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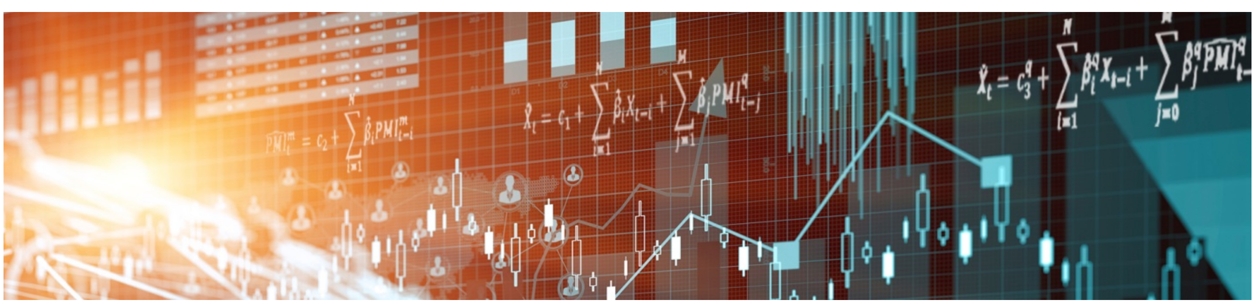
Credit Crunches from Occasionally Binding Bank Borrowing Constraints



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Credit Crunches from Occasionally Binding Bank Borrowing Constraints

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Abstract

We present a model in which banks and other financial intermediaries face both occasionally binding borrowing constraints and costs of equity issuance. Near the steady state, these intermediaries can raise equity finance at no cost through retained earnings. However, even moderately large shocks cause their borrowing constraints to bind, leading to contractions in credit offered to firms, and requiring the intermediaries to raise further funds by paying the cost to issue equity. This leads to the occasional sharp increases in interest spreads and the countercyclical, positively skewed equity issuance that are characteristic of the credit crunches observed in the data.

Bank topics: Business fluctuations and cycles; Credit and credit aggregates; Economic models; Financial markets
JEL codes: E22, E32, E51, G2

Résumé

Nous présentons un modèle dans lequel les banques et les autres intermédiaires financiers font parfois face à des conditions d'emprunt contraignantes et à des coûts d'émission d'actions. Près de l'état stationnaire, ces intermédiaires peuvent obtenir du financement par émission d'actions à coût nul, au moyen des bénéfices non répartis. Cependant, même des chocs d'ampleur modérée peuvent rendre les conditions d'emprunt de ces intermédiaires contraignantes, ce qui entraîne un resserrement de l'offre de crédit aux entreprises, et les obliger à amasser des fonds supplémentaires en assumant les coûts d'émission d'actions. Il en résulte des hausses occasionnelles marquées des écarts de taux d'intérêt et l'émission d'actions contracyclique présentant une asymétrie positive qui caractérisent les étranglements du crédit observés dans les données.

Sujets : Cycles et fluctuations économiques; Crédit et agrégats du crédit; Modèles économiques; Marchés financiers
Codes JEL : E22, E32, E51, G2

Non-technical summary

During the 2007–08 financial crisis, there was a widespread credit crunch, interest rates spreads increased dramatically and real economic activity contracted. Because such events are occasional phenomena, time series of interest spreads are very "spiky" with sharp rises occurring during downturns. This paper presents a macroeconomic model that explains this observation as arising from the possibility that financial intermediaries can become borrowing constrained.

In the model, banks face both the risk of being borrowing constrained and costs of issuing equity. Even if intermediaries leverage up as much as possible, because they can raise equity finance via retained earnings for free, financial intermediation is usually efficient. Moderately large adverse shocks, however, can cause banks to be borrowing constrained and, thanks to equity issuance costs, lead to an increase in interest spreads and a reduction in credit to the non-financial sector. The model can help interpret two stylized facts. First, as the credit crunches are occasional phenomena, increases in the interest spread during recessions are much greater than the decreases occurring during booms, as observed in the data. Second, due to equity issuance costs, debt is preferred to raising equity finance under normal circumstances and banks issue new equity only when financial conditions worsen. This leads to countercyclical equity issuance, with occasional large issuances, as observed in the data, but missed in other models.

1 Introduction

Economic downturns are usually accompanied by sharp increases in interest spreads as the effects of financial frictions worsen. This is particularly true during banking crises when the financing costs faced by intermediaries rise dramatically.[1] In this paper, we present a model in which financial intermediaries face occasionally binding borrowing constraints that cause spreads to rise when the value of assets declines sufficiently, thanks to the costs these intermediaries face in issuing new equity. The increased spread between the savings rate and the return on capital implies a drop in the marginal efficiency of investment, generating declines in aggregate investment relative to the efficient benchmark, and introducing asymmetries in macroeconomic time series. However, in our model, in the vicinity of the steady state, financial constraints are slack and financial intermediation is efficient. This allows for the characterization of *normal times* and *credit crunches*. Our model differs importantly from the existing literature in this dimension. Whereas the presence of occasionally binding constraints usually depends on calibration,[2] in our model, borrowing constraints are always occasionally binding, essentially irrespective of parameter values. This holds since financial intermediaries, henceforth simply known as "banks," choose to borrow to the edge of the constrained region.

The model is in the spirit of the banking model proposed in Gertler & Kiyotaki (2010) (henceforth GK), but while these authors prevent equity issuance to ensure banks are always financially constrained,[3] endogenous dividend payments and equity issuance costs in our model imply that the financial constraint is only occasionally binding. In order to raise funds when further debt finance is unavailable, banks can reduce dividend payments and use retained profits for free. However, if banks are unable to raise sufficient funds via retained earnings, they are restricted to costly equity issuance. This introduces a

[1]See Babihuga & Spaltro (2014) for a discussion of bank funding costs during the 2007–08 financial crisis.

[2]Compare, for example, Gertler & Kiyotaki (2010) with Bocola (2016). The constraint is always binding in the former but only binds occasionally in the latter due to different calibrations of banking sector parameters.

[3]There is an extension discussed in GK, pursued further in Gertler, Kiyotaki & Queralto (2012) (henceforth GKQ), that introduces bank equity issuance by extending the same agency problem in debt finance to equity finance by differentiating between inside and outside shareholders. But in doing so, the set-up generates counter-factual dynamics with respect to equity issuance. Specifically, equity issuance is procyclical whereas the data indicate that this is countercyclical.

spread between the risk-free saving rate, and the risky return to capital, often described as an investment or capital wedge.[4] Because the investment wedge appears only during downturns, the effects of the financial friction are inherently asymmetric. This enables our model to better explain a number of key facts as compared with other models such as GK. In particular, we are able to match the large positive skewness in spreads and to provide an explanation for the observation that crises are occasional phenomena during which the adverse effects of financial frictions worsen significantly. Furthermore, because it is desirable to issue equity only when all other sources of finance are exhausted, bank equity issuance is strongly countercyclical, consistent with the data,[5] but missed in other models of bank equity issuance, such as GKQ.[6] Additionally, modelling occasionally binding financial constraints eliminates the financial accelerator mechanism during normal times, in line with the evidence that models without a financial accelerator perform better in normal times (Del Negro, Hasegawa & Schorfheide 2016); in our model, only during sufficiently deep downturns do the financial constraints bind, further amplifying the recession.

As well as modelling the financial structure of banks more realistically, we improve upon the GK agency problem. The GK borrowing constraint emerges due to limited contract enforceability; banks have an outside option to divert assets and declare bankruptcy. By parameterizing the proportion of assets that can be reclaimed by creditors, the authors set the outside option to a fixed amount of the current value of bank assets. In our model, we carefully specify off-equilibrium play and use U.S. bankruptcy law to implement the amount recoverable by creditors. In particular, whereas GK place timing restrictions on when banks can choose to default in order to prevent banks making a large, unrecoverable dividend at the end of one period before defaulting in the next, the restriction is not required in our approach as, according to U.S. bankruptcy law, the amount paid out would also be liable to be reclaimed by the courts. This mechanism also gives an additional motive for dividend payments; since recent dividend payments are reclaimable during bankruptcy, dividend payments act to relax the present and fu-

[4]See Chari, Kehoe & McGrattan (2007) as an example of the former, and Hall (2010) of the latter.

[5]It is widely accepted that equity issuance by most non-financial firms is procyclical; however, recent studies have shown that bank equity issuance is countercyclical (see, e.g., Baron 2017).

[6]As mentioned in footnote 3. The introduction of differing costs of equity and debt is similar to that proposed in Jermann & Quadrini (2012) who include tax benefits of debt finance; however, in our model, the tightness of the borrowing constraint is endogenous, and only occasionally binding.

ture borrowing constraint, and consequently can sometimes be paid even if the bank is issuing equity, helping to explain a long-discussed puzzle (see, e.g., Myers 1984, Loderer & Mauer 1992, Fama & French 2005).

We compare our model both with the standard real business cycle (henceforth RBC) model, which provides an efficient benchmark, and with the always-binding borrowing constraints model of GK with the equity issuance extension. In their model, to ensure that the borrowing constraint is always binding, bankers exit with a fixed probability. This is set to 2.5 percent per quarter and is described as a turnover between workers and bankers. However, as this is treated as a payment to the representative household, it is equivalent to a fixed dividend rate, which, at 10 percent per annum, seems implausibly high.[7] This high dividend payment rate ensures debt is always the cheapest source of finance in GK, which, in combination with the calibration of the proportion of divertable assets, implies the borrowing constraint is always binding. By contrast, in our model, the borrowing constraint binds when demand for funds increases without an equivalent rise in the value of future discounted dividends. This can occur following an adverse supply-side shock to capital, which increases the demand for investment. Such a shock implies a reduction in the bank's future profit stream and so an increase in the marginal value of the bank cashing-out, i.e., diverting assets and defaulting.

We examine model dynamics in the presence of investment adjustment costs and capital quality shocks, which, following GK, may be thought of as modelling the economic obsolescence of capital, rather than its physical destruction. This introduces an exogenous variation to the value of capital. As a source of occasional disasters, this shock is particularly relevant given the events of late 2007 in the U.S., when a huge amount of value was knocked off bank assets, leading to the banking crisis.

We differ from the GK set-up with the use of the household preferences proposed in Jaimovich & Rebelo (2009). This allows for the parameterization of the strength of the short-run wealth effect on labour supply. By choosing a weak wealth effect, positive (negative) news about the future can generate a rise (fall) in labour supply, so producing co-movement in consumption and investment following capital quality shocks.[8] Further-

[7] Between 1965 and 2013, dividend payments made by the largest 20 U.S. banks averaged 5.15 percent (using the data set constructed in Baron 2017).

[8] We analyze various utility functions, forms of investment and capital adjustment costs, and habits in consumption and leisure, finding that the key results are unchanged.

more, a small short-run wealth effect can be motivated by the observation that a large proportion of households have very little or no net wealth, with just a small few owning a disproportionate share of total wealth (see, e.g., Mankiw 2000).[9]

In the remainder of the article, we discuss the support for the chosen model of financial constraints before briefly outlining the related literature on financial frictions and occasionally binding financial constraints. We then proceed to describe in detail the derivation of equilibrium conditions that characterize the behaviour of the economy and discuss some key analytical results. We end with a discussion of the main numerical results, and point to future research.

1.1 Our model of financial constraints

In order to generate crisis periods, our model must feature an aggregate occasionally binding constraint. We argue that the most appropriate location for this occasionally binding constraint is on debt finance, since under normal circumstances, debt is preferred to equity due to equity issuance costs. Prior to the financial crisis of 2007–08, the banking system had built up a reliance on short-term debt finance.[10] Following the bursting of the U.S. subprime mortgage bubble, there was a sharp contraction in the money markets cumulating in the collapse of the shadow banking system. While debt finance had been relatively unconstrained prior to the financial crises, bank borrowing constraints began to bind as the value of assets plummeted.

In a study of U.S. commercial banks between 1925 and 2012, Baron (2017) finds that bank equity issuance has been countercyclical. This observation seems self-evident in Figure 1, which plots new equity issuance for the largest U.S. commercial banks since the Great Depression.[11] The implication is that banks switch from debt finance to equity

[9]GKQ employ GHH preferences that are quantitatively very similar to our model but inconsistent with balanced growth.

[10]This issue is discussed at length in Shin (2009); explaining the financial crisis as a bank run, the author highlights the rising importance of alternative sources of debt finance such as money market funds.

[11]Data as described in Baron (2017) and kindly provided by the author. New equity issuance is derived from bank level net issuances, adjusting for dilutions and stock splits. Following Jagannathan, Stephens & Weisbach (2000), net issuances are decomposed onto new issuance = max(net issuance, 0) and repurchases = min(net issuance, 0). Baron (2017) hand-collects the 1930–1965 data from Moody's

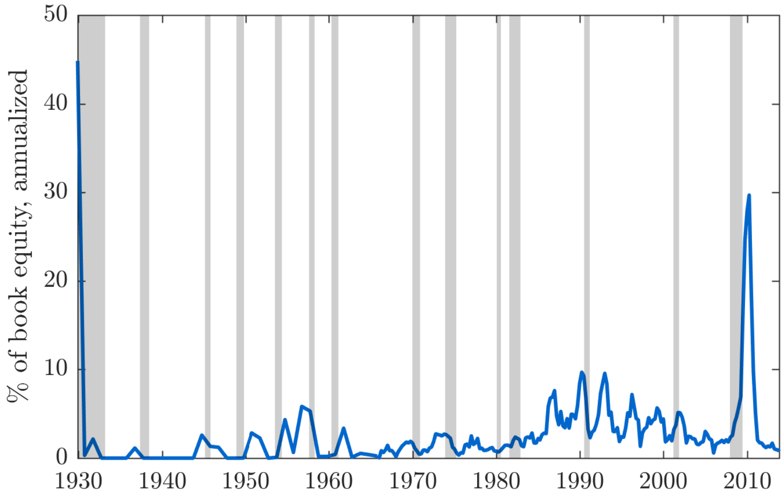

Figure 1: New equity issuance aggregated over the 20 largest U.S. commercial banks with NBER recession bands.

finance during periods of financial stress as the marginal value of finance rises higher than equity issuance costs. We propose that this is driven by constraints on debt finance beginning to bind.

One dimension in which debt finance is preferable to equity is due to a tax advantage (see, e.g., Jermann & Quadrini 2012), but a number of studies have also estimated the transaction costs associated with equity issuances (e.g., underwriter fees, legal costs). These estimates lie between 5 and 7 percent on average and fall in the size of offering (see Lee, Lochhead, Ritter & Zhao 1996, Altinkiliç & Hansen 2000, Hennessy & Whited 2007). As well as these explicit costs, raising equity finance is plagued by agency problems (see, e.g., Jensen & Meckling 1976, Myers & Majluf 1984, Miller & Rock 1985, Asquith & Mullins 1986).[12] These frictions result in implicit issuance costs that can be estimated by observing the change in share price following an offering. These observed declines in value have been estimated to be anything between 0.4 and 9.9 percent following offerings (Jensen 1986) with a mid-point of around 3 percent (Mann & Sicherman 1991, Altinkiliç

Bank and Finance Manuals for the largest 15 U.S. banks and takes 1965–2014 data from Compustat for the 20 largest banks.

[12] For example, costs resulting from asymmetric information leading to principle-agent problems and the implied dilution of current shareholders' value.

& Hansen 2003). Furthermore, whereas the transactional costs fall in the size of issuance, the implicit costs have been found to rise. Altinkiliç & Hansen (2000) find evidence in support of U-shaped total implicit and explicit issuance fees; the initial decline driven by falling transactional fees, and the subsequent rise due to the agency frictions. In this paper, the borrowing constraint is endogenous and equity issuance costs are exogenously imposed. Following Altinkiliç & Hansen (2000), these costs increase in aggregate equity issuance, acting as a congestion charge. This can be motivated by increases in agency costs following a large cross-sector equity issuance due, for instance, to costly monitoring and downward pressure on the issuance price as the market is flooded with new equity.

1.2 Related literature

A starting point for the model is the agency problem proposed in Kiyotaki & Moore (1997) and extended in the GK banking model. The authors introduce limited contract enforceability on bank borrowing that results in a financial friction between banks and households. It is assumed that banks can default on their debts and exit the market, so, as the courts can only reclaim a proportion of outstanding debts, endogenous borrowing limits arise. However, unlike other models of financial frictions, such as that of Bernanke, Gertler & Gilchrist (1999), there is no default in equilibrium, since households will only loan to a bank that has no incentive to default. This constraint on debt introduces a wedge between the risk-free rate and the expected discounted return on capital that fluctuates due to movements in the value of bank assets.

There is a growing literature looking at models with occasionally binding financial constraints. For instance, He & Krishnamurthy (2013) propose an occasionally binding constraint on equity, rather than on debt, in which interest premia rise sharply when the constraint binds, deepening downturns. The evidence, however, indicates that debt, rather than equity, is subject to occasionally binding constraints (see, e.g., Kashyap & Stein 2000, Calomiris & Mason 2003, Ivashina & Scharfstein 2010). In related work, Brunnermeier & Sannikov (2014) propose a model of constrained equity issuance that leads to non-linear dynamics; most fluctuations can be absorbed by the intermediaries' balance sheets but larger negative shocks might lead to unstable, volatile episodes. Akinci & Queralto (2014) and Bocola (2016) present occasionally binding extensions to GK, the latter to study the pass-through of sovereign risk. In both these papers, whether the constraint is occasionally binding depends crucially on model calibration, unlike our model. By removing the exogenous bank exit common to these studies, and allowing them to choose dividend payments, banks will borrow to the edge of the constraint, but can always raise equity finance for free in the vicinity of the steady state. It follows that credit crunches are occasional phenomena, in contrast to Akinci & Queralto (2014) and Bocola (2016), where it is implied that banks are constrained in the steady state. In related work, Jermann & Quadrini (2012) present a model that differentiates between the costs of debt and equity finance. This results in countercyclical equity issuance via a similar mechanism to our model, however, as with GK, the financial constraint is always binding and not subject to the endogenous variation that our model implies. There have been models of occasionally constrained household borrowing, including Guerrieri & Iacoviello (2017), who show that collateral constraints ceased to bind during the 2001–2006 U.S. housing boom but tightened during the crisis, exacerbating the recession that followed. Other related works include Dixon & Pourpourides (2016), who look at occasionally binding cash-in-advance constraints, and Abo-Zaid (2015), who imposes a collateral constraint on firms to guarantee promised wages to workers.

2 The model

The model features a household and firm sector common to the real business cycle literature, with the banking sector acting to intermediate funds between these two sectors.

2.1 Households

The representative household maximizes expected lifetime utility:

$$\max_{C_{t+s},H_{t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} U\left(C_{t+s}, H_{t+s}, X_{t+s}\right)$$

subject to the budget constraint:

$$C_t + B_t = W_t H_t + R_{t-1} B_{t-1} + D_t - E_t + \Pi_t - T_t,$$

where C_t is consumption, H_t is hours worked, X_t is a habit stock, W_t is the wage rate, and B_t is deposits with the bank that pay interest rate R_t in the following period. D_t and

Π_t are dividends paid and any other profits, respectively; E_t is bank equity purchased; and T_t represents lump-sum taxes. We assume that households cannot lend directly to firms, so the intermediation provided by banks is necessary to provide funding to firms.

To achieve co-movement between investment and consumption, we employ the preferences proposed in Jaimovich & Rebelo (2009), which allow for the control of the short-run wealth effect on labour supply. In particular, we suppose that the period utility takes the form:

$$U\left(C_t, H_t, X_t\right) = \frac{\left[C_t - \varrho H_t^{1+\psi} X_t\right]^{1-\sigma_c} - 1}{1-\sigma_c},$$

where:

$$X_t = C_t^{\gamma} X_{t-1}^{1-\gamma},$$

where $\sigma_c > 0$ is the intertemporal elasticity of substitution, $\varrho > 0$ is the utility weight on leisure, $\psi > 0$ controls the elasticity of labour supply, and $0 < \gamma \leq 1$ controls the wealth effect. When $\gamma = 0$, the preferences are equivalent to those of Greenwood, Hercowitz & Huffman (1988) (GHH) with no wealth effect on labour supply.[13]

Household optimization leads to the following Euler equation and labour supply condition:

$$1 = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}\right] R_t$$
$$-\frac{U_{H,t}}{\lambda_t} = W_t,$$

where $U_{H,t}$ is the marginal utility of labour, and λ_t^C is the Lagrange multiplier on the household budget constraint, i.e., the marginal value of income. λ_t^C is given by:

$$\lambda_t^C = U_{C,t} + \gamma \mu_t \frac{X_t}{C_t},$$

where μ_t is the Lagrange multiplier on equation (2.1), which is given by:

$$\mu_t = U_{X,t} + \beta\left(1-\gamma\right)\mathbb{E}_t\left[\mu_{t+1}\frac{X_{t+1}}{X_t}\right].$$

[13] Jaimovich & Rebelo (2009) preferences benefit from being compatible with balanced growth, unlike the GHH preferences used in GKQ, which are not.

2.2 The banking sector

Banks in the model are owned by households. As a result, they maximize their expected value, i.e., the expected present discounted value of net dividend payments, $D_t - E_t$. In treating equity issuance as a negative dividend payment, we are following, for example, Miller & Rock (1985). However, raising equity financing from households will be costly.

The relationship between banks and households is subject to an agency problem, which arises due to imperfect contract enforcement; banks are able to declare bankruptcy and exit with creditors able to reclaim only a proportion of the outstanding debt. This follows the collateral constraints model of Kiyotaki & Moore (1997), and more closely the extension to the banking sector by GK. However, whereas GK assume an exogenous bank exit rate that fixes the dividend rate and ensures the borrowing constraint is always binding, our model relaxes this assumption so that net dividend payments are endogenous and the borrowing constraint is only occasionally binding. While it is possible to parameterize the GK model to produce an occasionally binding borrowing constraint, the range of parameters for which this is true is narrow. Our approach avoids this problem, as, endogenously, in the steady state the bank will always be just on the edge of the constraint binding, irrespective of parameters. We also give the derivation of the borrowing constraint a more careful treatment, based on U.S. bankruptcy law. It turns out that this necessitates a role for government insurance of the banking sector against extreme tail events.

Bank j raises debt finance B_t^j promising to repay $R_t^j B_t^j$ the following period. A government guarantee on these savings mean that banks actually need repay only $\left(R_t^j - \mathcal{G}_{t+1}\right) B_t^j$ where $\mathcal{G}_{t+1}$ is only non-zero in the face of an extreme adverse shock that would otherwise cause a systemic banking collapse. The government funds this insurance via lump-sum taxes on households. The bank will pay dividends D_t^j and raise equity E_t^j. While making dividend payments is costless, we assume there are administrative costs involved in issuing equity. To bank j the cost is exogenous and linear in equity issuance, being equal to $\kappa_t E_t^j$. However, we model κ_t as an increasing function of aggregate equity issuance. This captures congestion externalities such as monitoring. Specifically, we let:

$$\kappa_t := \bar{\kappa} \left[1 - \exp\left(-\nu \frac{E_t}{\max\{0, V_t\}} \right) \right],$$

where V_t is the value of the entire banking sector, so E_t/V_t is the aggregate rate of equity issuance, and where $\bar{\kappa} \in (0,1)$ gives the maximum cost of equity issuance and ν is a parameter that determines the velocity at which κ_t converges to $\bar{\kappa}$.

Banks raise debt and equity finance in order to lend to the production sector. The lending channel is characterized by perfect monitoring and perfect contractual enforcement. Therefore, banks frictionlessly lend to firms against their future profits, and firms offer banks fully state-contingent debt, or, equivalently, equity. We denote by S_t^j the number of firm shares held by bank j at t, and we assume that each share delivers a gross return of R_t^K per unit. We will normalize the units of these shares such that one share entitles the owner to the gross return from the ownership of one unit of capital.

The book value of bank j at time t is given by:

$$\hat{V}_t^j \equiv \left[R_t^K S_{t-1}^j - \left(R_{t-1} - \mathcal{G}_t\right) B_{t-1}^j\right] \frac{1}{1-\kappa_t}. \tag{1}$$

This is the cost that households would have to pay in order to create a "copy" of bank j. Were equity issuance impossible (i.e., were $\kappa_t = 1$), then creating a "copy" of a bank with positive net worth would be impossible, or infinitely expensive. We assume that once equity is in the banking system, it may be transferred between banks without incurring additional costs. Thus $\hat{V}_t^j$ is also the maximum amount that another bank would be prepared to pay in order to purchase bank j. As such, $\hat{V}_t^j$ gives a "cash-out" value of the bank.

A bank that decides not to exit next period will face the budget constraint:

$$D_t^j + S_t^j + \left(R_{t-1} - \mathcal{G}_t\right) B_{t-1}^j \leq B_t^j + \left(1-\kappa_t\right) E_t^j + R_t^K S_{t-1}^j. \tag{2}$$

As previously stated, the objective of bank j is to maximize its expected value. Additionally, we suppose that where the household is strictly indifferent between dividends being paid today or in future, the bank has a preference toward paying dividends now. This may capture agency problems within the bank that lead to an excess focus on short-term returns, or it may reflect a remote fear of forced nationalization. In particular, the bank solves:

$$V_t^j = \max_{B_t^j, S_t^j, E_t^j, D_t^j} \left\{D_t^j - E_t^j + (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1} V_{t+1}^j\right]\right\}, \tag{3}$$

subject to the budget constraint (2) and the borrowing constraint, which is still to be derived, for $\iota \to 0^+$, where $\Lambda_{t,t+1} \equiv \beta \frac{\lambda_{t+1}^C}{\lambda_t^C}$ is the stochastic discount factor of the shareholders and V_t^j is the value of the bank. The term $(1-\iota)$ is superficially similar to the exogenous bank exit rate in Kiyotaki & Moore (1997) and GK but, since preferences are under the limit as $\iota \to 0^+$, its only impact is to capture banks' arbitrarily weak preference toward paying dividends sooner rather than later.[14] If the (arbitrarily small) additional discounting is interpreted as an idiosyncratic bank "death" shock, then a crucial difference between our approach and that of GK is that whereas the owners of our banks do not gain any value after the "death" shock (e.g., because the bank has been forcibly nationalized), in GK, dividends are paid only after the bank is hit with such a shock.

2.3 Bank exit and default

We now consider the default decision and other aspects of off-equilibrium play that are nonetheless critical for equilibrium outcomes. If bank j fails to repay outstanding debts in period t, the bank must file for chapter 7 bankruptcy. Following U.S. law (title 11 U.S.C. §548), any remaining assets are seized and sold at market value. If this is enough to repay $R_{t-1}B_{t-1}^j$, any remaining assets are paid to shareholders as a final dividend; otherwise, the court will examine the previous two years of dividend payments. If, when a dividend was paid, the value of the bank's liabilities were greater than the value of its assets, or the bank had "unreasonably small capital" at that point, then the dividend would be deemed fraudulent. In this model, we assume that all dividends paid within this two-year window would be considered fraudulent as at the point of default the bank was left insolvent.[15] Given that a fraudulent payment had been made, the court would then attempt to recover these dividends plus interest at the risk-free rate. It is assumed that this is a costly process due, for instance, to costs associated with tracking down shareholders, and so the court is able to recover only a fraction $(1-\theta)$ of the total amount sought, where $\theta \in (0,1)$. If the amount recovered is sufficient to cover $R_{t-1}B_{t-1}^j$ then any remaining funds are returned to shareholders; otherwise, the creditors take a

[14] $\iota > 0$ is required by our numerical strategy, as we take a perturbation approximation around the deterministic steady state, which would otherwise be indeterminate. Subject to numerical accuracy limits though, ι may be set arbitrarily small. This is discussed further in section 4.

[15] This is consistent with the legal definition of "unreasonably small capital" according to which payments would be considered fraudulent if it later transpired the firm was left with insufficient capital to repay creditors. See Wittstein & Douglas (2014) for further discussion.

haircut.

We first consider the bank's decision in period t whether to exit that same period. First, note that for the bank to fully meet its liabilities prior to an exit without default would require households to contribute $\max\{0, -\hat{V}_t^j\}$, since $\hat{V}_t^j$ includes the costs of equity issuance. Indeed, since bank j can always sell itself to another bank and receive $\hat{V}_t^j$, the bank can always receive $\hat{V}_t^j$ by a default-free exit in period t. As a result, it must always be the case that $V_t^j \geq \hat{V}_t^j$. Alternatively, the bank can decide to exit via default. Letting $\tau = 8$ (quarters) denote the horizon to which creditors can reclaim assets, then, the maximum amount that can be recouped from previous dividend payments is:

$$(1-\theta)\sum_{i=1}^{\tau}\left(\prod_{k=1}^{i} R_{t-k}\right) D_{t-i}^j.$$

Consequently, the value of a bank exiting in period t is:

$$\max\left\{\hat{V}_t^j, -(1-\theta)\sum_{i=1}^{\tau}\left(\prod_{k=1}^{i} R_{t-k}\right) D_{t-i}^j\right\}.$$

Thus, as $V_t^j \geq \hat{V}_t^j$, the bank will default if and only if:

$$V_t^j < -(1-\theta)\sum_{i=1}^{\tau}\left(\prod_{k=1}^{i} R_{t-k}\right) D_{t-i}^j.$$

If this occurs for bank j on the equilibrium path, then, by symmetry, all banks will default. So, to prevent a financial collapse, it would be rational for the government to bail out the banks in this extreme tail situation. In our model, we assume the government performs the smallest possible bail-out to avoid such a collapse, by choosing $\mathcal{G}_t$ such that the following complementarity condition holds:

$$\min\left\{\mathcal{G}_t, V_t + (1-\theta)\sum_{i=1}^{\tau}\left(\prod_{k=1}^{i} R_{t-k}\right) D_{t-i}\right\} = 0.$$

Thus, the government is effectively offering free insurance on firm equity to banks. Although this policy rules out bank default along the equilibrium path, it will lead to risk being underpriced relative to the efficient benchmark, since banks internalize the insurance against tail events that the government is providing. However, without artificial constraints on when banks can default, such insurance is inescapable, as banks are undertaking risky investments but promising safe returns.

We now move on to consider whether in period t a bank might like to plan to default in period $t+1$. Although the government insurance prevents unplanned default due to tail shock realizations, this is not sufficient to rule out defaults in which a bank deviates from the equilibrium path in advance of their eventual default. It is to avoid such planned defaults that households will restrict their lending to banks, leading to the borrowing constraint.

At this point, it is important to clarify the order of moves so as to correctly specify this off-equilibrium play. In particular, we assume that households observe all bank and aggregate variables from $t-1$ but only the period t aggregate shocks before choosing the maximum amount they are prepared to deposit at the bank in period t. The bank then chooses its individual variables subject to the implied borrowing constraint. The choice of this ordering is important; if households could observe bank behaviour in advance of borrowing decisions, then they would not lend to any bank that took an off-equilibrium action, as this would be interpreted as a preparation for default.

Now, the value of bank j at time t of preparing to default in $t+1$ is given by:

$$V_t^X = D_t^j - E_t^j - (1-\iota)\,\mathbb{E}_t\left[\Lambda_{t,t+1}\right](1-\theta)\sum_{i=1}^{\tau}\left(\prod_{k=1}^{i} R_{t+1-k}\right) D_{t+1-i}^j,$$

Suppressing the bank indices for neatness, we postulate that the borrowing constraint takes the form:

$$B_t \leq \mathcal{A}_t \hat{V}_t + \sum_{i=1}^{\tau-1} \mathcal{F}_{i,t} D_{t-i}, \tag{4}$$

for some values independent of the decisions of the bank in question $\mathcal{A}$, $\mathcal{F}_{1,t}$, $\ldots$, $\mathcal{F}_{\tau-1,t}$. The linearity of the borrowing constraint follows from the linearity of the objective function and the budget constraint in the state variables. The household will choose the limit on B_t so that the bank weakly prefers not to deviate from the equilibrium path by planning to default. Maximizing the value of exit subject to the borrowing constraint and the budget constraint implies that the borrowing constraint will bind, the bank will make no further investments (i.e., $S_t = 0$), and will issue no equity (i.e., $E_t = 0$). Again, because the budget constraint, objective function, and borrowing constraint are linear in the state and choice variables, the bank value function must be homogeneous of degree one in the state. Furthermore, as the bank can sell and then re-buy assets for the same price, the value function must have a linear representation in $\hat{V}_t$ and $\{D_{t-i}\}_{i=1}^{\tau-1}$, so:

$$V_t = \mathcal{M}_t \hat{V}_t + \sum_{i=1}^{\tau-1} \mathcal{N}_{i,t} D_{t-i}, \tag{5}$$

for some values independent of the decisions of the bank in question $\mathcal{M}_t$, $\mathcal{N}_{1,t}$, $\ldots$, $\mathcal{N}_{\tau-1,t}$.

To prevent default, the household must ensure that $V_t \geq V_t^X$. The weakest condition ensuring this implies:

$$\mathcal{A}_t = \frac{\mathcal{M}_t}{1-(1-\iota)(1-\theta)} - (1-\kappa_t), \tag{6}$$

$$\mathcal{F}_{i,t} = \frac{\mathcal{N}_{i,t} + (1-\iota)(1-\theta)\prod_{k=1}^{i} R_{t-k}}{1-(1-\iota)(1-\theta)}. \tag{7}$$

The bank maximizes objective (3) subject to the borrowing constraint (4), the budget constraint (2), and positivity constraints on D_t and E_t, where the value and book value of the bank are given by equations (5) and (1), respectively. By taking first-order conditions, substituting these first-order conditions back into the problem's Lagrangian and then matching the terms in each state variable, we arrive at:

$$(1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\tfrac{1-\kappa_t}{1-\kappa_{t+1}}\tfrac{\mathcal{M}_{t+1}}{\mathcal{M}_t}\left(R_t - \mathcal{G}_{t+1}\right)\right] = \left(1 - \tfrac{\lambda_t^B}{(1-\kappa_t)(1-(1-\iota)(1-\theta))}\right), \tag{8}$$

$$\mathcal{N}_{i,t} = \mathcal{Z}_{i,t}\frac{(1-\iota)(1-\theta)}{1-(1-\iota)(1-\theta)}\prod_{k=1}^{i} R_{t-k}, \quad i = 1,\cdots,\tau-1, \tag{9}$$

where:

$$\begin{aligned} \mathcal{Z}_{i,t} &\equiv \frac{\lambda_t^B}{1-\lambda_t^B}\frac{\mathcal{M}_t}{1-\kappa_t} + (1-\iota)\mathbb{E}_t\left[\mathcal{Z}_{i+1,t+1}\right], \quad i = 1,\cdots,\tau-2, \\ \mathcal{Z}_{\tau-1,t} &\equiv \frac{\lambda_t^B}{1-\lambda_t^B}\frac{\mathcal{M}_t}{1-\kappa_t}, \end{aligned} \tag{10}$$

and where λ_t^B is the Lagrange multiplier on the borrowing constraint. The first condition gives the law of motion for the marginal value of the bank book value; the second for the marginal value of past dividend payments. Defining:

$$\mathcal{H}_t \equiv \lambda_t^B + \mathcal{M}_t\left(1 - \frac{\lambda_t^B}{(1-\kappa_t)(1-(1-\iota)(1-\theta))}\right), \tag{11}$$

and:

$$\Xi_{t,t+1} \equiv (1-\iota)\Lambda_{t,t+1}\frac{1-\kappa_t}{1-\kappa_{t+1}}\frac{\mathcal{M}_{t+1}}{\mathcal{H}_t},$$

equation (8) and the first-order conditions for dividends, equity and shares can be written as:

$$\lambda_t^B = \mathcal{H}_t - \mathcal{H}_t \mathbb{E}_t \left[\Xi_{t,t+1} \left(R_t - \mathcal{G}_{t+1} \right) \right] \geq 0, \tag{12}$$

$$\lambda_t^D = \mathcal{H}_t - (1-\iota)(1-\kappa_t) \mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{N}_{1,t+1} \right] - (1-\kappa_t) \geq 0, \tag{13}$$

$$\lambda_t^E = 1 - \mathcal{H}_t \geq 0, \tag{14}$$

$$1 = \mathbb{E}_t \left[\Xi_{t,t+1} R_{t+1}^K \right], \tag{15}$$

where λ_t^D and λ_t^E are the Lagrange multipliers on the positivity constraints on dividend payments and equity issuance, respectively. The final equation implies that $\Xi_{t,t+1}$ is the pricing kernel (or stochastic discount factor) for firm equity.

2.4 Firms

The final good is produced by a perfectly competitive industry with access to the technology:

$$Y_t = \left(A_t H_t \right)^{1-\alpha} {K_{t-1}}^{\alpha},$$

where A_t is a stationary stochastic process. Firms producing the final good choose the amount of labour, H_t, and capital, K_{t-1}, to hire in order to maximize their profits, which are given by $Y_t - W_t H_t - Z_t K_{t-1}$, where Z_t is the rental rate of capital. Hence, from the first-order conditions, we have the usual marginal product conditions:

$$W_t = (1-\alpha) \frac{Y_t}{H_t},$$

$$Z_t = \alpha \frac{Y_t}{K_{t-1}}.$$

The capital stock is owned by firms in a perfectly competitive industry with access to the following technology for producing the next period's installed capital from investment and the previous period's capital:

$$K_t = \left[1 - \Phi \left(\frac{I_t}{I_{t-1}} \right) \right] I_t + (1-\delta) K_{t-1}, \tag{16}$$

where I_t is investment (of the final good), δ is the depreciation rate and Φ governs the Christiano, Eichenbaum & Evans (2005) style of investment adjustment costs, where

$\Phi(1) = \Phi'(1) = 0$ and $\Phi''(\cdot) = \phi > 0$. Since these capital-producing firms are owned by banks, they choose investment to maximize:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=0}^{s-1} \Xi_{t+k,t+k+1} \right] (Z_{t+s} K_{t+s-1} - I_{t+s}).$$

Therefore, from the capital producers' first-order conditions:

$$1 = Q_t \left(1 - \Phi\left(\frac{I_t}{I_{t-1}}\right) - \Phi'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \right) + \mathbb{E}_t \left[\Xi_{t,t+1} Q_{t+1} \Phi'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \right],$$

$$1 = \mathbb{E}_t \left[\Xi_{t,t+1} \frac{Z_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \right],$$

where Q_t is the Lagrange multiplier on equation (16), i.e., the value of a unit of installed capital. From comparing the second equation with equation (15), we see that the gross rate of return on shares in capital producers must be given by $R_t^K \equiv [Z_t + (1-\delta)Q_t] \frac{1}{Q_{t-1}}$ (i.e., the gross return on capital), since all capital producer returns are transferred to the bank in all states of the world.

Finally, the model is closed with the resource constraint

$$Y_t = C_t + I_t.$$

3 Theoretical results

Before turning to numerical results, we will discuss a few key theoretical properties of the model. All proofs are contained in Appendix A. We begin by focusing on the Lagrange multipliers and the coefficients of the bank's value function, as these offer some immediate insight into the importance of the financial constraints.

Proposition 1 $\forall t, \lambda_t^E = 0$*: that is, the positivity constraint on equity issuance never binds.*

This result brings computational benefits, as it means we can drop the inequality constraint $E_t \geq 0$, which will speed up simulation. Additionally, this result suggests that it can be optimal for banks to simultaneously issue equity and make dividend payments, thanks to the "signalling" value of dividend payments. Note that we are not using "signalling" in the typical asymmetric information sense here. Rather, the bank's decision to pay dividends communicates to households that they are unlikely to default in future, as dividend payments can be partially recovered following default, leading households to raise the borrowing limit. Without this channel, households would care only about $D_t - E_t$, and so, since issuing equity is costly, it could never be optimal to pay dividends while issuing equity.

To understand when simultaneous dividend and equity issuance might occur, recall that:

$$\lambda_t^D = \kappa_t - (1-\iota)(1-\kappa_t)\,\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{1,t+1}\right] \geq 0.$$

This tells us that if the marginal "signalling" value of paying a dividend is positive (i.e., if $\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{1,t+1}\right] > 0$), then it must be the case that $\kappa_t > 0$, which in turn implies that $E_t > 0$, as κ_t is an increasing function of E_t, with $\kappa_t = 0$ when $E_t = 0$. Furthermore, since issuing equity is costly, the total amount issued will be as low as possible. Therefore, if the bank has no other reason to issue equity, as the borrowing constraint is not binding, then it will be the case that $\lambda_t^D = 0$, implying that dividends payments are being funded by equity issuance. Such a situation is not implausible, as $\mathcal{N}_{1,t} > 0$ if $\Pr_t(\lambda_{t+k-1}^B > 0) > 0$ for any $k \in \{1, \ldots, \tau\}$. It follows that there is always a signalling value of making dividend payments, and as such equity will be issued every period. That said, if $\kappa_t'(E_t)$ is sufficiently high in the region of $E_t = 0$, then the amount of equity issued will be very low and could disappear entirely were there also fixed costs of issuance in our model.

Proposition 1 also implies that $\mathcal{H}_t = 1$, and so the stochastic discount factor applied to firms becomes:

$$\Xi_{t,t+1} \equiv (1-\iota)\Lambda_{t,t+1}\frac{1-\kappa_t}{1-\kappa_{t+1}}\mathcal{M}_{t+1}. \tag{17}$$

From this, it is easy to see that if the marginal value of an additional unit of funding is equal to one, and if the cost of equity issuance is constant, then in the limit as $\iota \to 0^+$, equation (17) will equal the household discount factor; that is to say, financial intermediation would be efficient.

Proposition 2 $\lambda_t^B = 0 \iff \mathcal{M}_t = 1$ *and* $\lambda_t^B > 0 \iff \mathcal{M}_t > 1$. *That is, the marginal value of bank finance is greater than one if and only if the borrowing constraint is binding.*

It follows that the borrowing constraint is slack only if $\mathcal{M}_t = 1$. We referred to $\mathcal{M}_t$ as the marginal value of the bank book value, but it can also be described as the shadow price of bank finance; it is intuitive that this increases above unity as the bank becomes financially constrained.

The spread between the savings rate and the expected return on equity gives a measure of the current strength of the financial friction. We are particularly interested in the component of the spread that emerges from the agency problem, rather than the risk premium component. This component is captured by the Lagrange multiplier on the borrowing constraint, λ_t^B. To see this, note that from equations (12) and (15), we have:

$$\lambda_t^B = \mathbb{E}_t \left[\Xi_{t,t+1} \left(R_{t+1}^K - (R_t - \mathcal{G}_{t+1}) \right) \right].$$

The size of the spread depends crucially on the cost of issuing equity; if the cost were always zero, there would be no financial friction as banks would issue equity until their borrowing constraint was slack. In the benchmark GK case, equity finance is ruled out entirely, which sets $\kappa_t = 1$ for all t. (GK also propose an extension in which equity finance can be issued but is subject to the same type of friction as debt finance.) Our approach highlights the role that costly equity issuance plays when debt finance is constrained. The marginal value of bank finance, $\mathcal{M}_t$, is the value of one extra dollar of finance on the balance sheet of the bank; if the bank can raise finance via reductions in dividend payment or increased borrowing, then this will equal one dollar. As equity is issued and κ_t increases, $\mathcal{M}_t$ rises above unity. An additional dollar of finance reduces the need to raise costly equity by one dollar today, and by lowering the leverage of the bank, will relax the borrowing constraint in this and future periods.

3.1 Deterministic steady state

The premise for our model of occasionally binding financial constraints is that financial intermediation is close to efficient in the vicinity of the steady state, but that sufficiently large adverse shocks can cause the financial constraints to bind. We can show that in the limit as $\iota \to 0^+$, banks are not financially constrained but just at the edge of the constrained region. It follows that financial intermediation is efficient in the limit, and in this region the borrowing constraints model replicates the standard RBC model. Throughout this paper, values without time subscripts will refer to steady-state values.

Proposition 3 *The borrowing constraint is slack in the steady state only if $\iota = 0$. The* *banking sector is at the edge of the constrained region in the steady state in the limit as* $\iota \to 0^+$.

Corollary 1 *If* $\iota > 0$*, then* $\mathcal{M} > 1$ *and* $\mathcal{N}_i > 0$ *for all* i*.*

Corollary 2 $\lim_{\iota \to 0^+} \mathcal{M} = 1$ *and* $\lim_{\iota \to 0^+} \mathcal{N}_i = 0$ *for all* i*.*

Corollary 3 *If* $\iota > 0$*,* $V > \hat{V}$ *and* $D > 0$*.* $\lim_{\iota \to 0^+} V = \hat{V}$*, and* $\lim_{\iota \to 0^+} E = 0$*.*

Corollary 4 *If* $\iota > 0$*,* $R^K > R$*.* $\lim_{\iota \to 0^+} R^K = R$*.*

These results indicate that, in the limit as $\iota \to 0^+$, the borrowing constraint becomes slack, the marginal value of dividend payments at any horizons i goes to zero, the marginal value of bank finance goes to unity, the value of the bank descends to its book value, equity issuance falls to zero, and the return on shares falls to the gross real interest rate. Together, these results tell us that steady-state financial intermediation is efficient in the limit as $\iota \to 0^+$, just as in a standard RBC model.

We conclude this section by noting that our model nests the standard RBC model for appropriate parameters.

Proposition 4 *If we take the limit as* $\iota \to 0^+$ *and either* $\kappa \to 0^+$ *or* $\theta \to 0^+$*, then the model converges to the standard real business cycle model.*

4 Numerical analysis

We calculate a second-order pruned perturbation approximation to the model, and then use news shocks to impose the inequality constraints, following the algorithm of Holden (2017*a*).[16] Following the basic algorithm of Holden (2017*a*), we treat the constraints in a perfect-foresight manner. That is, we approximate by assuming that the model's agents act today as if they were certain in which future periods the constraint would be binding.[17] We have experimented with more accurate simulations that do not make this

[16]The algorithm is implemented in the "DynareOBC" toolkit, which extends Dynare (Adjemian, Bastanie, Karamé, Juillard, Maih, Mihoubi, Perendia, Pfeifer, Ratto & Villemot 2011) to solve models featuring inequality constraints. This is available at `https://github.com/tholden/dynareOBC`. Holden (2017*b*) provides the theoretical foundations for this method.

[17]An identical perfect-foresight assumption is made in the solution algorithm of Guerrieri & Iacoviello (2015), but their algorithm works only with a first-order approximation to the underlying model, whereas

perfect-foresight approximation, and we found qualitatively similar results, suggesting that the precautionary effects associated with the bound are not overly important. However, performing calibration and producing average impulse responses at this higher level of accuracy are computationally difficult as the constraint is so close to binding in the steady state. Thus, for consistency we treat the bound in this perfect-foresight manner throughout. However, since we have a second-order solution to the underlying model, we will still capture precautionary effects stemming from the model's other non-linearities.

Because we perturb around the non-stochastic steady state, a strictly positive ι is necessary. To see this, suppose that both in this period and in the next, the borrowing constraints were slack. Then, a unit increase in dividend payments could be paid for by a unit increase in deposits now followed by a reduction in dividend payments of R_t in the next period. Thus, by the household Euler equation, households are indifferent about the level of dividends in this case.[18] Including $\iota > 0$ in the banker's discounting resolves this indeterminacy, and pins down the deterministic steady state. In practice, we set $\iota := 10^{-8}$ to minimize the departure from the $\iota \to 0^+$ world of our theoretical results, without introducing numerical problems.

4.1 Model parameters

We compare our numerical results to two benchmarks. A standard RBC model with $\mathbb{E}_t\left[\Lambda_{t,t+1}R^K_{t+1}\right] = \mathbb{E}_t\left[\Lambda_{t,t+1}\right]R_t$ so financial intermediation is efficient, and the GK borrowing constraints model with equity issuance, as outlined in Appendix B. These two benchmarks provide a never-binding financial friction in the case of the RBC model, and an always-binding financial friction in the case of the GK model.

Parameters common to the RBC literature are chosen to target a number of long-run ratios consistent with the literature. A discount factor $\beta = 0.995$ is chosen to achieve an average yearly real interest rate close to 2 percent; capital depreciates at 2.5 percent per quarter and the capital share is chosen to be $\alpha = 0.3$ as is standard in the literature.

the algorithm of Holden (2017*a*) can handle higher-order approximations. The Holden (2017*a*) algorithm also allows us to be sure that when there is multiplicity, we are choosing the solution that escapes the bound as soon as possible.

[18]More generally, households cannot be sure that the bank's borrowing constraint will be slack next period, and so they might strictly prefer one unit of dividends today to R_t units next period.

We choose $\varrho = 2.6$ to target a steady state value of hours to equal about one-third. Following Jaimovich & Rebelo (2009), we choose $\gamma = 0.001$, so it is small and positive, and choose $\psi = 0.4$, which corresponds to a Frisch elasticity of 2.5 when preferences take the GHH form. The second derivative of the investment adjustment cost is set as $\phi = 4$ and the (inverse) intertemporal elasticity of substitution is chosen as $\sigma_c = 2$, both within typical ranges from the literature. For the equity issuance costs, we choose a value for $\bar{\kappa}$ of 10 percent and set $\nu = 400$, which, in a fully non-linear solution, would imply that the costs would converge to the maximum for very small issuances. In our numerical simulations, the issuance costs typically fall in the 3 to 8 percent range.

The standard deviation of the total factor productivity shock, $\sigma_a = 0.0061$, is calibrated to hit a standard deviation of output of 1.015 percent,[19] and the persistence $\rho_a = 0.95$ is chosen to target a first-order output autocorrelation of 0.86.[20] We choose the proportion of assets that are unrecoverable after default, $\theta = 0.67$, to target a standard deviation of the spread between the deposit rate and the risky return on capital of 0.18 percentage points quarterly.[21] As the spread is close to zero in the unconstrained economy, the volatility of the spread is a natural choice for an additional target; in the absence of features such as liquidity premia, differing tax treatments and true default risk, the model inevitably underpredicts the mean spread.[22]

[19] This requires $\sigma_a = 0.0061$ in the RBC model and 0.0057 in the GK model.

[20] Non-banking data is 1983Q3–2016Q3 U.S. time series from `https://fred.stlouisfed.org`: GDP, FPI and PCEC for output, investment and consumption respectively, deflated using GDPDEF with CNP160V to convert to per capita. The Hodrick-Prescott filter is applied to these time series. The spread is that between Moody's Seasoned BAA and AAA Corporate Bond yields. New equity issuance is as described in Baron (2017) for the largest 20 U.S. commercial banks. For dividend payments, we sum dividends and share repurchases from Baron's (2017) data.

[21] θ is calibrated to 0.89 in the GK model with the same target.

[22] In the GK model, there are two additional parameters that control the survival rate of bankers and the amount transferred to new bankers, as well as parameters controlling outside equity issuance. The banker survival rate is equivalent to a dividend rate but has to be set high to ensure an always-binding constraint. We follow GK and set this to 0.975, which is equivalent to an expected survival rate of 10 years, and set the proportion of bank equity transferred to the new "start-ups" equal to 0.3. These allow a mean spread approximately equal to the observed 0.57 percentage points and a bank leverage ratio close to the average of 4, targeted in GK. We follow GKQ with our choice of equity issuance parameter values.

4.2 Impulse response functions and simulations

In order to assess the propagation of shocks and the role of the financial constraints, we compute the median impulse response functions for shocks to productivity and capital quality.[23] This follows GK, who argue that negative capital quality shocks should not be considered physical depreciation of capital, but rather represent some form of economic obsolescence; they also suggest a possible micro-foundation. As in GK, the inclusion of the capital quality shocks allows for the characterization of occasional "disaster" shocks. In particular, we will examine the impacts of a 5 percent unanticipated decline in capital quality.

Let us consider the role of the borrowing constraint following such a disturbance. When either the banks' demand for funding increases, or the borrowing constraint tightens due to a relative decline in the banks' continuation value, the banks must raise equity finance. If the bank is unable to raise sufficient finance through retained earnings, they must sell equity, paying issuance costs that rise in the volume of issuance. This causes the expected marginal value of bank finance, $\mathcal{M}_{t+1}$, to increase above unity. As dividend payments relax the borrowing constraint, it is optimal for the bank to keep paying dividends even as they begin to issue equity. Indeed, past dividends become particularly important to the bank once financially constrained; the lower the past dividend payments, the tighter the borrowing constraint. This is also true for the interest rate; the lower the interest rate over the previous two years, the tighter the constraint.

Now, recall that the households discount using the stochastic discount factor $\Lambda_{t,t+1}$, whereas equity is priced using $\Xi_{t,t+1}$. The latter augments the former with the marginal value of bank finance, implying that in the unconstrained case, $\Lambda_{t,t+1} = \Xi_{t,t+1}$, while $\Lambda_{t,t+1} < \Xi_{t,t+1}$ when there is a positive probability of financial constraints binding. The augmented stochastic discount factor is asymmetric as $\mathcal{M}_t \geq 1$, and has higher volatility than the household stochastic discount factor; if the expected marginal utility of future consumption increases relative to that of current consumption, as would be expected following an adverse shock, then $\Lambda_{t,t+1}$ would increase. Because the expected value of $\mathcal{M}_{t,t+1}$ is also likely to rise, $\Xi_{t,t+1}$ rises further still. This introduces a hedging value of debt finance that increases as the financial constraint tightens. Because of this, when a

[23] We take the median of the difference between 256 pairs of simulation runs, where each pair of runs has identical shocks, apart from one additional impulse in period 100 for the first of each pair.

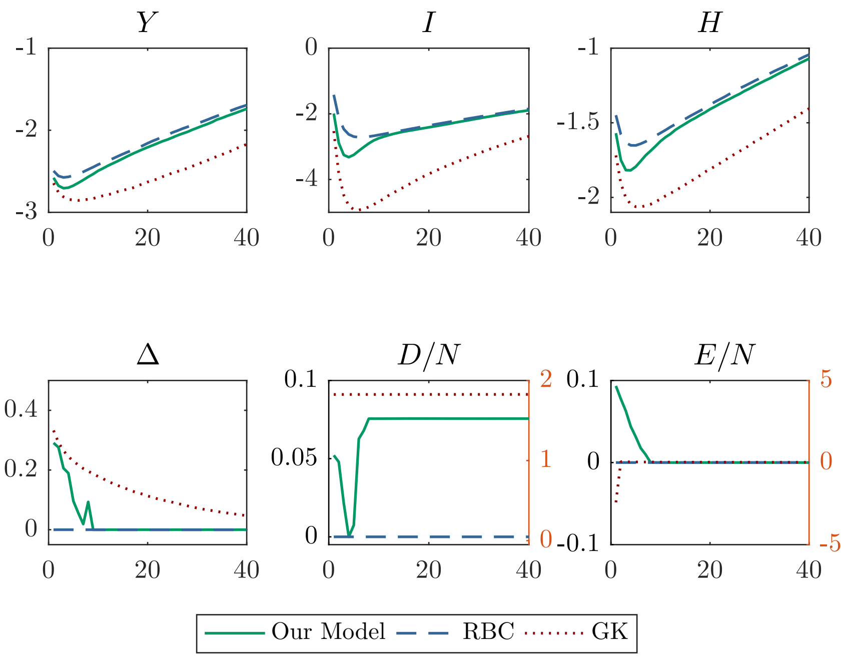

Figure 2: Median impulse responses to a negative 5 percent capital quality shock for our model, and the RBC and GK model. Plots show percent deviations from the ergodic median for Y, I, and H and level (percentage point) deviations for $\Delta = \mathbb{E}_t\left[R^K_{t+1} - R_t\right]$, D/N and E/N. The left axes of the last two plots correspond to our model, the right to GK.

bank experiences a balance sheet shock that reduces the value of assets, such as a capital quality shock, the value of equity falls relative to debt and the leverage of the bank will increase. This results in a further tightening of the borrowing constraint.

Figure 2 shows impulse response functions for output, investment, labour, the investment wedge ($\mathbb{E}_t[R^K_{t+1} - R_t]$), and rates of dividend payment and equity issuance to a 5 percent reduction in capital quality across the three models. Leisure is a normal good, so the presence of short-run wealth effects would imply that the adverse shock to household wealth would decrease demand for leisure and increase labour supply. Under most model specifications, this would imply an increase in investment following the shock as the poorer households consume less, and work and save more. This is overturned by reducing the short-run wealth effect on labour supply as the lower real wage rate causes a reduction in labour supply.[24] Banks in GK are always financially constrained but following the capital quality shock, the financial constraint tightens further, causing a larger decline in investment relative to the RBC model. Bank leverage increases when the value of assets falls, causing a reduction in both the borrowing limit and equity issuance. The contraction in available funds leads to a deeper decline in investment that remains below the RBC model into the long run. The effective dividend rate is fixed by the constant probability of banker exit. The decline in investment in our model is close to that of the GK model on impact, but begins to converge back to the RBC model after about five quarters. Nonetheless, the episode of constrained finance is persistent, with the investment wedge taking around three years to return to normal levels. Unlike in GK, banks increase equity issuance when the borrowing constraint tightens, and, due to the signalling role of dividend payments in relaxing the borrowing constraint, it is unnecessary for payments to cease before banks begin to issue equity. Indeed, for several periods, the banks simultaneously pay dividends and issue equity. Due to the costs of equity issuance, the marginal bank funding cost increases above the savings rate; this is the force behind the sharp rise in the interest spread and the deeper fall in investment relative to the RBC model.

A final point to note is that the impulse response functions are asymmetric and non-monotonic; the financial accelerator effects decline significantly as the size of adverse shocks falls, and are all but absent for shocks of the opposite sign. We illustrate this with further impulse responses in Appendix C.

4.2.1 Simulated moments

Table 1 reports simulated moments and cross-correlations for the three models together with those computed from the data. Our model introduces significant skewness in the interest spread that is entirely missing from the GK models, as well as skewness in equity issuance that arises due to occasional episodes of sharp issuances. Furthermore, when

[24]Investment does fall in both GK and our model on impact with standard King-Plosser-Rebelo (KPR) preferences as financial constraints tighten, but quickly rebounds, leading to an investment boom. The increase in investment can also be overturned with habits in consumption (see, e.g., Cochrane & Campbell 1999) as the substitution between consumption and savings become costly. We choose the Jaimovich-Rebelo approximation to GHH preferences, as both habits in consumption and KPR preferences imply a counterfactual increase in labour.

Table 1: Simulated and empirical moments. Standard deviation in percent except D, E and Δ, which are in percentage points.

		Y	I	C	D	E	Δ
Correlation with Y	Data	1	0.879	0.882	0.335	-0.279	-0.393
	Our model	1	0.956	0.984	0.344	-0.407	-0.405
	RBC	1	0.948	0.983	–	–	–
	GK	1	0.948	0.973	0.673	0.302	-0.651
Standard Deviation	Data	1.056	4.515	0.917	3.831	4.565	0.178
	Our model	1.056	1.659	0.894	0.631	0.013	0.179
	RBC	1.056	1.623	0.914	–	–	0
	GK	1.056	2.183	0.810	0.001	0.129	0.179
Skewness	Data	-0.240	-0.606	-0.315	0.343	3.599	1.670
	Our model	-0.013	-0.042	0.015	1.126	2.738	1.784
	RBC	-0.049	-0.085	-0.033	–	–	–
	GK	0.114	0.147	0.081	-0.193	0.029	-0.043

repurchases are included in the measure of gross dividends, as we do here, our model does well at predicting the cyclicality of both dividend payments and equity issuance. It also captures some of their volatility. In fact, without stock repurchases, dividend payments in the data are actually more stable than in our model, but the inclusion of stock repurchases substantially increases their volatility.[25]

Volatility in investment is lower than in the data due partly to the household preferences, and partly to the capital adjustment costs. This is higher in the GK model, resulting from the financial accelerator mechanism introduced by the borrowing constraint. Volatility of investment is between the RBC and GK models as the financial accelerator is in effect only when the borrowing constraints are binding.

[25] This discrepancy suggests the presence of additional factors, such as other agency problems, missed by the model. Dividend payments alone are acyclical or slightly countercyclical in the data and, given that banks appear to vary stock repurchases rather than dividend payments, the empirical time series suggest that dividends are used during downturns either as a signalling device to indicate the strength of the individual bank, or as a result of the reduced number of profitable investment opportunities.

5 Conclusion

This paper embeds a model of banking into a real business cycle framework, resulting in a model that generates occasional endogenous credit crunches. In the vicinity of the deterministic steady state, the model behaves much like a standard RBC model: financial intermediation is efficient and the interest rate spread is equal to the standard risk premium. Credit crunches are precipitated by sufficiently large adverse shocks that cause the bank financing constraint to bind. This is the result of an increased incentive for banks to divert funds and declare bankruptcy caused by a reduction in expected bank profits. Banks are able to issue equity when debt finance is constrained, but issuance costs introduce a wedge between the risk-free rate and the risky return to capital, resulting in reduced investment and output.

By removing the exogenous bank exit rate common to many similar models and allowing endogenous dividend payments, we find that the borrowing constraint is always occasionally binding, independent of calibration. Furthermore, in our model, credit crunches are truly an occasional phenomena in contrast to the majority of existing models in which financial constraints bind in steady state. A key contribution is a careful treatment of the Kiyotaki & Moore (1997) agency problem extended in GK. By modelling the U.S. law relating to bankruptcy, we reveal a potentially important signalling role for dividends in acting to relax the borrowing constraint. Finally, our model gives a number of improvements in the empirical fit of simulated time series. Notably, we capture the strong positive skewness in the interest spread and equity issuance that are missing in the standard RBC and GK models. We also replicate the countercyclical equity issuance observed in the data, contrary to other papers, such as GKQ, which predict procyclical equity issuance.

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Appendix A Proofs

Proof 1 (Proof of Proposition 1) *Substituting equation* (14) *into* (13) *gives*

$$\lambda_t^D = 1 - \lambda_t^E - (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(1-\kappa_t\right)\mathcal{N}_{1,t+1}\right] - \left(1-\kappa_t\right). \tag{18}$$

Suppose that $\lambda_t^E > 0$. *Then* $E_t = 0$ *by complementary slackness, so, from the definition of* κ_t*, the previous equation becomes:*

$$\lambda_t^D + \lambda_t^E + (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{1,t+1}\right] = 0, \tag{19}$$

and so $\lambda_t^D = \lambda_t^E = (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{1,t+1}\right] = 0$ *giving the required contradiction.* □

Proof 2 (Proof of Proposition 2) *Substituting* $\mathcal{H}_t = 1$ *into equation* (11) *leads to:*

$$\mathcal{M}_t = \frac{\left(1-\lambda_t^B\right)\left(1-\kappa_t\right)\left(1-\left(1-\iota\right)\left(1-\theta\right)\right)}{\left(1-\kappa_t\right)\left(1-\left(1-\iota\right)\left(1-\theta\right)\right)-\lambda_t^B} \tag{20}$$

Since $0 \le \left(1-\kappa_t\right)\left(1-\left(1-\iota\right)\left(1-\theta\right)\right) < 1$*, it follows that* $\mathcal{M}_t = 1$ *if and only if* $\lambda_t^B = 0$. *Given that* $\mathcal{M}_t \ge 1$ *as a bank can always sell itself to another bank for* $\hat{V}_t$*, independent of its history of dividend payments, this also implies that* $\mathcal{M}_t > 1$ *if and only if* $\lambda_t^B > 0$. □

Proof 3 (Proof of Proposition 3) *Using equation* (8)*, we have that the steady-state value of* λ_t^B *is given by:*

$$\lambda^B = \iota\left(1-\kappa\right)\left(1-\left(1-\iota\right)\left(1-\theta\right)\right) \in (0,1), \tag{21}$$

where throughout this document, values without time subscripts will refer to steady-states. This implies that the borrowing constraint binds with positive ι *but* $\lim_{\iota\to 0^+}\lambda^B = 0$. *As* λ^B *is the Lagrange multiplier on the borrowing constraint, the claim follows.* □

Proof 4 (Proof of Corollaries 1 and 2) *The results for* $\mathcal{M}$ *in Corollaries 1 and 2 follow immediately from Proposition 2. Indeed, from equation* (20)*, we find:*

$$\mathcal{M} = \frac{1-\iota\left(1-\kappa\right)\left[1-\left(1-\iota\right)\left(1-\theta\right)\right]}{1-\iota} > 1 \tag{22}$$

and so in the limit as $\iota \to 0^+$*, we have* $\mathcal{M} \to 1$. *The same is true for* $\mathcal{N}_i$ *as:*

$$\mathcal{N}_i = \mathcal{Z}_i\frac{\left(1-\iota\right)\left(1-\theta\right)}{1-\left(1-\iota\right)\left(1-\theta\right)}R^i, \quad i = 1,\cdots,\tau-1, \tag{23}$$

where:

$$\mathcal{Z}_i \equiv \frac{1-(1-\iota)^{\tau-i}}{\iota}\frac{\lambda^B}{1-\lambda^B}\frac{\mathcal{M}}{1-\kappa} > 0, \quad i = 1, \cdots, \tau-1. \tag{24}$$

So $\mathcal{N}_i > 0$ *for* $i = 1, \cdots, \tau - 1$*, but as* $\iota \to 0$*,* $\mathcal{Z}_i \to 0$ *and* $\mathcal{N}_i \to 0$*.* □

Proof 5 (Proof of Corollary 3) *The value of the bank is given by:*

$$V = \mathcal{M}\hat{V} + \sum_{i=1}^{\tau-1} \mathcal{N}_i D. \tag{25}$$

Hence, the value of a bank is always greater than its book value for $\iota > 0$*, but* $\lim_{\iota \to 0^+} V = \hat{V}$*.*

Now, banks must pay dividends in steady state, at least with $\iota > 0$*, for, suppose they did not. Then, their steady-state value would be zero, by the definition of bank value, and so since book value is always weakly below value, their steady-state book value would be non-positive. However, since equity issuance is always strictly positive with* $\iota > 0$*, steady-state book-value would be infinite without dividend payments, giving the required contradiction. Consequently:*

$$\lambda^D = \kappa - (1-\iota)\frac{\beta}{\Pi^*}\mathcal{N}_1 (1-\kappa) = 0, \tag{26}$$

so:

$$\kappa = \frac{(1-\iota)\frac{\beta}{\Pi^*}\mathcal{N}^1}{1 + (1-\iota)\frac{\beta}{\Pi^*}\mathcal{N}_1} > 0. \tag{27}$$

It follows from $\lim_{\iota \to 0^+} \mathcal{N}_1 = 0$*, that* $\lim_{\iota \to 0^+} \kappa = 0$ *and so there is no equity issuance in the limit.* □

Proof 6 (Proof of Corollary 4) *Note:*

$$R = (1 - \iota(1-\kappa)[1-(1-\iota)(1-\theta)])\, R^K, \tag{28}$$

so $R^K > R$ *but* $\lim_{\iota \to 0^+} R^K = R$*.* □

Proof 7 (Proof of Proposition 4) *First suppose that* $\kappa = 0$*. In this case, the first order condition with respect to dividend payments becomes:*

$$\lambda_t^D = -(1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{1,t+1}\right]. \tag{29}$$

Now, it follows from the definition of $\mathcal{N}_{1,t}$ in equations (9) *to* (10)*, that $\mathcal{N}_{1,t} \geq 0$ for all t, since $\lambda_t^B \in [0,1]$, by equation* (12)*. Hence, since $\lambda_t^D \geq 0$, equation* (29) *in fact implies that $\lambda_t^D = \mathcal{N}_{1,t} = 0$ for all t. Consequently, again by equations* (9) *to* (10)*, we must also have that $\lambda_t^B = 0$ for all t, which in turn implies that $\mathcal{M}_t = 1$ for all t, by Proposition 2. Using this in the definitions of the pricing kernels for bank and firm equity, we find that when $\iota = 0$ as well, $\Lambda_{t,t+1} = \Xi_{t,t+1}$ for all t, so financial intermediation is efficient. The bank is never financially constrained as they can always raise equity finance at no cost.*

Next, suppose that $\theta = 0$. Recall the borrowing constraint is of the form:

$$B_t \leq \mathcal{A}\hat{V}_t + \mathcal{F}_{1,t}D_{t-1} + \cdots + \mathcal{F}_{\tau-1,t}D_{t-\tau+1}. \tag{30}$$

If $\theta = 0$, then as $\iota \to 0$, it follows from the solutions of the coefficients in equations (6) *and* (7)*, that $\mathcal{A}_t \to \infty$ and $\mathcal{F}_{i,t} \to \infty$ for $i = 1, ..., \tau - 1$. So in the limit as $\iota \to 0$, borrowing becomes unlimited. As in the previous case, it follows that for all t, $\lambda_t^B = 0$, $\mathcal{M}_t = 1$ and $\Lambda_{t,t+1} = \Xi_{t,t+1}$ if $\iota = 0$, and so financial intermediation is efficient.*

Appendix B Gertler & Kiyotaki (2010) model

We describe a version of the GK model extension with equity issuance. The household and firm sectors are identical to our model, the difference is on the intermediation of funds between these two sectors. Every period, banks face a constant probability, $1-\sigma_B$, of exiting and paying the household a dividend. No dividend is paid if the bank continues, the bank decides on debt and outside equity finance and issues loans to non-financial firms. When a bank exits, a new bank takes is place and is transferred a fraction ξ_B of the exiting banks' net worth. Bank activity is subject to financial constraints as the inside shareholders can divert assets. In particular bank j solves

$$\begin{aligned} V_t^j = \max_{S_t,B_t,E_t} \; & \mathbb{E}_t\{\,(1-\sigma_B)\,N_{t+1}^j + \sigma_B\Lambda_{t,t+1}V_{t+1}^j\} \\ \text{s.t.} \quad & V_t^j \geq \Theta\left(x_t^j\right)S_t^j \\ & N_t^j = R_t^K S_{t-1}^j - R_t^E Q_{t-1}^E E_{t-1}^j - R_{t-1}B_{t-1}^j \\ & S_t^j = B_t^j + Q_t^E E_t^j + N_t^j \end{aligned}$$

where E_t is the stock of outside equity, rather than new issuance of inside equity as in our model, Q_t^E is the price of equity, and R_t^E is the rate of return on outside equity. Where each unit of $E_t Q_t^E$ is a claim on one unit of S_t, itself a claim on a unit of $Q_t K_t$. The proportion of divertable assets, θ is a quadratic function of $x_t \equiv Q_t^E E_t / S_t$:

$$\theta\left(x_t^j\right) = \bar{\theta}\left(1 + \epsilon x_t + \frac{\kappa^{GK}}{2} x_t^2\right)$$

Dropping bank indices, this leads to demand equations for debt and equity finance

$$\begin{aligned}
\nu_t^b &= \phi_t \left(\theta\left(x_t\right) - \left[\mu_t^s + \mu_t^e x_t\right]\right) \\
\mu_t^e &= \left[\mu_t^s + \mu_t^e x_t\right] \frac{\theta'\left(x_t\right)}{\theta\left(x_t\right)}
\end{aligned}$$

with $\phi_t \equiv \frac{S_t}{N_t}$ and where

$$\begin{aligned}
\Omega &\equiv 1 - \sigma_B + \sigma_B \theta\left(x_t\right) \phi \\
\mu_t^s &\equiv \mathbb{E}_t\left[\Lambda_{t,t+1} \Omega_{t+1}\left(R_{t+1}^K - R_t\right)\right] \\
\nu_t^b &\equiv \mathbb{E}_t\left[\Lambda_{t,t+1} \Omega_{t+1} R_t\right] \\
\mu_t^e &\equiv \mathbb{E}_t\left[\Lambda_{t,t+1} \Omega_{t+1}\left(R_t - R_{t+1}^E\right)\right]
\end{aligned}$$

Finally, the demand for outside equity must satisfy

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1} R_{t+1}^E\right].$$

Appendix C Additional impulse response functions

In addition to the impulse response function to an adverse capital quality shock in the paper, here we show a positive capital quality shock that highlights the asymmetry. Plots of the responses to a positive 5 percent capital quality shock are shown in figure 3. The same is true for shocks to total factor productivity. As a negative productivity shock decreases the continuation value of the bank, or the value of future profits, the constraint tightens. As there is also a decline in the value of bank assets, which acts in the opposite direction, a large shock is required to cause the borrowing constraint to bind sufficiently to have a large impact. There is a small financial accelerator for adverse shocks, but as shown in figure 4, this effect is not persistent and the model converges quickly to the RBC model. As shown in figure 5, for positive technology shock there is little difference between our model and the RBC model.

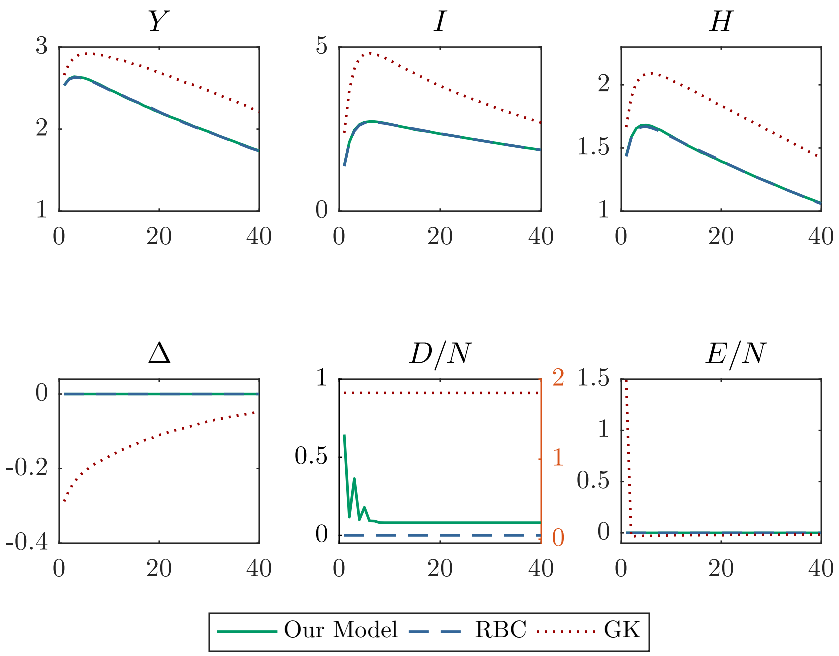

Figure 3: Median impulse response functions to a positive capital quality shock for our model, and the RBC and GK models. The shock is defined as a one time 5 percent increase in the capital stock. Plots show percent deviation from ergodic median for Y, I, and H and level deviation for $\Delta = \mathbb{E}_t\left[R^K_{t+1} - R_t\right]$, D/N and E/N. The left axis of the D/N plot corresponds to our model, the right to GK.

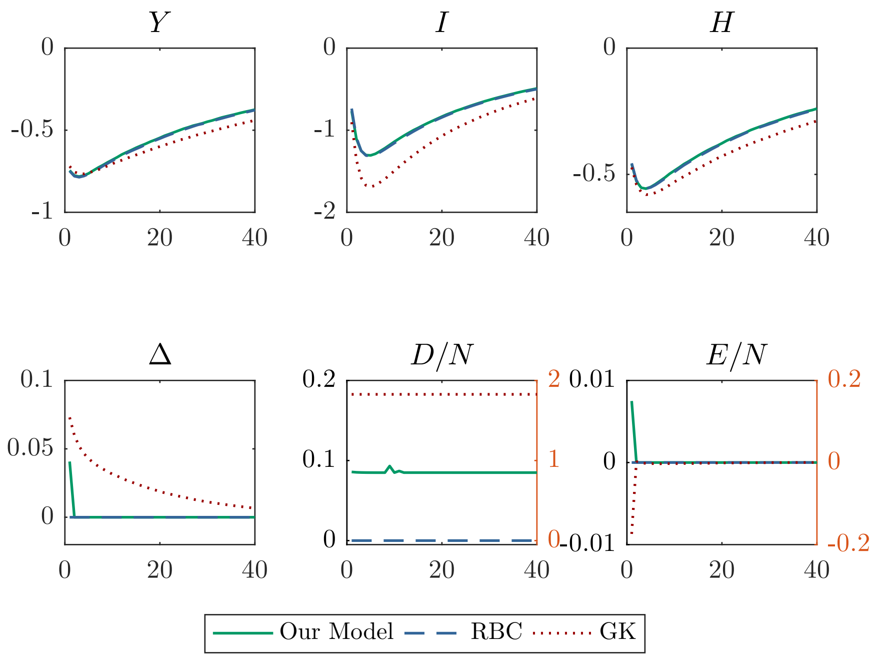

Figure 4: Median impulse response functions to a 1-standard deviation, negative productivity shock in our model, and the RBC and GK models. Plots show percent deviations from ergodic median for Y, I, and H and percentage point deviations for $\Delta = \mathbb{E}_t\left[R^K_{t+1} - R_t\right]$, D/N and E/N. The left axes of the last two plots correspond to our model, the right to GK.

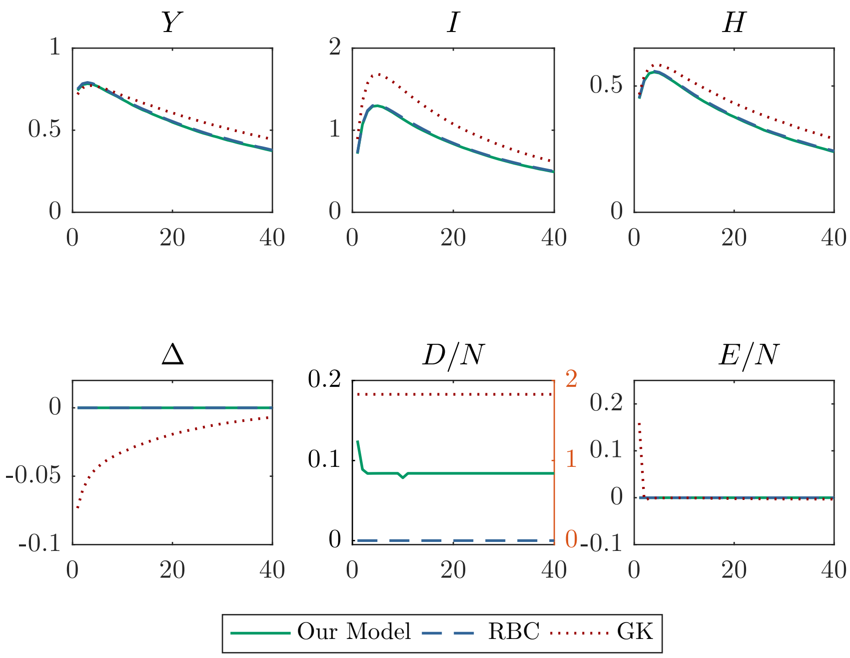

Figure 5: Median impulse response functions to a 1-standard deviation, positive productivity shock in our model, and the RBC and GK models. Plots show percent deviations from ergodic median for Y, I, and H and percentage point deviations for $\Delta = \mathbb{E}_t\left[R^K_{t+1} - R_t\right]$, D/N and E/N. The left axis of the D/N plot corresponds to our model, the right to GK.