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The Bank of Canada's Version of the Global Economy Model (BoC-GEM)

by René Lalonde and Dirk Muir

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René Lalonde and Dirk Muir

International Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
rlalonde@bankofcanada.ca
dmuir@bankofcanada.ca

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Contents

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Abstract

The Bank of Canada's version of the Global Economy Model (BoC-GEM) is derived from the model created at the International Monetary Fund by Douglas Laxton (IMF) and Paolo Pesenti (Federal Reserve Bank of New York and National Bureau of Economic Research). The GEM is a dynamic stochastic general-equilibrium model based on an optimizing representative-agent framework with balanced growth, and some additional features to help mimic the overlapping-generations' class of models. Moreover, there is a concrete role for fiscal policy (albeit not fully optimized) and monetary policy. At the Bank, the model has been extended beyond the standard version with tradable and non-tradable goods sectors to include both oil and non-oil commodities. Furthermore, the oil sector is decomposed into oil for production and oil for retail consumption. The authors provide a detailed technical description of the model's structure and calibration. They also describe the model's simulation properties for Canadian and U.S. domestic shocks, and describe how the model can be used to analyze issues that currently are at the forefront for the Canadian and global economies, such as trade protectionism, global imbalances, and increasing oil prices.

JEL classification: C68, E27, E37, F32, F47
Bank classification: Economic models; International topics; Business fluctuations and cycles

Résumé

Le modèle BOC-GEM de la Banque du Canada a été établi à partir du modèle de l'économie mondiale GEM (pour *Global Economy Model*), élaboré au Fonds monétaire international par Douglas Laxton (FMI) et Paolo Pesenti (Banque fédérale de réserve de New York et National Bureau of Economic Research). GEM est un modèle d'équilibre général dynamique et stochastique qui repose sur la présence d'agents représentatifs optimisateurs, suppose une croissance de l'activité économique et emprunte certaines caractéristiques aux modèles à générations imbriquées. De plus, la politique budgétaire (sans être pleinement optimisée) et la politique monétaire y jouent un rôle important. Le modèle de la Banque adjoint deux autres secteurs (les produits pétroliers et les produits de base autres que l'énergie) aux secteurs des biens échangeables et non échangeables inclus dans le modèle du FMI. Il établit également une distinction entre le pétrole utilisé pour la production et le pétrole destiné à la consommation. Les auteurs brossent un tableau détaillé de la structure et de l'étalonnage du modèle. Ils décrivent aussi comment se comporte le modèle lorsqu'on simule l'effet de chocs au sein des économies canadienne et américaine, ainsi que la façon dont le modèle peut servir à analyser des questions

d'actualité sur la scène économique nationale et internationale telles que le protectionnisme, les déséquilibres mondiaux et la hausse des prix du pétrole.

Classification JEL : C68, E27, E37, F32, F47
Classification de la Banque : Modèles économiques; Questions internationales; Cycles et fluctuations économiques

1. Introduction

The Bank of Canada has a rich history of modelling, focusing mainly on the economies of Canada and the United States. In the early 1990s, with advances in economic theory and increased computing power, the Bank developed a new Canadian projection model, the Quarterly Projection Model (QPM). This model reflected the state of the art in terms of theoretical structure and dynamic adjustment, and was the Bank's main model for Canadian economic projections and policy advice for the following decade. Recently, the Bank replaced QPM with a new projection and policy-analysis model of the Canadian economy, named ToTEM (Terms-of-Trade Economic Model). ToTEM was developed at the Bank of Canada and is considered to be at the cutting edge of the art and technology of macroeconomic modelling for projection and policy analysis (see Murchison and Rennison 2006).

Although the different generations of models of the Canadian economy have undergone significant changes in terms of theoretical underpinnings or macroeconomic structure, they have always relied on other models or sources of information for forecasts of external economic activity. Until the introduction of the Global Economy Model (GEM), described in this report, the Bank's modelling efforts regarding the external environment have concentrated mainly on the U.S. economy.[1] Over the past four years, staff have developed and used a new macroeconometric model, MUSE (Model of the United States Economy – see Gosselin and Lalonde 2005), to analyze and forecast the U.S. economy. This model is an estimated forward-looking model with stock-flow dynamics based on the polynomial-adjustment-cost framework of Tinsley (1993). The model describes the interactions among the principal macroeconomic variables, such as the gross domestic product and its components, inflation, interest rates, and the exchange rate.

With increasing openness to trade worldwide, however, the emergence of the economies of China and India, the rise of global imbalances, and the recent increase in the price of oil, it is necessary to view the external environment from a consistent global perspective. As a result, for many issues it has become unsatisfactory to take a stand on the current and future positions of the domestic economy by focusing mainly on the United States and Canada. With economic events of global importance occurring outside of our traditional spheres of interest, such as the Asian financial crisis of the late 1990s, and the role of emerging Asia and the oil-exporting countries in financing the large (and continuing) current account deficit of the United States, the Bank has come to recognize the need to complement its existing tools

[1] Bank staff also introduced a new, small, reduced-form model of the euro area and the U.K. economies, called NEUQ: the New European Quarterly model (Piretti and St-Arnaud 2006).

with a global model. This is the broad motivation for the project described in this report.

More specifically, in 2004, the senior staff in the Bank's International Department identified the following key requirements for a new global model:

(i) an appropriate decomposition of the world into different regions beyond simply Canada and the United States;

(ii) model properties that reflect both the Bank's analytical needs and the current state of economic theory and empirics;

(iii) a general-equilibrium framework;

(iv) technical support from the creators of the model, to help maintain and extend it over time, in conjunction with Bank staff;

(v) an independent possibility for Bank staff to extend or modify the model; and

(vi) ease of use, in that it fits with the Bank's current store of economic and technical knowledge.

After considering several alternatives, the GEM created at the IMF was judged to best meet the above criteria, and to be a good starting point for further elaboration. The Bank's version of the GEM is far from exclusively a work of Bank staff. Rather, this is a new extended version of a model that is the product of a network of central banks: the Bank of Canada, the originating institutions (the IMF and the Federal Reserve Bank of New York) and other central banks that have learned from and contributed to the experience of building a truly global, medium-scale dynamic stochastic general-equilibrium (DSGE) model, such as the Norges Bank, the Bank of Japan, and the Reserve Bank of New Zealand. It offers a theoretical coherence (yet flexibility) not found in many modelling tools.

The GEM belongs to the class of models known as DSGE models. This puts it in the same class as the Bank's main policy-analysis and projection tool for the Canadian economy, ToTEM. The GEM is a representative-agent model with a fully optimizing framework based on microfoundations.[2] However, it differs from ToTEM in that it fully utilizes the theory of new open-economy macroeconomics, which implies a full articulation of the world within

[2] Goodfriend and King (1997) provide a good starting point for a survey of the new neoclassical synthesis, the literature that best represents these models.

the GEM.[3] The model is multi-region, encompassing the entire world economy, modelling explicitly all the bilateral trade flows and their relative prices for each region, including exchange rates. The GEM, as a tool, is conducive to large-scale analysis of global issues, as well as country-specific issues. The Bank of Canada's version of the Global Economy Model (BoC-GEM) comprises five regional blocs: Canada (CA), the United States (US), emerging Asia (AS), the commodity exporter (CX), and the remaining countries (RC). Moreover, because of its composition, the BoC-GEM can analyze issues particular to Canada, or issues elsewhere in the world, and how they will affect Canada directly or via effects on another country, such as the United States.

The BoC-GEM incorporates three major extensions to the model of Faruqee et al. (2007a, b). First, Canada is included as a separate region and the country composition of the regions is different. Second, the structure of the model is richer, because two sectors important for the Canadian economy are added: oil and non-oil commodities. Therefore, like the other prices modelled, the prices of oil and non-oil commodities are endogenous. Third, the calibration is adjusted to reflect the views of Bank staff and the properties of ToTEM and MUSE. This last point is important for two reasons: (i) ToTEM (Murchison and Rennison 2006) and MUSE (Gosselin and Lalonde 2005) are good representations of the Canadian and U.S. data; (ii) for issues that need a global and/or a multi-sectoral perspective, the BoC-GEM is used to generate risk scenarios around the base-case staff projection generated using ToTEM and MUSE. Consequently, when applicable, the BoC-GEM's properties need to be consistent with those of ToTEM and MUSE.

One main feature that the BoC-GEM shares with its parent from the IMF is flexibility, in keeping with the philosophy that the GEM is supposed to be a toolbox and a framework for exploring the global macroeconomy using the latest theory and techniques (Pesenti 2007). Although the BoC-GEM is a large, complex DSGE model, it is not a monolith. It can be easily adapted to create other configurations, with either fewer regions or fewer sectors, with or without features such as a fully functioning fiscal sector, or a distribution sector for imported goods.[4] However, our main goal in this report is to describe the BoC-GEM in its entirety and document its properties. We also demonstrate how the model can be used to analyze some challenges that are currently facing the Canadian and global economies. These include:

[3] For more on new open-economy macroeconomics, see Lane (2001). For a good explanation of many of the theoretical concepts used in this report, see Corsetti and Pesenti (2005).

[4] Conceptually, there is nothing to prevent an augmentation in the number of regions and/or sectors; currently, the limitation in this direction is related to computational tractability.

- the impact of emerging Asia on the price of traded goods and on the prices of oil and non-oil commodities,
- the risks associated with the emergence of global imbalances,
- the factors underlying the recent increase in the price of oil, and
- the determinants of the Canadian real exchange rate over the medium and long run.

We provide a detailed description of the model's structure in section 2, and its calibration in section 3. Sections 4 and 5 examine a variety of the properties of the BoC-GEM, many of which are derived from previous versions of the GEM or overlap with other models of interest to the Bank. Other properties and shocks are unique to the BoC-GEM, and this is the first time that some of them have been presented in a published form. It is also the first time that the GEM has been applied to the Canadian economy. In section 6, we conclude by discussing different alternatives for the further development of the BoC-GEM and its use within the staff's economic projection.

2. The Model's Structure

This section outlines the entire structure of the BoC-GEM, moving from a general to a more detailed (but far from exhaustive) technical description. The description of the model that follows (excepting the commodities, oil, and gasoline sectors) relies heavily on Pesenti (2007); it should be referred to directly for a more complete understanding of the model's core structure.

2.1 Overview of the model

As stated in the introduction, the BoC-GEM comprises five regional blocs: Canada (CA), the United States (US), emerging Asia (AS), the commodity exporter (CX), and the remaining countries (RC).[5] Emerging Asia includes eight of the "Asian tigers": China, India, Hong Kong Special Administrative Region of China, the Republic of Korea, Malaysia, the Philippines, Singapore, and Thailand. The CX bloc includes the largest exporters of oil and non-oil commodities, such as the OPEC countries, Norway, Russia, South Africa, Australia, New Zealand, Argentina, Brazil, Chile, and Mexico. The RC bloc includes all the other

[5] Appendix A lists the countries within each of the regional blocs.

countries in the world, but effectively this means Japan and the members of the European Union (since Africa is very small economically).

Each of the five regions is modelled symmetrically, and consists of:

- a continuum of firms (to allow for monopolistic competition, and, by extension, price markups) that produce (and therefore supply and demand) raw materials, intermediate goods, and final goods;

- two types of households (to allow for a differentiation between liquidity-constrained and forward-looking consumers) that consume the (non-traded) final goods and supply differentiated labour inputs to firms; and,

- a government consisting of a fiscal authority that consumes non-tradable goods and services, financed through taxation or borrowing, and a monetary authority that manages short-term interest rates through monetary policy to provide a nominal anchor to the economy.

In general, firms supply goods to domestic and foreign consumers, and demand labour from domestic consumers. In addition, firms demand intermediate goods from other firms that supply them, both domestically and internationally. Consumers demand products from firms both at home and abroad, and supply labour to domestic firms. The entire model in its non-linear form can be thought of as a system of demand, supply, and pricing functions, as a general rule using the mathematical form of the constant elasticity of substitution (CES) function (and its associated Dixit-Stiglitz representation).

Five sectors produce goods from capital and labour and other factors. The production of each of the five sectors is assumed to be monopolistically competitive, which means that firms can still enter and exit the market, but, because each firm's good is slightly differentiated from those produced by other firms, each firm is able to set a price above its marginal cost, allowing a markup. The five sectors are non-tradable goods, tradable goods, oil and natural gas, non-oil commodities, and heating fuel and automobile fuel. Special emphasis is placed on oil and natural gas, and commodities, since the Canadian economy has been historically subject to terms-of-trade shocks from these sectors, and the Bank has no other tool that can deal with such shocks in a fully articulated global general-equilibrium framework. From this point forward in this report, commodities excluding oil and natural gas are referred to as commodities; oil and natural gas are referred to as oil; and heating and automobile fuels are referred to as gasoline.

Figures 1 to 3 show the production structure for a single region. We describe the model first in general, and then in detail, starting with the primary goods and moving upward in the production structure.

Figures 2 and 3 illustrate the production process for oil and commodities, respectively. Each region has firms that produce oil from capital, labour, and crude oil reserves.[6] Other firms produce commodities using capital, labour, and land. Oil can be further processed (along with capital and labour) into gasoline.

Oil and commodities can be traded across regions. Figure 1 shows that oil and commodities are further combined with capital and labour to produce a tradable good (mostly financial services, such as investment banking, or manufactured items, such as automobiles or semi-durables) and a non-tradable good (mostly services outside of the financial sector, such as retailing, education, or health care). The tradable good can also be traded across regions. There are then, three intermediate goods: gasoline, the tradable good, and the non-tradable good. All three are combined to form a final consumption good, while the tradable and non-tradable goods are combined to form a final investment good. The (non-traded) consumption and investment good can be consumed by private agents or by the fiscal agent (who also consumes some share of non-tradable goods directly).

All goods at all levels are assumed to be produced or aggregated using a CES technology. While a Cobb-Douglas form is, in some sense, more tractable, CES forms allow elasticities of substitution that differ from one between inputs in production, or between goods in formulating final demands.

Households consume the final goods, and provide labour to produce them. Moreover, only one class of consumers (forward-looking consumers) ultimately owns all firms and the capital stock used by firms for production; the other class of consumers (liquidity-constrained consumers) has no access to capital markets and depends solely on their labour income.

Regarding international trade, all the bilateral flows (across regions) of the exports and im-

[6] Conceptually, there are three kinds of crude oil reserves: proven and exploited reserves, proven and unexploited reserves, and unknown and/or unproven reserves. In the BoC-GEM, the first type of reserves are designated as crude oil reserves, although the second type can be easily approximated by shocks to the stock of crude oil reserves (as can the third type, if desired).

ports of oil, commodities, and tradable goods for consumption and investment are explicitly modelled as demands for imported goods from specific regions. In order to facilitate international trade, there is a single internationally traded bond, denominated in U.S. dollars; thus, financial markets are assumed to be incomplete. A region's bond holdings define its net foreign asset (NFA) position, which is maintained at some exogenously specified NFA-to-GDP ratio using a modified risk-adjusted uncovered interest rate parity (UIRP) condition to define all the bilateral real exchange rates vis-à-vis the United States (from which all other cross-bilateral exchange rates can be deduced). There is also an explicit link between the level of government debt the fiscal agent holds and the level of net foreign assets, meaning that the representative agent in this model is non-Ricardian. There are further non-Ricardian elements in the GEM: some consumers are liquidity constrained, as noted above (assumed to arise from a low labour skill set), and the government, as fiscal agent, raises revenues through distortionary taxation on labour income, capital income, and (possibly) tariffs on imports.[7] The monetary authority targets either core inflation (defined as the consumer price index excluding gasoline prices and the effects of tariffs), or headline CPI inflation (or a fixed nominal exchange rate) to achieve an objective related to price stability (or price certainty) with a standard reaction function.

Finally, as is generally the case in the DSGE modelling literature, in order to match the persistence observed in the data, the model includes real adjustment costs and nominal rigidities that are allowed to differ across regions.[8] We assume real adjustment costs in capital, investment, labour, and imports. We also suppose that the real adjustment costs are very important in the production and demand of oil and commodities. Nominal rigidities are found in wages and the prices of tradable and non-tradable goods (but not for the oil and commodities sectors).

2.2 Technical preliminaries

Before embarking on a comprehensive overview of the model, several technical matters need to be addressed.

First, in most sections that focus on region-specific equations that are independent of foreign variables and are thus qualitatively similar across countries, region indexes are dropped for notational simplicity. For the sections involving international transactions, region indexes

[7] The fiscal agent can also subsidize trade if it so chooses.

[8] Two practical examples are the Bank of Canada's ToTEM (Murchison and Rennison 2006) and the Board of Governors of the Federal Reserve's SIGMA model (Erceg, Guerrieri, and Gust 2005b).

are explicitly incorporated into the notation, where H is the domestic region and R is a representative region from the rest of the world.

Second, there is a common trend in productivity for the world economy ($TREND$), whose gross rate of growth between time t and time τ is $g_{t,\tau}$. All quantity variables in the model are expressed in detrended terms, as ratios relative to $TREND$. Productivity growth is but one component of growth. The other component is population growth, but, for now, we assume it is zero at all points in time. Furthermore, all prices in the model are stated as relative prices, where the *numéraire* price is the headline CPI. CPI is normalized to unity, and all other prices are stated in relation to it. The GEM is detrended in this way to allow for ease of computation.[9]

Third, variables that are not explicitly indexed to firms, households, or the government are understood to be expressed in per-capita (average) terms. For instance, with the final consumption good, A_t is the sum of the output of all firms over the continuum ss, $(1/ss)\int_0^{ss} A_t(x)dx$, where ss is the size of a region in terms of its labour force and $\sum_{ss} ss = 1$.

Fourth, as a general convention throughout the model, when we state that variable X follows a stochastic process, we mean that it uses an autoregressive formulation, in either levels or logarithmic levels:

$$X_t = (1 - \lambda_X) X + \lambda_X X_{t-1} + e_{X,t}, \tag{1}$$

where $0 < \lambda_X < 1$, X is the steady-state value of X_t, and $e_{X,t}$ is a noise term.

Note also that the following terminology is used in this report:

- a period is assumed to be one quarter;
- all variables are stated in annualized terms, unless otherwise noted;
- when there are superscripts for the regions in bilateral equations, a generic region is defined as R and the importing region is defined as H;
- "oil" means the commodities oil and natural gas, "commodities" means all other commodities, "raw materials" means the oil and commodities sectors together, and "gasoline" means automobile and heating fuels.

[9] Most DSGE models assume zero growth in the steady state or either (or both) the real and nominal sides of the economy. We hope that, by allowing such growth, we will be able in the future to better match the model to real data without having to rely on arbitrary data-detrending methods, such as the band-pass or Hodrick-Prescott (HP) filter.

2.3 The firms' problem

There are three levels of production by firms in the model. We will consider each in turn, starting with the lowest level, raw materials, which in the BoC-GEM are oil and commodities. We will then discuss the intermediate-goods sectors (tradables, non-tradables, and gasoline) and, lastly, the top level, final goods (the consumption good and the investment good).

2.3.1 Raw materials

For the two raw materials sectors, we will discuss the commodities sector only, with the understanding that this exposition is equally applicable to the oil sector.

In each region, there is a continuum of firms that can produce commodities in a monopolistically competitive framework. Each firm is indexed by $s \in [0, ss]$, where ss is the size of the region in the world ($0 < ss < 1$). Firm s produces $S_t(s)$ at period t of its variety of commodities by combining capital $K(s)$, labour $L(s)$, and a fixed factor that is a non-reproducible resource, $LAND(s)$, using a CES technology:

$$\begin{aligned} S_t(s)^{1-\frac{1}{\xi_S}} &= Z_{S,t}[\alpha_{LANDS,t}^{\frac{1}{\xi_S}}(Z_{LANDS,t}LAND_t(s))^{1-\frac{1}{\xi_S}} \\ &\quad + (1-\alpha_{KS,t}-\alpha_{LANDS,t})^{\frac{1}{\xi_S}}\left(Z_{LS,t}\ell_t(s)(1-\Gamma_{LS,t})\right)^{1-\frac{1}{\xi_S}} \\ &\quad + \alpha_{KS,t}^{\frac{1}{\xi_S}}(Z_{KS,t}K_t(s)(1-\Gamma_{KS,t}))^{1-\frac{1}{\xi_S}}], \end{aligned} \tag{2}$$

where Z_S is a shock that follows a stochastic process common to all firms producing commodities to the level of productivity in the entire sector; Z_{LS}, Z_{KS}, and Z_{LANDS} are productivity shocks following stochastic processes that are common to all firms producing commodities specific to the factors in that sector (labour, capital, and land, respectively); Γ_{KS} and Γ_{LS} are real adjustment costs incurred when changing the levels of capital and labour.

Because the production of commodities does not respond immediately to movements in demand, we need to differentiate between a short-run and a long-run supply curve, so that we can match price elasticities that are smaller in the short run than in the long run. Therefore, we model the use of labour and capital in the production process as being subject to real adjustment costs of a quadratic form. In the case of capital, we assume that real adjustment costs can be represented as:

$$\Gamma_{KS,t}[\frac{K_{S,t}(s)}{S_t(s)}/\frac{K_{S,t-1}}{S_{t-1}}] = \frac{\phi_{KS}}{2}\left[\left(K_{S,t}(s)/S_t(s)\right)/\left(K_{S,t-1}/S_{t-1}\right)-1\right]^2, \tag{3}$$

where $\Gamma_{KS}[1] = 0$. This functional form is also assumed for labour. These adjustment costs reduce the ability of firms to change the input composition of their production. Therefore, in the short run, the elasticity of substitution among inputs is lower than in the long run. The size of ϕ_{KS} determines by how much and for how long the elasticity of substitution will be lower than its permanent value, ε_s.

Defining w_t and r_t as the real prices of labour and capital (relative to the CPI) common across all sectors of production, the real marginal cost of commodities production is:

$$mc_t(s) = \frac{1}{Z_{S,t}}[(1 - \alpha_{KS,t} - \alpha_{LANDS,t})\left(\frac{w_t}{Z_{LS,t}}\right)^{1-\xi_S} + \alpha_{KS,t}\left(\frac{r_t}{Z_{KS,t}}\right)^{1-\xi_S} + \alpha_{LANDS,t}\left(\frac{p_{LAND,t}}{Z_{LANDS,t}}\right)^{1-\xi_S}]^{\frac{1}{1-\xi_S}}, \tag{4}$$

and the capital-labour ratio (subject to real adjustment costs) is:

$$\frac{K_t(s)}{\ell_t(s)} = \frac{Z_{LS,t}}{Z_{KS,t}}\frac{(1-\Gamma_{LS,t})}{(1-\Gamma_{KS,t})}\frac{\alpha_{KS,t}}{1-\alpha_{KS,t}-\alpha_{LANDS,t}} \times \left(\frac{Z_{LS,t}}{Z_{KS,t}}\frac{r_t}{w_t}\frac{(1-\Gamma_{KS,t}-K_t(s)\Gamma'_{KS,t})}{(1-\Gamma_{LS,t}-\ell_t(s)\Gamma'_{LS,t})}\right)^{-\xi_S}. \tag{5}$$

Labour inputs are differentiated and come in different varieties (skill levels). They are defined over a continuum of mass ss and indexed by $j \in [0, ss]$. Each firm s uses a CES combination of labour inputs:

$$\ell_t(s) = \left[\left(\frac{1}{ss}\right)^{\frac{1}{\psi_{L,t}}}\int_0^{ss}\ell(s,j)^{1-\frac{1}{\psi_{L,t}}}dj\right]^{\frac{\psi_{L,t}}{\psi_{L,t}-1}}, \tag{6}$$

where $\ell(s, j)$ is the demand for labour input of type j by the producer of good s, and $\psi_L > 1$ is the elasticity of substitution among labour inputs (differentiated by skill level). Cost minimization implies that $\ell(s, j)$ is a function of the relative wage:

$$\ell_t(s,j) = \left(\frac{1}{ss}\right)\left(\frac{w_t(j)}{w_t}\right)^{-\psi_{L,t}}\ell_t(s), \tag{7}$$

where $w(j)$ is the wage paid to the domestic labour input, j, and the average real wage, w, is defined as:

$$w_t = \left[\left(\frac{1}{ss}\right)\int_0^{s} w_t(j)^{1-\psi_{L,t}}dj\right]^{\frac{1}{1-\psi_{L,t}}}. \tag{8}$$

Finally, cost minimization implies that firm s's demand for the fixed factor of production, $LAND$, is:

$$LAND_t(s) = \frac{\alpha_{LANDS,t}}{Z_{LANDS,t}} \left(\frac{p_{LAND,t}}{Z_{LANDS,t}} / \frac{mc_t(s)}{Z_{S,t}} \right)^{-\xi_S} \frac{S_t(s)}{Z_{S,t}}, \tag{9}$$

which implies that, as the price of the fixed factor (p_{LAND}) diverges further above (below) from the real marginal cost of producing commodities, the demand for $LAND$ will fall (rise). The elasticity of substitution between input factors (ξ_S) determines how much demand will react. Because ξ_S is less than unity, the demand for $LAND$ is relatively inelastic, since the expression $\left(\frac{p_{LAND,t}}{Z_{LANDS,t}} / \frac{mc_t(s)}{Z_{S,t}} \right)^{-\xi_S}$ implies that the negative effect on demand will be diminished.

Having defined the supply of commodities, we have to consider the demands for those goods by the types of two intermediate-goods firms: the representative firm h which produces tradables (T), and the representative firm n which produces non-tradables (N). Therefore, we will consider, in turn, the demand for domestically produced and imported commodities.

Demand for domestically produced commodities The aggregate demand for domestically produced commodities by intermediate-goods-producing firms is summarized by:

$$\left(\frac{p_t(s)}{p_{SN,t}} \right)^{-\theta_{S,t}} S_{N,t} + \left(\frac{p_t(s)}{p_{ST,t}} \right)^{-\theta_{S,t}} S_{T,t} = \int_0^{ss} S_t(s,n) dn + \int_0^{ss} S_t(s,h) dh, \tag{10}$$

where $S(s,n)$ is the demand by firm n producing non-tradables for commodities produced by firm s, and $S(s,h)$ is the demand by firm h producing tradables for commodities produced by firm s. S_N is the amount of commodities produced for use by the non-tradables-producing firms, given the commodities' firms' price, $p_t(s)$, relative to the aggregate price, p_{SN}, that non-tradables-producing firms are willing to pay. S_T is defined similarly for firms producing tradables.

We will focus first on the basket $S(s,n)$, which is a CES index of all domestic varieties of commodities used in the production of non-tradable goods by firm n. Therefore, for $S_t(s,n)$:

$$Q_{S,t}(n) = \left[\left(\frac{1}{ss} \right)^{\frac{1}{\theta_{S,t}}} \int_0^{ss} S_t\left(s,n\right)^{1-\frac{1}{\theta_{S,t}}} ds \right]^{\frac{\theta_{S,t}}{\theta_{S,t}-1}}, \tag{11}$$

where $\theta_{S,t} > 1$ denotes the elasticity of substitution among commodities produced by different firms, and $Q_S(n)$ is the demand by all non-tradable-goods-producing firms for domestically produced commodities.

Any firm n takes as given the price of commodities, $p(s)$, produced domestically (Q), giving the price of domestically produced commodities, $p_Q(s)$. Cost minimization implies:

$$S_t(s,n) = \frac{1}{ss}\left(\frac{p_{Q,t}(s)}{p_{SN,t}}\right)^{-\theta_{S,t}} S_t(s), \tag{12}$$

where p_{SN} is the price of one unit of the commodities basket designated for the production of non-tradable goods. As the price $p_Q(s)$ for firm s becomes larger (smaller) than the aggregate price of commodities used in non-tradable-goods production (p_{SN}), demand for commodities from firm s by firm n ($S(s,n)$) will fall (increase), usually by an amount larger than the price difference, since the elasticity of substitution between the varieties of commodities produced by firms is greater than unity. The demand by all firms producing tradable goods for domestically produced commodities ($Q_S(t)$) is similarly characterized.

Demand for imported commodities The demand for imported commodities occurs at two different levels. First, the importing region, H, demands commodities from the other regions, R. Second, different sectors within the importing region have demands that must be met. The representative firm in the commodities sector is $s^H \in [0, ss^H]$. Its imports, $M_S^H(s^H)$, are a CES function of baskets of goods imported from the other regions, or:

$$M_{S,t}^H(s^H)^{1-\frac{1}{\rho_A^H}} = \sum_{R\neq H}\left(b_{S,t}^{H,R}\right)^{\frac{1}{\rho_S^H}}\left(M_{S,t}^{H,R}(s^H)(1-\Gamma_{MS,t}^{H,R}(s^H)\right)^{1-\frac{1}{\rho_S^H}}, \tag{13}$$

where:

$$0 \leq b_{S,t}^{H,R} \leq 1, \quad \sum_{R\neq H} b_{S,t}^{H,R} = 1. \tag{14}$$

In (13), ρ_S^H is the elasticity of import substitution across regions: the higher the ρ_S^H, the easier it is for firm s^H to substitute imports of commodities from one region with imports from another. The parameter $b_S^{H,R}$ is the per cent amount of the commodities imported by region H from region R, subject to the elasticity of import substitution, ρ_S^H. $M_S^{H,R}(s^H)$ denotes imports of commodities by region H's firm s from region R.[10]

Denoting $p_{MS}^{H,R}$ the price in region H of a basket of commodities imported from region R, cost

[10] The parameter $b_S^{H,R}$ can be time-varying; it is represented in the model by the stochastic process shown in equation (1).

minimization implies that:

$$M_{S,t}^{H,R}(s^H) = b_{S,t}^{H,R} \left(\frac{p_{MS,t}^{H,R}}{p_{MS,t}^{H}(s^H)} \right)^{-\rho_S^H} M_{S,t}^{H}(s^H), \tag{15}$$

so that, as $p_{MS}^{H,R}$ rises above (falls below) the aggregate price of imported commodities, $p_{MS}^{H}(s^H)$, region H will import less (more) from region R, and this amount will shift readily, since the elasticity of import substitution, ρ_S^H, is set quite high (well above unity) for commodities.

The import price for commodities in region H, p_{MS}^{H}, is defined as:

$$p_{MS,t}^{H}(s^H) = \left[\sum_{R \neq H} b_{S,t}^{H,R} \left(p_{MS,t}^{H,R} \right)^{1-\rho_S^H} \right]^{\frac{1}{1-\rho_S^H}}. \tag{16}$$

In principle, the cost-minimizing import price $p_{MS}^{H}(s^H)$ is firm-specific, since it depends on the import share of firm s^H. To the extent that all firms s^H are symmetric within the commodities sector, there will be a unique import price, p_{MS}^{H}.

We know that, in aggregate, each region's representative firm demands $M_{S,t}^{H}(s^H)$; it is distributed across the non-tradable and tradable sectors according to the following two demand equations:

$$\begin{aligned} M_{S,t}(n) &= (1-\nu_{SN,t}) \left(p_{MS,t}/p_{SN,t} \right)^{-\mu_{SN}} S_t(n) \times \\ &\quad (1-\Gamma_{MS,t}(n) - M_{S,t}(n)\Gamma'_{MS,t}(n))/(1-\Gamma_{MS,t}(n)), \qquad (17) \\ M_{S,t}(h) &= (1-\nu_{ST,t}) \left(p_{MS,t}/p_{ST,t} \right)^{-\mu_{ST}} S_t(h) \times \\ &\quad (1-\Gamma_{MS,t}(h) - M_{S,t}(h)\Gamma'_{MS,t}(h))/(1-\Gamma_{MS,t}(h)), \qquad (18) \end{aligned}$$

where ν_{SN} and ν_{ST} represent the bias of the home region towards commodities produced domestically. We also assume that, in the short run, there is an additional lag required for firms to shift from domestically produced commodities to imported commodities (because of fixed supply contracts, for example). This explains the presence of the real adjustment costs, $\Gamma_{MS}(n)$ and $\Gamma_{MS}(h)$:

$$
\begin{aligned}
& \Gamma_{MS,t}(n)\left[\frac{M_{S,t}(n)}{N_t(n)}/\frac{M_{SN,t-1}}{N_{t-1}}\right] = \\
& \frac{\phi_{MSN}}{2}\frac{\left[\left(M_{S,t}(n)/N_t(n)\right)/\left(M_{SN,t-1}/N_{t-1}\right)-1\right]^2}{\left(1+\left[\left(M_{S,t}(n)/N_t(n)\right)/\left(M_{SN,t-1}/N_{t-1}\right)-1\right]^2\right)}, \qquad (19)\\
& \Gamma_{MS,t}(h)[\frac{M_{S,t}(h)}{T_t(h)}/\frac{M_{ST,t-1}}{T_{t-1}}] = \\
& \frac{\phi_{MST}}{2}\frac{\left[\left(M_{S,t}(h)/T_t(h)\right)/\left(M_{ST,t-1}/T_{t-1}\right)-1\right]^2}{\left(1+\left[\left(M_{S,t}(h)/T_t(h)\right)/\left(M_{ST,t-1}/T_{t-1}\right)-1\right]^2\right)}, \qquad (20)
\end{aligned}
$$

such that $\Gamma^{H,R}_{MS}[1] = 0$, $\Gamma^{H,R}_{MS}[\infty] = \phi^{H,R}_{MS}/2$, and $\Gamma^{H,R}_{MS}[0] = \Gamma^{H,R}_{MS}[2] = \phi^{H,R}_{MS}/4$ for both firms h and n.

The price of imported commodities used in the production of non-tradables, $p_{SN,t}$, and the price of imported commodities used in the production of tradables, $p_{ST,t}$, are both CES aggregators of the price of domestically produced commodities, p_{QS}, and imported commodities, p_{MS}, differentiated by the real adjustment costs in the import sector, and the elasticity of substitution between domestically produced and imported commodities, μ_{SN} and μ_{ST}:

$$
\begin{aligned}
p_{SN,t} &= [\nu_{SN,t} p_{QS,t}^{1-\mu_{SN}} + (1-\nu_{SN,t})\, p_{MS,t}^{-\mu_{SN}} \times \\
&\quad (1-\Gamma_{MS,t}(n) - M_{S,t}(n)\Gamma'_{MS,t}(n))^{\mu_{SN}-1}]^{\frac{1}{1-\mu_{SN}}}, \qquad (21)\\
p_{ST,t} &= [\nu_{ST,t} p_{QS,t}^{1-\mu_{ST}} + (1-\nu_{ST,t})\, p_{MS,t}^{-\mu_{ST}} \times \\
&\quad (1-\Gamma_{MS,t}(h) - M_{S,t}(h)\Gamma'_{MS,t}(h))^{\mu_{ST}-1}]^{\frac{1}{1-\mu_{ST}}}. \qquad (22)
\end{aligned}
$$

Imported commodities can be expressed as a sum of commodities imported for use in the production of tradable and non-tradable goods, or as a sum of the imported commodities from the different regions, subject to real adjustment costs:

$$
\begin{aligned}
p^H_{MS,t}(s) M^H_{S,t}(s) &= p^H_{MSN,t} M^H_{SN,t} + p^H_{MST,t} M^H_{ST,t} \\
&= \sum_{R \neq H} p^{H,R}_{MS,t}(s) M^{H,R}_{S,t}(s) * \Gamma^H_{MSagg,t}, \qquad (23)
\end{aligned}
$$

where Γ_{MSagg} is the aggregated effects of $\Gamma_{MS}(n)$ and $\Gamma_{MS}(h)$ on both prices and volumes.

We can next equate the demand for commodities by firms producing non-tradable and tradable goods by aggregating across firms in the non-tradable and tradable sectors that use commodities produced domestically or imported from abroad:

$$\begin{aligned}
&\left(\frac{p_t(s)}{p_{SN,t}}\right)^{-\theta_{S,t}} S_{N,t} + \left(\frac{p_t(s)}{p_{ST,t}}\right)^{-\theta_{S,t}} S_{T,t} = \\
&\int_0^{ss} Q_{S,t}(s,n)dn + \int_0^{ss} Q_{S,t}(s,h)dh \\
&+ \int_0^{ss} M_{S,t}(s,n)dn + \int_0^{ss} M_{S,t}(s,h)dh. \qquad (24)
\end{aligned}$$

The oil sector For the oil sector, the technology of production can be quantitatively different from the commodities sector, but its formal characterization is very similar, with self-explanatory changes in notation. For instance, a firm $o \in [0, ss]$ that produces oil uses the production technology:

$$\begin{aligned}
O_t(o)^{1-\frac{1}{\xi_o}} &= Z_{O,t}[\alpha_{OILO,t}^{\frac{1}{\xi_O}}(Z_{OILO,t}OIL_t(o))^{1-\frac{1}{\xi_O}} \\
&+ (1-\alpha_{KO,t}-\alpha_{OILO,t})^{\frac{1}{\xi_O}}\left(Z_{LO,t}L_t(o)(1-\Gamma_{LO,t})\right)^{1-\frac{1}{\xi_O}} \\
&+ \alpha_{KO,t}^{\frac{1}{\xi_O}}(Z_{KO,t}K_t(o)(1-\Gamma_{KO,t}))^{1-\frac{1}{\xi_O}}], \qquad (25)
\end{aligned}$$

where, in this case, crude oil reserves, $OIL(o)$, is the fixed factor.[11] The demand for oil in the intermediate-goods firms, both domestically produced and imported, is also analogous to the commodities sector. The only difference from the demand for commodities is that there is a third type of firm in the intermediate-goods sector, firm g, that produces gasoline, GAS, which has a demand for oil as well; as a result, the aggregate constraint in the oil sector (analogous to equation (24)) becomes:

$$\begin{aligned}
&\left(\frac{p_t(o)}{p_{ON,t}}\right)^{-\theta_{O,t}} O_{N,t} + \left(\frac{p_t(o)}{p_{OT,t}}\right)^{-\theta_{O,t}} O_{T,t} + \left(\frac{p_t(o)}{p_{OGAS,t}}\right)^{-\theta_{O,t}} O_{GAS,t} = \\
&\int_0^{ss} Q_{O,t}(o,n)dn + \int_0^{ss} Q_{O,t}(o,h)dh + \int_0^{ss} Q_{O,t}(o,g)dg \\
&+ \int_0^{ss} M_{O,t}(o,n)dn + \int_0^{ss} M_{O,t}(o,h)dh + \int_0^{ss} M_{O,t}(o,g)dg. \qquad (26)
\end{aligned}$$

[11] The real marginal cost for the oil sector is the cost dual of this equation, with one addition: τ_{ROYAL} is the royalties paid on the holdings of crude oil reserves by the oil-producing firms to the government.

2.3.2 Intermediate goods

For intermediate goods, we consider the non-tradable good in detail, and treat the tradable and gasoline sectors as analogues. Intermediate inputs come in different varieties (brands) and are produced under conditions of monopolistic competition. In each region, there are three kinds of intermediate goods: non-tradables, tradables, and gasoline. Each kind is defined over a continuum of mass ss (the size of the region in the world according to its labour force). Each non-tradable good is produced by a domestic firm indexed by $n \in [0, ss]$, each tradable good is produced by a firm $h \in [0, ss]$, and each gasoline good is produced by a firm $g \in [0, ss]$.

Non-tradables sector The non-tradable N is produced by firm n with the following CES technology:

$$\begin{aligned} N_t(n) &= Z_{N,t}[(1-\alpha_{KN,t}-\alpha_{SN,t}-\alpha_{ON,t})^{\frac{1}{\xi_N}} \left(Z_{LN,t}\ell_t(n)\right)^{1-\frac{1}{\xi_N}} \\ &\quad +\alpha_{KN,t}^{\frac{1}{\xi_N}}(Z_{KN,t}K_t(n))^{1-\frac{1}{\xi_N}}+\alpha_{SN,t}^{\frac{1}{\xi_N}}(S_t(n)(1-\Gamma_{SN,t}))^{1-\frac{1}{\xi_N}} \\ &\quad +\alpha_{ON,t}^{\frac{1}{\xi_N}}(O_t(n)(1-\Gamma_{ON,t}))^{1-\frac{1}{\xi_N}}]^{\frac{\xi_N}{\xi_N-1}}. \end{aligned} \tag{27}$$

Firm n uses four variable factors in the production of its good, and there is no fixed factor present. It uses labour, $\ell(n)$, capital, $K(n)$, commodities, $S(n)$, and oil, $O(n)$, to produce $N(n)$ units of its variety. $0 < \xi_N < 1$ is the elasticity of substitution among factor inputs. As in the commodities and oil sectors, Z_N is a sector-wide productivity shock common to all firms n producing a non-tradable good, while Z_{LN} and Z_{KN} are productivity shocks that are specific to labour and capital, respectively, in the non-tradables sector. In the short run, the capacity of the non-tradable firms to adjust their demand for commodities and oil is very small; therefore, we assume that they are facing real adjustment costs, Γ. Using commodities as an example, the real adjustment costs take the form:

$$\Gamma_{SN,t}[\frac{S_t(n)}{N_t(n)}/\frac{S_{N,t-1}}{N_{t-1}}] = \frac{\phi_{SN}}{2}\left[\left(S_t(n)/N_t(n)\right)/\left(S_{N,t-1}/N_{t-1}\right)-1\right]^2. \tag{28}$$

Therefore, in the short run, the elasticity of substitution among factor inputs is lower than in the long run. The size of the parameter ϕ_{SN} determines by how much, and for how long, the effective elasticity of substitution of the use of commodities as a factor input with other factors will be lower than its long-run value of ε_N. The real marginal cost in non-tradables production is:

$$
\begin{aligned}
mc_t(n) &= \frac{1}{Z_{N,t}}[(1-\alpha_{KN,t}-\alpha_{SN,t}-\alpha_{ON,t})\,(\frac{w_t}{Z_{LN,t}})^{1-\xi_N}+\alpha_{KN,t}(\frac{r_t}{Z_{KN,t}})^{1-\xi_N} \\
&+\alpha_{SN,t}(p_{SN,t}^{1-\xi_N}(1-\Gamma_{S,t}(n)-S_t(n)\Gamma'_{S,t}(n))^{\xi_N-1})^{1-\xi_N} \\
&+\alpha_{ON,t}(((1+\tau_{OIL,t})p_{ON,t})^{1-\xi_N}(1-\Gamma_{O,t}(n)-O_t(n)\Gamma'_{O,t}(n))^{\xi_N-1})]^{\frac{1}{1-\xi_N}}, \qquad (29)
\end{aligned}
$$

and the capital-labour ratio is:

$$
\frac{K_t(n)}{\ell_t(n)} = \frac{Z_{KN,t}}{Z_{LN,t}}\frac{\alpha_{KN,t}}{1-\alpha_{KN,t}-\alpha_{SN,t}-\alpha_{ON,t}}\left(\frac{Z_{LN,t}}{Z_{KN,t}}\frac{r_t}{w_t}\right)^{-\xi_N}, \qquad (30)
$$

where the real marginal cost equation is simply the cost dual of equation (27).

As in the commodities sector, each firm n uses a CES combination of labour inputs:

$$
\ell_t(n) = \left[\left(\frac{1}{ss}\right)^{\frac{1}{\psi_{L,t}}}\int_0^s \ell(n,j)^{1-\frac{1}{\psi_{L,t}}}dj\right]^{\frac{\psi_{L,t}}{\psi_{L,t}-1}}, \qquad (31)
$$

where $\ell(n,j)$ is the demand for labour input of type j by the producer of good n. Cost minimization implies that $\ell(n,j)$ is a function of the relative wage:

$$
\ell_t(n,j) = \left(\frac{1}{ss}\right)\left(\frac{w_t(j)}{w_t}\right)^{-\psi_{L,t}}\ell_t(n), \qquad (32)
$$

where $w(j)$ is the wage as defined in equation (8).

Tradables sector Production of tradable goods follows the same line of reasoning as that of non-tradables. $T(h)$ is the supply of each intermediate tradable, h, produced by firm x in the following manner from labour, $\ell(h)$, capital, $K(h)$, commodities, $S(h)$, and oil, $O(h)$:

$$
\begin{aligned}
T_t(h) &= Z_{T,t}[(1-\alpha_{KT,t}-\alpha_{ST,t}-\alpha_{OT,t})^{\frac{1}{\xi_T}}(Z_{LT,t}\ell_t(h))^{1-\frac{1}{\xi_T}} \\
&+\alpha_{KT,t}^{\frac{1}{\xi_T}}(Z_{KT,t}K_t(h))^{1-\frac{1}{\xi_T}}+\alpha_{ST}^{\frac{1}{\xi_T}}(S_t(h)(1-\Gamma_{ST,t}))^{1-\frac{1}{\xi_T}} \\
&+\alpha_{OT,t}^{\frac{1}{\xi_T}}(O_t(h)(1-\Gamma_{OT,t}))^{1-\frac{1}{\xi_T}}]^{\frac{\xi_T}{\xi_T-1}}, \qquad (33)
\end{aligned}
$$

where Z_T, Z_{LT}, and Z_{KT} are productivity shocks that follow stochastic processes. The rest follows as above for the non-tradables sector.

Gasoline sector The supply (i.e., production) of gasoline is similar to that of non-tradables and tradables. However, there are two differences: there is no role for commodities, plus there are real adjustment costs of the form found in the oil and commodities sectors. Firm g supplies gasoline in the amount $GAS(g)$:

$$\begin{aligned} GAS_t(g) &= Z_{GAS,t}[(1-\alpha_{KGAS,t}-\alpha_{OGAS,t})^{\frac{1}{\xi_{GAS}}}(Z_{LGAS,t}\ell_t(g)(1-\Gamma_{LGAS,t}))^{1-\frac{1}{\xi_{GAS}}} \\ &\quad +\alpha_{KGAS,t}^{\frac{1}{\xi_{GAS}}}(Z_{KGAS,t}K_t(g)(1-\Gamma_{KGAS,t}))^{1-\frac{1}{\xi_{GAS}}} \\ &\quad +\alpha_{OGAS,t}^{\frac{1}{\xi_{GAS}}}(O_t(g)(1-\Gamma_{OGAS,t}))^{1-\frac{1}{\xi_{GAS}}}]^{\frac{\xi_{GAS}}{\xi_{GAS}-1}}, \end{aligned} \tag{34}$$

where Z_{GAS}, Z_{LGAS}, and Z_{KGAS} are productivity shocks following stochastic processes. Notice that real adjustment costs are present for all factor inputs (Γ_{LGAS}, Γ_{KGAS}, and Γ_{OGAS}), meaning that the short-run supply curve is much more inelastic than its long-run equivalent.

Now that we know the supply of intermediate goods, we have to consider the demands for those goods by the types of two final-goods firms, x and e, for consumption goods (A) and investment goods (E), respectively, and, if necessary, by the government (in the case of non-tradables). Therefore, we will consider, in turn, the demand for non-tradable goods, and then domestically produced goods and imported tradable goods. The demand for gasoline (which is a non-traded good in the BoC-GEM) will be covered in section 2.3.3.

Demand for non-tradable intermediate goods The aggregate demand for non-tradable intermediate goods can be summarized by:

$$p_{N,t}N_t = \int_0^{ss} N_{A,t}(n,x)dx + \int_0^{ss} N_{E,t}(n,e)de + G_{N,t}(n) = \left(\frac{p_t(n)}{p_{N,t}}\right)^{-\theta_{N,t}} (N_{A,t}+N_{E,t}+G_{N,t}), \tag{35}$$

where $N_{A,t}(n,x)$ is the demand from firms x for non-tradables for the consumption good, $N_{E,t}(n,e)$ is the demand for non-tradables from firms e for the investment good, and $G_{N,t}(n)$ is the demand for non-tradables by the government sector.

Focusing first on the basket N_A, this is a CES index of all domestic varieties of non-tradables. Denoting as $N_A(n,x)$ the demand by firm x for an intermediate good produced by firm n, the basket $N_A(x)$ is:

$$N_{A,t}(x) = \left[\left(\frac{1}{ss} \right)^{\frac{1}{\theta_{N,t}}} \int_0^{ss} N_{A,t}(n,x)^{1-\frac{1}{\theta_{N,t}}} dn \right]^{\frac{\theta_{N,t}}{\theta_{N,t}-1}}, \tag{36}$$

where $\theta_{N,t} > 1$ denotes the elasticity of substitution among intermediate non-tradables.

Firm x takes as given the prices of the non-tradable goods, $p(n)$. Cost minimization implies:

$$N_{A,t}(n,x) = \frac{1}{ss} \left(\frac{p_t(n)}{p_{N,t}} \right)^{-\theta_{N,t}} N_{A,t}(x), \tag{37}$$

where p_N is the price of one unit of the non-tradable basket, or:

$$p_{N,t} = \left[\left(\frac{1}{ss} \right) \int_0^{ss} p_t(n)^{1-\theta_{N,t}} dn \right]^{\frac{1}{1-\theta_{N,t}}}. \tag{38}$$

The basket N_E is similarly characterized.

Demand for domestically produced tradable goods Following the same steps, we can derive the domestic demand schedules for the intermediate goods, h:

$$\int_0^{ss} Q_{A,t}(h,x)dx + \int_0^{ss} Q_{E,t}(h,e)de = \left(\frac{p_t(h)}{p_{Q,t}} \right)^{-\theta_{T,t}} (Q_{A,t} + Q_{E,t}), \tag{39}$$

where $Q_{A,t}(h,x)$ is the demand from firms x for tradables for the consumption good and $Q_{E,t}(h,e)$ is the demand for non-tradables from firms e for the investment good.

Demand for imported tradable goods The derivation of the foreign demand schedule for good h from the home country is analytically more complex but, as we show later in equation (47), it shares the same functional form as equations (35) and (39), and can be written as a function of the relative price of good h (with elasticity $\theta_{T,t}$) and the total foreign demand for imports of goods from the home country.

We will focus first on import demand in the consumption-goods sector. Since we deal with goods produced in different regions, there are region indexes in the notation. The imports M_A^H by firm x^H for the home region H are a CES function of baskets of goods imported from the other regions, R, or:

$$M_{A,t}^{H}(x^{H})^{1-\frac{1}{\rho_A^H}} = \sum_{R\neq H} \left(b_{A,t}^{H,R}\right)^{\frac{1}{\rho_A^H}} \left(M_{A,t}^{H,R}(x^H)(1-\Gamma_{MA,t}^{H,R}(x^H))\right)^{1-\frac{1}{\rho_A^H}}, \tag{40}$$

where:

$$0 \leq \ b_{A,t}^{H,R} \leq 1, \quad \sum_{R\neq H} b_{A,t}^{H,R} = 1. \tag{41}$$

In equation (40), ρ_A^H is the elasticity of import substitution across countries: the higher the ρ_A^H, the easier it is for firm x^H to substitute away from importing goods from one region to importing goods from another region. The parameter $b_A^{H,R}$ helps to determine the percentage share of imports from a particular region, subject to the elasticity of substitution, ρ_A^H.[12]

The response of imports to changes in fundamentals and their price elasticities are typically observed to be smaller in the short run than in the long run. To model realistic dynamics of import volumes (such as delayed and sluggish adjustment to changes in relative prices), we assume that imports are subject to real adjustment costs, $\Gamma_{MA}^{H,R}$. These costs are specified as a function of the one-period change in import shares relative to the output of firm x^H, and can be different across exporters. They are zero in the steady state. Specifically, we adopt the parameterization:

$$\begin{aligned}
&\Gamma_{MA,t}^{H,R}[\frac{M_{A,t}^{H,R}(x^H)}{A_t^H(x)}/\frac{M_{A,t-1}^{H,R}}{A_{t-1}^H}] = \\
&\frac{\phi_{MA}^{H,R}}{2}\frac{\left[\left(M_{A,t}^{H,R}(x^H)/A_t^H(x)\right)/\left(M_{A,t-1}^{H,R}/A_{t-1}^H\right)-1\right]^2}{\left(1+\left[\left(M_{A,t}^{H,R}(x^H)/A_t^H(x)\right)/\left(M_{A,t-1}^{H,R}/A_{t-1}^H\right)-1\right]^2\right)},
\end{aligned} \tag{42}$$

such that $\Gamma_{MA}^{H,R}[1]=0$, $\Gamma_{MA}^{H,R}[\infty]=\phi_{MA}^{H,R}/2$, and $\Gamma_{MA}^{H,R}[0]=\Gamma_{MA}^{H,R}[2]=\phi_{MA}^{H,R}/4$.[13]

Denoting as $p_M^{H,R}$ the price in region H of a basket of intermediate inputs imported from

[12] The parameter $b_A^{H,R}$ can be time-varying; it is represented in the model by the stochastic process found in equation (1).

[13] This parameterization of import adjustment costs allows the non-linear model to deal with potentially large shocks relative to the quadratic specification adopted originally in Laxton and Pesenti (2003).

region R, cost minimization implies:

$$M_{A,t}^{H,R}(x^H) = b_{A,t}^{H,R} \left(\frac{p_{M,t}^{H,R}}{p_{MA,t}^{H}(x^H)} \right)^{-\rho_A^H} M_{A,t}^{H}(x^H) \frac{\left(1 - \Gamma_{MA,t}^{H,R}(x^H) - M_{A,t}^{H,R}(x^H)\Gamma_{MA,t}^{\prime H,R}(x^H)\right)^{\rho_A^H}}{\left(1 - \Gamma_{MA,t}^{H,R}(x^H)\right)}, \tag{43}$$

where $\Gamma_{MA}^{\prime H,R}(x^H)$ is the first derivative of $\Gamma_{MA}^{H,R}(x^H)$ with respect to $M_A^{H,R}(x^H)$. The import price in the consumption sector, p_{MA}^H, is defined as:

$$p_{MA,t}^{H}(x^H) = \left[\sum_{r \neq H} b_{A,t}^{H,R} \left(\frac{p_{M,t}^{H,R}}{1 - \Gamma_{MA,t}^{H,R}(x^H) - M_{A,t}^{H,R}(x^H)\Gamma_{MA,t}^{\prime H,R}(x^H)} \right)^{1-\rho_A^H} \right]^{\frac{1}{1-\rho_A^H}}. \tag{44}$$

In principle, the cost-minimizing import price $p_{MA}^H(x^H)$ is firm-specific, since it depends on the import share of firm x^H. To the extent that all firms x^H are symmetric within the consumption sector, however, there will be a unique import price, p_{MA}^H. This means that the aggregate level of nominal imports for the consumption good is the following function of imports from the differing regions, subject to the real adjustment costs of $\Gamma_{MA}^{H,R}$:

$$p_{MA,t}^{H}(x^H) M_{A,t}^{H}(x^H) = \sum_{R \neq H} p_{M,t}^{H,R} \frac{M_{A,t}^{H,R}(1 - \Gamma_{MA,t}^{H,R}(x^H))}{(1 - \Gamma_{MA,t}^{H,R}(x^H) - M_{A,t}^{H,R}(x^H)\Gamma_{MA,t}^{\prime H,R}(x^H))}. \tag{45}$$

$M_A^{H,R}(x^H)$ is the basket of imported consumption goods in region H imported from region R. It is a CES index of all varieties of tradable intermediate goods destined for consumption produced by firms h^R operating in region R and exported to region H (similar to equation (36)). The imported good $M_A^{H,R}\left(x^H\right)$ can also be defined as the sum of the demands by domestic consumption-good-producing firms x^H of an intermediate good produced by firms in region R producing the tradable good h^R (denoted as $M_A^{H,R}\left(h^R, x^H\right)$):

$$M_{A,t}^{H,R}(x^H) = \left[\left(\frac{1}{ss^R} \right)^{\frac{1}{\theta_{T,t}^R}} \int_0^{ss^R} M_{A,t}^{H,R}\left(h^R, x^H\right)^{1 - \frac{1}{\theta_{T,t}^R}} dh^R \right]^{\frac{\theta_{T,t}^R}{\theta_{T,t}^R - 1}}, \tag{46}$$

where $\theta_T^R > 1$ is the elasticity of substitution among intermediate tradables, the same elasticity entering equation (39) in region R.

The cost-minimizing firm x^H takes as given the prices of the imported goods $p^H(h^R)$ and determines its demand for good h^R according to:

$$M_{A,t}^{H,R}(h^R, x^H) = \frac{1}{ss^R} \left(\frac{p_t^H(h^R)}{p_{M,t}^{H,R}} \right)^{-\theta_{T,t}^R} M_{A,t}^{H,R}(x^H), \tag{47}$$

where $M_{A,t}^{H,R}(x^H)$ has been defined in (43). $p_M^{H,R}$ is:

$$p_{M,t}^{H,R} = \left[\left(\frac{1}{ss^R} \right) \int_0^{ss^R} (1 + \tau_{TRF}^{H,R}) p_t^H \left(h^R \right)^{1-\theta_{T,t}^R} dh^R \right]^{\frac{1}{1-\theta_{T,t}^R}}, \tag{48}$$

where τ_{TRF} is the tariff imposed by the fiscal agent in region H on the nominal value of imports (and hence their price $p_t^H \left(h^R \right)$) from region R.

Import demand in the investment-goods sector is derived in the same manner as above. As a last step, we can derive region R's demand for region H's intermediate good, h^H; that is, the analogue of (39). Aggregating across firms gives us the result that imports supplied to region R are equal to the demand for imports in region R:

$$\begin{aligned} &\int_0^{ss^R} M_{A,t}^{R,H}(h^H, x^R) dx^R + \int_0^{ss^R} M_{E,t}^{R,H}(h^H, e^R) de^R \\ &= \frac{ss^R}{ss^H} \left(\frac{p_t^R(h^H)}{p_{M,t}^{R,H}} \right)^{-\theta_{T,t}^H} (M_{A,t}^{R,H} + M_{E,t}^{R,H}). \end{aligned} \tag{49}$$

2.3.3 Final goods – consumption and investment

In each region, there is a continuum of symmetric firms producing the two non-traded final goods, A (the consumption good) and E (the investment good), under perfect competition. Consider first the consumption sector. Each firm producing the final consumption good is indexed by $x \in [0, ss]$. The output of firm x at time t is denoted $A_t(x)$. The consumption good is produced with the following double-nested CES technology:

$$\begin{aligned} &A_t(x)^{1-\frac{1}{\epsilon_{GAS}}} = \gamma_{GAS,t}^{\frac{1}{\epsilon_{GAS}}} GAS_t(x)^{1-\frac{1}{\epsilon_{GAS}}} \\ &+ (1 - \gamma_{GAS,t})^{\frac{1}{\epsilon_{GAS}}} \left[(1 - \gamma_{A,t})^{\frac{1}{\varepsilon_A}} N_{A,t}(x)^{1-\frac{1}{\varepsilon_A}} \right. \\ &\left. + \gamma_{A,t}^{\frac{1}{\varepsilon_A}} [\nu_{A,t}^{\frac{1}{\mu_A}} Q_{A,t}(x)^{1-\frac{1}{\mu_A}} + (1 - \nu_{A,t})^{\frac{1}{\mu_A}} M_{A,t}(x)^{1-\frac{1}{\mu_A}}]^{\frac{\mu_A}{\mu_A - 1}\left(1-\frac{1}{\varepsilon_A}\right)} \right]^{\frac{\varepsilon_A}{\varepsilon_A - 1}\left(1-\frac{1}{\epsilon_{GAS}}\right)}. \end{aligned} \tag{50}$$

Four intermediate inputs are used in the production of consumption good A: there is a combination of a basket (Q_A) of domestic tradable goods, and a basket (M_A) of imported goods to obtain a basket of tradable goods (notionally called T_A), which is then combined with a basket (N_A) of non-tradable goods to obtain a basket of non-gasoline goods (notionally called F_A), which is finally combined with a basket (GAS_A) of gasoline goods to produce the consumption good A. The double-nested CES technology generates more flexibility in the calibration by allowing different elasticities of substitution between the components of the consumption basket. The elasticity of substitution between tradables and non-tradables is $\varepsilon_A > 0$, the elasticity of substitution between domestic and imported tradables is $\mu_A > 0$, and the elasticity of substitution between gasoline and the composite good is $\mu_{GAS} > 0$. The biases towards the use of the four inputs in the production of the consumption good are: γ_{GAS} for gasoline $(1-\gamma_{GAS})(1-\gamma_A)$ for the non-tradable good, $(1-\gamma_{GAS})\gamma_A\nu_A$ for the domestically produced tradable good, and $(1-\gamma_{GAS})\gamma_A(1-\nu_A)$ for the imported tradable good, with $0 < \gamma_A, \gamma_{GAS}, \nu_A < 1$.

Firm x takes as given the prices of the four inputs and minimizes its costs subject to the supply function (50). Cost minimization implies that the demands of firm x for intermediate inputs are:

$$
\begin{aligned}
GAS_t(x) &= \gamma_{GAS,t}((1+\tau_{GAS,t})p_{GAS,t})^{-\varepsilon_{GAS}} A_t(x), && (51)\\
N_{A,t}(x) &= \left(1-\gamma_{GAS,t}\right)\left(1-\gamma_{A,t}\right) p_{N,t}^{-\varepsilon_A} p_{FA,t}^{\varepsilon_A-\varepsilon_{GAS}} A_t(x), && (52)\\
Q_{A,t}(x) &= \left(1-\gamma_{GAS,t}\right)\gamma_{A,t}\nu_{A,t} p_{Q,t}^{-\mu_A} p_{TA,t}^{\mu_A-\varepsilon_A} p_{FA,t}^{\varepsilon_A-\varepsilon_{GAS}} A_t(x), && (53)\\
M_{A,t}(x) &= \left(1-\gamma_{GAS,t}\right)\gamma_{A,t}\left(1-\nu_{A,t}\right) p_{MA,t}^{-\mu_A} p_{TA,t}^{\mu_A-\varepsilon_A} p_{FA,t}^{\varepsilon_A-\varepsilon_{GAS}} A_t(x), && (54)
\end{aligned}
$$

where τ_{GAS} is the tax rate on gasoline; p_{GAS}, p_N, p_Q, and p_{MA} are the relative prices of the inputs; p_{TA} is the shadow relative price of the composite basket of domestic and foreign tradables:

$$
p_{TA,t} \equiv \left[\nu_{A,t} p_{Q,t}^{1-\mu_A} + \left(1-\nu_{A,t}\right) p_{MA,t}^{1-\mu_A}\right]^{\frac{1}{1-\mu_A}}, \qquad (55)
$$

and p_{FA} is the shadow relative price of the composite basket of the non-gasoline final consumption good:

$$
p_{FA,t} \equiv \left[\gamma_{A,t} p_{TA,t}^{1-\epsilon_A} + \left(1-\gamma_{A,t}\right) p_{N,t}^{1-\epsilon_A}\right]^{\frac{1}{1-\epsilon_A}}. \qquad (56)
$$

The price of the consumption good is normalized to one, since the CPI is the *numéraire* of the economy:

$$1 \equiv \left[\gamma_{GAS,t}(1+\tau_{GAS,t})p_{GAS,t}^{1-\mu_{GAS}} + \left(1-\gamma_{GAS,t}\right) p_{FA,t}^{1-\mu_{GAS}}\right]^{\frac{1}{1-\mu_{GAS}}} . \tag{57}$$

CPI is the basis for the calculation of headline inflation. Core inflation excludes the direct effects of gasoline prices as well as tariffs. Therefore, core inflation is calculated from an analogue of equation (56) where its component price, p_{TAx} (instead of p_{TA}), excludes the direct effects of tariffs on the price of imports:

$$CPI_t^X \equiv \left[\gamma_{A,t}p_{TAx,t}^{1-\epsilon_A} + \left(1-\gamma_{A,t}\right) p_{N,t}^{1-\epsilon_A}\right]^{\frac{1}{1-\epsilon_A}} . \tag{58}$$

The formulation of the investment good is simpler, produced with only a single-nested CES technology:

$$\begin{aligned} E_t(e)^{1-\frac{1}{\mu_E}} &= [\left(1-\gamma_{E,t}\right)^{\frac{1}{\varepsilon_E}} N_{E,t}(e)^{1-\frac{1}{\varepsilon_E}} \\ &+\gamma_{E,t}^{\frac{1}{\varepsilon_E}}[\nu_{E,t}^{\frac{1}{\mu_E}} Q_{E,t}(e)^{1-\frac{1}{\mu_E}} + (1-\nu_{E,t})^{\frac{1}{\mu_E}} M_{E,t}(e)^{1-\frac{1}{\mu_E}}]^{\frac{\mu_E}{\mu_E-1}\left(1-\frac{1}{\varepsilon_E}\right)}]^{\frac{\varepsilon_E}{\varepsilon_E-1}}. \end{aligned} \tag{59}$$

Three intermediate inputs are used in the production of investment good E: a basket (N_E) of non-tradable goods is combined with the notional basket of tradable goods T_E (itself a combination of a basket (Q_E) of domestic tradable goods, and a basket (M_E) of imported investment goods). The elasticity of substitution between tradables and non-tradables is $\varepsilon_E > 0$, and the elasticity of substitution between domestic and imported tradables is $\mu_E > 0$. The biases towards the use of the three inputs in the production of the investment good are $(1-\gamma_E)$ for the non-tradable good, $\gamma_E \nu_E$ for the domestically produced tradable good, and $\gamma_E(1-\nu_E)$ for the imported tradable good, with $0 < \gamma_E, \nu_E < 1$.

Firm e takes as given the prices of the four inputs and minimizes its costs subject to the technological constraint (59). Cost minimization implies that the demands of firm e for intermediate inputs are:

$$N_{E,t}(e) = \left(1-\gamma_{E,t}\right)(\frac{p_{N,t}}{p_{E,t}})^{-\varepsilon_E} E_t(x), \tag{60}$$

$$Q_{E,t}(e) = \gamma_{E,t}\nu_{E,t}(\frac{p_{Q,t}}{p_{E,t}})^{-\mu_E}(\frac{p_{TE,t}}{p_{E,t}})^{\mu_E-\varepsilon_E} E_t(x), \tag{61}$$

$$M_{E,t}(e) = \gamma_{E,t}\left(1-\nu_{E,t}\right)(\frac{p_{ME,t}}{p_{E,t}})^{-\mu_E}(\frac{p_{TE,t}}{p_{E,t}})^{\mu_E-\varepsilon_E} E_t(x), \tag{62}$$

where p_N, p_Q, and p_{ME} are the relative prices of the inputs, p_{TE} is the shadow relative price of the composite basket of domestic and foreign tradables:

$$p_{TE,t} \equiv \left[\nu_{E,t} p_{Q,t}^{1-\mu_E} + (1-\nu_{E,t})\, p_{ME,t}^{1-\mu_E}\right]^{\frac{1}{1-\mu_E}}, \tag{63}$$

and p_E is the relative price of investment goods. The supply of investment goods will be considered in section 2.5.2.

2.4 Price-setting by the firms

Having outlined the real side of the five production sectors for the firms, we next consider the price-setting decisions they face. We examine, in turn, the raw materials sectors, the two non-traded sectors (non-tradables and gasoline), and the tradables sector.

2.4.1 The raw materials sectors

For the raw materials sectors, we examine the commodities sector. The arguments that follow, however, also apply exactly to the oil sector: firms in both oil and commodities sectors (o and s) face significant real adjustment costs, but they have market power (and hence fix a price with a markup over real marginal cost). They are flexible in their price-setting at each period, since they face no nominal rigidities and prices can adjust instantaneously. We consider this an accurate reflection of the behaviour of raw materials prices in the data. The oil sector's pricing differs slightly in form from that of the commodities sector, since oil is also used in the production of gasoline, and not only for tradable and non-tradable goods.

In the commodities sector, each firm s takes into account the demand (24) for its product and sets its nominal price by maximizing the present discounted value of real profits:

$$\max_{p_t^H(s)} \mathsf{E}_t \sum_{\tau=t}^{\infty} D_{t,\tau}^H \pi_{t,\tau}^H g_{t,\tau} \left[p_{Q,\tau}^H(s) - mc_\tau^H(s)\right] \left(\frac{p_\tau^H(s)}{p_{QS,\tau}^H}\right)^{-\theta_{S,t}^H} (Q_{ST,\tau}^H + Q_{SN,\tau}^H + \Sigma_R M_{S,\tau}^{R,H}), \tag{64}$$

where $D_{t,\tau}$ (with $D_{t,t} = 1$) is the appropriate discount rate, to be defined later in equation (91). As real variables are detrended and prices are deflated by the CPI, equation (64) includes $\pi_{t,\tau}$, the CPI inflation rate between time t and time τ, and $g_{t,\tau}$, the rate of growth of the global trend ($TREND$) between t and τ.

As domestic firms s are symmetric and charge the same equilibrium price, $p(s) = p_{QS}$, the first-order condition required for profit maximization can be written as:

$$p_{Q,t}(s) = \frac{\theta_{S,t}}{\theta_{S,t} - 1} mc_t(s), \tag{65}$$

where the gross markup is a negative function of the elasticity of input substitution, θ_S. As the varieties of commodities produced by firms s are more alike (i.e., θ_S is a higher value, since this implies a higher elasticity of substitution amongst commodities), the lower is the potential markup that a firm can charge over its real marginal cost.

2.4.2 The non-tradables sector

Consider next profit maximization in the intermediate non-tradables sector. Each firm n takes into account the demand (35) for its product and sets its nominal price by maximizing the present discounted value of real profits. There are costs of nominal price adjustment measured in terms of total profits foregone. The nominal rigidity is denoted $\Gamma_{PN,t}\left[p_t(n), p_{t-1}(n)\right]$.

The price-setting problem for the typical non-tradable-goods-producing firm n is:

$$\max_{p_t(n)} \mathsf{E}_t \sum_{\tau=t}^{\infty} D_{t,\tau}\pi_{t,\tau}g_{t,\tau}\left[p_\tau(n) - mc_\tau(n)\right]\left(\frac{p_\tau(n)}{p_{N,\tau}}\right)^{-\theta_{N,t}}\left(N_{A,\tau} + N_{E,\tau} + G_{N,\tau}\right)\left(1 - \Gamma_{PN,\tau}(n)\right). \tag{66}$$

Since firms n are symmetric and charge the same equilibrium price $p(n) = p_N$, the first-order condition can be written as:

$$\begin{aligned} 0 = {} & \left(1 - \Gamma_{PN,t}(n)\right)\left[p_t(n)\left(1 - \theta_{N,t}\right) + \theta_{N,t} mc_t(n)\right] - \left[p_t(n) - mc_t(n)\right]\frac{\partial \Gamma_{PN,t}}{\partial p_t(n)} p_t(n) \\ & - \mathsf{E}_t D_{t,t+1}\pi_{t,t+1}g_{t,t+1}\left[p_{t+1}(n) - mc_{t+1}(n)\right]\frac{N_{t+1}}{N_t}\frac{\partial \Gamma_{PN,t+1}}{\partial p_t(n)} p_t(n). \end{aligned} \tag{67}$$

Interpreting the previous equation, when prices are fully flexible ($\Gamma_{PN} = 0$), the optimization problem collapses to the standard markup rule:

$$p_t(n) = \frac{\theta_{N,t}}{\theta_{N,t} - 1} mc_t(n), \tag{68}$$

where the gross markup is a negative function of the elasticity of input substitution. As the varieties of non-tradable goods produced by firms n are more alike (i.e., θ_n is a higher value, since this implies a higher elasticity of substitution among varieties of non-tradable goods), the lower is the potential markup that a firm can charge over its real marginal cost. Deviations from markup pricing occur if firms face costs for modifying their prices in the short term. The speed of adjustment in response to shocks depends on the trade-off between current and future expected costs, making the price-setting process forward looking, but preferably also with a lag; this is the basis of the linearized formulation of the hybrid New Keynesian Phillips Curve, where price inflation is generally a function of its lag, its expected one-period-ahead level, and contemporaneous real marginal cost (Galí and Gertler 1999). Such a Phillips curve is implied by all the relative prices in the BoC-GEM.

Three types of adjustment costs associated with price-setting are generally used in DSGE models. First, there is the form originally stated by Calvo (1983), where some random share of firms are assumed to adjust their prices entirely each period. In this case, there is the concept of contract length for the prices, generally assumed to be four quarters. Second, and similarly based on contract length, is Taylor (1980), where some firms are assumed to adjust their prices fully each period. Taylor (1980) differs from Calvo (1983) in that all firms will adjust their prices at regular, but staggered, intervals.[14] The third type is based on Rotemberg (1982), where it is assumed that all firms partially adjust their prices each period towards the steady-state price level. We choose the Rotemberg formulation, since it allows the model to reproduce realistic nominal dynamics in an analytically tractable form; furthermore, its main parameter, ϕ_{PN1}, can be translated into contract lengths, as in the Calvo and Taylor pricing methodologies.

The original Rotemberg formulation was adopted in a model assuming zero steady-state inflation and therefore a constant price level; because the BoC-GEM contains a non-zero steady-state inflation rate (and hence a non-stationary price level), we assume that firms are attempting to stabilize the inflation rate at some combination of a constant inflation rate (i.e., the first difference of inflation) and the steady-state level of inflation (i.e., the actual versus targeted inflation gap). Achieving a constant inflation rate stabilizes price movements in the long run and allows a lag to enter the implied Phillips curve; however, using this method exclusively implies that prices follow a path with (excessively) long cycles before reaching the steady state. Trying to move the inflation rate directly to the steady-state level of inflation allows the entire nominal side of the model to converge smoothly without the excessive cycling implied by converging only to a stable inflation rate; but using this method exclusively would remove any backward-lookingness from the Phillips curve and impart what is generally agreed in the literature to be too high a degree of perfect foresight.

[14] Whereas in Calvo pricing, some firms could, theoretically, never adjust their prices (unless full indexation is assumed, which disallows price dispersion).

The nominal rigidities are formulated as follows:

$$\Gamma_{PN,t}(n) \equiv \frac{\phi_{PN1}}{2}\left(\pi_{t-1,t}\frac{p_t(n)/p_{t-1}(n)}{\pi_{Nt-2,t-1}^{\phi_{PN2}}\Pi_{t-4,t}^{0.25(1-\phi_{PN2})}}-1\right)^2, \tag{69}$$

where the nominal rigidities are related to changes of the nominal price of non-tradable n relative to a target that is a weighted average of last period's non-tradable price inflation (weighted by ϕ_{PN2}) and the quarterly version of the year-over-year inflation target, $\Pi_{t-4,t}$ (weighted by $(1-\phi_{PN2})$). This is the formulation for nominal rigidities used for all relative prices in the BoC-GEM.

2.4.3 The gasoline sector

Profit-maximizing behaviour in the gasoline sector, a non-traded good, is much like that in the non-tradables sector. However, there are no nominal rigidities in the price of gasoline (thereby resembling price-setting in the raw materials sectors).

The price-setting problem is then characterized as:

$$\max_{p_t(g)} \mathsf{E}_t \sum_{\tau=t}^{\infty} D_{t,\tau}\pi_{t,\tau}g_{t,\tau}\left[p_\tau(g)-mc_\tau(g)\right]\left(\frac{p_\tau(g)}{p_{GAS,\tau}}\right)^{-\theta_{GAS,t}} GAS_\tau. \tag{70}$$

Since domestic firms g are symmetric and charge the same equilibrium price, $p(g)=p_{GAS}$, the first-order condition required for profit maximization can be written as:

$$p_t(g)=\frac{\theta_{GAS,t}}{\theta_{GAS,t}-1}mc_t(g). \tag{71}$$

2.4.4 The tradables sector and exchange rate pass-through

Consider next the price-setting problem in the tradables sector. To the extent that the five regional blocs represent segmented markets in the global economy, each firm h has to set five prices, one in the domestic market and the other four in the export markets. Exports are invoiced (and prices are set) in the currency of the destination market.[15] As we discuss export markets, once again our notation needs to make explicit the regions' indexes.

[15] This is more commonly known in the literature as "local currency pricing."

Accounting for (49), the four price-setting problems of firm h in region H can be characterized as follows:

$$\max_{p_t^R(h^H)} \sum_R \mathsf{E}_t \sum_{\tau=t}^{\infty} D_{t,\tau}^H \pi_{t,\tau}^H g_{t,\tau} [\varepsilon_{\tau\tau}^{H,R} \bar{p}^R(h^H) - mc_\tau^H(h^H)]$$
$$* \frac{ss^R}{ss^H} \left(\frac{\overline{p_\tau^R}(h^H)}{\overline{p_{M,\tau}^{R,H}}} \right)^{-\theta_{T,\tau}^H} \left(M_{A,\tau}^{R,H} + M_{E,\tau}^{R,H} \right) \left(1 - \Gamma_{\overline{PM},\tau}^{R,H}(h) \right). \tag{72}$$

When $H \neq R$, $\overline{p^R}(h^H)$ is the wholesale price (i.e., before tariffs are applied by region R) of good h^H in region R, $\overline{p_M^{R,H}}$ is the wholesale price of region R's imports of consumption and investment goods from region H, and $M_A^{R,H} + M_E^{R,H}$ are region R's imports from region H. The term $\varepsilon^{H,R}$ is the real bilateral exchange rate between region H and region R (an increase in $\varepsilon^{H,R}$ represents a depreciation of region H's currency against region R), and $\Gamma_{\overline{PM}}^{H,R}(h^H)$ are adjustment costs related to changes in the price of good h^H in region R. These costs are the analogues of equation (69):

$$\Gamma_{\overline{PM},t}^{R,H}(h^H) \equiv \frac{\phi_{\overline{PM}1}^{R,H}}{2} \left(\pi_{t-1,t}^R \frac{\overline{p_t^R}(h^H)/\overline{p_{t-1}^R}(h^H)}{\left(\overline{\pi_{Mt-2,t-1}^R}\right)^{\phi_{\overline{PM}2}^{R,H}} (\Pi_{t-4,t}^R)^{0.25(1-\phi_{\overline{PM}2}^{R,H})}} - 1 \right)^2. \tag{73}$$

For the domestic prices of tradables $p^H(h^H)$,we still use equation (72) with $R = H$, adopting the notational conventions $\overline{p_M^{H,H}} = p_Q^H$, $M_A^{H,H} = Q_A^H$, and $M_E^{H,H} = Q_E^H$, as described in equation (39), and $\Gamma_{\overline{PM}}^{H,H} = \Gamma_{PQ}^H$.

Profit maximization in the tradables sector yields:

$$0 = \left(1 - \Gamma_{\overline{PM},t}^{R,H}(h^H) \right) \left[\varepsilon_t^{H,R} \overline{p_t^R}(h^H) \left(1 - \theta_{T,t}^H \right) + \theta_{T,t}^H mc_t^H(h^H) \right]$$
$$- \left[\varepsilon_t^{H,R} \overline{p_t^R}(h^H) - mc_t^H(h^H) \right] \frac{\partial \Gamma_{\overline{PM},t}^{R,H}}{\partial \overline{p_t^R}(h^H)} \overline{p_t^R}(h^H) - \mathsf{E}_t \{ D_{t,t+1}^H \pi_{t,t+1}^H g_{t,t+1}$$
$$* \left[\varepsilon_{t+1}^{H,R} \overline{p_{t+1}^R}(h^H) - mc_{t+1}^H(h^H) \right] \left(\frac{M_{A,t+1}^{R,H} + M_{E,t+1}^{R,H}}{M_{A,t}^{R,H} + M_{E,t}^{R,H}} \right) \frac{\partial \Gamma_{\overline{PM},t+1}^{R,H}}{\partial p_t^R(h^H)} \overline{p_t^R}(h^H) \}. \tag{74}$$

If real adjustment costs in the export market are strong (i.e., the parameter $\phi_{\overline{PM}1}^{R,H}$ is relatively large), the prices of region H's goods in the foreign markets are characterized by significant stickiness in local currency. In this case, the degree to which exchange rate movements (and other shocks to marginal costs in region H) pass through into import prices in region R is rather low. If, instead, the $\phi_{\overline{PM}1}^{R,H}$ coefficients are zero worldwide, equation (74) collapses to the typical markup rule with full and immediate exchange rate pass-through:

$$p_t^{H,H}(h^H) = p_{Q,t}^H = \varepsilon_t^{H,R}\overline{p_t^R}(h^H) = \varepsilon_t^{H,R}\overline{p_{M,t}^{R,H}} = \frac{\theta_{T,t}}{\theta_{T,t}-1}mc_t. \tag{75}$$

2.5 The consumers' problem

Having fully articulated the production side of the economy, we can address the consumers' problem in turn. In each region there is a continuum of households indexed by $j \in [0, ss]$, the same index as labour inputs. Some households have access to capital markets, and some do not. The latter finance their consumption by relying exclusively on their labour incomes. We refer to the first type as 'Ricardian' or 'forward looking'; they represent a share $(1 - s_{LC})$ of domestic households and are indexed by $j \in [0, ss\,(1 - s_{LC})]$. We refer to the second type as 'non-Ricardian' or 'liquidity constrained'; they represent a share (s_{LC}) of domestic households and are indexed by $j \in (ss\,(1 - s_{LC})\,, ss]$. Moreover, in order to make the labour market more tractable in the structure of the model, we associate liquidity-constrained consumers with low-skilled workers, and forward-looking consumers with highly skilled labour, as in Faruqee et al. (2007b). Therefore, we assume that $\psi_{L,t}$ is the elasticity substitution between the two classes of labour of the liquidity-constrained (low-skilled) households and the forward-looking (high-skilled) households.

2.5.1 *The utility function*

The specification of households' preferences uses the Greenwood, Hercowitz, and Huffman (1988) (GHH) utility function, adjusted for habit formation and preference shocks. A utility function must express the level of utility such that it grows at the same rate as the rest of the economy, in the steady state. In the case of the GEM, this means all the elements of the utility function must grow at the trend productivity growth rate, g_t. This is the case for consumption, but not the case for labour, which grows at the population growth rate (assumed to be zero in the BoC-GEM). Often, DSGE models use a utility function that is additively separable in consumption and leisure for reasons of computational simplicity when using the model. This type of utility function can be represented in a general fashion as:

$$u\ (C, \ell) = U\ (C) - V\ (\ell). \tag{76}$$

This is inappropriate for the GEM because labour does not grow at the same rate as productivity. One solution is to use a multiplicative utility function:

$$u\ (C, \ell) = U\ (C) * (-V\ (\ell)), \tag{77}$$

such as the King, Plosser, and Rebelo (1988) utility function used in ToTEM (Murchison and Rennison 2006). However, it can prove to be an intractable functional form when using the non-linear representation of the model.[16] Therefore, for the BoC-GEM, we continue the use of the GHH utility function, as in Faruqee et al. (2007a, b):

$$u\ (C, \ell) = U\ (C - V\ (z\ell)). \tag{78}$$

Note that the level of productivity, z (where $z = TREND$), is included directly in the utility function, with labour. As a result, the disutility of labour in the market increases with the level of productivity, which can be interpreted as representing gains in productivity associated with production at home, which is often not captured in the measurement of output (Correia, Neves, and Rebelo 1995). Although the GHH utility function is not separable in consumption and leisure, it retains some additive properties. Therefore, when the marginal utility of consumption and the marginal disutility of labour are derived, they will both grow at the same rate as the economy over time. Unlike other multiplicative utility functions (i.e., the King-Plosser-Rebelo form), the marginal disutility of labour depends on both labour and consumption, while the marginal utility of consumption depends solely on consumption. Consequently, the optimal real wage for consumers will depend solely on labour supplied by the consumer, and will not be affected by their consumption choice (Correia, Neves, and Rebelo 1995, and discussed further in section 2.5.3).

In the following discussion, we will consider the forward-looking household, FL, but the notation can be applied equally to the liquidity-constrained households, LC. We have $\mathcal{W}_{FL,t}(j)$, the lifetime expected utility of household j:

$$\mathcal{W}_{FL,t}(j) \equiv \mathsf{E}_t \sum_{\tau=t}^{\infty} \beta_{t,\tau} g_{t,\tau}^{1-\sigma} u_\tau(\ C_{FL,\tau}(j), \ell_{FL,\tau}(j)), \tag{79}$$

where the instantaneous felicity is a function of consumption demanded, C, and labour effort

[16] Although, to be fair, the problem of tractability does not exist in the version of ToTEM that is commonly used, since it is linearized around a steady state using a first-order Taylor approximation.

supplied, ℓ:

$$u_{FL,t}\ (\ C_{FL,t}(j), \ell_{FL,t}(j)\ ) = Z_{UFL,,t}(1 - \frac{b_{cfl}}{g_{t-1,t}})(\frac{1-b_{\ell fl}}{1-\sigma_{fl}}) \times$$
$$[\frac{C_{FL,t}(j) - b_{cfl}C_{FL,j,t-1}/g_{t-1,t}}{1-b_{cfl}/g_{t-1,t}} - \frac{Z_{VFL,t}}{1+\zeta_{fl}}(\frac{\ell_{FL,t}(j) - b_{\ell fl}\ell_{FL,j,t-1}}{1-b_{\ell fl}})^{1+\zeta_{fl}}]^{1-\sigma_{fl,}}. \quad (80)$$

In the expressions above, $\beta_{t,\tau}$ is the discount rate between time t and time τ, possibly different across regions. As mentioned in section 2.2, because of technological progress associated with home production activities (here related to the global trend), the term $g_{t,\tau}^{1-\sigma}$ in (79) implies that the disutility of labour effort increases with the common trend.[17] The parameter σ in equations (79) and (80) is the reciprocal of the elasticity of intertemporal substitution. The parameter ζ is the reciprocal of the Frisch elasticity of labour and affects the curvature of the labour disutility function. The terms Z_{UFL} and Z_{VFL} are shocks to consumption and labour, respectively, that are modelled as stochastic processes.

Since there is habit persistence in consumption demand, we include $C_{FL,j,t-1}$ in equation (80), which is the past per-capita consumption of household j's peers (i.e., forward-looking agents).[18] Habit persistence in consumption is governed by the parameter $0 < b_{cfl} < 1$. $b_{cfl} = 0$ implies that agents draw utility only from consuming at the current time in any given period, whereas $b_{cfl} = 1$ implies that utility increases (decreases) as the rate of growth of consumption increases (decreases). Similarly, there is habit persistence in labour supply, governed by the parameter $0 < b_{\ell fl} < 1$. Habit persistence means that agents place a large weight on their past behaviour in their amount of consumption (use of leisure time), which helps match the "hump-shaped response" of consumption demanded (labour effort supplied) that is a stylized fact in most economies, in the face of a large variety of shocks.

Therefore, households' preferences are symmetric within their respective categories (i.e., forward-looking consumers and liquidity-constrained consumers), but, because of the possibility of different degrees of habit formation in either consumption or leisure, households' preferences are not necessarily symmetric across categories.

[17] The restriction $\beta_{t,\tau}g_{t,\tau}^{1-\sigma} < 1$ is imposed to ensure that utility is bounded.

[18] This form is known as external habit persistence: an agent j informs their decision on consumption $C_{FL,t}(j)$ based on the economy-wide measure of consumption last period $C_{FL,t-1}$, instead of solely on their own measure of consumption $C_{FL,t-1}(j)$. When habit is formed using $C_{FL,t-1}(j)$, this is known as internal habit persistence, and is less analytically and computationally tractable than the external form (Laxton and Pesenti 2003).

2.5.2 Budget constraint for forward-looking consumers

The individual flow budget constraint for the forward-looking household, $j \in [0, (1 - s_{LC})\, ss]$, is:

$$
\begin{aligned}
&B_t(j) + \varepsilon_t B_t^*(j) \leq (1 + i_{t-1}) \frac{B_{t-1}(j)}{\pi_{t-1,t} g_{t-1,t}} + (1 + i_{t-1}^*) \left[1 - \Gamma_{B,t-1}\right] \frac{\varepsilon_t B_{t-1}^*(j)}{\pi_{t-1,t}^{US} g_{t-1,t}} \\
&+ (1 - \tau_{K,t})\, r_t K_t(j) + (1 - \tau_{L,t})\, w_{FL,t}(j) \ell_{FL,t}(j) \left(1 - \Gamma_{WFL,t}(j) \right) \\
&- C_{FL,t}(j) - p_{E,t} I_t(j) + \Phi_t(j) - TT_t(j). \qquad (81)
\end{aligned}
$$

Households hold two nominal bonds, denominated in domestic and U.S. currency, respectively. $B_t(j)$ is the holdings of the domestic (government-issued) bond by household j, expressed in terms of domestic consumption units; $B_t^*(j)$ is the holdings of the international bond, expressed in terms of U.S. consumption units; and ε_t is the CPI-based real exchange rate, expressed as the price of one U.S. consumption basket in terms of domestic consumption.[19]

Financial assets, financial intermediation, UIRP, and the link with government debt The short-term nominal rates i_t and i_t^* are paid at the beginning of period $t+1$ and are known at time t. The two rates are directly controlled by their respective national governments. Only the U.S.-currency bond is traded internationally: the U.S. bond is in zero net supply worldwide, while the domestic bond is issued by the domestic government.[20] It follows that the net financial wealth of forward-looking household j at time t is:

$$
F_t(j) \equiv (1 + i_{t-1}^*) \left[1 - \Gamma_{B,t-1}\right] \frac{\varepsilon_t B_{t-1}^*(j)}{\pi_{t-1,t}^{US} g_{t-1,t}}. \qquad (82)
$$

A financial friction, Γ_B, is introduced to guarantee that the five regions' net foreign asset positions follow a stationary process, allowing their economies to converge asymptotically to a well-defined steady state (Schmitt-Grohé and Uribe 2003). It also contains a differential between regions' rates of time preference β. Households that take a position in the international bond market must deal with financial intermediaries who charge a transaction fee, Γ_B, on sales/purchases of the international bond.[21] This transaction cost is a function of the

[19] It is understood that ε is shorthand for $\varepsilon^{H,US}$, where H denotes the country under consideration.

[20] If the country under consideration is the United States, $\varepsilon = 1$ and $i = i^*$.

[21] In our model, it is assumed that all intermediation firms are owned by a region's residents, and that their revenue is rebated to domestic households in a lump-sum fashion — part of Φ in equation (81). There are no

average net foreign asset position of the whole economy. Specifically, we adopt the following functional form:

$$
\begin{aligned}
1-\Gamma_{B,t} \quad &= \quad \left(1-\phi_{B1}\frac{\exp\left(\phi_{B2}\left[\varepsilon_t B_t^*/GDP_t - b_{DES,t}^*\right]\right)-1}{\exp\left(\phi_{B2}\left[\varepsilon_t B_t^*/GDP_t - b_{DES,t}^*\right]\right)+1} - Z_{B,t}\right) \\
&\quad * \frac{1}{\exp\left(\phi_{B3}\left[\Delta_{t,t+1}/\Delta_{t-1,t} - (\pi/\pi^{US})^{0.25}\right]\right)}\frac{\beta_t^{US}}{\beta_t},
\end{aligned} \tag{83}
$$

where $0 \leq \phi_{B1} \leq 1$, $\phi_{B2} > 0$, $0 \leq \phi_{B3} \leq 1$, and $\varepsilon_t B^* \equiv (1/ss)\,\varepsilon_t \int_0^{ss(1-s_{LC})} B^*(j)dj$ is the per-capita net foreign asset position of the region. The term b_{DES}^* is the 'desired' net asset position of the region expressed as a ratio of GDP.[22] This variable measures the degree of international exposure that financial intermediaries consider appropriate for the economy, based on their assessment of the economic outlook.

To understand the role played by the financial intermediation cost Γ_B, suppose first that b_{DES}^*, Z_B, and ϕ_{B3} equal zero and that the rates of time preference are the same across regions ($\beta^{US} = \beta$). In this case, when the NFA position of the region is equal to its 'desired' level of zero, it must be the case that $\Gamma_B = 0$ and the return on the international bond is equal to $1 + i^*$. If the region is a net creditor worldwide, then Γ_B rises above zero, implying that the region's households lose an increasing fraction of their international bond returns to financial intermediaries. When holdings of the international bond go to infinity, the return on the international bond approaches $(1 + i^*)(1 - \phi_{B1})$. By the same token, if the region is a net debtor worldwide, then Γ_B falls from zero to $-\phi_{B1}$, implying that households pay an increasing intermediation premium on their international debt. When net borrowing goes to infinity, the cost of borrowing approaches $(1 + i^*)(1 + \phi_{B1})$. The parameter ϕ_{B2} controls the flatness of the Γ_B function: if $\phi_{B2} = 0$, then $\Gamma_B = 0$ regardless of the net foreign asset position; if ϕ_{B2} tends to infinity, then $1-\Gamma_B = (1 - \phi_{B1})$ for any arbitrarily small net lending position, and $1 - \Gamma_B = (1 + \phi_{B1})$ for any arbitrarily small net borrowing position.

Consider the other components of equation (83). By including an international differential for rates of time preference $\frac{\beta_t^{US}}{\beta_t}$, there can be differences across regions in their real interest rates (defined in equation (95)) even in the long run. The variable $Z_{B,t}$ is a shock that follows a stochastic process that, in our framework of international financial intermediation, plays the same role as 'uncovered interest parity shocks' in other open-economy models.[23]

intermediation costs for U.S. residents entering the international bond market; that is, there is no difference between onshore and offshore U.S. interest rates.

[22] The concept of GDP in our model will be defined and discussed in equation (126).

[23] Fluctuations in Z_B cannot be large enough to push Γ_B, or else this would imply a negative financial

The term b^*_{DES} could be non-zero (either positive or negative) and the foregoing discussion would still be entirely valid. The desired net asset position in region H is characterized as:

$$b^{*H}_{DES,t} = b^{*H}_{NEUT} - \phi^H_{F1}\frac{B^H_t}{GDP^H_t} + \sum_{R\neq H}\phi^{R,H}_{F2}\phi^R_{F1}\frac{B^R_t}{GDP^R_t}, \tag{84}$$

where b^{*H}_{NEUT} is a region-specific (possibly time-varying) constant, adjusted to account for changes in the debt-to-GDP ratios in either the domestic economy (with a weight of $0 < \phi^H_{F1} < 1$ on $\frac{B^H}{GDP^H}$) or the rest of the world (with a weight of $0 < \phi^{R,H}_{F2} < 1$ on $\phi^R_{F1}\frac{B^R}{GDP^R}$). This specification provides a plausible link between domestic government debt and net foreign asset positions. If the targeted debt-to-GDP ratio increased in the United States, investors in the rest of the world would require a higher return on U.S. securities, leading to a higher share of U.S. assets in their portfolios or a reduction in net borrowing from the United States. If, however, the target debt increased in the home region as well, the U.S. premium would fall somewhat.[24]

Finally in equation (83), the term $\Delta_{t,t+1}/\Delta_{t-1,t} - (\pi/\pi^{US})^{0.25}$ is associated with the parameter ϕ_{B3}. It attempts to capture the forward premium puzzle – the empirical fact that there is a risk premium on exchange rate transactions that is negatively correlated with expected future depreciations (Duarte and Stockman 2005). It also implies that domestic investors will accept a lower return on their holdings of the international bond relative to their holdings of domestic debt, if the future real exchange rate is expected to depreciate in consecutive periods (and is therefore easier to predict). In such an instance, domestic investors expect, in domestic currency terms, that their holdings of the international bond will increase in value simply from a shift in the bilateral real exchange rate; hence, a lower return from the real interest rate is acceptable (Adolfson et al. 2007).

When we later derive the uncovered interest rate parity (UIRP) condition in equation (93), the term $1 - \Gamma_B$ will serve as the main channel for the effects of equation (83). This implies that the standard UIRP condition will be adjusted for risk (i.e., financial intermediation costs), and its dynamics will be modified by the inclusion of a lag of the exchange rate (from the representation of the forward premium puzzle derived from Adolfson et al. 2007). To better reflect these extra features, equation (93), equating the future one-period change in

intermediation cost (bondholders receive a fee for transacting in the market).

[24] Our approach should be viewed as only a crude approximation of the actual determinants of cross-country spreads and interest rate premia in response to macroeconomic imbalances. There are other models that endogenize this feature, usually in a Blanchard-Weil framework (for example, Botman et al. 2006), but many other features present in the GEM are not currently possible under those types of models; hence, we use a reduced-form approximation.

the exchange rate and the forward sum of the interest rate differentials vis-à-vis the U.S. short-term interest rate, is referred to as the modified risk-adjusted UIRP condition.

Physical capital Households accumulate physical capital $K(j)$, which they rent to domestic firms at the after-tax rental rate of $r(1-\tau_K)$. This is the aggregate of the capital stocks from all sectors of production (see equation (116)). The law of motion of capital is:

$$K_{t+1}(j)g_{t,t+1} = (1-\delta)\,K_t(j) + \Gamma_{I,t}K_t(j) \qquad 0<\delta\leq 1, \tag{85}$$

where δ is the region-specific depreciation rate of capital. To simulate realistic investment flows, capital accumulation is subject to adjustment costs. Therefore, capital is represented by $\Gamma_{I,t}K_t(j)$, where $\Gamma_I(.)$ is quadratic, with the properties of being an increasing, concave, and twice-continuously differentiable function of the investment/capital ratio $I_t(j)/K_t(j)$ that ensure no adjustment costs in steady state: $\Gamma_I(\delta+g-1)=\delta+g-1$ and $\Gamma_I'(\delta+g-1)=1$. The form of the real adjustment cost, $\Gamma_I(j)$, is:

$$\Gamma_{I,t}(j) \equiv \frac{I_t(j)}{K_t(j)} - \frac{\phi_{I1}}{2}\left(\frac{I_t(j)}{K_t(j)} - (\delta+g-1)\right)^2 - \frac{\phi_{I2}}{2}\left(\frac{I_t(j)}{K_t(j)} - \frac{I_{t-1}}{K_{t-1}}\right)^2, \tag{86}$$

where ϕ_{I1}, $\phi_{I2}\geq 0$.

Labour income Each forward-looking household j is the monopolistic supplier of a specific labour input, and sets the nominal wage for its labour variety j. Labour incomes are taxed at the rate τ_L. Wages are assumed to adjust sluggishly in the short run, either because of contracts or a general resistance to a change in the wage level by workers. The adjustment cost is denoted $\Gamma_{WFL,t}$ and its specification is the analogue of the nominal rigidities as found in equation (69):

$$\Gamma_{WFL,t}(j) \equiv \frac{\phi_{WFL1}}{2}\left(\frac{\pi_{W_{FL}t-1,t}}{\pi_{W_{FL}t-2,t-1}^{\phi_{WFL2}}(\Pi_{t-4,t}^{0.25}g_{t-1,t})^{(1-\phi_{WFL2})}} - 1\right)^2, \tag{87}$$

where

$$\pi_{W_{FL}t-1,t} = \frac{w_{FL,t}(j)}{w_{FL,t-1}(j)}\pi_{t-1,t}g_{t-1,t}, \tag{88}$$

and necessitating the presence of the growth term $g_{t-1,t}$, since real wages are nominal wages detrended by both inflation and productivity.

Other features of the budget constraint Forward-looking households own all domestic firms and there is no international trade in claims on firms' profits. The variable Φ includes all dividends accruing to shareholders, plus all revenue from nominal and real adjustment rebated as a lump sum to all forward-looking households, plus revenue from financial intermediation, which is assumed to be provided by domestic firms exclusively.

Finally, agents pay lump-sum (non-distortionary) taxes net of transfers $TT_t(j)$.

2.5.3 Consumer optimization for forward-looking households

The representative forward-looking household chooses bond holdings, capital, and consumption paths, and sets wages to maximize its expected lifetime utility (79) subject to (81) and (85), taking into account equations (116) and (117) in section 2.7 on market clearing.

For expositional convenience, it is worthwhile to write explicitly the maximization problem of agent $j \in [0, (1 - s_{LC})\, ss]$ in terms of the following Lagrangian:

$$\begin{aligned}
&\max_{\left\{C_{FL,\tau}(j), I_\tau(j), B_\tau(j), B^*_\tau(j), K_{\tau+1}(j), w_{FL,\tau}(j)\right\}_{\tau=t}^{\infty}} \mathsf{E}_t \beta_{t,\tau} g_{t,\tau}^{1-\sigma} \{\ u\Big(\ C_{FL,\tau}(j),\ \ w_{FL,\tau}^{-\psi_{L,\tau}}(j) w_\tau^{\psi_{L,\tau}} \ell_{FL,\tau},\ \Big) \\
&+ \mu_{FL,\tau}(j)\ \Big(\ -B_\tau(j) - \varepsilon_\tau B^*_\tau(j) + \frac{(1 + i_{\tau-1}) B_{\tau-1}(j)}{\pi_{\tau-1,\tau} g_{\tau-1,\tau}} + \frac{(1 + i^*_{\tau-1})(1 - \Gamma_{B,\tau-1}) \varepsilon_\tau B^*_{\tau-1}(j)}{\pi^{US}_{\tau-1,\tau} g_{\tau-1,t}} \\
&+ (1 - \tau_{K,\tau})\, r_\tau K_\tau(j) + (1 - \tau_{L,\tau})\, w_{FL,\tau}(j)^{1-\psi_{L,\tau}} w_\tau^{\psi_{L,\tau}} \ell_\tau (1 - \Gamma_{WFL,\tau}[w_{FL,\tau}(j), w_{FL,\tau-1}(j)]) \\
&- C_{FL,\tau}(j) - p_{E,\tau} I_\tau(j) + \Phi_\tau(j) - TT_\tau(j)\ \ \Big) \\
&+ \lambda_{FL,\tau}(j)\ \Big(\ -K_{\tau+1}(j) g_{\tau,\tau+1} + (1 - \delta)\, K_\tau(j) + \Gamma_{I,\tau}\left[I_\tau(j)/K_\tau(j)\right] K_\tau(j)\ \Big)\ \},
\end{aligned} \tag{89}$$

where μ and λ are the multipliers associated with, respectively, the budget constraint and capital accumulation.

The first-order conditions with respect to $C_{FL,t}(j)$ and $I_t(j)$ yield:

$$\mu_{FL,t}(j) = \partial u_{FL,t}(j)/\partial C_{FL,t}(j) = \lambda_{FL,t}(j) \Gamma'_{I,t}(j)/p_{E,t}. \tag{90}$$

In a symmetric set-up, $\partial u_{FL,t}(j)/\partial C_{FL,t}(j)$ is the same across forward-looking agents j. Their stochastic discount rate and pricing kernel is therefore the variable $D_{t,\tau}$, which is defined as:

$$D_{t,\tau} \equiv \beta_{t,\tau} g_{t,\tau}^{1-\sigma} \frac{\mu_{FL,\tau}}{\mu_{FL,t}} \frac{1}{\pi_{t,\tau}} \frac{1}{g_{t,\tau}}. \tag{91}$$

Accounting for the above expressions, the first-order conditions with respect to $B_t(j)$ and $B_t^*(j)$ are, respectively:

$$1 = (1 + i_t)\, \mathsf{E}_t D_{t,t+1}, \tag{92}$$

$$1 = (1 + i_t^*)\, (1 - \Gamma_{B,t})\, \mathsf{E}_t \left(D_{t,t+1} \Delta_{t,t+1} \right), \tag{93}$$

where Δ denotes the rate of nominal exchange rate depreciation against the U.S. dollar:

$$\Delta_{t,\tau} = \frac{\varepsilon_\tau}{\varepsilon_t} \frac{\pi_{t,\tau}}{\pi^*_{t,t}}. \tag{94}$$

Combining equations (92) and (93) yields the modified risk-adjusted UIRP, recalling that the return on international bond holdings is augmented to account for the costs of intermediation, Γ_B. In the steady state, the interest rate differential $(1 + i) / [(1 + i^*)(1 - \Gamma_B)]$ is equal to the steady-state nominal depreciation rate of the currency vis-à-vis the U.S. dollar, and relative purchasing power parity holds.

In the steady-state, equations (91) and (92) imply that:

$$\frac{(1 + i)}{\pi} = \frac{g^\sigma}{\beta}, \tag{95}$$

such that the 'natural' interest rate of the economy is equivalent to $(1 + i) / \pi$, the real interest rate (the nominal interest rate divided by the gross steady-state quarterly inflation rate π), and defined as g^σ / β, a function of the gross steady-state quarterly rate of growth of the world economy, g, and the rate of time preference (the inverse of the discount factor, β).[25]

The first-order condition with respect to $K_{t+1}(j)$ is:

$$\begin{aligned} \frac{p_{E,t}}{\Gamma'_{I,t}(j)} \mathsf{E}_t g_{t,t+1} &= \mathsf{E}_t \{ \; D_{t,t+1} \pi_{t,t+1} g_{t,t+1} (\; (1 - \tau_{K,t+1})\, r_{t+1} \\ &+ \frac{p_{E,t+1}}{\Gamma'_{I,t+1}(j)} \left[1 - \delta + \Gamma_{I,t+1}(j) - \Gamma'_{I,t+1}(j) \frac{I_{t+1}(j)}{K_{t+1}(j)} \right] \;) \; \}. \end{aligned} \tag{96}$$

Expression (96) links capital accumulation to the behaviour of the after-tax price of capital, $(1 - \tau_K)\, r$. In the steady state, $1 + (1 - \tau_K)\, r / p_E$ is equal to the sum of the natural real rate of interest, g^σ / β, and the rate of capital depreciation, δ.

[25] International differences in natural rates of interest can arise from asymmetric rates of time preference across regions. They are accounted for in the definition of the risk premium Γ_B in equation (83).

The first-order condition with respect to forward-looking agents' real wage, $w_{FL}(j)$, determines the wage dynamics specific to the forward-looking households, since $w_{FL}(j)$ is not the same as the average wage, $w(j)$, of the economy:

$$
\begin{aligned}
&-\psi_{FL,t}\frac{u_{\ell,t}(j)}{u_{C,t}(j)}\frac{1}{w_t(j)} = \left(\psi_{FL,t}-1\right)\left[1-\Gamma_{WFL,t}(j)\right](1-\tau_{L,t}) + \frac{\partial \Gamma_{WFL,t}(j)}{\partial w_{FL,t}(j)} w_{FL,t}(j)\,(1-\tau_{L,t}) \\
&+\mathsf{E}_t D_{t,t+1}\pi_{t,t+1}g_{t,t+1}\frac{\left(w_{FL,t+1}(j)/w_{t+1}\right)^{-\psi_{L,t+1}}}{\left(w_{FL,t}(j)/w_t\right)^{-\psi_{L,t}}}\frac{w_{FL,t+1}(j)}{w_{FL,t}(j)}\frac{\ell_{FL,t+1}}{\ell_{FL,t}}\frac{\partial \Gamma_{WFL,t+1}(j)}{\partial w_t(j)} w_{FL,t}(j)\,(1-\tau_{L,t+1})\,.
\end{aligned}
\tag{97}
$$

In a steady state where nominal rigidities have fully resolved, the real wage, $w_{FL}(j)$, is equal to the marginal rate of substitution, $MRS_{FL}(j)$, between consumption and leisure, augmented by the markup (which reflects the monopoly power of forward-looking agents in the labour market):

$$
w_{FL,t}(j) \;=\frac{\psi_{FL,t}}{\left(\psi_{FL,t}-1\right)}\frac{-u_{\ell FL,t}(j)}{u_{CFL,t}(j)} \equiv \frac{\psi_{FL,t}}{\left(\psi_{FL,t}-1\right)} MRS_{FL,t}(j), \tag{98}
$$

where an agent j's marginal rate of substitution between consumption and leisure, $MRS_{FL,t}(j)$, is solely a function of the labour supplied by forward-looking agents, and is invariant to the outcome of the consumption-savings decision – there is no trade-off in their wage decision between consumption and leisure. This is a property of the GHH utility function (Greenwood, Hercowitz, and Huffman 1988), which implies that labour effort will exhibit strong procyclical movements.

2.5.4 Consumer optimization for liquidity-constrained households

Liquidity-constrained households have no access to capital markets. They can optimize their labour supply based solely on their labour income. Similar to forward-looking households, they can optimally set their wages to exploit their market power. The maximization problem of agent $j \in ((1-s_{LC})\,ss, ss]$ can be written as follows:

$$
\begin{aligned}
&\max_{C_t(j),w_t(j)} u_t\ (\ C_{LC,t}(j), \ell_{LC,t}(j)\ ) + \mu_t(j)\ (\ -C_{LC,t}(j) - TT_t(j) \\
&\quad + (1-\tau_{L,t})\, w_{LC,t}(j)^{1-\psi_{L,t}} w_t^{\psi_{L,t}} \ell_{LC.t}(1-\Gamma_{WLC,t}[w_{LC,t}(j), w_{LC,t-1}(j)])).
\end{aligned}
\tag{99}
$$

Their resulting level of consumption is:

$$C_{LC,t}(j) = (1 - \tau_{L,t})\, w_{LC,t}(j)\ell_{LC,t}(j). \tag{100}$$

The first-order conditions with respect to liquidity-constrained agents' consumption, $C_{LC}(j)$, and their real wage, $w_{LC}(j)$, determines the partial adjustment of wages:

$$-\psi_{LC,t}\frac{u_{\ell LC,,t}(j)}{u_{CLC,,t}(j)}\frac{1}{w_{LC,t}(j)} = (1 - \tau_{L,t})\left[\left(\psi_{LC,t} - 1\right)(1 - \Gamma_{WLC,t}(j)) + \frac{\partial \Gamma_{WLC,t}(j)}{\partial w_{LC,t}(j)} w_{LC,t}(j)\right], \tag{101}$$

and their real wage, $w_{LC}(j)$, is a function of a unique marginal rate of substitution, $MRS_{LC}(j)$, between consumption and leisure, augmented by a markup, in a manner similar to the forward-looking agents.

2.5.5 Aggregating across categories of consumers

To discuss economy-wide measures of labour and wages, we need to aggregate across categories of consumers, of which we have two: liquidity constrained and forward looking.

Equation (102) determines the aggregate wage rate, w:

$$w_t^{1-\psi_{L,t}} = s_{LC} w_{LC,t}^{1-\psi_{L,t}} + (1 - s_{LC})\, w_{FL,t}^{1-\psi_{L,t}}, \tag{102}$$

where ψ_L is the degree of substitutability between lower-skilled and higher-skilled workers.

The aggregate labour supply function (ℓ) is implied by the labour supply of forward-looking agents, ℓ_{FL}, and liquidity-constrained agents, ℓ_{LC}:

$$\ell_t^{1-\frac{1}{\psi_{L,t}}} = (1 - s_{LC})^{\frac{1}{\psi_{L,t}}} \ell_{FL,t}^{1-\frac{1}{\psi_{L,t}}} + s_{LC}{}^{\frac{1}{\psi_{L,t}}} \ell_{LC,t}^{1-\frac{1}{\psi_{L,t}}}. \tag{103}$$

2.6 Government

The government is the third type of agent in the model, and it fulfills two roles: as the fiscal agent that collects and distributes tax revenues, and as the monetary authority that provides (at a minimum) a nominal anchor for the domestic economy.

2.6.1 Fiscal agent

Government expenditures, while part of GDP, do not form part of the productive capital stock, nor do they enter the consumers' utility functions. Public spending falls into three categories: government consumption, G_C; government investment, G_I; and public purchases of intermediate non-tradables, G_N. In the data, G_C can be considered as the purchase of goods, G_I is government investment in fixed capital, and G_N is government spending on wages and services. There are seven sources of (net) tax revenue: a distortionary tax on capital income, τ_K; a distortionary tax on labour income, τ_L; lump-sum taxes, TT, net of transfers to households; tariffs, τ_{TRF}, imposed on imported goods by region H on region R; royalties, τ_{ROYAL}, from the extraction of crude oil reserves; a distortionary *ad-valorem* tax on oil, τ_{OIL}, used in the production of tradable and non-tradable goods; and a distortionary *ad-valorem* tax on gasoline, τ_{GAS}.[26] The government finances the excess of public expenditure over net taxes by issuing debt denominated in nominal currency, denoted B in per-capita terms. All national debt is held exclusively by domestic (forward-looking) agents. The budget constraint of the government is:

$$B_t \geq (1 + i_{t-1}) \frac{B_{t-1}}{\pi_{t-1,t} g_{t-1,t}} + G_t - G_{REV,t}, \tag{104}$$

where:

$$G_t = G_{C,t} + p_{E,t} G_{I,t} + p_{N,t} G_{N,t}, \tag{105}$$

and government revenues are defined as:

$$\begin{aligned} G_{REV,t} &= \frac{1}{ss} \left(\int_0^{ss} TT_t(j) dj + \tau_{K,t} r_t \int_0^{ss(1-s_{LC})} K_t(j) dj + \tau_{L,t} \int_0^{ss} w_t(j) \ell_t(j) dj \right) \\ & \quad + G_{TRF,t} + G_{OIL,t} + G_{ROYAL,t} + G_{GAS,t}, \end{aligned} \tag{106}$$

[26] An additional tax not yet in the model is a value-added tax (VAT), such as the Goods and Services Tax (GST) in Canada. Such a tax rate will be introduced in the near future.

which have the following component revenues:

$$G^H_{TRF,t} = \sum_{R \neq H} \frac{1}{ss^H} \int_0^{ss^H} \tau^{H,R}_{TRF,t} p^{H,R}_{M,t} [M^{H,R}_t(x^H) + M^{H,R}_t(e^H)], \tag{107}$$

$$G_{OIL,t} = \frac{1}{ss} \int_0^{ss} \tau_{OIL,t} [p_{OT,t} O_t(h) + p_{ON,t} O_t(n)], \tag{108}$$

$$G_{ROYAL,t} = \frac{1}{ss} \int_0^{ss} \tau_{ROYAL,t} p_{OIL,t} OIL_t(s), \tag{109}$$

$$G_{GAS,t} = \frac{1}{ss} \int_0^{ss} \tau_{GAS,t} p_{GAS,t} GAS_t(g). \tag{110}$$

We can define the average tax rate for the economy, τ, as:

$$\tau_t \equiv G_{REV,t}/GDP_t. \tag{111}$$

The deficit-to-GDP ratio is:

$$\frac{DEF_t}{GDP_t} = \left(B_t - \frac{B_{t-1}}{\pi_{t-1,t} g_{t-1,t}} \right) / GDP_t. \tag{112}$$

From (104), in steady state we obtain:

$$\frac{B}{GDP} = \frac{\pi g}{\pi g - (1+i)} \left(\frac{G}{GDP} - \tau \right) = \frac{\pi g}{\pi g - 1} \frac{DEF}{GDP}. \tag{113}$$

The prior three equations define the relations between the debt-to-GDP ratio, the average tax rate, and the deficit-to-GDP ratio that are sustainable in the long term. In what follows, we treat the long-run debt-to-GDP ratio as a policy parameter set by the government, and let τ and DEF/GDP be determined by (113).

It is assumed that the government controls all of the component tax rates directly, with the exception of the labour tax, τ_L. The labour tax is the residual tax rate that allows the aggregate tax rate, τ, to respect a fiscal rule based on the stable long-run debt-to-GDP ratio in equation (113). The fiscal rule for τ is specified as:

$$\begin{aligned} \tau_t \quad = \quad & (\tau_{t-1} + \tau_t + \mathsf{E}_t \tau_{t+1})/3 + \phi_{\tau 1} \left(\frac{B_t}{GDP_t} - \phi_{\tau 2} B_{TAR,t} - (1 - \phi_{\tau 2}) \frac{B_{t-1}}{GDP_{t-1}} \right) \\ & + \phi_{\tau 3} \left(\frac{DEF_t}{GDP_t} - \frac{DEF}{GDP} \right), \end{aligned} \tag{114}$$

where B_{TAR} is the debt-to-GDP target, which is specified as a stochastic process. Therefore, the aggregate tax rate is a smoothed function of past and expected future tax rates, adjusted upward when the current debt-to-GDP ratio ($\phi_{\tau 1}$) is different from the average of its current target and its past observed level ($\phi_{\tau 2}$), or when the current deficit-to-GDP ratio differs from its sustainable steady-state level ($\phi_{\tau 3}$). Having $\phi_{\tau 3}$ greater than zero smooths the short-run development of the deficit-to-GDP ratio in face of any movement in the debt-to-income ratio.

2.6.2 Monetary authority

The government, in its role as the monetary authority, is assumed to define an objective for its monetary policy, and it controls the short-term nominal interest rate, i_t, as its instrument. The monetary authority can then specify some target to hit using its instrument. In the BoC-GEM, monetary policy is specified usually as an inflation-forecast-based rule. That is, we use an annualized interest rate rule of the form:[27]

$$(1+i_t)^4 = (1+i_{t-1})^{4\omega_i} \left(1+i_t^{neut}\right)^{4(1-\omega_i)} \mathsf{E}_t \left(\pi^X_{t-1,t+3} - \Pi_{t-1,t+3}\right)^{\omega_1} \left(GDP_t/GDP_{POT,t}\right)^{\omega_2}, \tag{115}$$

where the current interest rate, i_t, is a function of the average of the lagged rate, i_{t-1},and the current 'neutral' interest rate, i_t^{neut}[28], as well as

- a weight of ω_1 on $\mathsf{E}_t\left(\pi^X_{t-1,t+3} - \Pi_{t-1,t+3}\right)$, the expected year-over-year core inflation gap three quarters in the future, to return the economy to its target inflation rate by looking at the core year-over-year rate of inflation, $\pi^X_{t-4,t}$;
- a weight of ω_2 on $GDP_t/GDP_{POT,t}$, the output gap (output (GDP) divided by potential output (GDP_{POT})), to return the economy to its potential level of output in the steady state. GDP can be defined as either the model definition or a definition consistent with measured GDP.[29] Potential output, GDP_{POT}, is the rate of output that prevails under the current capital stock and the steady-state level of labour inputs and technological process.[30]

[27] Recall that $\Pi_{t-\tau,t-\tau+4}$ is the year-over-year gross CPI inflation target at time t for the four-quarter period between $t-\tau$ and $t-\tau+4$.

[28] The 'neutral' rate is the interest rate at which the targeted variables are stabilized at their target values, meaning that the real interest rate is a constant related to the growth rate of the economy and the rate of time preference (see equation (95)).

[29] For the measured concept of GDP (referred to as "national accounts real GDP"), see Appendix B. The measured concept of GDP is our preference for the sake of realism, since data are not measured accurately with continually shifting relative prices across time, as in the model.

[30] This is not to be confused with any concept of the flexible-price output gap (see Neiss and Nelson 2003 or Woodford 2003 for two differing examples). Instead, this is an extension of the older idea of potential

Note that the inflation gap mentioned here is not total CPI inflation (derived from CPI in equation (57)), but core inflation (derived from CPI^X in equation (58)), which excludes the effects of fuel prices (i.e., gasoline) and indirect taxes (in this version of the BoC-GEM, only tariffs).

The rule in equation (115) can be modified to include policy responses to a set of other variables (such as the nominal exchange rate) expressed as deviations from their targets. For this application of the model, such a modification is indeed put into practice. In the case of emerging Asia (AS), we model an exchange rate targeting regime by introducing the component $\omega_3^{AS}\Delta_t^{AS}$ in (115), where Δ^{AS} is defined by equation (94). We choose a very high value of ω_3^{AS}, so that the AS currency moves in tandem with the U.S. dollar – the bilateral nominal exchange rate is 'pegged.' At any rate, monetary policy has effects only in the short to medium term, and, after all the targets have been achieved, the monetary policy reaction function reduces to the neutral rate of interest.

2.7 Market clearing

The model is closed by imposing the following resource constraints and market-clearing conditions.

For each region H, the domestic resource constraints for capital and labour are, respectively:

$$\int_0^{ss^H\left(1-s_{LC}^H\right)} K_t^H(j^H)dj^H \geq \int_0^{ss^H} K_t^H(n^H)dn^H + \int_0^{ss^H} K_t^H(h^H)dh^H$$
$$+\int_0^{ss^H} K_t^H(s^H)ds^H + \int_0^{ss^H} K_t^H(o^H)do^H + \int_0^{ss^H} K_t^H(g^H)dg^H, \tag{116}$$

and:

$$\ell_t^H(j^H) \geq \int_0^{ss^H} \ell_t^H(n^H,j^H)dn^H + \int_0^{ss^H} \ell_t^H(h^H,j^H)dh^H$$
$$+\int_0^{ss^H} \ell_t^H(s^H,j^H)ds^H + \int_0^{ss^H} \ell_t^H(o^H,j^H)do^H + \int_0^{s^H} \ell_t^H(g^H,j^H)dg^H, \tag{117}$$

output used at the Bank of Canada, as expressed in Butler (1996) and still used in the Bank of Canada's *Monetary Policy Report*.

where these constraints imply that labour and capital can move freely across sectors, subject to any short-run real adjustment costs they face, such as in the production of commodities or oil.[31]

The resource constraint for the non-tradable good n^H is:

$$N_t^H(n^H) \geq \int_0^{ss^H} N_{A,t}^H(n^H, x^H)dx^H + \int_0^{ss^H} N_{E,t}^H(n^H, e^H)de^H + G_{N,t}^H(n^H), \tag{118}$$

while the tradable h^H can be used by domestic firms or be imported by foreign firms:

$$\begin{aligned} T_t\left(h^H\right) \geq & \int_0^{ss^H} Q_{A,t}(h^H, x^H)dx^H + \int_0^{ss^H} Q_{E,t}(h^H, e^H)de^H \\ & + \sum_{R \neq H} \left(\int_0^{ss^R} M_{A,t}^{R,H}(h^H, x^R)dx^R + \int_0^{ss^R} M_{E,t}^{R,H}(h^H, e^R)de^R \right). \end{aligned} \tag{119}$$

The same follows for the commodities and oil goods (s^H and o^H, respectively). For example, for commodities:

$$\begin{aligned} S_t\left(s^H\right) \geq & \int_0^{ss^H} Q_{SN,t}(s^H, n^H)dn^H + \int_0^{ss^H} Q_{ST,t}(s^H, h^H)dh^H \\ & + \sum_{R \neq H} (\int_0^{ss^R} M_{SN,t}^{R,H}(s^H, n^R)dn^R + \int_0^{ss^R} M_{ST,t}^{R,H}(s^H, h^R)dh^R). \end{aligned} \tag{120}$$

The final good A can be used for private (by both liquidity-constrained and forward-looking households) or public consumption:

$$\int_0^{ss^H} A_t^H(x^H)dx^H \geq \int_0^{ss^H(1-s_{LC}^H)} C_{FL,t}^H(j^H)dj^H + \int_{ss^H(1-s_{LC}^H)}^{ss^H} C_{LC,t}^H(j^H)dj^H + ss^H G_{C,t}^H, \tag{121}$$

and similarly for the investment good E:

$$\int_0^{ss^H} E_t^H(e^H)de^H \geq \int_0^{\left(1-s_{LC}^H\right)ss^H} I_t^H(j^H)dj^H + ss^H G_{I,t}^H. \tag{122}$$

All profits, adjustment costs, and intermediation revenues are assumed to accrue to forward-looking households as $\Phi_t(j)$.

[31] See equation (2) for the case of commodities.

Market clearing in the asset market requires:

$$\int_0^{ss^H(1-s^H_{LC})} B^H_t(j^H)dj^H = ss^H B^H_t, \tag{123}$$

for each of the five regions' government bond markets, and:

$$\sum_R \int_0^{ss^R(1-s^R_{LC})} B^{*R}_t(j^R)dj^R = 0, \tag{124}$$

for the international bond market (net foreign assets). Finally, aggregating the budget constraints across private and public agents after imposing the appropriate transversality conditions, we obtain the law of motion for financial wealth, where the present value of the next period's financial wealth is equal to the sum of this period's financial wealth, plus the financial transactions costs incurred by the holdings of U.S.-dollar-denominated international bonds, plus the domestic production of the economy, plus the net export positions of commodities ($TBAL_S$) and oil ($TBAL_O$), less this period's consumption, investment, and government expenditures:

$$\begin{aligned}\mathsf{E}_t D^H_{t,t+1}\pi^H_{t,t+1}g_{t,t+1}F^H_{t+1} &= F^H_t + \Gamma^H_{B,t-1}\frac{\left(1+i^*_{t-1}\right)\varepsilon^{H,US}_t B^{*H}_{t-1}}{\pi^{US}_{t-1,t}g_{t-1,t}} \\ &+ p^H_{N,t}N^H_t + p^H_{T,t}T^H_t + (1+\tau^H_{GAS,t})p^H_{GAS,t}GAS^H_t \\ &+ TBAL^H_{S,t} + TBAL^H_{O,t} - C^H_t - p^H_{E,t}I^H_t - G^H_t. \end{aligned} \tag{125}$$

2.8 Definition of the gross domestic product

The gross domestic product (in consumption units – i.e., deflated by the consumer price index) can be stated as either the sum of all goods consumed domestically once the net flows of final goods (EX and IM) are accounted for, or all goods produced domestically once net cross-border flows of all intermediate goods ($TBAL_S$ and $TBAL_O$) are accounted for:

$$\begin{aligned} GDP^H_t &= A^H_t + p^H_{E,t}E^H_t + p^H_{N,t}G^H_{N,t} + EX^H_t - IM^H_t \\ &= p^H_{N,t}N^H_t + p^H_{T,t}T^H_t + (1+\tau^H_{GAS,t})p^H_{GAS,t}GAS^H_t + TBAL^H_{S,t} + TBAL^H_{O,t}, \end{aligned} \tag{126}$$

where total exports, EX, are:

$$EX^H_t = p^H_{T,t}T^H_t + p^H_{S,t}S^H_t + p^H_{O,t}O^H_t - p^H_{Q,t}\left(Q^H_{A,t} + Q^H_{E,t}\right) - p^H_{QS,t}Q^H_{S,t} - p^H_{QO,t}Q^H_{O,t}, \tag{127}$$

total imports, IM, are:

$$IM_t^H = \sum_{R \neq H} p_{M,t}^{H,R} \left(M_{A,t}^{H,R} + M_{E,t}^{H,R} \right) + p_{MS,t}^{H,R} M_{S,t}^{H,R} + p_{MO,t}^{H,R} M_{O,t}^{H,R}, \tag{128}$$

the trade balance in commodities, $TBAL_S$, is:

$$TBAL_{S,t}^H = p_{S,t}^H S_t^H - p_{QS,t}^H Q_{S,t}^H - \sum_{R \neq H} p_{MS,t}^{H,R} M_{S,t}^{H,R}, \tag{129}$$

and the trade balance in oil, $TBAL_O$, is similarly stated.

Appendix B deals with the conventional measures of volumes typically found in the national accounts, and issues related to that topic.

3. Calibrating the Model

Given the large and complex nature of the BoC-GEM, it is readily apparent that, at this point in time, full estimation of the parameters of the model is not possible. Therefore, the model must be calibrated, and we have relied on previous work on the GEM (such as Laxton and Pesenti 2003; Bayoumi, Laxton, and Pesenti 2004; Faruqee et al. 2007a,b) to guide our calibration work. Work done in tandem with this technical report has also provided useful insights for the calibration – see Coletti, Lalonde, and Muir (2007) and Elekdag et al. (2007). We also rely on previously published work for particular economies, namely

- Canada (Murchison and Rennison 2006 – the reference work for the Bank of Canada's projection and policy analysis model for Canada, ToTEM; also Perrier 2005);
- the euro area (Coenen, McAdam, and Straub 2007 – refers to the NAWM (New Area-Wide Model), a DSGE model; also, de Walque, Smets, and Wouters 2006 and Smets and Wouters 2005, for a Bayesian-estimated DSGE model of both the U.S. and the European economies); and
- the United States (Brayton et al. 1997 – a published reference for the Board of Governors of the Federal Reserve System's PAC model of the United States, FRB/US;[32] Erceg, Guerrieri, and Gust 2005a,b – the references for the SIGMA DSGE model used

[32] "PAC" stands for "polynominal adjustment cost." See the seminal work by Tinsley (1993) for more on the methodology.

at the Board of Governors of the Federal Reserve System; Gosselin and Lalonde 2005 – the source for the Bank's PAC model of the U.S. economy, MUSE; Juillard et al. 2006 – a small Bayesian-estimated DSGE model of the U.S. economy).

This section consists of seven parts. Sections 3.1 and 3.2 present the steady-state parameters and ratios. Section 3.3 examines the calibration of the international linkages of the model (both the steady state and their dynamics). Sections 3.4 and 3.5 deal with the steady-state and dynamic calibrations of the oil and commodities sectors, respectively. Section 3.6 describes the real adjustment costs and the nominal rigidities in the dynamic model of the BoC-GEM. Finally, section 3.7 describes the calibration of the fiscal and monetary policy rules.

3.1 Key steady-state parameters

In general, the key parameters of the steady state are the same across the five regions of the world (Table 1). At steady state, in all the regions of the world, the growth rate of GDP per capita is equal to 1.9 per cent per annum and the real interest rate is equal to 3 per cent. Other key parameters are associated with the consumers' utility function and their utility maximization problem, of which there are two: one for forward-looking agents (equation (89)) and one for liquidity-constrained consumers (equation (99)). The intertemporal elasticity of substitution in consumption is set to 0.7 for all the regions. The chosen value is in the mid-range of the values used in the DSGE literature (between 0.5 and 1.0). This parameter drives the amplitude of the effect of the interest rate on consumption. This value of the elasticity of intertemporal substitution, combined with habit persistence in consumption equal to 0.85, generates the expected gradual hump-shaped response of consumption to shocks. The parameter for habit persistence in consumption is relatively high compared with the literature. However, Juillard et al. (2006) estimate a DSGE model of the United States similar in structure to the GEM, and they obtain a result close to our calibration (0.83 vs. 0.85) using a similar intertemporal elasticity of substitution in consumption (0.8 vs. 0.7). With the exception of emerging Asia, we set the share of liquidity-constrained consumers to 20 per cent. This value, combined with the existing link between the government debt and the net foreign assets, allows the model to generate a reasonable impact of debt shock on output. Because of the limited access to credit in newly industrializing economies, we set the share of liquidity-constrained consumers in emerging Asia to 50 per cent, as in Faruqee et al. (2007a). Like Erceg, Guerrieri, and Gust (2005b), we set the Frisch elasticity of labour supply to 0.2 for all regions, which is consistent with results obtained in microeconomic studies (between 0.05 and 0.35). The labour habit persistence parameter is 0.75 – a value almost identical to the estimation result of Juillard et al. (2006).

All the production functions of the model use a CES technology (Table 2). For all the regions, the elasticity of substitution between factors of production of the tradable, non-tradable, and gasoline sectors is 0.7. This value is below one, which would correspond to a Cobb-Douglas production function.[33] For the sectors that include a fixed factor of production (oil and commodities), we assume even less substitution between the remaining factors of production, setting the elasticity of substitution to 0.6. In the short and medium terms, the effective elasticities of substitution in the oil, gasoline, and commodities sectors are even lower because of short-term real adjustment costs in the factors of production. We address the calibration of these adjustment costs in section 3.5. For all the regions, we assume that the tradable sector is more capital intensive than the non-tradable sector. With the exception of emerging Asia, the calibration implies a capital-to-output ratio between 1.8 and 1.9 (Table 4). For emerging Asia, it is 2.3.

The calibration of the price markup over marginal cost in the tradable and non-tradable sectors is based on the estimates obtained by Martins, Scarpetta, and Pilat (1996) for prices, and by Jean and Nicoletti (2002) for wages.[34] According to the calibration shown in Table 3, the level of competitiveness is higher in the United States and in emerging Asia than in the other three regions.

3.2 Composition of aggregate demand

Table 5 outlines the calibration of the composition of aggregate demand for the five regions. At steady state, the United States and Canada are the only regions that have negative net-foreign-asset-to-GDP positions. This ratio is set to -50 per cent of GDP for the United States and -7.5 per cent of GDP for Canada. Because of their negative NFA positions, the United States and Canada must generate a small trade surplus in the long run. On the other hand, at steady state, AS, RC, and CX must exhibit a small trade deficit to maintain their positive NFA-to-GDP ratios (derived from the data underlying Lane and Milesi-Ferretti 2006).

In the United States, the investment-to-GDP and the government-expenditure-to-GDP ratios

[33] Perrier (2005) shows that the Cobb-Douglas assumption is rejected by the data for Canada.

[34] Even though the study done by Martins, Scarpetta, and Pilat (1996) is over ten years old, it is the only study that provides a consistent calibration of the markups across most of the regions of the world.

are set to 16.0 per cent and 17.2 per cent, respectively, which yields a consumption-to-GDP ratio of 66.3 per cent of GDP. These ratios are very similar to those found in Juillard et al. (2006). They are also compatible with Smets and Wouters (2005). In Canada, the investment-to-GDP ratio is set to 16.6 per cent and the ratio of government expenditures relative to GDP is 26.0 per cent. Therefore, the consumption-to-GDP ratio is 57.3 per cent of GDP, much lower than in the United States.

For the other regions, the calibration of the decomposition of aggregate demand follows Faruqee et al. (2007a). It is interesting to note that emerging Asia has the largest investment share (29.9 per cent), while RC has the next-highest government-expenditure-to-GDP ratio (23.2 per cent) because it includes the European Union.

3.3 International linkages

Figures 4 to 6 illustrate the calibration of all the bilateral trade flows between the regions in all types of tradable goods (i.e., consumption, investment, oil and commodities), and then the oil sector and the commodities sector alone. The calibration of these trade matrices is central to the properties of the model, especially for the steady-state movements and dynamics of the different bilateral exchange rates, as well as the spillover effects of any shock in one region to another. The calibration of the trade links is based on the current trends in trading patterns observed in the COMTRADE database, maintained by the United Nations. As expected, 82 per cent of Canadian exports are sent to the United States. The three largest trading partners of the United States are Canada, emerging Asia (including China), and the commodity exporter bloc (which includes Mexico). Emerging Asia exports worldwide, but most notably to the United States and RC (where Japan depends heavily on their commodities exports). Also note the patterns for the aggregate trade volumes: Canada, a small open economy, has the highest trade volumes, with an export-to-GDP ratio of 37 per cent, while RC (which includes the largely self-contained trading bloc of the European Union) is the smallest, with an export-to-GDP ratio of 9 per cent.

The substitutability between domestically produced goods and imports are governed by the elasticities of substitution for each traded sector. As in Erceg, Guerrieri, and Gust (2005b) and Murchison and Rennison (2006), we set the elasticity of substitution between domestically produced tradable goods and imported tradable goods at 1.5, which is lower than the value (2.5) assumed in previous published work using the GEM (Table 6). However, when one region imports from the four other regions, the elasticity of its demand for goods from one particular region versus goods from another region is considerably higher (i.e., 3.0). Combined with the trade matrices for extra sectors, we obtain the bias of each region towards imports from another region (Table 6).

The adjustment in the trade sector of the economies is largely governed by the speed of adjustment of the NFA-to-GDP ratio to its desired ratio. This is the determinant for the financial intermediation costs (equation (83)) that form part of the modified risk-adjusted UIRP condition (equation (93)). While we do not have any reliable econometric estimates of the speed of adjustment of the NFA-to-GDP ratio, we set the two necessary parameters such that the NFA-to-GDP ratio converges to its desired level (i.e., the NFA gap) within 15 to 20 years after a shock to the desired level. This speed of adjustment is a compromise. Faster convergence of the NFA gap implies that the bilateral U.S.-dollar exchange rate deviates too strongly from the standard UIRP condition even in the short run. However, a speed of adjustment that is too low eliminates, in practice, the stock-flow dynamics between the current account and the net foreign asset position, and creates extremely long-lived gaps throughout each economy. This would generate a disequilibrium in the current account for an implausibly long period of time. Given the current context of global imbalances, this result would be an undesirable property for the model.

3.4 The oil sector

3.4.1 The supply of oil

The CES production function of the oil sector depends on capital, labour, and crude oil reserves. Therefore, real marginal cost in the oil sector depends on the real wage, the real rental price of capital, and the real price of crude oil reserves. Figure 7 illustrates how the market for crude oil reserves behaves in terms of demand and supply curves. The supply curve is vertical because the reserves are assumed to be a fixed factor of production (relative to GDP). All else being equal, and for a given demand curve, a region with a larger endowment of reserves, like the commodity-exporting region (CX), will have a lower real price of crude oil reserves and a lower marginal cost than a region with fewer reserves, such as the United States. This is even more important if the share of crude oil reserves in the production of oil is large relative to capital and labour. Table 7 shows the distribution of crude oil reserves around the world, based on data published by the U.S. Department of Energy. Overall, the calibration is consistent with the data. In order to better align the price of oil across the regions, we make some minor adjustments to slightly reduce the dispersion of marginal costs across them. Nevertheless, the commodity exporter (CX) holds 75 per cent of the world oil reserves.

Because the reserves of crude oil are modelled as a fixed factor of production in equation (25), the more intensively a region's production of oil uses its reserves of crude oil, the more vertical is the supply curve (as shown in Figure 7). In reality, the crude oil reserves are partly endogenous and are a function of the price of oil. The Athabasca tar sands in Western Canada are a good example. As oil prices move higher, more of the tar-sands fields can be profitable in the long run; therefore, the producers increase the level of active crude oil reserves for Canada. Higher oil prices also induce more exploration, with the (usual) consequence that new reserves of crude oil will be found and become active.[35]

Regions where oil production depends more on the amount of capital (such as Canada, which depends on tar sands and offshore oil fields) have a supply curve that is less vertical, because capital is an endogenous and variable factor of production. However, the level of capital used to exploit the tar sands and offshore oil fields is very slow to adjust, because of the complexity of the capital involved in extracting oil from those two types of oil fields. Therefore, we introduce strong real adjustment costs in the short run that greatly reduce the effective elasticity of substitution between factors of production in the oil sector during the first five years. Section 3.5 addresses the calibration of these real adjustment costs.

The calibration of the relative shares of capital and crude oil reserves in the oil sector's production function (equation (25)) greatly affects the slope of the supply curve for oil. The bias parameters of the oil-production function of each of the regions are calibrated according to the following hypotheses:

- The OPEC production process is simpler and more reserve-intensive than offshore oil fields and tar sands. For instance, Norway, the Gulf of Mexico (28 per cent of U.S. production), and the Gulf of St. Lawrence offshore oil fields and the Athabasca tar-sands fields (both in Canada) are much more capital intensive than the oil-well fields of the Arabian peninsula. The Norges Bank assumes that reserves, capital, and labour represent, respectively, 50 per cent, 40 per cent, and 10 per cent of the costs of production for the Norwegian state oil producer, Statoil. At the opposite end of the spectrum,

[35] Recall that the level of crude oil reserves, the fixed factor OIL, is exogenous to the model. However, since it is formulated as a stochastic process, an exogenously specified permanent shock can approximate this behaviour when and where it is necessary.

Hunt (2005) assumes that reserves represent 96 per cent of OPEC costs of production (essentially calibrated with Saudi Arabia).

- Roughly half of Canadian oil production relies on the Athabasca tar sands. Therefore, we assume that Canada has the most capital-intensive oil-production process of the five regions.

- For every region, oil production is more reserve intensive than it is intensive in variable factors of production (i.e., capital and labour).

Table 2 shows the calibration of the bias towards each of the oil sector's factors of production for the five regions. The bias parameters are calibrated on the assumption that for the tar sands and offshore oil fields, the capital share is between 40 per cent and 50 per cent. For the oil-well fields, capital represents only between 5 per cent and 10 per cent of the production cost. We also assume that the labour share is small for all the regions.

Canada is the region where the marginal cost depends the most on the rental price of capital, whereas the marginal cost for CX depends almost exclusively on the price of crude oil reserves. Because CX has 75 per cent of the world's crude oil reserves and its production is not as capital intensive, it has the lowest marginal cost. This is consistent with anecdotal evidence that in Saudi Arabia the marginal cost of producing a barrel of oil is between US$2 and US$3. To account for the wedge between marginal cost in CX and the observed price of oil, we assume that firms in CX have strong market power. This assumption allows us to have an oil price that is similar across regions, despite the fact that CX has a marginal cost that is much lower than the other regions. Furthermore, in order to generate an oil price that moves uniformly across all the regions, we assume a very high elasticity of substitution of 10 between the demand for domestically produced and imported oil in all regions, and an elasticity of substitution of 5 between the different exporters of oil, in all the regions (Table 6).

3.4.2 The demand for oil and gasoline

The demand for oil comes from three sources: refineries that produce gasoline, firms that produce non-tradable goods, and firms that produce tradable goods. These firms can buy oil on the domestic market or import it from abroad. The trade flows of oil are calibrated based on the United Nations COMTRADE database for 2003. Figure 5 shows that Canada and the commodity-exporting region CX are the only net exporters of oil, at 3.6 per cent and 8.0 per cent of domestic GDP, respectively.

In order to calibrate the demand for oil for the refineries producing gasoline, we need to know the demand generated by consumers. Recall that the consumption basket is composed of gasoline and tradable and non-tradable goods. Since consumption is represented by a nested CES aggregator, we need to set the values of the bias parameter towards gasoline in the consumption aggregator to fit the share of gasoline in consumption. To do so, we rely on the weight of fuel in the CPI (Table 8). For CX and AS, data are not readily available, so we make the assumption that the gasoline share of consumption is lower than in the industrialized countries. Therefore, when we calibrate the aggregate consumption equation (50), we set the elasticity of substitution between tradable and non-tradable goods to 0.5, as did previous work with GEM (Faruqee et al. 2007a). We also assume a lower elasticity of substitution between gasoline and the rest of consumption (i.e., 0.3).

On the supply side of the gasoline sector (equation (34)), we assume a similar technology of production of gasoline around the world. Therefore, for the five regions, we set the oil, capital, and labour shares of production to around 60 per cent, 25 per cent, and 15 per cent, respectively.

We also need to decide on the distribution of oil between the gasoline refineries and the tradable and non-tradable sectors. We find this by first calculating the ratio of oil and natural-gas production in GDP for the five regions using data from the U.S. Department of Energy in 2003. These data, combined with the net export positions, give the ratio of domestic oil demand to GDP. From this ratio we subtract the demand for oil by gasoline refineries, and then assume that the tradable sector is more oil intensive than the non-tradable sector. In general, we find that 60 per cent of oil demand is from the gasoline refineries (Tables 4 and 5).

3.5 The commodities sector

The calibration of the commodities sector is analogous to that of the oil sector. However, data availability issues and the fact that this sector is actually a basket of many different commodities make the calibration less precise. We assume that commodities include the products of the agricultural, fisheries, forestry, and mining sectors of the economy, as well as some fuel sources such as coal. The major source of data for the share of the commodities sector in total production is the United Nations COMTRADE database. These data, combined with production data for some countries, allow us to calibrate the bias parameters of commodities in the production of tradable and non-tradable goods. Turning to the supply of commodities, we make the assumption that the share of land, capital, and labour are, respectively, around 55 per cent, 25 per cent, and 20 per cent for all the regions (Table 2).

3.6 Rigidities and adjustment costs

3.6.1 Real adjustment costs in capital, investment, and imports

As explained in section 2.5.2, capital and investment are subject to quadratic adjustment costs. As in Faruqee et al. (2007a) and Juillard et al. (2006), for all the regions but RC, we assume that the adjustment costs related to a change in the level of capital are relatively small whereas those related to the change in the level of investment are large, around 100. The latter value is larger than the value of 78 estimated by Juillard et al. (2006), but in the context of our model it is necessary to assume greater adjustment costs to better match the properties of the ToTEM, MUSE, and FRB/US models. Based on the estimation results obtained by de Walque, Smets, and Wouters (2006), for the euro area we set the values of the investment adjustment cost in RC (which includes the European Union) at 160 instead of 100. For all the regions, we set the real adjustment cost parameters associated with the share of imported consumption and investment goods to 0.95, as in Faruqee et al. (2007a,b).

3.6.2 Real adjustment costs in the oil, gasoline, and commodities sectors

In the oil sector, we assume strong real adjustment costs related to changing the capital share and labour share of production, particularly for offshore oil fields and the Athabasca tar sands in Canada. We calibrate these real adjustment costs based on the following principles:

- In general, several years are necessary for a new oil-producing facility to be running at capacity. Therefore, during the initial years following a permanent increase in the demand for oil, oil production should barely increase (i.e., the short-run supply curve is almost vertical).
- For the regions that rely heavily on tar sands (Canada) and offshore oil fields (Norway, the United States, and China), we assume that the adjustment costs are greater.

For the gasoline sector, we assume the same sort of strong real adjustment costs on capital and labour, while assuming that the technology for gasoline refineries is similar across regions. In the commodities sector, we assume that the production process is somewhat more flexible than in the oil sector. This allows the BoC-GEM to generate a smaller reaction of commodities prices to world demand fluctuations than for oil prices, as implied by the data (Lalonde, Zhu, and Demers 2003). Table 9 presents the calibration of the parameters that define the amplitude of the real adjustment costs in the oil, gasoline, and commodities sectors.

Similar to the supply side of the oil market, we assume that it is costly for firms to adjust the share of oil and commodities used in the production of tradable and non-tradable goods. These real adjustment costs dampen the reaction of the demand for oil and commodities during the first two years for commodities, following a permanent supply shock, decreasing the price elasticity of the short- and medium-run demand curves for oil and commodities. We assume that these real-adjustment-cost parameters are equal to 300 for oil demand and 200 for commodities demand.

3.6.3 Nominal rigidities

In the BoC-GEM, we assume that it is costly to adjust the level and the first difference of inflation and the growth rate of nominal wages relative to their steady states. Juillard et al. (2006) arrive at a value of 700 for parameters of the nominal rigidities.[36] Moreover, these values map well the properties of ToTEM (based on Calvo 1983 pricing), MUSE, and FRB/US (both based on polynomial adjustment costs for pricing – see Tinsley 1993). Therefore, for all the regions but RC, we set the parameters of the nominal rigidities to 700 (see Table 10). In RC, which is represented mainly by the European Union and Japan, we assume somewhat larger nominal rigidities than elsewhere, consistent with the estimation results obtained by de Walque, Smets, and Wouters (2006).

We also assume that the nominal rigidities are larger in the labour market than in the goods market (usually 800). Finally, to have plausible properties concerning the exchange rate pass-through in the short run, we need very strong nominal rigidities on import prices (around 4,000).[37] Murchison and Rennison (2006) make a similar assumption.

[36] Each increment of 100 in the nominal rigidity is roughly equivalent to a one-quarter-long contract in the Calvo-contracting framework (Pesenti 2007). Therefore, a nominal rigidity of 700, for example, would result in a contract length of roughly seven quarters.

[37] This allows the model dynamics to behave as if the law of one price does not hold in the short run. To

3.7 Fiscal and monetary policy rules

In Table 11, we characterize the behaviour of the government, which acts as both the fiscal agent and the monetary authority.

As the fiscal agent, the governments of all five regions are assumed to target an explicit level of government debt, as shown in Table 5 using the tax-targeting rule of equation (114), which determines the labour income tax rate. The smoothing parameter required to impart the degree of sluggishness found in ToTEM (for CA), MUSE (for US), or NAWM (for the euro area as part of RC) is 0.005. The weight in the government-debt-to-GDP ratio gap on the debt-to-GDP target (as opposed to the lagged debt-to-GDP ratio) is unity for all regions.[38] This parameterization ensures that the tax rate will change at a speed that will guarantee that the observed debt-to-GDP ratio returns to its targeted level at a horizon similar to the estimated tax rate rule found in MUSE (Gosselin and Lalonde 2005).

For monetary policy, we assume that emerging Asia follows a fixed nominal exchange rate vis-à-vis the U.S. dollar.[39] Therefore, the Asian monetary authority effectively imports its monetary policy from the United States. The nominal interest rate in AS moves such that the modified risk-adjusted UIRP condition always maintains a constant interest rate differential between AS and US for a given nominal exchange rate.

The other four regions' monetary authorities target year-over-year core inflation, defined using the price index of equation (58). We can achieve this by using several different parameterizations. One candidate parameterization is from estimated rules for the United States such as English, Nelson, and Sack (2003), or Erceg, Guerrieri, and Gust (2005a), where the smoothing parameter is around 0.75 and the weight on the forward-looking inflation gap is around 0.65. Another option is to use the optimal rule found in ToTEM, which is more consistent with the optimal rule literature (Cayen, Corbett, and Perrier 2006). In this case, the smoothing parameter is set to a high value of 0.95 and the forward-looking inflation-gap parameter is 0.90. The latter option gives more reasonable properties for structural and permanent shocks in the BoC-GEM, and is the basis for the model used in the simulations described in sections 4 and 5.

break the law of one price in the long run, and to disentangle the purpose of nominal rigidities on import prices, we could introduce a distribution sector in future work, as in Corsetti and Pesenti (2001), and as found in the original two-country version of the GEM of Laxton and Pesenti (2003).

[38] Future work on fiscal issues will probably lead to a differentiation of these parameters across the regional blocs.

[39] It is important to note that the model does not account for the sterilization of capital flows that often accompanies the maintenance of fixed (or strongly managed) exchange rate regimes in emerging Asia.

4. Model Properties for Canadian and U.S. Domestic Shocks

In this section, we examine the basic model properties of the BoC-GEM for the two major regions of interest: Canada and the United States. For both regions, we look at a temporary rise in consumption (a "demand shock"), and a temporary 100-basis-point increase in interest rates (a "monetary policy shock").

4.1 Domestic Canadian shocks

We first focus on the response of the Canadian economy to domestic shocks and compare the responses of the BoC-GEM with those of ToTEM (the Bank of Canada's new one-country small-open-economy DSGE model for forecasting and policy analysis).

4.1.1 A temporary shock to Canadian consumption

We trigger a temporary increase in Canadian consumption through a rise in the marginal utility of consumption (Figure 8). The size of the shock is calibrated such that, after two quarters, GDP will increase 0.5 per cent relative to control. This shock increases the demand for gasoline, tradable goods, and non-tradable goods for roughly two years. It is interesting to note that part of the increase in consumption is financed from abroad, so imports rise by one per cent relative to control. Canadian firms meet the increase in demand by increasing their demand for labour, capital, oil, and commodities. This increases the real wage, the rental price of capital, the price of oil, and the price of commodities. Therefore, we see a rise in real marginal cost and consequently inflation. The peak response of year-over-year core inflation is 0.07 percentage points, four quarters after the shock, which is two quarters after the peak response of GDP. The monetary policy authority raises the nominal interest rate by 16 basis points in response to the increased inflationary pressures, which creates a 0.2 percentage point appreciation of the real effective exchange rate. The increase in the interest rate and the appreciation of the Canadian dollar help return the economy to its steady-state equilibrium. With the exception of the real exchange rate, the amplitude and the timing of all of these responses are very similar to responses in ToTEM. In ToTEM, the appreciation of the exchange rate is larger than in the BoC-GEM. This difference is due to the fact that in the BoC-GEM, financial intermediation costs play a larger role than in ToTEM, where exchange rate dynamics are almost exclusively determined by a modified UIRP condition, with a very small role for adjustment of the country-specific risk premium.[40]

4.1.2 A temporary shock to the Canadian interest rate

To demonstrate the monetary policy transmission mechanism in the economy, we introduce a temporary increase of 100 basis points in the Canadian short-term interest rate (Figure 9). Inertia in monetary policy ensures that interest rates stay above control for around two years. The shock increases the rental price of capital and therefore reduces investment. Forward-looking consumers increase their savings and reduce their consumption. The increase in the interest rate induces a 1.7 per cent appreciation of the real effective exchange rate, which increases the price of Canadian goods abroad and decreases the price of foreign tradable goods in Canada, thereby reducing demand for Canadian goods abroad and increasing Canadian imports. Overall, GDP drops by 0.34 percentage points, reaching its trough after four quarters. The reduction in domestic demand induces firms to reduce their demand for the variable factors of production. The real wage falls (as does the real rental price of capital in the medium term), and, by extension, so does real marginal cost. Consequently, year-over-year core inflation decreases by 0.26 percentage points four to five quarters after the initial impact of the shock. The magnitudes and timing of the responses are similar to those of ToTEM.

4.2 Domestic U.S. shocks

We next examine U.S. domestic shocks and their impact on the world economy, with an emphasis on Canada. We also compare the responses of the U.S. economy in the BoC-GEM with those from MUSE, the Bank of Canada's forecasting model of the U.S. economy.

4.2.1 A temporary shock to U.S. consumption

The shock that occurs in the United States is a rise in the marginal utility of consumption, causing a temporary increase in domestic consumption (Figures 10 and 11). As in the Canadian case, the size of the shock is calibrated to create a peak response of around 0.5 per cent of GDP after two quarters. This shock increases the demand for gasoline, and tradable and non-tradable goods for roughly two years. Year-over-year core inflation increases by 0.16 percentage points (after four quarters). The U.S. monetary authority raises interest rates

[40]This is ToTEM's analogue of the financial intermediation costs found in the GEM.

by 32 basis points, which generates an appreciation of the real effective exchange rate of 0.37 percentage points. The amplitude and the timing of the responses are similar to that in MUSE.

Since the United States is a large economy and is a key player in world trade, any U.S. shock has spillover effects on the other four regions of the world. This occurs through four channels. We consider the role of these four channels in the Canadian economy in detail:

- U.S. demand for foreign consumption and investment goods increases; therefore, Canadian exports increase (the income effect).
- The Canadian real effective exchange rate depreciates, which reduces the price of Canadian goods and increases Canadian exports (the price effect). It has the opposite effect on imports.
- The U.S. share of world demand for oil and commodities is large enough to increase the real prices of oil and commodities by 2.0 per cent and 0.4 per cent, respectively. Given that Canada is a net exporter of these goods, this increase implies a positive terms-of-trade shock and a positive wealth effect, thereby increasing Canadian consumption.
- The increase in domestic demand drives up real marginal cost, while the depreciation of the exchange rate increases real import prices. Therefore, the year-over-year core inflation rate rises by 0.05 percentage points and the monetary authority increases the nominal interest rate by 12 basis points, helping to re-equilibrate the economy.

These effects increase Canadian economic activity, with the exception of the response of U.S. monetary policy. The peak response of Canadian GDP is 0.17 percentage points after three quarters – one quarter after the peak response of U.S. GDP. The rise in Canadian GDP is almost entirely due to an increase in Canadian exports to the United States of roughly 0.5 per cent relative to control. Similar effects occur in RC, CX, and AS, but the effects are much smaller than those in Canada. As shown in section 3.3, of the four regions, Canada depends the most on U.S. trade. Consequently, because of the modified risk-adjusted UIRP condition, the Canadian dollar is actually appreciating relative to the RC and CX currencies.

4.2.2 A temporary shock to the U.S. interest rate

Figures 12 and 13 show an exogenous 100 basis point increase in the U.S. interest rate, which then stays above control for seven quarters. U.S. GDP gradually drops by 0.40 percentage points after four quarters. The U.S. economy is slightly more sensitive to the interest rate than the Canadian economy, mainly because the GDP share of government expenditures is larger in Canada and these expenditures are less sensitive to the interest rate. The shock reduces the demand for domestic and imported goods (which falls by 0.8 percentage points). The increase in the interest rate causes an appreciation of the U.S. real effective exchange rate of 0.8 per cent after three quarters. Relative to the Canadian dollar, the U.S. currency appreciates 1.5 per cent in real terms. U.S. year-over-year core inflation decreases by 0.33 percentage points. These responses are almost identical to those of MUSE.

As in the U.S. consumption shock, we see spillover effects around the world. The four channels behave for Canada in the following way:

- The drop of U.S. demand for foreign consumption and investment goods reduces Canadian exports (the income effect).
- The depreciation of the Canadian dollar reduces the price of Canadian goods and increases Canadian exports (the price substitution effect).
- The U.S. share of world demand for oil and commodities is large enough to reduce the real prices of oil and commodities by 2.0 per cent and 1.1 per cent, respectively. Given that Canada is a net exporter of these goods, there is a negative terms-of-trade shock and a negative wealth effect, reducing Canadian consumption.
- The fall in the demand for factors of production decreases marginal cost, while the depreciation of the real exchange rate increases real import prices. Therefore, the year-over-year core inflation rate barely reacts (since the two effects are offsetting), which requires little action on the part of the monetary authority.

Three of the four effects are negative for Canadian economic activity, while the exchange rate effect is positive. In Canada, the first effect (the income effect) is very important, because the United States absorbs 82 per cent of Canadian exports. This effect, combined with the fall of oil and commodities prices, dominates the exchange rate effect so that Canadian exports and GDP drop by 0.39 and 0.10 percentage points, respectively. Therefore, the impact of the U.S. interest rate shock on Canadian GDP is about one quarter of that on U.S. GDP.

In RC (which is mainly the European Union and Japan), GDP increases slightly, because the effect of the depreciation of their currency dominates the income effect. The income effect is less important because the U.S. share of RC exports is only 40 per cent, compared with 82 per cent for Canada. Furthermore, RC is a net importer of oil and commodities. Therefore, shocks that decrease the prices of oil and commodities imply positive terms-of-trade and wealth effects, thereby stimulating consumption – the opposite of their effects in Canada.

Emerging Asia, because of their fixed nominal exchange rate relative to the United States, are importing U.S. monetary policy. Therefore, the nominal interest rate increases by the same amount as in the United States. Moreover, the effects of the shock are similar to those found in the United States.

5. Some Applications of the Model

This section describes some applications that require either a global model, a multi-sector model, or both. We examine four issues. First, we examine the effect of the multi-sector dimension on the link between productivity and the exchange rate (i.e., the Balassa-Samuelson effect). Second, in the context of the recent rise in the price of oil, we analyze the responses of the oil and commodities sectors to demand and supply shocks. Third, we use the model to analyze the impact of the economic emergence of China and India on imported goods, oil, and commodities prices. Fourth, in the context of the recent emergence of the global imbalances, we analyze shocks such as a rise in protectionism (as in Faruqee et al. 2007b) – and describe policies that could contribute to the resolution of global imbalances – a fiscal consolidation in the United States. Table 12 summarizes the key findings of the simulations.

5.1 Permanent productivity shocks in the United States, and the Balassa-Samuelson effect

In this section, we analyze the effect of the model's multi-sector dimension on the link between productivity and the exchange rate. In a one-good model, an increase in domestic productivity increases supply, reduces the prices of domestic goods, and induces a depreciation of the real effective exchange rate. This depreciation is necessary to sell abroad the additional supply of goods from the increase in domestic production. When the economy is multi-sector, the impact of a productivity shock on the real effective exchange rate depends on its source. If the productivity shock occurs in all the sectors of the domestic economy, then we get the same result as in a one-good model (a depreciation of the real exchange rate). In contrast, if the productivity shock is specific to the tradable sector of the domestic economy, then we expect the opposite effect in the long run – an appreciation of the real effective exchange rate. This effect is known as the Balassa-Samuelson effect (Balassa 1964; Samuelson 1964).

The Balassa-Samuelson effect works as follows. If labour is mobile across sectors (but not mobile across countries), a positive shock to the level of productivity in the tradable sector will raise the real wage in all sectors of the domestic economy, including the non-tradable sector. Consequently, the shock creates an increase in marginal cost and therefore in the price of the non-tradable good. On the one hand, the shock creates a drop in the price of tradable goods, but, on the other hand, the shock induces an increase in the price of non-tradables. If the elasticity of substitution between domestically produced and imported tradable goods is large enough, and if the non-tradable sector is more labour intensive than the tradable sector (as is normally the case in the data, and is the case in the BoC-GEM), then the increase in the price of the non-tradable good should dominate and the real effective exchange rate should appreciate.

Figure 14 compares the responses of the U.S. economy with two different domestic shocks: a permanent increase in productivity in all the sectors of the economy (with the exception of the oil sector), and a permanent increase in the productivity of the traded sectors (tradable goods and commodities).[41] The two sets of shocks are calibrated to have a similar output response. Both shocks reduce real marginal cost and the core inflation rate. Thus, the U.S. monetary authority decreases the nominal interest rate which, through the modified risk-adjusted UIRP condition, creates an initial depreciation of the U.S. dollar in both shocks. This is in spite of the Balassa-Samuelson effect for the shock exclusively in the traded-goods sectors. Nevertheless, after a delay of about two years, the Balassa-Samuelson effect dominates and, as expected, a permanent increase in the domestic productivity specific to the traded sectors induces a permanent appreciation of the domestic currency, while an increase in productivity in the entire economy creates a depreciation of the real effective exchange rate at every time horizon.

These results are mirrored in Figure 15 for Canada. Over the medium term, we see that Canada's bilateral exchange with the United States appreciates by about 0.4 per cent when

[41] We exclude the oil sector from the productivity shock to the traded sectors in order to clarify the results of the shocks. A change in productivity in the oil sector would induce large, temporary swings in oil prices (because of the very strong real adjustment costs in the short run) that would obscure and unnecessarily complicate the resulting conclusions made in this section.

the U.S. productivity shock is economy-wide, but it depreciates 0.4 per cent when the shock is only in U.S. traded goods. Canada's real effective exchange rate moves by less (around 0.2 per cent) in both cases. Because the United States is wealthier, it demands more Canadian exports in both shocks, but fewer under the U.S.-traded-goods-only shock, since the appreciating bilateral exchange rate in Canada serves to dampen its export demand. Overall, Canadian GDP increases, but in different amounts, because of the different responses of the exchange rate (over the medium term, 0.20 per cent in the U.S.-traded-goods-only productivity shock, and about 0.15 per cent when the U.S. productivity shock is economy-wide).

5.2 The oil and commodities sectors: demand and supply shocks

This section analyzes the responses of the oil and commodities sectors to demand and supply shocks. We use three different scenarios which exploit the links between the world economy and the prices of oil, gasoline, and commodities to illuminate the response of the model to an increase in the demand for oil and commodities. The first scenario is a temporary worldwide increase in consumption. It can be viewed as a world demand shock that creates a temporary increase in the demand for oil, gasoline, and commodities. The second scenario is a permanent increase in tradable and non-tradable productivity in the commodity-importing regions (US, RC, and AS), which generates a permanent increase in the demand for oil, gasoline, and commodities. Finally, we analyze the response of the model to a permanent reduction in the supply of oil of the commodity-exporting region (CX), and examine the effects on Canada, where the supply of crude oil reserves is not subject to an exogenous shock.

5.2.1 A temporary shock to world consumption

First we will consider a temporary increase in the level of consumption worldwide, as shown in Figure 16. The size of the shock is the same across all the regions and is calibrated to create an initial increase of about one percentage point of GDP in each region, increasing the demand for the components of consumption goods (gasoline, tradable goods, and non-tradable goods) for roughly two years. Because tradable and non-tradable goods are produced using oil and commodities, the demand for raw materials also increases. The demand for oil increases even more than the demand for commodities, because the rise in the demand for gasoline means that refineries need more oil for their production.

The increased demand for oil and commodities causes an upward movement along their supply curves, each of which has a very steep slope in the short run. In the case of oil, during the first two years, the slope of the short-run supply curve is almost vertical. Recall that the steep slopes are explained by the following factors:

- There is a fixed amount of crude oil reserves and land. The more the production of oil or commodities depends on its fixed factor, the less elastic is its supply curve.
- The long-run elasticity of substitution between factors of production is relatively low, limiting the speed and capability of the oil- and commodities-producing firms to adjust their levels of production.
- The presence of strong real adjustment costs in the production of oil and commodities reduces the elasticity of substitution between factors of production in the short and medium run. Adjustment costs are greater in the oil sector than in the commodities sector. The effect of these adjustment costs on the price of oil is even more pronounced, because oil-producing firms know in advance that the increase in oil demand is only temporary.

As a result, the 1 per cent increase in world GDP induces a rise in the real price of oil that peaks at close to 14 per cent, and it induces an increase of 3 per cent in the real price of commodities. The amplitude of these links between the world output gap and the real prices of oil and commodities is consistent with the estimation results of Lalonde, Zhu, and Demers (2003). Despite the increase in the real price of oil, the level of production of oil in Canada and CX increases by less than 0.1 per cent. The increase in commodities production is much larger (between 0.8 and 1.0 per cent, depending on the region). In general, the real price of gasoline follows closely the real price of oil, increasing by 10 per cent.

5.2.2 A permanent shock to the productivity of the tradable and non-tradable sectors of the commodity-importing regions (US, AS, and RC)

Another mechanism for a permanent rise in the real prices of oil and commodities is a permanent increase in the productivity of the tradable and non-tradable sectors of the three commodity-importing regions: US, AS, and RC (mainly the European Union and Japan). Figure 17 shows the effects of this shock on the two commodity-exporting regions (CX and CA).

In the commodity-importing regions, the shock is calibrated to create a permanent increase in GDP of 1.5 per cent. These regions increase their demand for all factors of production, including oil and commodities. Over the first two years, because of the real adjustment costs, there is a lack of response in the level of oil production, but a 9 per cent increase in the price of oil. Unlike a temporary demand shock, a permanent demand shock induces a gradual increase in the level of production, especially in Canada and CX, which limits the rise in the real price of oil and eventually contributes to a gradual reduction to a higher steady-state level. The shock elicits a similar, but more muted, short-run response in the commodities sector, because of the weaker real adjustment costs (relative to the oil sector). The real price of commodities increases by only 2.9 per cent.

In the commodity-exporting regions, the rise in the real prices of oil and commodities implies a positive terms-of-trade shock, an appreciation of their currency, and a reduction in the price of imported capital goods that increase potential output. The increase in the level of production is concentrated in the oil and commodities sectors. Furthermore, the positive wealth effect leads to higher consumption.

5.2.3 A permanent reduction in the supply of crude oil in the commodity exporter (CX)

The shock is an exogenous reduction in oil production of 6 percentage points in the main oil-exporting region (CX), but not in Canada (Figures 18 and 19). In other words, the near-vertical supply curve for oil in CX is shifting to the left. During the first years after the shock, the inelasticity of the demand for oil creates a 20 per cent increase in the price of oil. Outside CX, oil-producing firms respond to the increase in the internationally determined price of oil by increasing their level of production. In other words, given their marginal cost curves, some fields become profitable at the new price and can be exploited more fully by increasing capital and labour inputs. The response of the production of oil is particularly important for Canada, because it is the only oil exporter other than CX. The gradual and slow response of oil production outside CX, combined with some substitution of the production processes away from oil and consumption away from gasoline (i.e., switching from sport-utility vehicles to small cars), contributes, in the long run, to a reduction in the real price of oil relative to its peak.

In the oil-importing regions such as the United States, the long-lasting increase in the real price of oil implies a deterioration in the terms of trade, a depreciation of the real effective exchange rate, and a sizable negative wealth effect. Therefore, the shock reduces consumption, increases the relative price of imported investment goods, and reduces the general level of production. The increase in oil prices also raises the value of oil imports and leads to a deterioration in the U.S. current-account-to-GDP ratio by 0.20 percentage points. U.S. GDP falls by 0.23 percentage points. Given that we see the same sort of reaction in the European Union, Japan, and emerging Asia, world GDP falls by 0.25 percentage points (net of the gains in the oil-exporting regions).

In Canada, the shock leads to a positive terms-of-trade increase and an appreciation of its real effective exchange rate. This appreciation generates a positive wealth effect and reduces the relative price of imported goods, thereby raising consumption and imports. On the other hand, exports are affected negatively by the economic slowdown in the United States and in the other oil-importing regions. On net, Canadian GDP increases only slightly. While the increase in GDP partly reflects the increase in oil production, it is mainly driven by the drop in the relative price of imported investment goods. This is in line with previous quantitative research for Canada (for example, Stuber 2001).

As previously mentioned, the endogenous reaction of the world economy is very important for understanding the impact of the shock on Canadian GDP. The fall in exports cancels out a large share of the increase in consumption that results from the positive wealth effect. To illustrate this point, consider a similar shock in the commodities sector (Figure 20). This shock is a permanent reduction in the supply of commodities in the commodity exporting region (CX). Because the United States is a net exporter of commodities (mainly agricultural goods), in this scenario we see a slight increase in U.S. aggregate demand, instead of a reduction, as was the case with a negative shock to the world supply of oil. Thus, Canadian exports do not fall and, consequently, GDP increases substantially more than in the case of the oil-supply shock.

5.3 The impact of emerging Asia on the prices of imports, oil, and commodities

Figures 21 and 22 show the impact of rapid growth in emerging Asia on the prices of oil and commodities worldwide, and on the price of imported goods in the other regions. We focus on the effect on the U.S. economy (as a major importer of oil and commodities) and on the Canadian economy (as a major net exporter of oil and commodities). We generate a strong profile for growth in emerging Asia with a permanent increase in productivity of 5 per cent in both the tradable and non-tradable sectors. The shock creates a depreciation of emerging Asia's real effective exchange rate.

This productivity shock affects the other regions mainly through the following channels:

- In emerging Asia, the shock increases the demand for factors of production, including oil, investment goods, and commodities. Because of the important real adjustment costs in the production of oil and commodities, these prices increase substantially on impact. In the medium term, the production of oil and commodities adjusts upwards and generates a gradual reduction in their relative prices from their peaks.

- The shock reduces the relative price of consumption and investment goods in emerging Asia, which in turn decreases the relative prices of imported goods in the other regions. The permanent drop in the price of imported investment goods worldwide generates a permanent increase in potential GDP in the other regions. In addition, the reduction in the price of imported consumption goods generates a positive wealth effect, leading to higher consumption worldwide.

- The commodity-exporting regions export more oil and commodities to AS. This helps generate a positive wealth effect in CA and CX, allowing them to import more consumption goods from US and RC.

As shown in the previous section, the impact of the oil and commodities prices channel depends on whether a region is a net importer or a net exporter of these goods. For instance, for RC, this channel implies a negative terms-of-trade shock, a depreciation of the exchange rate, and a negative wealth effect that reduces consumption. In CA and CX, increases in the real prices of oil and commodities cause opposite effects. In contrast, for all the regions, the reduction in the relative price of imports (consumption and investment goods) generates a positive wealth effect, an increase in consumption, and a rise in potential output.

For oil-importing regions such as the United States, there are two wealth channels that have opposite effects on consumption and GDP. Initially, the negative wealth effect induced by the increase in the price of oil cancels out the positive wealth effect associated with the reduction in imported goods. As the price of oil gradually falls over time, the reduction in the prices of imported consumption and investment goods dominates, and both consumption and GDP increase permanently.

In Canada, the two wealth effects are positive for consumption and GDP. Therefore, the increase in consumption and GDP is greater and occurs more rapidly than in the United States. The permanent rise in Canadian production is explained by the fall of investment goods prices (i.e., an increase in potential GDP) and some increase in oil production.

As seen in section 5.1, because the shock affects both the tradable and the non-tradable sectors, it creates a depreciation in the real effective exchange rate in emerging Asia. In all the other regions, the real exchange rate is appreciating. As expected, the appreciation is larger in the oil-exporting regions (CA and CX) than in the oil-importing regions (US and RC). Therefore, the Canadian dollar appreciates relative to the U.S. dollar.

It is interesting to note that, outside emerging Asia, in spite of the reduction in the relative price of imported goods, the negative impact of the shock on the year-over-year core inflation rate is very small. Recall that the monetary authority is forward looking, and there is no uncertainty regarding the shock's source, persistence, and effect.

5.4 Shocks related to global imbalances

Next we analyze two scenarios related to the recent emergence of global imbalances: a rise in trade protectionism, and the extent to which a fiscal consolidation in the United States could help to eliminate global imbalances.

5.4.1 An increase in trade protectionism worldwide

One possibility that could arise in the current environment of global imbalances is increased protectionism: some in the United States argue in favour of raising tariffs against emerging Asia, since they use export-promotion policies to finance their holdings of U.S. assets (Faruqee et al. 2007b). The trade literature suggests that increases in tariffs by one region against another will benefit the region that imposes the tariff, but harm that region which is its target – the so-called "beggar-thy-neighbour policy." As past experience has shown (particularly the Great Depression), a beggar-thy-neighbour policy eventually escalates into a worldwide tariff war, and theory suggests that everyone loses in such an outcome. Figures 23 and 24 demonstrate that outcome in the BoC-GEM. However, we also use the BoC-GEM to demonstrate another possibility. Since Canada and the United States are separate regions in the world, we examine two cases. In the first case, all trading relationships break down worldwide, and there is a generalized increase in tariffs against tradable goods of 10 percentage points.[42] In the second case, we assume that NAFTA, the North American Free Trade Agreement, (or at least the Canada-United States portion of the agreement) survives unscathed, and the 10 percentage point hike in tariffs by Canada and the United States is against only the other three regions.

As we expect, in the first case we see that the real GDP of all regions falls, as does their level of imports. In the second case, we begin to see a difference for Canada and the United States. Since both countries still maintain trade with one another without any tariffs, GDP falls by less in both regions. Moreover, while overall trade in Canada and the United States falls, trade between the two regions actually increases, because import prices have less negative effects on consumption (since they rise less in CA and US when NAFTA continues to be in force). The real imports of CA from US are stronger if NAFTA is not broken in a worldwide tariff war, as are the real imports of US from CA (at least in the short to medium run). For Canada, the GDP loss is much smaller (0.9 per cent versus 3.5 per cent relative to control), since 80 per cent of Canadian trade is with the United States: the amount of its GDP affected by tariffs is much less if NAFTA remains in effect (about 7 per cent of its GDP, instead of around 37 per cent of its GDP for either imports or exports).[43] GDP in the United States falls less, as well, if NAFTA is maintained, but to a lesser extent than Canada (1.0 per cent with NAFTA versus 1.1 per cent). In this case, the change is much less – first, imports account only for 13.7 per cent of GDP, and second, only about 30 to 40 per cent of U.S. trade is with Canada This pattern also holds for consumption in both countries. In the case where NAFTA is maintained, the U.S. real effective exchange rate depreciates less (0.7 versus 0.8 per cent in the long run), and the Canadian real effective exchange rate appreciates substantially less (1.4 versus 2.3 per cent in the long run). This is almost exclusively a result of the much smaller movement in the bilateral Canadian–U.S.-dollar exchange rate.

For both scenarios of a global tariff war, we see similar effects across the other three regions: CX, AS, and RC. GDP falls substantially as consumption drops from higher import prices. Their net trade positions are not greatly changed, but this masks a huge fall in their real export and import volumes (around 6.5 per cent after the tariff war begins in CX, and around 10 per cent after the tariff war begins in both AS and RC). However, notice that their exports (and hence their levels of GDP) are almost identical whether or not NAFTA is maintained between Canada and the United States during a global tariff war. Maintaining NAFTA does

[42] Note that we do not impose any tariffs on raw materials, but that the results would not be substantively different in the long run. We also assume that all the new government revenues generated will go into government spending, and not domestic tax cuts.

[43] See section 3.3 on the calibration of international linkages, and Table 5.

not divert trade – it seems to create it between Canada and the United States, and both nations are definitely better off.[44]

This shock therefore demonstrates that a tariff war would be highly destructive, and that any maintenance of freer trade among any of the regions would leave the regions involved better off, as is the case when NAFTA remains in force.

5.4.2 A fiscal consolidation in the United States

In this section, we examine the impact of a fiscal consolidation in the United States on the current account imbalances (Figures 25 and 26). We consider a permanent and credible reduction in the U.S. government-debt-to-GDP ratio by 20 percentage points, which translates into a reduction in the U.S. deficit-to-GDP ratio by about 4 percentage points. The fiscal consolidation considered is financed through higher taxes on labour income. The link between the government debt and the desired net foreign asset position in the BoC-GEM means that the NFA position improves by 10 percentage points of GDP, which is an improvement in the U.S. current-account-to-GDP ratio of about 0.7 percentage points in the short run and 0.5 percentage points at the steady state. This leads to a depreciation of the real effective exchange rate of roughly 1.1 per cent at its peak seven to eight years after the beginning of the consolidation. In the short run, we see a large shift in the real net export position of the United States in order to finance the shift in the NFA position. After five years, export volumes are up by 2.2 per cent relative to control, while import volumes are down about 3.2 per cent.

The U.S. fiscal consolidation imposes an important cost for the U.S. economy, since the increase in the labour income tax rate reduces labour effort and, consequently, consumption and output. In fact, U.S. real GDP falls, on average, by about 1 per cent relative to control over the first eight to ten years. These factors lead to a fall in the world real U.S.-dollar price of oil, despite the offsetting effect of the depreciation of the U.S. dollar. Real commodity prices rise in U.S.-dollar terms, but are actually falling in value in other currencies. The different behaviour of the raw materials prices reflects the degree of real adjustment costs in demand (commodities demand adjusts more rapidly), and the fact that the United States has a smaller portion of world commodities consumption relative to its share of world oil consumption.

[44] This would be true even if we used a welfare measure based on consumption and leisure (and stated in terms of consumption), as in Faruqee et al. (2007b).

Despite the reduced U.S. demand, the world is, for the most part, better off. Canada and the commodity exporter gain less than the other regions, because of an increase in domestic currency earnings from commodities (the real U.S.-dollar commodity price increases coupled with notable currency appreciations in CA and CX) and oil (where currency appreciations in CA and CX overwhelm the small short-run fall in the real U.S.-dollar oil price).

Overall, although a fiscal consolidation in the United States helps to reduce the size of the U.S. current account imbalance, an attempt to reduce the U.S. federal debt by around 50 per cent relative to its current level has only a relatively small impact of 0.7 per cent of GDP, compared with the actual current account deficit of around 6 per cent of GDP, with notable short-run costs for the United States.

6. Conclusion

The BoC-GEM is a useful and necessary complement to the other models used at the Bank of Canada (ToTEM and MUSE). In particular, for issues that need a global and/or a multi-sectoral perspective, the BoC-GEM can be used to generate risk scenarios around the base-case projection generated by Bank staff using ToTEM and MUSE. Eventually, we would like to use the BoC-GEM annually or semi-annually to validate the consistency of the world projection underlying the staff's base-case projection scenario. More specifically, the projections for Canada, the United States, Europe, Japan, and the rest of the world are constructed using a mix of different models and consensus forecasts from outside the Bank of Canada. Up until the introduction of the BoC-GEM, the Bank staff did not have a formal tool with which to evaluate the consistency of the different projections done with different models. For issues that require an increasingly integrated global perspective, the BoC-GEM, with its fully articulated and consistent world framework, will be a useful tool to verify the consistency of the staff's global outlook. For instance, the BoC-GEM could be useful to make sure that the trade flows implied by the staff's projections are consistent with the forecasts for domestic demand and bilateral exchange rates of the different regions, while accounting for a global stock-flow perspective. This validation exercise is very ambitious, because it implies the (at least partial) use of the model in a real environment (i.e., with actual time-series data). Currently, all our work with the BoC-GEM has been around a steady state constructed with calibrated, but artificial, data, or a generated state of disequilibrium as the base case that approximates some salient features of the global and national economies.

The BoC-GEM is also a powerful and flexible tool for research, particularly for issues directly relevant to the baseline global projection used at the Bank of Canada. The number of sectors of production and regions can be easily reduced or adjusted to better fit the needs of a particular research project. Recently, the Bank staff have used smaller versions of the model to analyze issues like the cause and the effects of the recent increase in the world price of oil, and risks regarding global imbalances originating with the large current account deficits of the United States in recent years. In the near future, the International Department of the Bank of Canada plans to use the model to address issues such as the effect of emerging Asia on traded-goods prices, the relative merits of inflation and price-level-path targeting in a global framework, the relative merits of targeting headline versus core CPI inflation, exchange rate pass-through to inflation, and the implications of differing forms of trade protectionism in different regions of the world.

During the process of the above research agenda, we will be able to further improve the model along various dimensions. This includes fully operationalizing the distribution sector (introduced in Laxton and Pesenti 2003), which will allow us to break the long-run law of one price and thus be consistent with the data, and refining the calibration by adding more differentiation between regions. Eventually, it may be worthwhile to estimate a scaled-down Canada–United States version of the model using the Bayesian approach, building on Juillard et al. (2006).

References

Adolfson, M., S. Laséen, J. Lindé, and M. Villani. 2007. "Evaluating An Estimated New Keynesian Small Open Economy Model." CEPR Discussion Paper No. 6027.

Balassa, B. 1964. "The Purchasing Power Parity Doctrine: A Reappraisal." *Journal of Political Economy* 72(6): 584–96.

Bayoumi, T., D. Laxton, and P. Pesenti. 2004. "Benefits and Spillovers of Greater Competition in Europe: A Macroeconomic Assessment." NBER Working Paper No. 10416.

Botman, D., D. Laxton, D. Muir, and A. Romanov. 2006. "A New-Open-Economy-Macro Model for Fiscal Policy Evaluation." IMF Working Paper 06/45.

Brayton, F., E. Mauskopf, D. Reifschneider, P. Tinsley, and J. Williams. 1997. "The Role of Expectations in the FRB/US Macroeconomic Model." *Federal Reserve Bulletin* 83(4): 227–45.

Butler, L. 1996. *The Bank of Canada's New Quarterly Projection Model (QPM) Part 4 – A Semi-Structural Method to Estimate Potential Output: Combining Economic Theory With a Time-Series Filter.* Technical Report No. 77. Ottawa: Bank of Canada.

Calvo, G. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12(3): 383–98.

Cayen, J.-P., A. Corbett, and P. Perrier. 2006. "An Optimized Monetary Policy Rule for ToTEM." Bank of Canada Working Paper No. 2006–41.

Coenen, G., P. McAdam, and R. Straub. 2007. "Tax Reform and Labour-Market Performance in the Euro Area: A Simulation-Based Analysis Using the New Area-Wide Model." ECB Working Paper No. 747. *Journal of Economic Dynamics and Control.* Forthcoming.

Coletti, D., R. Lalonde, and D. Muir. 2007. "Inflation Targeting (IT) and Price-Level-Path Targeting (PLPT) in the GEM: Some Open Economy Considerations." *IMF Staff Papers.* Forthcoming.

Correia, I., J. Neves, and S. Rebelo. 1995. "Business Cycles in A Small Open Economy." *European Economic Review* 39: 1089–1113.

Corsetti, G. and P. Pesenti. 2001. "Welfare and Macroeconomic Interdependence." *Quarterly Journal of Economics* 116(2): 421–45.

Corsetti, G. and P. Pesenti. 2005. "The Simple Geometry of Transmission and Stabilization in Closed and Open Economies." NBER Working Paper No. 11341.

De Walque, G., F. Smets, and R. Wouters. 2006. "An Estimated Two-Country DSGE Model for the Euro Area and the U.S. Economy." Paper presented at the Bank of Canada Workshop on Commodity Price Issues in Ottawa, Canada, July 2006.

Duarte, M. and A. Stockman. 2005. "Rational Speculation and Exchange Rates." *Journal of Monetary Economics* 52(1): 3–29.

Elekdag, S., R. Lalonde, D. Laxton, D. Muir, and P. Pesenti. 2007. "Oil Price Movements and the Global Economy: A Model-Based Assessment." Bank of Canada Working Paper No. 2007-34.

English, W.B., W.R. Nelson, and B.P. Sack. 2003. "Interpreting the Significance of the Lagged Interest Rate in Estimated Monetary Policy Rules." *Contributions to Macroeconomics* 3(1): Article 5.

Erceg, C., L. Guerrieri, and C. Gust. 2005a. "Expansionary Fiscal Shocks and the Trade Deficit." Board of Governors of the Federal Reserve System International Finance Discussion Paper No. 825.

Erceg, C., L. Guerrieri, and C. Gust. 2005b. "SIGMA: A New Open Economy Model for Policy Analysis." Board of Governors of the Federal Reserve System International Finance Discussion Paper No. 835.

Faruqee, H., D. Laxton, D. Muir, and P. Pesenti. 2007a. "Smooth Landing or Crash? Model-Based Scenarios of Global Current Account Rebalancing." In *G7 Current Account Imbalances: Sustainability and Adjustment,* edited by R. Clarida, NBER conference volume: University of Chicago Press.

Faruqee, H., D. Laxton, D. Muir, and P. Pesenti. 2007b. "Would Protectionism Defuse Global Imbalances and Spur Economic Activity? A Scenario Analysis." *Journal of Economic Dynamics and Control* Forthcoming.

Galí, J. and M. Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics* 44(2): 195–222.

Goodfriend, M. and R. King. 1997. "The New Neoclassical Synthesis and the Role of Monetary Policy." In *NBER Macroeconomics Annual 1997* edited by B. Bernanke and J. Rotemberg. Cambridge, Mass.: MIT Press.

Gosselin, M.-A. and R. Lalonde. 2005. *MUSE: The Bank of Canada's New Projection Model of the U.S. Economy*. Technical Report No. 96. Ottawa: Bank of Canada.

Greenwood, J., Z. Hercowitz, and G. Huffman. 1988. "Investment, Capacity Utilization, and the Business Cycle." *American Economic Review* 78(3): 402–17.

Hunt, B. 2005. "Oil Price Shocks: Can They Account for Stagflation in the 1970s?" IMF Working Paper 05/215.

Jean, S. and G. Nicoletti. 2002. "Product Market Regulation and Wage Premia in Europe and North America: An Empirical Investigation." OECD Economics Department Working Paper No. 318.

Juillard, M., P. Karam, D. Laxton, and P. Pesenti. 2006. "Welfare-Based Monetary Policy Rules in an Estimated DSGE Model of the U.S. Economy." ECB Working Paper No. 613.

King, R., C. Plosser, and S. Rebelo. 1988. "Production, Growth and Business Cycles: 1. The Basic Neoclassical Model." *Journal of Monetary Economics* 21: 195–232.

Lalonde, R., Z. Zhu, and F. Demers. 2003. "Forecasting and Analyzing World Commodity Prices." Bank of Canada Working Paper No. 2003-24.

Lane, P. 2001. "The New Open Economy Macroeconomics: A Survey." *Journal of International Economics* 54(2): 235–66.

Lane, P. and G.M. Milesi-Ferretti. 2006. "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004." IMF Working Paper 06/69.

Laxton, D. and P. Pesenti. 2003. "Monetary Policy Rules for Small, Open, Emerging Economies." *Journal of Monetary Economics* 50(2): 1109–46.

Martins, J.O., S. Scarpetta, and D. Pilat. 1996. "Markup Pricing, Market Structure and the Business Cycle." *OECD Economic Studies* 27 (II): 71–106.

Murchison, S. and A. Rennison. 2006. *ToTEM: The Bank of Canada's New Projection and Policy Analysis Model.* Technical Report No. 97. Ottawa: Bank of Canada.

Neiss, K. and E. Nelson. 2003. "The Real-Interest-Rate Gap as an Inflation Indicator." *Macroeconomic Dynamics* 7(2): 239–62.

Perrier, P. 2005. "La fonction de production et les données canadiennes." Bank of Canada Working Paper No. 2005-20.

Pesenti, P. 2007. "The Global Economy Model (GEM): Theoretical Framework." *IMF Staff Papers.* Forthcoming.

Piretti, A. and C. St-Arnaud. 2006. "Launching the NEUQ: The New European Union Quarterly Model: A Small Model of the Euro Area and the U.K. Economies." Bank of Canada Working Paper No. 2006-22.

Rotemberg, J. 1982. "Sticky Prices in the United States." *Journal of Political Economy* 90(6): 1187–1211.

Samuelson, P. 1964. "Theoretical Notes on Trade Problems." *Review of Economics and Statistics* 46(2): 145–54.

Schmitt-Grohé, S. and M. Uribe. 2003. "Closing Small Open Economy Models." *Journal of International Economics* 61(1): 163–85.

Smets, F. and R. Wouters. 2005. "Comparing Shocks and Frictions in U.S. and Euro Area Business Cycles: A Bayesian DSGE Approach." *Journal of Applied Econometrics* 20 (2): 161–83.

Stuber, G. 2001. "The Changing Effects of Energy-Price Shocks on Economic Activity and Inflation." *Bank of Canada Review* (Summer): 3–14.

Taylor, J. 1980. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88(1): 1–23.

Tinsley, P.A. 1993. "Fitting Both Data and Theories: Polynomial Adjustment Costs and Error-Correction Decision Rules." Board of Governors of the Federal Reserve System Finance and Economics Discussion Series No. 93-21.

Whelan, K. 2000. "A Guide to the Use of Chain Aggregated NIPA Data." Board of Governors of the Federal Reserve System Finance and Economics Discussion Series No. 2000-35.

Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton: Princeton University Press.

Table 1: Key Parameters of the GEM

Parameter	CA	US	CX	AS	RC
Rate of time preference $\left(1/\beta^4 - 1\right) * 100$	1.9	1.9	1.9	1.9	1.9
Depreciation rate δ	0.02	0.02	0.02	0.02	0.02
Intertemporal elasticity of substitution $1/\sigma$	0.70	0.70	0.70	0.70	0.70
Habit persistence in consumption b_c	0.85	0.85	0.85	0.85	0.85
Frisch elasticity of labour $1/\zeta$	0.20	0.20	0.20	0.20	0.20
Share of liquidity-constrained consumers SLC	0.20	0.20	0.20	0.50	0.20
Habit persistence in labour b_ℓ	0.75	0.75	0.75	0.75	0.75

CA = CAnada, US =United States, CX = Commodity-eXporter,
AS = emerging ASia, and RC = Remaining Countries (including Japan and E.U.)

Table 2: Parameterization of the Production Functions

Parameter	CA	US	CX	AS	RC
Tradable Intermediate Goods					
Substitution between factors of production ξ_T	0.70	0.70	0.70	0.70	0.70
Bias towards capital α_{KT}	0.67	0.71	0.72	0.82	0.73
Bias towards oil α_{OT}	0.06	0.04	0.07	0.03	0.04
Bias towards commodities α_{ST}	0.15	0.08	0.16	0.07	0.10
Non-tradable Intermediate Goods					
Substitution between factors of production ξ_N	0.70	0.70	0.70	0.70	0.70
Bias towards capital α_{KN}	0.68	0.59	0.65	0.73	0.68
Bias towards oil α_{ON}	0.04	0.03	0.06	0.03	0.02
Bias towards commodities α_{SN}	0.11	0.05	0.12	0.05	0.06
Oil					
Substitution between factors of production ξ_O	0.60	0.60	0.60	0.60	0.60
Bias towards capital α_{KO}	0.68	0.53	0.12	0.44	0.59
Bias towards crude oil reserves (OIL) α_{OILO}	0.32	0.46	0.87	0.53	0.40
Gasoline					
Substitution between factors of production ξ_{GAS}	0.70	0.70	0.70	0.70	0.70
Bias towards capital α_{KGAS}	0.45	0.47	0.42	0.43	0.46
Bias towards oil α_{OGAS}	0.54	0.51	0.57	0.52	0.53
Commodities					
Substitution between factors of production ξ_S	0.60	0.60	0.60	0.60	0.60
Bias towards capital α_{KS}	0.50	0.38	0.23	0.37	0.36
Bias towards natural resources $(LAND)$ α_{LANDS}	0.49	0.60	0.77	0.61	0.63

Table 3: Price and Wage Markups (Per cent)

Parameter	CA	US	CX	AS	RC
Tradables					
Markup $\theta_T/(\theta_T-1)$	1.20	1.15	1.18	1.14	1.20
θ_T	6.00	7.67	7.67	8.00	5.89
Non-tradables					
Markup $\theta_N/(\theta_N-1)$	1.42	1.28	1.23	1.25	1.38
θ_N	3.38	4.58	4.58	5.00	3.63
Oil					
Markup $\theta_O/(\theta_O-1)$	1.01	1.01	476	1.01	1.01
θ_O	100	100	1.21	100	100
Gasoline					
Markup $\theta_{GAS}/(\theta_{GAS}-1)$	1.16	1.17	1.17	1.17	1.16
Markup and Taxes	1.50	1.34	1.34	1.34	1.50
θ_{GAS}	7.15	7.00	7.00	7.00	7.15
Wages					
Markup $\psi/(\psi-1)$	1.20	1.16	1.30	1.16	1.30
ψ	6.00	7.30	4.30	7.30	4.30

Table 4: Steady-State National Accounts - Production Side

Ratio of GDP	CA	US	CX	AS	RC
Tradables	43.3	40.1	35.3	50.9	41.1
Non-tradables	53.5	56.0	62.3	46.5	56.1
Oil	7.3	2.0	11.9	2.8	2.2
Commodities	8.0	3.3	6.6	4.1	2.1
Factor Incomes (% share of oil)					
Capital	27.6	19.6	10.8	20.5	25.2
Labour	14.0	11.2	10.1	24.5	11.5
Crude oil	58.4	69.2	79.1	55.0	63.3
Factor Incomes (% share of commodities)					
Capital	25.1	21.9	19.0	23.3	22.2
Labour	20.8	23.1	21.8	22.0	25.0
Land	54.0	55.1	59.2	54.7	52.9

Note: Columns will not sum to 100, since the measures include both final goods (included in GDP) and intermediate goods (not included in GDP).

Table 5: Steady-State National Accounts - Expenditure Side (Per cent of domestic GDP)

Ratio of GDP	CA	US	CX	AS	RC
Total Consumption $A + P_N G_N$	80.8	81.4	83.4	69.0	81.1
Private C	57.3	66.3	66.3	54.7	60.5
Public $G_C + P_N G_N$	23.5	15.2	17.1	14.3	20.7
Total Investment $P_E E$	19.1	18.0	16.9	31.4	19.1
Private $P_E I$	16.6	16.0	14.9	29.9	16.6
Public $P_E G_I$	2.5	2.0	2.0	1.5	2.5
Trade balance $TBAL$	0.1	0.5	-0.2	-0.4	-0.2
Imports IM	37.0	13.7	24.0	26.5	9.0
Consumption Goods $P_{MA} MA$	19.9	7.0	10.7	8.1	3.2
Investment Goods $P_{ME} ME$	11.6	4.0	8.4	12.5	3.2
Oil $P_{OT} MOT + P_{ON} MON + P_{OGAS} MOGAS$	2.2	1.7	1.7	2.8	1.2
Commodities $P_{MS}(MST + MSN)$	3.3	1.0	3.1	3.0	1.4
Oil Demand $P_{OT} OT + P_{ON} ON + P_{OGAS} OGAS$	3.7	3.5	3.9	4.8	2.7
Gasoline Demand $P_{GAS}(1 + \tau_{GAS}) GAS$	3.1	3.3	2.7	3.0	3.0
Commodities Demand $P_{ST} ST + P_{SN} SN$	4.6	2.6	4.5	5.2	2.9
Government Debt B	25.0	50.0	15.0	24.0	67.0
Net Foreign Assets B^*	-7.5	-50.0	21.4	35.0	20.0
Share of World GDP (per cent)	2.4	30.1	9.3	10.6	47.7

Table 6: Parameterization of the Final Demand Functions

Parameter	CA	US	CX	AS	RC
Final Consumption Goods					
Substitution between domestic and imported goods μ_A	1.5	1.5	1.5	1.5	1.5
Bias towards domestic goods ν_A	0.36	0.80	0.50	0.34	0.92
Substitution between domestic tradables and non-tradables ε_A	0.5	0.5	0.5	0.5	0.5
Bias towards tradable goods γ_A	0.65	0.66	0.67	0.47	0.66
Substitution between gasoline and the rest ε_{GAS}	0.3	0.3	0.3	0.3	0.3
Bias towards gasoline γ_A	0.13	0.15	0.11	0.06	0.16
Final Investment Goods					
Substitution between domestic and imported goods μ_E	1.5	1.5	1.5	1.5	1.5
Bias towards domestic goods ν_E	0.17	0.69	0.13	0.23	0.80
Substitution between domestic tradables and non-tradables ε_E	0.5	0.5	0.5	0.5	0.5
Bias towards tradable goods γ_E	0.81	0.76	0.81	0.84	0.76
Demand for Oil in Production					
Substitution between domestic and imported oil μ_{OT}, μ_{ON}, μ_{OGAS}	10.0	10.0	10.0	10.0	10.0
Bias towards domestic oil for producing tradables ν_{OT}	0.07	0.00	0.00	0.83	0.96
Bias towards domestic oil for producing non-tradables ν_{ON}	0.04	0.00	0.00	0.56	0.90
Bias towards domestic oil for producing gasoline ν_{OGAS}	0.22	0.00	0.00	0.97	0.99
Demand for Commodities in Production					
Substitution between domestic and imported commodities μ_{ST}, μ_{SN}	5.0	5.0	5.0	5.0	5.0
Bias towards domestic commodities for producing tradables ν_{ST}	0.41	0.79	0.01	0.01	0.78
Bias towards domestic commodities for producing non-tradables ν_{SN}	0.50	0.70	0.02	0.00	0.65

Table 7: Distribution of Crude Oil Reserves Around the World (Per cent)

Parameter	CA	US	CX	AS	RC
Calibration	14	6	75	4	2
Data	11	4	81	3	1

Table 8: Per Cent Share of Gasoline in Consumption and GDP

Gasoline as a % share of	CA	US	CX	AS	RC
Consumption	5.4	4.9	4.1	4.9	4.1
GDP	3.1	3.3	2.7	3.0	3.1

Table 9: Real Adjustment Costs and Rigidities

Parameter	CA	US	CX	AS	RC
Real Adjustment Costs					
Capital accumulation ϕ_{I1}	1.00	1.00	1.00	1.00	1.00
Investment changes ϕ_{I2}	100	100	100	100	160
Imports of consumption goods ϕ_{MA}	0.95	0.95	0.95	0.95	0.95
Imports of investment goods ϕ_{ME}	0.95	0.95	0.95	0.95	0.95
Real Adjustment Costs in the Oil and Gasoline Sectors					
Capital for producing oil ϕ_{KO}	400	300	200	300	300
Capital for producing gasoline ϕ_{KGAS}	500	500	500	500	500
Labour for producing oil ϕ_{LO}	400	300	200	300	300
Labour for producing gasoline ϕ_{LGAS}	500	500	500	500	500
Demand for oil in production. ϕ_{OT}, ϕ_{ON}, ϕ_{OGAS}	300	300	300	300	300
Real Adjustment Costs in the Commodities Sector					
Capital for producing commodities ϕ_{KS}	200	200	200	200	200
Labour for producing commodities ϕ_{LS}	200	200	200	200	200
Demand for commodities in production. ϕ_{ST}, ϕ_{SN}	200	200	200	200	200

Table 10: Nominal Rigidities

Parameter	CA	US	CX	AS	RC
Weight on past versus steady-state inflation	0.5	0.5	0.5	0.5	0.5
Wages for liquidity-constrained consumers ϕ_{WLC}	900	800	800	800	1050
Wages for forward-looking consumers ϕ_{WFL}	900	800	800	800	1050
Price of domestically produced tradables ϕ_{PQ}	700	700	700	700	900
Price of non-tradables ϕ_{PN}	700	700	700	700	900
Price of imported intermediate goods ϕ_{PM}	4000	4000	4000	4000	4000

Table 11: Monetary Policy Reaction Function Parameters

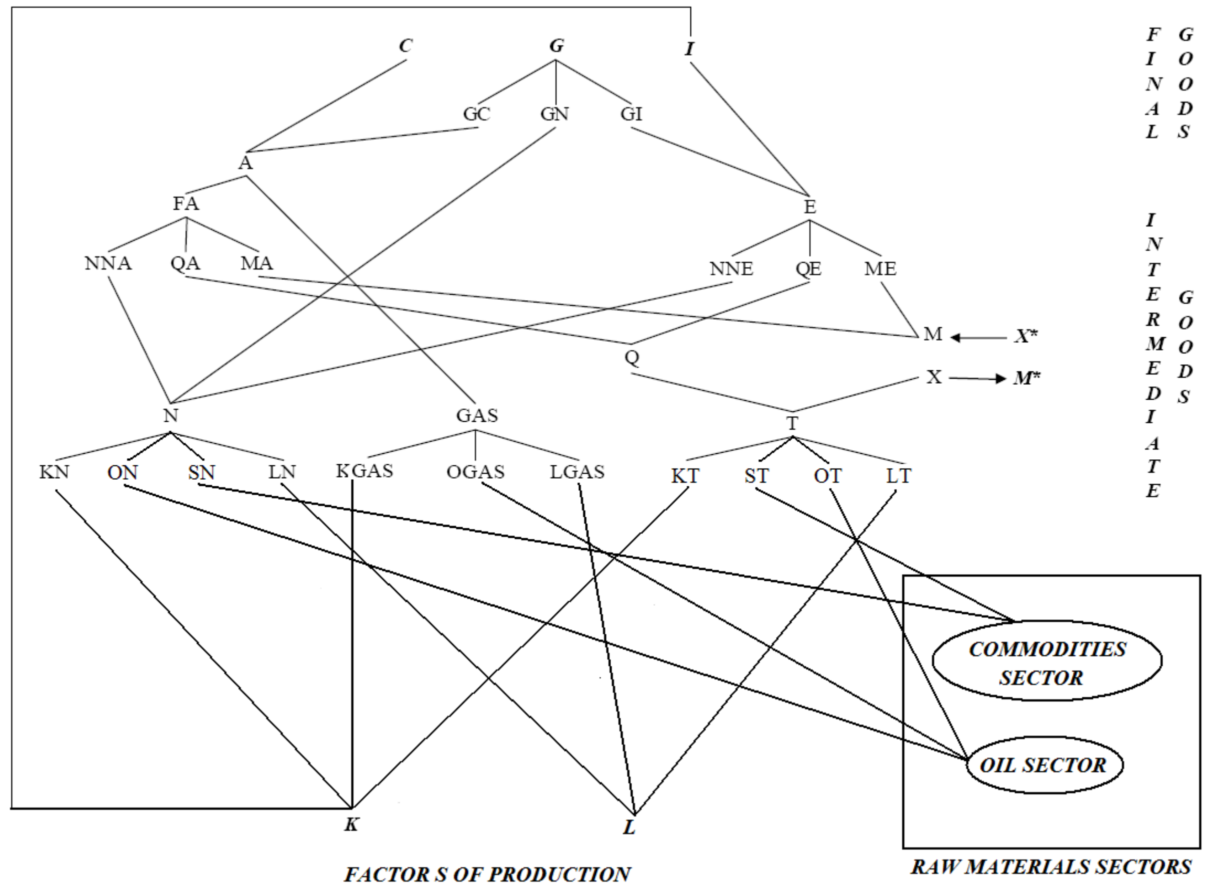
Parameter	IFB Rule	Fixed Exchange Rate Regime
Lagged interest rate at $t-1$ ω_i	0.95	1.00
Year-over-year core inflation gap at $t+3$ ω_1	0.90	0.00
Contemporaneous output gap ω_2	0.00	0.00
Change in the nominal exchange rate at t ω_3	0.00	1000000 (proxy for ∞)

Table 12: Summary of the Key Findings of the Simulations Using BoC-GEM

Simulations	*Key findings*
5.1 - Permanent productivity shocks in the United States	Illustrates the Balassa-Samuelson effect.
Permanent increase in productivity in the traded and non-traded sectors	Depreciation of the U.S. dollar.
Permanent increase in productivity in the traded sectors only	Appreciation of the U.S. dollar (because of the Balassa-Samuelson effect).
5.2 - Demand and supply shocks to the oil and commodities sectors	Outlines the properties of two sectors unique to BoC-GEM (relative to other versions of GEM).
Temporary demand shock (1% world consumption shock)	Temporary increase in the real prices of oil (13%) and commodities (3%).
Permanent demand shock (permanent increase in productivity in the commodity-importing regions)	Peak increase of 9% and 2.9% in the real prices of oil and commodities, respectively. Some long-run adjustment in the supply of oil and commodities.
Permanent reduction in the supply of oil in the commodity exporter	Real price of oil peaks at +20%. In Canada, a positive wealth effect and an increase in consumption; the opposite occurs for the United States. A drop of 0.3% in world GDP.
5.3 - Impact of emerging Asia on world prices (via a permanent increase in productivity)	Drop of consumption and investment prices worldwide. Increase in oil and commodity prices creates a positive wealth effect in Canada.
5.4 - Shocks related to global imbalances	Explores some topics of current interest in a global context.
Permanent increase in tariffs worldwide (including a collapse of NAFTA)*	Large permanent loss in output worldwide.
Permanent increase in tariffs worldwide, except that NAFTA is maintained	Large permanent loss in output worldwide, except in Canada, where the fall in GDP is relatively small.
Impact of a U.S. fiscal consolidation (permanent 50% reduction in the U.S. federal debt)	U.S. current-account-to-GDP ratio improves by 0.7 percentage points.

*NAFTA: North American Free Trade Agreement

Figure 1: Structure of the BoC-GEM - Overview of the Production Sectors



C is private consumption; G is government spending; I is business investment;

A is aggregate consumption; E is aggregate investment;

N is non-tradable goods; GAS is gasoline; T is tradable goods;

Q is domestically produced goods; M is imported goods; X is exported goods;

K is the capital stock; L is labour; S is commodities; O is oil.

Other symbols are combinations; i.e., QE (Q and E together) is domestically produced investment goods.

For Figures 2 and 3, OIL is crude oil reserves and LAND is the fixed factor for commodities.

Figure 2: Structure of the BoC-GEM — The Oil Sector

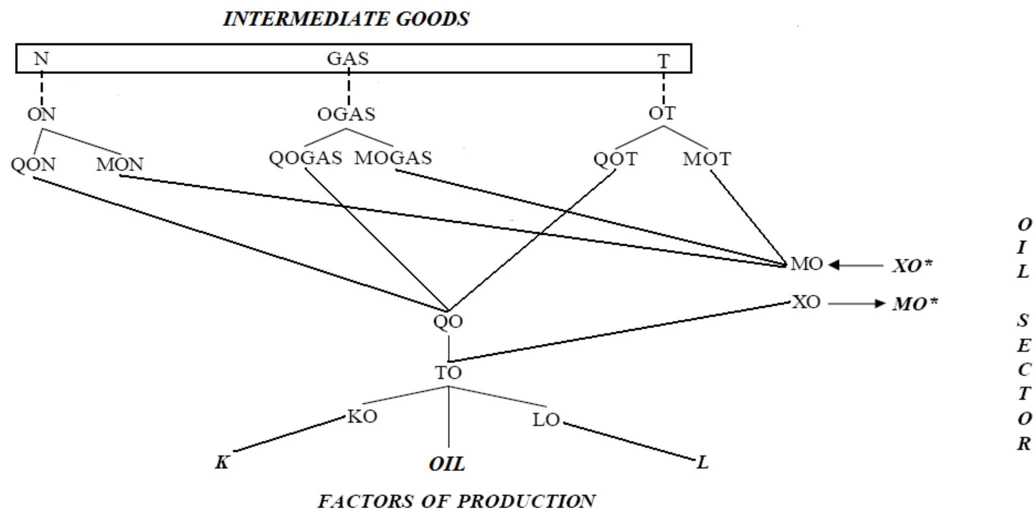


Figure 3: Structure of the BoC-GEM — The Commodities Sector

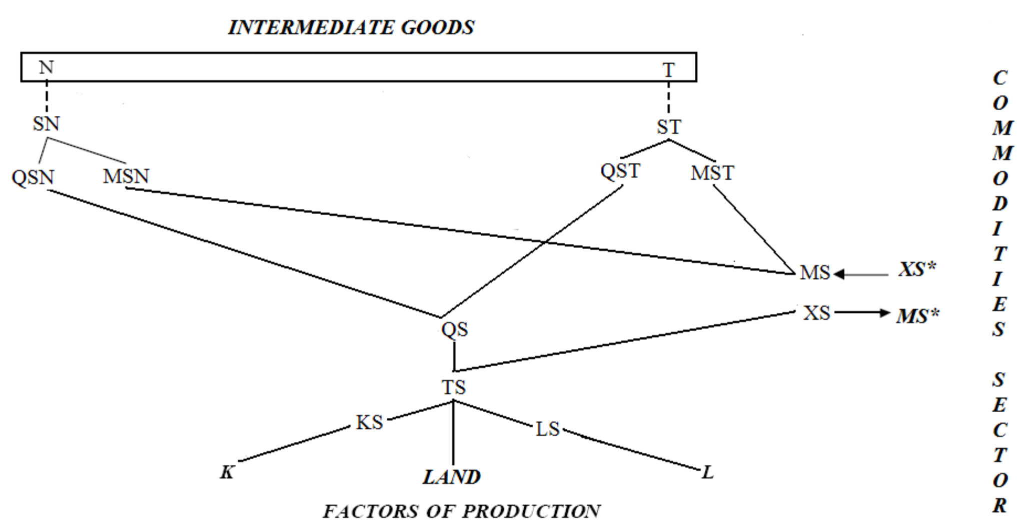
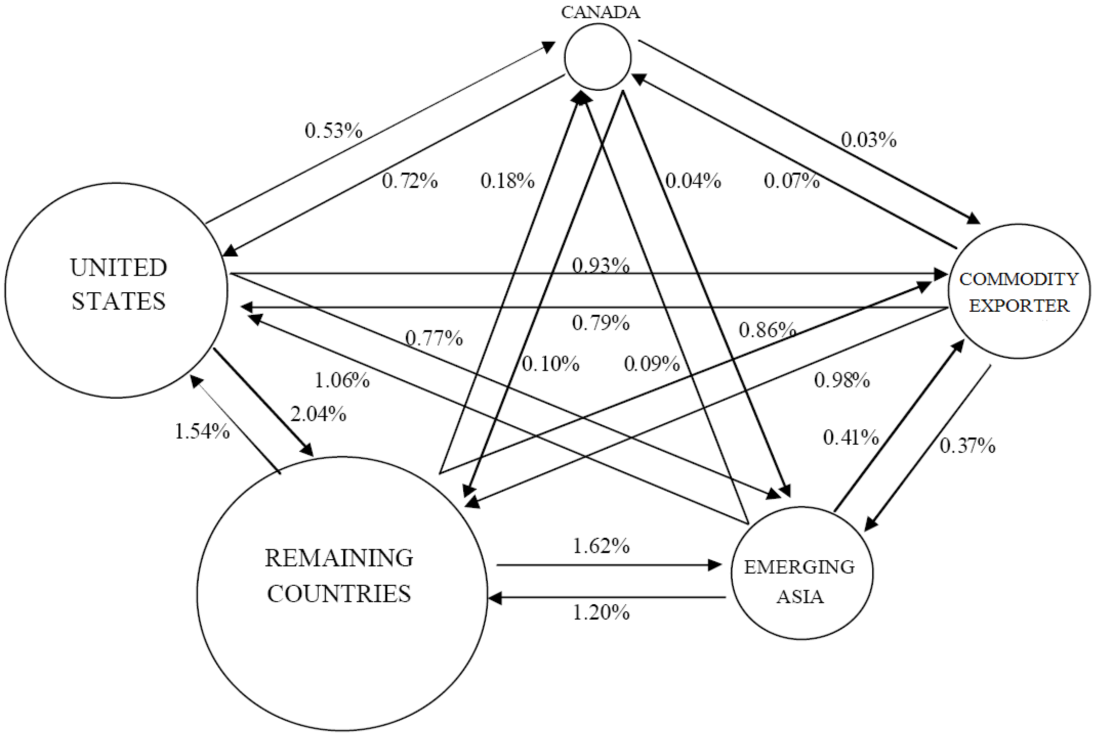
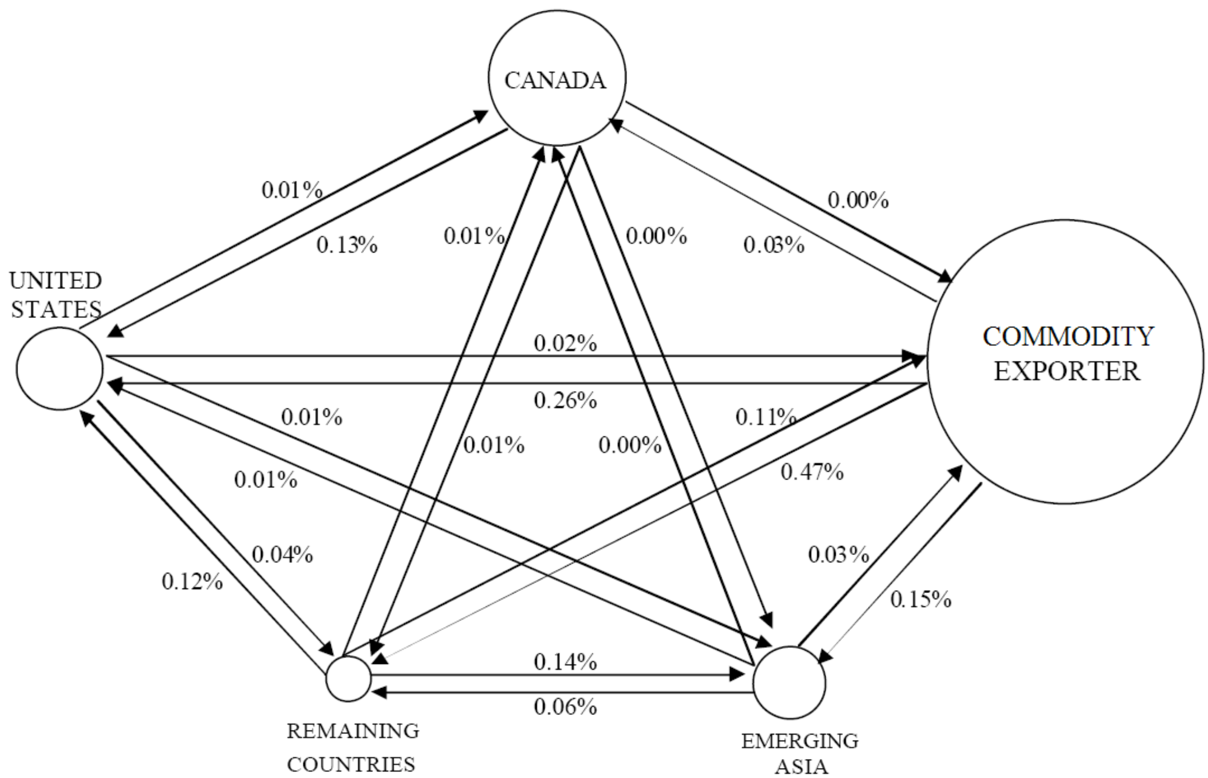


Figure 4: Global Bilateral Trade Flows — All Goods (Percentage of World GDP)



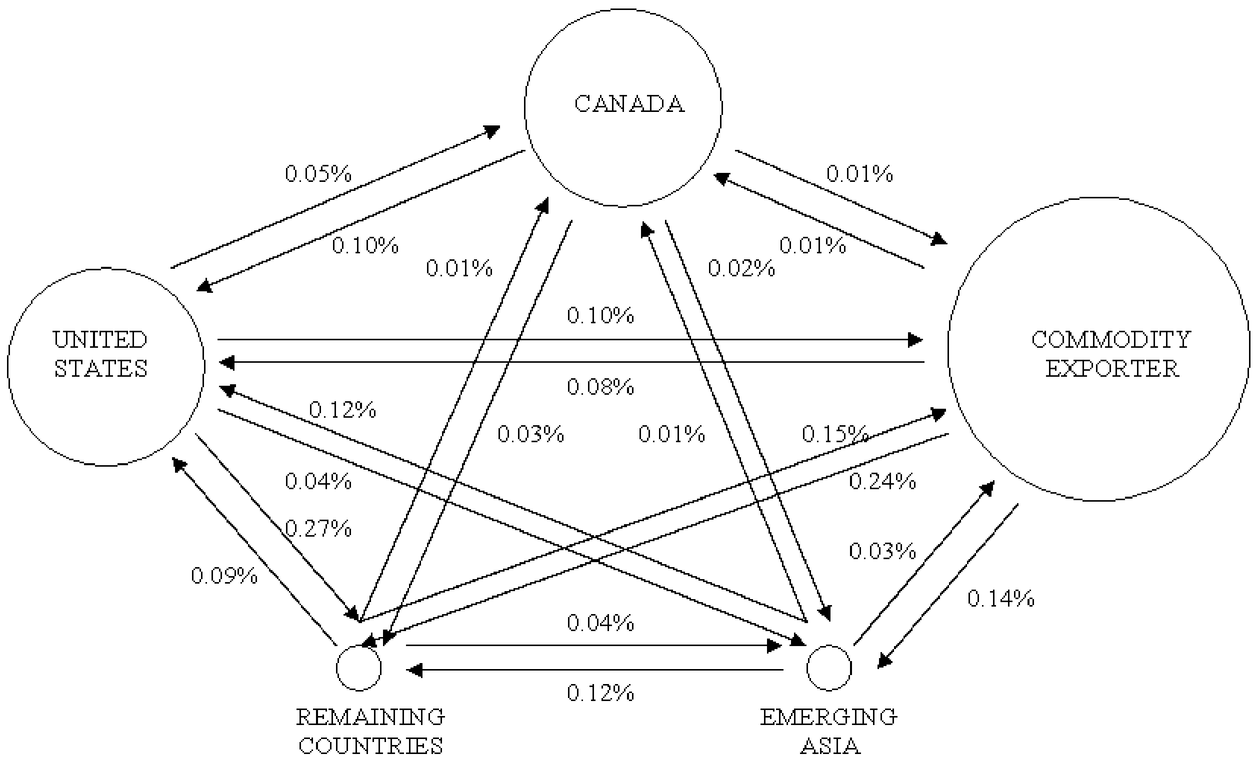
Circle sizes represent the share of global GDP that is held by each region.

Figure 5: Global Bilateral Trade Flows — The Oil Sector (Percentage of World GDP)



Circle sizes represent the share of world crude oil reserves held in each region.

Figure 6: Global Bilateral Trade Flows — The Commodities Sector (Percentage of World GDP)



Circle sizes represent the share of world commodities (mining resources, forests, fish stocks, arable land, etc.) notionally held in each region.

Figure 7: A Stylized Representation of the Crude Oil Reserves Market

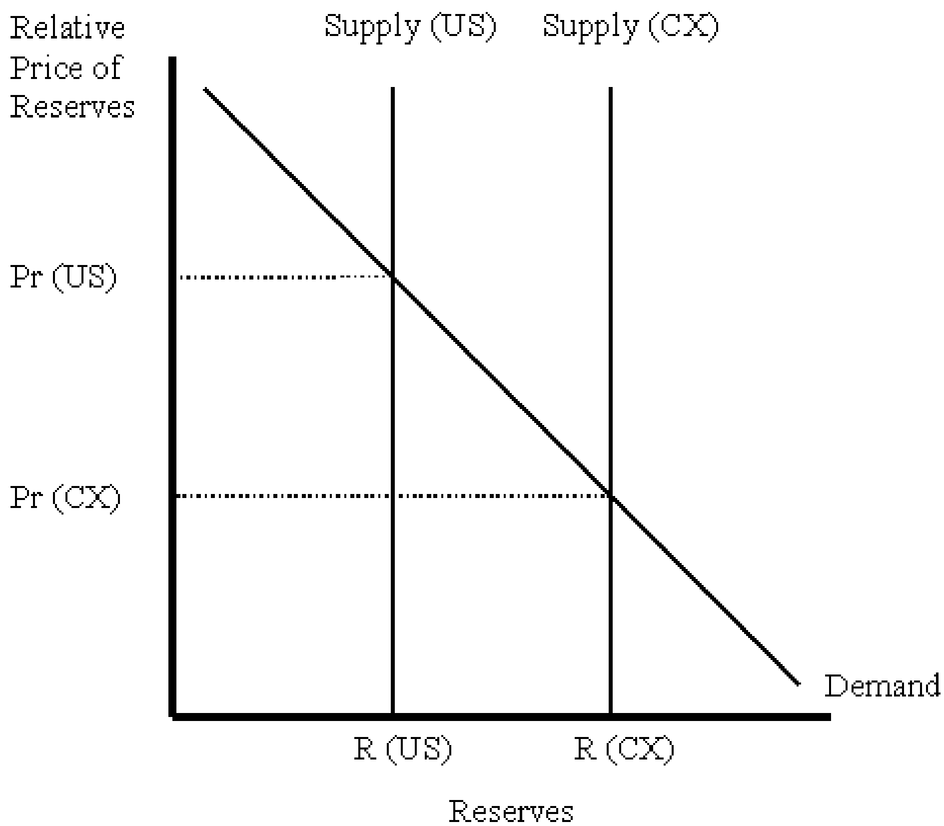


Figure 8: A Temporary Increase in Consumption in Canada

(Deviation from control, in per cent)

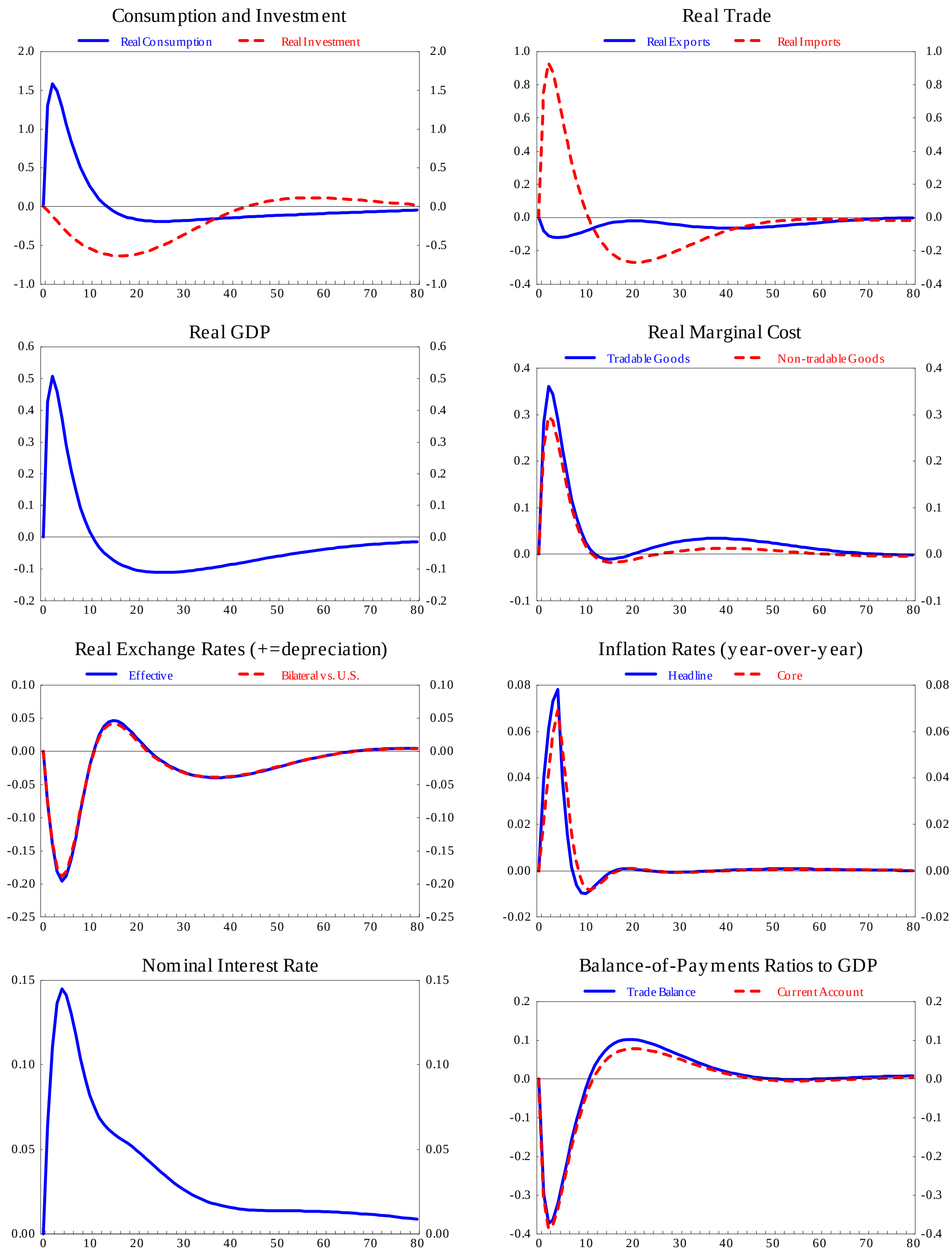


Figure 9: A Temporary 100 Basis Point Increase in the Canadian Interest Rate

(Deviation from control, in per cent)

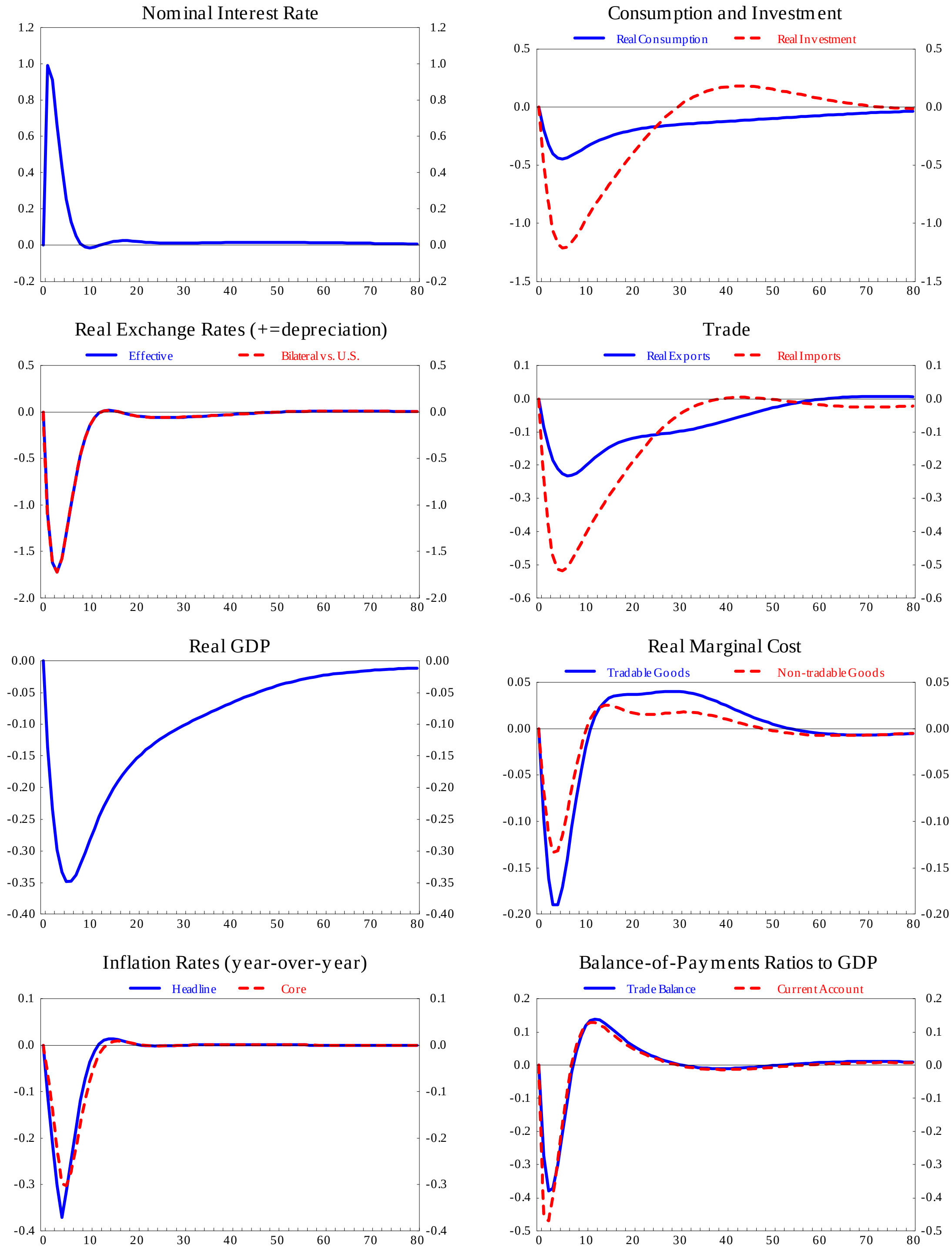


Figure 10: A Temporary Increase in Consumption in the United States — U.S. Effects

(Deviation from control, in per cent)

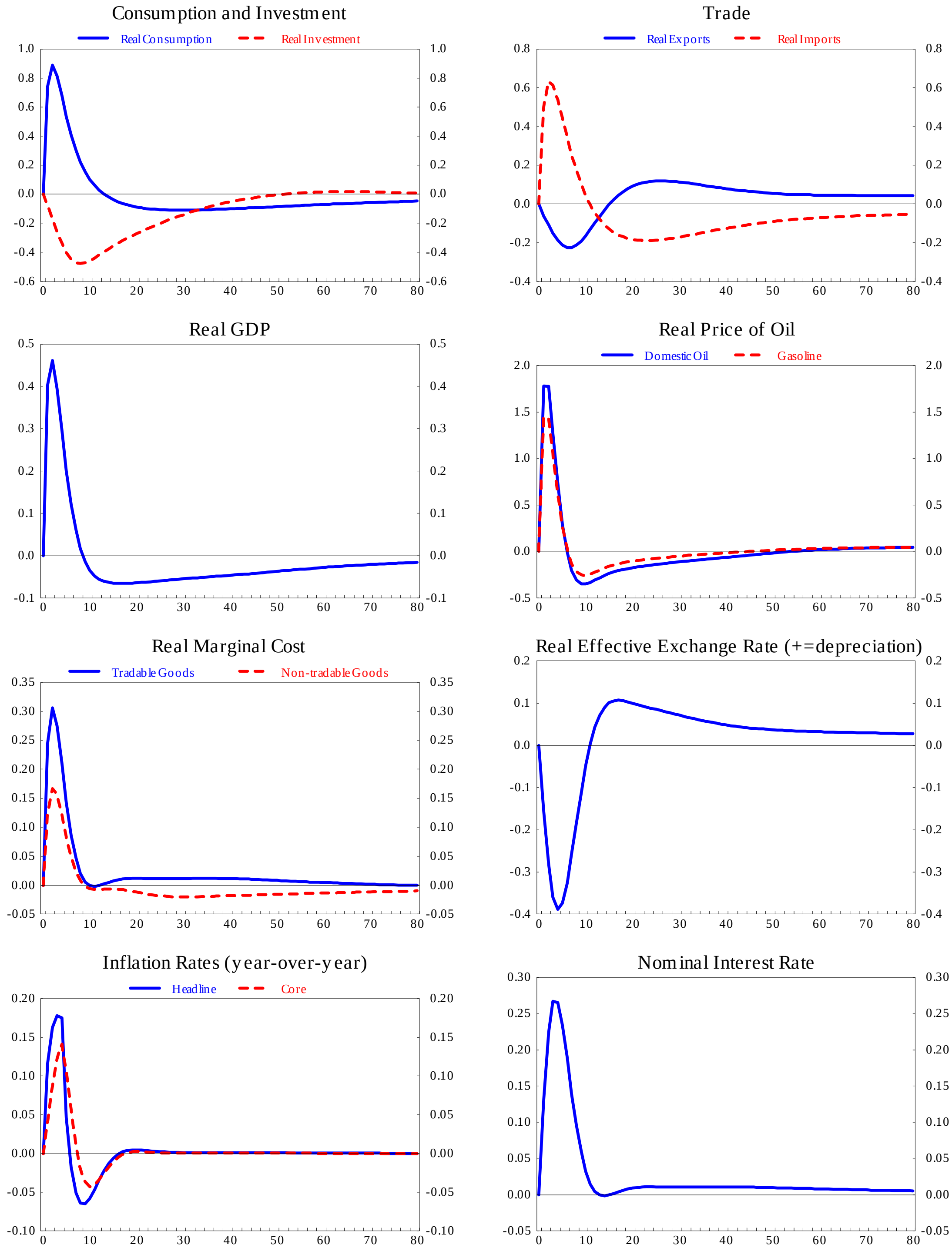


Figure 11: A Temporary Increase in Consumption in the United States — Effects on Canada

(Deviation from control, in per cent)

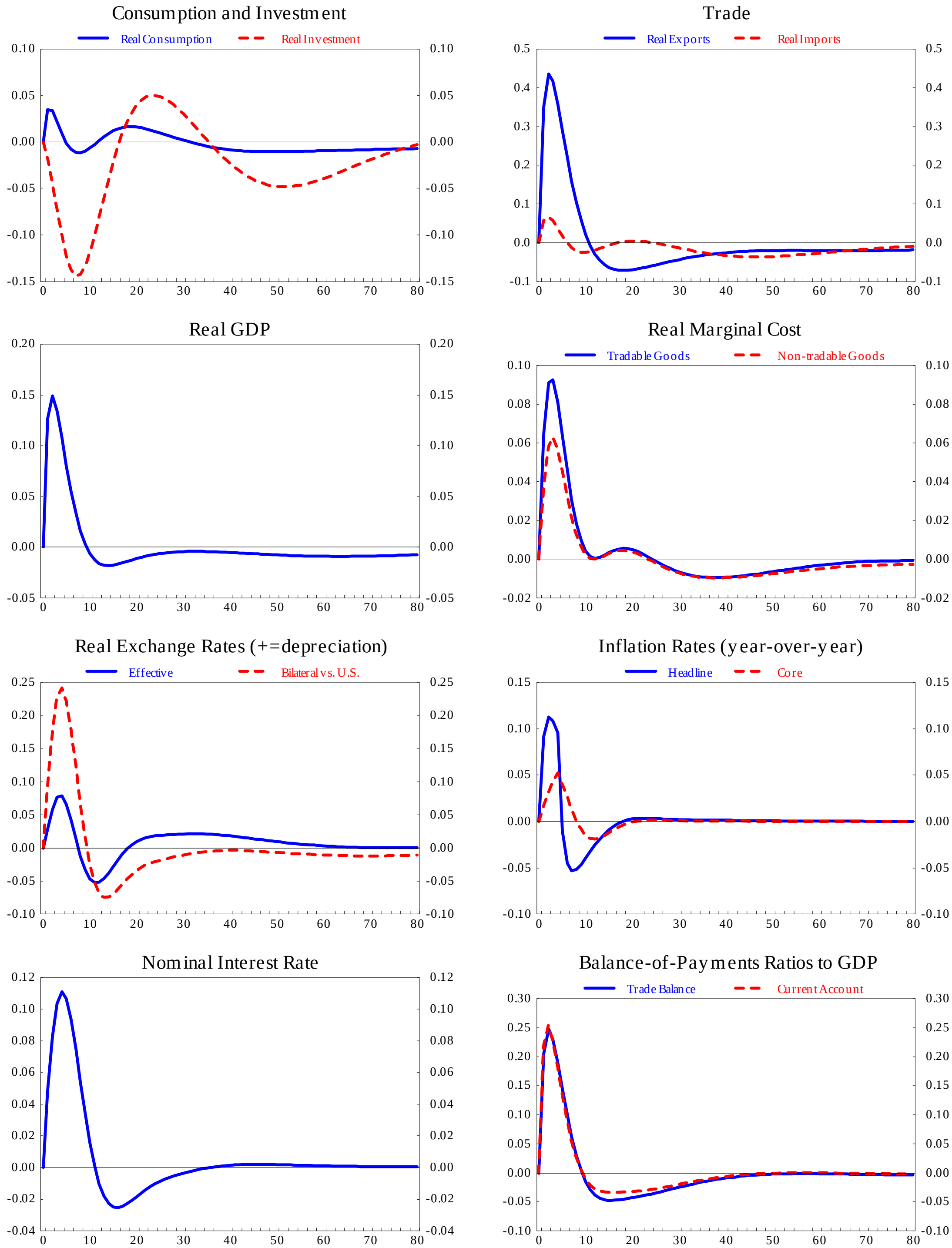


Figure 12: A Temporary 100 Basis Point Increase of the Interest Rate in the United States — U.S. Effects

(Deviation from control, in per cent)

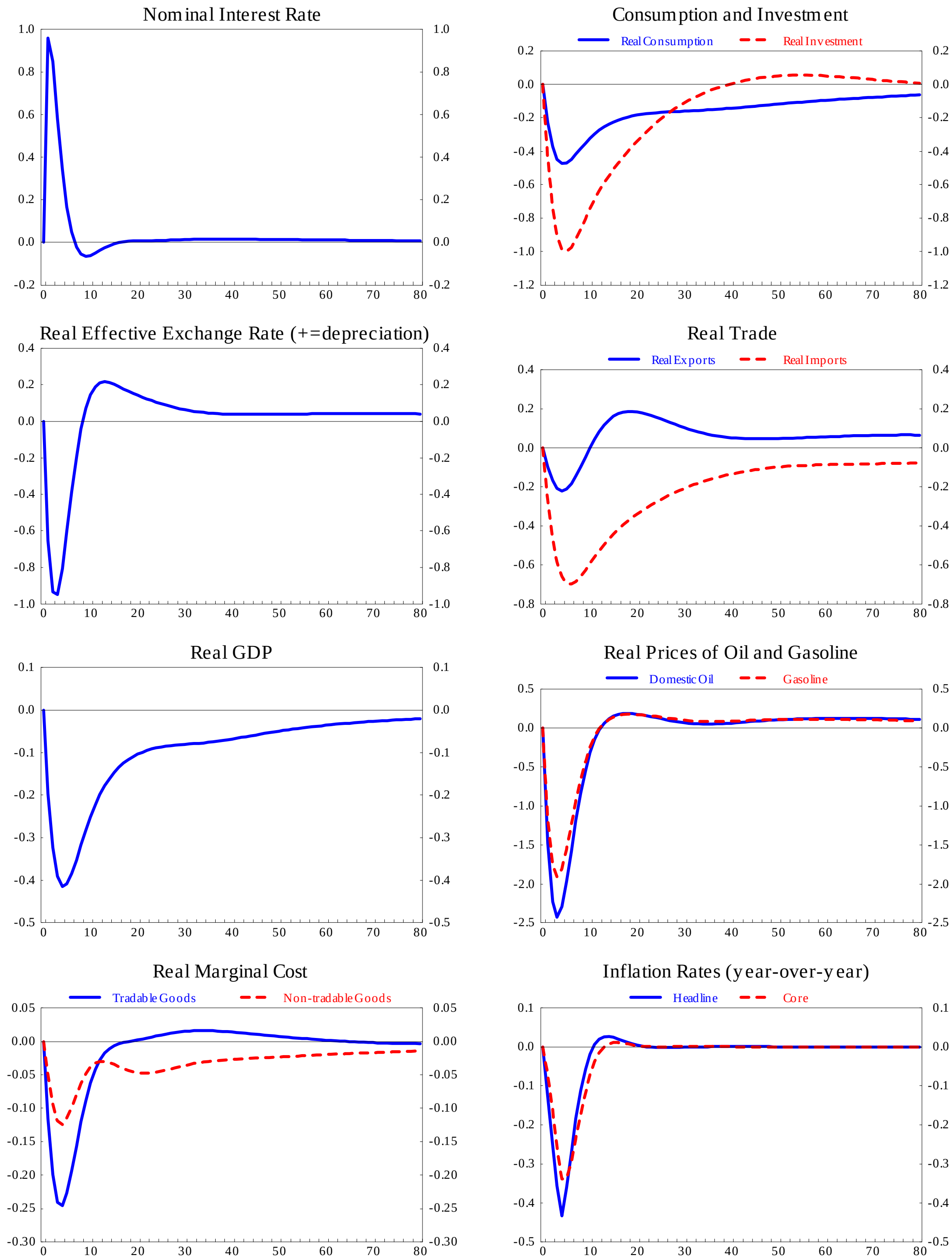


Figure 13: A Temporary 100 Basis Point Increase in the Interest Rate in the United States — Effects on Canada

(Deviation from control, in per cent)

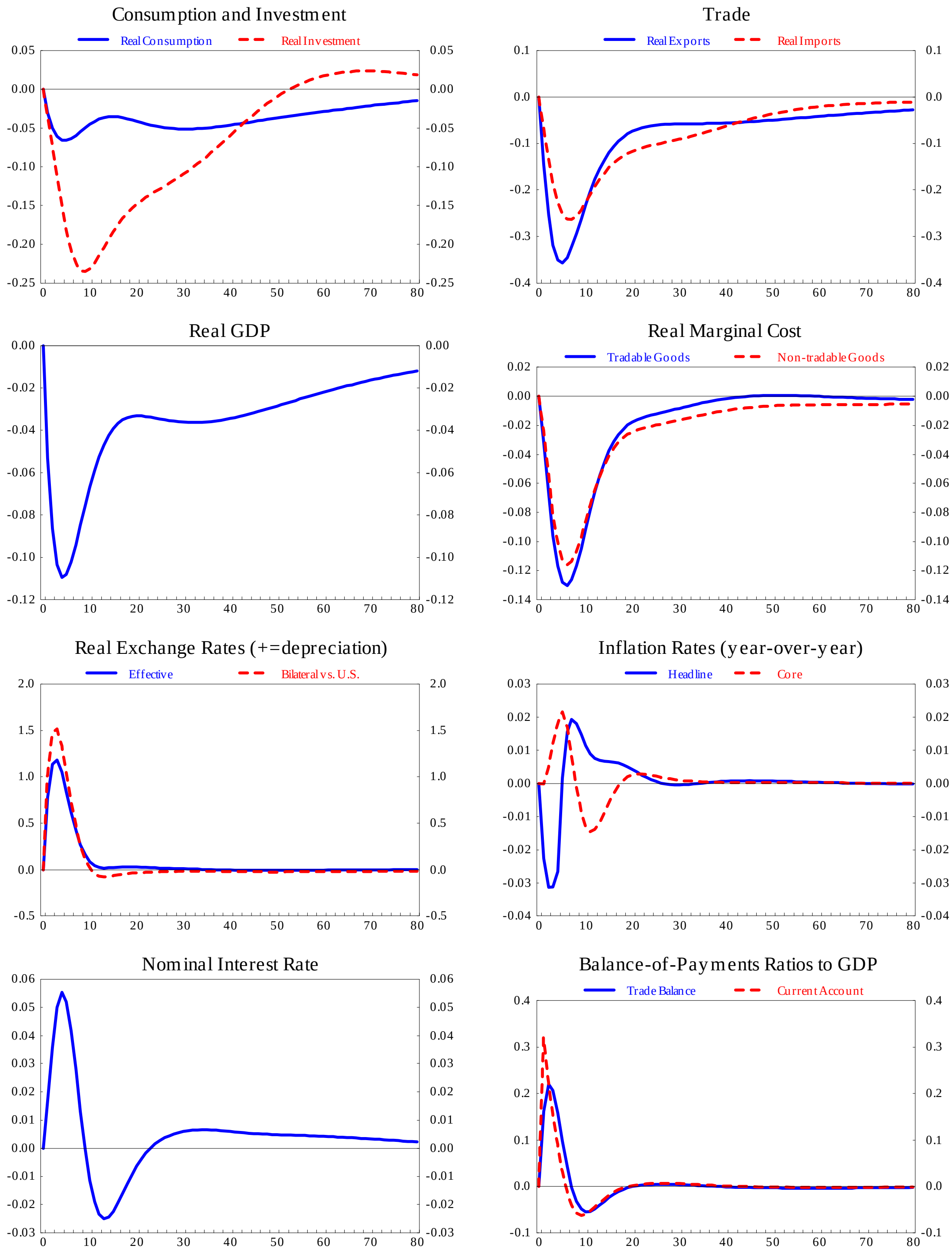


Figure 14: U.S. Productivity and the Balassa-Samuelson Effect — U.S. Effects

Permanent Increase in Productivity in All the Sectors Except Oil (Solid)
Permanent Increase in Productivity in the Traded Sectors (Dashed)
(Deviation from control, in per cent)

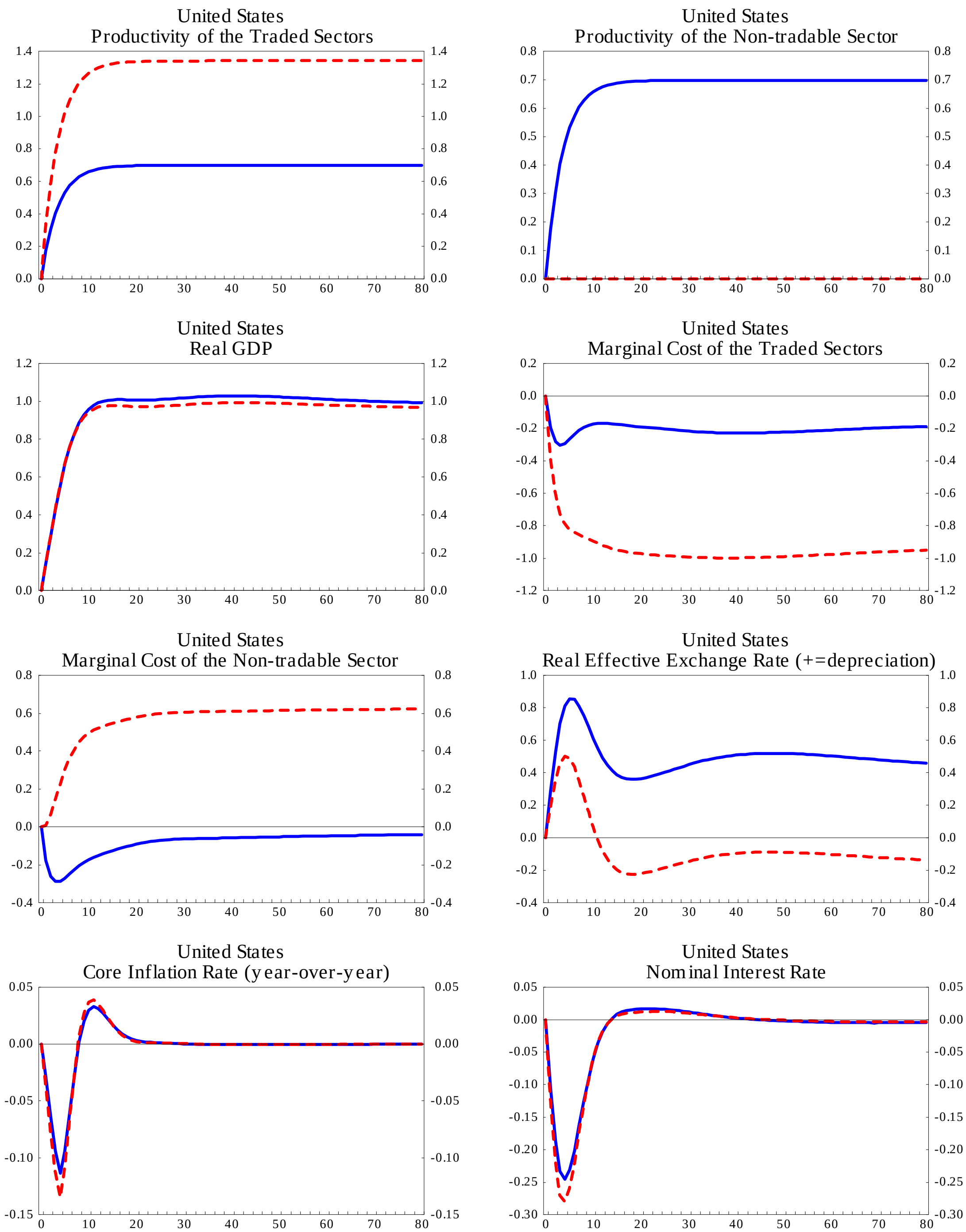


Figure 15: U.S. Productivity and the Balassa-Samuelson Effect — Effects on Canada

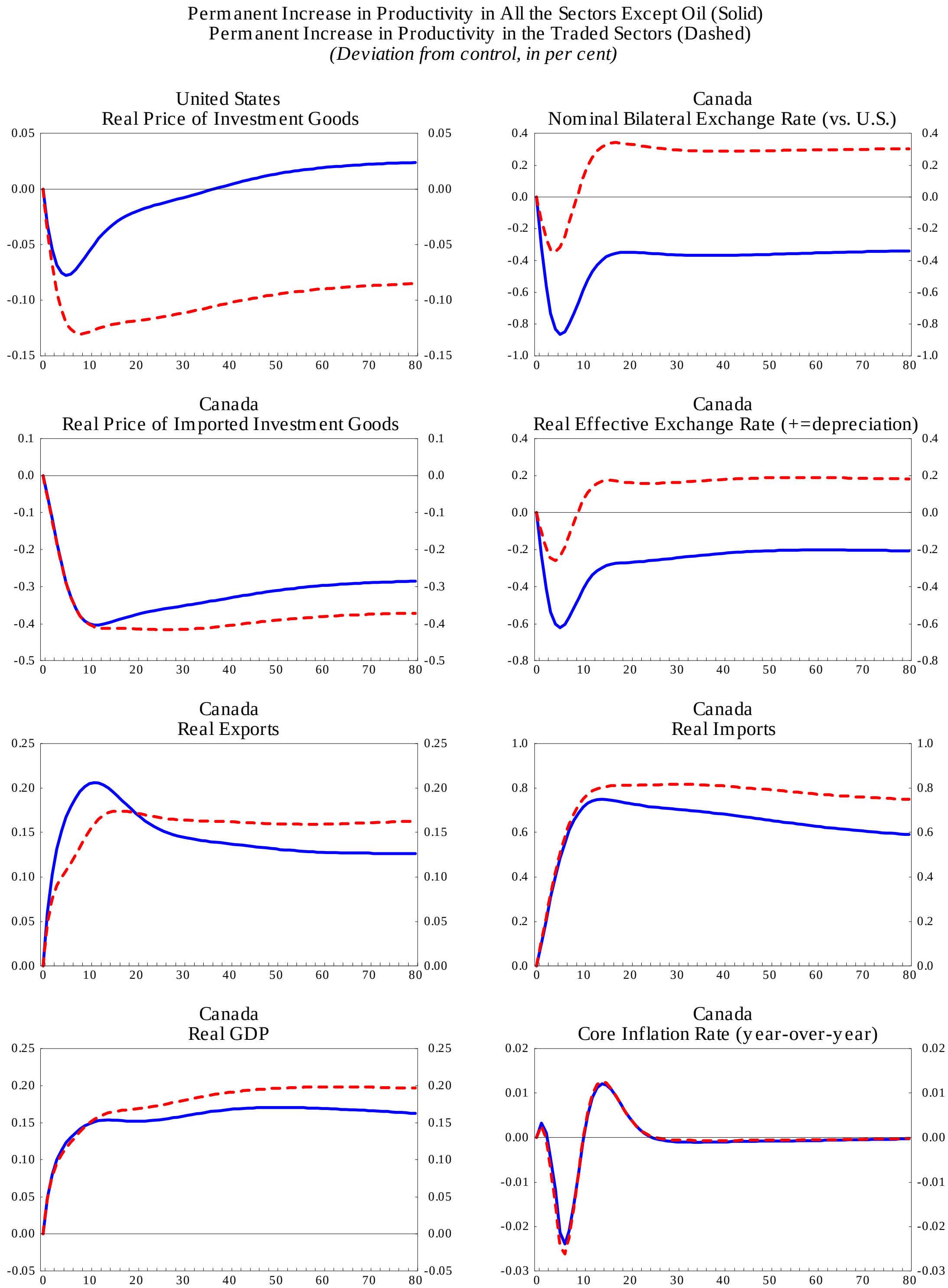


Figure 16: A Temporary World Consumption Shock and its Effects on Oil and Commodities Prices

(Deviation from control, in per cent)

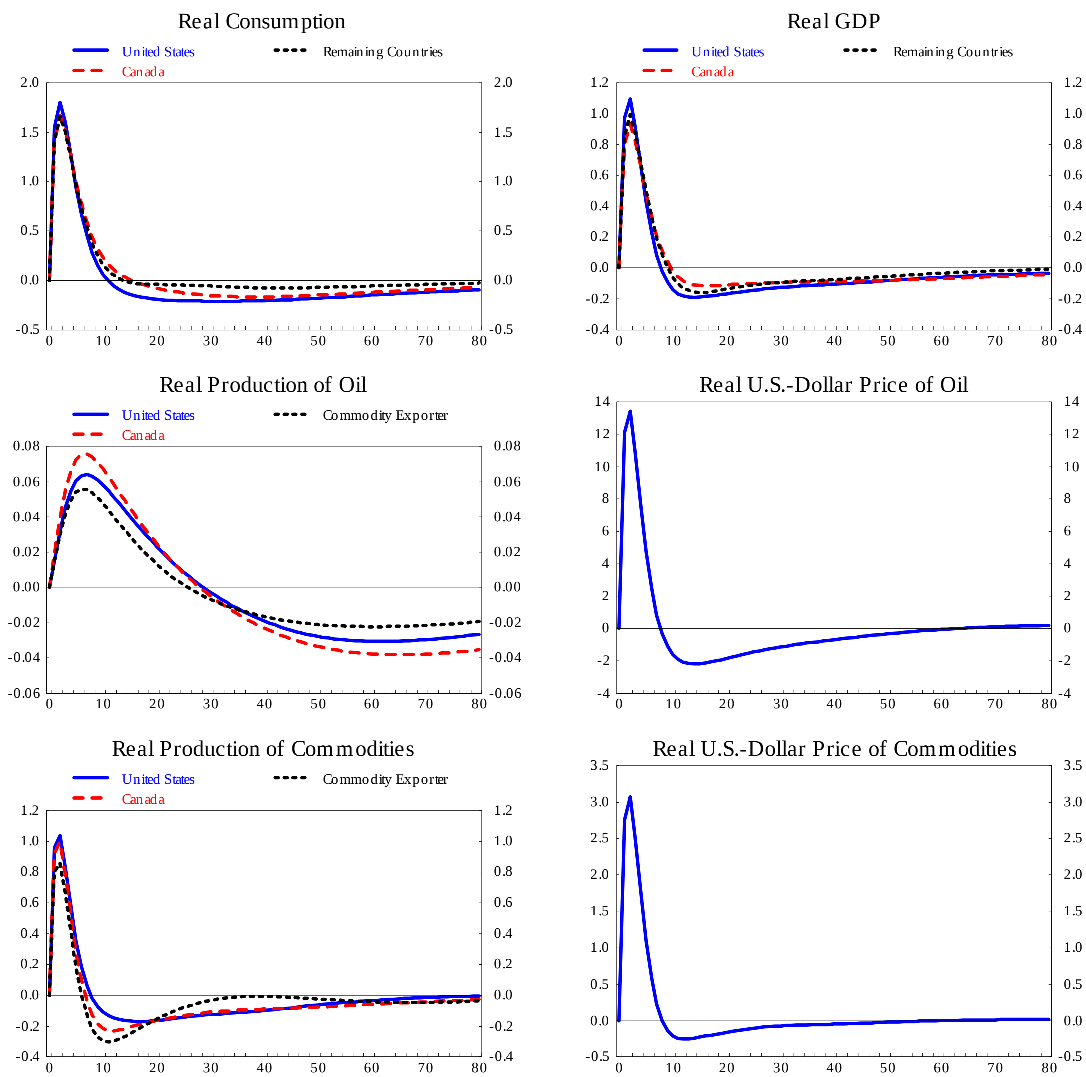


Figure 17: A Permanent Increase in the Productivity of the Commodity-Importing Regions

(Deviation from control, in per cent)

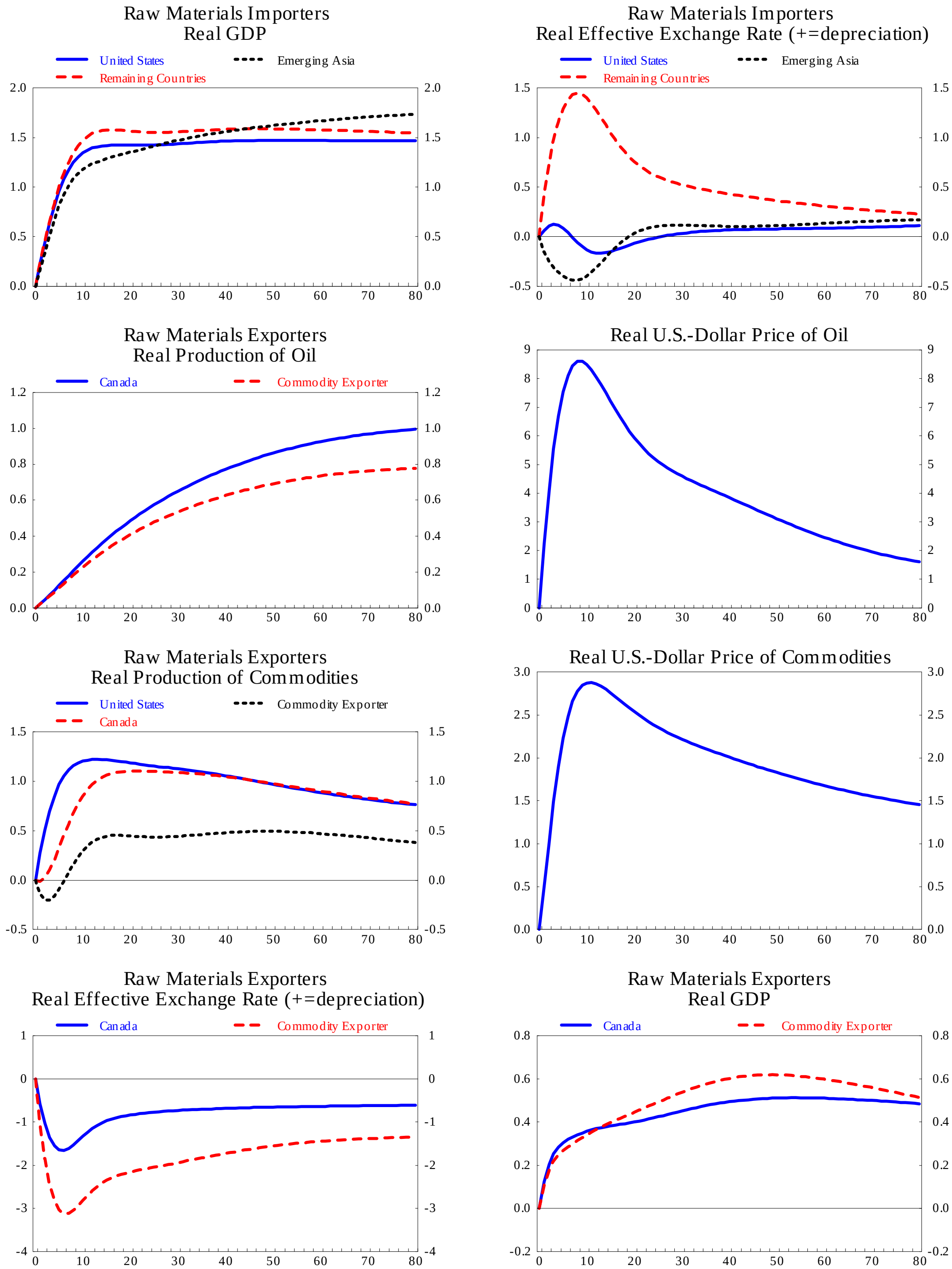


Figure 18: A Permanent Decrease in the Oil Production of the Commodity Exporter — Part I

(Deviation from control, in per cent)

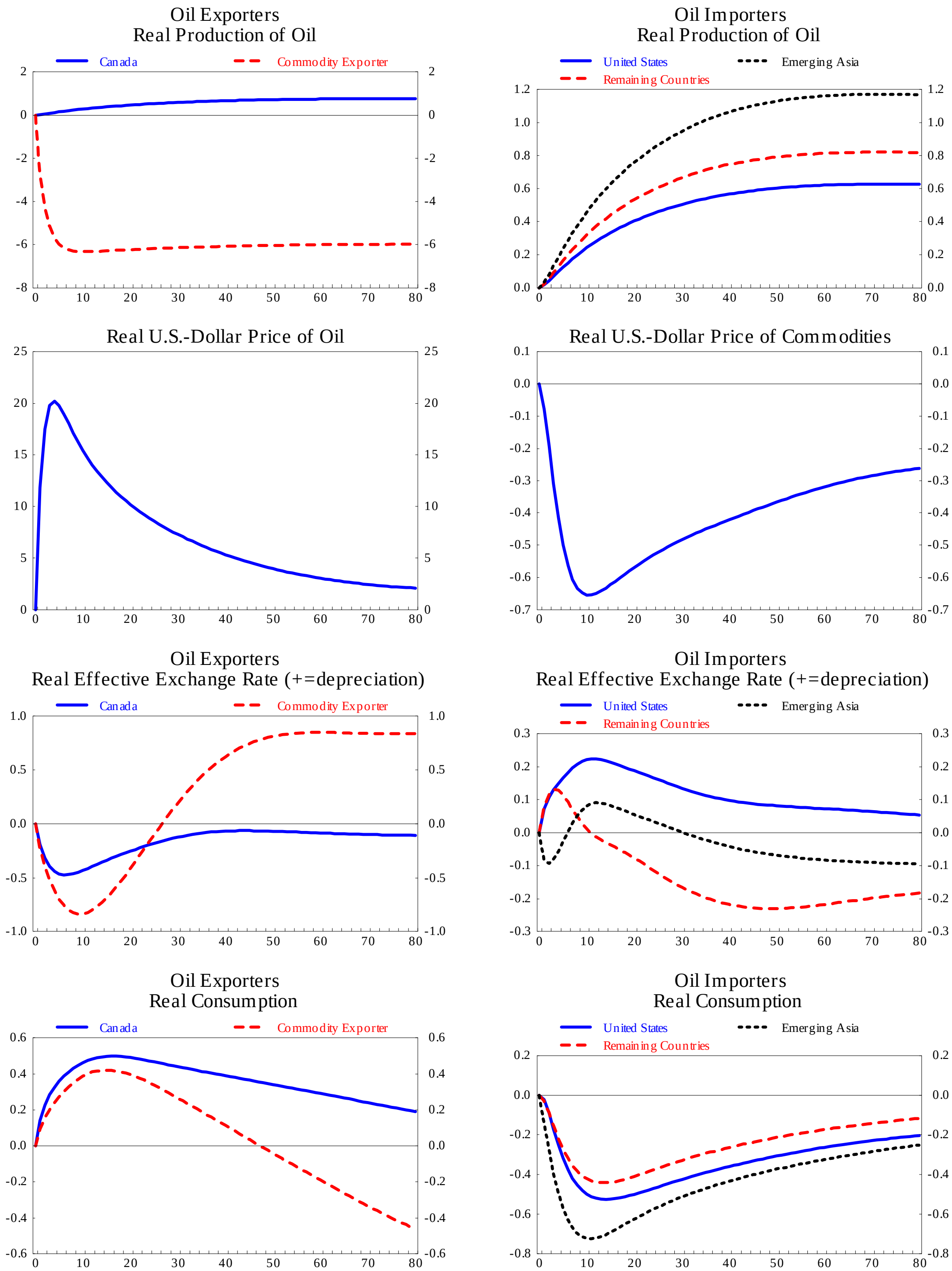


Figure 19: A Permanent Decrease in the Oil Production of the Commodity Exporter — Part II

(Deviation from control, in per cent)

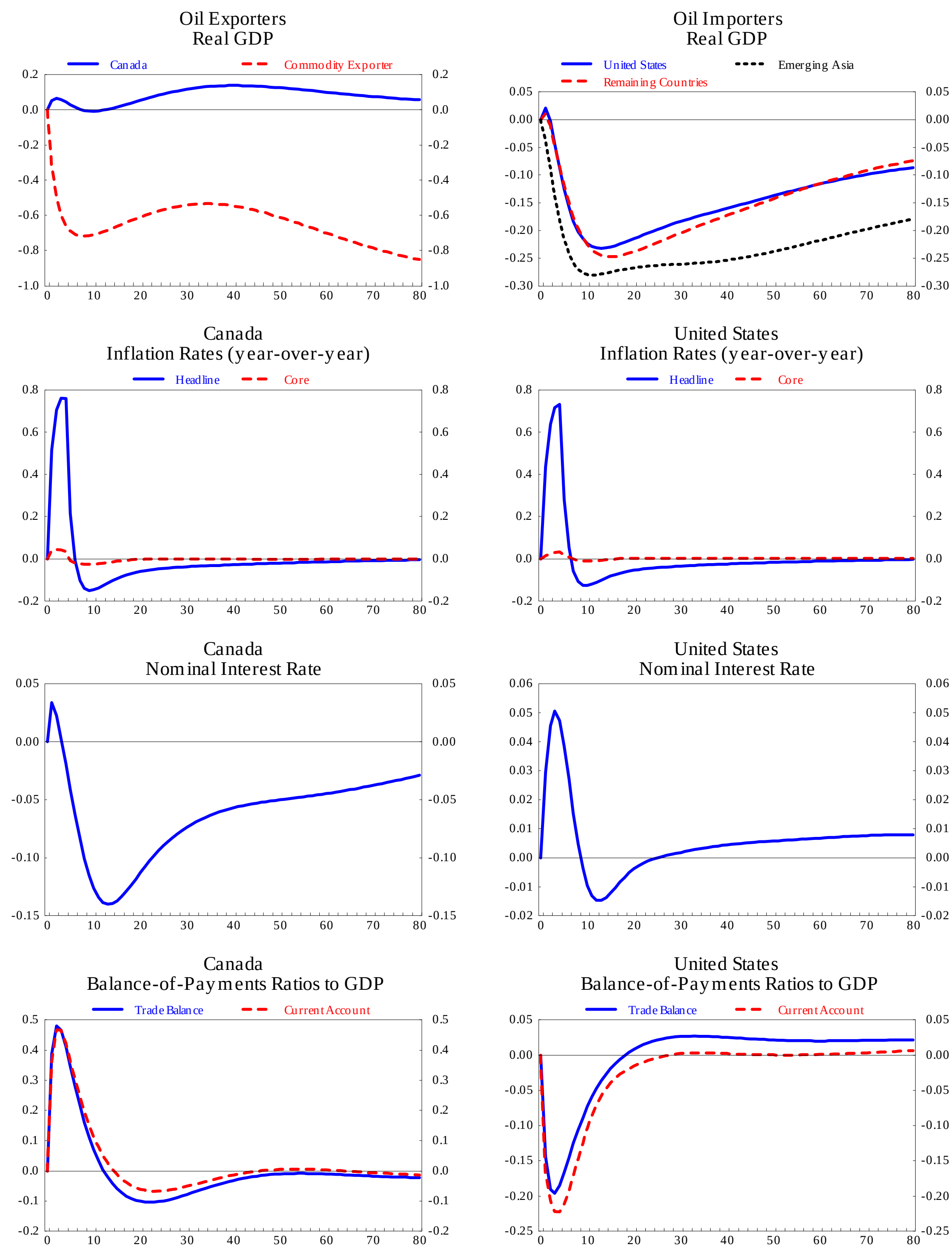


Figure 20: Permanent Decrease in the Commodities Production of the Commodity Exporter

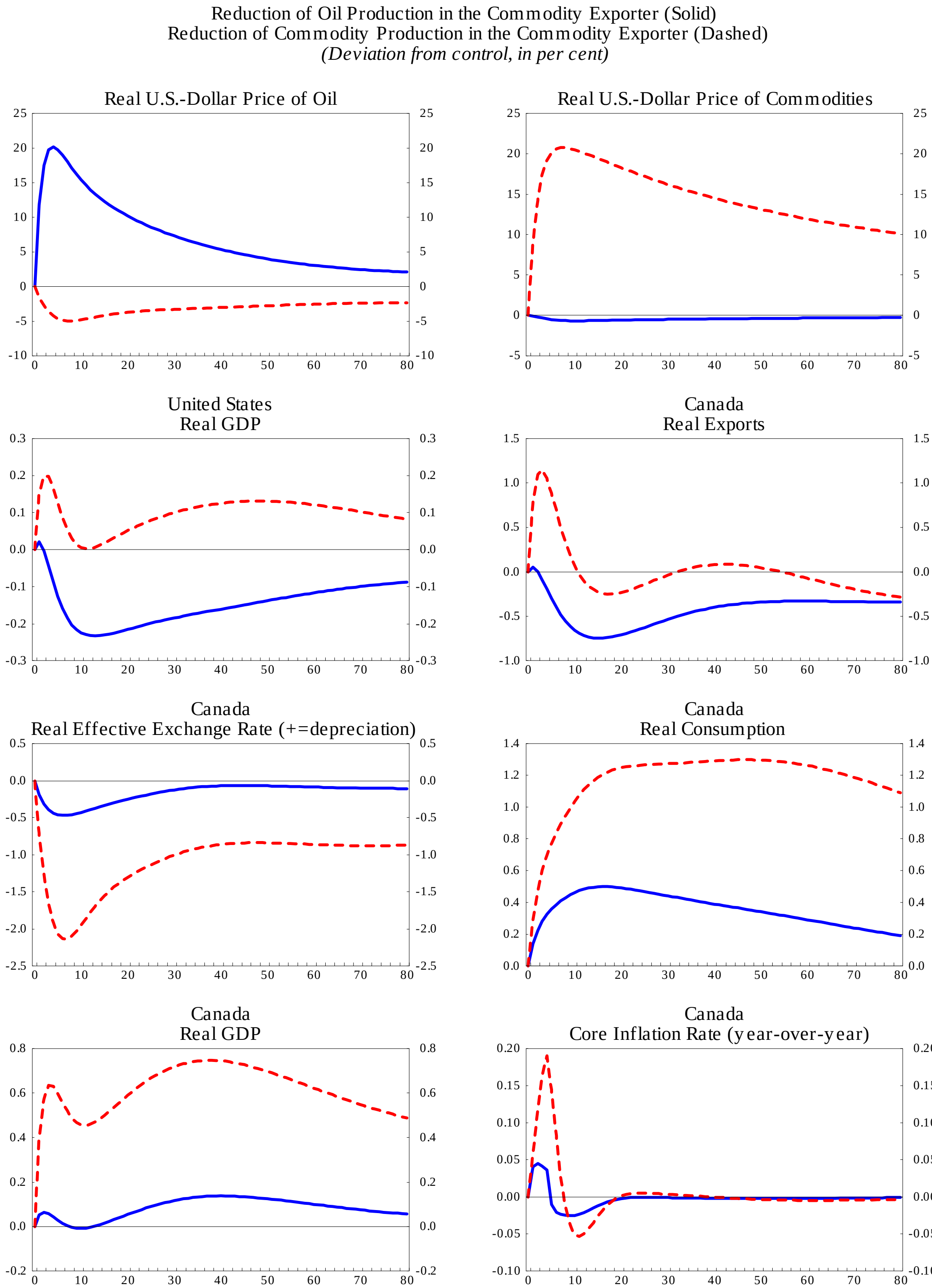


Figure 21: Permanent Increase in Productivity in Emerging Asia — Part I

(Deviation from control, in per cent)

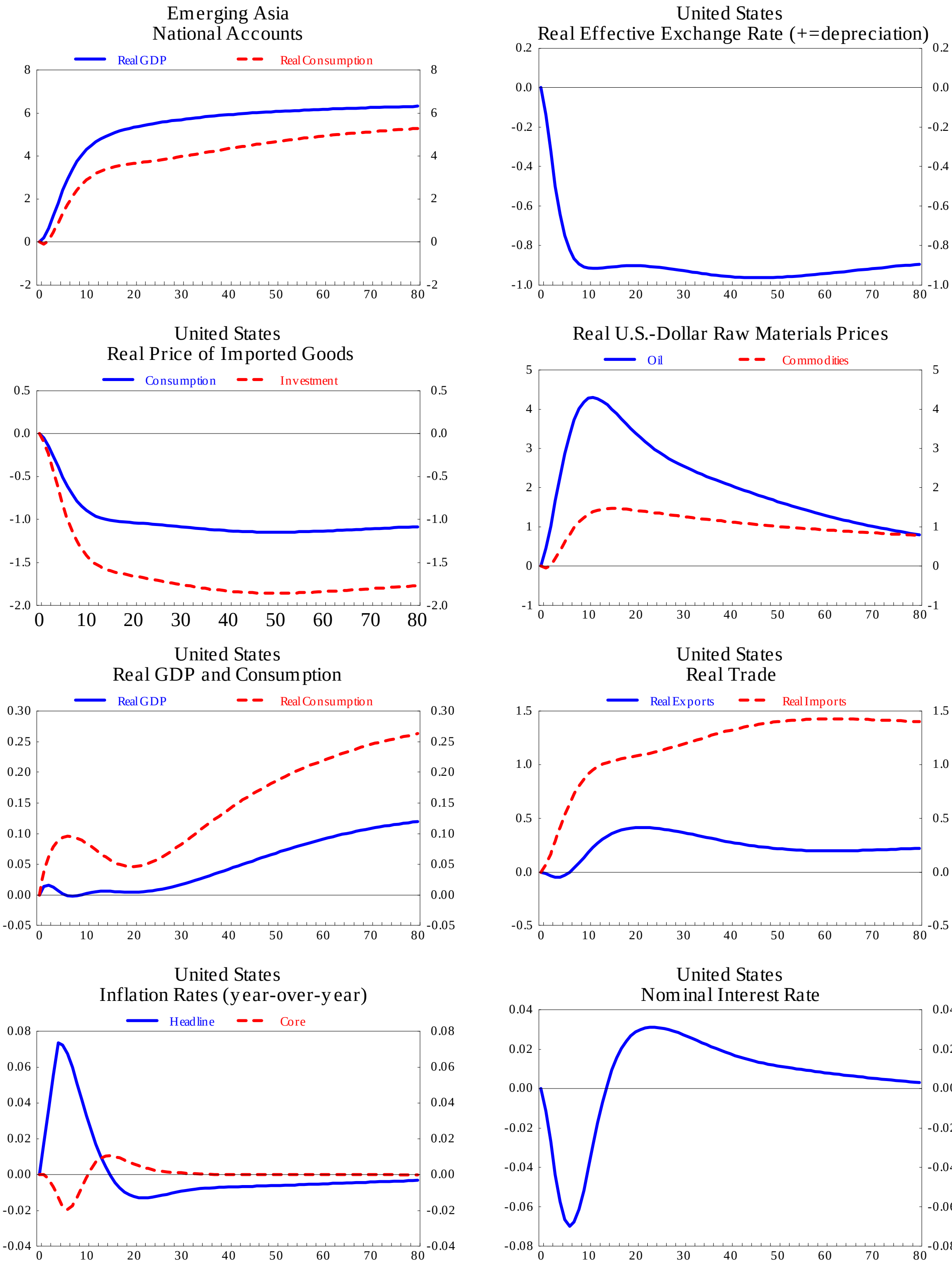


Figure 22: Permanent Increase in Productivity in Emerging Asia — Part II

(Deviation from control, in per cent)

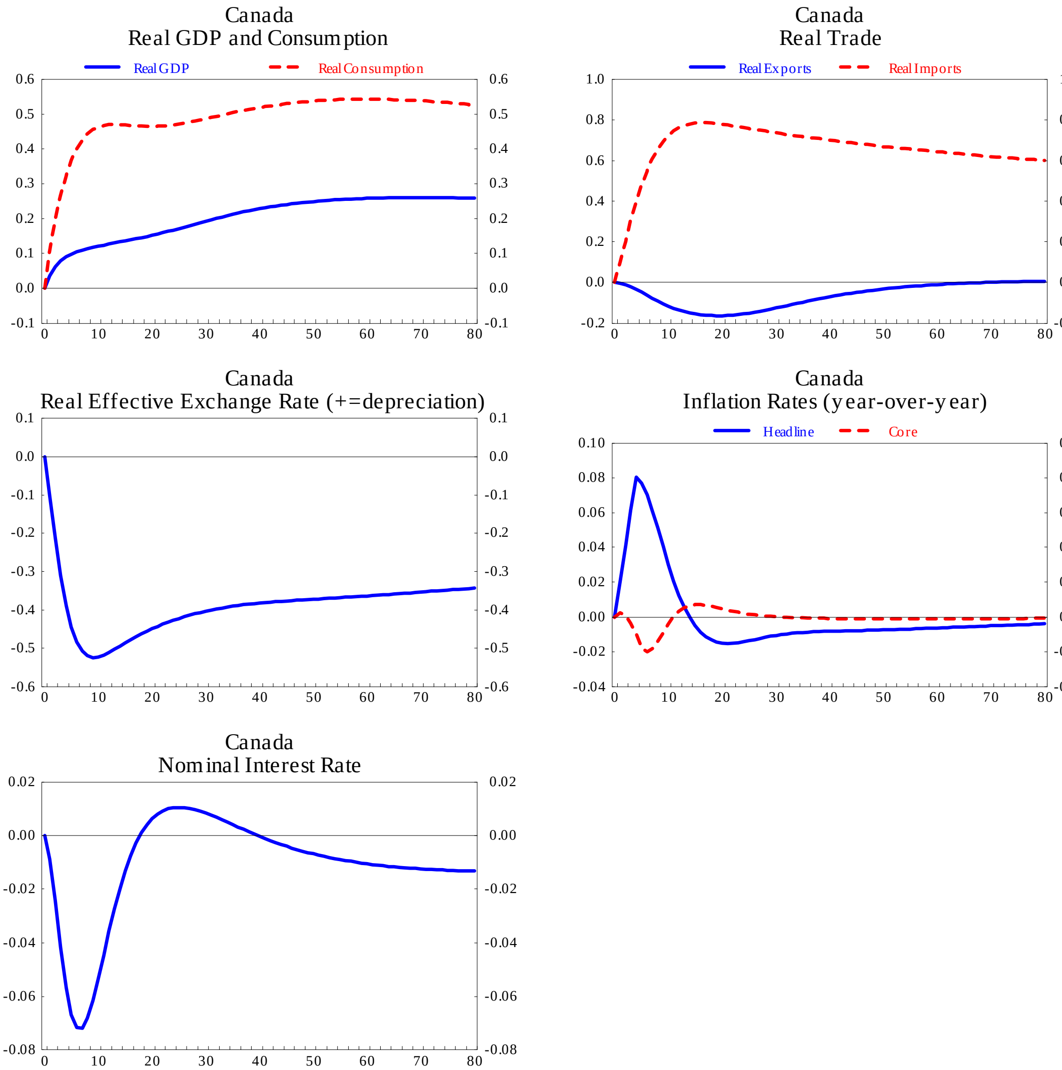


Figure 23: An Increase in Trade Protectionism Worldwide — Part I

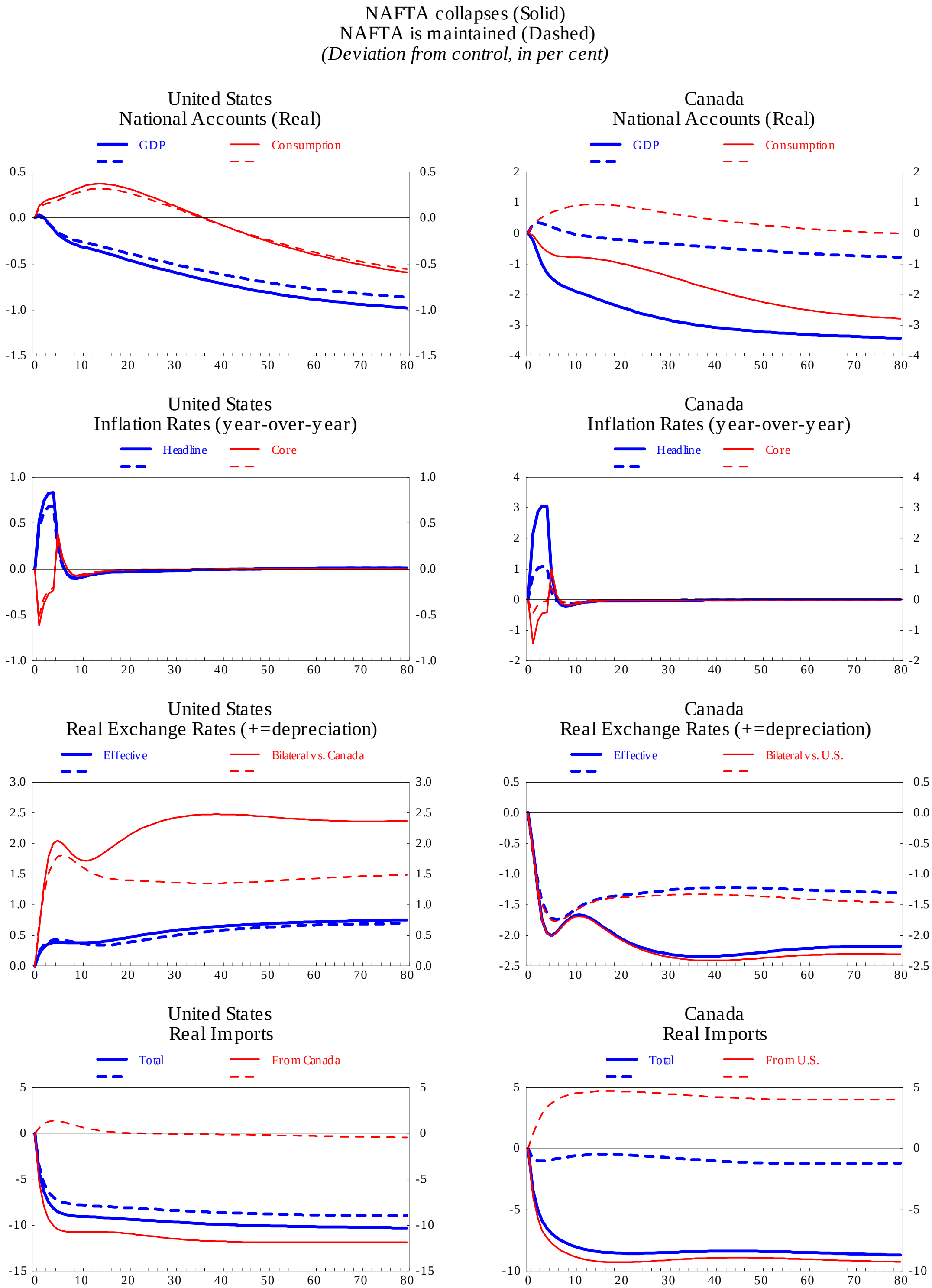


Figure 24: An Increase in Trade Protectionism Worldwide — Part II

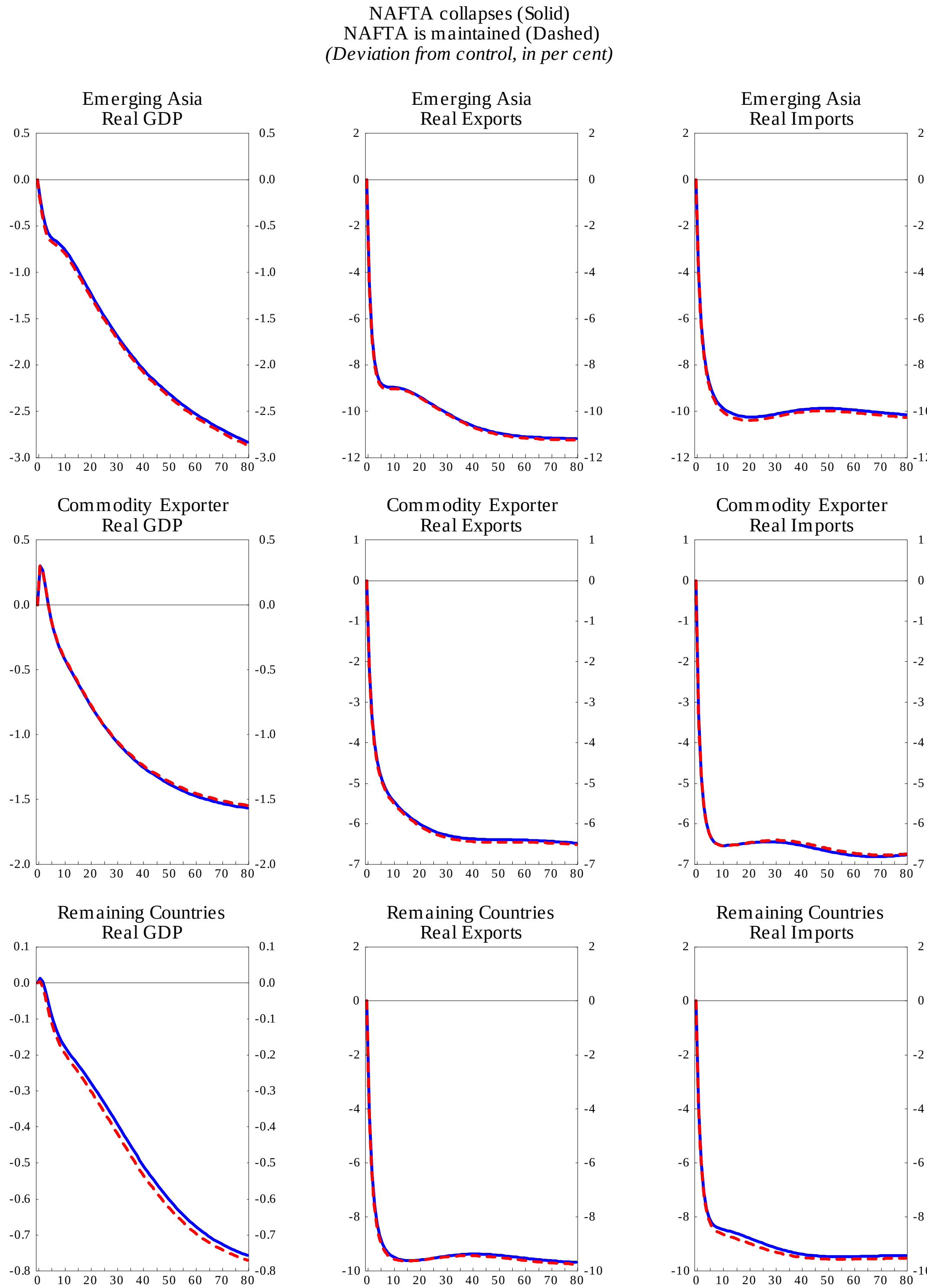


Figure 25: The Effects of a Fiscal Consolidation in the United States — U.S. Effects

(Deviation from control, in per cent)

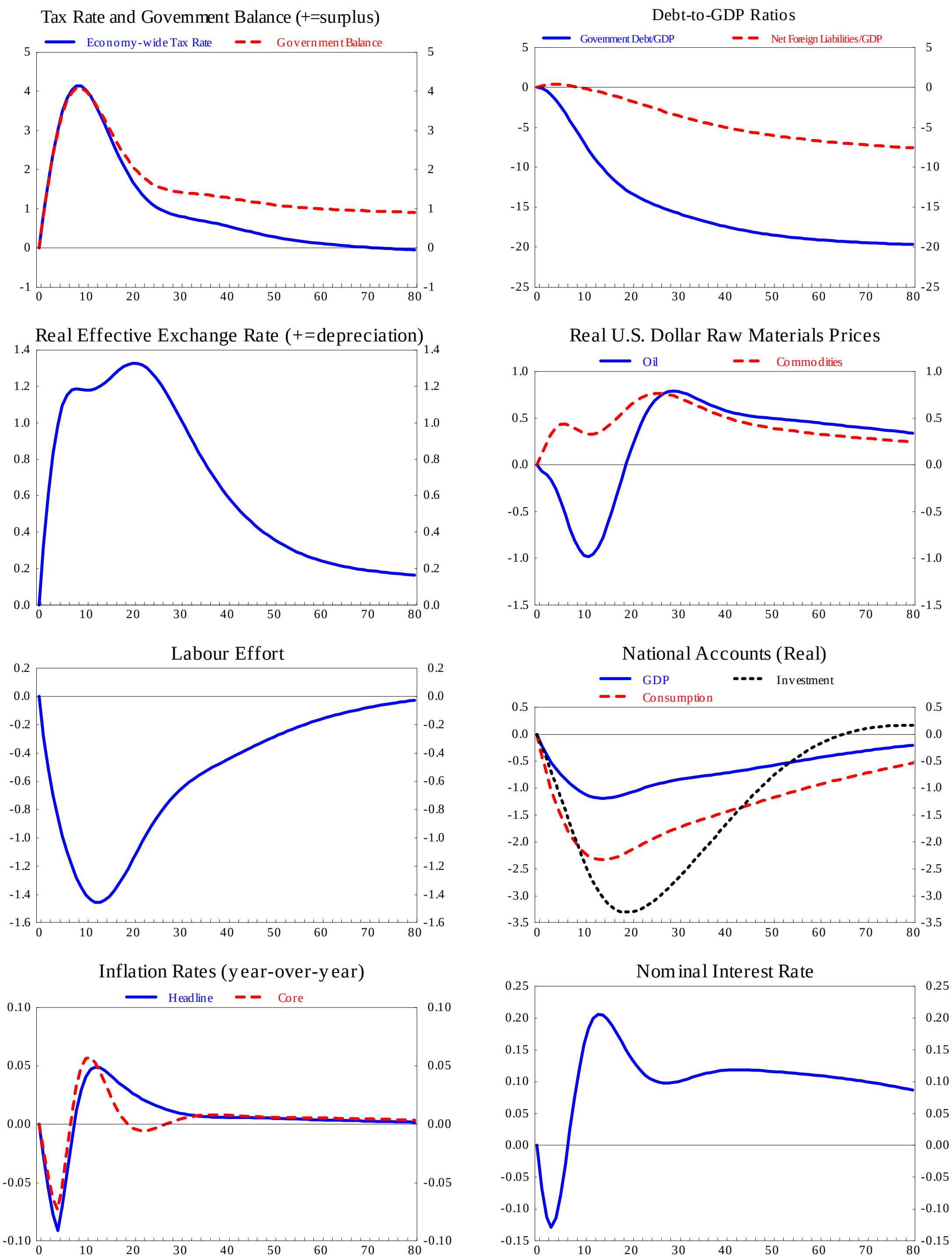
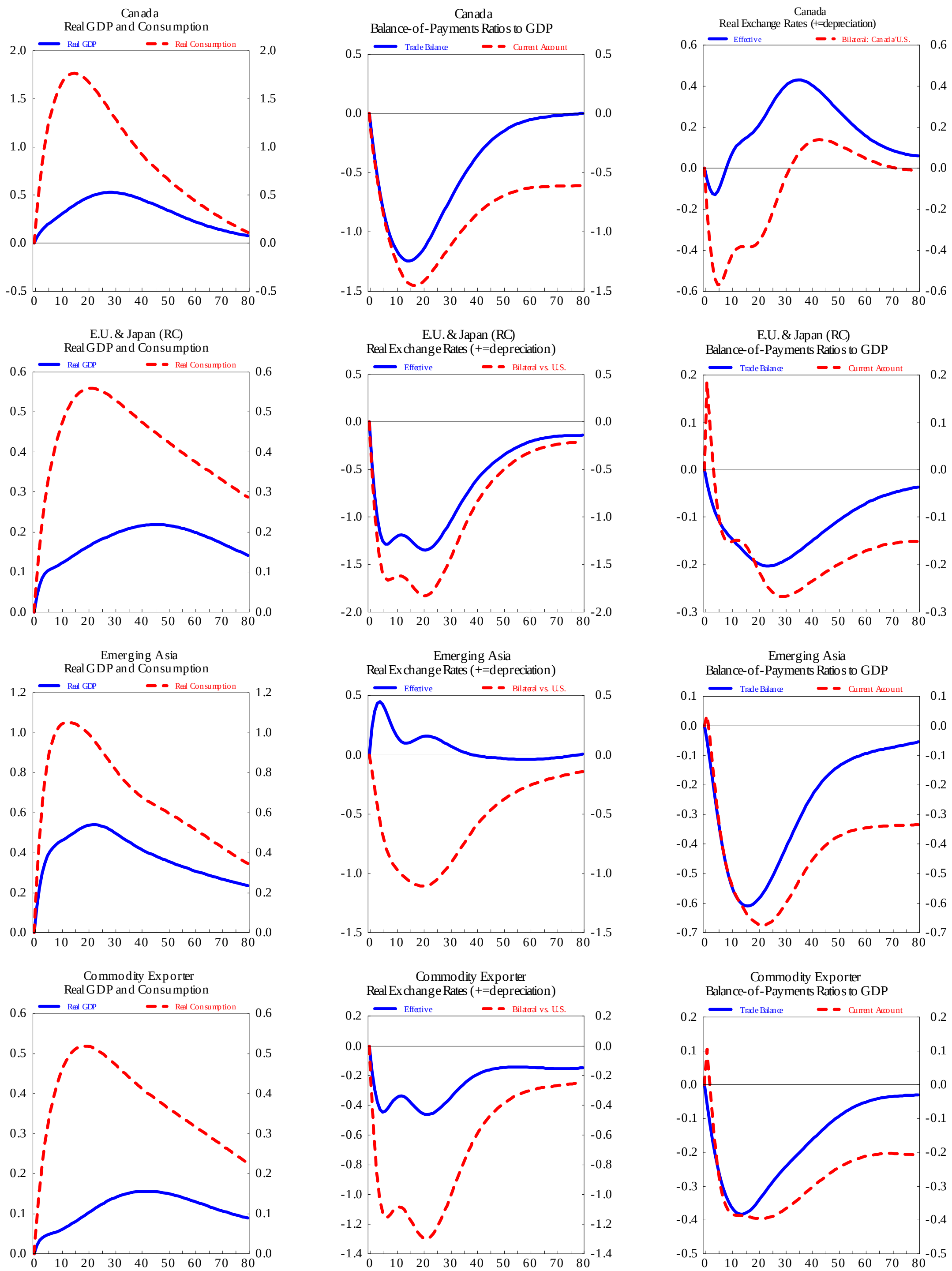


Figure 26: The Effects of a Fiscal Consolidation in the United States — Effects on the Rest of the World

(Deviation from control, in per cent)



Appendix A: Composition of the Regions in the BoC-GEM

The five regions are as follows:

(i) CA – CAnada

(ii) US – United States

(iii) CX – Commodity eXporter = 22 countries

OPEC = Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, Venezuela (Note: data of good quality are not available for the United Arab Emirates)

Algeria; Argentina; Australia; Azerbaijan; Bahrain; Brazil; Chile; Indonesia; Mexico; New Zealand; Norway; Oman; Russia; South Africa

(iv) AS – emerging ASia (IMF definition, excluding Indonesia) = 8 countries

China; Hong Kong Special Administrative Region of China; India; the Republic of Korea; Malaysia; the Philippines; Singapore; Thailand

(v) RC – Remaining Countries

Includes all the other countries in the world, but its properties in the model are focused mainly on members of the entire European Union (=25 countries) and Japan.

Appendix B: Volume, Price, and Current Dollar Measures of the National Accounts

To complete our examination of the BoC-GEM's accounting structure, we will consider once again the current account.

Expression (125) can be rewritten as:

$$CURBAL_t^H = \varepsilon_t^{H,US}\left(B_t^{*H} - \frac{B_{t-1}^{*H}}{\pi_{t-1,t}^{US} g_{t-1,t}}\right) = \frac{i_{t-1}^{*}\varepsilon_t^{H,US} B_{t-1}^{*H}}{\pi_{t-1,t}^{US} g_{t-1,t}} + TBAL_t^H. \qquad \text{(B1)}$$

The left-hand side of (B1) is region H's current account (in nominal terms, in consumption units). The first term on the far right-hand side of the equation is net factor payments from the rest of the world to region H, and $TBAL$ is the trade balance:

$$TBAL_t = EX_t - IM_t, \qquad \text{(B2)}$$

where total exports, EX, are defined by equation (127) and total imports, IM, are defined by equation (128). We define the model-based GDP (in consumption units) as before:

$$\begin{aligned} GDP_t &= A_t + p_{E,t}E_t + p_{N,t}G_{N,t} + EX_t - IM_t \\ &= p_{N,t}N_t + p_{T,t}T_t + (1+\tau_{GAS,t})p_{GAS,t}GAS_t + TBAL_{S,t} + TBAL_{O,t}, \end{aligned} \qquad \text{(B3)}$$

so that:

$$\begin{aligned} CURBAL_t^H &= TBAL_t^H + \frac{i_{t-1}^{*}\varepsilon_t^{H,US} B_{t-1}^{*H}}{\pi_{t-1,t}^{US} g_{t-1,t}} \\ &= GDP_t^H - \left(C_t^H + p_{E,t}^H I_t^H + G_t^H\right) + \frac{i_{t-1}^{*}\varepsilon_t^{H,US} B_{t-1}^{*H}}{\pi_{t-1,t}^{US} g_{t-1,t}}. \end{aligned} \qquad \text{(B4)}$$

While theoretically sound, this measure of output (and those measures for its associated components) bears little similarity to real GDP as measured by the system of national accounts in countries such as the United States and Canada. The problem is particularly severe for relatively open economies facing large swings in real exchange rates and relative prices. To approximate real GDP as closely as possible, we construct 'national accounts' concepts for GDP and its components, whereby we create new definitions of GDP, consumption, investment, government spending, exports, and imports (GDP, C, I, G, EX, and IM, respectively), along with their associated price indexes. We construct the concepts using the Tornqvist approximation to chain-weighted Fisher indexes for the data.[1] For each region, we have the following relationships between model concepts, current dollars, and prices multiplied by volumes:

$$CPI_t GDP_t = GDP_t^{NOM} = p_{GDP,t}^{NAT} GDP_t^{NAT}, \tag{B5}$$

$$CPI_t C_t = C_t^{NOM} = p_{C,t}^{NAT} C_t^{NAT}, \tag{B6}$$

$$CPI_t p_{I,t} I_t = I_t^{NOM} = p_{I,t}^{NAT} I_t^{NAT}, \tag{B7}$$

$$CPI_t p_{G,t} G_t = G_t^{NOM} = p_{G,t}^{NAT} G_t^{NAT}, \tag{B8}$$

$$CPI_t EX_t = EX_t^{NOM} = p_{X,t}^{NAT} EX_t^{NAT}, \tag{B9}$$

$$CPI_t IM_t = IM_t^{NOM} = p_{M,t}^{NAT} IM_t^{NAT}. \tag{B10}$$

[1] Whelan (2000) provides a straightforward and clear explanation of the use of chain-weighted Fisher indexes by the U.S. system of national accounts.

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