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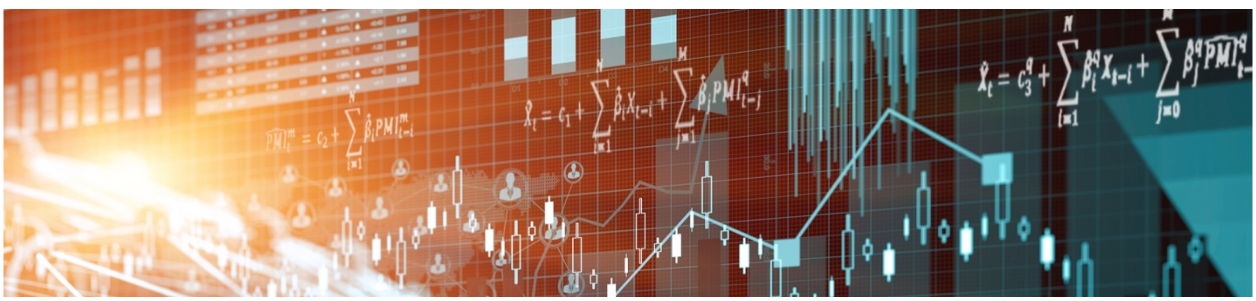
Quantitative Easing in a Small Open Economy: An International Portfolio Balancing Approach



by Serdar Kabaca

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Quantitative Easing in a Small Open Economy: An International Portfolio Balancing Approach

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Serdar Kabaca

International Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
kaba@bankofcanada.ca

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Abstract

This paper studies the effects of quantitative easing (QE) in a small open economy dynamic stochastic general-equilibrium model with international portfolio balancing. Portfolios are classified as imperfectly substitutable short-term and long-term sub-portfolios, each including domestic and foreign bonds. Unlike in standard small open economy models, both domestic and foreign bonds may be traded internationally. The model links the domestic term premium to the global term premium, and the implication of the model on the effectiveness of QE policies in reducing the domestic term premium depends crucially on the degree of substitutability between domestic and foreign bonds. The estimated model implies that QE in small open economies is expected to be much less effective on long-term yields because of the high substitutability between home and foreign assets found in the data. In the model, this causes the effect on the exchange rate to be limited. The paper also shows that foreign investors' access to the domestic debt market is essential when evaluating the QE policy; ignoring foreign investors' access would mistakenly make the policy look more effective.

Bank topics: Transmission of monetary policy; International topics
JEL codes: E52; F41

Résumé

Cette étude analyse les effets de l'assouplissement quantitatif dans un modèle d'équilibre général dynamique et stochastique adapté à une petite économie ouverte qui inclut des rééquilibrages de portefeuilles internationaux. Les portefeuilles sont classés comme des sous-portefeuilles de court et de long terme non parfaitement substituables, chacun comportant des obligations nationales et étrangères. Contrairement aux modèles standards de petite économie ouverte, les obligations nationales et étrangères peuvent faire l'objet d'échanges internationaux. Le modèle lie la prime de terme nationale à la prime de terme mondiale, et les résultats du modèle en ce qui concerne l'efficacité des politiques d'assouplissement quantitatif à réduire la prime de terme nationale dépendent fortement du degré de substituabilité entre les obligations nationales et étrangères. Le modèle estimé implique que l'assouplissement quantitatif dans les petites économies ouvertes devrait avoir une efficacité beaucoup moindre sur les rendements à long terme en raison de la grande substituabilité entre les actifs nationaux et étrangers observée dans les données. Dans le modèle, l'incidence sur le taux de change s'en trouve ainsi limitée. L'étude montre également que l'accès des investisseurs étrangers au marché intérieur de titres d’emprunt constitue un facteur essentiel à considérer lorsqu’on évalue la politique d’assouplissement quantitatif : ignorer ce facteur ferait faussement paraître la politique plus efficace.

Sujets : Transmission de la politique monétaire; Questions internationales
Codes JEL : E52, F41

Non-Technical Summary

Facing the zero lower bound constraint on nominal short-term interest rates, many advanced economies have performed unconventional monetary policy measures such as quantitative easing (QE) to alleviate recessionary pressures after the global financial crisis. This paper investigates the effectiveness of QE policies on long-term interest rates, the exchange rate and the aggregate demand in a small open economy, and compares the effects with those that would result from a large-economy set-up.

The paper proposes a small open economy, dynamic stochastic general-equilibrium model with nominal and real rigidities, and international portfolio balance effects. Portfolios are classified as short- and long-term sub-portfolios, each including domestic and foreign bonds. All assets are allowed to be traded internationally; however, bond holders face portfolio adjustment costs, making assets imperfectly substitutable with each other. Having an imperfect substitution between short- and long-term sub-portfolios, the model links the domestic term premium to the global term premium, and the implication of the model on the effectiveness of QE policies in reducing the domestic term premium depends crucially on the degree of substitutability between domestic and foreign bonds in each maturity.

The model is then estimated to gauge the substitution between domestic and foreign bonds using the Canadian and US data. The estimated model implies a high substitution between domestic and foreign bonds in each maturity, and thus, a strong link between the global and domestic term premium. Quantitatively, the QE effect on the domestic term premium is found to be one-third of what a QE policy would imply in a large economy. In addition, the paper shows that foreign investors' access to the domestic debt market is essential when evaluating the QE policy; ignoring foreign investors' access would mistakenly make the policy look more effective.

Finally, the paper also analyzes the effects of QE on the exchange rate. The model implies an exchange rate depreciation following a QE policy even without a short-term interest rate differential with the world. This is due to a slightly smaller substitutability between domestic and foreign bonds at the short end. Nevertheless, the exchange rate effect is not enough to cover the losses stemming from a smaller drop in long-term interest rates. Even with the stimulatory effects of the exchange rate on the external demand, QE is about half as effective at stimulating the total aggregate demand in a small open economy relative to a large economy.

1 Introduction

Following the global financial crisis, many advanced economies have used unconventional policy measures such as quantitative easing (QE; also known as large-scale asset purchases or LSAP). Several studies have found stimulatory effects from QE in terms of lowering long-term yields and strengthening the economic activity (see, among others, D'Amico et al., 2012; Gagnon et al., 2011; and Krishnamurthy and Vissing-Jorgensen, 2011). These studies concentrate on systemic and large advanced economies such as the euro area, Japan, the United Kingdom and the United States, since QE policies were implemented mainly in those economies due to the severity of the zero lower bound (ZLB) problem that they faced. Recently, the policy rates in small advanced economies, too, have approached near-zero levels (Figure 1), making alternative policies such as asset purchases a possibility to consider in the near future. This paper addresses the effects of large-scale asset purchases in a small open economy (SOE) model and underscores the differences from those results that are drawn in a large, approximately closed economy set-up.

In order to evaluate large-scale asset purchases, I propose an SOE version of a dynamic stochastic general-equilibrium (DSGE) model with nominal and real rigidities as well as international portfolio balance effects. Portfolios are classified as short- and long-term sub-portfolios, each including domestic and foreign internationally traded bonds. Portfolio balance effects are modelled via portfolio adjustment costs in the household's optimization problem. Thus, households incur costs when adjusting portfolio shares between short- and long-term portfolios as well as between home and foreign assets. The imperfect substitution among bonds leads to deviations from classical arbitrage conditions for asset prices such as the expectation hypothesis for long-term interest rates and the uncovered interest rate parity (UIRP) condition. The model shows that imperfect substitution between short- and long-term bonds, the known requirement for the portfolio balancing channel of QE, is not sufficient for QE to lower the domestic term premium in a small open economy. An effective QE also requires an imperfect substitution between domestic and foreign bonds, either in the short- or long-term markets.[1] In the case where home and foreign bonds are perfect substitutes in both maturities, households can perfectly rebalance their portfolios using foreign bonds to maintain their short- to long-term asset positions, resulting in no stimulatory effects from a QE policy on long-term interest rates and the exchange rate.

Portfolio adjustment costs in the household's problem can be motivated by financial institutions' relative portfolio preferences with respect to the different types of government bonds (i.e., their "preferred habitat"). For instance, pension funds may prefer to hold relatively more long-term

[1] The substitution between home and foreign bonds might differ in different maturities, too. For instance, home and foreign bonds might be less substitutable in long-term maturities due to uncertainties in the integration of economies in the long run. On the other hand, the particular use of short-term home government liabilities in the interbank market or in daily transactions for consumption might make home and foreign government liabilities less substitutable in short-term maturities, too. In the model, I take into account possible differences of substitution between home and foreign assets in different maturities.

government bonds to match their future projected cash flows, and may be less willing to rebalance their portfolios when there is a change in the relative price of short- to long-term assets (Andres et al., 2004). In addition, home and foreign assets could be imperfect substitutes because of the differential convenience and safety benefits generated by these assets, or because of the asymmetric information cost that investors might face in international markets (Huberman, 2001).[2] Hau and Rey (2004) find some evidence in support of the international portfolio balance channel affecting exchange rates using a vector autoregressive framework. Moreover, Valchev (2015) argues that imperfect substitution between home and foreign short-term bonds helps solve the interest parity puzzle.[3]

Another important feature of this model is that it allows foreigners' access to domestic debt markets, unlike many SOE models that assume perfect substitution between assets (see, for instance, Adolfson et al., 2008; Gali and Monacelli, 2005; Justiniano and Preston, 2010a). Under perfect substitution, when both assets are allowed to be traded internationally, an indeterminacy problem arises for gross holdings of home and foreign assets. On the other hand, the imperfect substitution among assets in the model presented here pins down a unique solution for gross holdings of home and foreign assets by both domestic and foreign investors. Foreign investors' access to the domestic debt market, thus modelling gross asset positions, is shown to be essential for understanding the effectiveness of QE policy. In the absence of foreigners' access, a QE policy would lead to a rebalancing only in domestic residents' portfolios. Facing a larger decrease (increase) in long-term (short-term) bond holdings in their portfolios, domestic residents would require a larger drop in long-term interest rates, implying a bigger stimulus to the economy. Therefore, ignoring foreigners' access to the domestic debt market would overstate the effect of QE in a small open economy.

In the empirical section, I estimate the model using the Canadian data and find that a QE policy in a small open economy can lower the domestic term premium by only about one-third the size of the reduction in the large-economy term premium following the same QE shock. The smaller QE effect in the model arises as a result of a large supply of foreign substitutes in the world. Thus, home and foreign bonds are estimated to be highly substitutable, albeit not perfectly. The small effect on long-term rates also implies a limited effect on the exchange rate depreciation following a QE policy. As a result, the exchange rate channel, a key channel in small open economies relative to large economies, is not stimulatory enough to cover the losses stemming from a smaller drop in the domestic term premium.

Highly substitutable home and foreign bonds in the estimated model imply a domestic term premium that is likely to move with the global term premium, which is taken as exogenous by

[2]See Krishnamurthy and Vising-Jorgensen (2012), and Valchev (2015) for a discussion on the convenience and safety benefits provided by government bonds.

[3]The literature on imperfect substitution between home and foreign assets dates back to Hendersson and Rogoff (1982) and Kouri (1983). Both papers and, more recently, Blanchard et al. (2005) relax the interest parity condition by assuming an imperfect substitution between home and foreign bonds. These papers mainly focus on the current account determination and do not explore QE policies.

the small open economy. The strong co-movement between domestic and foreign term premia is also consistent with the evidence presented in literature. The left panel in Figure 2 shows term premium estimates over time for several small open economies, obtained from Wright (2011). For comparison, the US term premium is also added as a benchmark global term premium. Term premia across countries generally move together over time, although deviations among them are still possible. In addition, term premia in countries that are economically more integrated, such as Canada and the United States, or Australia and New Zealand, tend to follow each other more closely. Lastly, model-free, survey-based estimates for the Canadian and US term premium shown in the right panel of Figure 2 also confirm the strong co-movement between two premia.[4]

To my knowledge, this paper is the only paper that incorporates the ideas of Tobin (1969) on imperfect asset substitution into an SOE-DSGE model and provides estimates on portfolio balance effects between not only short- and long-term but also domestic and foreign assets. Andres et al. (2004) incorporate the portfolio balancing channel into a closed economy DSGE model, generating imperfect substitution between long-term assets and money through liquidity concerns in preferences. Chen et al. (2012) use a similar set-up to study the effects of QE in a closed economy context. Dorich et al. (2012) also consider a similar set-up and analyze the effects of QE within an SOE model featuring the exchange rate channel. However, these models do not take into account the substitutability between home and foreign bonds or a link between the global and domestic term premia, nor foreigners' access to domestic debt markets. Recently, in Alpanda and Kabaca (2015), we evaluate international spillovers of QE policies in a calibrated two-country DSGE model with portfolio balance effects. The model in that paper studies term premium spillovers across two regions; however, it is not suitable for a small open economy QE analysis. In addition, this paper contributes to the literature by providing estimates on portfolio balance effects. Lastly, Chin et al. (2015) introduce term premia into an estimated SOE-DSGE model; however they do not explore substitution across home and foreign bonds, nor QE effects in such models.

The remainder of the paper is organized as follows. Section 2 presents the SOE-DSGE model with international portfolio balance effects. Section 3 describes the data and discusses the estimation of the model. Section 4 discusses the quantitative implications of a QE simulation in the estimated model. Section 5 concludes.

2 Small Open Economy Model

This section details an SOE-DSGE model with real and nominal rigidities, as well as portfolio balance effects. Real rigidities are represented by a habit formation in consumption, while nominal rigidities are represented by a Calvo-style price adjustment and an indexation of prices. These rigidities

[4] In addition, empirical studies such as Hofmann and Takáts (2015) and Obstfeld (2015) also estimate larger spillovers from long-term than short-term US interest rates to the rest-of-the-world yields, even after controlling for domestic and global factors.

are standard in the DSGE literature and capture the persistence in inflation and consumption in line with the vector autoregression (VAR) evidence. Portfolio balance effects are introduced through a utility cost in the households' problem when adjusting their financial portfolio. The household's portfolio includes short- and long-term sub-portfolios, each including internationally traded home and foreign government bonds. Besides households, the country is populated by final-goods producers, intermediate-goods producers, retailers, and fiscal and monetary policy authorities. QE policy is modelled via a negative shock to the market supply of long-term bonds that increases the supply of short-term bonds through a consolidated government budget constraint. Foreign bloc is exogenous to the small open economy. Thus, the economy takes foreign goods and asset prices as given. When necessary, foreign variables are denoted with a (*) superscript.

2.1 Households

The representative household's intertemporal preferences over consumption, c_t, labour supply, n_t, and financial asset adjustment, Θ_t, are described by the following expected utility function:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \varepsilon_t^g \left[\log\left[c_\tau - h_\tau\right] - \xi_n \frac{{n_\tau}^{1+\vartheta}}{1+\vartheta} - \Theta_t \right], \tag{1}$$

where t indexes time, $\beta < 1$ is the time-discount parameter, $h_t \equiv \zeta c_{t-1}$ is the external habit taken as exogenous by the household, ϑ is the inverse of the Frisch elasticity of labour supply, and ξ_n is the level parameter that determines the relative importance of labour in the utility function.[5] ε_t^g denotes a preference shock that follows an autoregressive AR(1) process in logs.

Imperfect substitution across assets is captured using a portfolio adjustment cost, Θ_t. In particular, following Andres et al. (2004), the adjustment cost is defined as quadratic cost functions across different asset classes:

$$\Theta_t = \frac{\xi_a}{2} \left(\frac{1-\gamma_a}{\gamma_a} \frac{A_{S,t}}{A_{L,t}} - 1 \right)^2 + \frac{\xi_S}{2} \left(\frac{1-\gamma_S}{\gamma_S} \frac{B_{HS,t}}{e_t B_{FS,t}} - 1 \right)^2 + \frac{\xi_L}{2} \left(\frac{1-\gamma_L}{\gamma_L} \frac{q_{L,t} B_{HL,t}}{e_t q^*_{L,t} B_{FL,t}} - 1 \right)^2 \tag{2}$$

$$A_{S,t} = B_{HS,t} + e_t B_{FS,t} \tag{3}$$

$$A_{L,t} = q_{L,t} B_{HL,t} + e_t q^*_{L,t} B_{FL,t}, \tag{4}$$

where $A_{S,t}$ is the short-term sub-portfolio that consists of home and foreign short-term government bonds, $B_{HS,t}$ and $e_t B_{FS,t}$, and $A_{L,t}$ is the long-term sub-portfolio that consists of home and foreign long-term government bonds, $q_{L,t} B_{HL,t}$ and $e_t q^*_{L,t} B_{FL,t}$.[6] [7] The variables $q_{L,t}$ and $q^*_{L,t}$ denote the

[5]Households are a continuum of measure one; therefore, the variables in the utility function require an index i except for the external habit formation. For simplicity, the indices are not presented.

[6]Note that modelling imperfect substitutability is kept as simple and tractable as possible, using a quadratic adjustment cost function in preferences similar to Andres et al. (2004). Alternatively, when introducing imperfect substitutability across the four types of assets, one can consider portfolio adjustment costs in the budget constraint of

relative prices of real domestic and foreign long-term bonds, and e_t denotes the nominal exchange rate between home and foreign currencies (in units of domestic currency per unit of foreign currency). The parameter $0 < \gamma_a < 1$ is the steady-state share of the short-term assets in the aggregate portfolio, while $0 < \gamma_S < 1$ and $0 < \gamma_L < 1$ are the share parameters for home assets in short- and long-term sub-portfolios, respectively. The coefficients $\xi_a \geq 0$, $\xi_S \geq 0$ and $\xi_L \geq 0$ denote the degree of adjustment costs for the financial portfolio. In the extreme case where these coefficients are zero, portfolio balance effects disappear, making assets perfectly substitutable with each other. The model in this case turns into a standard frictionless SOE model, where assets are priced using the arbitrage conditions.

The above cost function in preferences is motivated by the fact that households perceive entering long-term or foreign bond markets to be "riskier." The first expression in the cost function represents preferences for liquidity, as discussed in Andres et al. (2004) and recently in Krishnamurthy and Vising-Jorgensen (2012).[8] Thus, agents are concerned about a loss of liquidity when they hold long-term bonds relative to the same investment in short-term bonds. Different from Andres et al. (2004), agents here trade in international financial markets and need liquidity in foreign currency as well; therefore, the liquidity concerns also include foreign short-term bonds. In addition, foreign bonds, regardless of their maturity, are perceived to have a different risk structure and convenience benefits from home bonds, and are not perfectly substitutable with home bonds. This captures asymmetric information costs between home and foreign assets, second-order effects such as the exchange rate volatility, or the convenience and safety benefits of home assets in transactions at home (Huberman, 2001; Valchev, 2015). As agents increase their holdings of foreign bonds, they also increase the holdings of home bonds to compensate themselves for the extra risk taken.

households (Chen et al., 2012) or in the objective function of the portfolio manager's or bankers' problem (Harrison, 2011 and Ellison and Tischbirek, 2014). Recently, Valchev (2015) captures imperfect substitutability between domestic and foreign bonds in transaction services for consumption that appears in the household's budget constraint. All these approaches yield similar behavioural equations for portfolio dynamics. This is akin to equivalent approaches to money in the utility models while considering money demand decisions.

[7] Note that in Alpanda and Kabaca (2015), we introduce imperfect substitution among assets via constant-elasticity-of-substitution (CES) portfolio specification in preferences. Thus, there is an intrinsic value of holding financial wealth. Since the QE policy does not alter the total supply of bonds but changes only the composition between short- and long-term assets, modelling portfolio balance effects via CES portfolios or quadratic adjustment costs in preferences leads to similar results. Because the model aims to be taken to the data for portfolio balance effects, I prefer here quadratic adjustment costs that directly address portfolio balancing issues abstracting the intrinsic value of wealth.

[8] Note that in Andres et al. (2004), only money and long-term bond holdings appear in preferences; thus, short-term bonds do not enter the household's utility. Here, I consider both money and short-term Treasury bills in the same basket entering household preferences and do not solve for money holdings separately. To the extent that the monetary base and Treasury bills are perfect substitutes at the ZLB, adding money holding decisions to the model does not change the results on financial or real variables following QE.

The households' period budget constraint is given by

$$
\begin{aligned}
& c_t + \frac{B_{HS,t}}{P_t} + \frac{e_t B_{FS,t}}{P_t}\varepsilon_t^{cr} + \frac{q_{L,t} B_{HL,t}}{P_t}\varepsilon_t^{dr} + \frac{e_t q_{L,t}^* B_{FL,t}}{P_t}\varepsilon_t^{cr}\varepsilon_t^{dr} \\
& \leq \frac{W_t}{P_t} n_t + \frac{R_{t-1} B_{HS,t-1}}{P_t} + \frac{e_t R_{t-1}^* B_{FS,t-1}}{P_t} + \frac{\left(1+\kappa q_{L,t}\right) B_{HL,t-1}}{P_t} \\
& \quad + \frac{e_t \left(1+\kappa q_{L,t}^*\right) B_{FL,t-1}}{P_t} + \frac{\Pi_{H,t}}{P_t} + \frac{\Pi_{R,t}}{P_t} + \frac{\Pi_{FI,t}}{P_t} - \frac{TAX_t}{P_t},
\end{aligned} \tag{5}
$$

where W_t represents the wage rate in the competitive labour market, $\Pi_{H,t}$ and $\Pi_{R,t}$ denote the profits of monopolistically competitive domestic producers and retailers; and TAX_t represents lump-sum taxes paid by households to the government. Short-term domestic and foreign bonds pay predetermined interest rates of R_{t-1} and R_{t-1}^*, respectively, while long-term bonds are perpetuities that pay a coupon payment of 1 unit in the first period after issuance and have coupon payments decaying at a rate of κ for each period after that, as in Woodford (2001). Since these long-term bonds are tradable, we can write them in recursive form in the budget constraint above. The yields on domestic and foreign long-term bonds are defined, respectively, as

$$
R_{L,t} = \frac{1+\kappa q_{L,t}}{q_{L,t}} \text{ and } R_{L,t}^* = \frac{1+\kappa q_{L,t}^*}{q_{L,t}^*}. \tag{6}
$$

Households also pay transaction costs, ε_t^{dr} and ε_t^{cr}, per units of long-term and foreign bonds purchased. These service fees are paid to a financial intermediary that distributes its profits, $\Pi_{FI,t}$, to households in a lump-sum fashion. The varying transaction cost on long-term bonds, ε_t^{dr}, is standard in estimated DSGE models with portfolio balance effects (Andres et al., 2004; Chen et al., 2012) and represents the exogenous variation in the duration risk premium. Similarly, the varying transaction cost on foreign bonds, ε_t^{cr}, represents the exogenous variation in the country risk premium as commonly used in many estimated SOE models (Adolfson et al., 2008; Justiniano and Preston 2010a). A fall in ε_t^{cr} can be interpreted as an increase in the country risk premium. Note that in frictionless SOE models, country risk premium generally also include a foreign debt stock to ensure the stationarity in the model (Schmitt-Grohé and Uribe, 2003). Since portfolio balancing terms in the utility function here ensures the stationarity in the model, country risk premium is kept at its simplest case with only an exogenous term. Both ε_t^{dr} and ε_t^{cr} follow AR(1) processes in logs.

2.1.1 Domestic and global term premia

The households' objective is to maximize their utility subject to the budget constraint, and appropriate no-Ponzi conditions. The first-order condition for consumption is standard, and is given by

$$
\frac{\varepsilon_t^g}{c_t - \zeta c_{t-1}} = \lambda_t, \tag{7}
$$

where λ_t is the Lagrange multiplier on the budget constraint. Similarly, the optimal labour supplied by households can be written as:

$$\varepsilon_t^g n_t^\vartheta = \lambda_t w_t, \tag{8}$$

meaning that households supply labour at which marginal disutility of labour is equal to the marginal value from working.

The optimality conditions with respect to domestic short- and long-term bonds are given by

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] - \frac{\partial \Theta_t}{\partial b_{HS,t}} \varepsilon_t^g, \tag{9}$$

$$q_{L,t} \lambda_t \varepsilon_t^{dr} = \beta E_t \left[\lambda_{t+1} \frac{1 + \kappa q_{L,t+1}}{\pi_{t+1}} \right] + \frac{\partial \Theta_t}{\partial b_{HL,t}} \varepsilon_t^g, \tag{10}$$

where $b_{HS,t} = B_{HS,t}/P_t$, and $b_{HL,t} = B_{HL,t}/P_t$. The two expressions above can be log-linearized and combined to generate an expression for the yield on long-term bonds as

$$\widehat{R}_{L,t} = \left(1 - \frac{\kappa}{R_L}\right) E_t \sum_{s=0}^{\infty} (\frac{\kappa}{R_L})^s \left\{ \widehat{R}_{t+s} + \frac{(1-\zeta)\, c/y}{a/y} \widehat{T}_{t+s} \right\}, \tag{11}$$

where

$$\begin{aligned} \widehat{T}_t = {} & \frac{\xi_a}{\gamma_a (1-\gamma_a)} \left(\widehat{a}_{L,t} - \widehat{a}_{S,t} \right) - \frac{\xi_S}{\gamma_a \gamma_S} \left(\widehat{b}_{HS,t} - \widehat{rer}_t - \widehat{b}_{FS,t} \right) \\ & + \frac{\xi_L}{(1-\gamma_a)\gamma_L} \left(\widehat{q}_{L,t} + \widehat{b}_{HL,t} - \widehat{rer}_t - \widehat{q}^*_{L,t} - \widehat{b}_{FL,t} \right) + \widehat{\varepsilon}_t^{dr}, \end{aligned} \tag{12}$$

and R, R_L, π, and a are the steady-state levels for short- and long-term interest rates, consumer price inflation, and aggregate real portfolio holdings ($\frac{A_S + A_L}{P}$). The hatted variables denote log-deviations of variables from their steady-state values. Equation (11) implies that the yield on long-term bonds, $R_{L,t}$, is a combination of current and expected short-term rates as well as a premium that households require for switching their portfolios to home bonds from alternatives due to the imperfect substitution across bonds. Note that the expression $\widehat{T}_t$ governs the term premium component in long-term yields, and can be considered as the relative demand function between long- and short-term home bonds.[9] The expression suggests that an increase in the term premium leads households to demand more long-term home bonds and fewer short-term home bonds. The extent of the increase in the demand for long-term home bonds depends on adjustment cost and share parameters, consumption habit, and the relative importance of steady-state consumption over financial assets (c/a).

Similar to equations (9) and (10), optimality conditions for foreign short- and long-term bonds

[9] One can show that $\frac{(1-\zeta)c}{a}\widehat{T}_t$ is equal to $R^e_{L,t} - R$, where $R^e_{L,t}$ is the one-period holding return for long-term home bonds.

can be written as:

$$rer_t \lambda_t = \beta E_t \left[\lambda_{t+1} rer_{t+1} \frac{R_t^*}{\pi_{t+1}^*} \right] + \frac{\partial \Theta_t}{\partial b_{FS,t}} \varepsilon_t^g, \tag{13}$$

$$rer_t q_{L,t}^* \lambda_t = \beta E_t \left[\lambda_{t+1} rer_{t+1} \frac{1 + \kappa q_{L,t+1}^*}{\pi_{t+1}^*} \right] + \frac{\partial \Theta_t}{\partial b_{FL,t}} \varepsilon_t^g, \tag{14}$$

where $b_{FS,t} = B_{FS,t}/P_t$, $b_{FL,t} = B_{FL,t}/P_t$, and $rer_t = e_t P_t^*/P_t$ denotes the real exchange rate. After log-linearization, the two equations above can be combined to generate a demand schedule for foreign long-term bonds:

$$\widehat{R}_{L,t}^* = \left(1 - \frac{\kappa}{R_L}\right) E_t \sum_{s=0}^{\infty} (\frac{\kappa}{R_L})^s \left\{ \widehat{R}_{t+s}^* + \frac{(1-\zeta)\, c/y}{a/y} \widehat{T}_{t+s}^* \right\}, \tag{15}$$

where

$$\begin{aligned} \widehat{T_t}^* = & \frac{\xi_a}{\gamma_a(1-\gamma_a)} \left(\widehat{a}_{L,t} - \widehat{a}_{S,t}\right) - \frac{\xi_S}{\gamma_a(1-\gamma_S)} \left(\widehat{rer}_t + \widehat{b}_{FS,t} - \widehat{b}_{HS,t}\right) \\ & + \frac{\xi_L}{(1-\gamma_a)(1-\gamma_L)} \left(\widehat{rer}_t + \widehat{q}_{L,t}^* + \widehat{b}_{FL,t} - \widehat{q}_{L,t} - \widehat{b}_{HL,t}\right) + \widehat{\varepsilon}_t^{dr}. \end{aligned} \tag{16}$$

The expression above, $\widehat{T_t}^*$, demonstrates a link between the term premium component in foreign long-term yields, which is taken as given by the small open economy, and the relative demand between short- and long-term foreign bonds. An increase in $\widehat{T_t}^*$ leads home agents to increase (decrease) the share of long-term (short-term) foreign bonds in their portfolio. Combining equations (12) and (16), one can write down an expression for the excess premium on home long-term yields:

$$\begin{aligned} \widehat{T_t} = \widehat{T_t}^* & + \frac{\xi_L}{(1-\gamma_a)\gamma_L(1-\gamma_L)} \left(\widehat{q}_{L,t} + \widehat{b}_{HL,t} - (\widehat{rer}_t + \widehat{q}_{L,t}^* + \widehat{b}_{FL,t})\right) \\ & - \frac{\xi_S}{\gamma_a \gamma_S (1-\gamma_S)} \left(\widehat{b}_{HS,t} - (\widehat{rer}_t + \widehat{b}_{FS,t})\right), \end{aligned} \tag{17}$$

which implies that deviations of the domestic term premium from the global term premium depend on the substitutability between home and foreign assets in both short- and long-term maturities. As home and foreign assets become more substitutable (lower ξ_S or ξ_L), an increase in the excess return on long-term home bonds ($\widehat{T_t} > \widehat{T_t}^*$) leads to a larger portfolio switching between foreign and home bonds. In the extreme situation when home and foreign bonds are perfect substitutes in both short- and long-term sub-portfolios—i.e., when $\xi_S = \xi_L = 0$—the domestic term premium cannot deviate from the global term premium. Note that, on the other hand, even when home and foreign long-term bonds are perfect substitutes, $\xi_L = 0$, the domestic term premium can still differ from the global term premium if home bonds are not substitutable with foreign bonds in the short-term sub-portfolio ($\xi_S > 0$). Thus, frictions in the short-term market (currency markets, at the extreme case) would affect household long-term portfolio decisions as well.

Equation (17) also implies that for QE policies to be effective in a small open-economy in reducing domestic term premium, it is necessary for home and foreign bonds to be imperfect substitutes with each other in either short- or long-term maturities ($\xi_S > 0$ or $\xi_L > 0$). Particularly, following a QE policy in the home country, the market supply of long-term home bonds will fall and that of short-term home bonds will rise (see section 2.3 for a formal definition of QE policy). To the extent that home bonds are imperfect substitutes for foreign global bonds, agents will not be able to compensate their home long-term bonds with foreign counterparts, leading to an increase in the price of long-term bonds, and thus, a fall in the domestic term premium. As home and foreign assets become less substitutable, the effectiveness of QE policies would increase.

Figure 3 illustrates the model's implications of QE policies in a two-dimensional framework under imperfect and perfect substitution between home and foreign assets. The x-axes denote the quantity of short-term bonds relative to the quantity of long-term bonds in the home bond market, while the y-axes denote the term premium. Relative demand schedules can be considered as the sum of relative demand from home households and external demand from foreign investors, which will be detailed in section 2.7. In the left panel, the relative demand schedule is downward sloping due to the imperfect substitution between home and foreign assets. As a result, following a QE shock that increases the relative supply of home short-term bonds in the market, agents require a fall in the domestic term premium to hold more short-term bonds relative to long-term at the equilibrium. Therefore, the domestic term premium falls below the global term premium. On the other hand, the relative demand schedule is perfectly elastic in the right panel due to the perfect substitution between home and foreign assets in the eyes of households. In this case, the domestic term premium is always equal to the global term premium and a QE shock can alter only the composition of portfolios without any change in the term premium.

Note that, as in the case of closed economy models, it is still necessary for short- and long-term portfolios to be imperfect substitutes for a QE to be effective ($\xi_a > 0$). When short- and long-term portfolios are perfect substitutes—i.e., when $\xi_a = 0$—equation (16) simplifies to:

$$\frac{\xi_S}{\gamma_a(1-\gamma_S)}\left(\widehat{rer}_t + \widehat{b}_{FS,t} - \widehat{b}_{HS,t}\right) = \frac{\xi_L}{(1-\gamma_a)(1-\gamma_L)}\left(\widehat{rer}_t + \widehat{q}^*_{L,t} + \widehat{b}_{FL,t} - \widehat{q}_{L,t} - \widehat{b}_{HL,t}\right)$$

since $\widehat{T}_t^* = 0$ following a QE in the small open economy. An increase in home short-term bonds and a decrease in long-term bonds in portfolios after a QE policy implies that $\widehat{rer}_t + \widehat{b}_{FS,t} = \widehat{b}_{HS,t}$ and $\widehat{rer}_t + \widehat{q}^*_{L,t} + \widehat{b}_{FL,t} = \widehat{q}_{L,t} + \widehat{b}_{HL,t}$ for the above equality to hold. Therefore, $\widehat{T}_t = 0$, following equation (17). Note that in this case, households adjust their home and foreign positions in each maturity by the same percentage, thereby keeping home-to-foreign ratios constant in both short- and long-term portfolios. Intuitively, it means that when $\xi_a = 0$, households can perfectly adjust their portfolios following a QE policy in a way that rebalancing does not lead to any utility costs even when $\xi_S > 0$ or $\xi_L > 0$. Thus, QE would not have any effect on asset prices.

2.1.2 Exchange rate determination

The effect of asset purchases on the exchange rate can be illustrated by considering the optimality conditions of households with respect to foreign and home short-term bonds. These conditions, when combined, yield a relative demand schedule for short-term domestic and foreign bonds. After log-linearization, this condition can be written as

$$E_t\widehat{d}_{t+1} = \widehat{R}_t - \widehat{R}_t^* - \frac{\widetilde{\Theta}\xi_S}{\gamma_a\gamma_S(1-\gamma_S)}\left(\widehat{b}_{HS,t} - \widehat{rer}_t - \widehat{b}_{FS,t}\right) + \hat{\varepsilon}_t^{cr}, \tag{18}$$

where $\widehat{d_t} = \widehat{e}_t - \widehat{e}_{t-1}$ denotes the nominal depreciation rate of the home currency and $\widetilde{\Theta} = \frac{(1-\zeta)c/y}{a/y}$. The above equation can be interpreted as a modified short-term UIRP condition, where the country risk premium is determined by the relative holdings of short-term domestic and foreign bonds plus an exogenous term. Thus, even when the short-term rate differentials cannot change due to the ZLB, QE policies can still affect the exchange rate through the country risk premium. Specifically, asset purchases at home increase the short-term home bonds in portfolios, making them less attractive than their foreign counterparts and leading to an exchange rate depreciation.

The above condition can further be combined with the term premium expression in equation (17), as follows:

$$E_t\widehat{d}_{t+1} = \underbrace{\widehat{R}_t - \widehat{R}_t^* + \widetilde{\Theta}(\widehat{T}_t - \widehat{T_t^*})}_{\widehat{R}^e_{L,t} - \widehat{R}^{e^*}_{L,t}} - \frac{\widetilde{\Theta}\xi_L}{(1-\gamma_a)\gamma_L(1-\gamma_L)}\left(\widehat{q}_{L,t} + \widehat{b}_{HL,t} - (\widehat{rer}_t + \widehat{q}^*_{L,t} + \widehat{b}_{FL,t})\right) + \hat{\varepsilon}_t^{cr}, \tag{19}$$

which shows the link between the depreciation of the currency and the term premium differential along with the relative holdings of domestic and foreign long-term bonds. Note that the first three terms in the right-hand side of the above equation represent the one-period return differential between home and foreign long-term bonds, making this equation a long-term UIRP condition modified with portfolio terms. However, the portfolio term here moves in the opposite direction, dampening the effect on the exchange rate depreciation following a QE policy. This is because a QE at home lowers the supply of long-term home bonds, making them more attractive relative to foreign long-term bonds, which in turn has a positive effect on the value of the exchange rate. Therefore, a higher adjustment cost between home and foreign long-term bonds (higher ξ_L) leads to a smaller depreciation following a QE at home, whereas a higher adjustment cost between home and foreign short-term bonds (higher ξ_S) leads to a bigger depreciation.

Equations (18) and (19) imply no exchange rate effect following a QE policy when domestic and foreign assets are perfect substitutes in both short- and long-term maturities; i.e., when $\xi_S = \xi_L = 0$.

Note that this is also the case in which QE cannot affect the domestic term premium and long-term interest rates. Therefore, the model implies no exchange rate effect when long-term rates do not move following a QE program.[10] On the other hand, the exchange rate can still depreciate after a QE policy when $\xi_L = 0$, as long as the short-term UIRP condition does not hold; thus, when $\xi_S > 0$. The resulting expected appreciation in the home currency also implies a drop in domestic long-term interest rates through the arbitrage condition between home and foreign long-term bonds; i.e., through the long-term UIRP condition. This is why QE is still effective on domestic long-term interest rates even when home long-term bonds are quite substitutable with foreign counterparts as long as there exist frictions in the short-term debt market.

2.2 Production side

The production side of the economy has standard features as in open economy New Keynesian models (Gali and Monacelli, 2005; Monacelli, 2005). I summarize below the environment for firms and highlight the key outcomes of firms' maximization problems, namely New Keynesian Phillips curves. Details on firms' optimization problems can be found in Appendix A.

Firms include final-goods producers, intermediate-goods producers and retailers that import foreign goods from world markets. Final-goods producers are perfectly competitive, and they use home and foreign intermediate goods, $c_{h,t}$ and $c_{f,t}$ with a technology $c_t = \left[\gamma_c^{\frac{1}{\lambda}} c_{h,t}^{\frac{\lambda-1}{\lambda}} + (1-\gamma_c)^{\frac{1}{\lambda}} c_{f,t}^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}}$, where γ_c denotes the share of domestic goods, and λ is the elasticity of substitution between home and foreign goods. Using the demand for domestic and imported goods from their maximization problem, the following consumer price index (CPI) inflation can be obtained:

$$\widehat{\pi}_t = \gamma_c \widehat{\pi}_{h,t} + (1-\gamma_c)\,\widehat{\pi}_{f,t}, \tag{20}$$

where $\pi_{h,t} = P_{h,t}/P_{h,t-1}$ and $\pi_{f,t} = P_{f,t}/P_{f,t-1}$ are the domestic and import price inflation, respectively.

Domestic producers are monopolistically competitive firms indexed by j. These firms produce differentiated intermediate goods, which are then aggregated into a homogeneous domestic good using a standard Dixit-Stiglitz aggregator. They use a technology described by the following production function: $y_t(j) = \varepsilon_t^z n_t(j)$, where ε_t^z is the productivity level normalized to 1 and follows an AR(1) process. In determining the price of the domestic good, $P_{h,t}$, a Calvo-style price setting is assumed, allowing for indexation to past domestic good price inflation. Specifically, in any period t, a fraction $1-\theta_h$ of firms set prices optimally, while a fraction $0 < \theta_h < 1$ of goods prices are

[10]Note that, if purchases are targeted toward foreign assets instead of domestic long-term bonds, the exchange rate could depreciate without any change in the term premium.

adjusted according to the indexation rule:

$$\log P_{h,t}(j) = \log P_{h,t-1}(j) + \varsigma_h \pi_{h,t-1}, \tag{21}$$

where $0 < \varsigma_h < 1$ measures the degree of indexation to the previous period's inflation rate for domestic good prices. Firms' optimization problem combined with the Dixit-Stiglitz aggregate price index yields a forward-looking Phillips curve for the domestic good inflation:

$$\widehat{\pi}_{h,t} = \frac{\varsigma_h}{1+\varsigma_h\beta}\widehat{\pi}_{h,t-1} + \frac{\beta}{1+\varsigma_h\beta}E_t\widehat{\pi}_{h,t+1} + \frac{(1-\theta_h)(1-\beta\theta_h)}{\theta_h\,(1+\varsigma_h\beta)}\left(\widehat{mc}_t\right), \tag{22}$$

where $\widehat{mc}_t = \vartheta y_t - (1-\vartheta)\widehat{\varepsilon}_t^z + \frac{1}{1-\zeta}\left(\widehat{c}_t - \zeta\widehat{c}_{t-1}\right) - \widehat{p}_{h,t}$ and $p_{h,t} = P_{h,t}/P_t$ is the relative price of home goods. Note that in the absence of indexation, $\varsigma_h = 0$, the above expression can be simplified to a typical forward-looking Phillips curve. Note also that marginal cost depends on past consumption as well, since real wages are affected by past consumption because of the habit formation (see equation (8)).

There are also retailers importing foreign differentiated goods, for which law of one price holds at customs. Retailers are monopolistically competitive (indexed by j), which leads to a violation of the law of one price in the short run. In determining the domestic currency price of imported goods, $P_{f,t}$, a Calvo-style price setting is assumed, similar to that for domestic producers. In any period t, a fraction $1-\theta_f$ of retail firms set prices optimally, while a fraction $0 < \theta_f < 1$ of goods prices are adjusted according to the indexation rule:

$$\log P_{f,t}(j) = \log P_{f,t-1}(j) + \varsigma_f \pi_{f,t-1}, \tag{23}$$

where $0 < \varsigma_f < 1$ measures the degree of indexation to the previous period's inflation rate for imported good prices. Firms' optimization problem combined with the Dixit-Stiglitz aggregate price index and the marginal cost of the production yield a forward-looking Phillips curve for the imported good inflation:

$$\widehat{\pi}_{f,t} = \frac{\varsigma_f}{1+\varsigma_f\beta}\widehat{\pi}_{f,t-1} + \frac{\beta}{1+\varsigma_f\beta}E_t\widehat{\pi}_{f,t+1} + \frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f\,(1+\varsigma_f\beta)}\left(\widehat{rer}_t - \widehat{p}_{f,t}\right) + \widehat{\varepsilon}_t^f, \tag{24}$$

where $p_{f,t} = P_{f,t}/P_t$ is the relative price of imported goods. A cost-push shock, $\widehat{\varepsilon}_t^f$, is added to capture inefficient variations in markups (Justiniano and Preston, 2010a; 2010b).

2.3 Monetary and fiscal policy

The central bank targets the nominal interest rate using a Taylor rule:

$$\log R_t = \rho \log R_{t-1} + (1-\rho)\left(\log R + r_\pi \log\frac{\pi_{h,t}}{\pi} + r_y \log\frac{y_t}{y} + r_{\Delta y}\log\frac{y_t}{y_{t-1}} + r_d \log d_t\right) + \varepsilon_t^r, \tag{25}$$

where R is the steady-state value of the (gross) nominal policy rate, ρ determines the extent of interest rate smoothing, and the parameters r_π, $r_y, r_{\Delta y}$, and r_d determine the importance of domestic inflation, the output gap, output growth and depreciation of the currency in the Taylor rule, respectively. y is the steady-state level of output, and ε_t^r is a monetary policy shock that follows an independent and identically distributed (i.i.d.) process.

The consolidated government budget constraint is given by

$$\frac{R_{t-1}}{\pi_t} b_{S,t-1} + \frac{R_{L,t}}{\pi_t} q_{L,t} b_{L,t-1} = \frac{TAX_t}{P_t} + b_{S,t} + q_{L,t} b_{L,t}, \tag{26}$$

where $b_{S,t}$ and $b_{L,t}$ represent real short- and long-term government debt, respectively. Lump-sum taxes adjust with the level of lagged long-term government debt to rule out a Ponzi scheme for the government as in Chen et al. (2012) and Davig and Leeper (2006):

$$\frac{TAX_t}{P_t} = \Xi y \left(\frac{b_{S,t-1} + q_{L,t-1} b_{L,t-1}}{b_S + q_L b_L} \right)^{\tau_b} \varepsilon_t^\tau, \tag{27}$$

where Ξ is a level parameter and determines the steady-state ratio of tax to GDP. Note that taxes are in net terms and represent the net borrowing needs for the government. Moreover, $\tau_b > 0$ determines the response of taxes to the long-term government debt and ε_t^τ follows an AR(1) process in logs.

Finally, the government controls the supply of long-term bonds:

$$q_{L,t} b_{L,t} = \Gamma \varepsilon_t^b b_{S,t}, \tag{28}$$

where Γ is the steady-state relative supply of long-term bonds and ε_t^b represents an unconventional monetary policy shock (i.e., a QE shock), which follows an AR(1) process in logs.

2.4 Market clearing conditions and external demand for goods and assets

The domestic goods are used in the final-goods production for consumption, $c_{h,t}$, and exports, $c_{f,t}^*$:

$$c_{h,t} + c_{f,t}^* = y_t. \tag{29}$$

The model is closed with respect to the home-goods market by assuming the following functional form for the external demand for home goods:

$$c_{f,t}^* = \left(\frac{P_{f,t}^*}{P_t^*}\right)^{-\lambda^*} c_t^*, \tag{30}$$

where λ^* determines the foreign elasticity of substitution across goods in the foreign economy. The functional form is standard in SOE models and assumes that aggregate foreign consumption follows a CES function (see Kollman, 2002 and Monacelli, 2005). Following Monacelli (2005), I assume that, for simplicity, the price of the home good in the foreign country, $P^*_{f,t}$, is flexible and determined by the law of one price. I also assume $\lambda^* = \lambda$, implying the same elasticity of substitution between home and foreign goods in both regions of the model.

Furthermore, bond markets clear in both short- and long-term maturities; i.e.,:

$$b_{S,t} = b_{HS,t} + b^*_{FS,t} \tag{31}$$

$$b_{L,t} = b_{HL,t} + b^*_{FL,t}. \tag{32}$$

In order to close the model with respect to the home-bonds market, I assume external demand functions for home assets as follows:

$$\frac{b^*_{FS,t}}{rer_t} = \left(\frac{R_t}{R^*_t E_t d_{t+1} \varepsilon^{cr}_t}\right)^{-\eta^*_S} a^*_{S,t} \tag{33}$$

$$\frac{q_{L,t} b^*_{FL,t}}{rer_t} = \left(\frac{R^e_{L,t}}{R^{e*}_{L,t} E_t d_{t+1} \varepsilon^{cr}_t}\right)^{-\eta^*_L} a^*_{L,t}. \tag{34}$$

where $\eta^*_S = \frac{(1-\zeta)c}{a} \frac{\xi_S}{\gamma_a \gamma_S (1-\gamma_S)}$ and $\eta^*_L = \frac{(1-\zeta)c}{a} \frac{\xi_L}{(1-\gamma_a)\gamma_L(1-\gamma_L)}$. Thus, as in the case of goods markets, I also assume the same elasticity of demand for assets across domestic and foreign investors. The functional form assumes that foreigners observe the same country risk premium shock that domestic residents face. These equations represent foreigners' relative demand between home and foreign assets in different maturities and can be shown as foreign counterparts of equations (18) and (19), after log-linearization. Since $a^*_{S,t} = b^*_{S,t}$ and $a^*_{L,t} = b^*_{L,t}$ (due to the large-economy effect), the above equations can be further simplified when combined with equations (18) and (19), after log-linearization:

$$\begin{aligned} \widehat{b}^*_{FS,t} - \widehat{b}^*_{S,t} &= \widehat{b}_{HS,t} - \widehat{b}_{FS,t} \\ \widehat{b}^*_{FL,t} - \widehat{b}^*_{L,t} &= \widehat{b}_{HL,t} - \widehat{b}_{FL,t}, \end{aligned} \tag{35}$$

which suggests that both foreigners and domestic residents have the same relative demand schedule between home and foreign assets.

The external asset demand functions do not arise in standard SOE models because of an indeterminacy for gross holdings of home and foreign assets when assets are perfectly substitutable. On the other hand, the imperfect substitutability of assets in this paper allows me to solve for a unique equilibrium for gross portfolio holdings when all assets are traded internationally. In section 4.3, I will show that foreigners' access to the domestic debt market is crucial to understanding the effects of QE in small open economies.

Combining market clearing conditions with the household's and government's budget constraints, the balance-of-payments identity in the model can be written as:

$$\left(\frac{e_t B_{FS,t}}{P_t} - \frac{e_t R^*_{t-1} B_{FS,t-1}}{P_t}\right) + \left(\frac{e_t q^*_{L,t} B_{FL,t}}{P_t} - \frac{e_t R^*_{L,t} q^*_{L,t} B_{FL,t-1}}{P_t}\right)$$
$$- \left(\frac{B^*_{FS,t}}{e_t P_t} - \frac{R_{t-1} B^*_{FS,t-1}}{e_t P_t}\right) - \left(\frac{q_{L,t} B^*_{FL,t}}{e_t P_t} - \frac{R_{L,t} q_{L,t} B^*_{FL,t-1}}{e_t P_t}\right) = \frac{P_{h,t}}{P_t} c^*_{f,t} - \frac{e_t P^*_t}{P_t} c_{f,t}, \quad (36)$$

where the right-hand side denotes the trade balance, while the left-hand side captures the corresponding net change in foreign bond holdings.

Finally, the model's equilibrium is defined as prices and allocations such that households maximize the discounted present value of utility; all firms maximize the discounted present value of profits, subject to their constraints; and all markets clear.

2.5 Foreign bloc

The paths of foreign variables in the model—foreign output, inflation, short-term interest rate, long-term interest rate, short-term bond supply and long-term bond supply—are assumed to be exogenously given. This reflects the assumption that the home economy is small relative to the foreign and therefore any changes in the home economy cannot move foreign variables. Let $X_t = \left(\widehat{y}^*_t \ \ \widehat{\pi}^*_t \ \ \widehat{R}^*_t \ \ \widehat{R}^*_{L,t} \ \ \widehat{b}^*_{S,t} \ \ \widehat{q^*_{L,t} b^*_{L,t}}\right)$. Then the foreign economy is modelled as a vector autoregressive process of order 2:

$$X_t = \Upsilon_1 X_{t-1} + \Upsilon_2 X_{t-2} + e^X_t \quad \text{where} \quad e^X_t \backsim N(0, \Sigma_x).$$

3 Estimation

The system of log-linearized equations is shown in Appendix B. The log-linearized version of the model is then taken to the data by using Bayesian estimation techniques, which have recently been commonly used in the DSGE literature because of the scarcity of macroeconomic data or identification problems (An and Schorfheide, 2007). I have used seven quarterly domestic economic variables for the small open economy: output, inflation, GDP deflator, short-term interest rate, long-term interest rate, the real exchange rate, and the relative supply between long- and short-term government bonds. Note that relative to the standard 3-equation, closed economy New Keynesian DSGE model, we have four more domestic variables in the observation set: terms of trade (as a result of having both consumer price inflation and GDP deflator), real exchange rate, long-term interest rates and relative supply of long-term bonds. Terms of trade and real exchange rate represent two key macroeconomic variables for small open economies as in the estimation performed by Justiniano and Preston (2010a; 2010b). In addition, the relative bond supply and long-term interest rates represent key macro variables used in the portfolio balancing DSGE literature (Chen et al., 2012).

The data for Canada are obtained from Statistics Canada's Canadian socioeconomic database via Haver Analytics. I first construct the real GDP per capita using the chained GDP series and population. Real GDP per capita is then presented in log-deviations from a linear trend, consistent with the model (see also Justiniano and Preston, 2010b). I then construct the real exchange rate using the nominal exchange rate between the Canadian and US dollar, and inflation rates for Canada and the United States. The rate is expressed in log differences. For CPI inflation, I use the Bank of Canada's core CPI excluding food, energy and indirect taxes. A measure excluding indirect taxes is used so that inflation series are not affected by the outlier in 1991, stemming from a change in tax code that year.[11] Short- and long-term rates are proxied by the overnight money market financing rate and five-year benchmark bond yields, respectively. Quarterly inflation and both interest rates are expressed in percentages. Lastly, I use the ratio of long-term to short-term Canadian federal government bonds outstanding as the measure of the quantity of debt similar to the measure used in Chen et al. (2012). The measure is expressed as the log difference between long-term and short-term government bonds. Note that since the data on the remaining maturity are not observed, I used the original maturity on bonds to classify bonds as short- and long-term. Short-term bonds represent the federal government debt with an original maturity of less than three months plus the monetary base, while long-term bonds have a maturity of more than three months.

For the estimation of foreign bloc, I use quarterly US economic variables: output, GDP deflator, short- and long-term interest rates, and short- and long-term government bond supply as they enter the VAR. The data for US variables are obtained from the National Income and Product Accounts (NIPA; Bureau of Economic Analysis) and the Flow of Funds Accounts (FOF; Federal Reserve Board) via Haver Analytics. Real GDP per capita measures output and is expressed as log deviations from a linear trend, similar to the transformation in the Canadian counterpart. Inflation corresponds to the log difference in the GDP deflator, and the effective federal funds rate and five-year Treasury constant maturity yield are taken to measure short- and long-term interest rates, respectively. The Treasury debt supply variables in short-term and long-term maturities are expressed in first-differenced. As in the Canadian case, I use the original maturity to classify bonds for consistency.

The sample period in the estimation runs from 1990Q1 to 2015Q4. The starting date is chosen to be 1990Q1 since the data for the Canadian bond supply go back only to this date. In addition, the first six quarters are used to initialize the Kalman filter; thus, the estimation effectively starts at 1991Q3. All series are seasonally adjusted (if needed) and demeaned before estimation. First, the mode of the posterior distribution is estimated by maximizing log posterior function, which is a product of prior information on the parameters and the likelihood of the data. The likelihood function is evaluated for any given set of parameters using the Kalman filter. Second, the Metropolis–Hastings

[11] There are also series excluding only food and energy. Two series are almost identical except for the first quarter of 1991. In their estimation for Canada, Justiniano and Preston (2010b) also adjust the inflation rate in 1991 using the series excluding food, energy and taxes to cope with the outlier.

algorithm is used to obtain the complete posterior distribution (An and Schorfheide, 2007).[12] Following Adolfson et al. (2008), the exogenous foreign VAR is estimated before the estimation of SOE-DSGE parameters since it does not use any information from the Canadian variables. The parameters of VAR are kept fixed at their posterior mode estimates throughout the estimation of DSGE parameters.

3.1 Calibrated parameters

The parameters that determine the steady state of the model are calibrated since the data used in the estimation do not contain such information. Table 1 summarizes the calibrated parameters. The discount factor, β, is calibrated to 0.9941 using the Euler equation that has steady-state (annualized) interest and inflation rates, 4.4% and 2%, respectively. The portfolio share parameters γ_a, γ_S, γ_L are calibrated based on Canadian residents' domestic and US government bond holdings in short- and long-term maturities. The Balance of International Payments data from CANSIM provide information on foreigners' holdings of Canadian short- and long-term government bonds. The home asset holdings by Canadian residents are then constructed using those two series. In addition, annual data on Canadian holdings of US long-term and short-term government bond holdings are obtained from the US Treasury. One caveat is that the data go back only to 2002; however, they still give us information about the US bond holdings over half of the sample period. Using the constructed portfolio, the share of total short-term bonds, γ_a, is calibrated to 0.42. The shares of home assets in short-term and long-term sub-portfolios are then calibrated to 0.93 and 0.92, respectively, matching home biasness for Canadian assets in the portfolio. The coupon rate, κ, is calibrated to 0.98, implying a duration of 20 quarters, similar to the average duration of Canadian long-term government bonds outstanding in the secondary market. Lastly, the home biasness parameter in consumption goods is set to 0.70 to match the average ratio of 30% for imports-to-GDP in the data.

3.2 Prior choice

Table 2 reports the prior distribution of each parameter estimated. For parameters that the economic theory suggests should be non-negative, I use gamma distributions, constraining their support on the interval $[0, \infty]$. I use beta distributions for those parameters that span between 0 and 1. For the standard deviation of shock innovations, I use the inverse gamma distribution. For the foreign bloc, I use normal distributions in the estimation of VAR coefficients.

The key parameters for QE results are the portfolio adjustment cost parameters. For those parameters, I set priors with high values of standard deviation so that the QE shock in Canada, a priori, covers a large set of possibilities. For the adjustment cost between short- and long-term sub-portfolios, ξ_a, the prior mean is chosen based on the portfolio balance estimates in Gagnon et al.

[12] I use Dynare for estimation of the model. For the Metropolis–Hastings algorithm, I use two chains of 250,000 draws, dropping the first 45% of the draws. The acceptance ratio was about 26%.

(2011) for the United States. In particular, using the closed economy version of the model with US short- and long-term portfolio shares, the prior mean is set to 0.60/100, which implies a reduction of about 30 basis points (bps) in the US term premium following a $600 billion asset purchase in the United States. This corresponds to about 27% shock to bond supply shock, which lowers the supply of long-term bonds by about 10.5% and increases the supply of short-term bonds by about 16.5%. The standard deviation for the prior is set high enough that the QE effect spans the range of estimates found in the literature from 3 bps to 15 bps for $100 billion asset purchases.[13]

The parameters governing portfolio balancing costs between home and foreign bonds, ξ_S and ξ_L, are crucial in determining the QE effectiveness in a small open economy relative to a large economy. I assume a prior mean for these parameters so that the QE in the small open economy is almost as effective as that in the large economy, and a prior standard deviation that covers a range of values implying from much smaller to much larger effects than in a large economy. In doing so, I am agnostic as to which direction the effectiveness will go for the small open economy. In particular, the prior means for these parameters are set to be 0.1/100, which implies about 35 bps following the same shock explained above.[14]

The other structural DSGE parameters have standard priors used widely in the literature (see Smets and Wouters, 2007; Justiniano and Preston, 2010a; 2010b). For the habit parameter, ζ, I use a beta prior with mean 0.60 and standard deviation 0.10. For the inverse Frisch labour supply elasticity, ϑ, I use a gamma prior with a mean 1.50 and standard deviation 0.25. Calvo probabilities and indexation parameters are centred at 0.50 with a standard deviation 0.15 and 0.10, respectively, as in Smets and Wouters (2007). The elasticity of substitution between home and foreign goods has a gamma prior with a mean 1.50 and a standard deviation 0.25, similar to that in Justiniano and Preston (2010a, 2010b).

Turning to the policy parameters, I use a beta prior with a mean 0.60 and a standard deviation 0.10 for the smoothing parameter, ρ, in the Taylor rule. For the inflation reaction coefficient, r_π, I use a gamma prior with a mean 1.5 and a standard deviation 0.20. For parameters governing the sensitivity of monetary policy to output, output growth and depreciation, I use gamma priors with a mean 0.20 and a standard deviation 0.05. Note that those parameters are set to be low a priori, reflecting the inflation-targeting regime performed by the Bank of Canada over the sample period. For the elasticity parameter in tax policy, τ_b, I use a gamma prior with a mean 1.50, similar to the that in Chen et al. (2012) and the estimate found in Andres et al. (2004). As in the cited literature, the standard deviation is set to be large since we do not have a strong prior belief for this parameter.

The shock persistence parameters have beta priors with a mean 0.75 and a standard deviation

[13]See Table 1 in Chen et al. (2012), which summarizes the empirical estimates for asset purchases conducted by the Federal Reserve. For LSAP2 in particular, Bernanke (2012) reports a range of 2.5 to 7.5 bps decline in long-term yields per $100 billion asset purchases.

[14]For a 30 bps effect on term premium, the parameters have to be set to 0.08. The mean is rounded for presentation purposes. Assuming the smaller prior does not change the results much.

0.10, and the shock standard deviations have inverse gamma priors with a mean 0.5 and a standard deviation as large as 2. Finally, the foreign bloc VAR(2) is assumed to have priors centred close to those suggested by individual autoregressions. For the first-order autoregressive coefficients, I specify an $N(0.60, 0.20^2)$ for inflation, $N(0.90, 0.10^2)$ for output and short- and long-term interest rates, an $N(0.90, 0.10^2)$ for foreign short- and long-term bond supply. Second-order own lags have an $N(0, 0.25^2)$ prior, while the off-diagonal elements of the first and second lag matrices have priors $N(0, 0.30^2)$ and $N(0, 0.15^2)$, respectively.

3.3 Posterior estimates

Table 2 reports the posterior mode, median and mean estimates along with the 5th and 95th percentiles of the posterior distribution. The data are quite informative about most of the parameters, since the posterior distributions are fairly different from the priors; and the estimation results, including those related to portfolio adjustment costs, are not driven by the priors assumed. The portfolio adjustment cost parameters are estimated to be lower than prior means. Those governing the substitution between Canadian and US bonds especially are quite low, suggesting a large substitution between those bonds.[15] These relatively small estimates for the portfolio adjustment costs between home and foreign bonds are consistent with the empirical literature on portfolio balance effects on exchange rates, which typically find limited effects of central banks' foreign exchange interventions on their exchange rates (Lewis, 1995; Engel, 2014). Note that the cost between home and foreign bonds in the long-term sub-portfolio is estimated to be lower than that in the short-term sub-portfolio. This implies a smaller deviation in the long-term UIRP condition, also found in the empirical literature (Chinn and Meredith, 2004; and Chinn and Quayyum, 2012).

Estimates for the other DSGE parameters are by-and-large quite standard and close to those estimates found in Justiniano and Preston (2010a, 2010b). In particular, the habit parameter is estimated to be around 0.37, while the inverse Frisch elasticity is estimated to be around 1.70. The median estimate for the elasticity of substitution between home and foreign goods is around 0.52. Calvo parameters are estimated to be 0.54 and 0.34 for home and imported good prices, implying 2-quarter and 1.5-quarter price rigidities, respectively. The indexation parameters are estimated to be around 0.27 and 0.38 for home and imported goods, respectively. The parameters governing price rigidities and the consumption habit are found to be somewhat lower than those found in papers that study the US economy; however, they are consistent with those found in papers that study small open economies such as Canada (Justiniano and Preston, 2010a and 2010b).

[15] A large substitution between Canadian and US bonds implies a large effect of US long-term rates on Canadian long-term yields through equation (17). This is consistent with recent studies that found that US long-term rates, rather than short-term rates, have a larger effect on advanced economies' long-term yields (Hoffman and Takáts, 2015; and Obstfeld, 2015). The estimated model at the posterior implies that 100 bps exogenous increase in US long-term rates increases the Canadian long-term rate by 70 bps, which is close to the findings in those papers and consistent with the high co-movement between Canadian and U.S. term premia presented in Figure 2.

Taylor rule parameters are found to be closer to those estimates found in the literature. Particularly, the smoothing parameter is estimated to be around 0.76 and the coefficients on inflation, output, output growth and depreciation are estimated to be 2.62, 0.07, 0.25, 0.11, respectively. The elasticity parameter in the tax policy is estimated to 1.55 at the median of the posterior. The estimates for the persistence parameters range from 0.76 to 0.97, implying that the data are informative and suggest a higher persistence for these parameters than priors. The estimates for shock standard deviations are also larger than priors for productivity, cost-push, bond supply and duration risk premium shocks.

4 Results

This section performs stochastic simulations following a QE shock in the model. The QE shock is calibrated to match the change in the relative supply (in terms of percentages) between short-term and long-term government bonds following the large-scale asset program announced in the United States in 2010. By doing this, I take the QE in the United States as a benchmark case to compare with a Canadian QE using an SOE model. In 2010, the Federal Reserve announced a $600 billion purchase of US long-term government bonds, which lowered private holdings of US long-term government bonds by about 10.5% and increased the short-term government liabilities by 16.5%. Therefore, the scenario assumes a 27% change in the relative supply for Canadian long-term bonds in the market; thus, $\varepsilon_t^b = 0.27$.[16] The QE path, and thus the persistence of the QE shock, is assumed to be consistent with market expectations of the Federal Reserve's balance sheet in 2010: purchases occur over the first four quarters, after which the balance sheet remains unchanged over the next eight quarters and then converges to its steady-state level in five years. All agents know the whole path of the aforementioned QE policy at the impact period. The model uses the posterior median estimates of parameters, except that I assume a high coefficient for the smoothing parameter in the Taylor rule to ensure no significant increase in the current or expected short-term interest rates. Thus, agents expect no change in the path of policy rates. This reflects the signalling channel of QE, which states that QE signals low short-term rates even after the economy recovers (i.e., lower than what a Taylor rule may call for; see Krishnamurthy and Vissing-Jorgensen, 2011).

4.1 The impulse responses to a QE shock

Figure 4 shows the impulse responses to the QE shock, details of which are described above. For comparison, the responses of QE in the large economy version of the model are also added. First, I

[16] Note that since the Canadian bond market is slightly less weighted toward long-term maturities than the US bond market, the same QE shock lowers the supply of Canadian long-term bonds by 9% and increases that of short-term bonds by 16%. Using the relative supply shock rather than an absolute supply shock for long-term bonds isolates the small-open-economy effect from the asset supply differences between countries when comparing QE effects in a small versus large economy.

discuss the effects of QE on financial variables and then analyze the effect on the real economy. The shock rebalances portfolios toward Canadian short-term bonds, lowering the amount of Canadian long-term bonds in portfolios. Since the rebalancing is not perfect, the resulting excess supply of short-term Canadian bonds relative to long-term bonds generates an upward pressure on the price of long-term bonds, lowering the term premium. However, some of this effect on the term premium is dampened by the large supply of foreign substitutes; namely, US long-term bonds in the world. At the equilibrium, the domestic term premium drops by about 10 bps, and Canadians hold fewer (more) domestic long-term (short-term) bonds while they hold more (fewer) US long-term (short-term) bonds. Similarly, foreign investors hold fewer Canadian long-term bonds and more Canadian short-term bonds. Note that even though Canadian and US bonds are estimated to be almost perfect substitutes, the term premium effect is different from zero.[17] This is due to a non-zero portfolio cost parameter between Canadian and US bonds in the short-term portfolio, requiring a compensation for investors to hold a larger share of Canadian bonds in the short-term sub-portfolio than before.

The decline in the term premium generates a return differential between long-term Canadian and global bonds. As a result, the Canadian dollar falls by 0.55% through the long-term UIRP condition (equation (19)). The arbitrage condition among short-term bonds also validates this result (equation (18)): A higher supply of Canadian short-term bonds relative to foreign short-term bonds makes Canadian short-term bonds less attractive, lowering the value of the Canadian dollar. The fall in the nominal exchange rate also leads to a fall in the real exchange rate due to the nominal rigidities in the model.

Both the lower domestic term premium and the depreciation of currency help stimulate the aggregate demand in Canada through domestic and external demand. The decline in the domestic term premium reduces long-term rates in Canada, increasing domestic demand and raising inflationary pressures. Moreover, the depreciated currency increases external demand for Canadian goods, raising real exports and putting upward pressures on inflation. As a result of these expansionary effects of QE, GDP rises by slightly higher than 0.16% and the inflation rate increases by 1.1%. Note that the effect on inflation is larger than that on the aggregate demand due to the low estimates of nominal rigidities from the estimated model. When I increase the rigidities (perhaps due to rigid wages that are assumed to be flexible in the model for simplicity), the effect on the aggregate demand increases while inflation increases by a smaller amount. Note that such an exercise would not change the effect on term premium.

Figure 4 also compares the stimulatory effect of QE in a small open economy with that implied by the large-economy bloc in the model calibrated to the Canadian bond market. Had Canada been a large (closed) economy, the same QE shock would lower the term premium by 20 bps more than in the case of a small open economy (30 bps compared with 10 bps). Thus, the QE effect on long-term rates is about one-third of that in a large economy. The difference stems from the dampening effect

[17] Note that when ξ_L is set to be zero instead of 1.3e-05/100, the simulation results remain virtually the same.

of the substitutable foreign asset supply in the case of the small open economy. In the absence of such substitutable assets, agents require a larger drop in the term premium to hold more short-term bonds and fewer long-term bonds following a QE shock. As a result, QE is more stimulatory in the large economy, increasing aggregate demand by 0.40%, while it increases aggregate demand by only 0.16% in the small open economy. Note that although the effect on term premium in the small open economy is one-third of that in a large economy, the effect on the aggregate demand is relatively larger in the small open economy due to the additional stimulatory effect of exchange rate depreciation, which is not present in the case of the large economy. However, the additional stimulus coming from the exchange rate effect is not enough to cover the losses stemming from a smaller drop in the term premium.

4.2 QE shock without ZLB commitment

Previous analysis assumes that policy rates do not respond to the expansionary effects of QE. As discussed previously, this assumption reflects the signalling channel that is found to be effective in the literature. I now discuss the contribution of this channel by letting policy rates respond through the estimated Taylor rule. Thus, the policy has no ZLB commitment in this case.

Figure 5 shows the responses to the QE shock without the ZLB commitment as well as the results from the baseline scenario (blue line). An upward shift in the path for policy rates can be interpreted as a faster normalization of policy rates.[18] As a result of higher-than-initial current and expected short-term rates, long-term interest rates do not fall significantly. In fact, the immediate impact on long-term rates is positive. Therefore, aggregate demand increases much less (0.01%) than the baseline case (0.16%). A smaller stimulus leads to a lower inflation as well. Note that in this scenario, the aggregate demand still increases even though both short- and long-term nominal rates increase. This is due to the inflation, lowering the "real" long-term borrowing costs.

Without ZLB commitment, the QE shock is less effective not only on long-term rates but also on the exchange rate, as a result of a smaller return differential between Canadian and US long-term bonds. Consequently, the external demand increases less, contributing to the smaller effect on the aggregate demand.

4.3 Open vs. closed domestic debt market

Many SOE-DSGE models in the literature assume perfect substitution among assets. In addition, these models are commonly solved by linear techniques, especially when they are estimated and used for forecasting purposes. When solved linearly, these models cannot pin down agents' gross holdings of foreign and domestic bonds. This is why the domestic debt market is usually assumed to be closed

[18]Note that, since the simulation starts from the steady-state levels, the policy rate converges to the initial point; thus, its steady-state level. Had we started from an initial point where a negative shock generated a recessionary output gap, the rates would monotonically go up.

in such models, implying no international trade in domestic assets. On the other hand, the model presented here assumes imperfect substitution, which allows domestic assets to be internationally traded as well. I now discuss the contribution of this additional channel of foreigners' access to the domestic debt market.

To understand the contribution of foreigners' access to the domestic debt market, I assume a closed domestic debt market and compare the results from this version of the model with those implied by the baseline model. Specifically, now all domestic bonds are held by domestic residents; i.e., $b_{HS,t} = b_{S,t}$ and $b_{HL,t} = b_{L,t}$. Figure 6 shows the results (red line) from the model without an external demand for Canadian assets, along with the results from the baseline scenario. In the absence of foreigners' access to the Canadian debt market, QE in Canada leads to a portfolio rebalancing only within the Canadian residents' portfolio. Thus, all the additional Canadian short-term bonds after QE are held by Canadian residents. To hold more Canadian short-term bonds and, in return, fewer Canadian long-term bonds than in the baseline case, Canadian residents require a smaller term premium at equilibrium. Therefore, the term premium drops more than in the baseline scenario (27 bps compared with 10 bps), making the QE effect on long-term interest rates closer to the one that a large economy model would apply (30 bps).

A larger drop in the term premium in the absence of foreigners' access to the Canadian debt market further depreciates the Canadian dollar. With lower long-term interest rates and more depreciated currency, QE becomes more effective in stimulating aggregate demand and generating more inflationary pressures in Canada. These results underscore the importance of modelling foreigners' access when evaluating QE programs in small open economies. Ignoring foreigners' access to the domestic debt market would mistakenly make QE look more effective in these economies.

4.4 Sensitivity analysis on the substitution between assets

This section discusses the sensitivity of elasticity-of-substitution parameters between domestic and foreign assets. Figure 7 shows the results when the portfolio cost parameters between home and foreign assets, ξ_S and ξ_L, are set to a higher (0.1) and a lower value (an extreme case of zero) while the cost parameter between short- and long-term sub-portfolios, ξ_a, is kept constant at its estimated value. The figure shows that as assets become more substitutable, the QE effect on both financial and real variables gets smaller. In the extreme case where Canadian and US assets are perfect substitutes in both short-term and long-term sub-portfolios (green line), $\xi_S = \xi_L = 0$, QE in Canada has no effect on the economy. In this case, the Canadian term premium cannot deviate from the global term premium. Conversely, when the substitutability between Canadian and US assets is reduced, and a larger portfolio cost parameter is therefore assumed, a Canadian QE becomes more effective (red line).

5 Conclusion

This paper introduces QE policies in a small open economy model with international portfolio balance effects and foreigners' access to the domestic debt market. The main finding is that the effectiveness of QE policies in a small open economy depends on the extent of the integration of the economy. The effect is smaller as home and global bonds become more substitutable. The paper also provides evidence from the estimated model on the large substitutability between home and foreign bonds using Canadian data, which implies less effective QE policies for small open economies. The stimulatory effects of QE become even smaller in the absence of a clear signal on policy rates. The paper also emphasizes the importance of modelling foreigners' access to domestic bond markets. Ignoring foreigners' access to the domestic debt market would mistakenly make QE look more effective.

Note that the analysis in the paper assumes a constant degree of substitution between home and foreign assets. A large negative shock (akin to a domestic financial crisis) or a significant deterioration in fiscal stance might break the substitutability between home and foreign assets in both domestic and global portfolios. In such scenarios, where the domestic term premium significantly deviates from the global term premium, QE policies might become more effective in reducing the domestic term premium and altering asset prices in the economy.

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Table 1: Calibrated parameter

Parameter	Value	Explanation
Discount factor, β	0.985	matching the st.-st. interest rate (1.1%, quarterly)
Home share (short), γ_S	0.93	st-st ratio of home bonds in short-term portfolio
Home share (long), γ_L	0.92	st-st ratio of home bonds in long-term portfolio
Short share, γ_a	0.42	st-st ratio of short-term bonds in overall portfolio
Coupon, κ	0.98	calibrated matching the average duration of bonds (5 years)
Home biasness, γ_c	0.70	matching the st-st ratio of imports-to-GDP (30%)

Notes: The structural parameters for the global bloc are the same as in the home country except where otherwise noted in the table. See the section 3.2 for details.

Figure 1: Policy rates in selected small open economies

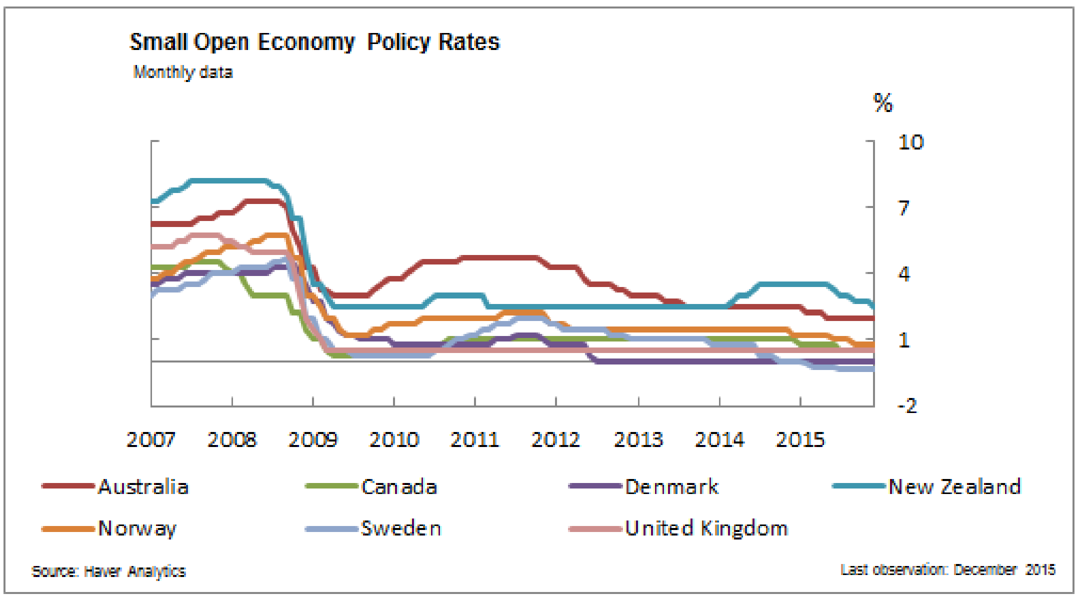
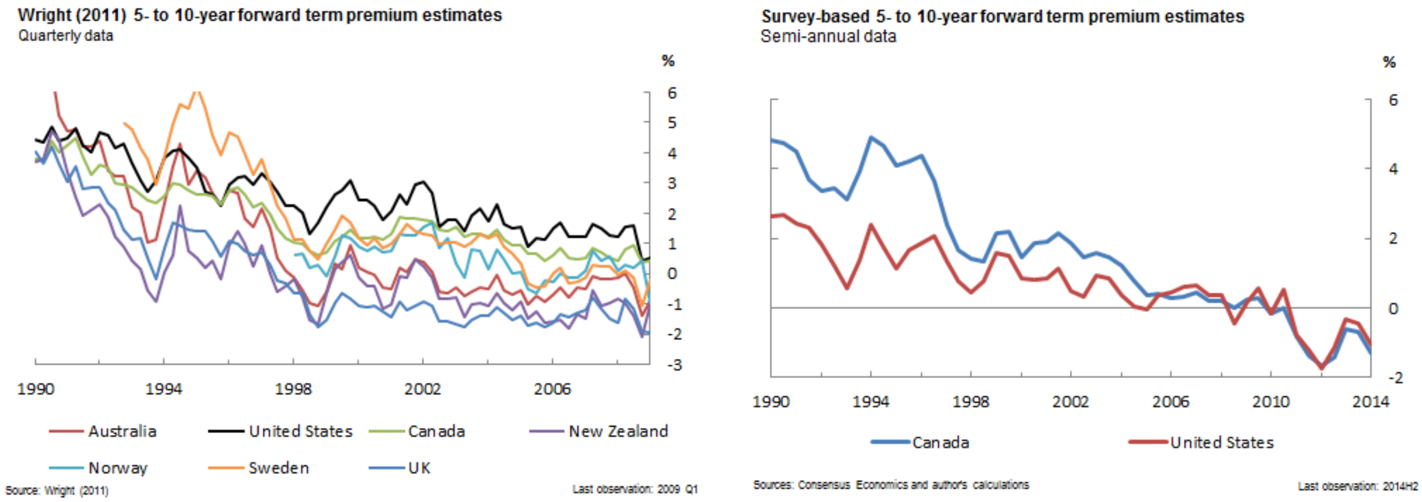


Table 2: Prior densities and posterior distribution of model parameters

Parameter		*Prior* Density	Mean	Std	*Posterior* Mode	Median	Mean	Std	5%	90%
Inverse elas. ST-LT portfolios	$100\xi_a$	G	0.60	0.30	0.2584	0.3750	0.4034	0.1475	0.1256	0.6632
Inverse elas. H-F Short-term bonds	$100\xi_S$	G	0.10	0.10	0.0030	0.0042	0.0045	0.0017	0.0014	0.0074
Inverse elas. H-F Long-term bonds	$100\xi_L$	G	0.10	0.10	1.0e-05	1.3e-05	1.3e-05	2.5e-08	1.2e-05	1.4e-05
Inverse Frisch elas.	ϑ	G	1.50	0.25	1.6648	1.6908	1.7047	0.2548	1.2919	2.1240
Habit	ζ	B	0.60	0.10	0.3640	0.3728	0.3735	0.0483	0.2927	0.4539
Calvo domestic prices	θ_h	B	0.50	0.15	0.5267	0.5356	0.5340	0.0475	0.4543	0.6122
Calvo import prices	θ_f	B	0.50	0.15	0.3545	0.3427	0.3446	0.0535	0.2633	0.4327
Indexation domestic prices	ς_h	B	0.50	0.10	0.2646	0.2707	0.2784	0.0816	0.1405	0.4064
Indexation import prices	ς_f	B	0.50	0.10	0.3562	0.3839	0.3888	0.0967	0.2309	0.5384
Elas. H-F goods	λ	G	1.50	0.25	0.5137	0.5207	0.5268	0.0449	0.4575	0.5985
Taylor rule, smoothing	ρ	B	0.60	0.10	0.7550	0.7594	0.7572	0.0312	0.7024	0.8074
Taylor rule, inflation	r_π	G	1.50	0.20	2.6005	2.6169	2.6217	0.2096	2.2880	2.9601
Taylor rule, output	r_y	G	0.20	0.05	0.0681	0.0713	0.0722	0.0146	0.0469	0.0958
Taylor rule, output growth	$r_{\delta y}$	G	0.20	0.05	0.2448	0.2533	0.2571	0.0584	0.1582	0.3483
Taylor rule, nominal exchange rate	r_d	G	0.20	0.05	0.1046	0.1108	0.1122	0.0213	0.0743	0.1460
Elas. in tax policy	τ_b	G	1.50	0.50	1.3348	1.5524	1.6316	0.4721	1.0002	2.1849
Pers. productivity	ρ_z	B	0.75	0.10	0.9578	0.9512	0.9494	0.0135	0.9257	0.9748
Pers. preferences	ρ_d	B	0.75	0.10	0.7911	0.7879	0.7770	0.0993	0.6352	0.9296
Pers. cost-push imports	ρ_f	B	0.75	0.10	0.9744	0.9712	0.9700	0.0084	0.9546	0.9854
Pers. country risk premium	ρ_{cr}	B	0.75	0.10	0.8305	0.7870	0.7732	0.0739	0.6310	0.9081
Pers. duration risk premium	ρ_{dr}	B	0.75	0.10	0.8712	0.8660	0.8644	0.0277	0.8155	0.9121
Pers. bond supply	ρ_b	B	0.75	0.10	0.9525	0.9515	0.9509	0.0148	0.9277	0.9752
Pers. tax	ρ_τ	B	0.75	0.10	0.7818	0.7568	0.7475	0.1041	0.5993	0.9146
Std. productivity	σ_z	IG	0.50	2.00	0.9493	0.9861	0.9955	0.0927	0.8261	1.1497
Std. preferences	σ_d	IG	0.50	2.00	0.2319	0.3221	0.4668	0.0918	0.1059	1.0243
Std. cos-push imports	σ_f	IG	0.50	2.00	2.8929	3.0479	3.1406	0.7023	1.9111	4.3183
Std. country risk premium	σ_{cr}	IG	0.50	2.00	0.1960	0.2178	0.2248	0.0465	0.1394	0.3073
Std. duration risk premium	σ_{dr}	IG	0.50	2.00	0.7693	0.8112	0.8242	0.1603	0.5302	1.1256
Std. bond supply	σ_b	IG	0.50	2.00	4.8607	4.9175	4.9406	0.3460	4.3649	5.5371
Std. tax	σ_τ	IG	0.50	2.00	0.2351	0.3451	0.4696	0.0954	0.1182	0.6935
Std. monetary policy	σ_r	IG	0.50	2.00	0.2178	0.2212	0.2236	0.0233	0.1850	0.2639

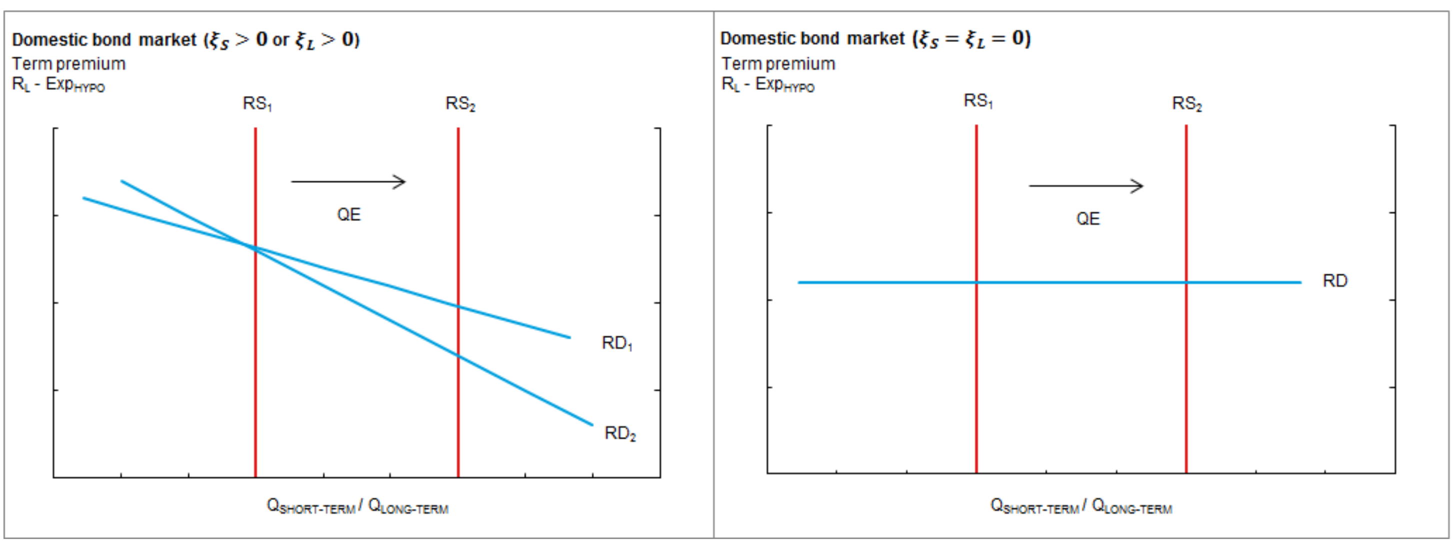
Distributions: B: Beta, G: Gamma, IG: Inverse Gamma. ST and LT stand for short-term and long-term, respectively.

Figure 2: Term premium estimates over time



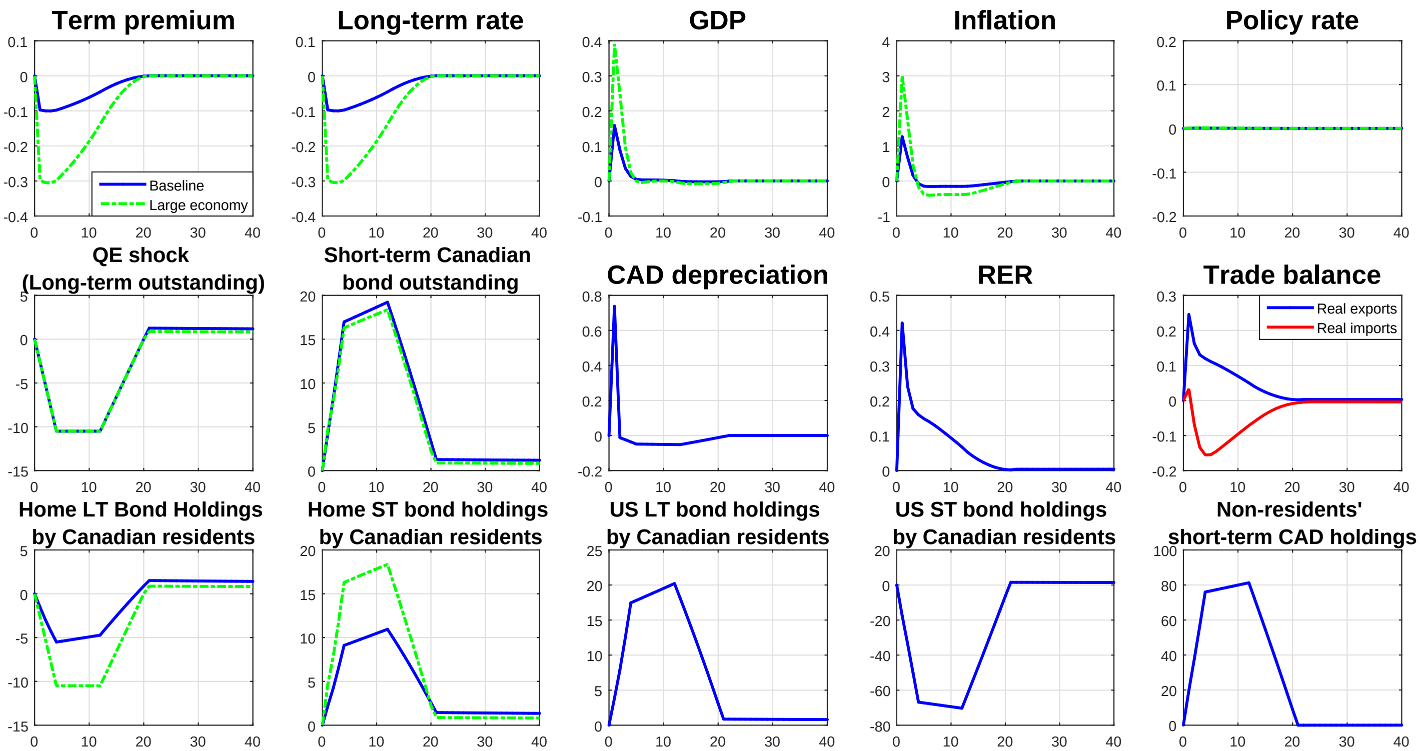
Notes: The figure shows term premium estimates on 5- to 10-year forward rates for selected small advanced economies. The estimates in the left panel are obtained from Wright (2011), who studies an affine term-structure model. Alternatively, the right panel shows survey-based estimates for Canada and the United States. The estimates are obtained using forecasts on long-run real GDP, inflation and short-term interest rates from Consensus Economics and Blue Chip Surveys for Canada and the United States, respectively.

Figure 3: Illustration of QE in a small open economy



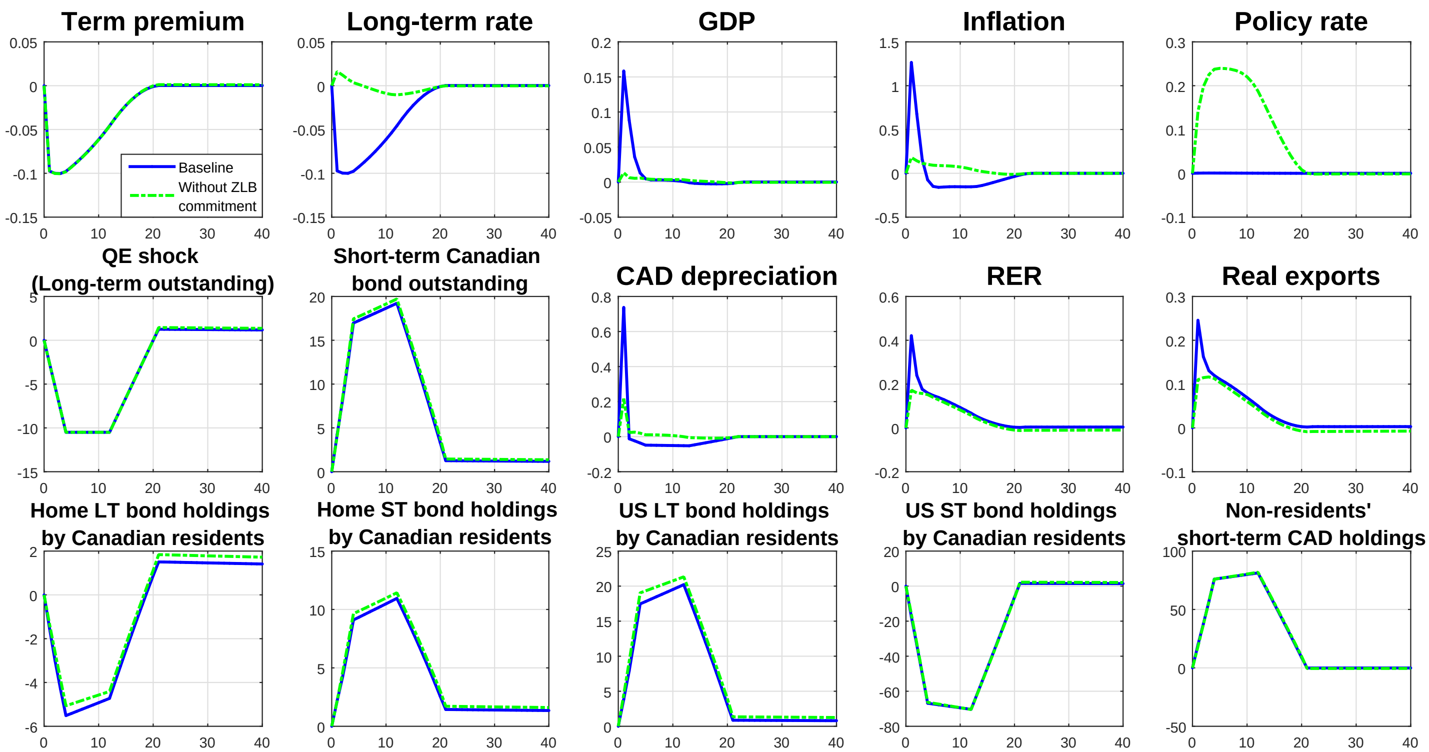
Notes: The vertical and horizontal axes represent the term premium and the quantity of short-term bonds relative to long-term bonds in the home bond market. "RD" and "RS" denote relative demand and relative supply, respectively. ξ's denote the coefficients in portfolio adjustment costs. RD_1 assumes a smaller portfolio adjustment cost between home and foreign bonds (either ξ_S or ξ_L) than RD_2.

Figure 4: Responses to a QE shock



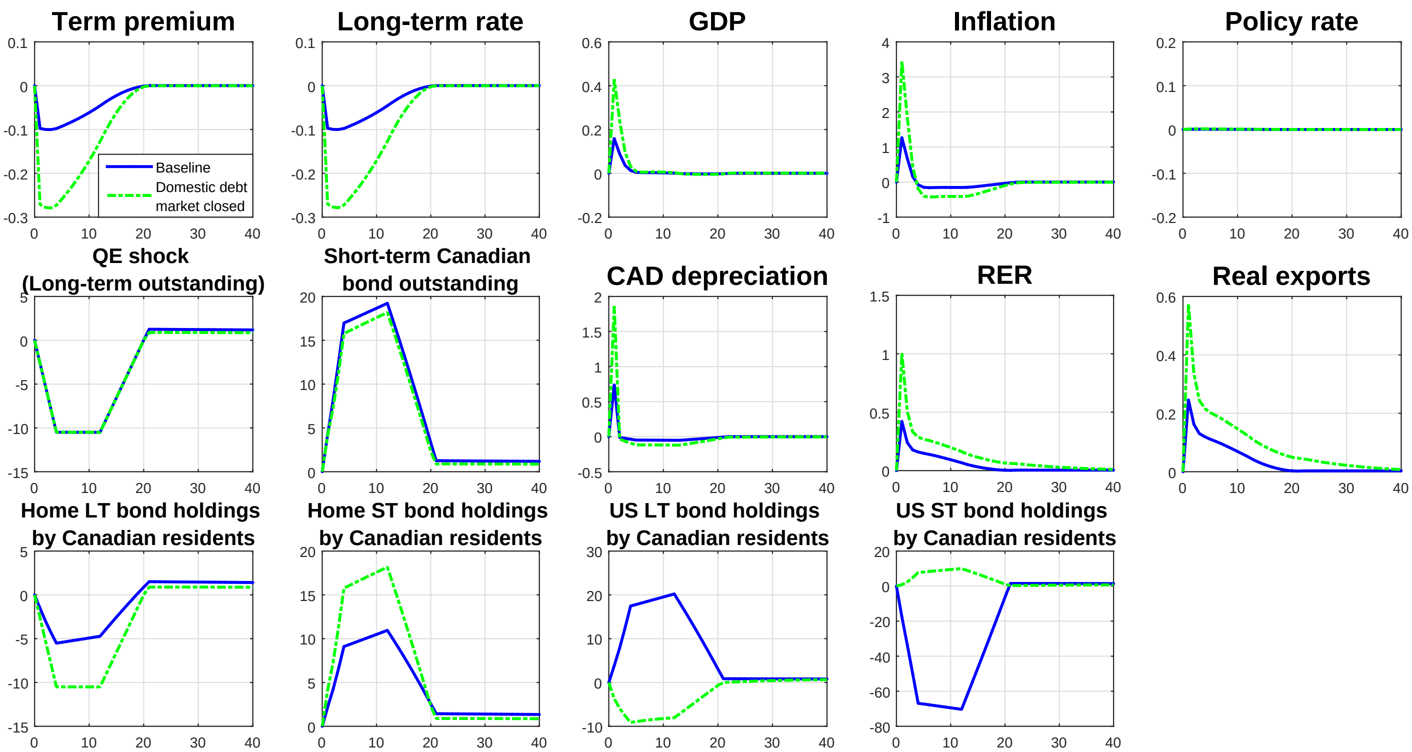
Notes: Impulse responses use posterior modes of parameters except the smoothing parameter in the Taylor rule, which is set to be high to mimic commitment at the initial short-term interest rate. The responses are presented as percentage point deviations (%) from the steady-state level for each variable. The "Large economy" line shows the responses of QE in the case of a global economy; thus, $\gamma_S = \gamma_L = \gamma_c = 1$.

Figure 5: Responses without policy rate commitment



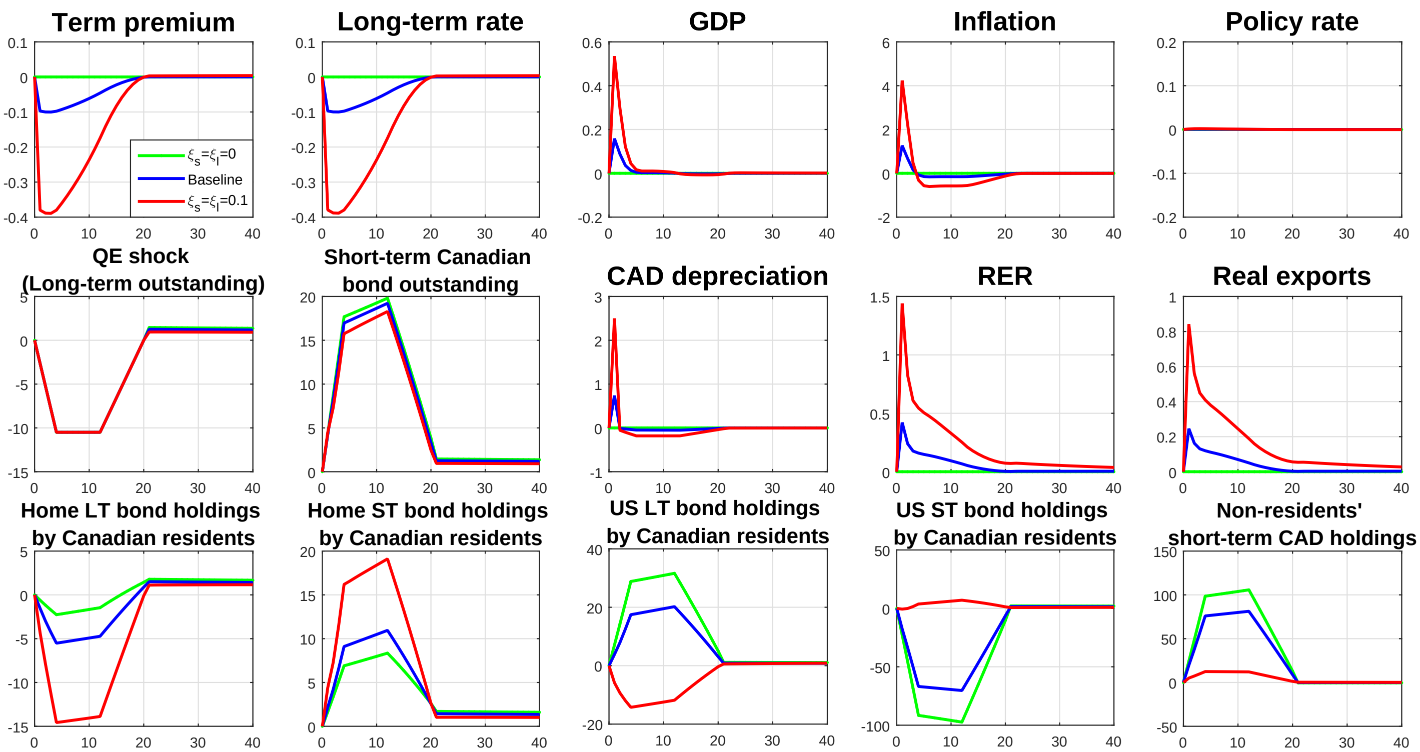
Notes: Impulse responses are presented as percentage point deviations (%) from the steady-state level for each variable.

Figure 6: Domestic debt market: closed vs. open



Notes: Impulse responses are presented as percentage point deviations (%) from the steady-state level for each variable.

Figure 7: Sensitivity on the elasticity of substitution between home and foreign bonds



Notes: Impulse responses are presented as percentage point deviations (%) from the steady-state level for each variable.

A Price-Setting Problems

A.1 Final-goods producers

The final-goods producers' problem is to maximize their profits at each period:

$$\max P_t c_t - P_{h,t} c_{h,t} - P_{f,t} c_{f,t}$$

subject to

$$c_t = \left[\gamma_c^{\frac{1}{\lambda}} c_{h,t}^{\frac{\lambda-1}{\lambda}} + (1-\gamma_c)^{\frac{1}{\lambda}} c_{f,t}^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}} .$$

Solving the problem, the following consumption demand functions are obtained:

$$c_{h,t} = \left(\frac{P_{h,t}}{P_t} \right)^{-\lambda} \gamma_c c_t \qquad \text{and} \qquad c_{f,t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\lambda} (1-\gamma_c) c_t,$$

where the CPI price index (defined as the minimum expenditure required to buy one unit of c_t) is given by:

$$P_t = \left[\gamma_c P_{h,t}^{1-\lambda} + (1-\gamma_c) P_{f,t}^{1-\lambda} \right]^{\frac{1}{1-\lambda}} .$$

Log-linearization leads to:

$$\widehat{\pi}_t = \gamma_c \widehat{\pi}_{h,t} + (1-\gamma_c)\, \widehat{\pi}_{f,t} \ .$$

A.2 Domestic intermediate-goods producers

Given nominal rigidities defined in the text, the Dixit-Stiglitz aggregate price index evolves as follows:

$$P_{h,t} = \left[(1-\theta_h) P_{h,t}^{new\ (1-\varepsilon_h)} + \theta_h \left(P_{h,t-1} \left(\frac{P_{h,t-1}}{P_{h,t-2}} \right)^{\varsigma_h} \right)^{1-\varepsilon_h} \right]^{1/(1-\varepsilon_h)} , \tag{37}$$

where ε_h denotes the elasticity of substitution between differentiated home goods. Firms setting new prices at period t face a demand curve:

$$y_T(j) = \left(\frac{P_{h,t}^{new}}{P_{h,s}} \left(\frac{P_{h,s-1}}{P_{h,t-1}} \right)^{\varsigma_h} \right)^{-\varepsilon_h} (c_{h,t} + c_{f,t}^*) \tag{38}$$

for all $s > t$ and take aggregate prices and consumption bundles as parametric.

The firms that are allowed to reoptimize their prices in period t choose a price $P_{h,t}^{new}(j)$, to maximize their expected present value of profits:

$$E_t \sum_{s=t}^{\infty} \theta_h^{s-t} q_{t,s} y_s(j) \left[P_{h,t}^{new}(j) \left(\frac{P_{h,s-1}}{P_{h,t-1}} \right)^{\varsigma_h} - P_{h,s} MC_s \right] \tag{39}$$

subject to the demand curve, (38). $MC_s = \frac{W_s}{P_{h,s}\varepsilon_s^z}$ is the real marginal cost function for each firm.The factor $q_{t,s}$ is interpreted as a stochastic discount factor evaluated at aggregate income and θ_h^{s-t} denotes the probability that the firm will not be able to adjust its price in the next $(s-t)$ periods. The optimization problem implies the first order condition:

$$E_t \sum_{s=t}^{\infty} \theta_h^{s-t} q_{t,s} y_s(j) \left[P_{h,t}^{new}(j) \left(\frac{P_{h,s-1}}{P_{h,t-1}} \right)^{\varsigma_h} - \frac{\theta_h}{\theta_h - 1} P_{h,s} MC_s \right] = 0. \tag{40}$$

By combining (37) and (40), we can obtain the New Keynesian Phillips Curve for domestic good prices (after log-linearization):

$$\widehat{\pi}_{h,t} = \frac{\varsigma_h}{1+\varsigma_h \beta} \widehat{\pi}_{h,t-1} + \frac{\beta}{1+\varsigma_h \beta} E_t \widehat{\pi}_{h,t+1} + \frac{(1-\theta_h)(1-\beta\theta_h)}{\theta_h (1+\varsigma_h \beta)} (\widehat{mc}_t) \, .$$

A.3 Retail firms

The Dixit-Stiglitz aggregate price index for imported goods evolves as follows:

$$P_{f,t} = \left[(1-\theta_f) P_{f,t}^{new\ (1-\varepsilon_f)} + \theta_f \left(P_{f,t-1} \left(\frac{P_{f,t-1}}{P_{f,t-2}} \right)^{\varsigma_f} \right)^{1-\varepsilon_f} \right]^{1/(1-\varepsilon_f)}, \tag{41}$$

where ε_f denotes the elasticity of substitution between differentiated home goods. The retailers setting new prices at period t face a demand curve:

$$c_{f,s}(j) = \left(\frac{P_{f,t}^{new}}{P_{f,s}} \left(\frac{P_{f,s-1}}{P_{f,t-1}} \right)^{\varsigma_f} \right)^{-\varepsilon_f} (c_{h,t} + c_{f,t}^*) \tag{42}$$

for all $s > t$ and take aggregate prices and consumption bundles as parametric.

The retail firms that are allowed to reoptimize their prices in period t choose a price $P_{f,t}^{new}(j)$, to maximize their expected present value of profits:

$$E_t \sum_{s=t}^{\infty} \theta_f^{s-t} q_{t,s} c_{f,s}(j) \left[P_{f,t}^{new}(j) \left(\frac{P_{f,s-1}}{P_{f,t-1}} \right)^{\varsigma_f} - e_s P_{f,s}^* \right] \tag{43}$$

subject to the demand curve, (42). The factor θ_f^{s-t} denotes the probability that the firm will not be able to adjust its price in the next $(s-t)$ periods. The optimization problem implies the first order condition:

$$E_t \sum_{s=t}^{\infty} \theta_f^{s-t} q_{t,s} c_{f,s}(j) \left[P_{f,t}^{new}(j) \left(\frac{P_{f,s-1}}{P_{f,t-1}} \right)^{\varsigma_f} - \frac{\theta_f}{\theta_f - 1} P_{f,s} MC_s \right] = 0. \tag{44}$$

By combining (41) and (44), we can obtain the New Keynesian Phillips Curve for imported good prices (after log-linearization):

$$\widehat{\pi}_{f,t} = \frac{\varsigma_f}{1+\varsigma_f \beta} \widehat{\pi}_{f,t-1} + \frac{\beta}{1+\varsigma_f \beta} E_t \widehat{\pi}_{f,t+1} + \frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f (1+\varsigma_f \beta)} \left(\widehat{rer}_t - \widehat{p}_{f,t} \right).$$

B The System of Log-Linearized Equations

This section provides log-linearized equations, along with exogenous processes, used in the estimation:

$$\widehat{\varepsilon}_t^g - \frac{1}{1-\zeta} (\widehat{c}_t - \zeta \widehat{c}_{t-1}) = \widehat{\lambda}_t$$

$$\widehat{\lambda}_t = \left(E_t \widehat{\lambda}_{t+1} + \widehat{R}_t - E_t \widehat{\pi}_{t+1} \right) - \frac{(1-\zeta)\, c/y}{\gamma_a a/y} \left(\xi_a \left(\widehat{a}_{S,t} - \widehat{a}_{L,t} \right) + \frac{\xi_S}{\gamma_S} \left(\widehat{b}_{HS,t} - \widehat{rer}_t - \widehat{b}_{FS,t} \right) \right)$$

$$\frac{R_L}{R_L - \kappa} \widehat{R}_{L,t} = \left(\widehat{R}_t + \frac{\kappa}{R_L - \kappa} \widehat{R}_{L,t+1} \right) + \frac{(1-\zeta)\, c/y}{a/y} \left(\begin{array}{c} \frac{\xi_a}{(1-\gamma_a)\gamma_a} \left(\widehat{a}_{L,t} - \widehat{a}_{S,t} \right) - \frac{\xi_S}{\gamma_a \gamma_S} \left(\widehat{b}_{HS,t} - \widehat{rer}_t - \widehat{b}_{FS,t} \right) + \\ \frac{\xi_L}{(1-\gamma_a)\gamma_L} \left(\widehat{q}_{L,t} + \widehat{b}_{HL,t} - \widehat{rer}_t - \widehat{q}_{L,t}^* - \widehat{b}_{FL,t} \right) \end{array} \right) + \widehat{\varepsilon}_t^{dr}$$

$$\widehat{R}_t - \widehat{R}_t^* = E_t \widehat{d}_{t+1} + \frac{\xi_S (1-\zeta)\, c/y}{\gamma_S b_{FS}/y} \left(\widehat{b}_{HS,t} - \widehat{rer}_t - \widehat{b}_{FS,t} \right) - \widehat{\varepsilon}_t^{cr}$$

$$\frac{R_L \widehat{R}_{L,t} - \kappa E_t \widehat{R}_{L,t+1}}{R_L - \kappa} - \frac{R_L \widehat{R}_{L,t}^* - \kappa E_t \widehat{R}_{L,t+1}^*}{R_L - \kappa} = E_t \widehat{d}_{t+1} + \frac{\xi_L (1-\zeta)\, c/y}{\gamma_L q_L b_{FL}/y} \left(\widehat{q}_{L,t} + \widehat{b}_{HL,t} - \left(\widehat{rer}_t + \widehat{q}_{L,t}^* + \widehat{b}_{FL,t} \right) \right) - \widehat{\varepsilon}_t^{cr}$$

$$\widehat{a}_{S,t} = \gamma_S \widehat{b}_{HS,t} + (1-\gamma_S) \left(\widehat{rer}_t + \widehat{b}_{FS,t} \right)$$

$$\widehat{a}_{L,t} = \gamma_L \left(\widehat{q}_{L,t} + \widehat{b}_{HL,t} \right) + (1-\gamma_L) \left(\widehat{rer}_t + \widehat{q}_{L,t}^* + \widehat{b}_{FL,t} \right)$$

$$\widehat{R}_{L,t} = -\left(1 - \frac{\kappa}{R_L} \right) \widehat{q}_{L,t}$$

$$\widehat{R}^*_{L,t} = -\left(1 - \frac{\kappa}{R_L}\right)\widehat{q}^*_{L,t}$$

$$\widehat{rer}_t - \widehat{rer}_{t-1} = \widehat{d}_t + \widehat{\pi}^*_t - \widehat{\pi}_t$$

$$\widehat{\pi}_t = \gamma_c \widehat{\pi}_{h,t} + (1-\gamma_c)\,\widehat{\pi}_{f,t}$$

$$\widehat{\pi}_{h,t} - \widehat{\pi}_t = \widehat{p}_{h,t} - \widehat{p}_{h,t-1}$$

$$\widehat{\pi}_{f,t} - \widehat{\pi}_t = \widehat{p}_{f,t} - \widehat{p}_{f,t-1}$$

$$\widehat{c}_{h,t} - \widehat{c}_{f,t} = \lambda\left(\widehat{p}_{f,t} - \widehat{p}_{h,t}\right)$$

$$\widehat{c}_t = \gamma_c \widehat{c}_{h,t} + (1-\gamma_c)\,\widehat{c}_{f,t}$$

$$\widehat{\pi}_{h,t} = \frac{\varsigma_h}{1+\varsigma_h\beta}\widehat{\pi}_{h,t-1} + \frac{\beta}{1+\varsigma_h\beta}E_t\widehat{\pi}_{h,t+1} + \frac{(1-\theta_h)(1-\beta\theta_h)}{\theta_h\,(1+\varsigma_h\beta)}\left(\vartheta y_t - (1-\vartheta)\widehat{\varepsilon}^z_t + \frac{1}{1-\zeta}\left(\widehat{c}_t - \zeta\widehat{c}_{t-1}\right) - \widehat{p}_h\right)$$

$$\widehat{\pi}_{f,t} = \frac{\varsigma_f}{1+\varsigma_f\beta}\widehat{\pi}_{f,t-1} + \frac{\beta}{1+\varsigma_f\beta}E_t\widehat{\pi}_{f,t+1} + \frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f\,(1+\varsigma_f\beta)}\left(\widehat{p}_{f,t} - \widehat{rer}_t\right) + \varepsilon^f_t$$

$$\widehat{r}_t = \rho\widehat{r}_{t-1} + (1-\rho)\left[r_\pi\widehat{\pi}_t + r_y\widehat{y}_t + r_{\Delta y}\left(\widehat{y}_t - \widehat{y}_{t-1}\right)\right] + \widehat{\varepsilon}_{r,t}$$

$$\frac{R}{\pi}\frac{b_S}{y}(\widehat{R}_{t-1} - \widehat{\pi}_t + \widehat{b}_{S,t-1}) + \frac{R_L}{\pi}\frac{q_L b_L}{y}\left(\widehat{R}_{L,t} - \widehat{\pi}_t + \widehat{q}_{L,t} + \widehat{b}_{L,t-1}\right) = \frac{tax}{y}\widehat{tax}_t + \frac{b_S}{y}\widehat{b}_{S,t} + \frac{q_L b_L}{y}\left(\widehat{q}_{L,t} + \widehat{b}_{L,t}\right)$$

$$\widehat{q}_{L,t} + \widehat{b}_{L,t} = \widehat{b}_{S,t} + \widehat{\varepsilon}^b_t$$

$$\widehat{tax}_t = \tau_b(\widehat{q}_{L,t-1} + \widehat{b}_{L,t-1}) + \widehat{\varepsilon}^\tau_t$$

$$\widehat{y}_t = \gamma_c\frac{c}{y}\widehat{c}_{h,t} + (1-\gamma_c\frac{c}{y})\widehat{c}^*_{f,t}$$

$$\frac{b_S}{y}\widehat{b}_{S,t} = \frac{b_{HS}}{y}\widehat{b}_{HS,t} + (\frac{b_S}{y} - \frac{b_{HS}}{y})\widehat{b}^*_{FS,t}$$

$$\frac{b_L}{y}\widehat{b}_{L,t} = \frac{b_{HL}}{y}\widehat{b}_{HL,t} + (\frac{b_L}{y} - \frac{b_{HL}}{y})\widehat{b}^*_{FL,t}$$

$$\frac{b_{FS}}{y}\left[\left(\widehat{rer}_t + \widehat{b}_{FS,t}\right) - \frac{R}{\pi}\left(\widehat{rer}_t + \widehat{R}^*_{t-1} + \widehat{b}_{FS,t-1} - \widehat{\pi}^*_t\right)\right] + \frac{q^*_L b_{FL}}{y}\left[\begin{array}{c}\left(\widehat{rer}_t + \widehat{q}^*_{L,t} + \widehat{b}_{FL,t}\right) - \\ \frac{R_L}{\pi}\left(\widehat{rer}_t + \widehat{R}^*_{L,t} + \widehat{q}^*_{L,t} + \widehat{b}_{FL,t-1} - \widehat{\pi}^*_t\right)\end{array}\right]$$

$$-\frac{b^*_{FS}}{y}\left[\left(-\widehat{rer}_t + \widehat{b}^*_{FS,t}\right) - \frac{R}{\pi}\left(-\widehat{rer}_t + \widehat{R}_{t-1} + \widehat{b}^*_{FS,t-1} - \widehat{\pi}_t\right)\right] - \frac{q_L b^*_{FL}}{y}\left[\begin{array}{c}\left(-\widehat{rer}_t + \widehat{q}_{L,t} + \widehat{b}^*_{FL,t}\right) - \\ \frac{R_L}{\pi}\left(-\widehat{rer}_t + \widehat{R}_{L,t} + \widehat{q}_{L,t} + \widehat{b}^*_{FL,t-1} - \widehat{\pi}_t\right)\end{array}\right] \ldots$$

$$= (1-\gamma_c\frac{c}{y})\left(\widehat{p}_{h,t} + \widehat{c}^*_{f,t}\right) - (1-\gamma_c)\frac{c}{y}\left(\widehat{rer}_t + \widehat{c}_{f,t}\right)$$

$$\widehat{c}^*_{f,t} = \lambda(\widehat{rer}_t - \widehat{p}_{h,t}) + \widehat{y}^*_t$$

$$\widehat{b}^*_{FS,t} - \widehat{b}^*_{S,t} = \widehat{b}_{HS,t} - \widehat{b}_{FS,t} \quad \text{and} \quad \widehat{b}^*_{FL,t} - \widehat{b}^*_{L,t} = \widehat{b}_{HL,t} - \widehat{b}_{FL,t}.$$

Exogenous processes:

$$\widehat{\varepsilon}^g_t = \rho_g\widehat{\varepsilon}^g_{t-1} + e^g_t$$

$$\widehat{z}_t = \rho_z\widehat{z}_{t-1} + e^z_t$$

$$\widehat{\varepsilon}^f_t = \rho_f\widehat{\varepsilon}^f_{t-1} + e^f_t$$

$$\widehat{\varepsilon}^{cr}_t = \rho_{cr}\widehat{\varepsilon}^{cr}_{t-1} + e^{cr}_t$$

$$\widehat{\varepsilon}^{dr}_t = \rho_{dr}\widehat{\varepsilon}^{dr}_{t-1} + e^{dr}_t$$

$$\widehat{\varepsilon}^b_t = \rho_b\widehat{\varepsilon}^b_t + e^b_t$$

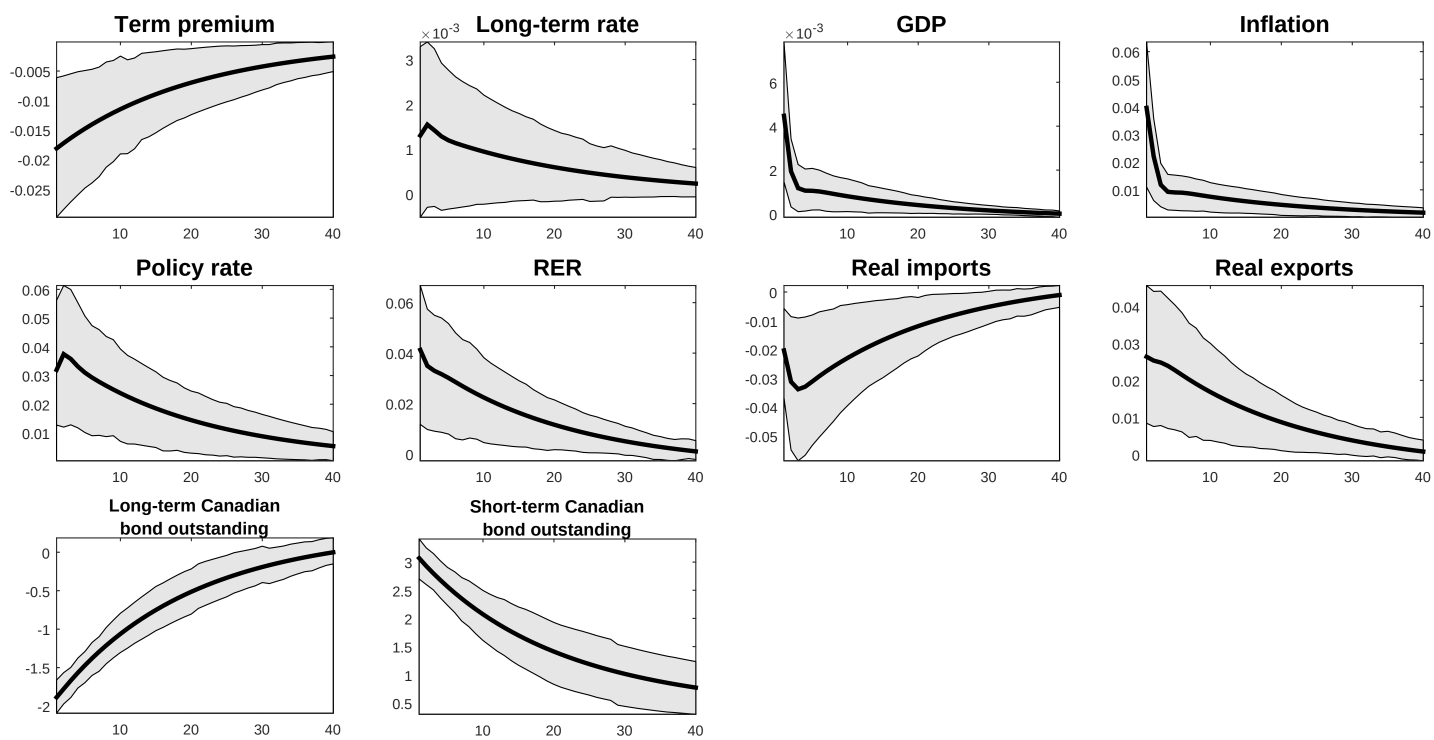
$$\widehat{\varepsilon}^\tau_t = \rho_\tau\widehat{\varepsilon}^\tau_t + e^\tau_t$$

Foreign bloc: $X_t = \Upsilon_1 X_{t-1} + \Upsilon_2 X_{t-2} + e^X_t$ where $X_t = \left(\widehat{y}^* \quad \widehat{\pi}^* \quad \widehat{R}^* \quad \widehat{R}^*_L \quad \widehat{b}^*_S \quad \widehat{q}^*_L + \widehat{b}^*_L\right)$

C Estimated Impulse Responses to a QE Shock

Figure 8 presents mean impulse responses, along with 95% confidence bands, to a one-standard deviation QE shock (about 5%) in Canada from the estimated model. The shock lowers the long-term outstanding shock by about 2% and increases short-term outstanding bonds by about 3%. The responses have the expected signs and are significant at the 95% confidence level. Specifically, following a QE shock, the term premium falls, putting downward pressures on long-term borrowing costs. As a result, aggregate demand increases, leading to an inflationary output gap. Note that nominal long-term interest rates tend to increase even though the term premium falls. This is due to the monetary policy response to inflation. Since the monetary policy response is estimated to be largely sensitive to inflation, the increase in short-term rates offsets the effect of term premium on long-term rates. Note also that the increase in short-term bond supply and the fall in term premium leads to an exchange rate depreciation, resulting in an increase (decrease) in real exports (imports).

Figure 8: Bayesian impulse responses to a QE shock



Notes: The mean impulse responses are generated from the estimated model. Impulse responses are presented as percentage point deviations (%) from the steady-state level for each variable. The grey shaded areas provide 95 % confidence bands for responses.