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Abstract

The authors assess the performance of the Canadian economy under a variety of interest rate rules when the zero bound on nominal interest rates can bind. Their assessment is based on numerical simulations of a dynamic stochastic general-equilibrium model in a stochastic environment. Consistent with the literature, the authors find that the probability and consequences of the zero bound depend strongly on the targeted rate of inflation and that price-level targeting generally leads to better outcomes. Their results show that a non-linear rule is preferable to a linear rule under both inflation and price-level targeting, because of the zero-bound issue. This suggests that central banks should be pre-emptive and adopt an aggressive monetary policy when expected inflation falls below its desired level. The authors' results also show that the monetary authority must be much more forward looking under price-level targeting than under inflation targeting.

JEL classification: E43, E47, E52
Bank classification: Inflation: costs and benefits; Interest rates; Monetary policy framework

Résumé

Les auteurs évaluent comment l'économie canadienne se comporte en présence de diverses règles de taux dans un contexte où la borne limitant les taux d'intérêt nominaux à zéro peut avoir un effet contraignant. Leur évaluation se fonde sur la réalisation de simulations numériques au moyen d'un modèle d'équilibre général dynamique et stochastique. En accord avec la littérature, les auteurs constatent que la probabilité et les conséquences d'une baisse des taux d'intérêt jusqu'à zéro dépendent fortement du taux d'inflation visé et que la poursuite de cibles définies par rapport au niveau des prix donne de meilleurs résultats. Ils montrent qu'à cause de la borne du zéro, il vaut mieux opter pour une règle non linéaire, que les cibles soient exprimées en fonction de l'inflation ou du niveau des prix. Les résultats présentés donnent donc à penser que les banques centrales devraient prendre les devants et intervenir avec vigueur lorsque l'inflation descend au-dessous du niveau souhaité. Ils révèlent en outre que l'autorité monétaire a avantage à adopter un horizon plus long quand elle poursuit une cible de niveau des prix.

Classification JEL : E43, E47, E52
Classification de la Banque : Inflation : coûts et avantages; Taux d'intérêt; Cadre de la politique monétaire

1 Introduction

Macroeconomists generally agree that a low-inflation environment is desirable. There is still a debate, however, on how low the optimal rate of inflation is. The ability of monetary authorities to conduct countercyclical policy is an important question in this debate. As noted by Summers (1991), because the nominal short-term interest rate cannot be lowered below zero, the monetary authority could be constrained in its ability to conduct countercyclical policy in a low-inflation environment. The non-negativity constraint on nominal rates is more likely to be binding with a lower average rate of inflation and/or greater variability of the interest rate. Recent experience in Japan, which resembled the experience of the 1930s in North America, is evidence that the zero interest rate bound remains a significant and relevant practical issue for monetary policy.

Our study examines the implications of the zero bound for the conduct of monetary policy in Canada. We use the Terms-of-Trade Economic Model (ToTEM), an open-economy dynamic stochastic general-equilibrium model developed at the Bank of Canada. We examine the implications of the zero bound under various regimes and policy rules, and seek to determine the optimized monetary policy conduct in a context where the zero bound can bind.

Section 2 describes the mechanisms of the zero bound and reviews existing studies. The model and the simulation methods are described in sections 3 and 4, respectively. Section 5 examines the implications of hitting the zero bound when the monetary authority is assumed to behave as it did over recent history. Section 6 discusses the issue of optimized policy rules in the context of the zero bound, and section 7 examines how sensitive the implications of the zero bound are to certain characteristics of the economy. Section 8 offers some conclusions.

It is important to stress that our analysis considers only the effects of the zero bound on nominal interest rates, and not other factors that may affect the choice of the optimal rate of inflation. For example, we do not address the issue of possible downward nominal-wage rigidities or the benefits of low inflation, such as those associated with a reduction in distortions.

2 Review of Existing Studies

There are four key factors that determine the risks and the costs of hitting the zero bound: the variance of shocks in the different inflation regimes; the equilibrium real interest rate; the policy rule; and the model, particularly the inflation process in the model. Studies in the literature use different sets of assumptions regarding these four factors, but all conclude that there is only a small probability (5 per cent or less) of hitting the zero bound when targeting inflation rates of

2 per cent or above (see, for example, Babineau, Lavoie, and Moreau 2001; Reifschneider and Williams 2000; Orphanides and Wieland 1998; Fuhrer and Madigan 1997; Cozier and Lavoie 1994). Each of the studies concludes that reducing the targeted inflation to zero per cent would lead to a greater variability of output and inflation if the economy was to continue to be subject to stochastic shocks similar in magnitude to those observed since the 1970s. The zero bound constraint thus introduces a long-run trade-off between inflation and output.

The severity of this deterioration in macroeconomic performance significantly diverges across studies. Some studies suggest that the zero bound constraint would lead to a severe deterioration in economic conditions if the inflation target were reduced to zero. Others conclude that the implications of the zero bound would be small. The difference in the results can be traced to the different assumptions regarding the policy rule and the structure of the price setting.

In studies where inflation is assumed to be highly persistent and the monetary authority is assumed to follow a simple Taylor rule (e.g., Cozier and Lavoie 1994; Fuhrer and Madigan 1997), the zero bound constraint produces a quantitatively greater variability of output and inflation when the targeted inflation is reduced to zero. In that regime, when the zero bound is binding following a negative shock, the monetary policy cannot bring output to its equilibrium, and inflation remains below its targeted level. If inflation is assumed to be persistent (not well anchored), this leads to a fall in inflation expectations. Since the nominal interest rate is stuck at zero, this yields an increase in the real interest rate. The increase in the real interest rate generates additional negative pressure on output and inflation. This leads to a deflationary spiral, which persists until a positive shock hits the economy.

Several studies (Reifschneider and Williams 2000; Orphanides and Wieland 1998) point out the possibility that the design of policy can alter the effects of the zero bound. For example, Eggertsson and Woodford (2003) show that a central bank can significantly alleviate the impact of the zero bound because it can exploit forward-looking expectations. Price-level targeting represents one special case of policy rules that have the property of reducing the negative effects of the zero bound. For example, Wolman's (1998, 2003) results suggest that the consequences of the zero bound would be small in a low-inflation regime if the monetary authority targets the price level. In a price-level targeting regime, when the zero bound is binding, the monetary policy cannot bring output to its equilibrium and the price level remains below its targeted level. The increased difference between the price level and its target leads to a rise in inflation expectations and a decrease in the real interest rate. The decrease in the real interest rate generates positive pressure on output and inflation, which pushes the economy towards its equilibrium. Obviously, as Maclean and Pioro (2001) note, these results hinge on the assumptions that monetary policy is perfectly credible, and that agents continue to believe that monetary policy can eventually bring the price level back to its equilibrium level even after the zero bound has been hit. If agents start doubting that the central bank cannot bring the economy back to its equilibrium, inflation expectations will not rise and the economy will fall into a liquidity trap, as under an inflation-targeting regime.

Our studies first compute the unconditional probabilities of falling into a liquidity trap, using historical and optimized rules for Canada. We use a recent structural, optimizing-agent, open-economy, dynamic stochastic general-equilibrium (DSGE) model calibrated for Canada. Existing studies for Canada (Babineau, Lavoie, and Moreau 2001; Cozier and Lavoie 1994) use reduced-form models and examine only the quantitative importance of the zero bound under historical rules. In computing our probabilities, we also ensure that agents in the economy factor the chance that the zero bound might bind. Many existing studies allow private sector agents to believe that the nominal interest rate can be negative, which leads to an underestimation of the importance of the zero bound.

We then assess the performance of a variety of optimized rules when the threat of a liquidity trap exists, including non-linear feedback rules. This expands on Reifschneider and Williams (2000) and Orphanides and Wieland (1998) by explicitly computing optimized rules.

3 The Model

Our analysis is conducted with ToTEM, a model developed at the Bank of Canada. ToTEM is an optimizing-agent, monopolistic-competition, open-economy DSGE model with wage and price rigidities and habit formation in consumption.

In this model, firms combine labour, capital, imports, and commodities to produce goods using a constant elasticity of substitution (CES) technology, but they face costs in adjusting labour and capital. Once built, capital is assumed to be specific to each firm. Firms produce four distinct finished products: consumption goods and services, investment goods, government goods, and commodity or non-commodity export goods. Products are sold on monopolistically competitive markets for consumption, investment, public consumption, and non-commodity exports. Commodity producers are assumed to be price-takers on world markets, and commodities are exported or sold to domestic producers as an intermediate input.

Utility-maximizing households supply labour and purchase consumption goods with labour income, dividends, and interest from foreign-bond holdings. Consumption behaviour shows habit persistence.

Prices and wages are rigid because of the existence of multi-period contracts. When given the opportunity to renegotiate their contracts, firms realign their wages and prices with their equilibrium marginal rate of substitution and marginal costs. However, this realignment may generate inflationary or deflationary pressures that the monetary authority must fight. Because prices and wages are not equal to their equilibrium in the short run, the level of production diverges from its equilibrium (potential output), yielding an output gap. The model adopts the standard Calvo contract for prices and wages. The equations and other characteristics of the model are described in Murchison and Rennison (2006).

In ToTEM, monetary policy moves the one-period bond nominal interest rate in proportion to the difference between the current level of inflation and its targeted value. The monetary authority is also assumed to smooth the interest rate.

The model is calibrated to match a large set of conditional and unconditional moments over the 1982–2004 period. The values of the parameters imply average price and wage contract lengths of two and six quarters, respectively, with 20 per cent of the price and 40 per cent of the wage contracts being indexed to last-period inflation. The policy rule is calibrated based on empirical results.[1] The policy rule used is the following:

$$r1n_t = 0.8r1n_{t-1} + (1-0.8)\left(r1^* + p^* + 3.5(p^{e}_{t+2} - p^*)\right) + e_t \, .$$

We examine the empirical performance of the model by conducting stochastic simulations of the estimated model and then comparing moments of simulated and actual data to evaluate how closely the model can replicate key features of the historical data. We find that the estimated model is able to reproduce key aggregate stylized facts, including several key bivariate relationships. Judging from the autocorrelations, the artificial data appear to capture historical persistence quite well (Appendix A). The model is, however, less successful in replicating historical variances. In general, the variances are higher in the artificial data than in the actual

1. When matching the empirical moments, we assume that monetary policy was not perfectly credible. The difference between the observed interest rate and that prescribed by the rule is interpreted by some agents as a shock to the inflation target.

data. The variance of the interest rate more closely replicates the historical data than do the variances of output and inflation.

4 Simulation Methods

The model is non-linear and contains unobserved expectations of future state variables. We log-linearize the model to get a linear forward-looking version of it. We impose the zero bound on the nominal interest rate in this linearized version of the model and simulate it using stack simulations. This ensures that agents in the economy factor in the chance that the zero bound might bind. This is a technical issue, but an important one to bear in mind when examining the policy implications of hitting the zero bound. As noted earlier, in several existing studies on the zero bound, models are solved conditioning on the linearity of the model, but are simulated in response to shocks with the zero bound constraint binding. This implicitly means that the models are solved under the assumptions that private sector agents do not factor in the chance that the zero bound may bind. As a result, these studies underestimate the implications of the zero bound constraint. Our simulation technique ensures that our results do not suffer from this bias.[2] Studies by Wolman (2003) and Eggertsson and Woodford (2003) are also immune from this bias, since they use non-linear solution methods.

The model is simulated stochastically 20 times for 30 years (2,500 periods) under different inflation regimes,[3] using the variance-covariance matrix of the historical shocks.[4] The probability of hitting the zero bound, and the economic implications of it, are then computed. To correct for the fact that the model tends to generate higher variances than in actual data, we scale down the estimated variance of the shocks that are used to simulate the model stochastically. Since the variance of the interest rate is key to examining the zero bound issue, we scale the shocks such that the variance of the interest rate produced by the model is close to the historical variance.[5] The variances of inflation and output generated by the model remain slightly higher than the

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2. The imposition of a terminal condition in the forward-looking version of the model would bias the probability of a liquidity trap if the terminal condition were set at a period relatively close to the starting condition. We seek to minimize this bias by setting the terminal condition 250 years after the starting point. We believe that this length of simulation time renders the bias practically non-existent.
3. The properties of the linearized version of the model are invariant to the choice of the steady-state inflation.
4. We use the historical shocks estimated over the 1982–2004 period. If we were to use a different period, the variance of the interest rate and the probability of hitting the zero bound would be different.
5. We exclude the monetary policy shocks in our computation of the variance.

historical variances, but most of the relative variances are within a 95 per cent confidence band of the historical ones.

5 Results under Historical Policy Rules

To compare our results with those in the literature, and to isolate the effect of the zero bound constraint, we first examine the implications of the zero bound under a 2 and 0 per cent inflation-targeting regime assuming that the monetary authority behaves in both regimes as it did in the past. Our results suggest that if the economy is subject to the same stochastic shocks that have characterized the Canadian economy over the past 25 years, the impact of the zero bound is non-linear with respect to the inflation target, with the zero bound producing distortions in the economy in low-inflation regimes.

The relative frequency of zero interest rate binding increases substantially when inflation drops to zero. For an inflation target of 2 per cent, the zero bound is reached about 4 per cent of the time, and the average duration is about four quarters (Table 1). As the inflation target is lowered, both the frequency of hitting the constraint and the average duration of periods at the constraint rises. In the case of the zero per cent inflation target, the rate is bounded at zero about 20 per cent of the time, and the average duration is six quarters. The probability of falling into a liquidity trap increases, but it remains small, passing from 0 per cent at 2 per cent inflation to 0.2 per cent at zero inflation. We define a liquidity trap as a situation where the economy enters a deflationary spiral and cannot escape from it, which leads the stochastic simulation to fail.[6]

The existence of the zero bound influences the distribution of output and inflation. When inflation is zero, the frequency of mild recessions is lower but the likelihood of severe contractions is higher than with a 2 per cent inflation target (Chart 1). Similarly, the distribution of inflation is skewed to the left under a 0 per cent inflation target, but evenly distributed under a 2 per cent inflation target.

6. With stack simulation, one needs to have a long lead stack to correctly capture the possibility of a trap. To have a short lead period is to implicitly assume that agents believe that such a trap cannot occur. We set the lead time to 1,000 periods. This significantly increases the computing time.

Chart 1

Distribution of Output Gap under 2 and 0 Per Cent Inflation Targets

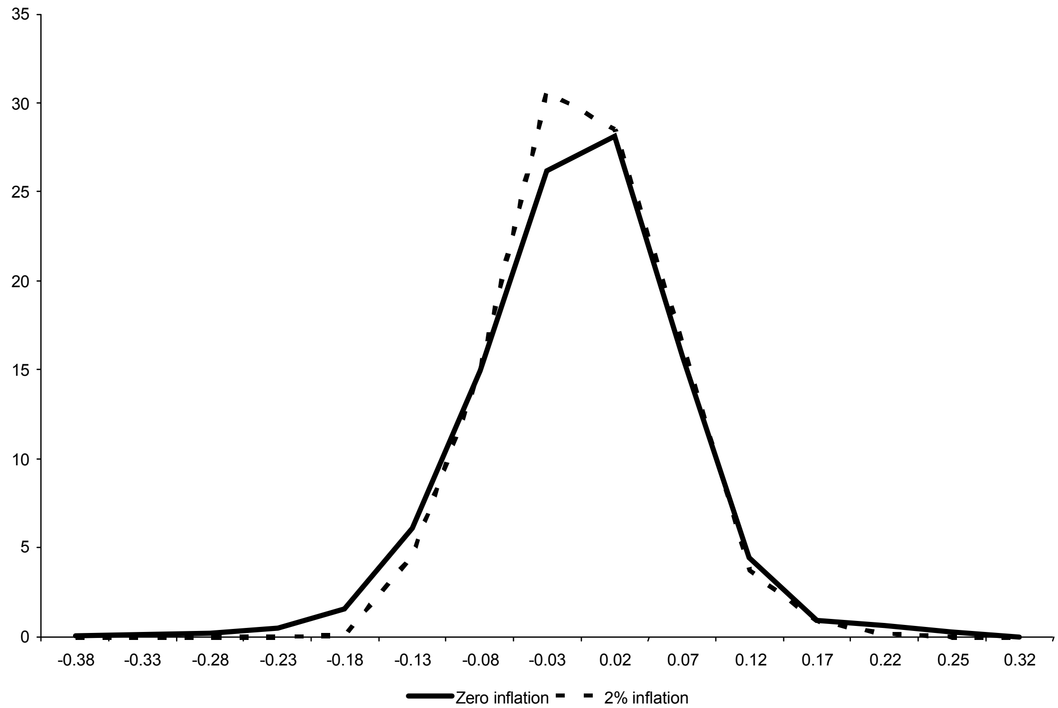

Table 1
Comparing Macroeconomic Performance Under 2 and 0 Per Cent Inflation

	2 per cent inflation	0 per cent inflation
Variance of output gap	0.36	0.48
Variance of inflation (deviation from target)	0.07	0.10
Average output gap	0.0	-1.15
Average inflation (deviation from target)	0.0	-0.68
Probability of hitting zero bound (per cent)	3.8	19.3
Average time that the zero bound constraint binds (quarters)	3.4	5.7
Probability of falling into a liquidity trap (per cent)	0.0	0.2

The zero bound constraint invalidates the long-run superneutrality. The relationship between the average level of output and the average level of inflation that is due to the zero bound implies the existence of a non-vertical long-run Phillips curve. This gives rise to a trade-off between inflation and output. Under a zero-inflation target, the output gap and inflation fall short of their means, on average, by 0.7 and 1.2 percentage points, respectively (Table 1).

The implications of the zero bound in our study appear slightly more severe than what other studies using structural models have found. This may be partly because we ensure that economic agents factor in the chance that the zero bound might bind. For example, if agents do not explicitly incorporate the zero bound in forming their expectations, as in Reifschneider and Williams (2000), the probability of hitting the zero bound at zero per cent inflation would be only about 12 per cent, rather than 20 per cent.

6 Implications for Optimized Monetary Policy Rules

In this section, we investigate how the design of monetary policy can be improved in light of the zero bound. The optimized monetary policy rule is likely to be affected by the zero bound and the non-linearities induced by the zero bound constraint, as shown in section 4. This will likely lead the optimized rule to be different across inflation regimes. Also, we examine whether price-level targeting is preferable in terms of output and inflation stabilization over inflation targeting when the zero bound constraint is taken into account.

To address these issues, we compute the optimized policy rule at 2 and 0 per cent inflation in our model where the zero bound constraint is taken into account. We simulate the forward-looking version of our model under several combinations of parameters for the policy rule, and under different inflation regimes. Given the computational complexity of this process, due to our model's large size, we concentrate on a very small representative grid search. For computational purposes, we restrict our analysis to a specific functional form for the policy rule and to a given monetary policy loss function.[7]

7. An alternative would be to focus on an optimal targeting rule, rather than an instrument rule. Doing so would allow us to derive an analytical characterization of optimal policy, thereby avoiding the costly grid search. Derivation of the optimal targeting rule when the zero bound may bind is more complicated than in the standard case, but it is still feasible in a smaller model (see, for example, Eggertsson and Woodford 2003).

Explicitly, we assume that, under inflation targeting, the monetary authority wants to have a simple inflation forecast-based rule with the following functional form:

$$r1n_t = r r1n_{t-1} + (1 - r)\left(r1^* + p^* + b(p^e_{t+k} - p^*)\right),$$

and that the monetary authority wants to minimize the following loss function:

$$\text{Min } E(Y - Y^*)^2 + E(p - p^*)^2 = s_y + s_p + (E(Y))^2 + (E(p) - p^*)^2, \tag{1}$$

where r is the degree of smoothing, b the degree of aggressiveness of the monetary policy, $r1^*$ the real equilibrium interest rate, $r1n$ the nominal interest rate, p^* the targeted inflation, p_{t+k} the expected rate of inflation in k periods, s_y the variance of the output gap, and s_p the variance of inflation.

This is the usual standard problem found in the optimized policy rule literature (e.g., Cayen, Corbett, and Perrier 2006), with the exception of the third and fourth terms in the loss function. These terms seek to capture the fact that the zero bound, as shown above, may cause the average output gap to be different from zero and the average rate of inflation to be different from the targeted inflation. Without the zero bound constraint, the average output gap is zero; the average inflation is equal to the targeted inflation and the above loss function collapses to the standard loss function.[8]

As noted, we also want to examine whether price-level targeting is preferable in terms of output and inflation stabilization. Under price-level targeting, we assume that the monetary authority desires to have a simple rule with the following functional form:

$$r1n_t = r r1n_{t-1} + (1 - r)\left(r1^* + p^* + b(P^e_{t+k} - P^*_t)\right).$$

We consider two possible loss functions for price-level targeting. First, we assume that the central monetary authority targets the price level in order to minimize the quadratic deviation of inflation and output from their desired levels, as in the inflation-targeting problem described above. However, because of the zero bound, the monetary authority will not be able to keep the

8. The standard loss function is: $(Y\text{-}Y^*)^2+(p\text{-}p^*)^2$. However, without the zero bound, $Y^*=E(Y)$ and $p^*=E(p)$. We thus have decomposed our loss function in this way: $(Y\text{-}E(Y))^2+(p\text{-}E(p))^2+(E(Y)\text{-}Y^*)^2+(E(p)\text{-}p^*)^2$.

price level on its desired target, on average. Because this may lead to credibility loss, we also assume the possibility that the monetary authority may want to minimize the following loss function under price-level targeting:

$$s_y + s_p + (E(Y))^2 + (E(P) - P^*)^2 . \quad (2)$$

We compute the value of these loss functions under a 2 and 0 per cent inflation target, and price-target regimes under a predetermined set of combinations for the values of r, b, and k. We also allow the rule to be non-linear, with the value of b varying depending on whether inflation is above or below its target. This follows the work of Kato and Nishiyama (2005) and Orphanides and Wieland (1998), who demonstrate that the optimized policy under the zero bound constraint is a non-linear function. Their results suggest that central banks must adopt an aggressive monetary policy as the nominal interest rate approaches the zero lower bound.

6.1 The optimal behaviour of the monetary authority under a 2 per cent inflation target

We find that under a 2 per cent inflation regime, an aggressive policy rule with a large degree of smoothing and a value for k of 2 is desirable (Table 2). Although they do not take into consideration the zero bound problem, Cayen, Corbett, and Perrier (2006) also find that an aggressive rule that looks at expected inflation two to three quarters ahead is optimal in ToTEM. However, they find that interest rate smoothing is optimal only when the variance of the change in the interest rate is included in the monetary authority's loss function.

In contrast, we find that smoothing is optimal even when the interest rate variance is not included in the loss function, since interest rate smoothing reduces the probability of hitting the zero bound and hitting the zero bound leads to more pronounced downturns. Moreover, if the nominal interest rate is lowered to zero, a high smoothing parameter leads agents to expect that rates will stay at zero for a longer period. This stimulates aggregate demand more, resulting in less average time spent at the zero bound than under a low smoothing parameter. This is interesting, since Cayen, Corbett, and Perrier (2006) introduce the variance of interest rates in their loss function to indirectly take into account the zero bound constraint.

Table 2
Various Policy Rule Performances under a 2 Per Cent Inflation-Target Regime[1]

Rules	s_y	s_p	*E(y)*	*E(p)*	Tot. loss	*P*(*r*1*n*=0)	*P* of trap
r=0.8, *b*=3.5, *k*=2	0.36	0.07	0.0	-0.0	0.47	3.8	0
r=0.8, *b*=1.5, *k*=2	0.49	0.10	0.1	-0.4	0.76	1.1	0
r=0.8, *b*=5.0, *k*=2	0.33	0.06	-0.2	-0.1	0.44	7.7	0
r=0.3, *b*=3.5, *k*=2	0.34	0.07	-0.2	-0.3	0.54	16.8	0
r=0.6, *b*=3.5, *k*=2	0.34	0.07	-0.1	-0.2	0.46	10.8	0
r=0.8, *b*=3.5, *k*=4	0.42	0.1	0	-0.2	0.56	2.3	0
r=0.8, *b*=20.0, *k*=2	0.19	0.04	-0.1	-0.3	0.33	17.6	0

1. We have also examined a few other policy rules, but we show only the best performing ones.

6.2 The optimal behaviour of the monetary authority under a 0 per cent inflation target

Consistent with results from Kato and Nishiyama (2005), we find that the optimized policy rule under a 0 per cent inflation target is non-linear (Table 3). This evidence suggests that central banks should adopt an aggressive monetary policy when expected inflation falls below its desired level and the nominal interest rate approaches the zero lower bound. Under a 0 per cent inflation-target regime, monetary policy must thus be pre-emptive to prevent the constraint from binding. Adopting such a policy significantly limits the implications of the zero bound constraint. For example, the probability of falling into a liquidity trap and the value of the loss function are significantly reduced. The implications of the zero bound on the distribution of the output gap and inflation are also very limited, but the variance of output remains greater than under a 2 per cent inflation target. Interestingly, we find that the optimal amount of leads on inflation in the policy rule is the same under a 2 and 0 per cent inflation-targeting regime (two to three quarters).

Our results contribute to the debate on whether we should 'conserve the ammunition' or take aggressive action in face of the zero lower bound constraint. The results show that being aggressive when the expected inflation is below its desired level reduces the risk of falling into a deflationary spiral.

Table 3
Comparing Inflation-Targeting Policy Rules under a 0 Per Cent Inflation-Target Regime

Rules	s_y	s_p	*E(y)*	*E(p)*	Tot. loss	***P*(*r*1*n* = 0)**	***P* of trap**
r=0.8, b=3.5, k=2	0.48	0.10	-1.15	-0.68	2.39	19.3	0.2
r=0.8, b=1.5, k=2	0.61	0.14	-0.5	-0.82	1.66	12.1	0.2
r=0.8, b=5.0, k=2	0.45	0.09	-1.46	-0.69	3.14	24.6	0.2
r=0.3, b=3.5, k=2	0.48	0.12	-1.58	-0.89	3.89	35.4	0.2
r=0.8, b=3.5, k=4	0.57	0.14	-0.81	-0.64	1.78	15.5	0
r=0.8, b=10 if $p < p^*$ and =3.5 if $p > p^*$, k=2	0.44	0.09	-0.6	-0.2	0.93	26.3	0
r=0.8, b=5 if $p < p^*$ and =3.5 if $p > p^*$, k=2	0.48	0.10	-0.95	-0.49	1.72	21.9	0
r=0.8, b=5 if $p < p^*$ and =1.5 if $p > p^*$, k=2	0.50	0.10	-0.1	0.2	0.65	18.2	0

If we were to consider only the class of linear policy rules, our results suggest that under a zero-inflation target a less aggressive policy rule is preferable. This is contrary to what we found under a 2 per cent inflation-target regime. Because of the zero bound constraint, a linear aggressive rule would lead monetary policy to be too contractionary, on average, yielding significantly skewed distributions of inflation and the output gap, and an average negative output gap.

6.3 The zero bound and price-level targeting

Some studies (e.g., Wolman 2003) suggest that the consequences of the zero bound are negligible when inflation is not persistent and the monetary authority follows a price-level targeting rule. In a price-level targeting regime, when the zero bound is binding and the price level remains below its targeted level, inflation expectations rise, yielding a decrease in the real interest rate.[9] This decrease in the real rates pushes the economy back towards its equilibrium.

Under the assumptions that monetary policy is credible and expectations remain anchored, price-level targeting will thus guard against falling into a liquidity trap. However, it does not necessarily imply that the zero interest rate bound would have no implications.

9. See Maclean and Pioro (2001) for an example of this using the Quarterly Projection Model.

We find that the economy never falls into a liquidity trap under a price-level targeting regime. Our results suggest that the optimized policy is non-linear, but with the monetary authority being much more forward looking than under inflation targeting (Table 4). Our simulations suggest that the monetary authority should look at the expected price level at least 10 quarters ahead in order to minimize the value of the loss function. Under the optimized rule, price-level targeting appears to be slightly dominating zero-inflation targeting under the specific considered loss function described, mainly because it yields lower output variability. These results hinge on the assumptions that monetary policy is perfectly credible and inflation expectations remain well anchored, even when the zero bound is hit.[10,11]

Table 4
Comparing Price-Level Targeting Policy Rules

Rules	s_y	s_p	s_p	*E(y)*	*E(p)*	Loss (1)	Loss (2)	*P*(*r*1*n* ‹ 0)
r=0.8, *b*=3.5, *k*=2	0.4724	0.0272	0.0301	-1.00	-0.65	1.50	1.92	39.5
r=0.8, *b*=1.5, *k*=2	0.4764	0.0287	0.0327	-0.73	-0.49	1.04	1.25	27.8
r=0.8, *b*=1.5, *k*=10	0.4080	0.0553	0.0465	-0.39	-0.31	0.61	0.71	14.3
r=0.8, *b*=5.0, *k*=2	0.4721	0.0273	0.0295	-1.12	-0.72	1.75	2.27	43.8
r=0.3, *b*=3.5, *k*=2	0.4494	0.0278	0.0290	-1.15	-0.71	1.80	2.30	50.2
r=0.8, *b*=3.5, *k*=4	0.4372	0.0279	0.0390	-0.82	-0.54	1.15	1.42	34.7
r=0.8, *b*=3.5, *k*=10	0.3890	0.0436	0.0398	-0.46	-0.34	0.55	0.76	20.3
r=0.8, *b*=1.5 if *p* ‹ *p** and =5 if *p* › *p**, *k*=2	0.4783	0.0287	0.0314	-1.23	-0.88	2.02	2.83	33.8
r=0.8, *b*=5 if *p* ‹ *p** and =1.5 if *p* › *p**, *k*=10	0.4001	0.0489	0.0424	-0.12	0.05	0.45	0.46	11.8
r=0.8, *b*=5 if *p* ‹ *p** and =1.5 if *p* › *p**, *k*=2	0.4703	0.0271	0.0309	-0.6	-0.32	0.86	0.96	28.4

10. These assumptions ensure that a liquidity trap cannot occur. Under the setting in which the simulations are done, a liquidity trap could not occur. Obviously, in the real world there is a high chance that monetary policy credibility will fall and that expectations will shift when the zero bound is hit. We, however, believe that a liquidity trap will still be more rare under price-level than under inflation targeting.

11. Interestingly, our results also suggest that, for the same parameters in the policy rule, the average output is lower under price-level targeting than under inflation targeting. This may need to be investigated further.

7 Sensitivity Analysis

As noted in section 2, the implications of the zero bound vary with model assumptions. For example, the distortions generated by the zero bound constraint in low-inflation regimes are higher when inflation is assumed to be highly persistent than when inflation is assumed to be well anchored to the target. In this section, we examine the impacts of the zero bound in a 2 and 0 per cent inflation-targeting regime under various model parameterizations. We then compare the relative costs of reducing the inflation target over these various parameterizations.[12] Table 5 shows the results of the sensitivity analysis.

First, consistent with the literature, we find that a higher degree of indexation (or a larger degree of adaptive expectations) worsens the implications of the zero bound, because it makes inflation more persistent (Table 5). We assume that this degree of indexation is low, based on empirical evidence and price surveys, and is not state dependent. As noted, our results hinge on the assumption that monetary policy remains credible and expectations remain anchored even when the zero bound is hit.

However, it is not clear that the degree of indexation would remain constant if the zero bound were hit. For example, agents may start to believe that the monetary authority may not have the required lever to bring the economy and inflation back to equilibrium when the zero bound is binding for an extended period. In that case, monetary policy credibility could be reduced and inflation expectations could cease to be anchored and become more adaptive. Inflation would then become more persistent and the negative implications of the zero bound would be greater than in our base case.

Second, we find that, although they also make inflation more persistent, longer contracts for consumer prices reduce the implications of the zero bound. This is because longer contracts make inflation and prices less responsive to shocks. For example, the price would not fall as much in the near term following a negative shock, reducing the probability of deflation and a deflationary spiral. In contrast, when contracts are short, prices respond immediately to a negative shock, increasing the chance for deflation to occur and the need for large interest rate cuts. If interest rates are constrained by the zero bound, deflation leads to higher real rates and a deflationary spiral.

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12. Comparing absolute loss functions across different parameterizations is not appropriate, since we are comparing different economies.

Third, we find that the degree to which the exchange rate is allowed to rapidly respond to a change in the interest rate significantly affects the consequences of the zero bound. When the interest rates are bounded at zero, monetary policy can still use the exchange rate to affect monetary conditions. Therefore, the greater the ability of monetary policy to affect monetary conditions through the exchange rate channel, the lower the implications of the zero bound constraint. In our model, in order to better match the data, the exchange rate fully responds to a shock only with some lag. The model incorporates a hybrid version of the uncovered interest rate parity, which takes the following form:

$$z_t = a z_{t+1}^e + (1-a) z_{t-1} + r1n_t - r1n_t^f ,$$

where z is the nominal exchange rate, $r1n$ the nominal domestic interest rate, and $r1n^f$ the nominal foreign interest rate. Increasing a makes the exchange rate respond more rapidly to a rate change, and it reduces the impact of the zero bound.

Fourth, we find that the greater the impact of the interest rate on the economy, the lower the consequences of the zero bound, since the monetary authority has a greater ability to control the economy and to offset negative shocks, limiting the probability of hitting the zero bound for an extended period.

Table 5
Sensitivity of the Costs of Reducing the Inflation Target that Are Induced by the Zero Bound

Sensitivity	Parameter change	Relative costs of reducing the inflation target
Higher degree of indexation	Higher share of contracts for the consumption good and service prices that are indexed	Higher
Longer contracts	Lower share of contracts that are renegotiated at each period	Lower
More responsive exchange rate	Lower weight on the lagged exchange rate in the hybrid uncovered interest rate parity condition	Lower
Larger impact of monetary policy	Higher intertemporal substitution of consumption	Lower
More persistent economic cycle	Higher degree of habit formation	Higher

Finally, the more persistent the economic cycle, the higher the consequences of the zero bound—for example, when the economy is hit by a negative shock that forces the zero bound constraint to bind, the more persistent the shock, the greater the consequences of the zero bound.

Changes in other parameters examined have little impact on the implications of the zero bound.

8 Conclusion and Future Work

Our results suggest that the zero bound does not have important implications when the monetary authority targets inflation of 2 per cent, except that it leads the optimized policy rule to have a larger degree of interest rate smoothing than it would otherwise. Our results also suggest that, when the monetary authority targets zero inflation, the zero bound constraint yields an asymmetric distribution of output and inflation, which leads the optimized policy rule to be non-linear. This suggests that central banks should be pre-emptive and adopt an aggressive monetary policy when expected inflation falls below its desired level and the nominal interest rate approaches the zero lower bound. We find that the optimal amount of lead on inflation in the policy rule is two to three quarters under both a 2 and a 0 per cent inflation-targeting regime.

Under price-level targeting, the optimized policy is also non-linear, but the monetary authority must be much more forward looking than under inflation targeting. Our simulations suggest that the monetary authority should look at the expected price level at least 10 quarters ahead in order to minimize the value of the loss function.

Overall, our results suggest that it is costly (greater loss according to our loss function) to reduce the inflation target, because of the problems associated with the zero bound. However, if the benefits were proven to outweigh the costs, it would be preferable for the monetary authority to adopt a non-linear price-level targeting rule than to adopt a zero-inflation target.

Our results are sensitive to the assumed specification of the economy. In particular, the costs of reducing the inflation target induced by the zero bound are significantly larger when we assume that inflation expectations are less well anchored, expectation formation is more adaptive, and inflation is more persistent. The costs of reducing the inflation target are also larger when the monetary authority has less control over the economy, including the exchange rate, than assumed in our base case.

There are several caveats to our work, and issues that need to be examined further. First, our model was calibrated in order to reproduce the key moments observed over the 1982–2005 period. It is possible that these moments are not invariant to the inflation regime. We have observed, for example, a decline in the variability of output and inflation, and in the persistence of inflation, in Canada after the Bank adopted its current 1 to 3 per cent inflation-target regime.[13]

We have not examined the implications of the zero bound for the transition from one inflation regime to another. For example, if the Bank decided to lower its inflation target band or to adopt a price-level targeting regime, it is not clear what impact the zero bound constraint would have on the transition costs.

Finally, because of the small size of our grid search, we have not correctly identified the optimized policy rule under each regime. A more refined grid would yield a different parameterization for the optimized rules, but we do not think the differences would be large and our qualitative results would not change.

13. The reduction in the variability of these variables may be due, however, to the adoption of a target for inflation, rather than to the reduction in inflation itself. If this were the case, lowering the current target range would not reduce further the variability of inflation. This needs to be investigated.

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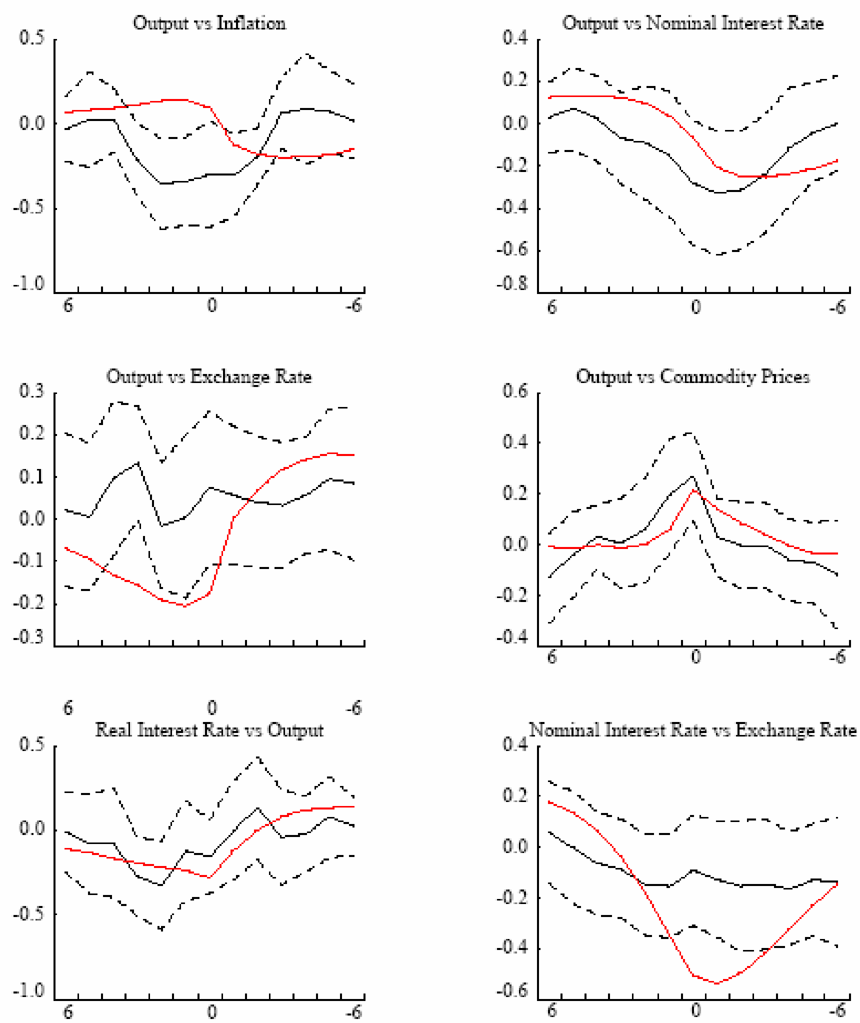
Appendix A: Charts of Model-Generated Cross-Correlation and Autocorrelation Functions

Cross-Correlation and Autocorrelation Functions

red: model calibration

black: correlations and 95% confidence bands – historical values



Cross-Correlation and Autocorrelation Functions

red: model calibration

black: correlations and 95% confidence bands – historical values