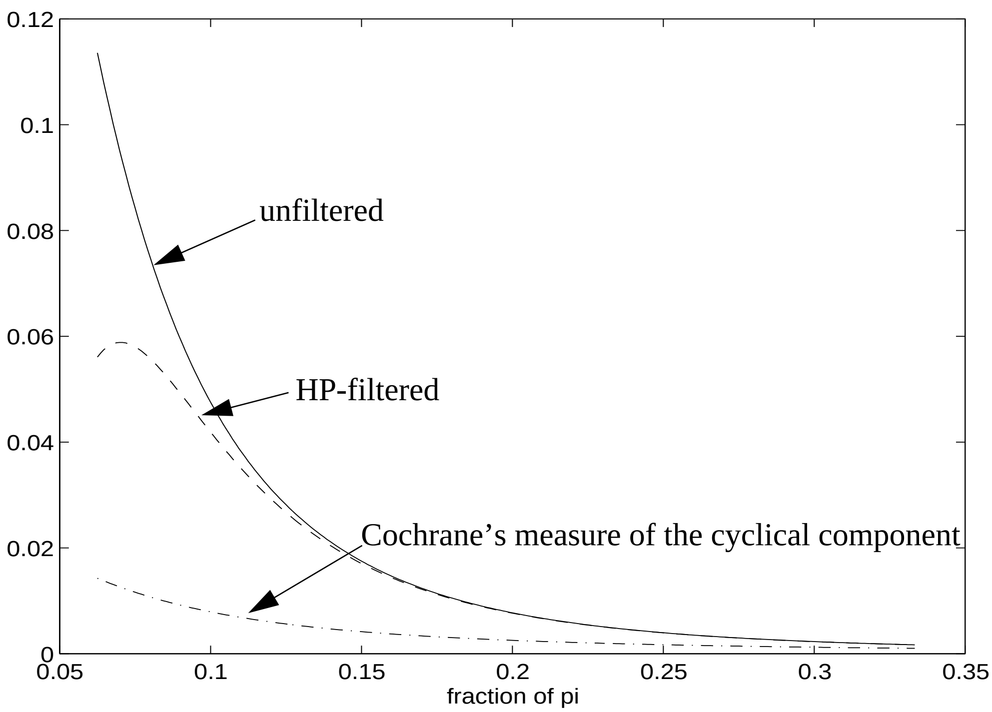
Cochrane (1994) proposes a simple detrending method for output based on the permanent-income hypothesis. This implies (for a constant real interest rate) that consumption is a random walk with drift that is

12. We chose Kuttner's estimates because he uses a larger sample than Watson. The use of Watson's estimates would not change our conclusions, however.

cointegrated with total income. Thus, any fluctuations in GDP with unchanged consumption must be transitory. Cochrane uses these assumptions to decompose U.S. real GDP into permanent and transitory components. Chart 12 presents the spectra for U.S. real GDP, for the same series with HP-filtering, and for Cochrane's cyclical component.

Using Cochrane's measure for comparison, the HP cyclical component greatly amplifies business-cycle frequencies. Also, while the peak of the spectrum of the HP-filtered cyclical component is located at business-cycle frequencies, the peak of Cochrane's measure is at zero frequency. The correlation between the two cyclical components is 0.57. To the extent that Cochrane's method provides a good approximation of the cyclical component of U.S. real GDP, the HP-filtered measure appears inadequate.

CHART 12: Spectrum of U.S. real GDP



6 CONCLUSIONS

This paper shows that two mechanical filters, the HP and BK filters, do relatively well when applied to series with peaks in their spectra at business-cycle frequencies. However, most macroeconomic time series have the typical Granger shape; in other words, most of their power lies at low frequencies and their spectra decrease monotonically at higher frequencies. Consequently, the conditions required to obtain a good approximation of the cyclical component with the HP and BK filters are rarely met in practice.

What are the alternatives for a business-cycle researcher interested in measuring the cyclical properties of macroeconomic series? For evaluating business-cycle models, researchers are often interested only in the second moments of the cyclical component. In that case, there is no need to extract a cyclical series. King and Watson (1996) show how to obtain correlations and cross-autocorrelations without filtering the observed and simulated series. The strategy involves calculating these moments from the estimated spectral-density matrix for business-cycle frequencies. We can obtain an estimator of the spectral-density matrix with a parametric estimator, such as that used by King and Watson, or with a non-parametric estimator. The cyclical component can also be obtained in a univariate or a multivariate representation using the Beveridge-Nelson (1981) decomposition method. Economic theory also provides alternative methods of detrending, such as Cochrane's (1994) method, based on the permanent income theory, or the Blanchard and Quah (1989) structural decomposition method.[13] The authors are currently investigating the properties of these alternative methodologies.

13. Cogley (1996) compares the HP and BK filters with the univariate Beveridge-Nelson decomposition and Cochrane's method, using a real-business-cycle model with different exogenous processes.

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