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Household Risk Assessment Model

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Abstract

Household debt can be an important source of vulnerability to the financial system. This technical report describes the Household Risk Assessment Model (HRAM) that has been developed at the Bank of Canada to stress test household balance sheets at the individual level. In addition to stress testing, HRAM is flexible enough to analyze the effects of a variety of shocks (such as an increase in mortgage rates) and changes to policy, including both monetary policy and macroprudential regulation. The model's strength is its ability to exploit information from survey microdata on the distribution of debt, assets and income across Canadian households.

JEL classification: C0, C6, C63, C65, D0, D1, D14
Bank classification: Financial stability; Housing; Sectoral balance sheet

Résumé

La dette des ménages peut constituer une importante source de vulnérabilité pour le système financier. Le présent rapport technique se propose de décrire le modèle d'évaluation des risques dans le secteur des ménages (modèle HRAM) conçu par la Banque du Canada pour tester, au niveau microéconomique, la capacité de résistance des bilans des ménages. Le modèle HRAM offre en outre la souplesse nécessaire pour permettre l'analyse des effets de chocs divers (tels qu'une hausse des taux hypothécaires) et de l'évolution des politiques publiques, tant la politique monétaire que la réglementation macroprudentielle. Le modèle se distingue par sa capacité à exploiter le contenu informatif de microdonnées d'enquête utilisées pour établir la distribution des dettes, des actifs et des revenus des ménages canadiens.

Classification JEL : C0, C6, C63, C65, D0, D1, D14
Classification de la Banque : Stabilité financière; Logement; Bilan sectoriel

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1 Introduction

Recent international experience, most notably the U.S. subprime crisis of 2007-2008, has reinforced that financial vulnerability in the household sector can be an important contributing cause of domestic financial instability. One essential key to understanding household vulnerabilities is the analysis of individual-level household income and balance sheets to capture the distribution of household debt and other financial characteristics. To that end, the Bank of Canada has continued to develop its micro-simulation stress-testing model, which has appeared in the *Financial System Review* since 2008, as one of its tools to assess the risks to financial stability emanating from the elevated debt burdens of Canadian households. This report provides a technical description of the current version of this model, HRAM, to promote a broader understanding of this approach.[1]

The primary two inputs into HRAM are household microdata, and a set of assumptions, exogenous to HRAM, which characterize the macroeconomic scenario. Econometric models, with stochastic error terms, simulate individual household paths for income, assets and debt over a horizon of 3 to 5 years. By construction, HRAM's aggregate results are consistent with the scenario assumptions that are exogenous to HRAM – however, the distribution of household financial characteristics changes throughout the simulation. These distributional changes are the key to assessing current household vulnerabilities. Indeed, the main outputs from the model, the household arrears rate, the fraction of households with high debt-service ratios, and the fraction of households with low financial asset-coverage ratios, are summary measures of this micro-heterogeneity.

HRAM contributes to an emerging area of interest at central banks focusing on household-sector vulnerability analysis based on microdata, as illustrated by Fuenzalida and Ruiz-Tagle (2010), Herrela and Kauko (2007), and Johansson and Persson (2006). HRAM contributes to this work by extending the analysis to a multi-year horizon, by allowing risks to evolve according to the scenario in question. It does so while maintaining a high degree of micro-detail, including, for example, the probability of unemployment and a first-time homebuyer decision, that is conditional on household characteristics.

While no single model can provide a comprehensive account of all possible risks, HRAM is an important part of the Bank's ongoing development of complementary approaches to monitoring risks in the household sector, and it has evolved into a

[1] This report combines, updates, and expands on content from reports and discussion papers that have previously been published on this topic (Faruqui et al. (2012) and Djoudad (2010,2012)), by the Bank of Canada.

flexible tool that can consider a wide array of alternative scenarios.

This report is organized as follows. Section 2 provides an overview of the approach used, covering key modelling features and the steps involved in an HRAM simulation. Section 3 describes the conceptual model structure, emphasizing the role of the exogenous scenario assumptions as inputs for the model. Section 4 provides the details of the specific equations that make up HRAM. The data used to initialize the model are covered in Section 5, while Section 6 discusses calibration. Some examples of simulation results are illustrated in Section 7. Section 8 concludes.

2 Overview of HRAM Approach

The goal of HRAM is to simulate the impacts of large macroeconomic shocks at the micro household level, where non-linearities, financial frictions, and heterogeneity play a large role. All three features are essential to stress testing the household sector since we are trying to capture default on debt, by a subset of households, which is inherently a non-linear financial outcome. The inputs into the model are a scenario for the macroeconomic environment, called a "macro scenario", and data at the household level that incorporate a significant amount of household-level heterogeneity. The outputs are aggregate statistics of observations at the micro level, such as the arrears rate on household debt and the percentage of households that have a "vulnerable" balance sheet owing to high debt-to-income ratios or large debt-servicing burdens relative to income.

By construction, HRAM is meant to complement other policy models, especially rational forward-looking dynamic stochastic general-equilibrium (DSGE) models. Such DSGE models, while elegant and with good predictive powers, also come with strong restrictions that limit their ability to incorporate non-linearities, financial frictions and heterogeneity. HRAM is constructed in order to address these limitations. However, there is a trade-off in that HRAM deviates from the rational forward-looking general-equilibrium environment, and so there is no explicit optimizing behaviour. In the rest of this section, we present the basic framework of HRAM, highlighting the advantages and shortcomings of this approach.

2.1 Key Modelling Features

HRAM has three key modelling features that are instrumental in its structure for analyzing household vulnerabilities:

(i) HRAM incorporates a significant amount of *household heterogeneity*. For instance, households are heterogeneous in terms of income, assets and age. More importantly, some households may face higher costs of carrying debt and have fewer financial assets, leaving them more vulnerable to a loss of income than others. In contrast, DSGE models are limited in their ability to capture this level of heterogeneity due to computational costs.

(ii) HRAM allows for a rich variety in the *financial frictions* that households face, particularly in the types of mortgages that are available.[2] For instance, in HRAM, households who wish to buy a house face two constraints: a constraint on a minimum down payment and a constraint on the maximum size of monthly mortgage payments relative to income.

(iii) HRAM is not an equilibrium model. It allows for the capturing of dynamics that are *non-linear at the household level.* By contrast, DSGE models typically use a solution method that log-linearizes around a steady state. This limits the impact that a shock can have on the variables in a system, since the system always tries to return to the steady state and stabilize.[3] Additionally, HRAM captures the inherently non-linear default outcome at the household level.

In the rational expectations literature, the "heterogeneous agent" class of models, starting with Huggett (1993) and Aiyagari (1994), followed later by Krusell and Smith (1998), which allowed for aggregate uncertainty, has tried to address all three of the limitations of standard DSGE models. For instance, these models have allowed for household heterogeneity in education, income, family size and asset holdings. Financial frictions have been added so that households face down payment constraints on buying a house, or households cannot completely insure themselves against certain types of shocks, such as shocks to income or medical needs. The models have also been solved using complex techniques that allow for large non-linearities at the household level.

However, these types of models, particularly those with aggregate shocks, have proven very difficult to fully analyze. A particular limitation is the curse of dimensionality, which limits the complexity of the current environment in which a household has to make complicated decisions.[4] For instance, a typical household

[2] Financial frictions were first introduced into representative-agent DSGE models beginning with Bernanke and Gertler (1989) and have become more complex over time, allowing for an increase in the types of agents facing some kind of "micro-friction". However, within each class of agent is a representative agent.

[3] For instance, in HRAM, the magnitude and the duration of a stress episode can be chosen by the analyst, without necessarily having to restrict oneself to a path from a mean-reverting DSGE model.

[4] More formally, the state space of the model gets too large to handle given current computational power.

decides how much to save, in what assets, whether to borrow, whether to buy a house, with what type of mortgage, etc. The result is that these models, while significantly more complex than standard DSGE models, involve limiting the complexity of the environment and limiting the scope for policy analysis.

Therefore, in order to provide analysis of the issues that often interest policymakers, such as whether households will default on their debt, the structure of HRAM necessarily deviates from the elegance of rational forward-looking decision-making general-equilibrium modelling. In particular, there are two key modelling features that depart from traditional modelling. The first feature relates to how we model decision making at the household level.

Model Feature 1. *In lieu of rational forward-looking behaviour, we* approximate behaviour at the micro level *using a combination of restrictions guided by economic theory, and econometric estimation.*

This is akin to the traditional models used for the macroeconomy in the 1960s before the rational expectations revolution placed considerable importance on rational, optimizing agents. A difference here is that we approximate behaviour at the micro level instead of at the macro level. The second key modelling feature is meant to partially address the concerns related to approximating behaviour, by the imposition of a "macro scenario".

Model Feature 2. *In lieu of general-equilibrium, restrictions are imposed on the aggregate behaviour of households by using a* ***macro scenario*** *that imposes a deterministic path for certain variables at the aggregate level, such as income growth, asset prices, interest rates, and income. The approximated behaviour at the micro level is made consistent with the use of a set of* ***consistency factors*** *that adjust behaviour at the micro level in order to ensure consistency with the macro scenario.*

These consistency factors are described in detail in the following section. The combined role of the macro scenario and consistency factors is to be able to capture some of the restrictions from a general-equilibrium model. For instance, the macro scenario could be generated by a DSGE model.

The key benefit of this modelling approach is that it allows for a large amount of flexibility to include heterogeneity, financial frictions, and non-linearities at the micro level, by only approximating behaviour but then imposing restrictions at the macro level. These restrictions capture some of the benefits from the restrictions of rational, forward-looking decision-making, general-equilibrium models. We define such a modelling approach as a **macro-consistent micro-simulation**. We provide a very general definition of such an approach in Section 3. We then use this approach to formally cover the specific details of HRAM in Section 4.

Before going into the full details of the methodology, in the following subsection we provide a high-level overview of the steps in an HRAM simulation.

2.2 Steps in an HRAM Simulation

Four main steps are involved in running a simulation with HRAM: (i) the design of a macroeconomic scenario exogenous to the model; (ii) the initialization of household balance sheets using microdata; (iii) the simulation itself; and (iv) the calculation of the output of the model. Figure 1 shows these steps, where steps one and two are the inputs into the model. HRAM includes steps three and four, and leads to the final calculation of aggregate measures of household vulnerabilities and arrears.

In the first step, the design of an exogenous macroeconomic scenario includes a coherent set of assumptions for the paths of key macroeconomic variables such as household debt growth, income growth, interest rates, and unemployment. Typically, an HRAM stress testing exercise includes both the shock scenario, and a control case that provides a stable scenario as a reference point. The formulation of the control-case assumptions can draw on projections from the Bank's *Monetary Policy Report* (MPR), market expectations for the overnight rate, and additional judgment, for example. A shock scenario could be historically based, or not, and could instead be more severe than what has been experienced in the past.[5] For other analysis, assumptions can be based on forecasts from other models such as the Bank's dynamic stochastic general-equilibrium model, ToTEM. Paths for the assumptions are typically defined at a quarterly frequency for the simulation horizon of 3-5 years. As highlighted earlier, the macro scenario can be designed to indirectly capture general-equilibrium effects that cannot be explicitly modelled in HRAM.

The second step consists of initializing the balance sheets and other characteristics (such as income and age) of a large number of heterogeneous households that serve as the starting point of the simulation. The initialization is done using the latest data from the Canadian Financial Monitor (CFM), which is a large micro-level data set, covering over 12,000 households in a year.[6] Some degree of simplification is involved: for example, CFM survey respondents can list up to eight mortgages; the HRAM set-up sums up any outstanding mortgage debt into one primary mortgage. Preliminary calculations are then performed – for example, the calculation of household-specific risk premiums and missing values are addressed.

[5]Note that HRAM scenarios are separate from any evaluation of event probability and are illustrative of possible outcomes conditional on the scenario materializing – the focus generally being the stress testing of tail events that could pose a risk to financial and macroeconomic stability.

[6]This data set is discussed in more detail in Section 5.

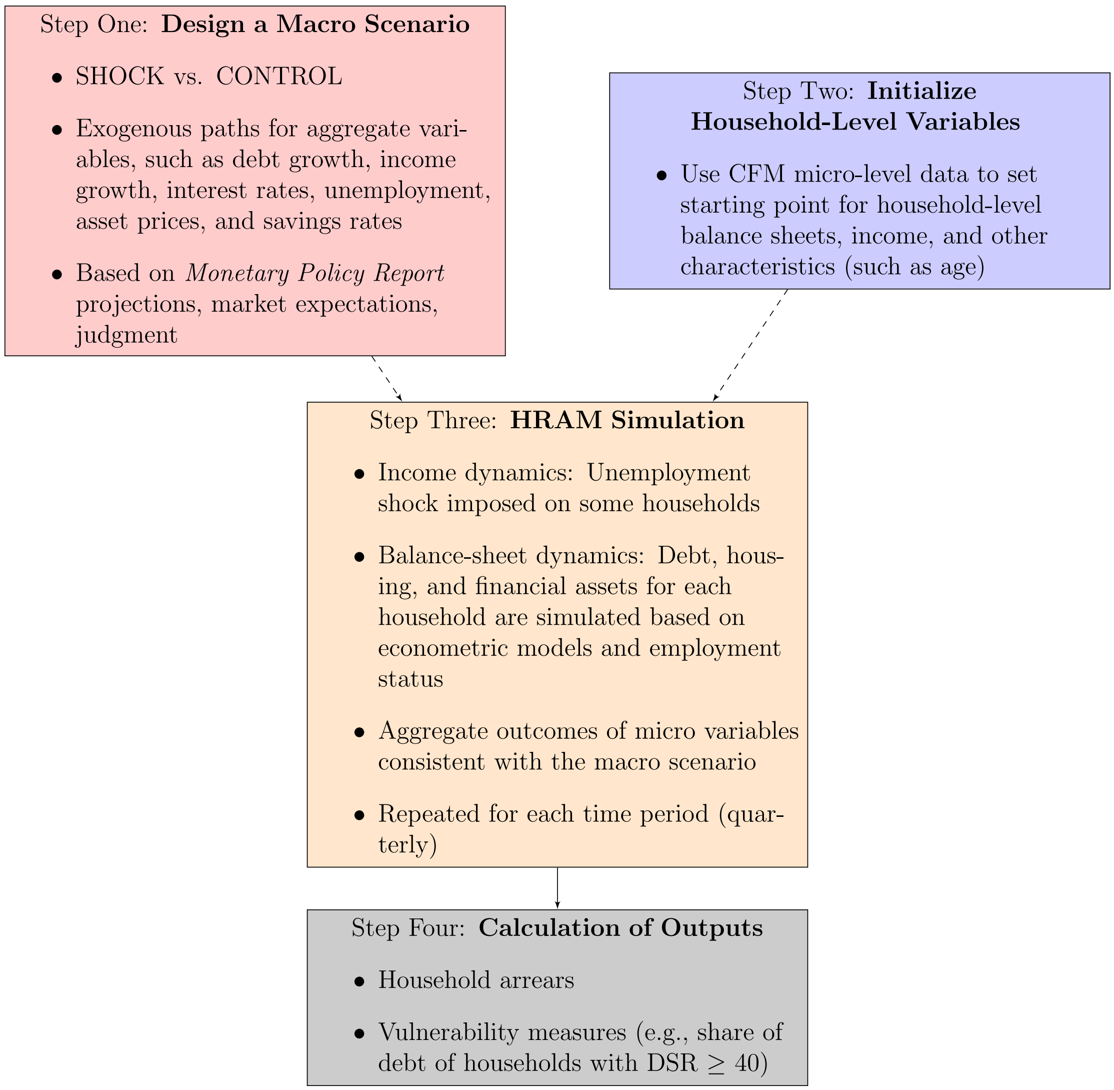


Figure 1: Flow Chart of HRAM Structure

Third, taking the microdata and the macro scenario as given, an HRAM simulation consists of two main components that function primarily at the household level: a set of rules (laws of motion) governing income dynamics; and a set of rules governing balance-sheet dynamics. The income dynamics include a shock process that assigns unemployment to individual households, and an income-growth shock process for employed households. The balance-sheet dynamics consist of home purchases by first-time homebuyers (with the creation of a corresponding mortgage), changes to household debt, and savings decisions that increase household financial assets. A key feature is that the evolution of a household's balance sheet depends partly on the evolution of income, particularly since unemployed households are at risk of depleting their savings.

Each period of the simulation involves an iteration through both the income and balance-sheet dynamics of each individual household. To ensure consistency with the macro scenario, which is exogenous to HRAM, consistency factors are calculated in each period to adjust household-level variables (such as income and debt) – these factors are explained in detail in the following section. This ensures that the relevant aggregate variables from the model match the exogenous macro-scenario assumptions. This puts the focus on the micro-level heterogeneity in the results and the potential emergence of financial vulnerability for a share of households.

Last, the model's risk-metric outputs are calculated. The key outputs are the rate of household debt in arrears and household vulnerability measures, such as the percentage of households with a debt-service ratio over 40 per cent.

3 General Methodology: Macro-Consistent Micro-Simulation

This section provides a general overview of the methodology used in HRAM.[7]

Time is discrete, with a finite horizon given by T. Index time by

$$t \in \mathcal{T} = \{0, 1, 2, \dots, T-1, T\}.$$

There is a discrete set of **households**, $\mathcal{I}$. Index each household by

$$i \in \mathcal{I} = \{1, 2, 3, \dots, I-1, I\}.$$

[7] This section is more technical, and readers who are more interested in the specific assumptions of HRAM are encouraged to first read Section 4.

3.1 Households

A household i is defined as

$$i = \left(\Omega_i, \{X_{i,t}\}_{t=0}^{t=T}\right).$$

The vector Ω_i is a $J \times 1$ vector of **fixed household characteristics**, such as age, education, and region.[8] Refer to an element in Ω_i as $\omega_{j,i}$, so that Ω_i is given by

$$\Omega_i = \begin{bmatrix} \omega_{1,i} \\ \vdots \\ \omega_{j,i} \\ \vdots \\ \omega_{J,i} \end{bmatrix}.$$

The fixed household characteristics for all households are contained in the matrix Ω, given by

$$\Omega = \begin{bmatrix} \begin{bmatrix} \omega_{1,1} \\ \vdots \\ \omega_{j,1} \\ \vdots \\ \omega_{J,1} \end{bmatrix} & \ldots & \begin{bmatrix} \omega_{1,i} \\ \vdots \\ \omega_{j,i} \\ \vdots \\ \omega_{J,i} \end{bmatrix} & \ldots & \begin{bmatrix} \omega_{1,I} \\ \vdots \\ \omega_{j,I} \\ \vdots \\ \omega_{J,I} \end{bmatrix} \end{bmatrix}.$$

The vector $X_{i,t}$ is a $K \times 1$ vector of time-varying **household variables**, such as income and assets. Refer to an element in $X_{i,t}$ as $x_{k,i,t}$, so that $X_{i,t}$ is given by

$$X_{i,t} = \begin{bmatrix} x_{1,i,t} \\ \vdots \\ x_{k,i,t} \\ \vdots \\ x_{K,i,t} \end{bmatrix}.$$

Note that there is a vector of household variables for each household i for each time period t. At any point in time t, the variables for all households are contained in

[8] Please refer to the Appendix for a complete list of macro and household variables in an HRAM simulation.

the matrix X_t, given by

$$X_t = \begin{bmatrix} \begin{bmatrix} x_{1,1,t} \\ \vdots \\ x_{k,1,t} \\ \vdots \\ x_{K,1,t} \end{bmatrix} & \dots & \begin{bmatrix} x_{1,i,t} \\ \vdots \\ x_{k,i,t} \\ \vdots \\ x_{K,i,t} \end{bmatrix} & \dots & \begin{bmatrix} x_{1,I,t} \\ \vdots \\ x_{k,I,t} \\ \vdots \\ x_{K,I,t} \end{bmatrix} \end{bmatrix}.$$

Therefore, at any point in time t, household i is completely described by $(\Omega_i, X_{i,t})$, while all households at time t are captured by (Ω, X_t).

3.2 Evolution of Household Variables: Macro-Consistent Micro-Simulation

The objective of the model is to shock households in order to simulate paths for future individual-household variables, given by $\{X_t\}_{t=1}^{t=T}$, taking as given household fixed characteristics, Ω, and an initial allocation of time-varying household variables, X_0. A key feature of the model is that the simulation is constrained by a **macro scenario** that imposes a path on the aggregate of the individual-household variables.[9] We refer to such a simulation that combines micro-level uncertainty with aggregate-level restrictions as a **"macro-consistent micro-simulation"**. This subsection is devoted to describing the process for the simulation and providing an explicit mathematical definition for such a simulation.

To provide an overview, for each period $t > 0$, the following steps are taken in order to simulate household variables:

(i) Each household receives **idiosyncratic shocks**. The process for the shocks to a variable, in terms of the aggregate mean of the shock distribution, is driven by the macro scenario.

(ii) Given the idiosyncratic shocks, fixed household characteristics and household variables from $t-1$, a vector of **intermediate household variables** is determined using a set of **laws of motion**. A law of motion defines how a household variable transitions from one period to the next. No explicit optimizing economic behaviour is used to determine this transitioning. Instead,

[9] For example, a simulation gives a household an income shock, in each period, that is drawn from a distribution – the mean of this distribution is essentially determined by the macro scenario, such that the aggregate sum of all individual income shocks produces the desired aggregate path for income.

restrictions guided by economic theory, and estimated equations, are used to approximate household decision making.

(iii) Given the intermediate household variables, the consistency of the path for household variables with the macro scenario is ensured by solving for a vector of endogenous **consistency factors** that linearly adjust the variables to restore consistency with the set of **macro restrictions**. The factors are defined so as to adjust the variables proportionally for the relevant group of households.[10]

(iv) The **final household variables** for period t are computed by updating the intermediate household variables via an **allocation function** that uses the consistency factors for this linear adjustment.

For instance, one household variable is income. Each household receives an idiosyncratic shock to its income, but through the macro scenario there is a restriction imposed on overall income growth across all households.

In the remainder of this subsection, we define a macro scenario, go over each step in simulating household variables, and finally define a macro-consistent micro-simulation.

3.2.1 Macro scenario

Define the **macro scenario** for period t as a $M \times 1$ vector Z_t with element $z_{m,t}$ so that

$$Z_t = \begin{bmatrix} z_{1,t} \\ \vdots \\ z_{m,t} \\ \vdots \\ z_{M,t} \end{bmatrix}.$$

All of the variables in the macro scenario are deterministic and exogenous to the model. Examples of variables in the macro scenario are prices, such as interest rates,

[10]As an example, the income process involves both gradual transitions across time for employed households, in addition to much sharper discontinuous transitions in income, for some households, from an employed status to an unemployed status, and vice versa. To ensure that the aggregate path for income is maintained, some adjustment is usually necessary for the set of households that stay employed. In the case of rising unemployment, and aggregate income growth of zero, employed households would in fact be experiencing positive income growth, to compensate for the negative income growth of the households transitioning to an unemployed status. If there was zero unemployment in the scenario, the process would be much simpler, and no consistency factors would be necessary.

and other macroeconomic variables such as aggregate nominal income growth and the unemployment rate.[11] The complete macro scenario for all time periods is given by the following matrix:

$$\mathcal{Z} = \left[Z_0, Z_1, Z_2, \ldots, Z_{T-1}, Z_T \right].$$

The macro scenario must be chosen before the simulation is performed and is part of the initialization of the simulation. Typically, the macro scenario is constructed to be consistent with a dynamic general-equilibrium model. Mathematically, the macro scenario is a set of M restrictions for each period t on the path of household variables given by $\{X_t\}_{t=1}^{t=T}$.

3.2.2 Idiosyncratic shocks

Let $\varepsilon_{i,t}$ be a $K \times 1$ vector of **idiosyncratic shocks** with element $\varepsilon_{k,i,t}$, so that

$$\varepsilon_{i,t} = \begin{bmatrix} \varepsilon_{1,i,t} \\ \vdots \\ \varepsilon_{k,i,t} \\ \vdots \\ \varepsilon_{K,i,t} \end{bmatrix}.$$

The shocks are stochastic and are drawn from a distribution (e.g., normal or gamma). Typically, the mean and variance of the shocks will vary with the fixed household characteristics of household i, the macro scenario for both t and $t-1$, the distribution of households across household variables in the previous period, and a set of parameters Θ^{ε} that affect the distribution functions. Therefore, we write the shocks as being drawn from a cumulative distribution function, F, that depends upon household i fixed characteristics, the past aggregate economy via X_{t-1}, and the macro scenario via Z_t and Z_{t-1}:

$$\varepsilon_{i,t} \sim F\left(\Omega_i; X_{t-1}, Z_t, Z_{t-1}; \Theta^{\varepsilon}\right). \tag{1}$$

The underlying distributions for each element of $\varepsilon_{i,t}$ (i.e., such as normal or uniform), as well as the vector of parameters, Θ^{ε}, are determined in the calibration stage and are fixed throughout the simulation. The matrix for all the shocks across all

[11] Please refer to the Appendix for a complete list of macro and household variables in an HRAM simulation.

households at time t is expressed as

$$\varepsilon_t = \left[\begin{bmatrix} \varepsilon_{1,1,t} \\ \vdots \\ \varepsilon_{k,1,t} \\ \vdots \\ \varepsilon_{K,1,t} \end{bmatrix} \quad \cdots \quad \begin{bmatrix} \varepsilon_{1,i,t} \\ \vdots \\ \varepsilon_{k,i,t} \\ \vdots \\ \varepsilon_{K,i,t} \end{bmatrix} \quad \cdots \quad \begin{bmatrix} \varepsilon_{1,I,t} \\ \vdots \\ \varepsilon_{k,I,t} \\ \vdots \\ \varepsilon_{K,I,t} \end{bmatrix} \right].$$

3.2.3 Intermediate household variables: Law of motion

Given the idiosyncratic shocks, intermediate values for all household variables for household i, denoted as $\tilde{X}_{i,t}$, are updated via the following function:

$$\tilde{X}_{i,t} = \tilde{G}\left(X_{i,t-1}, \varepsilon_{i,t}\right). \tag{2}$$

Note that equation (2) depends only upon household-level variables, and through the idiosyncratic shocks, the aggregate of the variables, and the macro scenario. Refer to the function $\tilde{G}$ as the **law of motion** for household variables. A central element of the model is the construction of the law of motion for each household variable, detailed in Section 4. For certain specific household variables, such as income, the law of motion involves the interaction of more than one idiosyncratic shock per period, i.e., there is a process for fully employed households, as well as transitions from an employed status to an unemployed status, and vice versa.

3.2.4 Consistency factors: Macro restrictions

The evolution of the household variables is refined by a set of time-varying $M \times 1$ vector C_t of **consistency factors**. These are endogenous factors that ensure that the evolution of the household variables is consistent with the macro scenario. Formally, the $M \times 1$ restrictions from the macro scenario are captured in the function H, given by

$$C_t = H\left(\tilde{X}_t, X_{t-1}, Z_t, Z_{t-1}\right). \tag{3}$$

Note that equation (3) consists of M restrictions on the aggregate household variables. Refer to H as the function of **macro restrictions**. The specific definitions of the consistency factors for each specific household variable will be explained in the relevant subsections.

3.2.5 Final household variables: Allocation function

Given the consistency factors, the temporary household variables for a household i at time t are updated by the function G to arrive at the **final household variables**:

$$X_{i,t} = G\left(\tilde{X}_{i,t}, \tilde{X}_t, C_t; \varepsilon_{i,t}\right). \tag{4}$$

Refer to the function G as the **allocation function**. If the functions H and G have been appropriately constructed, then

$$H\left(X_t, X_{t-1}, Z_t, Z_{t-1}\right) = 0.$$

3.2.6 Definition of a macro-consistent micro-simulation

A **macro-consistent micro-simulation** is defined as follows.

Definition 1. *A **macro-consistent micro-simulation** consists of sequences,* $\{\varepsilon_t\}_{t=1}^{t=T}$, $\left\{\tilde{X}_t\right\}_{t=1}^{t=T}$, $\{C_t\}_{t=1}^{t=T}$ *and* $\{X_t\}_{t=1}^{t=T}$ *such that, given* X_0, Ω,*and* $\mathcal{Z}$*:*

(i) (Idiosyncratic shocks) $\varepsilon_{i,t}$ *satisfies equation (1) for each* $i \in \mathcal{I}$ *and* $t \in \{1, ..., T\}$*;*

(ii) (Evolution of intermediate household variables) $\tilde{X}_{i,t}$ *satisfies equation (2) for each* $i \in I$ *and* $t \in \{1, ..., T\}$*;*

(iii) (Macro consistency) C_t *satisfies equation (3) for each* $t \in \{1, ..., T\}$*; and*

(iv) (Evolution of final household variables) $X_{i,t}$ *satisfies equation (4) for each* $i \in I$ *and* $t \in \{1, ..., T\}$*.*

4 The Model

This section covers the primary equations behind HRAM. The first subsection discusses the evolution of income and employment at the household level and illustrates how the aggregate evolution of income and employment in the model is constrained by the macro scenario. The second subsection covers the evolution of individual household balance sheets, including the determination of whether a household will purchase a house for the first time and whether a household will go into arrears on its debt. Once again, a key feature is how the evolution of the aggregate balance sheet for the total household sector is constrained by the macro scenario. A complete list of all of the variables used is provided in Appendix A.

4.1 Evolution of Income and Employment

The **disposable nominal labour income** of household i in period t, denoted as $X_{i,t}^{DLY}$, is a function of a household's fully employed **permanent nominal labour income** in period t, $X_{i,t}^{PLY}$, and whether a household is **unemployed** in period t.[12] Letting $X_{i,t}^{U} = 0$ denote that household i is employed in period t, and $X_{i,t}^{U} = 1$ denote that household i is unemployed in period t, the disposable labour income of household i in period t is given by:

$$X_{i,t}^{DLY} = (1-\tau)\left(X_{i,t}^{PLY} - (1-b)\, X_{i,t}^{U} X_{i,t}^{PLY}\right), \tag{5}$$

where b denotes the percentage of permanent labour income that a household receives while unemployed (from unemployment compensation) and τ denotes the tax rate on labour income.

An individual household i faces three sources of uncertainty in its income process: (i) whether a household is employed or unemployed; (ii) while unemployed, the duration of unemployment; and (iii) uncertainty in its permanent labour income, which denotes income while employed.

In the aggregate, the macro scenario dictates the overall **unemployment rate** in period t, Z_t^U; the **expected duration of unemployment** for a household that becomes unemployed in period t, Z_t^{UD}; and the overall **nominal growth rate in labour income** from $t-1$ to t, Z_t^{LYG}.

4.1.1 Unemployment and unemployment duration

When a household becomes unemployed in period t, it receives an **unemployment duration shock**, $\varepsilon_{i,t}^{UD}$, that determines the duration[13] (number of periods) of unemployment for household i. Let $X_{i,t}^{UD}$ denote the expected remaining **unemployment duration** for household i at the end of period t. Therefore, at the start of period t, the number of households that continue to be unemployed is given by

$$I_{t-1}^{UD} = \sum_{i=1}^{i=I} Ind\left(X_{i,t-1}^{UD} > 0\right),$$

[12]With some abuse of terminology, we define permanent labour income to be what a household would earn in a fully employed state, ignoring any temporary effects from unemployment, which is the main source of the transitory component to current disposable income, in the model.

[13]More precisely, this is the maximum duration of the unemployment episode for the household since it can receive a shock and become employed before its remaining period of unemployment goes to zero. This possibility can exist if the unemployment rate in the macro scenario falls abruptly.

where Ind denotes an indicator function that is equal to one if, and only if, the argument is true, and zero otherwise. The macro scenario dictates that the aggregate unemployment rate should be equal to Z_t^U. The difference between the unemployment rate in the macro scenario and the implied unemployment rate in period t from households with continuing unemployment is given by[14]

$$\tilde{p}_t = Z_t^U - \frac{I_{t-1}^{UD}}{I}.$$

Let $p_{i,t}$ denote the probability that household i, employed in period $t-1$, $X_{t-1}^{UD} = 0$, will become unemployed in period t. Let q_t denote the probability that a household unemployed in period $t-1$ with a strictly positive remaining duration of unemployment, $X_{t-1}^{UD} > 0$, will become employed.[15] The values for the two probabilities are determined by $\tilde{p}$, and, in the case of $p_{i,t}$, a relative layoff risk factor $\varphi_{i,t}$, which is a function of the vector of fixed household-specific characteristics, Ω_i, and the employment status of households in period $t-1$, $X_{i,t-1}^{UD}$[16,17]:

$$p_{i,t} = \max\left\{0, \frac{\tilde{p}_t I}{I - I_{t-1}^{UD}} \varphi(\Omega_{i,t}, X_{i,t-1}^{UD})\right\} \qquad \text{and} \qquad q_t = \max\left\{0, -\frac{\tilde{p}_t I}{I_{t-1}^{UD}}\right\},$$

where φ is defined such that it has mean equal to one for households employed in period $t-1$.

Formally, let $\varepsilon_{i,t}^U$ denote a shock uniformly distributed over $[0,1]$. Whether a household is employed or unemployed in period t depends upon $X_{i,t-1}^{UD}$, $\varepsilon_{i,t}^U$, $p_{i,t}$ and

[14]Note that we describe the unemployment rate as relative to the total number of households, I, for simplicity of exposition. In the actual model, we examine the employment of heads of household relative to the labour force, controlling for labour force status when we initialize the model from data.

[15]Note that even though q_t is typically zero, there are still gross flows of unemployment owing to the duration shock, since the households that end their unemployment spell have to be replaced with other households in order to maintain a constant rate of unemployment. For instance, in a typical labour-search model, q_t refers to the probability that an unemployed agent finds work in a given period and is always strictly positive. Here, q_t refers to households that unexpectedly end their unemployment spell early.

[16]The estimation of $\varphi_{i,t}$ is explained further in Section 6.

[17]As can be seen from equation (6), q_t, and thus the duration of a household unemployment spell, does not depend on Ω_i in the model.

q_t by the following function, with $X^U_{i,t} = 1$ indicating unemployment:

$$X^U_{i,t} = \begin{cases} 1 & \text{if } X^{UD}_{i,t-1} = 0 \text{ and } \varepsilon^U_{i,t} \leq p_{i,t} \\ 0 & \text{if } X^{UD}_{i,t-1} = 0 \text{ and } \varepsilon^U_{i,t} > p_{i,t} \\ 1 & \text{if } X^{UD}_{i,t-1} > 0 \text{ and } \varepsilon^U_{i,t} > q_t \\ 0 & \text{if } X^{UD}_{i,t-1} > 0 \text{ and } \varepsilon^U_{i,t} \leq q_t \end{cases} . \tag{6}$$

By construction, the process for unemployment ensures that the unemployment rate from the macro scenario is exactly matched (assuming that there is a sufficient number of households to ensure that the law of large numbers holds) so that consistency factors are not needed in the unemployment process for households.

After the employment status for a household is determined, the remaining unemployment duration for unemployed households has to be updated. For those households that are in the middle of an unemployment spell, the remaining duration is simply reduced by a period. Households that are newly unemployed draw a shock that determines the duration of their unemployment spell. Formally, the law of motion for $X^{UD}_{i,t}$ is given by

$$X^{UD}_{i,t} = \begin{cases} X^{UD}_{i,t-1} - 1 & \text{if } X^U_{i,t} = 1 \text{ and } X^{UD}_{i,t-1} > 0 \\ \varepsilon^{UD}_{i,t} - 1 & \text{if } X^U_{i,t} = 1 \text{ and } X^{UD}_{i,t-1} = 0 \\ 0 & \text{if } X^U_{i,t} = 0 \end{cases} . \tag{7}$$

The idiosyncratic shock $\varepsilon^{UD}_{i,t}$ is the unemployment duration shock for a household that became unemployed in period t. The distribution of this shock is given by F^{UD} and the **expected duration of unemployment** depends upon the macro-scenario variable Z^{UD}_t, so that

$$\varepsilon^{UD}_{i,t} \sim F^{UD}\left(Z^{UD_t}\right),$$

where the mean of the distribution is determined by Z^{UD}_t. The cumulative distribution function, F^{UD}, captures the fact that many households that become unemployed find work quickly,[18] while other households remain unemployed for an extended period of time. The specifics are covered in Section 6 on estimation and calibration.

[18] The structure allows for some households to be unemployed for only one period.

4.1.2 Permanent labour income

The law of motion for the permanent labour income of household i at time t is given by

$$X_{i,t}^{PLY} = X_{i,t-1}^{PLY}\left(1+\varepsilon_{i,t}^{PLY}\right) + C_{i,t}^{LYG}, \tag{8}$$

where $\varepsilon_{i,t}^{PLY}$ is an idiosyncratic shock to permanent labour income for household i at time t. A consistency factor, $C_{i,t}^{LYG}$, ensures that the aggregate growth from the idiosyncratic labour-income shocks is consistent with the nominal growth rate of aggregate labour income from the macro scenario, Z_t^{LYG}. The next paragraphs cover first the idiosyncratic shock followed by the determination of the consistency factor.

Households that are employed in period t receive a shock that affects the growth rate of their income, $\varepsilon_{i,t}^{PLY}$. This shock is distributed normally, with the mean given by the macro scenario and standard deviation σ_i^{PLY} that is a function of the household's income quintile, ω_i^{YQ}.[19] Unemployed households' permanent labour income remains unchanged. Formally, the process for household i's idiosyncratic shock to permanent labour income at time t is given by

$$\varepsilon_{i,t}^{PLY} = \begin{cases} \sim N\left(Z_t^{LYG}, \sigma^{PLY}(\omega_i^{YQ})\right) & \text{if } X_{i,t}^{U}=0 \\ 0 & \text{if } X_{i,t}^{U}=1 \end{cases}. \tag{9}$$

Turning to the determination of the consistency factor, the idiosyncratic shock generates an intermediate household variable for permanent labour income:

$$\tilde{X}_{i,t}^{PLY} = X_{i,t-1}^{PLY}\left(1+\varepsilon_{i,t}^{PLY}\right).$$

The aggregate nominal labour income in period $t-1$ is given by

$$LY_{t-1} = \sum_{i}^{I}\left[X_{i,t-1}^{PLY} - (1-b)\,X_{i,t-1}^{U}X_{i,t-1}^{PLY}\right].$$

From the macro scenario, Z_t^{LYG} denotes the growth rate in aggregate nominal labour income from period $t-1$ to t, so that aggregate nominal labour income in period t should be equal to $\left(1+Z_t^{LYG}\right)LY_{t-1}$. Given $\tilde{X}_t^{PLY}$ and X_t^{U}, a consistency factor C_t^{LYG} is calculated that ensures that aggregate nominal income growth matches the

[19] See Djoudad (2012) for further details.

macro scenario:

$$C_t^{LYG} = \left(1 + Z_t^{LYG}\right) LY_{t-1} - \sum_i^I \left[\tilde{X}_{i,t}^{PLY} - (1-b)\, X_{i,t}^{U} \tilde{X}_{i,t}^{PLY}\right].$$

This captures the aggregate amount of income that has to be added to (or subtracted from if negative) employed households in order to match the macro scenario. The consistency factor C_t^{LYG} is allocated across employed households in proportion to their share of the income-growth shock, such that the share of the income-growth shock does not change for any household, to arrive at a final consistency adjustment to household permanent nominal income for period t:

$$C_{i,t}^{LYG} = \frac{Ind\left(X_{i,t}^{U} = 0\right) X_{i,t-1}^{PLY} \varepsilon_{i,t}^{PLY}}{\sum_i^I Ind\left(X_{i,t}^{U} = 0\right) X_{i,t-1}^{PLY} \varepsilon_{i,t}^{PLY}} C_t^{LYG}. \tag{10}$$

4.2 Evolution of Household Balance Sheet

Given the shocks to unemployment, permanent labour income, and the return on financial assets, the total financial resources available to a household, which we refer to as a household's budget, is the sum of realized gross income (minus tax payments) and financial assets (with the return) less debt. In addition, households are assumed to make their required debt payments if they have sufficient resources to cover them (while still maintaining a minimal level of consumption if a household is unemployed).

To add clarification, the allocation of a household's assets, debt, and consumption is captured by the household budget constraint in period t:

$$\underbrace{X_{i,t}^{FA} - X_{i,t}^{D} + X_{i,t}^{C}}_{\text{Assets, debt, consumption}} = \underbrace{X_{i,t}^{PLY}(1-\tau) + X_{i,t-1}^{FA}\left(1 + Z_t^{RFA}\right) - X_{i,t-1}^{D}}_{\text{Available financial resources}} - \underbrace{X_{i,t}^{DP}}_{\text{Required debt payments}},$$

where τ is the tax rate on income, and Z_t^{RFA} is the return on financial assets, which is part of the macro scenario.

To determine the allocation across consumption, and next period's debt and assets, no explicit modelling of behaviour is used. Instead, stochastic processes that provide a reduced-form representation of economic decision making by incorporating household variables and fixed household characteristics determine debt and assets for subsequent periods, with household consumption being given by the residual term. In the rest of this subsection, we go over the determination of debt, assets (both financial and whether a renting household buys a house), and whether a household

Table 1: Balance Sheet for Household i in Period t

Assets	Debt and Equity
Housing assets, $X_{i,t}^{HA}$	Mortgage debt, $X_{i,t}^{MD}$
	Housing equity, $X_{i,t}^{HEq}$
Financial assets, $X_{i,t}^{FA}$	Consumer debt, $X_{i,t}^{CD}$
	Net worth minus housing equity
Total Assets, $X_{i,t}^{A}$	Total Debt plus Total Equity, $X_{i,t}^{D} + X_{i,t}^{Eq}$

goes into arrears.

4.2.1 Debt payments

The model simulates household debt payments by using a profile of interest rates from the macro scenario and simulating household-level debt. A household has two types of debt, mortgage debt, $X_{i,t}^{MD}$, and consumer debt, $X_{i,t}^{CD}$. The total debt payments owed by household i in period t is given by

$$X_{i,t}^{DP} = \left(\omega_i^{PPCD} + X_{i,t}^{CDRATE}\right) X_{i,t-1}^{CD} + \left(\omega_i^{PPMD} + X_{i,t}^{MDRATE}\right) X_{i,t-1}^{MD}, \tag{11}$$

where ω_i^{PPCD} denotes the principal payment as a fraction of consumer debt, while ω_i^{PPMD} denotes the principal payment as a fraction of mortgage debt. The principal payments as a fraction of debt outstanding are fixed throughout the simulation and are estimated from the household-level data. The interest rates on the debt are denoted $X_{i,t}^{CDRATE}$ for consumer debt and $X_{i,t}^{MDRATE}$ for mortgage debt.

The interest rate on consumer debt consists of an individual household-risk premium (that varies across households and is constant throughout the simulation) and a market interest rate that is part of the macro scenario, $Z^{RFSHORT}$:

$$X_{i,t}^{CDRATE} = \omega_{i,t}^{RPCD} + Z^{RFSHORT},$$

where $\omega_{i,t}^{RPCD}$ is a household-specific risk premium on consumer debt.

Turning to mortgages, mortgages are heterogeneous along four dimensions:

(i) The original term of a mortgage, $X_{i,t}^{MTERM}$. The term is given by the micro-data, but to illustrate the model, we will consider 1-, 3- and 5-year terms, $X_{i,t}^{MTERM} \in \{1, 3, 5\}$. When a household renews a mortgage, we assume the household renews at the same term, except for variable-rate mortgages (see below). Note that a household is allowed to renew its mortgage irrespective of its employment status.

(ii) The remaining number of periods in the term of a mortgage, $X_{i,t}^{REMMTERM}$. This simply falls by one each period[20]:

$$X_{i,t}^{REMMTERM} = X_{i,t}^{REMMTERM} - 1.$$

(iii) Whether a mortgage has a fixed rate, $X_{i,t}^{MVAR} = 0$, or a variable rate, $X_{i,t}^{MVAR} = 1$.[21] When the term on a mortgage has expired, $X_{i,t}^{REMMTERM} = 0$, then we assume that a household with a fixed-rate mortgage renews with a fixed-rate mortgage, while only a fraction, ρ, of households with variable-rate mortgages renew with variable-rate mortgages.[22] All households that do renew with a variable-rate mortgage are assumed to have a 5-year term.

(iv) The interest rate on the mortgage:

- For fixed-rate mortgages, the rate is fixed as long as the mortgage term has not reached zero. When a mortgage is renewed, the rate is set to the current discounted mortgage rate, depending upon the term of the mortgage. Formally, if $X_{i,t}^{MVAR} = 0$, the process for the mortgage rate is given by

$$X_{i,t}^{MDRATE} = \begin{cases} Z_t^{R-5YRDISC} & \text{if } X_{i,t}^{REMMTERM} = 0 \text{ and } X_{i,t}^{MTERM} = 5 \\ Z_t^{R-3YRDISC} & \text{if } X_{i,t}^{REMMTERM} = 0 \text{ and } X_{i,t}^{MTERM} = 3 \\ Z_t^{R-1YRDISC} & \text{if } X_{i,t}^{REMMTERM} = 0 \text{ and } X_{i,t}^{MTERM} = 1 \\ X_{i,t-1}^{MRATE} & \text{if } X_{i,t}^{MTERM} > 0 \end{cases}.$$

- For a variable-rate mortgage, the rate is equal to the current short rate plus a household-specific risk premium, so that if $X_{i,t}^{MVAR} = 1$

$$X_{i,t}^{MDRATE} = \omega_{i,t}^{RPMD} + Z^{RSHORT},$$

where $\omega_{i,t}^{RPMD}$ is a household-specific risk premium on a mortgage. A fixed-rate mortgage impedes us from calculating a household-specific risk premium, owing to a lack of data on when the loan was originated.[23]

[20]For no mortgage, we set $X_{i,t}^{REMMTERM} = 0$.

[21]For no mortgage, we set $X_{i,t}^{MVAR} = -1$.

[22]The model can easily be adapted to allow households with fixed-rate mortgages to renew with a variable rate.

[23]However, such a risk premium could be estimated in a future version of the model.

Finally, the debt-service ratio for household i, $X_{i,t}^{DSR}$, is defined as the ratio of debt payments to disposable labour income:

$$X_{i,t}^{DSR} = \frac{X_{i,t}^{DP}}{X_{i,t}^{DLY}}.$$

4.2.2 Housing purchases: First-time homebuyers

A key feature of the model is the purchase of a house by first-time homebuyers. To be eligible to be a first-time homebuyer, a household must first satisfy certain demographic conditions and then be able to afford to purchase a starter home in the region in which it lives, ω_i^{REG}. Formally, the following conditions must hold:

- *Demographic conditions*: In order to be eligible to be a first-time homebuyer in period t, household i must
  - (i) be employed, $X_{i,t}^{U} = 0$,
  - (ii) have zero housing assets, $X_{i,t}^{HA} = 0$, and
  - (iii) be younger than 50 years old, $X_{i,t}^{AGE} < 50$.
- *Affordability conditions*: A household faces two constraints on the size of house it can afford to purchase:
  - (i) *Down payment*: A household must make a minimum down payment,[24] dp_{MIN}. Given the financial assets of household i, $X_{i,t-1}^{FA}$, the largest house it can purchase is
  $$X_{i,t}^{HPMAX,dp} = \frac{1}{dp_{min}} X_{i,t-1}^{FA}. \tag{12}$$
  - (ii) *Debt-service ratio*: In addition, a household faces a constraint from the debt-service ratio, which constrains the ratio of debt payments (both interest and principal) to income to be less than an exogenous limit given by DSR_{max}. Given a household's current debt payments and income, the largest monthly payment a household can make on a mortgage is given by
  $$X_{i,t}^{MPAYMAX,DSR} = \frac{1}{12} \max\left\{ DSR_{max} - \frac{X_{i,t}^{DP}}{X_{i,t}^{Y}}, 0 \right\} X_{i,t}^{YA},$$

[24] One optional configuration of the model allows for a fraction of first-time homebuyers to borrow for their down payments.

where $X_{i,t}^{YA}$ denotes the annnualized income of household i in period t. Given the current qualifying interest rate, Z_t^{R-5yr}, and the maximum amortization (in years), amt_{MAX}, the largest loan a household can take out is given by

$$X_{i,t}^{LOANMAX,DSR} = \Lambda X_{i,t}^{MPAYMAX,DSR},$$

where

$$\Lambda = \frac{1-(1+Z_t^{R-5yr})^{-amt_{MAX}\times 12}}{Z_t^{R-5yr}}.$$

Assuming that a household makes the largest down payment possible, equal to $X_{i,t}^{FA}$, then the largest house a household can purchase given the debt-service ratio constraint is given by

$$X^{HPMAX,DSR} = \Lambda X_{i,t}^{MPAYMAX,DSR} + X_{i,t-1}^{FA}. \tag{13}$$

(iii) *Combining* the two constraints, a household is eligible to become a first-time homebuyer, $\tilde{X}_{i,t}^{FTHB} = 1$, only if it can afford a starter home[25]

$$Z_t^{HPSTARTER}(\omega_i^{REG}) \leq \min\left\{X_{i,t}^{HPMAX,dp}, X^{HPMAX,DSR}\right\}. \tag{14}$$

If the above conditions hold, then household i is eligible to become a first-time homebuyer, $\tilde{X}_{i,t}^{FTHB} = 1$. When a household buys a house, it purchases the regional entry-level house, but its down payment, $dp_i Z_t^{HPSTARTER}(\omega_i^{REG})$, depends on whether its maximum affordable house price would have been constrained by its down-payment capacity or its debt-service capacity.

If the down payment is the household's binding constraint, then the household makes the minimum down payment, with $dp_i = dp_{MIN}$. If the household is either unconstrained, or if the debt-servicing capacity is the household's binding constraint, then $dp_i = f(\frac{X_{i,t}^{FA}}{X_{i,t}^{HP.Max}}) > dp_{MIN}$, a function of the household's financial assets relative to its maximum affordable house price. In this respect, a household makes a down payment based on its financial capacity: households with fewer financial assets make the minimum required down payment, dp_{MIN}.

To match the macro scenario, total growth in mortgage debt from first-time homebuyers is given by $\phi Z_t^{MDG} MD_{t-1}$. The parameter ϕ can be changed according to the design of the macro scenario . Given households with $\tilde{X}_{i,t}^{FTHB} = 1$, households

[25] In the calibration, we use the typical price for a starter home in the region in which a household lives. Currently, the regional breakdown is by province.

are randomly drawn, setting $X^{FTHB} = 1$ until

$$\sum_i X_{i,t}^{FTHB} Z_t^{HPSTARTER}(\omega_i^{REG})(1 - dp_i) \geq \phi Z_t^{MDG} MD_{t-1}.$$

If an eligible household is assigned as a first-time homebuyer, $X_{i,t}^{FTHB} = 1$, then its debt payments are based on the discounted 5-year mortgage rate, $Z_t^{R-5yr-disc}$, rather than the qualifying (posted) rate, Z_t^{R-5yr}.[26]

4.2.3 Mortgage debt: Non-first-time homebuyers

- If a household is employed, then an intermediate variable for mortgage debt is generated by the following law of motion:

$$\log \tilde{X}_{i,t}^{MD} = \log X_{i,t-1}^{MD} + \alpha_{MD}^{CNS} + \left(1 - \lambda_{MD} Ind\left(X_{i,t}^{DSR} > 40\right)\right) \times \left[\alpha_{MD}^{YG} \Delta log X_{i,t}^{PLY} + \alpha_{MD}^{R} \Delta Z_t^{R-5YRDISC} + \alpha_{MD}^{HP} Z_t^{HPG} X_{t-1}^{HA}\right] + \varepsilon_{i,t}^{MD}, \quad (15)$$

  where Z_t^{HPG} is the nominal growth in aggregate house prices given by the macro scenario. The distribution of the error term is given by

$$\varepsilon_{i,t}^{MD} \sim F^{MD}.$$

  Therefore, mortgage debt depends upon the change in income for a household, as well as the change in interest rates and the increase in house prices. The λ coefficient captures a non-linearity where a debt-service ratio over 40 lowers the growth in mortgage debt. In addition, there is an idiosyncratic shock that affects mortgage debt. The details for the estimation behind the calibrated values for the parameters are covered in Djoudad (2012). This specification captures the ability of households to borrow against the rising value of their homes.

- If a household is unemployed and has strictly positive mortgage debt, then

$$\tilde{X}_{i,t}^{MD} = X_{i,t-1}^{MD}.$$

- If a household has no mortgage debt, then they may accumulate mortgage debt according to the process for first-time homebuyers detailed in the previous section.

[26] In practice, the qualifying rate can be defined to equal the discounted rate, which is the case in Canada for borrowers that opt for a 5-year fixed-rate mortgage.

- The aggregate realized mortgage debt in period $t-1$ is given by

$$MD_{t-1} = \sum_{i}^{I} X_{i,t-1}^{MD}.$$

From the macro scenario, Z_t^{MDG} denotes the growth rate in mortgage debt from period $t-1$ to t, so that mortgage debt in period t should be equal to $\left(1 + Z_t^{MDG}\right) MD_{t-1}$. However, only $1-\phi$ per cent of mortgage debt growth is assumed to be due to households with an existing mortgage. Therefore, given $\tilde{X}_t^{MD}$, a consistency factor C_t^{MDG} is calculated that will be used to make sure that aggregate mortgage debt growth matches the macro scenario:

$$C_t^{MDG} = \left(1 + (1-\phi)\, Z_t^{MDG}\right) MD_{t-1} - \sum_{i}^{I} \tilde{X}_{i,t}^{MD}.$$

This captures the aggregate amount of mortgage debt that has to be added to (or subtracted from if negative) employed households that previously had mortgage debt in order to match the macro scenario.

- The consistency factor C_t^{MDG} is allocated across employed households proportionally based upon holdings of mortgage debt to arrive at final household mortgage debt for period t:

$$X_{i,t}^{MD} = \tilde{X}_{i,t}^{MD} + \frac{Ind\left(X_{i,t}^{U} = 0\right)\left(\tilde{X}_{i,t}^{MD} - X_{i,t-1}^{MD}\right)}{\sum_i^I Ind\left(X_{i,t}^{U} = 0\right)\left(\tilde{X}_{i,t}^{MD} - X_{i,t-1}^{MD}\right)} C_t^{MDG}.$$

4.2.4 Consumer debt

- If a household is employed and had consumer debt in the previous period $\left(X_{i,t-1}^{CD} > 0\right)$, then an intermediate variable for consumer debt is generated by the following law of motion:

$$\begin{aligned} \log \tilde{X}_{i,t}^{CD} = \log X_{i,t-1}^{CD} + \alpha_{CD}^{CNS} + \left(1 - \lambda_{CD} Ind\left(X_{i,t}^{DSR} > 40\right)\right) \times \\ \left[\alpha_{CD}^{YG} \Delta log X_{i,t}^{Y} + \alpha_{CD}^{R} \Delta Z_t^{RCD} + \alpha_{MD}^{HP} Z_t^{HPG} X_{t-1}^{HA}\right] + \varepsilon_{i,t}^{CD}, \quad (16) \end{aligned}$$

with the distribution of the error term given by

$$\varepsilon_{i,t}^{CD} \sim F^{CD}.$$

The λ_{CD} coefficient captures a non-linearity where a debt-service ratio over 40 lowers the growth in consumer debt. In addition, there is an idiosyncratic shock that affects consumer debt. The details for the estimation behind the calibrated values for the parameters are covered in Djoudad (2012):

- Unemployed households are not allocated additional consumer debt through the law of motion in equation 16,

$$\tilde{X}_{i,t}^{CD} = X_{i,t-1}^{CD}.$$

- If a household has no consumer debt, then a household does not acquire any consumer debt:

$$\tilde{X}_{i,t}^{CD} = 0.$$

An exception to this in the model is that an unemployed household can access existing lines of credit to lessen the impacts of a temporary spell of unemployment. See subsection 4.2.7.

- The aggregate realized consumer debt in period $t-1$ is given by

$$CD_{t-1} = \sum_{i}^{I} X_{i,t-1}^{CD}.$$

From the macro scenario, Z_t^{CDG} denotes the growth rate in consumer debt from period $t-1$ to t, so that consumer debt in period t should be equal to $\left(1 + Z_t^{CDG}\right) CD_{t-1}$. Given $\tilde{X}_t^{CD}$, a consistency factor C_t^{CDG} is calculated to make sure that aggregate consumer debt growth matches the macro scenario:

$$C_t^{CDG} = \left(1 + Z_t^{CDG}\right) CD_{t-1} - \sum_{i}^{I} \tilde{X}_{i,t}^{CD}.$$

This captures the aggregate amount of consumer debt that has to be added to (or subtracted from if negative) employed households in order to match the macro scenario.

- The consistency factor C_t^{CDG} is allocated across employed households proportionally, based upon holdings of consumer debt to arrive at final household consumer debt for period t:

$$X_{i,t}^{CD} = \tilde{X}_{i,t}^{CD} + \frac{Ind\left(X_{i,t}^{U} = 0\right) \cdot (\tilde{X}_{i,t}^{CD} - X_{i,t-1}^{CD})}{\sum_{i}^{I} Ind\left(X_{i,t}^{U} = 0\right) \cdot (\tilde{X}_{i,t}^{CD} - X_{i,t-1}^{CD})} C_t^{CDG}.$$

4.2.5 Evolution of housing assets

For households with strictly positive housing assets, housing assets grow with house prices:

$$X_{i,t}^{HA} = \left(1 + Z_t^{HPG}\right) X_{i,t-1}^{HA},$$

where Z_t^{HPG} denotes aggregate growth in nominal house values.

4.2.6 Savings: Accumulation of financial assets

Employed households are assumed to consume a fraction ω_i^{MPC} of their disposable labour income. This generates an intermediate value for savings, after also subtracting debt payments:

$$\tilde{X}_{i,t}^{SAV} = \left(1 - \omega_i^{MPC}\right) X_{i,t}^{DLY} - X_{i,t}^{DP}.$$

Unemployed households are assumed to consume a minimum, based on their permanent labour income, $X_{i,t}^{C} = \kappa X_{i,t}^{PLY}$. Therefore, savings for unemployed households are given by

$$\tilde{X}_{i,t}^{SAV} = X_{i,t}^{DLY} - \kappa X_{i,t}^{PLY} - X_{i,t}^{DP}.$$

Note that unemployed households can have negative savings. We assumed earlier that unemployed households do not increase borrowing, except in the special case of existing lines of credit (see subsection 4.2.7), so that the negative savings of unemployed households will result in a drawdown of their financial assets.

The consistency factor for savings is given by

$$C_t^{SAV} = Z_t^{SAV} \sum_i^I X_{i,t}^{DLY} - \sum_i^I \tilde{X}_{i,t}^{SAV},$$

where Z_t^{SAV} is the aggregate savings rate coming from the macro scenario. The consistency factor is then reallocated to employed households:

$$X^{SAV} = \tilde{X}_{i,t}^{SAV} + \frac{Ind\left(X_{i,t}^{U} = 0\right) X_{i,t}^{SAV}}{\sum_i^I Ind\left(X_{i,t}^{U} = 0\right) X_{i,t}^{SAV}} C_t^{SAV}.$$

Financial assets are then given by the combination of savings and asset returns:

$$X_{i,t}^{FA} = X_{i,t-1}^{FA}\left(1 + Ind\left(X_{i,t-1}^{FA} > 0\right) Z_t^{RFA}\right) + X_{i,t}^{SAV}.$$

We allow for the asset returns to depend upon whether financial assets are positive or negative by using an indicator function. This is done because we allow for the possibility of financial assets to be negative, which we then use as an indicator of a household being in arrears on its debt payments (see below). Note that because of the way savings are structured, a household can only have negative savings, and negative financial assets, if it cannot afford its debt payments. Therefore, the amount by which debt is in arrears is given by negative financial assets.

4.2.7 Lines of credit

If a household is unemployed and has depleted its financial assets to zero, it can draw on any unused credit room under its lines of credit to cover any dissavings during the period. The initial availability of lines of credit and any unused room are determined from the microdata. If credit room is insufficient, or if financial assets have not yet reached zero, there is no draw:

$$X_{i,t}^{LC-Draw} = \begin{cases} -X_{i,t}^{SAV} Ind\left(X_{i,t-1}^{SAV} < 0\right) & \text{if } X_{i,t-1}^{LC} < X_i^{LC-Max} + X_{i,t-1}^{SAV} \\ & \text{and } X_{i,t-1}^{FA} = 0 \text{ and } X_{i,t-1}^{UD} > 0 \\ 0 & \text{if } X_{i,t-1}^{LC} \geq X_i^{LC-Max} + X_{i,t-1}^{SAV} \\ & \text{or } X_{i,t-1}^{FA} > 0 \end{cases}.$$

Consumer debt and credit room under the lines of credit are updated accordingly:

$$X_{i,t}^{CD} = X_{i,t-1}^{CD} + X_{i,t}^{LC-Draw}.$$

$$X_{i,t}^{LC} = X_{i,t-1}^{LC} + X_{i,t}^{LC-Draw}.$$

When sufficient credit room to cover any dissavings is not available, a household is susceptible to going into arrears.

4.2.8 Arrears

If financial assets turn negative, then a household is in arrears:

$$X_{i,t}^{ARR} = \begin{cases} 1 & \text{if } X_{i,t}^{FA} < 0 \\ 0 & \text{if } X_{i,t}^{FA} \geq 0 \end{cases}.$$

The number of consecutive periods that a household has been in arrears in its debt payments is denoted by $X_{i,t}^{CONARR}$. The law of motion for $X_{i,t}^{CONARR}$ is given by

$$X_{i,t}^{CONARR} = \begin{cases} X_{i,t}^{CONARR} + 1 & \text{if } X_{i,t}^{FA} < 0 \\ 0 & \text{if } X_{i,t}^{FA} \geq 0 \end{cases}.$$

This variable can be used to construct measures of how long a household has been in debt. For instance, in a quarterly model, if $X_{i,t}^{CONARR} \geq 1$, then a household is 90 days or more in arrears, whereas in a monthly model, $X_{i,t}^{CONARR} \geq 3$ would indicate 90 days or more in arrears. Given the structure of the model, a household can come out of arrears only if it raises its financial assets back to positive.[27]

5 Data

Data to set up the starting point of HRAM's household variables come from the Canadian Financial Monitor (CFM), a survey compiled by Ipsos Reid. The data include information on balance sheets, income, debt payments, and other financial characteristics for about 12,000 households per year. The CFM data are weighted to be cross-sectionally representative of the Canadian population, but most households have taken part in the survey on more than one occasion.[28] Given HRAM's goal of assessing household vulnerabilities over the simulation horizon, a discussion of vulnerabilities as they appear at a relatively recent point in time is an apt starting point for describing the data set. We use 2013 data to avoid any confusion with more recent analyses of vulnerabilities that have appeared in the *Financial System Review*, such as in Cateau, Roberts, and Zhou (2015).

A household's debt payment burden is typically characterized relative to its income, to give a debt-service ratio (DSR) of debt payments divided by gross income. A DSR measure gives an intuitive sense of the current burden of debt payments for a household. However, a DSR indicator might not accurately characterize a household's vulnerability to a stress event that impairs income.

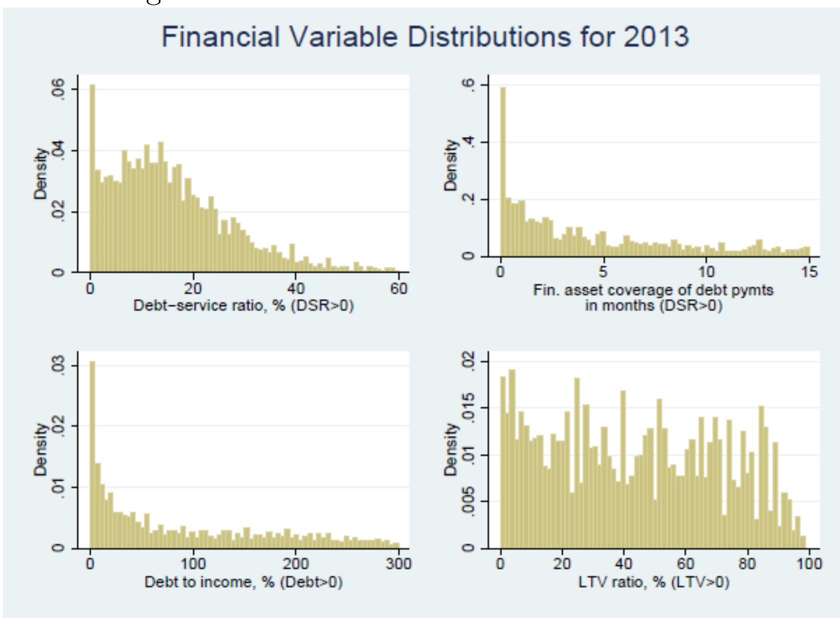
In 2013, the mean DSR for households with debt was around 16.7 per cent (about

[27]In addition to paying off debt owed, in the real world, households can restructure their debt or file for bankruptcy. In the current version of HRAM, this is not allowed for. However, when the aggregate stock of arrears is calculated, we make an exogenous assumption about the flow out of arrears. The model is agnostic about the extent that this flow out of arrears is written off or paid off. See Section 6, concerning the calibration of arrears.

[28]In the 2013 survey, 43.2 per cent of surveyed households appear for the first time, 50.9 per cent were surveyed in both 2012 and 2013, 34.4 per cent were surveyed in each year for 2011-2013, 24.4 per cent were surveyed each year for 2010-2013, and so forth.

68 per cent of households had debt, and 50 per cent of these held a mortgage). The standard deviation for the distribution is fairly pronounced at 14 percentage points, with asymmetry evident in an extended right tail, where households have higher debt payment burdens (Figure 2).

Figure 2: Household Financial Characteristics



Dey, Djoudad and Terajima (2008), using Statistics Canada's Survey of Financial Security and CFM data for 1999 to 2006, identify a DSR of 35 per cent as a critical threshold, above which there is a significant increase in households' propensity to be delinquent on their mortgages. In addition, the 40 per cent threshold is a common industry standard for loan eligibility, above which a household is expected to have more difficulty making loan payments.[29] Using thresholds to characterize which households are the most vulnerable, in 2013, about 8.7 per cent of households had a DSR equal to or greater than 35 per cent, and 5.6 per cent had a DSR equal to or greater than 40 per cent.[30]

The economy's risk exposure to vulnerable households is arguably more closely related to the proportion of debt held by these households. In 2013, 15.5 per cent of household debt was held by households with DSRs equal to or greater than 35 per

[29]Financial institutions often include a household's other financial obligations apart from debt payments when testing this threshold. As a result, a 40 per cent threshold calculated only with debt payments might under-identify households as vulnerable, relative to industry practices.

[30]While these thresholds are helpful for describing the extent of vulnerability, the entire distribution is important and is taken into account by simulations.

cent, while this figure was 9.9 per cent for household debt held by households with DSRs equal to or greater than 40 per cent.[31]

The debt-service burden is determined in part by interest rates, however, and could understate vulnerabilities, given the current low interest rate environment. As discussed in Cateau, Roberts, and Zhou (2015), the share of household debt held by highly indebted households (which they define as having a debt-to-gross-income ratio of 350 per cent and above) has increased from 12.7 per cent over 2005-2007 to 20.7 over 2012-2014. Cateau et al. provide a more comprehensive description of these highly indebted households and possible implications for Canadian financial system vulnerabilities from this increase.

Financial assets are also of particular interest, since these assets would be the most readily available alternative as a buffer in the event of a temporary loss of income. To put financial assets into perspective, we divide these holdings by a household's reported monthly debt payment obligations to give the number of months that a household could meet its debt obligations without recourse to any income (either labour or non-labour income). The top-right quadrant of Figure 2 illustrates that many indebted households have only enough financial assets to cover a couple of months of payments. The chart is truncated at 15 months because of the extended right tail of households with high levels of financial assets relative to debt payments.

To give a rough indication of the share of households that could have insufficient financial assets in the event of an income shock, threshold measures can help to describe the tail of the distribution. One month and four months of coverage are chosen as thresholds because the average complete unemployment episode lasts about four months. Abstracting from minimum consumption requirements and any alternative resources from which to make debt payments, one month of coverage would bring a household just to the point of the three-month arrears threshold, subject to an average unemployment episode; four months would bring a household to the point of a total drawdown of financial assets. The overall proportions for households that met or fell below these two thresholds in 2013 were 7.4 per cent and 15.8 per cent, respectively.[32]

[31] These measures describe the tail of the distribution and so are particularly affected by sampling variability. In a given year, about 450 households might have a DSR $\geq$ 40 per cent (accounting for missing survey responses). Higher sampling weights for certain observations can potentially add to this variability. Bootstrapping the share of households with a DSR $\geq$ 40 for 2013 roughly gives 5.6 per cent $\pm$ 0.5 percentage points at a 90 per cent confidence interval. For the share of debt held by households with a DSR $\geq$ 40 per cent, the confidence interval is roughly 9.9 per cent $\pm$ 1.1 percentage points, at a 90 per cent confidence interval.

[32] These measures should be interpreted as only being indicative. Estimates of minimum consumption requirements and any alternative sources of income, such as employment insurance, may

Table 2: Percentage of Indebted Households with a Vulnerable Debt-Service Ratio (DSR), or Financial Asset Coverage of Debt Payments (ACOV), by Income Quartile - 2013

Quartiles (low-income to high-income)	DSR $\geq$ 35%	DSR $\geq$ 40%	ACOV $\leq$ 1 Month	ACOV $\leq$ 4 Months
1	15.4	11.9	8.6	14.7
2	12.7	7.9	8.9	19.8
3	6.1	3.5	7.1	17.1
4	4.4	2.4	5.4	12.0
Overall	8.7	5.6	7.4	15.8

Two subgroups that we might expect to appear disproportionately in these vulnerability categories are lower-income households and younger households. Indebted households in the lowest income quartile are somewhat more likely to have low levels of asset coverage and are more likely to be in the high-DSR categories (Table 2). For younger households, defined as having a head of household younger than 35 years of age, the average DSR for indebted households is similar to the rest of the population, but the proportion of these households with low asset coverage is higher (Table 3).[33]

Figures 3, 4, and 5 illustrate the distributions with the same breakdown. The frequency of Figure 3 is relative to all households, rather than only indebted households, emphasizing that older households and lower-income households tend to carry less debt, although lower-income households are still well-represented in the vulnerable tail. When considering only indebted households, age differences appear to be less significant, but the lower-income quartiles have DSR distributions more skewed to the right and a somewhat greater incidence of low levels of asset coverage.

5.1 Missing Values

Response rates in the CFM vary by group and question. In many cases, common sense suggests that non-responses correspond to cases where the household does not have any debt, assets, or other amount pertaining to the question at hand. In other cases, though, data are clearly incomplete. For example, the household might hold debt, but it has not provided an interest rate for this debt – this type of missing response occurs for 1-2 per cent of households. Although these households could

help to improve these measures, although these factors would be partially offsetting.

[33]Using the findings of a survey of homebuyers done for CAAMP (now called Mortgage Professionals Canada), Dunning (2015) finds that the 25-34 age group accounts for roughly 57 per cent of first-time homebuyers, while the 35-44 age group accounts for roughly another 18 per cent.

Table 3: Percentage of Indebted Households with a Vulnerable Debt-Service Ratio (DSR), or Financial Asset Coverage of Debt Payments (ACOV), by Age Group - 2013

Age	DSR $\geq$ 35%	DSR $\geq$ 40%	ACOV $\leq$ 1 Month	ACOV $\leq$ 4 Months
25-34	8.6	5.7	11.3	26.5
35-49	9.8	5.5	9.9	19.0
50-64	7.4	5.4	6.2	12.4
65+	8.9	6.1	3.2	7.0
Overall	8.7	5.6	7.5	15.7

Figure 3: Debt-Service Ratio Distribution, 2013

Probability relative to all households; DSR>0 and <60

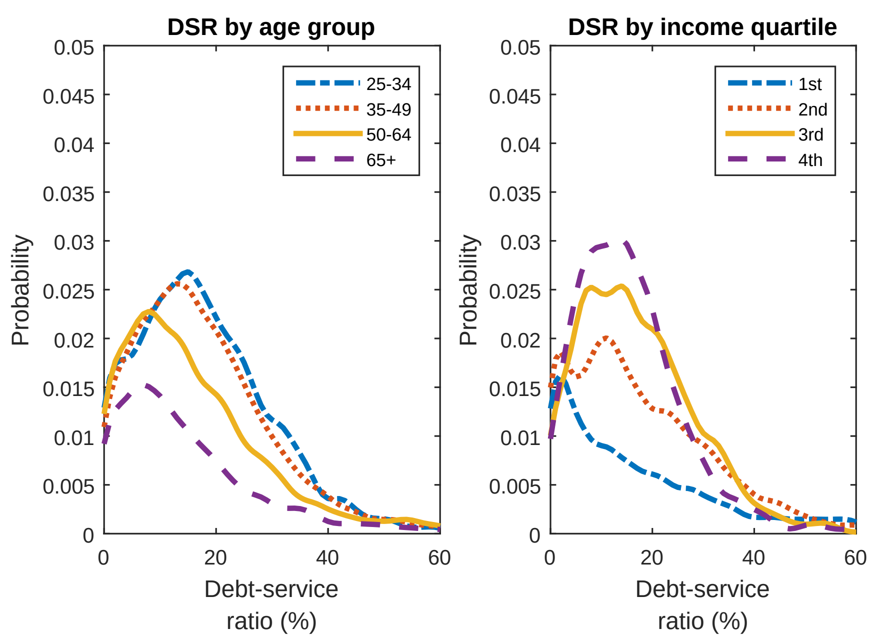


Figure 4: Debt-Service Ratio Distribution, 2013

Only indebted households; DSR>0 and <60

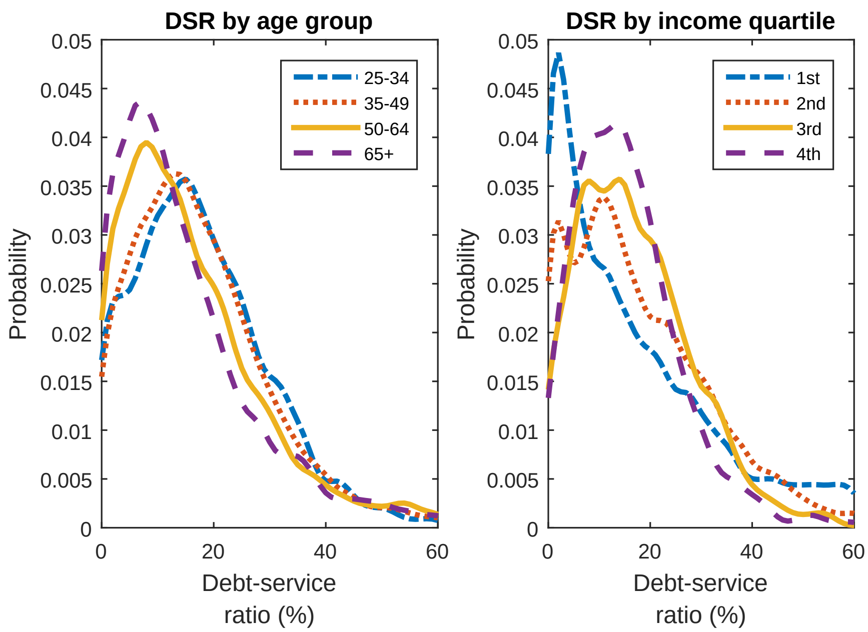
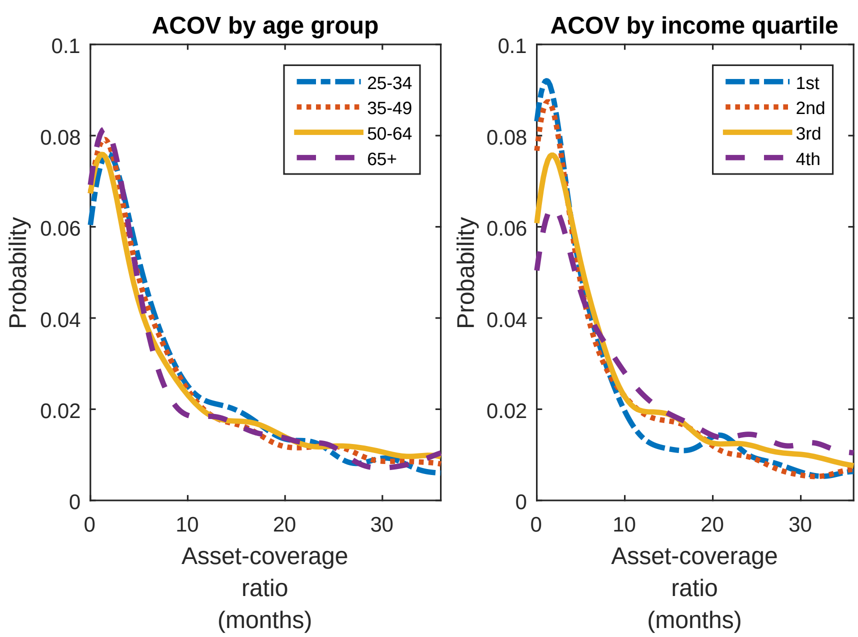


Figure 5: Asset-Coverage Distribution, 2013

Only indebted households; ACOV<=36 Months



simply be excluded, this presents the risk of biasing the sample (Djoudad 2012).

For missing interest rates and risk premiums, a household is assigned the average value from households in its income group for that same loan type. For the mortgage term and rate type (variable or fixed), a random draw is made from the existing distribution of available responses. Missing balance amounts are not replaced – related debt payments are assumed to stay unchanged over the simulation.

Other assumptions:

- All consumer debt, except for credit cards, is assumed to be at variable rates.
- For this consumer debt, households pay the overnight rate, plus an individual premium calculated at the starting point. This individual premium stays unchanged throughout the simulation, although a stress scenario could involve an increase to the general household-sector premium.
- There is full pass-through of changes to the overnight rate and aggregate changes to risk premiums, to all variable-rate debt, in each period. There is pass-through to fixed-rate mortgages only at renewal.
- Variable-rate mortgages are renewed as either fixed or variable, in proportions corresponding to current trends.[34]
- The CFM has information on the rate types and terms of mortgages, but does not say when mortgages are due for renewal. Renewal is staggered across periods, so that households with a specific mortgage term renew their mortgages evenly across simulation periods.

[34]The Bank of Canada's *Financial System Review* periodically discusses trends in the residential mortgage market. For instance, about 32 per cent of new mortgages in April 2015 had variable rates, a significant increase from about 9 per cent two years earlier. More than one-quarter of the outstanding stock of mortgages were at variable rates (Bank of Canada, 2015).

6 Estimation and Calibration

HRAM includes a number of parameters or distributional assumptions that must be calibrated to, or informed by, available data:

(i) The parameters in the equations for income and debt growth, equations (9), (15) and (16).

(ii) The distribution of unemployment.

(iii) The distribution of unemployment duration.

(iv) The likelihood that unemployed households will receive employment insurance, and if so, the amount.

(v) The parameters for a first-time homebuyer to qualify for a mortgage, such as minimum down payment, dp_{MIN}, and maximum amortization period, amt_{MAX}.

(vi) The arrears process.

(vii) The proportion of aggregate credit growth that is attributed to first-time homebuyers.

(viii) Tax rates.

An overview of the parameters is shown in Table 4.

Table 4: Parameters

Parameter	Notation	Value	Source
First-time homebuyers (FTHBs)			
Minimum down payment	dp^{MIN}	5%	Government regulation
Maximum amortization	amt^{MAX}	25 years	Government regulation
Share of debt growth attributable to FTHBs	ϕ	0.4-0.6	Survey data
Regional entry-level house price, by province	$Z^{HP.Starter}$	varies	Royal LePage

Table 4: Parameters

Parameter	Notation	Value	Source
Unemployment			
Average period that a loan stays in arrears		2.5 quarters	Calibrate to outstanding arrears data
Minimum consumption level	κ	0.45	Calibrate to outstanding arrears data
Employment insurance benefits	b	Max of \$501/week	Service Canada
Duration distribution	$\varepsilon^{UD} \sim F^{UD}$	Log-normal	Authors' Calculation
Average duration	$E(\varepsilon^{UD})$	17-25 weeks	StatsCan
Variance of duration	σ^{UD}	25 weeks	Authors' Calculation
Probability of layoff	β_{layoff}	–	Chan, Morissette, and Frenette (2011)
Income dynamics			
Income-growth equation	Equation (9)	$\sigma_{i,t}^{PLY} = \begin{cases} 0.04 & \text{if } \omega_i^{YQ} = 1 \\ 0.03 & \text{if } \omega_i^{YQ} = 2 \\ 0.025 & \text{if } \omega_i^{YQ} = 3 \\ 0.006 & \text{if } \omega_i^{YQ} = 4, 5 \end{cases}$	Djoudad (2012)
Balance-sheet dynamics			
Mortgage debt growth equation	Equation (15)	–	Djoudad (2012)
Constant	α_{MD}^{CNS}	0.0155	"
Elasticity with respect to income	α_{MD}^{YG}	0.5282	"
Elasticity with respect to the mortgage interest rate	α_{MD}^{R}	-0.0538	"
Elasticity with respect to house prices	α_{MD}^{HP}	0.001	"

Table 4: Parameters

Parameter	Notation	Value	Source
Coefficient for indicator of high debt-service ratio for a household	λ_{MD}	0.3367	''
Shock distribution	$\varepsilon_{i,t}^{MD} \sim F^{MD}$	$N(0, \sigma_{MD})$	''
Consumer debt growth equation	Equation (16)	–	Djoudad (2012)
Constant	α_{MD}^{CNS}	0.005	''
Elasticity with respect to income	α_{CD}^{YG}	0.8030	''
Elasticity with respect to the consumer interest rate	α_{CD}^{R}	-0.0266	''
Elasticity with respect to house prices	α_{CD}^{HP}	0.0007	''
Coefficient for indicator of high debt-service ratio for a household	λ_{CD}	0.2163	''
Shock distribution	$\varepsilon_{i,t}^{CD} \sim F^{CD}$	$N(0, \sigma_{CD})$	''

6.1 Income and Debt Equations

The specifications for the income and debt equations are taken directly from Djoudad (2012).

6.2 Unemployment

HRAM allows the incidence of unemployment to vary according to household characteristics. This is implemented by multiplying the unconditional transition rate from employment into unemployment by a relative layoff probability for each employed household. Chan, Morissette, and Frenette (2011) estimate a logit model that allows for the calculation of this relative probability, based on age, gender, education, job tenure, region, and job occupation. Although Chan et al. estimate this equation using data from Statistics Canada's Labour Force Survey, the CFM data provide a rough correspondence for most of the regressors in Chan et al. An exception is job tenure, which is proxied by age.

The initial probability-of-layoff estimate for household i, with fixed characteristics Ω_i, is provided by

$$p_{\text{layoff},i} = \frac{e^{\Omega_i \beta_{\text{layoff}}}}{1 + e^{\Omega_i \beta_{\text{layoff}}}},$$

where β_{layoff} is the vector of regression coefficients obtained from Chan, Morissette, and Frenette (2011).[35]

To integrate the estimated layoff probability into a particular scenario, which imposes the aggregate unemployment rate Z_t^U for period t, the estimated probability is converted into a relative probability for each employed household i in period $t-1$, with mean equal to one, and is set to zero for unemployed households:

$$\varphi_{i,t} = \begin{cases} \frac{p_{\text{layoff},i}}{\sum_{i=1}^{I} p_{\text{layoff},i} Ind\left(X_{i,t-1}^{UD}=0\right)/(I-I_{t-1}^{UD})} & \text{if } X_{i,t-1}^{UD} = 0 \\ 0 & \text{if } X_{i,t-1}^{UD} > 0 \end{cases}.$$

The specification results in higher layoff rates for workers in primary, construction, and manufacturing industries, those without a university education, and workers in the Atlantic provinces. The implication is that not only might underlying financial vulnerabilities correlate with the probability of layoff, as a result of common factors, but the incidence of unemployment, and thus cumulative financial strain over the simulation, is more concentrated among these higher-risk households. Chan et al. find that the relative pattern of layoffs is similar in recessionary and expansionary times – for example, workers in primary industries and construction would typically face a higher degree of job turnover regardless of the stage of the business cycle. In this case, the conditionality of the unemployment shock will not greatly affect the shock-versus-control results, after calibration. However, the model does allow for the examination of a shock scenario where the pattern of layoff risk changes. For added insight, Cateau, Roberts, and Zhou (2015) describe the distribution of household debt according to the relative risk of layoff that is calculated in HRAM.

This simplification of unemployment dynamics does not address job quits and new entrants as other components of unemployment. HRAM does not incorporate changes in labour force participation, but the distinction between voluntary (job quit) and involuntary (layoff) transitions from employment into unemployment could be important. We would not expect that voluntary transitions into unemployment would lead to financial distress to the same extent as layoffs. In addition, the rate of layoffs is countercyclical, whereas the rate of quits is procyclical (Cam-

[35]Chan, Morissette, and Frenette (2011) provide the marginal effects from the logit estimation. The regression coefficients were obtained through correspondence with the authors.

polieti 2011), which could exacerbate household vulnerabilities during a downturn, if a greater share of unemployment was involuntary. However, Campolieti (2011) finds that the increase in unemployment duration during downturns is the greatest for quits, so the net effect on the overall composition of unemployment is unclear.

6.3 Unemployment Duration

The duration of unemployment for each unemployed household is assigned at random and is limited to a range of 1 to 99 weeks. The assignment of duration requires a choice about distribution, but it will help to first discuss average duration.

Commonly cited statistics on average duration, such as those from the Labour Force Survey, refer to average incomplete duration – the average amount of time that a household has spent unemployed at the time it was surveyed. In contrast, the assignment of duration to an unemployed household in HRAM represents the complete period of unemployment that the household experiences for that jobless episode. To relate a measure of incomplete average duration to the underlying complete average duration, one must contend with two offsetting biases: a length bias and a sampling bias.

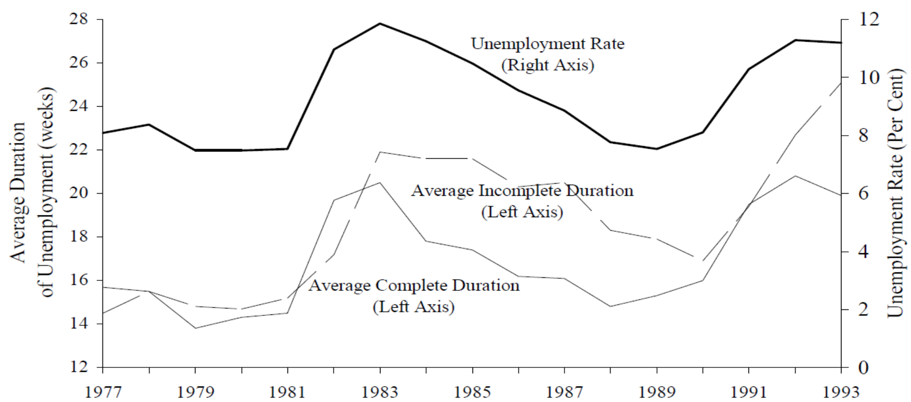
The length bias results from the fact that, conditional on an individual unemployed household having been chosen for the survey, the household could be surveyed at any point during this episode with equal probability. On average, the household will be surveyed at the middle of its jobless episode, so the length bias on its own would cut the measured average duration in half.

The sampling bias, however, arises because households with long spells of unemployment are more likely to be unemployed at the time of the survey and so are overrepresented in the calculation of average incomplete duration, as it would appear in the Labour Force Survey. In contrast, short jobless episodes might often be missed (undersampled) by the survey. Corak and Heisz (1995b) find that the sampling bias outweighs the length bias over the 1977 to 1993 period, such that average complete duration is about 16.9 weeks, compared with an average incomplete duration of 18.7 weeks. These measures are shown in Figure 6. For the purposes of HRAM's simulations, this average difference is relatively minor, and Statistics Canada's figures for incomplete duration, which are readily available, typically form the basis for a scenario's starting-point average duration.

The assumed distribution of duration in HRAM should reflect empirical evidence that, while episodes of unemployment are generally short, there is a significant share of long-term episodes. For example, among the unemployed households questioned by Statistics Canada in 2011 for the Labour Force Survey, about 62 per cent had been unemployed for 13 weeks or less, while 22 per cent had been unemployed for more than 26 weeks. Figure 7, from Corak and Heisz (1995a), illustrates the distribution of reported (incomplete) durations for 1977 to 1993, where this skewed right tail is evident.[36]

Another insight from the flattening slope in the right tail of the distribution is that as a household's unemployment spell persists, the empirical probability of exiting the unemployed state diminishes. This may be due to loss of human capital or loss of job-search motivation. One choice for a distribution that reflects the shape seen in Figure 7 is a lognormal distribution. HRAM adopts this as its duration distribution assumption, with a location parameter of $\mu = ln(E[\varepsilon^{UD}]) - \frac{1}{2}\sigma^2$, and a scale parameter σ given by $\sigma^2 = ln(1 + \frac{Var[\varepsilon^{UD}]}{E[\varepsilon^{UD}]^2})$.

Figure 6: Complete vs. Incomplete Duration



Source: Corak and Heisz (1995b)

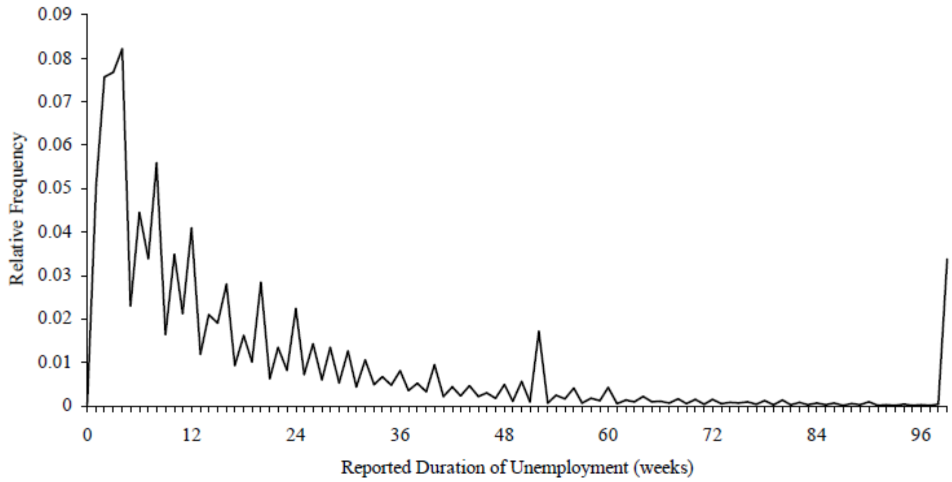
6.4 Employment Insurance

An unemployed income-earner in HRAM receives employment insurance benefits of 55 per cent of their labour income, up to a maximum (for example, the maximum was \$501 per week under 2013 rules), for a maximum period of 45 weeks.[37] For part-time workers, this maximum period is assumed to be shorter, since the period for receiving benefits depends on the number of hours of insurable employment accumulated in the qualifying period, usually the previous 52 weeks. Given that some households

[36]Figure 7 also shows a reporting bias towards certain numbers of weeks.

[37]In reality, this maximum period depends on the regional unemployment rate and a worker's accumulation of insured hours during the period, usually equal to the previous year.

Figure 7: Reported Incomplete Duration



Source: Corak and Heisz (1995a)

might not qualify for EI, for various reasons (e.g., insufficient insured hours), a fraction of workers do not receive benefits.

6.5 Mortgage Parameters for a First-Time Homebuyer

The parameters for a first-time homebuyer are calibrated as follows:

(i) The minimum down payment, dp_{MIN}, is set to the minimum allowed down payment for insured mortgages in Canada, which is currently 5 per cent.

(ii) The maximum amortization, amt_{MAX}, is set to the maximum allowed amortization for insured mortgages in Canada, which is currently 25 years.

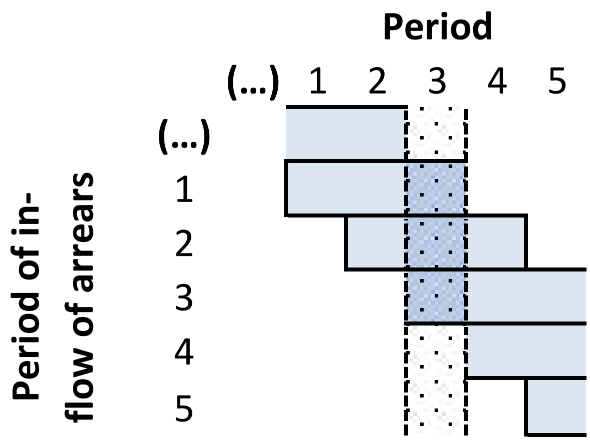
(iii) The cost of a starter home, $HA_{Starter}$, is set to the value of a bungalow for the region in which a household resides, as reported by Royal LePage. Currently, the regional breakdown is by province.

(iv) The maximum allowed debt-service ratio, DSR_{MAX}, is set to 40 per cent.

6.6 Arrears Process

One of the primary model outputs is the flow of arrears for household debt. However, in the data, arrears is reported as the stock of outstanding debt that is in arrears. The stock of arrears at any point in time is the sum of previous inflows of arrears that have not yet been restored as fully current or written off (see Figure 8).[38] The model is therefore calibrated to match the current stock of arrears on total household debt.[39] To match the stock of arrears: (i) all debt in arrears is assumed to be extinguished in one quarter for consumer debt and an average of 2.5 quarters for mortgage debt (owing to time lags involved in mortgages exiting the stock of arrears – the model is agnostic as to whether this occurs through foreclosure or some other resolution such as a house sale); and (ii) the parameter on minimum consumption, κ, is adjusted to match the current rate of arrears.

Figure 8: Stock vs. Flow of Arrears



Stock of arrears at $T = 3$ is equal to the sum of all inflows of arrears that have not been restored as fully current or written off.

6.7 Credit Growth Allocation to First-Time Homebuyers

First-time homebuyers have been important contributors to the growth of mortgage credit, with approximately 45 per cent of new mortgages from 2013 to April 2015 extended to first-time homebuyers (Dunning, 2015).

[38] At typical savings rates, and in the absence of debt-restructuring or asset sales, most households in HRAM would not recover quickly from financial distress. Even at a high rate of savings, unemployed households often deplete their financial assets more quickly than the rate at which they can recover from arrears once re-employed The model shortcut in this context is to not explicitly track the financial recovery of households, but to instead assume a fixed rate at which distressed households exit out of arrears.

[39] The arrears data are from OSFI regulatory filings and include both consumer debt and mortgage debt.

6.8 Taxes

A tax rate is calculated for each individual income-earner based on rules from the Canada Revenue Agency; the CFM data identify whether households have one or two income-earners (in some cases, there could be more, but we do not consider this). The federal government and each province have established marginal tax rates that increase according to incremental income ranges (with the exception of Alberta, which has a flat tax), so an individual's tax rate is based on their income and province. To calculate disposable income available for debt payments, taxes are subtracted from gross income.

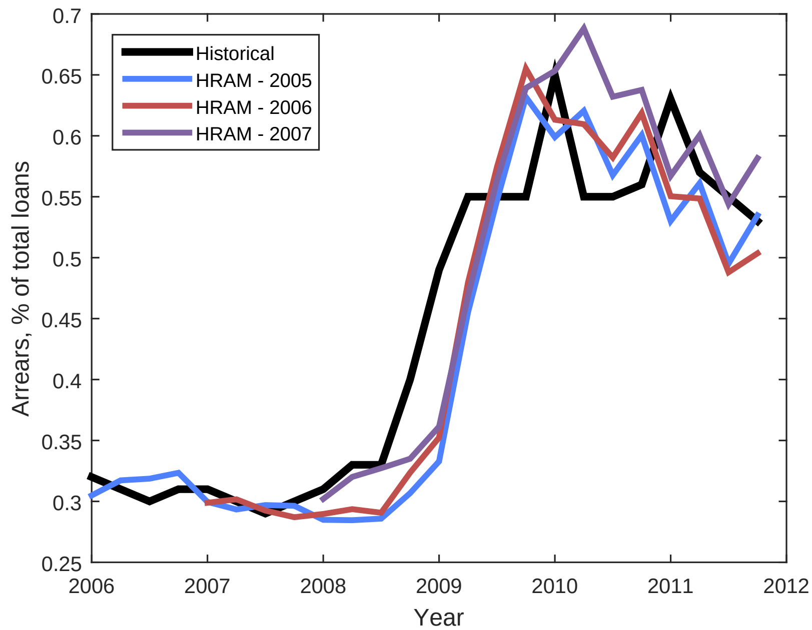
7 Illustrative Scenarios and Comparative Statics

This section describes results for three sets of illustrative scenarios. One type of validation exercise is to use past CFM data to set the starting point for the model and then to run a scenario comprising the historical paths for the required macroeconomic variable inputs. A second set of scenarios is based on the stress tests that appear in the *Financial System Review*. The third set of scenarios shows comparative statics where one or two assumptions or parameters are changed at a time.

7.1 Back-Testing Exercise

In 2008-09, Canadian households were moderately stressed as a result of the recession that followed the financial crisis. For example, unemployment increased from 6.3 per cent in 2006 to a peak of 8.6 per cent in 2009. The results of back testing HRAM with CFM data from 2005, 2006, and 2007, and historical macrodata from 2006 to 2011 (see Appendix B, Table B.1) are shown in Figure 9. While HRAM can produce an increase in financial distress of a similar magnitude to this historical episode, this increase is delayed by a couple of quarters and subsides somewhat after peaking early on relative to the historical series. The delayed increase could reflect the fact that HRAM does not account for forward-looking behaviour, which might otherwise contribute to a certain extent of strategic default, while the early tapering might suggest the model does not incorporate enough forbearance for distressed households in a major downturn, as lenders try to limit outright foreclosures, which would contribute to a more sustained level of loans in arrears. But overall the exercise provides evidence of the validity of the main mechanism of the model, whereby shocks to the labour market are an important explanatory factor for household arrears.

Figure 9: Back-Testing Results



Caution is warranted in interpreting such an exercise, however. HRAM is generally intended as a stress-testing tool rather than as a model that forecasts results within the bounds of non-extraordinary conditions. Because the model is calibrated to match the low starting point of arrears, there is a scaling-down effect on the peak arrears results. Further development will improve on the model's capacity to explain cyclical movements in arrears levels.

7.2 Other Illustrative Scenarios

To illustrate HRAM's flexibility, we consider the model's response under the same three scenarios that appeared in Faruqui et al. (2012) in the Bank's June 2012 *Financial System Review*: a control scenario, a short-lived unemployment shock, and a persistent unemployment shock. A subset of these assumptions is highlighted in Table 5, and the top-left quadrant of Figure 10.

The control scenario represents a stable macroeconomic environment in which the unemployment rate and the duration of the period of unemployment are unchanged throughout the simulation period. A gradual increase in the overnight rate is assumed, with a somewhat higher pickup in the 5-year household borrowing rate, as household borrowing premiums converge to a more normal historical level.

In the other scenarios, unemployment increases by 3 percentage points, and the

Table 5: Assumptions for an Illustrative Stress-Test Scenario

	Debt and income (quarter-over-quarter annualized growth - per cent)						**Interest rates (bps)**						**Unemployment (per cent)**					
	Control		**Persistent shock**		**Short-lived shock**		**Control**		**Persistent shock**		**Short-lived shock**		**Control**		**Persistent shock**		**Short-lived shock**	
	Growth of total household credit	Growth of disposable income	Growth of total household credit	Growth of disposable income	Growth of total household credit	Growth of disposable income	Overnight rate*	Effective household borrowing rate	Overnight rate	Effective household borrowing rate	Overnight rate	Effective household borrowing rate	Rate	Average Complete Duration (weeks)	Rate	Average Complete Duration (weeks)	Rate	Average Complete Duration (weeks)
2012Q1	4.7	1.1	4.7	3.9	4.7	1.1	100	343	100	343	100	343	7.4	16.5	7.4	16.5	7.4	16.5
2012Q2	5.3	3.8	5.3	3.8	5.3	3.8	100	354	100	361	100	354	7.4	16.5	7.4	16.5	7.4	16.5
2012Q3	5.3	3.8	5.3	3.8	5.3	3.8	109	375	100	379	109	375	7.4	16.5	7.4	16.5	7.4	16.5
2012Q4	5.3	3.8	1.0	0.0	1.0	1.0	122	399	100	453	122	620	7.4	16.5	8.2	18.0	10.4	23.9
2013Q1	5.0	3.8	1.0	0.0	1.0	1.0	131	419	100	527	131	640	7.4	16.5	8.9	19.5	10.4	23.9
2013Q2	5.0	3.8	1.0	0.0	1.0	1.0	143	443	100	600	143	664	7.4	16.5	9.7	21.0	10.4	23.9
2013Q3	5.0	3.8	1.0	0.0	1.0	1.0	148	459	100	655	148	680	7.4	16.5	10.4	22.5	10.4	23.9
2013Q4	5.0	3.8	2.5	2.0	5.0	3.8	152	475	100	655	152	475	7.4	16.5	10.4	22.5	7.4	16.5
2014Q1	4.5	⋮	⋮	⋮	4.5	⋮	155	489	⋮	⋮	155	489	⋮	⋮	⋮	⋮	⋮	⋮
2014Q2	4.5	⋮	⋮	⋮	4.5	⋮	158	492	⋮	⋮	158	492	⋮	⋮	⋮	⋮	⋮	⋮
2014Q3	4.5	⋮	⋮	⋮	4.5	⋮	160	494	⋮	⋮	160	494	⋮	⋮	⋮	⋮	⋮	⋮
2014Q4	4.5	3.8	2.5	2.0	4.5	3.8	162	496	100	655	162	496	7.4	16.5	10.4	22.5	7.4	16.5

*Based on market expectations of the 1-week rate in late March, 2012.

average duration of unemployment rises by six weeks (similar to the usual assumptions in past FSR stress-testing exercises). Under the scenario with a short-lived unemployment shock, unemployment rises for only one year and then returns to control. In the persistent-shock scenarios, unemployment increases gradually but remains elevated (Table 5).

Under the persistent-shock scenario, the prices of stocks and mutual funds decline from their starting point by a cumulative total of 28 per cent and 20 per cent, respectively. Given that near-cash assets are left unaffected by the scenario's financial asset-price shock, the total average cumulative effect of a change in asset prices in this scenario is a decline of about 15 per cent in household financial wealth. Aggregate annual savings in all scenarios are kept at about 3 per cent of disposable labour income. The shock scenarios hold the policy rate constant to permit an assessment of the impact of these shocks on household vulnerability in the absence of mitigating policy actions. Nonetheless, the effective household borrowing rate increases in response to a rise in risk premiums of about 220 basis points.

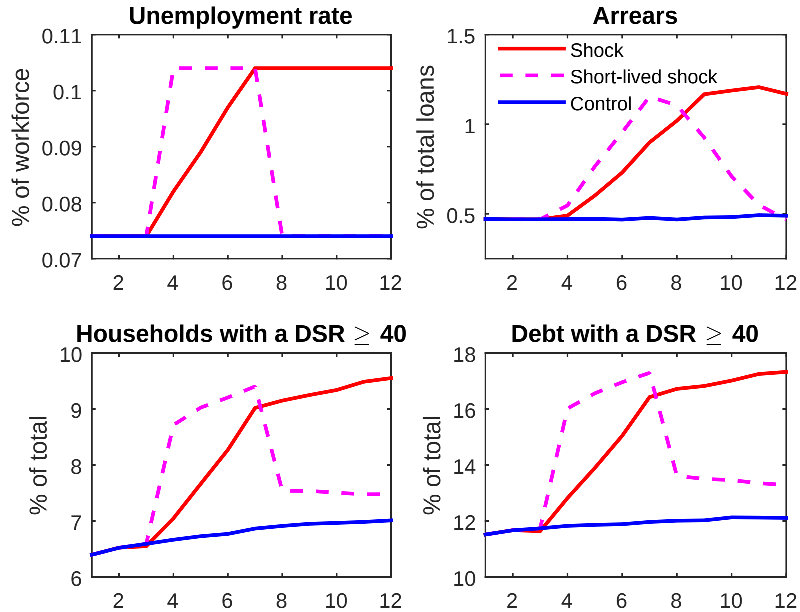
Under the control scenario, the share of households with a debt-service ratio greater than or equal to 40 per cent of gross income, as well as their share of total debt, increases slightly (Figure 10). Arrears increase only slightly, owing to the marginal change in vulnerabilities and stable unemployment.

Increases in unemployment lead to greater vulnerabilities and arrears. Although the short-lived unemployment shock leads to a sharp rise in the two vulnerability measures and in loans in arrears, most of these effects are temporary. These measures nonetheless return to a level that is somewhat higher than the control case as some households renew fixed-rate mortgages during the higher-rate period of the shock. For the persistent-shock scenario, both measures of household vulnerability rise to a higher level, as do loans in arrears.

Financial asset prices have a relatively small impact in the model, which can be explained by examining the balance sheets of households that go into arrears. These households typically start with low levels of financial assets that are generally in the form of less-risky investments and, therefore, are little affected by changes in equity prices. Conversely, households with high levels of risky assets also tend to have sizable amounts of cash or near-cash liquid financial assets (e.g., chequing and savings accounts or money market funds), which are left unaffected by the asset-price shock. Even a significant shock is unlikely to push such households into immediate distress. On the other hand, a relatively modest improvement in the levels of financial assets could help to prevent financial distress for the most-at-risk groups.

For a sensitivity analysis, we modify the unemployment and interest rate assump-

Figure 10: Results for an Illustrative Stress-Test Scenario



tions without making any further changes to the other assumptions in the control scenario. Table 6 summarizes the impact of these changes on the rate of loan arrears by the third year of the simulation. The effect on arrears is somewhat greater than the proportional change in unemployment. For example, a 2-percentage-point increase in unemployment corresponds to an 84 per cent increase in arrears, relative to the historical observation of 0.47 per cent at the start of the simulation. The multi-year unemployment shock leads to a cumulative financial strain on households, contributing to the greater-than-proportional effect.

The impact of a given increase in interest rates becomes more pronounced when it is combined with higher unemployment. In the most extreme example shown in Table 6 – where unemployment rises by 6 percentage points and household borrowing rates increase by 400 basis points – the rate of arrears almost quadruples, to a level of about 1.8 per cent (a 272 per cent increase from the starting point of 0.47 per cent). While credit growth would, in reality, slow down in response to the change in interest rates, we would still expect the increase in arrears to be significant, given the severity of the scenario.

Table 7 shows how loan arrears would increase if households had fewer available assets for making debt payments. The large increase in arrears as availability decreases from 20 per cent to 0 per cent illustrates that many households have a significant buffer of savings such that they would not go into arrears under normal

Table 6: Percentage Increase in Arrears as a Result of Changes in Unemployment and Interest Rates*

Unemployment (percentage points)	Interest rates (basis points) +0	+200	+400
+ 0	0	24	55
+ 2	84	120	173
+ 4	118	161	221
+ 6	153	202	272

*Measured as the average effect in the third year, relative to the starting-point arrears of 0.47 per cent in 2012Q1, with other assumptions taken from the control scenario.

Table 7: Percentage Increase in Arrears as a Result of Changes in Asset Availability*

Asset availability (%)	Increase in arrears (%) relative to an availability of 100%
100	0
80	11
60	24
40	47
20	91
0	282

*Measured as the average effect in the third year, relative to the starting-point arrears of 0.47 per cent in 2012Q1, with other assumptions taken from the control scenario.

circumstances, even when faced with a lengthy unemployment shock. In general, reductions in asset availability have an effect on arrears that increases non-linearly. When we consider that an important share of financial savings is held in less-risky asset classes (e.g., chequing and savings accounts), which would be unaffected by market shocks, Table 7 confirms that a financial asset-price shock would have to be significant to have a large effect on arrears outcomes.

7.3 Describing Households in Arrears

The starting-point characteristics of households that do go into arrears during a simulation, contrasted with those that do not, can reveal which factors pre-condition a household to a heightened risk of financial distress. Figure 11 (left panel) shows the average rate of arrears for the persistent-shock scenario in Table 5 according to a household's starting-point debt-service ratio (DSR). This rate steadily increases, up to about 6.5 per cent, cumulatively over three years, for households at a DSR

Table 8: Mean Characteristics of Indebted Households, by Arrears Status

	Debt-Service Ratio (%)	Income ($000s)	Debt ($000s)	Age	Two-Income Household (%)	Home Ownership Rate (%)	Unem–ployment Rate (%)
No Arrears	16.4	88.3	120.7	43.29	58	77	7.3
Arrears	44.6	62.0	175.0	41.21	57	87	21.1

threshold of 40 per cent. Beyond this point, owing to the scarcity of households in this part of the DSR distribution, and possibly other atypical characteristics of these households, the rate of arrears generally increases, but not consistently. In the right panel, the rate of arrears increases when a household starts with lower financial savings, measured in the number of weeks that the household can cover their debt payments solely through recourse to these savings. The thresholds of one month and four months of coverage broadly capture these increased levels of risk.

Figure 12 reflects the same pattern. The starting-point DSR distribution for those households that eventually go into arrears has a higher mean, and is more skewed to the right. In addition, a greater concentration of households that go into arrears started with particularly low levels of financial asset coverage.

Table 8 shows a similar rate of households having dual incomes, among indebted households that experience arrears versus those that do not. The model also has a higher rate of home-ownership among those that go into arrears, at 87 per cent versus 77 per cent. Although the comparison is not entirely suitable, this result contrasts with Allen and Damar (2011), who find that 79 per cent of individuals filing for bankruptcy and 61 per cent filing for restructuring were renters.[40] Households that go into arrears in the simulation tend to have lower incomes, higher debt, higher debt-service ratios, and younger heads. In the simulation, households that go into arrears had an overall unemployment rate of 21.1 per cent (and 100 per cent at the time of entering into arrears), whereas Allen and Damar find that 16 to 19 per cent of households that filed for bankruptcy were unemployed (though it seems possible that an additional share of such households had experienced some other loss of income apart from complete unemployment). Households that filed for debt restructuring were close to the national average of the unemployment rate, of between 7 and 8 per cent. Notwithstanding the significant difference between arrears and bankruptcy or insolvency, the comparison suggests that the model's job-loss mechanism does not yield a complete picture of the dynamics leading to arrears.

[40] One key aspect of mortgage arrears is that mortgages are collateralized by property, which would give homeowners in arrears more options for avoiding bankruptcy or insolvency, either through a house sale or as collateral for a line of credit.

Figure 11: Household Arrears Rate, by Starting-Point Vulnerabilities

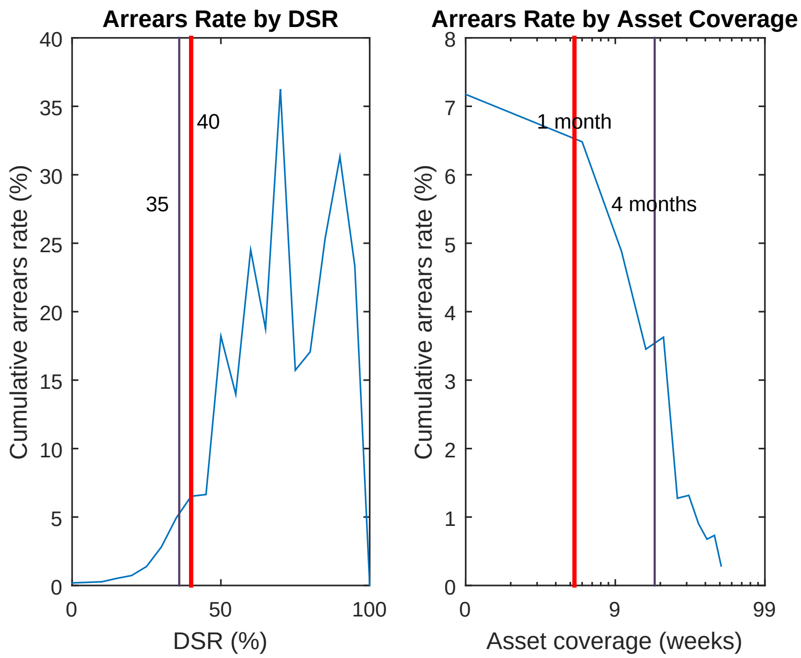


Figure 12: Distributions for Starting-Point Vulnerabilities

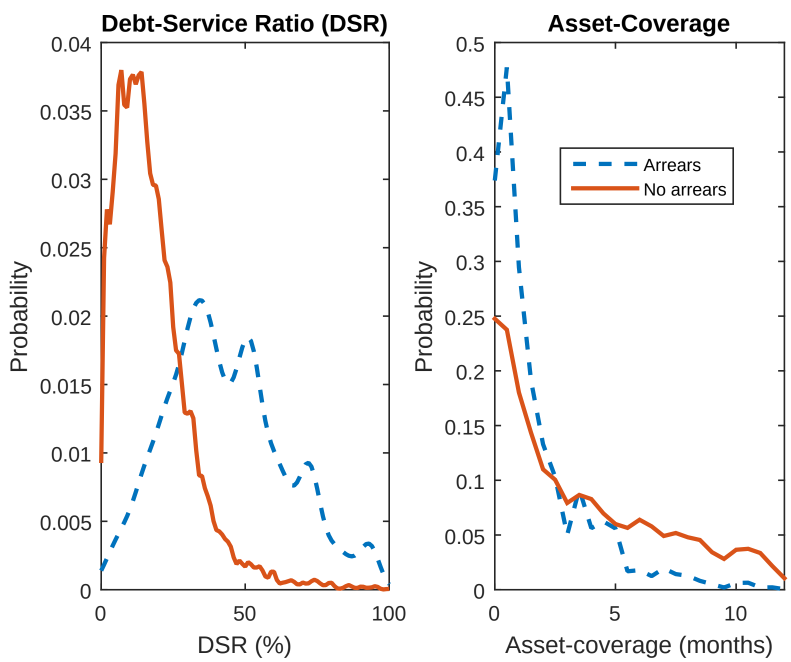


Table 9: Mean Characteristics of Indebted Households, by Region

	Debt-Service Ratio (%)	Income ($000s)	Debt ($000s)	Age	Home Ownership Rate (%)	Unem–ployment Rate (%)	Arrears Rate* (Cumulative, %)
BC	17.8	87.2	149.1	44.0	75	7.3	2.12
AB	18.5	106.2	156.9	42.4	80	5.5	1.98
SK	16.6	88.5	116.4	39.4	77	7.7	1.76
MB	14.7	89.3	97.8	44.1	78	5.3	1.75
ON	17.1	91.1	129.3	44.2	80	6.9	1.48
QC	15.2	77.7	90.1	42.3	72	8.4	2.78
Atlantic	16.5	77.6	90.7	43.1	82	11.3	2.23
Canada	16.7	88.0	121.4	43.3	77	7.5	1.95

* Rather than being a stock measure, this is measured as the cumulative rate at which debt has been in arrears at some point over the entire three-year simulation.

Simulation results can also be disaggregated by region. The resulting province- or region-level unemployment rate for the control scenario in Table 9 shows that a logit estimation of layoff probabilities can do a reasonable job of reflecting some observed regional patterns of unemployment.[41] For example, the Atlantic region has a substantially higher unemployment rate than the national average and, as expected, the model predicts a rate of arrears inflow that is higher than the national average.

Additional issues could have added relevance in a regional comparison. For example, regional conditions could affect the ability to deleverage when encountering financial distress. Housing equity, or lack thereof, could play a role in influencing household behaviour with respect to jobless spells. Households that know themselves to be in a precarious job situation could be less likely to take on large amounts of debt. And unemployment composition, in terms of quits versus layoffs, or short-term versus long-term unemployment, could vary by province.

[41] Discrepancies between simulation results and observed regional unemployment rates will arise from sampling variability in the CFM, as well as any changes in unemployment dynamics over time, relative to the model estimation of Chan, Morissette, and Frenette (2011), in addition to other factors.

8 Conclusions

Stress tests using microdata are a significant component in the assessment of the financial stability risk related to household balance sheets. While aggregate measures can describe important sectoral trends, it is at the micro level that we can better assess the potential change in loan arrears under an adverse-shock scenario.

Further work is planned to strengthen HRAM's empirical foundation. The modelling of household-specific income and credit growth will be refined to better account for the empirical determinants of these dynamics. In addition, the behavioural detail in the model could be extended further. For example, the simulation does not currently allow distressed households to sell their houses. If they are allowed to do so, they can potentially avoid default, leading to lower arrears. This extension would need to consider how the feasibility of this option would change in a severe housing-market downturn with slower market turnover and falling house prices.

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Appendix A List of Variables

In this subsection, we provide a list of the variables in the model: fixed household characteristics (Ω_i), household variables ($X_{i,t}$), household idiosyncratic shocks ($\varepsilon_{i,t}$), the macro scenario (Z_t) and the consistency factors (C_t). We further split the analysis into two large groups of variables:

(i) variables related to employment and income, and

(ii) variables related to the household balance sheet.

A.1 Household Fixed Characteristics (Ω_i)

Variables related to the assignment of relative layoff risk:

- ω_i^{REG}: the region in which a household lives.
- $\omega_i^{F} \in \{0,1\}$: equal to one if the head of household is female.
- $\omega_i^{UNIV} \in \{0,1\}$: equal to one if the head of household has a university degree.
- ω_i^{OCPPN}: categorical variable for the head of household's job occupation. Categories include:
  - Primary industries and construction
  - Manufacturing
  - Retail trade, accommodation, and food services
  - High-skill services
  - Public services
  - Other service-producing industries
- ω_i^{AGE} the age of household i. Though clearly not "fixed" in a literal sense, for the relatively short period of a simulation, it is currently treated as such.
- ω_i^{TENURE}: categorical variable for the job tenure of a head of household. Since this information does not exist in the CFM microdata, it is proxied by age:

$$\omega_i^{TENURE} = \begin{cases} \text{24 months or less} & \text{if } \omega_i^{AGE} < 27 \\ \text{25 months to 60 months} & \text{if } \omega_i^{AGE} \geq 27 \text{ and } \omega_i^{AGE} < 32 \\ \text{More than 60 months} & \text{if } \omega_i^{AGE} \geq 32 \end{cases}$$

Variables related to employment and income:

- ω_i^{YQ}: the **income quintile** of household i.

Variables related to the household balance sheet

- ω_i^{PPCD}: the size of a **principal payment on consumer debt** for household i as a percentage of total consumer debt for household i.
- ω_i^{PPMD}: the size of a **principal payment on mortgage debt** for household i as a percentage of total mortgage debt for household i.
- ω_i^{RPCD}: the household-specific **risk premium on consumer debt** on consumer debt for household i.
- ω_i^{RPMD}: the household-specific **risk premium on consumer debt** on mortgage debt for household i.
- ω_i^{MPC}: the marginal propensity to consume out of income for household i.

A.2 Household Variables ($X_{i,t}$)

Variables related to employment and income:

- $X_{i,t}^{U} \in \{0, 1\}$: whether a household is **unemployed** in period t, with $X_{i,t}^{U} = 1$ denoting that a household is unemployed.
- $X_{i,t}^{UD}$: the remaining **unemployment duration**, in number of periods, that a household unemployed in period t will remain unemployed in future periods.
- $X_{i,t}^{PLY}$: the **permanent labour income** of household i in period t.
- $X_{i,t}^{DLY}$: the **disposable labour income** of household i in period t, which is a function of a household's permanent labour income, and whether a household is unemployed:

$$X_{i,t}^{DLY} = (1 - \tau) \left(X_{i,t}^{PLY} - (1 - b) X_{i,t}^{U} X_{i,t}^{PLY} \right),$$

  where b denotes the percentage of permanent labour income that a household receives while unemployed and τ denotes the tax rate on labour income.

Variables related to the household balance sheet

- $X_{i,t}^{FA}$: the **financial assets** of household i at the end of period t.

- $X_{i,t}^{HA}$: the **housing assets** of household i at the end of period t.
- $X_{i,t}^{A}$: the **total assets** of household i at the end of period t, given by

$$X_{i,t}^{A} = X_{i,t}^{FA} + X_{i,t}^{HA}.$$

- $X_{i,t}^{CD}$: the remaining balance on **consumer debt** at the end of period t for household i.
- $X_{i,t}^{MD}$: the remaining balance on **mortgage debt** at the end of period t for household i.
- $X_{i,t}^{D}$: the remaining balance on **total debt** at the end of period t for household i, given by

$$X_{i,t}^{D} = X_{i,t}^{CD} + X_{i,t}^{MD}.$$

- $X_{i,t}^{HEQ}$: the **housing equity** of household i at the end of period t, given by

$$X_{i,t}^{HEQ} = X_{i,t}^{HA} - X_{i,t}^{MD}.$$

- $X_{i,t}^{NW}$: the **net worth** of household i at the end of period t, given by

$$X_{i,t}^{NW} = X_{i,t}^{A} - X_{i,t}^{D}.$$

- $X_{i,t}^{DP}$: the **debt payments** of household i in period t, given by

$$X_{i,t}^{DP} = \left(\omega^{PPCD} + X_{i,t}^{CDRATE}\right) X_{i,t-1}^{CD} + \left(\omega^{PPMD} + X_{i,t}^{MDRATE}\right) X_{i,t-1}^{CD}.$$

- $X_{i,t}^{DSR}$: the **debt-service ratio** for household i in period t, given by

$$X_{i,t}^{DSR} = \frac{X_{i,t}^{DP}}{X_{i,t}^{PLY}}.$$

- $X_{i,t}^{MTERM}$: the **original term of a mortgage**. Can be either one, two or three years.
- $X_{i,t}^{REMMTERM}$: the **remaining term of a mortgage**.
- $X_{i,t}^{MVAR}$: whether a household has a variable-rate mortgage, $X_{i,t}^{MVAR} = 1$, or a fixed-rate mortgage, $X_{i,t}^{MVAR} = 0$.

- $X_{i,t}^{C}$: **consumption** of household i in period t.
- $X_{i,t}^{ARR}$: the number of consecutive periods that household i has been in **arrears** on their debt at the end of period t.
- $X_{i,t}^{HPMAX,DP}$: the maximum house value a household can purchase, given a household's financial assets and the down payment constraint.
- $X_{i,t}^{MPAYMAX,DSR}$: the **largest monthly mortgage payment** a household can make, given a household's income.
- $X_{i,t}^{LOANMAX,DSR}$: the **maximum loan a household** can purchase, given mortgage regulation on amortization, posted mortgage rates, and the household's largest monthly mortgage payment.
- $X_{i,t}^{HPMAX,DSR}$: the maximum house value a household can purchase given a household's financial assets and the loan constraint from the debt-service ratio.
- $X_{i,t}^{FTHB}$: whether a household is a first-time homebuyer in period t, $X^{FTHB} = 1$, zero otherwise.
- $X_{i,t}^{ARR}$: whether a household is in **arrears** in period t, $X_{i,t}^{ARR} = 1$.
- $X_{i,t}^{CONARR}$: the consecutive number of periods that a household has been in arrears in its debt payments.

A.3 Household Idiosyncratic Shocks ($\varepsilon_{i,t}$)

Shocks related to employment and income:

- $\varepsilon_{i,t}^{U} \in \{0, 1\}$: **unemployment shock**.
- $\varepsilon_{i,t}^{UD}$: **unemployment duration shock**.
- $\varepsilon_{i,t}^{PLY}$: the log of a **permanent labour-income shock**.

Shocks related to the household balance sheet:

- $\varepsilon_{i,t}^{CD}$: **consumer debt shock**.
- $\varepsilon_{i,t}^{MD}$: **mortgage debt shock**.
- $\varepsilon_{i,t}^{FTHB}$: **first-time homebuyer shock**.
- $\varepsilon_{i,t}^{SAV}$: **savings shock**.

A.4 Macro Scenario (Z_t)

Macro-scenario variables related to employment and income:

- Z_t^U: the **unemployment rate** in period t.
- Z_t^{UD}: the mean **unemployment duration** of a household that becomes unemployed in period t.
- Z_t^{LYG}: the **growth rate of aggregate labour income**.

Macro-scenario variables related to the household balance sheet:

- $Z_t^{RFSHORT}$: the **risk-free rate on consumer debt** in period t.
- $Z_t^{R-5YRDISC}$: the **discounted rate on a 5-year fixed-rate mortgage** in period t.
- $Z_t^{R-3YRDISC}$: the **discounted rate on a 3-year fixed-rate mortgage** in period t.
- $Z_t^{R-1YRDISC}$: the **discounted rate on a 1-year fixed-rate mortgage** in period t.
- Z_t^{RFA}: the **return on financial assets** in period t.
- $Z_t^{HPSTARTER}\left(\omega_i^{REG}\right)$: the value of a starter home by region in period t.
- Z_t^{HPG}: the **growth rate in house prices** in period t.
- Z_t^{CDG}: the **growth rate of consumer debt**.
- Z_t^{MDG}: the **growth rate of mortgage debt**.
- Z_t^{SAV}: the aggregate **savings rate**.

A.5 Consistency Factors (C_t)

Consistency factors related to employment and income:

- C_t^{LYG}: the **labour income growth consistency factor** at time t to ensure that aggregate labour income growth is consistent with Z_t^{LYG}.

Consistency factors related to the household balance sheet:

- C_t^{CDG} the **consumer debt growth consistency factor** at time t to ensure that aggregate consumer debt growth is consistent with Z_t^{CDG}.
- C_t^{MDG} the **mortgage debt growth consistency factor** at time t to ensure that aggregate mortgage debt growth is consistent with Z_t^{MDG}.
- C_t^{SAV} the **savings rate consistency factor** at time t to ensure that the aggregate savings rate is consistent with Z_t^{SAV}.

A.6 Other Parameters

- DSR_{max}: the maximum debt-service ratio, incorporating all debt, for a new mortgage.
- dp_{min}: the minimum down payment to purchase a home.
- β_{layoff}: parameters for assignment of layoff-risk probability.

Parameters for balance-sheet dynamics:

- α_{MD}^{CNS}: Constant in mortgage debt growth equation.
- α_{MD}^{YG}: Elasticity of mortgage debt with respect to income.
- α_{MD}^{R}: Elasticity of mortgage debt with respect to the mortgage interest rate.
- α_{MD}^{HP}: Elasticity of mortgage debt with respect to house prices.
- λ_{MD}: Coefficient for indicator of high debt-service ratio for a household.
- F^{MD}: Mortgage debt growth shock distribution.
- α_{CD}^{CNS}: Constant in consumer debt growth equation.
- α_{CD}^{YG}: Elasticity of consumer debt with respect to income.
- α_{CD}^{R}: Elasticity of consumer debt with respect to the consumer interest rate.
- α_{CD}^{HP}: Elasticity of consumer debt with respect to house prices.
- λ_{CD}: Coefficient for indicator of high debt-service ratio for a household.
- F^{CD}: Consumer debt growth shock distribution.

Appendix B Back-Test Scenario

Table B.1: Back-Test Scenario, 2006-2011

(Growth rates are Q/Q annualized)

	Total HH credit growth	Personal disposable income growth	Unemployment rate	Incomplete unemployment duration	Overnight rate	Stock market growth	Total bond return	Mutual fund growth	Liquid asset growth	Savings rate	Term premium, 5-year mortgages (bps)	Risk premium, 5-year mortgages (bps)
2006Q1	9.7	15.4	6.5	14.9	3.75	21.5	-1.4	12.3	0.5	4.2	29	99
2006Q2	10.0	0.3	6.2	14.9	4.25	-7.1	-2.9	-7.5	0.5	2.9	16	88
2006Q3	9.8	7.0	6.4	14.0	4.25	16.0	19.7	13.8	0.5	3.3	-20	137
2006Q4	10.0	5.6	6.2	14.5	4.25	34.0	3.5	22.0	0.5	3.7	-29	123
2007Q1	10.7	10.0	6.2	14.1	4.25	1.8	4.1	0.3	0.5	4.1	-24	118
2007Q2	11.8	1.7	6.1	14.3	4.25	29.5	-7.2	16.3	0.5	2.5	17	102
2007Q3	13.0	6.0	6.0	13.7	4.5	9.1	2.8	4.9	0.5	2.8	-9	137
2007Q4	11.7	3.5	6.0	14.1	4.25	-12.9	6.2	-9.2	0.5	1.9	-21	182
2008Q1	12.7	14.0	6.0	13.6	3.5	-24.8	7.4	-17.0	0.5	4.1	-22	263
2008Q2	10.1	3.7	6.1	14.1	3	-0.3	-0.8	-2.4	0.5	3.9	27	208
2008Q3	9.2	0.6	6.1	13.3	3	-34.7	-8.0	-27.5	0.5	3	18	218
2008Q4	7.4	1.4	6.4	13.5	1.5	-62.3	2.1	-43.8	0.5	4.7	82	311
2009Q1	5.0	0.2	7.8	13.5	0.5	-31.3	11.8	-20.4	0.5	5.2	153	258
2009Q2	8.2	1.1	8.4	15.3	0.25	76.9	27.2	57.1	0.5	5	213	154
2009Q3	7.7	2.8	8.5	15.8	0.25	64.2	19.5	46.5	0.5	4.6	238	130
2009Q4	8.5	2.2	8.4	17.8	0.25	23.2	4.2	14.9	0.5	3.6	237	121
2010Q1	6.2	5.7	8.2	17.8	0.25	16.2	9.2	11.4	0.5	3.6	238	108
2010Q2	6.8	16.4	8.0	18.6	0.5	-32.2	8.9	-21.6	0.5	6.8	215	160
2010Q3	5.7	-5.3	8.1	18.0	1	47.5	12.5	33.4	0.5	4.3	118	177
2010Q4	5.7	6.9	7.7	19.2	1	39.9	-0.9	25.0	0.5	4.4	133	132
2011Q1	7.2	3.4	7.7	18.7	1	24.8	1.4	15.2	0.5	4.3	163	132
2011Q2	5.5	2.4	7.5	19.3	1	-6.8	9.2	-4.3	0.5	4.1	145	143
2011Q3	5.6	1.1	7.2	18.5	1	-42.7	14.9	-27.3	0.5	3.5	76	189
2011Q4	6.7	4.7	7.4	18.7	1	40.5	6.5	27.2	0.5	3.1	45	199