Should Central Banks Adjust Their Target Horizons in Response to House-Price Bubbles?

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Acknowledgements

Abstract

The authors investigate the implications of house-price bubbles for the optimal inflation-target horizon using a dynamic general-equilibrium model with credit frictions, house-price bubbles, and small open-economy features. They find that, given the distribution of shocks and inflation persistence over the past 25 years, the optimal target horizon for Canada tends to be at the lower end of the six- to eight-quarter range that has characterized the Bank of Canada's policy since the inception of the inflation-targeting regime. The authors' results also suggest that it may be appropriate to take a longer view of the inflation-target horizon when the economy faces a house-price bubble.

Résumé

Les auteurs analysent les implications de l'existence de bulles immobilières sur le marché résidentiel pour la détermination de l'horizon optimal d'une cible d'inflation. Ils ont recours pour ce faire à un modèle d'équilibre général dynamique de petite économie ouverte qui intègre des frictions sur le marché du crédit et des bulles immobilières. À la lumière de la distribution des chocs et du degré de persistance de l'inflation observés au cours des 25 dernières années, les auteurs concluent que l'horizon optimal dans le cas du Canada tend à se situer près de la limite inférieure de la fourchette de six à huit trimestres privilégiée par la Banque du Canada depuis l'adoption de cibles d'inflation. Leurs résultats donnent également à penser qu'un horizon plus long pourrait être approprié en présence de bulles immobilières.

1   Introduction

The lags inherent in the transmission mechanism of monetary policy imply that inflation-targeting central banks, such as the Bank of Canada, require an operating strategy for achieving their targets. A key element of an inflation-targeting regime is the inflation-target horizon. The *target horizon* is defined as the horizon over which policy aims to return inflation to target following a shock. Inflation-targeting central banks have an interest in communicating this horizon to the public, since it might help to anchor inflation expectations. A short horizon, whereby inflation returns to target quickly, may entail large interest rate movements that generate excessive volatility in the real economy. A long horizon, on the other hand, may return inflation to target too slowly, causing an unnecessarily high level of inflation variability. An *optimal target horizon* balances these two opposing considerations, and permits the central bank to react to anticipated inflationary pressures early and in a relatively gradual manner.

Research on optimal target horizons is fairly new and its findings are limited. Using a vector autoregressive model and a forward-looking structural model, Batini and Nelson (2000) conclude that the optimal target horizon for the United Kingdom is roughly 8–19 quarters. Their analysis is based on shocks to aggregate demand, aggregate supply, and the exchange rate. The Bank of Canada currently conducts monetary policy with the goal of returning inflation to target within a six- to eight-quarter horizon. With an increasing number of booms and busts in equity and housing markets worldwide, policy-makers are becoming increasingly interested in whether the target horizon should be adjusted in response to major movements in asset prices. This issue has not been investigated previously, although research abounds on whether monetary policy should respond to asset-price bubbles (Bernanke and Gertler 1999; Cecchetti, Genberg, and Wadhwani 2003; Gruen, Plumb, and Stone 2005). This paper contributes to the existing literature by assessing the implications of house-price bubbles for the optimal target horizon in Canada.[1] Our focus on housing is motivated by its significance on household balance sheets. In Canada, housing assets make up about 40 per cent of total household wealth, and roughly 70 per cent of total household debt is secured by real estate.

We conduct our analysis using a calibrated dynamic general-equilibrium model with small open-economy features, house-price bubbles, and a variety of real, nominal, and financial frictions. The real and nominal frictions, which help generate a degree of inflation

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1. In a companion paper, Cayen, Corbett, and Perrier (2006) study the optimal target horizon in ToTEM, the Bank of Canada's main projection model.

persistence that is consistent with Canadian data, follow Christiano, Eichenbaum, and Evans (CEE) (2005).

Financial frictions, in the form of a mortgage finance premium that increases with leverage, similar to Aoki, Proudman, and Vlieghe (2002), introduce a financial accelerator effect and allow monetary policy to affect output through a credit channel.[2] The financial frictions also offer an avenue through which to study problems of a financial nature, such as house-price bubbles. To gain insights into the mechanics of the financial accelerator effect, suppose there is a positive shock to house prices, *ceteris paribus*. This initial increase in the value of housing improves households' leverage ratio and reduces the mortgage finance premium, stimulating borrowing and aggregate demand. This causes a further increase in housing demand and house prices, which further lowers the mortgage finance premium. A self-reinforcing boom then emerges, with increases in house prices supporting stronger demand, and continuing until rising debt levels undo the decline in the mortgage finance premium.

House-price bubbles, defined as a sustained and growing gap between the market price of housing and its fundamental value, are modelled along the lines of Bernanke and Gertler (1999). In their framework, all agents know when there is a bubble as well as the *ex ante* stochastic process for the bubble, although they do not know when it will burst. This is a major caveat, since, in practice, it is difficult for economic agents to ascertain the presence and size of bubbles.

To calculate the optimal target horizon, the central bank's objectives are defined in terms of a quadratic loss function, penalizing deviations of inflation from target and departures of output from potential. The policy reaction function is specified as a smoothed feedback rule with responses to expected inflation and output. The model is then simulated using the historical distribution of shocks, and the parameters of the policy rule are chosen to minimize the loss function. We simulate the model using the optimal rule and calculate the time it takes for inflation to return to target. This process is repeated many times to obtain a distribution of target horizons, which characterizes the optimal target horizon.

Our main findings can be summarized as follows. Given the distribution of shocks over the past 25 years, the optimal target horizon for Canada tends to be at the lower end of the six- to eight-quarter range that has characterized the Bank's policy since the inception of the

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2. Iacoviello and Minetti (2000) document the empirical existence of a credit channel in the residential investment and consumption decisions of households.

inflation-targeting regime in 1991. This horizon extends to 13 quarters in the presence of a house-price bubble. Similar results are obtained if we take into account the central bank's aversion to interest rate volatility, which stems from concerns about financial stability and the risk of hitting the zero bound on nominal interest rates.

The rest of this paper is organized as follows. Section 2 describes the model. The calibration and solution of the model are discussed in sections 3 and 4, and its properties in section 5. Section 6 outlines the methodology used to calculate the optimal target horizon and discusses the results. Section 7 offers some conclusions.

2 The Model

2.1 The household sector

Households supply labour, purchase consumption goods, accumulate housing and capital, and participate in credit markets. They are divided into two groups: patient and impatient. Impatient households are symmetric to patient households, except that they are characterized by a relatively higher rate of time preference and they face a mortgage finance premium that is an increasing function of leverage. While the difference in discount rates generates borrowing and lending in equilibrium, the mortgage finance premium serves to ensure that borrowing by impatient households is not unbounded in the steady state.

2.1.1 Patient households

Patient households, indexed by $j \in (0, 1)$, are standard except that housing services appear in their utility function. We assume habit formation in the households' preferences for consumption, which implies that the marginal utility of consumption responds more acutely to changes in the level of consumption. This leads households to smooth consumption demand more than they would with purely time-separable preferences. In addition, each household is a monopoly supplier of a differentiated labour service, and sets its wage subject to Calvo-style frictions. In principle, this would imply household heterogeneity with respect to wage rates, hours worked, consumption, and asset holdings. However, we assume that households can trade securities with payoffs that depend on whether the household can reoptimize its wage. Following Erceg, Henderson, and Levin (2000) and Woodford (1999), it can be shown that, when such securities exist, households will be homogeneous with respect to consumption and asset holdings. The preferences of the jth patient household are

therefore given by:

$$E_t \sum_{l=0}^{\infty} \beta_p^l \left[\log \left(C_{t+l}^p - \gamma C_{t-1+l}^p \right) + \zeta_{H,t+l} \log \left(H_{t+l}^p \right) - \frac{\zeta_{L,t+l}}{2} \left(L_{j,t+l}^p \right)^2 \right],$$

(1)

where β_p is the discount factor for patient households, C_t^p is time t consumption, H_t^p is the stock[3] of housing held at the beginning of time t, and $L_{j,t}^p$ denotes time t hours worked. The superscript p indicates that the variable is associated with the patient households. In addition, $\zeta_{H,t}$ and $\zeta_{L,t}$ are a housing-demand shock and a labour-supply shock, respectively.

The period t budget constraint for patient household j is:

$$
\begin{aligned}
&P_t \left(C_t^p + a \left(u_t \right) \overline{K}_t \right) + R_{t-1}^n B_t^p + \kappa_{t-1} R_{t-1}^{n*} e_t B_t^* \\
&+ Q_{K,t} \left[\overline{K}_{t+1} - (1 - \delta_K) \overline{K}_t \right] \\
&+ S_{H,t} \left[H_{t+1}^p - (1 - \delta_H) H_t^p + \Phi \left(\frac{H_{t+1}^p}{H_t^p} \right) H_t^p \right] \\
&= W_{j,t}^p L_{j,t}^p + R_t^k u_t \overline{K}_t + B_{t+1}^p + e_t B_{t+1}^* + \Pi_t + A_{j,t}^p,
\end{aligned}
$$

(2)

where P_t is the price level, u_t is the capital utilization rate, e_t is the nominal exchange rate (the domestic currency price of foreign currency), $\overline{K}_t$ is the capital stock, and $Q_{K,t}$ and $S_{H,t}$ are the nominal prices of capital and housing. As will be discussed below, $S_{H,t}$ is the observed market price of housing, which is made up of a fundamental and a bubble component. δ_K and δ_H are the depreciation rates for capital and housing, B_t denotes the nominal quantity of domestic borrowing, and R_{t-1}^n is the nominal interest rate paid on domestic bonds held from $t-1$ to t. Similarly, B_t^* is the nominal quantity of foreign borrowing by domestic patient households, and R_{t-1}^{n*} is the associated foreign nominal interest rate. In addition, $W_{j,t}^p$ is the period t nominal wage of patient household j, R_t^k is the nominal rental rate for capital services, $A_{j,t}^p$ is the net payoff from state-contingent securities, and Π_t is profits from firms.

$a(\bullet)$ is the capital utilization cost function, which denotes the cost, in units of consumption goods, of setting the utilization rate to u_t. Variable capacity utilization allows for quick temporary changes in the effective level of the capital stock, even though the actual capital stock evolves only slowly over time. This serves to attenuate the requisite movements in the rental rate of capital in response to exogenous disturbances. Smoothing the rental rate of

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3. Our specification of utility assumes that the flow of housing services is proportional to the stock of housing held by a given household.

capital is key to ensuring an inertial behaviour of marginal cost, since the rental rate feeds directly into marginal cost. The specification of $a(\bullet)$, which is increasing and convex, follows Uribe and Schmitt-Grohe (2005). More precisely, $a(\bullet)$ is defined as follows:

$$a\left(u_t\right) = \gamma_1\left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2. \tag{3}$$

The housing-stock adjustment cost function is a standard quadratic adjustment cost function:

$$\Phi\left(\frac{H_{t+1}^p}{H_t^p}\right) = \frac{\phi}{2}\left(\frac{H_{t+1}^p}{H_t^p} - 1\right)^2. \tag{4}$$

The country-specific risk premium, κ_t, takes a form similar to much of the small open-economy literature, including Dib (2003):

$$\kappa_t = \exp\left(\frac{\varphi e_t B_{t+1}^*}{P_{d,t} Y_t}\right) + \varepsilon_t^\kappa, \tag{5}$$

where $P_{d,t}$ is the price level of domestic goods and Y_t is domestic output. Note that the household takes κ_t as given, since it is determined by aggregate variables. This risk premium is assumed to be an increasing function of the net foreign indebtedness of Canada. This assumption, which is standard in the literature, prevents agents from making unbounded arbitrage profits by borrowing in foreign markets and lending in domestic markets, or vice versa.

In each period, the household chooses C_t^p, H_{t+1}^p, $\overline{K}_{t+1}$, u_t, B_{t+1}, B_{t+1}^*. In periods in which the household reoptimizes its wage, it also chooses $W_{j,t}^p$. Labour supply is determined by the requirement that the household meet demand at the prevailing wage. As mentioned above, the jth household is a monopoly supplier of a differentiated labour service, $L_{j,t}^p$. It sells its service to a representative, competitive, price-taking firm that transforms it into an aggregate labour input, L_t^p, using the following technology:

$$L_t^p = \left[\int_0^1 \left(L_{j,t}^p\right)^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w}. \tag{6}$$

The demand curve for $L_{j,t}^p$ is:

$$L_{j,t}^p = \left(\frac{W_{j,t}^p}{W_t^p}\right)^{\frac{\lambda_w}{1-\lambda_w}} L_t^p, \tag{7}$$

where W_t^p is the aggregate wage rate; that is, the nominal price of L_t^p. It can be shown that W_t^p is related to $W_{j,t}^p$ by the Dixit-Stiglitz index:

$$W_t^p = \left[\int_0^1 \left(W_{j,t}^p\right)^{\frac{1}{1-\lambda_w}} dj\right]^{1-\lambda_w}. \tag{8}$$

Households take L_t^p and W_t^p as given and set their nominal wage à la Calvo. That is, they face a constant probability, $1 - \xi_w$, of being able to reoptimize their wage every period. In case they are unable to reoptimize their wage in period t, they index their nominal wage to lagged inflation as follows:

$$W_{j,t}^p = \pi_{t-1}^{\chi_w} \bar{\pi}^{1-\chi_w} W_{j,t-1}^p, \tag{9}$$

where χ_w measures the degree of wage indexation to past inflation. In particular, if $\chi_w = 1$, the wage is fully indexed to past inflation, and if $\chi_w = 0$, the wage is indexed to steady-state inflation. This indexation of wages to lagged inflation is key to ensuring that monetary policy actions have their full impact on inflation only with a lag.

Let $\widetilde{W}_t^p$ denote the value of $W_{j,t}^p$ chosen by a household that can reoptimize its wage in period t. The household index, j, is suppressed because all patient households that can reoptimize their wage in period t choose the same wage.[4] Ignoring irrelevant terms in the objective, the household chooses $\widetilde{W}_t^p$ to maximize:

$$E_t \sum_{l=0}^{\infty} \left(\beta_p \xi_w\right)^l \left\{ -\frac{\zeta_{L,t+l}}{2} \left(L_{j,t+l}^p\right)^2 + \Lambda_{t+l}^p W_{j,t+l}^p L_{j,t+l}^p \right\}, \tag{10}$$

subject to labour demand, equation (7). The presence of ξ_w in the discount factor serves to isolate future states of the world in which the household is not able to reoptimize its wage. It is only in these states that the choice of $\widetilde{W}_t^p$ affects utility.

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4. The proof is simply that the first-order condition for $\widetilde{W}_t^p$ does not depend on any variable with a j subscript; that is, it does not depend on any variable that differs across patient households.

2.1.2 Impatient households

As mentioned earlier, impatient households, indexed by $j \in (0,1)$, are symmetric to patient households except that they discount the future more heavily than patient households, so that $\beta_i < \beta_p$. This difference in discount rates implies that impatient households will want to borrow from patient households.

In equilibrium, impatient households will want to short sell all assets other than housing (because housing directly yields utility). We assume that short selling physical assets like capital is not possible, and so we omit these assets from the impatient household's budget constraint. Like their patient counterparts, each impatient household is a monopoly supplier of a differentiated labour service, and sets its wage subject to Calvo-style frictions. The preferences of the jth impatient household are given by:

$$E_t \sum_{l=0}^{\infty} \beta_i^l \left[\log \left(C_{t+l}^i - \gamma C_{t-1+l}^i \right) + \zeta_{H,t+l} \log \left(H_{t+l}^i \right) - \frac{\zeta_{L,t+l}}{2} \left(L_{j,t+l}^i \right)^2 \right], \tag{11}$$

where β_i is the discount factor for impatient households, C_t^i is time t consumption, H_t^i is the stock of housing held at the beginning of time t, and $L_{j,t}^i$ denotes time t hours worked. The superscript i indicates that the variable is associated with the impatient households. In addition, $\zeta_{H,t}$ and $\zeta_{L,t}$ are a housing-demand shock and a labour-supply shock, respectively.

In the spirit of Bernanke, Gertler, and Gilchrist (1999), we assume the existence of a mortgage finance premium that is increasing in household leverage. In the absence of this assumption, the desired borrowing by impatient households would be unbounded in steady state. However, unlike Bernanke, Gertler, and Gilchrist, we do not explicitly model the underlying agency problem. The period t budget constraint for impatient household j is:

$$P_t C_t^i + S_{H,t} \left[H_{t+1}^i - (1 - \delta_H) H_t^i + \Phi \left(\frac{H_{t+1}^i}{H_t^i} \right) H_t^i \right] + \Psi_t R_{t-1}^n B_{j,t}^i$$
$$= W_{j,t}^i L_{j,t}^i + B_{j,t+1}^i + A_{j,t}^i, \tag{12}$$

where $B_{j,t}^i$ denotes the nominal quantity of borrowing from period $t-1$ to t and Ψ_t is the mortgage finance premium facing impatient households. Households take the mortgage

finance premium as given;[5] however, in equilibrium, it is an increasing function of leverage:

$$\Psi_t = \Psi\left(\frac{B_t^i}{S_{H,t-1}H_t^i}\right), \qquad \Psi'(\bullet) > 0. \tag{13}$$

In each period, the household chooses C_t^i and H_{t+1}^i. As for the wage-setting decision of impatient households, it is symmetric to the wage decision of patient households.

The first-order condition with respect to $B_{j,t+1}^i$, which is the key equation for the financial amplification of exogenous shocks, is:

$$1 = \beta_i E_t\left[\frac{\lambda_{t+1}^i}{\lambda_t^i}\Psi\left(\frac{B_t^i}{S_{H,t-1}H_t^i}\right)\frac{R_t^n}{\pi_{t+1}}\right], \tag{14}$$

where λ_t^i is the marginal utility of consumption in period t. This is just a standard Euler equation, except that the effective real interest rate is augmented by the mortgage finance premium. In practice, fluctuations in the mortgage finance premium that these impatient households face may best be thought of in the following way. A decline in house prices translates into a reduction in home equity, and therefore a deterioration in the net worth position of households. This implies that, when these households refinance their mortgages, they face less favourable mortgage rates and have a smaller scope for extracting additional home equity to finance consumption. By affecting the collateral value of houses, fluctuations in house prices have an important impact on the borrowing conditions of households.

2.2 Capital producers

Capital production is undertaken using a technology similar to that proposed by CEE (2005). There are a large number of identical price-taking capital producers. Capital producers are owned by patient households and any profits (losses) are transferred in a lump-sum fashion to the patient households. Capital producers purchase investment goods in the final-goods market, which they transform into capital goods. Capital producers purchase existing capital, $x_{K,t}$, and investment goods, I_t^K, and combine these to produce new capital, $x'_{K,t}$, using the following technology:

$$x'_{K,t} = x_{K,t} + F\left(I_t^K, I_{t-1}^K\right). \tag{15}$$

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5. This assumption makes the associated first-order conditions resemble those that would arise if we explicitly modelled the agency problem.

New capital produced in period t can be used in productive activities in period $t + 1$. Let $Q_{K,t}$ be the nominal price of new capital. Since the marginal rate of transformation between new and old capital is unity, the price of old capital is also $Q_{K,t}$. Then the representative capital producer's period t profits are:

$$\Pi_t^K = Q_{K,t} \left[x_{K,t} + F \left(I_t^K, I_{t-1}^K \right) \right] - Q_{K,t} x_{K,t} - P_t I_t^K. \tag{16}$$

The profit-maximization problem of the capital producer is intertemporal, because the period t choice of I_t^K affects profits in period $t + 1$. Thus, the profit-maximization problem is:

$$\max_{I_t^K} E_t \left\{ \sum_{j=0}^{\infty} \beta_p^j \Lambda_{t+j}^p \left(\begin{array}{c} Q_{K,t+j} \left[x_{K,t+j} + F \left(I_{t+j}^K, I_{t+j-1}^K \right) \right] \\ -Q_{K,t+j} x_{K,t+j} - P_{t+j} I_{t+j}^K \end{array} \right) \right\}, \tag{17}$$

where P_t is the price of final output and Λ_{t+j}^p is the marginal value of a dollar to the patient household in period $t + j$. Note that, from this problem, it is obvious that any value of $x_{K,t+j}$ is profit maximizing. Thus, the market-clearing condition will determine the level of $x_{K,t+j}$:

$$x_{K,t+j} = (1 - \delta_K) \overline{K}_{t+j}, \tag{18}$$

where $\overline{K}_{t+j}$ is the aggregate capital stock in period $t + j$.

The functional forms follow Uribe and Schmitt-Grohe (2005):

$$F \left(I_t^K, I_{t-1}^K \right) = \left[1 - S \left(\frac{I_t^K}{I_{t-1}^K} \right) \right] I_t^K, \tag{19}$$

$$S \left(\frac{I_t^K}{I_{t-1}^K} \right) = \frac{\psi_K}{2} \left(\frac{I_t^K}{I_{t-1}^K} - 1 \right)^2. \tag{20}$$

Thus, the first-order condition for I_t^K is:

$$\lambda_t^p q_{K,t} F_1 \left(I_t^K, I_{t-1}^K \right) - \lambda_t^p + \beta_p E_t \left[\lambda_{t+1}^p q_{K,t+1} F_2 \left(I_{t+1}^K, I_t^K \right) \right] = 0. \tag{21}$$

Investment adjustment costs cause the marginal product of investment to respond sharply to changes in the level of investment. For example, a decline in the level of investment leads to a sharp increase in the marginal product of investment. This induces capital and housing producers to smooth their investment demand. The smoothing in investment demand, like

the smoothing in consumption caused by habit formation, translates into inertial aggregate-demand movements.

2.3 Housing producers

Housing producers are symmetric to capital producers. New housing, $x'_{H,t}$, is produced using the following technology:

$$x'_{H,t} = x_{H,t} + F\left(I_t^H, I_{t-1}^H\right),$$
(22)

where the definitions are analogous to the capital production technology. New housing produced in period t yields a flow of housing services in period $t+1$. The representative housing producer's period t profits are:

$$\Pi_t^H = Q_{H,t}\left[x_{H,t} + F\left(I_t^H, I_{t-1}^H\right)\right] - Q_{H,t}x_{H,t} - P_t I_t^H.$$
(23)

Thus, the profit-maximization problem is:

$$\max_{I_t^H} E_t \left\{ \sum_{j=0}^{\infty} \beta_p^j \Lambda_{t+j}^p \left(\begin{array}{c} Q_{H,t+j}\left[x_{H,t+j} + F\left(I_{t+j}^H, I_{t+j-1}^H\right)\right] \\ -Q_{H,t+j}x_{H,t+j} - P_{t+j}I_{t+j}^H \end{array} \right) \right\}.$$
(24)

As with the capital producers, the market-clearing condition will determine the level of $x_{H,t+j}$:

$$x_{H,t+j} = (1 - \delta_H) H_{t+j},$$
(25)

where H_{t+j} is the aggregate housing stock in period $t + j$.

Let the housing-investment adjustment cost function be given by:

$$F\left(I_t^H, I_{t-1}^H\right) = \left[1 - S\left(\frac{I_t^H}{I_{t-1}^H}\right)\right] I_t^H,$$
(26)

$$S\left(\frac{I_t^H}{I_{t-1}^H}\right) = \frac{\psi_H}{2}\left(\frac{I_t^H}{I_{t-1}^H} - 1\right)^2.$$
(27)

Thus, the first-order condition for I_t^H is:

$$\lambda_t^p q_{H,t} F_1\left(I_t^H, I_{t-1}^H\right) - \lambda_t^p + \beta_p E_t\left[\lambda_{t+1}^p q_{H,t+1} F_2\left(I_{t+1}^H, I_t^H\right)\right] = 0.$$
(28)

2.3.1 The observed market price of housing

Events in the Japanese land and stock markets in 1989 and in the U.S. equity markets in 1999–2000 remind us that destabilizing behaviour in asset markets can have far-reaching implications for asset prices and the macroeconomy. The theory of "rational bubbles" demonstrates that, even with rational expectations and behaviour, "rational" deviations in asset prices from their fundamental value are possible (Blanchard 1979; Blanchard and Watson 1982). A rational bubble involves an asset price that deviates progressively more quickly from the path dictated by its economic fundamentals, reflecting the presence of self-confirming expectations about the future appreciation of asset prices.

In the standard rational-bubble model, there is a fixed probability, p_{start}, of a bubble arising in any given period and a random initial magnitude. A bubble is defined as the difference between the actual market price of housing, $s_{h,t}$, and its fundamental value, $q_{h,t}$. Once a bubble has started, it evolves according to the following process:

$$s_{h,t+1} - q_{h,t+1} = \frac{a}{p}\left(s_{h,t} - q_{h,t}\right) R_{h,t+1} \quad \text{with probability } p, \tag{29}$$

$$= 0 \quad \text{with probability } 1-p,$$

with $p < a < 1$, where p is the probability of the bubble persisting in the next period, given that a bubble already exists, and $R_{h,t+1}$ is the fundamental return on housing. The assumption that $\frac{a}{p} > 1$ ensures that: (i) the bubble always grows unless it bursts, and (ii) investors receive an extraordinary return on floating days to compensate them for the risk of capital loss on bursting days. The higher the probability that the bubble will burst the next period (i.e., the lower the p), the greater the supranormal return on floating days.

Using equation (29), the average return that agents expect on the bubble is:

$$E_t \left(\frac{s_{h,t+1} - q_{h,t+1}}{R_{h,t+1}} \right) = a\left(s_{h,t} - q_{h,t}\right). \tag{30}$$

To be clear, all agents know when there is a bubble, and also the *ex ante* stochastic process for the bubble. However, they have no information on when a bubble will burst once it arises. The notion of a rational bubble can be captured by setting $a = 1$. However, a cannot be set equal to 1 in our model, given that non-stationarity violates the rank condition required to solve the model. It is common to set a close to 1 in the existing literature (for instance, Bernanke and Gertler 1999 set a equal to 0.98), in which case the

deviation from its fundamental value is referred to as a "near-rational" bubble. This assumption implies that, once a bubble arises, agents would expect it to start shrinking one period after the shock.[6]

Given the presence of housing-investment adjustment costs in this framework, agents would not undertake housing investment during a bubble period unless they expected a growing divergence from fundamentals at least for some time. We therefore deviate slightly from the existing literature to ensure that, when a bubble arises, agents in the model expect it to grow during its first few quarters. Thus, we assume that bubbles evolve according to:

$$(s_{h,t+1} - q_{h,t+1}) = a (s_{h,t} - q_{h,t}) R_{h,t+1} + \xi_{bub,t+1,} \tag{31}$$

$$\xi_{bub,t} = \rho_{bub}\xi_{bub,t-1} + \varepsilon_{bub,t,} \tag{32}$$

If no bubble exists in a given period, then a bubble can start with probability p_{start}. In this case, $\varepsilon_{bub,t}$ is drawn from a normal distribution. If a bubble already exists, then it can burst with probability $1 - p$. Thus, under this set-up, a one-time positive shock (i.e., when $\varepsilon_{bub,t}$ is non-zero) would yield a series of positive values for $\xi_{bub,t}$ in subsequent quarters, which in turn causes a series of *expected* increases in the bubble, $s_{h,t} - q_{h,t}$. Note that, even though the bubble is impervious to policy, the actual price of housing remains highly sensitive to monetary policy, since the latter influences the fundamental value of houses.

2.4 Open-economy features

The small open-economy features of the model are necessary for a plausible description of the Canadian economy. As in Dib (2003), we model foreign output, inflation, and interest rates (Y_t^*, π_t^*, R_t^{n*}) as exogenous AR(1) processes. The assumed exogeneity of these variables reflects our belief that the impact of Canadian variables on the rest of the world is sufficiently small that it can be safely ignored. This belief seems reasonable in light of the relatively small size of the Canadian economy.

The model also implies a balance-of-payments condition that must be respected in equilibrium. This condition can be derived from the budget constraints for the patient and impatient households. Under the assumption of no direct intervention in the foreign exchange market by the central bank, the balance-of-payments condition requires that the current account deficit (surplus) be equal to the capital account surplus (deficit). It should

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6. For the purpose of impulse-response analysis in those cases, the model is typically subject to growing exogenous house-price shocks, to ensure a growing deviation in house prices from fundamentals.

be noted that the effective interest rate on foreign currency securities faced by domestic agents is the sum of the foreign risk-free rate and a country-specific risk premium for Canada.

As mentioned in section 2.1.1, the risk premium is assumed to be an increasing function of the net foreign indebtedness of Canada. This assumption is standard in the literature. It is a technical requirement for stationarity.

2.5 Aggregating intermediate goods into final goods

The aggregation of intermediate goods into a final good, which can be thought of as a two-step process, is done by a perfectly competitive firm. The first step entails using a standard constant elasticity of substitution (CES) aggregator to transform differentiated domestic intermediate goods into domestic composite goods. A similar process applies to imported goods. In a second step, domestic and imported composite goods are combined into a final good using a CES production technology.

2.5.1 From intermediate goods to composite goods

The domestic composite good, Y_{dt}, is produced using a continuum of domestic intermediate goods, $Y_{dt}(j)$, and the CES aggregation technology:

$$Y_{dt} = \left(\int_0^1 Y_{dt}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \tag{33}$$

where $\theta > 1$ is the CES.

Given the domestic output price, P_{dt}, and the domestic intermediate-good price, $P_{dt}(j)$, the firm is faced with the following maximization problem:

$$\max_{\{Y_{dt}(j)\}} P_{dt}Y_{dt} - \int_0^1 P_{dt}(j)Y_{dt}(j)dj, \tag{34}$$

subject to (33).

Composite imported goods, Y_{mt}, are produced according to an analogous technology.

2.5.2 Production of the final good

The domestic and imported composite goods from step 1 are turned into a final good, Z_t, using the following aggregation technology:

$$Z_t = \left[(1 - \omega_m)^{\frac{1}{\nu}} Y_{dt}^{\frac{\nu-1}{\nu}} + \omega_m^{\frac{1}{\nu}} Y_{mt}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \tag{35}$$

where $\omega_m > 0$ denotes a positive share of imported goods in the final-good production, and $\nu > 0$ is the elasticity of substitution between domestic and imported goods.

The fact that the final good is used for domestic consumption (patient and impatient), capital investment, housing investment, utilization costs, and adjustment costs implies that the following market-clearing condition has to be satisfied:

$$Z_t = C_t^p + C_t^i + I_t^K + I_t^H + a(u_t)\overline{K}_t. \tag{36}$$

Given the final-good price level, P_t, and given P_{dt} and P_{mt}, the firm is faced with the following maximization problem:

$$\max_{\{Y_{dt}, Y_{mt}\}} P_t Z_t - P_{dt} Y_{dt} - P_{mt} Y_{mt}, \tag{37}$$

subject to (35).

2.6 Intermediate goods

2.6.1 Domestic intermediate goods

The domestic intermediate-goods market is modelled as per standard closed-economy models. The domestic producer j uses $K_t(j)$ and $L_t(j)$ to produce a differentiated domestic intermediate good, $Y_t(j)$, according to the following constant-returns-to-scale technology:

$$Y_t(j) = K_t(j)^\alpha \left[A_t L_t^p(j)^\eta L_t^i(j)^{1-\eta} \right]^{1-\alpha}, \quad \alpha \in (0,1), \tag{38}$$

where A_t is the level of labour-augmenting technology in the economy. It evolves according to the following process:

$$\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_{At}, \tag{39}$$

where ρ_A controls the persistence of A_t and ε_{At} is the serially uncorrelated technology shock with mean zero and standard deviation σ_A.

This domestic intermediate good can be allocated for domestic use, $Y_{dt}(j)$, or for export, $Y_{xt}(j)$:

$$Y_t(j) = Y_{dt}(j) + Y_{xt}(j). \tag{40}$$

The export price is $P_{dt}(j)/e_t$, given the assumption that domestic producers may not price-discriminate. It is assumed that the total foreign demand for exports is:

$$Y_{xt} = \left(\frac{P_{dt}}{e_t P_t^*}\right)^{-\tau} Y_t^*, \tag{41}$$

where $\tau > 0$ is the price elasticity of the home country's aggregate exports, and P_t^* is the world price level (denominated in foreign currency). The foreign-price index evolves according to:

$$\log(\pi_t^*) = \rho_{\pi^*} \log(\pi_{t-1}^*) + \varepsilon_{\pi^*t}, \tag{42}$$

where the world inflation rate is $\pi_t^* = P_t^*/P_{t-1}^*$. The shock term ε_{π^*t} is normally distributed with zero mean and standard deviation σ_{π^*}. To ensure that the nominal exchange rate is stationary, we assume that the foreign steady-state rate of inflation is the same as the domestic steady-state rate of inflation, $\pi^* = \pi$.

The domestic intermediate-goods producer sells its output at price $P_{dt}(j)$, on monopolistically competitive domestic and foreign markets. Following Calvo (1983), domestic intermediate-goods producers cannot change their prices unless they receive a random signal. The probability that a given price can be reset in any period is constant and is given by $(1 - \xi_p)$. Therefore, on average, the price is not reoptimized for $1/(1 - \xi_p)$ periods. In periods when the price is not reoptimized, the firm sets
$P_{d,t}(j) = \pi_{t-1}^{\chi_p} \bar{\pi}^{1-\chi_p} P_{d,t-1}(j)$.

2.6.2 Imported intermediate goods

In the home country, a continuum of importers indexed by $j \in (0, 1)$ import a homogeneous intermediate good produced abroad for the foreign price P_t^*. Each importer uses this imported good to produce a differentiated good, $Y_{m,t}(j)$, which is sold in a home

monopolistically competitive market to produce the imported composite good $Y_{m,t}$. As in the domestic intermediate-goods sector, importers can change their prices only when they receive a random signal. The constant probability of receiving such a signal is $(1 - \xi_p)$.

2.7 Monetary authority

We assume that the central bank conducts monetary policy by using the nominal interest rate as its instrument. It responds to deviations of inflation and output from their steady-state values. We also allow for responses to deviations of house prices from steady state in some versions of the model. This rule can be expressed as follows:

$$\log\left(\frac{R_t^n}{\overline{R}^n}\right) = \rho_R \log\left(\frac{R_{t-1}^n}{\overline{R}^n}\right) + \eta_\pi E_t \log\left(\frac{\pi_{t+k}}{\overline{\pi}}\right) + \eta_y \log\left(\frac{Y_t}{\overline{Y}}\right) + \varepsilon_{R,t}, \qquad (43)$$

where ρ_R is the interest rate smoothing parameter, η_π and η_y are the response coefficients to log-deviations of inflation and output, and k is the *feedback horizon* (that is, the optimal horizon for which the central bank should form a forecast for inflation to include in that rule). $\overline{R}^n$, $\overline{\pi}$, and $\overline{Y}$ refer to the steady-state values of interest rate, inflation, and output, respectively. $\varepsilon_{R,t}$ is a zero-mean, serially uncorrelated interest rate/monetary policy shock with standard deviation σ_{R^n}.[7]

3 Calibration

In setting parameter values for the model, the strategy has been partly to choose standard values from the literature and partly to calibrate to match selected targets for the Canadian economy. We base our calculations on quarterly data spanning the 1980–2004 period.[8]

The discount rate of patient households, β_p, is set to 0.9928. This implies a real interest rate of about 3 per cent per annum, consistent with the historical *ex post* real rate for Canada over the 1980–2004 period. The discount rate for impatient households, β_i, is set to 0.9890 to yield a steady-state mortgage finance premium of 150 basis points on the annualized rate. This is the average spread between the effective 5-year mortgage rate and 5-year Government of Canada bond yield over the 1980–2004 period. We choose the 5-year term, since it continues to be the most popular term for mortgages in Canada, although

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7. The steady-state nominal interest rate and inflation rate are assumed to be 5 per cent and 2 per cent, respectively.
8. This is the longest period for which we have a comprehensive data set. In particular, quarterly house-price data are available only from 1980Q1.

using the 1-year term would yield similar results.

Owing to a lack of data, it is difficult to estimate the elasticity of the mortgage finance premium with respect to leverage (the mortgage debt-to-housing assets ratio) for Canada. We set it to 0.05, consistent with Bernanke and Gertler (1999) for the business sector. This implies that the mortgage finance premium would rise by about 67 basis points if leverage increased from 0.75 to 0.80. Given its significance for the financial accelerator mechanism, we conduct sensitivity analyses, varying this parameter between 0.05, as in Bernanke and Gertler (1999), and 0.1, as in Aoki, Proudman, and Vlieghe (2002). These variations have little impact on our results.

The habit persistence parameter, γ, is set to 0.75, which is approximately the mid-point of the estimates of 0.63, reported by CEE (2005), and 0.9, reported by Fuhrer (2000). ζ_H takes a value of 0.12 in steady state, which is consistent with an annualized consumption-to-housing ratio of 40 per cent for Canada.

The housing-stock adjustment cost parameter, ϕ, takes a value of 10. Near the steady state, this implies a marginal transaction cost of 5 per cent per unit of housing. Since we are working with a model that is linearized around the steady state, we are effectively applying this 5 per cent transaction cost away from steady state also. This is roughly in line with actual real estate transaction costs. We set $\delta_H = 0.00375$, in line with Statistics Canada's estimate of 1.5 per cent for the annual depreciation rate of housing. The capital stock is assumed to depreciate at an annual rate of 10 per cent, as estimated by Statistics Canada.

Following CEE (2005), we set the investment adjustment cost parameters, ψ_K and ψ_H, equal to 2.48. Similarly, ξ_w takes a value of 0.5, implying an average wage contract length of two quarters. λ_w is set to 1.05. As for χ_w, it takes a value of 1, implying that wages are fully indexed to past inflation. We assume that the wage share of impatient households is 0.36, which allows us to match an overall mortgage debt-to-housing asset ratio of 0.33, consistent with national balance-sheet data for Canada.

The probability that the price of a good can be reset in any given period, $1 - \xi_p$, is set to 0.5, implying an average price contract length of two quarters, which is roughly consistent with survey evidence for Canada (Amirault, Kwan, and Wilkinson 2004–2005). We report sensitivity analyses with respect to ξ_p below. In the base case, we assume that prices are indexed to steady-state inflation. The elasticity of substitution that characterizes the aggregation technology for the domestic composite good takes a value of 6, which is in line

with estimates by CEE (2005).

The parameters for the open-economy features are largely drawn from Dib (2003). The price elasticity of the home country's aggregate exports, τ, and the elasticity of substitution between domestic and imported goods are both set to 0.8. The share of imported goods in the final-good production is assumed to be 0.28.

As is standard in the literature, we set α equal to 0.36, which corresponds to a steady-state capital share of income of 36 per cent.

Because little is known about the dimensions of bubbles in practice, the initial size of the bubble is drawn from a normal distribution and we assume ρ_{bub} to be 0.9. Drawing on stylized facts for house-price bubbles (Helbing and Terrones 2003), we calibrate the probability of a bubble occurring, p_{start}, to be 0.025. This means that a house-price boom tends to occur once every 10 years. The probability of this shock persisting from one period to the next, p, is set to be 11/12. This implies that the bubbles have an average duration of three years. To assess the robustness of the results, we carry out sensitivity analyses with respect to these bubble parameters.

As a baseline, we set the parameters of the policy rule to the following values: $\rho_R = 0.9$, $\eta_\pi = 1.5$, $\eta_y = 0$, and $k = 2$. We refer to this as the historical policy rule, and use it to evaluate model properties.

4   Solution

The model is solved by initially computing a first-order Taylor series approximation to the structural equations. This yields a linear approximation to the model that can be solved by standard methods for linear rational-expectations models. The Taylor series approximation is taken around the non-stochastic steady state. The steady-state values are computed by solving the non-linear structural equations with all aggregate uncertainty shut down.

5   Model Properties

To be able to use the model to inform policy issues, such as the inflation-target horizon, we first need to convince ourselves that the model can replicate key features of the data. In particular, the relationships among aggregate variables, including output, inflation, interest rates, and the exchange rate, are viewed as important. Given our focus on the housing

sector, it is also important for the behaviour of house prices, residential investment, and mortgage credit to match the data. We assess the validity of the model based on impulse-response functions as well as on key relative variances, and cross- and autocorrelations. Impulse responses generally have the expected sign and hump shape (Charts 1 to 7). In addition, key relative variances and correlations match the 1980Q1–2004Q4 data reasonably well (Tables 1 to 3).[9]

5.1 Impulse responses to stylized shocks

We next analyze the model's impulse responses, which are presented as the percentage deviation from steady state for all variables, except inflation and nominal interest rates, which are reported as the deviation from steady state in basis points. The horizontal axis refers to the number of quarters after the shock.

The responses of key variables to a temporary 1 per cent shock to the annualized nominal interest rate are summarized in Chart 1. As expected, a rise in interest rates is associated with reductions in inflation and output, reflecting declines in consumption, capital, and residential investments and exports. The speed of the response of output is due to a rapid movement in net exports, triggered by an appreciation in the real exchange rate. The combination of the rise in interest rates, which increases the cost of borrowing, and the contraction in output leads to the pro-cyclicality of house prices. As for mortgage credit, it remains persistently below its steady-state level, a pattern noted also in the case of a productivity shock.

Chart 2 shows that a temporary positive productivity shock causes output to rise and inflation to fall, as expected. To combat disinflationary pressures in the economy, the monetary authority reduces the nominal interest rates. Falling rates work in the same direction as output to increase the demand for housing, leading to a positive deviation of housing prices from the steady-state level. This increase in the demand for housing is also reflected in the hump-shaped rise in residential investment. As in the case of the policy shock, the movement in mortgage credit is very persistent, reflecting a larger stock of housing.

Chart 3 reports the economy's response to a house-price bubble that lasts one year and causes house prices to deviate by about 35 per cent from the steady-state level at its peak.

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9. Because the model does not include a government sector, we construct a measure of output that excludes government.

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The bursting probability for the bubble is $p = 0.75$. Thus, the expected duration is one year. This is shorter than the 3-year average duration used to generate the target horizons. We use a shorter bubble for impulse-response analysis because it facilitates the graphical presentation.

As one might expect, output and inflation both rise in the boom phase of the bubble. The inflation response is somewhat muted: although the bubble causes output and marginal cost to rise in the near term, price-setters recognize that the bubble will eventually burst, causing output and marginal cost to fall below trend. Agents, including the central bank, incorporate the fact that the bubble will eventually burst into their expectations, leading to a muted impact on inflation. Rising house prices fuel debt-financed housing investment, so that the peak of the cycle is associated with excess housing-stock accumulation and a general state of overindebtedness. A direct consequence of this overinvestment at the peak of the cycle is a protracted period of low investment after the bubble collapses, as the housing stock is allowed to depreciate to its normal level. This partly explains why output shoots below its steady state following the house-price correction. Note that, contrary to what one might expect, total consumption falls below steady state in the expansion phase of the bubble: the rise in impatient consumption during this phase is outweighed by a decline in consumption by patient households as they increase their savings to be able to lend to their impatient counterparts.

The responses of key variables when the economy is subject to a positive housing-demand shock are similar to the expansion phase of a house-price bubble (Chart 4).

As Chart 5 shows, a negative labour-supply shock causes output to fall by about 0.4 per cent. We also note that inflation shows little movement in response to the labour-supply shock, which can be explained as follows: the upward pressure on inflation stemming from a rise in real wages and marginal cost is more than neutralized by the effects of a fall in import prices caused by an appreciation of the real exchange rate.

Chart 6 reports the impulse responses for a country-specific risk-premium shock. A risk-premium shock translates into a depreciation of the real exchange rate, which causes inflation to rise via its effects on import prices. The monetary authority, in turn, raises interest rates to suppress these inflationary pressures. As a result, we observe a fall in output and its major components a few quarters after the shock.

The economy's response to a price-markup shock is summarized in Chart 7. Lower price

markups lead directly to a fall in contemporaneous inflation. The monetary authority responds by lowering interest rates, which causes output to rise. The lower price markups are also associated with higher house prices and increases in residential investment and consumption.

5.2 Key relative variances, and cross- and autocorrelations from the model

Table 1 presents the standard deviation of key variables in the model relative to the standard deviation of output. This table suggests that the model generally performs well in generating relative variances that lie within the 95 per cent confidence band in the data. In particular, note that, for key aggregate variables, such as inflation and nominal interest rates, the model produces relative variances that are almost identical to the average for the 1980–2004 period. The model, however, generates relative variances that lie outside the 95 per cent confidence band for consumption and business investment.

As shown in Table 2, the first-order autocorrelation of inflation is 0.57 in the model, compared with 0.63 in the data. This feature deserves to be highlighted, since the inflation-target horizon is heavily influenced by the degree of inflation persistence. The persistence of nominal interest rates in the model matches the historical mean exactly. The persistence of real house-price growth in the model is too low, a problem that is quite common with this class of models.[10] With respect to other variables, the model tends to generate somewhat more persistence than is observed in the data.

It is also important for the cross-correlation among key variables to be consistent with empirical data. Table 3 shows that the model generally performs well in this respect. The model does particularly well on correlations related to credit and house prices. There are, however, some weaknesses. For example, the model generates too much co-movement between consumption and residential investment, and between residential investment and mortgage credit. On balance, we believe that the model's strengths outweigh its weaknesses. Hence, we use it in the policy-analysis exercises that follow.

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10. For completeness, we also looked into the autocorrelation of the level of real house prices, which is 0.84 in the model, compared with a mean of 0.88 in the data, and upper and lower bounds of 0.76 and 1.0, respectively. However, we refrain from interpreting the reported bounds in this case, since the series is characterized by a unit root.

6 The Optimal Target Horizon

Given that the model behaves fairly well in matching certain stylized facts, we employ it to investigate optimal policy horizons for Canada. Similar to Batini and Nelson (2000), the methodology applied to the model to determine the optimal target horizon consists of three steps:

Step 1: The central bank's objectives are defined in terms of a standard loss function, which assumes that policy-makers try to minimize deviations of inflation from target, departures of output from potential, and the volatility of interest rates. The assumption that the central bank is averse to interest rate volatility is based on the following premises: first, large unanticipated movements in interest rates may cause problems for financial stability (Cukierman 1990; Smets 2003); second, the central bank might be concerned about the risk of hitting the zero bound on nominal interest rates (Rotemberg and Woodford 1997; Woodford 1999; Smets 2003). The loss function characterizing the central bank's preferences is:

$$L_t = \widehat{\pi}_t^2 + \omega_y \widehat{Y}_t^2 + \omega_{\triangle R}(\triangle \widehat{R}_t)^2, \tag{44}$$

where "hatted" variables are log deviations from steady state, $\triangle \widehat{R}_t$ refers to the change in the level of the policy instrument between period $t-1$ and t, and ω_y and $\omega_{\triangle R}$ are the relative weights on output fluctuations and interest rate volatility, respectively. The intertemporal loss function for the central bank can be written as:

$$\mathcal{L}_t = (1 - \beta) E_t \sum_{j=0}^{\infty} \beta^j L_{t+j}, \tag{45}$$

where β is the rate at which the central bank discounts future losses. As $\beta \to 1$, the value of the intertemporal loss function approaches the unconditional mean of the period loss function, given as:

$$\overline{\mathcal{L}} = \sigma_\pi^2 + \omega_y \sigma_y^2 + \omega_{\triangle R} \sigma_{\triangle R}^2, \tag{46}$$

where σ_π^2, σ_y^2, and $\sigma_{\triangle R}^2$ are the unconditional variances of the deviations of inflation from target, the departures of output from potential, and the volatility of interest rates. We choose to work with the unconditional loss function owing to the computational ease that it offers. For cases 1 and 2 below, we set ω_y to 1 and $\omega_{\triangle R}$ to 0, which means that the central bank assigns equal weight to inflation and output stability, but does not put any weight on

smoothing its policy instrument. In cases 3 and 4, we set ω_y to 1 and $\omega_{\triangle R}$ to 0.5, which means that the central bank cares equally about inflation and output stability, but less about smoothing interest rates.

Step 2: The parameters of the policy rule described earlier are optimized to minimize the central bank's loss function. More specifically, the model is simulated over the historical distribution of shocks to choose $\rho_R, \eta_\pi, \eta_y, \eta_s$, and k (i.e., the feedback horizon). It is important to note that the simulations are conducted under full commitment; that is, policy is credible given the parameterization of the rule. When policy is highly credible, inflation expectations remain well anchored to the target in the medium term, which implies that interest rates and output need to move less to counter movements in inflation away from target.[11]

Step 3: To calculate the time it takes for inflation to return to target, we draw from the historical distribution of shocks in the first period of the simulation. No shocks are drawn after the first period. We then solve the model under the optimized policy rule obtained in step 2, and calculate the time it takes to get inflation within 0.1 percentage points of the target (i.e., we use an absolute criterion). We repeat this 10,000 times to build up a distribution of target horizons and then use this distribution to calculate the range of target horizons, as well as the average.[12]

We consider a number of different scenarios in order to assess the importance of house-price bubbles for the optimal policy horizon, as well as to understand the sensitivity of our results to model specifications. Our main cases are as follows:

Case 1: Our base case uses the calibrated structure described so far, and a policy rule that is optimized with house-price bubbles as part of the historical mix of shocks.[13] In this case, we do not allow for a direct response to house prices in the policy rule. We set the

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11. An alternative approach to the question of optimal policy horizons would have been to use an optimal targeting rule. This entails choosing a targeting rule to explicitly maximize the central bank's objectives without specifying a particular form for the policy reaction function. This would have permitted an analysis of the implications of discretionary policy, and is an area for potential future research.
12. An alternative approach would be to carry out this exercise for each type of shock, rather than focus on the historical shock, which would yield an optimal target horizon for each type of shock. This approach is associated with two difficulties: (i) at any given point in time, the economy is subject to multiple shocks and it is difficult for the central bank to diagnose the exact nature of each shock, and (ii) it is difficult to compare the results for optimal target horizons across models, unless the models include the same set of shocks.
13. This is in line with the general view that Canada experienced major house-price misalignments in the late 1980s.

loss-function parameters ω_y to 1 and $\omega_{\triangle R}$ to 0, which means that the central bank assigns equal weight to inflation and output stability, but does not put any weight on smoothing its policy instrument.

Case 2: Case 1, but the rule is optimized with no active house-price bubbles. Because it is difficult to establish the existence of house-price bubbles with reasonable certainty, some may dispute their inclusion in the historical mix of shocks. It is therefore important to assess the sensitivity of our main results to the assumption of house-price bubbles in the past.

Case 3: Case 1, but the central bank is assumed to care about smoothing its policy instrument ($\omega_{\triangle R} = 0.5$).

Case 4: Case 2, but, as in Case 3, the central bank is assumed to care about smoothing its policy instrument.

6.1 Results

Table 4 reports the optimal target and feedback horizons for the different cases outlined above. Recall that, to calculate an optimal target horizon, we draw a set of shocks from the historical distribution in the first period of the simulation. No shocks are drawn after the first period. We then let these initial-period shocks run through the model with policy characterized by an optimized rule. We report two sets of numbers in Table 5: the first set corresponds to simulations *in the absence of a house-price bubble,* and the second set (in parentheses) refers to simulations *in the presence of a house-price bubble* using the same optimized rule. The simulations in the presence of a house-price bubble are conducted by assuming that, in addition to the other shocks, the bubble starts in the initial period. The length of the bubble is random and is governed by the bursting probability, p. The difference between the simulations with and without a bubble yields the implied extension to the average target horizon when there is a house-price bubble.

The results for Case 1 suggest that, given the distribution of shocks and inflation persistence over the past 25 years, the optimal target horizon for Canada would average around six quarters. We also conduct our simulations using another rule that is optimized without house-price bubbles as part of the historical mix of shocks, which yields similar results (Case 2). This fairly robust result would suggest that the optimal target horizon tends, on average, to be at the lower end of the six- to eight-quarter range that has characterized the

Bank's policy since the inception of its inflation-targeting regime in 1991. It is plausible that increased credibility since the introduction of the targeting regime has acted to reduce the lags in the transmission mechanism, and thus the target horizon. It is important to note that this horizon extends to 13 quarters in the presence of a house-price bubble. Thus, it may be appropriate to take a longer view of the inflation-target horizon when the financial accelerator is triggered by a relatively persistent shock, such as a house-price bubble.

Because policy-makers may also care about the financial instability that might be associated with volatile interest rates and the risk of hitting the zero bound on nominal interest rates, we also consider cases where the central bank is averse to the volatility of its policy instrument (Cases 3 and 4). Although there is little change in the average target horizons, we note that the range of target horizons increases appreciably. This increase in the range may reflect the tension that exists when the central bank faces a cost of adjusting its policy instrument.[14]

6.2 Sensitivity analysis

As mentioned in section 5.2, the degree of inflation persistence in Case 1 is 0.57, consistent with empirical estimates for Canada over the 1980–2004 period.[15] A number of recent studies, however, point to a significant decline in inflation persistence in Canada during the inflation-targeting regime (Amano and Murchison 2005; Longworth 2002). In fact, Amano and Murchison estimate inflation persistence over the 1993–2004 period to be around 0.14. We therefore consider a case where inflation persistence is reduced to 0.3 to assess the impact of lower persistence on the target horizon.[16] Lowering the degree of inflation persistence in the model greatly reduces the average target horizon, as well as the range (Table 5). It also reduces the length of the extension associated with house-price bubbles.

We also analyze the robustness of our results to the inclusion of house prices in the rule. We find that the optimal response to house prices is very small. It follows that the impact on the average inflation-target horizon is trivial.

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14. We also find that, if the central bank cares only about inflation stability (i.e., $\omega_y = \omega_{\triangle R} = 0$), then the average optimal target horizon drops to zero. In other words, the differing trade-offs across the shocks become irrelevant and the optimal policy involves the central bank setting policy rates in order to keep inflation on target in all periods, despite the resulting increase in output volatility.

15. For example, using the implicit inflation target from Amano and Murchison (2005) suggests that the persistence of the inflation gap is roughly 0.6.

16. The lower degree of inflation persistence is achieved by increasing the probability that wages and prices can be reset in any given period, reducing the degree of wage indexation to lagged inflation from 1 to zero, and increasing the degree of capacity utilization.

Sensitivity analysis shows that the extent to which house-price bubbles affect the average target horizon depends on the average duration of the bubble, as well as its size. For instance, house-price bubbles with an average duration of a year would extend the optimal target horizon by three quarters, rather than by seven quarters as in case 1 (Table 5). House-price bubbles with variances that are twice as large as in our base case extend the optimal target horizon by about 11 quarters, rather than 7 quarters (Table 5).

7   Conclusion

Research in the area of optimal target horizons is still in its infancy. This paper adds to the existing literature by investigating whether central banks should adjust the time frame for returning inflation to target in response to house-price bubbles. Our findings suggest that, in normal times, the optimal target horizon for Canada tends to be at the lower end of the six- to eight-quarter range that has characterized the Bank's policy since the inception of the inflation-targeting regime. However, we find that it may be appropriate to take a longer view of the inflation-target horizon when the financial accelerator is triggered by a relatively persistent shock, such as a house-price bubble. In fact, our results suggest an average extension of about seven quarters when there is a house-price bubble.

It is important to note that the point estimates of target horizons are model specific and sensitive to changes in model specification. The results should therefore be interpreted with caution. One important caveat is that the bursting of a bubble is exogenously determined in the model. This rules out the possibility that monetary policy can surgically eliminate a bubble (although the actual price of housing remains highly sensitive to monetary policy, since policy influences the fundamental component of house prices).[17] Furthermore, in the current framework, all agents know the *ex ante* stochastic process of the bubble, as well as its current state. In practice, it is difficult for all agents, including the central bank, to ascertain the presence and size of a bubble.

It is also important to note that these experiments likely underestimate the real impact of house-price bubbles. Since we solve a linear approximation of the underlying non-linear model, we may not be accounting for the full extent of financial disruption that accompanies the collapse of a house-price bubble. For example, while the financing

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17. The implications of endogenizing the bubble are not straightforward. Presumably, an endogenous bubble could arise only if there were confusion about fundamentals, or some lack of common knowledge. This, however, adds another dimension to the monetary policy problem: the central bank must identify the presence of a bubble prior to determining the appropriate policy action.

premium increases in response to falling house prices, quantity restrictions that may occur when financial imbalances unwind are not modelled. In fact, Tkacz and Wilkins (2005) find evidence in the Canadian data of important threshold effects in the relationship between house prices and real activity, suggesting that the bias from using a linear model may be important. In that sense, the model likely underestimates the downward pressure on output and inflation in the aftermath of the bubble. Note also that the only balance-sheet effects in the model are those that link the cost of mortgage financing to household balance sheets. Bank balance sheets, which are not modelled, could also play an important role.

The caveats discussed above, as well as other extensions, are left for future research. The results reported in this paper provide a useful starting point for further discussions about optimal target horizons for Canada. We believe that future work should focus on endogenizing the bubble and improving the modelling of monetary policy. In particular, we believe that the latter could be achieved by moving from the simple instrument rules studied in this paper to an optimal targeting rule. As for endogenizing the bubble, this would likely involve introducing confusion about the fundamentals, or some lack of common knowledge. This would allow for a richer analysis of the role of monetary policy, but it poses formidable technical challenges.

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Table1

Relative Standard Deviations (relative to SD of Output)

	Data (1980Q1–2004Q4)			Model
	Lower bound	Mean	Upper bound	
Inflation	0.28	0.36	0.46	0.38
Real house-price growth	1.11	1.41	1.79	1.27
Nominal interest rates	0.46	0.59	0.75	0.59
Exchange rate	1.50	1.91	2.43	2.50
Consumption	1.11	1.41	1.80	0.71
Residential investment	2.97	3.77	4.79	3.24
Business investment	2.14	2.72	3.46	3.93
Exports	1.83	2.33	2.96	2.22
Imports	2.16	2.74	3.49	2.24

Table 2

First-Order Autocorrelations

	Data (1980Q1–2004Q4)			Model
	Lower bound	Mean	Upper bound	
Inflation	0.39	0.63*	0.87	0.57
Output growth	-0.12	0.10	0.33	0.36
Nominal interest rates	0.78	0.90	1.01	0.90
Real house-price growth	0.69	0.83	0.97	0.17
Consumption	0.16	0.41	0.67	0.67
Residential investment	0.32	0.50	0.67	0.73
Business investment	0.11	0.35	0.59	0.74
Exports	-0.16	0.04	0.23	-0.11
Imports	0.04	0.27	0.49	0.69

* Amano and Murchison (2005)

Table 3

Cross-Correlations*

	Data (1980Q1–2004Q4)			Model
	Lower bound	**Mean**	**Upper bound**	**Model**
Output–Inflation	-0.73	-0.47	-0.21	-0.20
Output–Nominal interest rates	-0.69	-0.40	-0.11	-0.22
Output–Consumption	-0.21	0.11	0.42	0.41
Output–Residential investment	0.15	0.41	0.67	0.48
Output–Mortgage credit	-0.12	0.18	0.49	0.35
Output–Real exchange rate	0.02	0.25	0.49	0.52
Inflation–Nominal interest rate	-0.06	0.40	0.86	0.57
Inflation–Mortgage credit	-0.72	-0.44	-0.16	-0.53
Nominal interest rate–Mortgage credit	-0.67	-0.15	0.38	-0.12
Consumption–Residential investment	0.05	0.37	0.69	0.93
Consumption–Real house prices	-0.17	0.28	0.73	0.73
Consumption–Mortgage credit	0.07	0.40	0.73	0.61
Residential investment–Real house prices	-0.17	0.28	0.74	0.65
Residential investment–Mortgage credit	-0.07	0.17	0.41	0.67
Real house prices–Mortgage credit	0.22	0.51	0.80	0.69

* All variables specified in growth terms, except for inflation, and nominal interest rates

Table 4

Optimal Target Horizons - Main Cases

Case	Description	k	Average target horizon	Range of target horizons
1	$\overline{\mathcal{L}} = \sigma_\pi^2 + \sigma_y^2$ Policy optimized with bubbles	2	6 (13)	$3-9$ $(3-15)$
2	$\overline{\mathcal{L}} = \sigma_\pi^2 + \sigma_y^2$ Policy optimized with no bubble	2	6 (13)	$2-9$ $(4-38)$
3	$\overline{\mathcal{L}} = \sigma_\pi^2 + \sigma_y^2 + 0.5\sigma_{\triangle R}^2$ Policy optimized with bubbles	2	6 (13)	$3-9$ $(4-48)$
4	$\overline{\mathcal{L}} = \sigma_\pi^2 + \sigma_y^2 + 0.5\sigma_{\triangle R}^2$ Policy optimized with no bubble	2	6 (14)	$2-9$ $(3-51)$

Table 5

Optimal Target Horizons - Sensitivity Analysis

Description	k	Average target horizon	Range of target horizons
Inflation persistence reduced to 0.3	2	3 (7)	$2-4$ $(3-10)$
Policy responds directly to house prices	2	4 (15)	$3-5$ $(3-29)$
Average duration of bubble $= 1$ year	2	5 (8)	$3-9$ $(3-9)$
Bubbles are twice as large as in base case	2	5 (16)	$3-8$ $(3-75)$

Chart 1

Annualized Nominal Interest Rate Shock
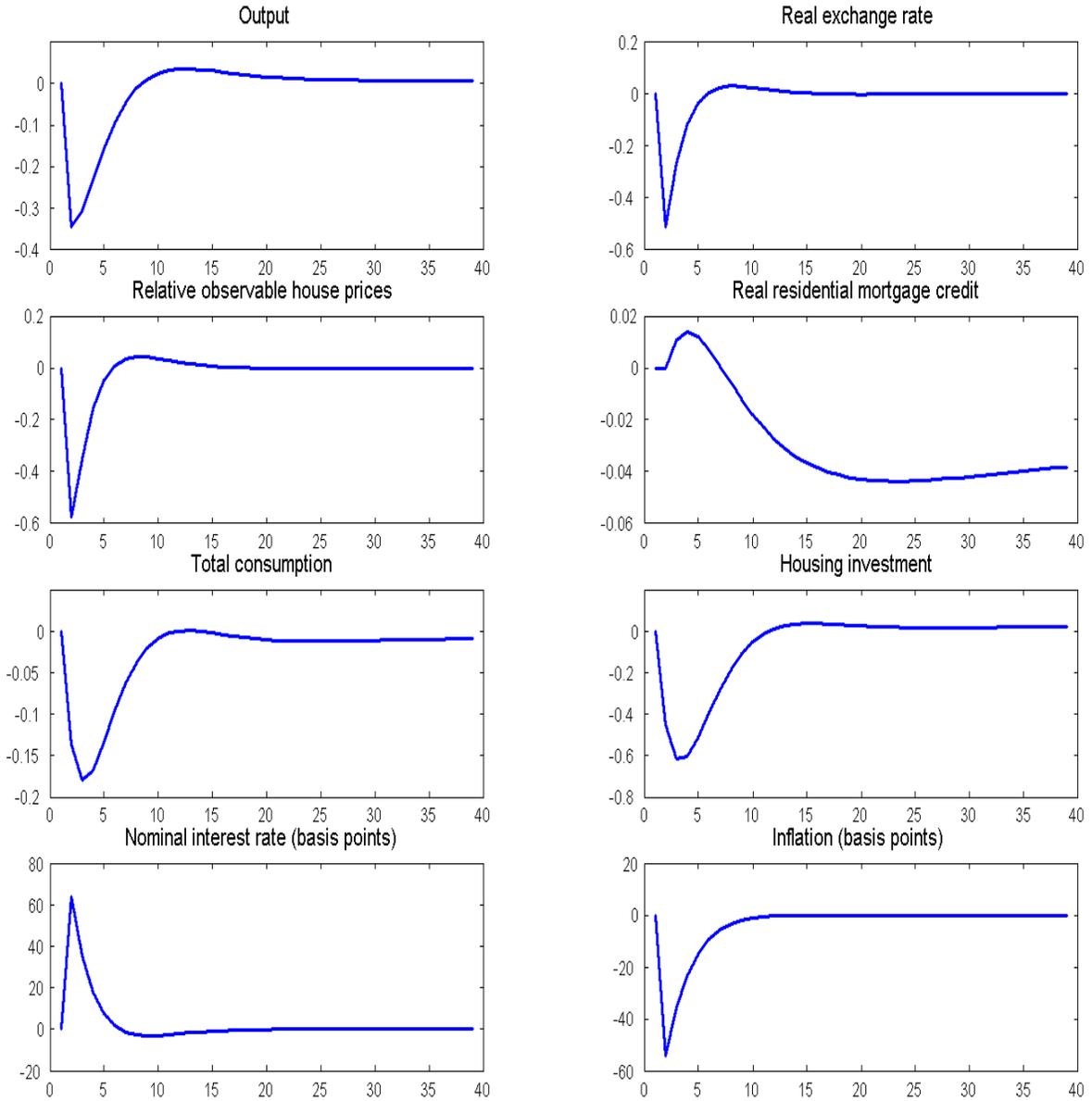
Chart 2

Temporary Productivity Shock

Output

Real exchange rate

Relative observable house prices

Real residential mortgage credit

Total consumption

Housing investment

Nominal interest rate (basis points)

Inflation (basis points)
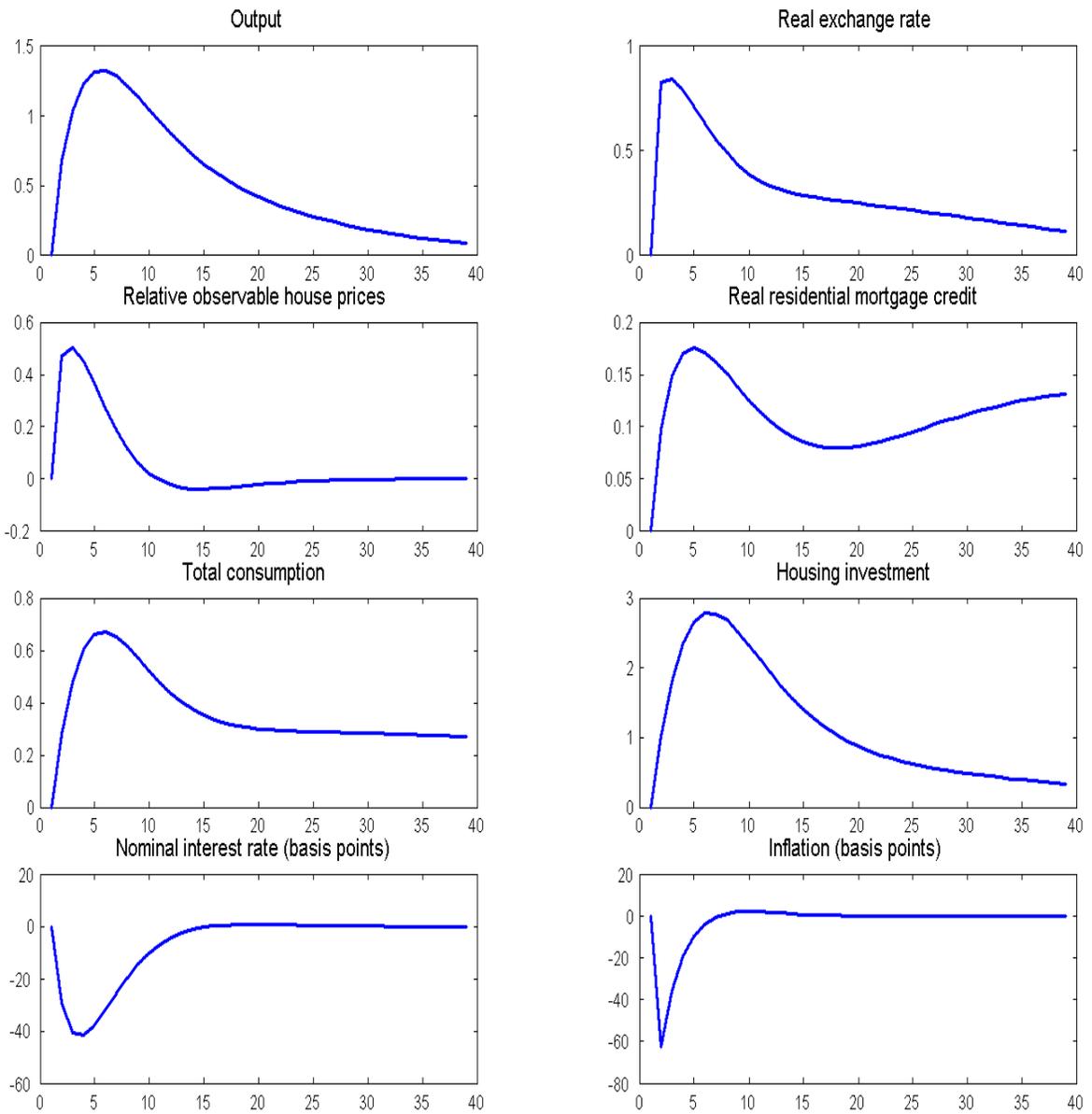
Chart 3

House-Price Bubble Shock
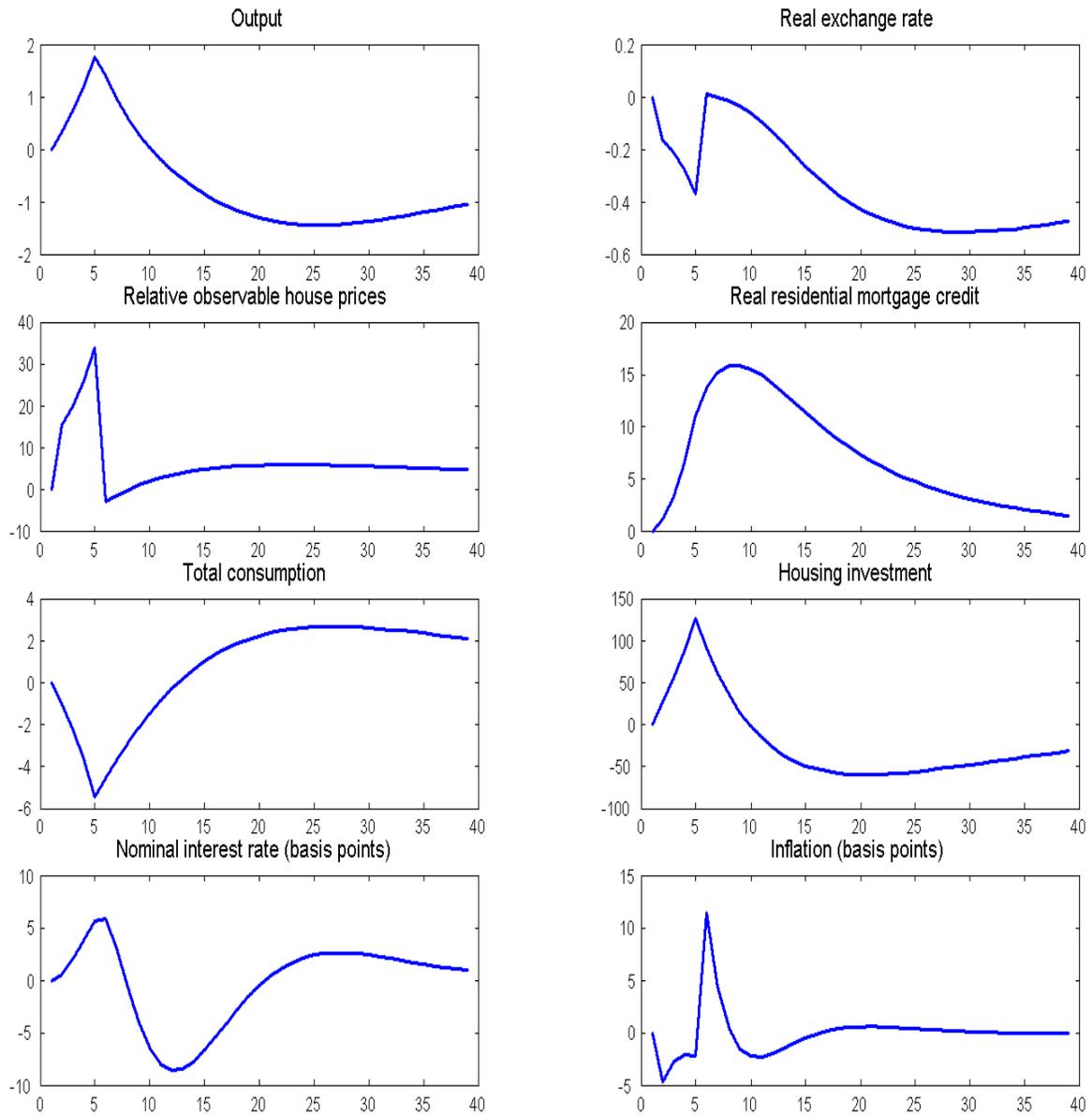
Chart 4

Housing-Demand Shock

Output

Real exchange rate

Relative observable house prices

Real residential mortgage credit

Total consumption

Housing investment

Nominal interest rate (basis points)

Inflation (basis points)
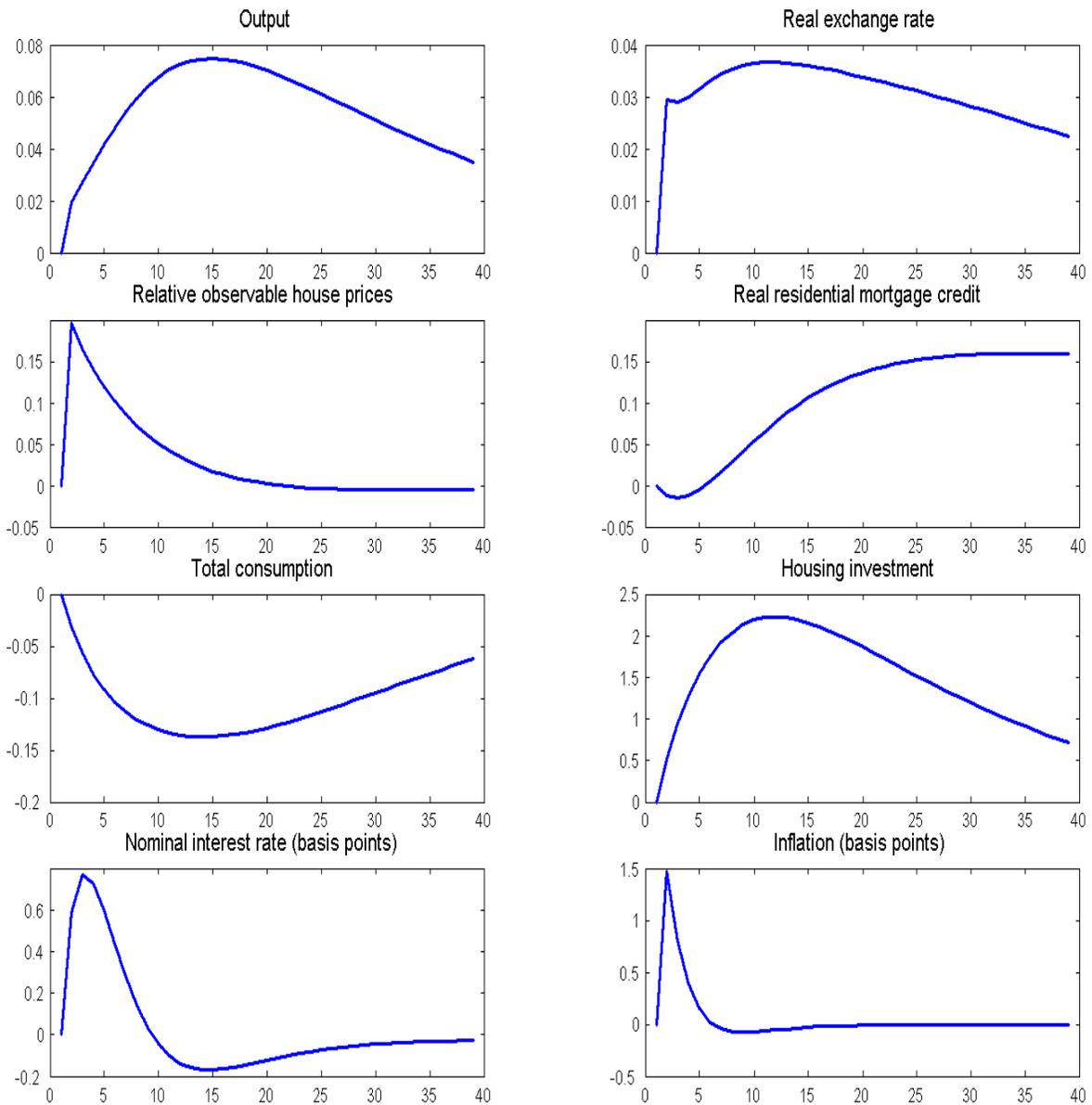
Chart 5

Negative Labour-Supply Shock

Output

Real exchange rate

Relative observable house prices

Real residential mortgage credit

Total consumption

Housing investment

Nominal interest rate (basis points)

Inflation (basis points)
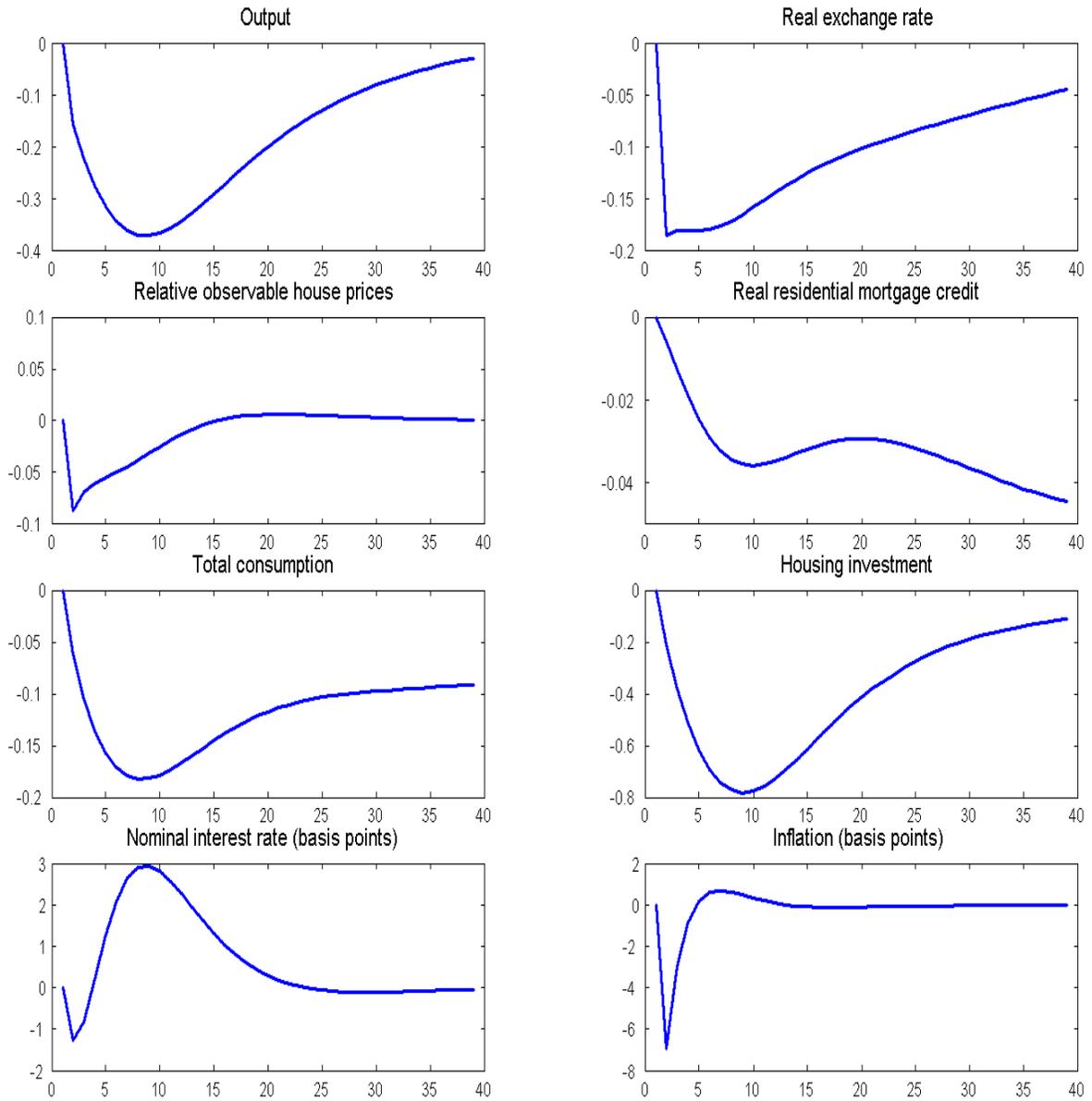
Chart 6

Country-Specific Risk-Premium Shock

38
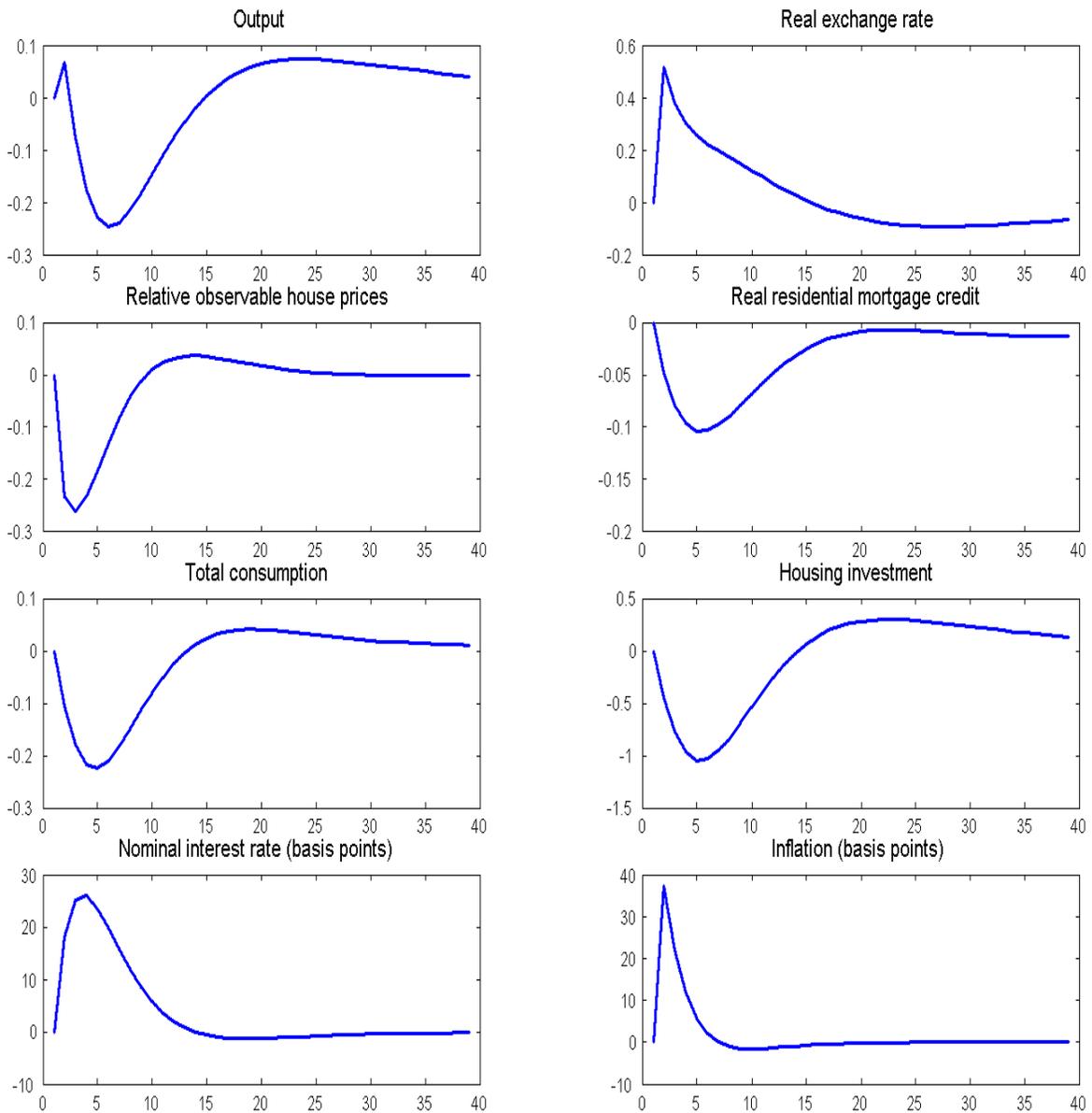
Chart 7
Price-Markup Shock

Output

Real exchange rate

Relative observable house prices

Real residential mortgage credit

Total consumption

Housing investment

Nominal interest rate (basis points)

Inflation (basis points)