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by

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Abstract

We decompose the variance risk premium into upside and downside variance risk premia. These components reflect market compensation for changes in good and bad uncertainties. Their difference is a measure of the skewness risk premium (SRP), which captures asymmetric views on favorable versus undesirable risks. Empirically, we establish that the downside variance risk premium (DVRP) is the main component of the variance risk premium. We find a positive and significant link between the DVRP and the equity premium, and a negative and significant relation between the SRP and the equity premium. A simple equilibrium consumption-based asset pricing model supports our decomposition.

JEL classification: G, G1, G12
Bank classification: Asset pricing

Résumé

Nous décomposons la prime de risque de la variance en primes de risque à la hausse et à la baisse. Ces composantes reflètent la rémunération, par le marché, des risques liés aux variations de la « bonne » et de la « mauvaise » incertitude. La différence entre les deux représente une mesure de la prime de risque d'asymétrie, laquelle rend compte de l'asymétrie des opinions au sujet des risques favorables ou défavorables. Nous déterminons de façon empirique que la prime de risque de la variance à la baisse est le principal élément de la prime de risque de la variance. Nous constatons qu'il existe une relation positive significative entre la prime de risque de la variance à la baisse et la prime de risque sur actions, et une relation négative significative entre la prime de risque d'asymétrie et la prime de risque sur actions. Un modèle simple d'équilibre des actifs fondé sur la consommation étaye notre décomposition.

Classification JEL : G, G1, G12
Classification de la Banque : Évaluation des actifs

Non-Technical Summary

The proper assessment of risk is of paramount importance to investment decisions, given the basic trade-off between risk and reward. The variance risk premium (VRP) is a measure of risk compensation used by investors and policy-makers to gauge investors' sentiments on uncertainty. The VRP is the difference between the forward-looking market variance implied by option prices and the actual variance realized over time. Since option-implied (risk-neutral) variance is, on average, higher than realized (physical) variance, the seller of a variance swap contract, where the fixed leg is the former quantity and the floating leg is the latter, demands compensation from the buyer for taking on the position.

Yet, while the VRP can be a valuable tool to appraise the uncertainty around future variation (including extreme events), this measure does not take into consideration one important point: not all uncertainties are bad. Intuitively, investors like good uncertainty (since it increases the potential of substantial gains) but dislike bad uncertainty (since it increases the likelihood of severe losses). Thus, we believe that it is not enough to merely look at the VRP as a whole. To improve our understanding of the distribution of future stock returns, we need to further dissect this information and scrutinize both the upside and the downside pieces. Indeed, failure to decompose implies mixing two opposing views of the market and risks not getting the complete picture.

For example, the VRP was found to be lower than expected during the 2008 crisis period characterized by extreme economic and financial uncertainty, fueled by the subprime market meltdown. Interestingly, the proposed decomposition tells us that a small positive VRP does not necessarily mean that the market is less concerned with future uncertainty. Rather, it could reflect a small asymmetry in the market's assessment of good versus bad uncertainty. In other words, in highly uncertain times with large swings in returns, the magnitude of the positive-valued DVRP could be sizeable, yet just slightly higher than the absolute magnitude of its negative-valued upside counterpart. Overall, the VRP, which sums up the two components with opposing signs, will yield a small positive value.

Thus, building on this intuition, we dissect the VRP in terms of upside (UVRP) and downside (DVRP) variance risk premia. The DVRP is the main component of the VRP, and the most important to assess, since investors tend to hedge against downward movements to avoid losing money. Conversely, investors often gravitate toward upside movements and are willing to pay to get exposure to it and the potential for higher profits.

In the finance literature, this highlights the pivotal role of asymmetry in the assessment of risk (Chabi-Yo, 2008), and echoes the directional jump analysis of Bandi and Reno (2015) and jump-tail risk concerns in Bollerslev and Todorov (2011). Thus, the DVRP should be positive (reflecting the compensation required by an agent to bear the downside risk), whereas the UVRP should be negative (viewed as the discount given by an agent to secure a positive return on an investment).

An interesting byproduct of this decomposition is the skewness risk premium, or SRP (simply defined as SRP = UVRP-DVRP), which will be negatively valued by construction. Kozhan, Neuberger, and Schneider (2014) show that compensation for variance and skewness risks are tightly linked.

In addition, this work explores the link between the DVRP and the equity risk premium, or ERP. Current asset pricing research considers that, over shorter time horizons, the VRP provides superior forecasts for the ERP; these periods are less than a year, typically one quarter, ahead (Bollerslev, Tauchen, and Zhou, 2009). To further this exploration, the study also considers the link between the SRP and the ERP at various prediction steps.

We also rationalize these observations within a general equilibrium model by introducing some asymmetry into Bollerslev, Tauchen, and Zhou's (2009) specification, in the spirit of Segal, Shaliastovich, and Yaron (2015).

This knowledge would greatly benefit researchers in the fields of finance and financial econometrics. It is also a procedure that central banks and regulatory bodies monitoring the financial system can implement to accurately predict the ERP or gauge the level of risk aversion in the market. And what will these findings on dissecting the VRP mean for practitioners in the industry? Borrowing from a Wall Street adage, we believe that ''when the DVRP is high, it's time to buy, when the DVRP is low, it's time to go.'' Beyond its theoretical and empirical appeal, the proposed decomposition of the VRP offers an easy-to-compute risk assessment tool of real-life good and bad uncertainties, as perceived by investors.

1 Introduction

The proper assessment of risk is of paramount importance to investment decisions, given the fundamental risk and return trade-off. The variance risk premium (henceforth, VRP) formalized and studied by Bollerslev, Tauchen, and Zhou (2009) (henceforth, BTZ) as the difference between option-implied and realized variances is a measure of risk compensation that reflects investors' appraisal of changes in future volatility. We propose a new decomposition of the VRP in terms of upside and downside variance risk premia (VRP^U and VRP^D, respectively).[1] We subsequently uncover a common component shared by the VRP and the skewness risk premium (henceforth, SRP), which is the VRP^D. We show that this component is the basis for many empirical regularities in aggregate market returns uncovered by recent studies and document novel theoretical and empirical findings.

Intuitively, this work is motivated by a simple observation: Investors like good uncertainty as it increases the potential of substantial gains, but dislike bad uncertainty, as it increases the likelihood of severe losses.[2] Given that investors tend to hedge against downward movements to avoid losses, the VRP^D is expected to be generally positive-valued and the main driver of the VRP. Conversely, investors often find upside movements desirable. They are willing to pay for exposure to such movements and the potential for higher profits. Thus, we expect a mostly negative-valued VRP^U. Theoretically, we support our empirical findings with a simple endowment equilibrium asset pricing model, where the representative agent is endowed with Epstein and Zin (1989) preferences, and where the consumption growth process is affected by distinct upside and downside shocks. Our model shares some features with Bansal and Yaron (2004), BTZ, and Segal, Shaliastovich, and Yaron (2015), among others. The model-implied equity, upside variance, downside variance, and skewness risk premia (derived in closed form) support the empirical findings presented in the paper.

[1] We define the down(up)side variance as the realized variance of the stock market returns for negative (positive) returns. The down(up)side variance risk premium is the difference between option-implied and realized down(up)side variance. Decomposing variance in this way was pioneered by Barndorff-Nielsen, Kinnebrock, and Shephard (2010). We define the difference between upside and downside variances as the relative upside variance. Feunou, Jahan-Parvar, and Tédongap (2014) show that relative upside variance is a measure of skewness. Based on their work, we use the difference between option-implied and realized relative upside variances as a measure of the skewness risk premium.

[2] Similar to Segal, Shaliastovich, and Yaron (2015) and Bekaert, Engstrom, and Ermolov (2014), we define "good uncertainty" and "bad uncertainty" as volatility associated with positive or negative shocks to fundamentals such as consumption of dividend growth.

This study highlights the importance of asymmetry in the assessment of risk. As mentioned previously, we find VRP^D to be positive (reflecting the compensation required by an investor to bear the downside risk), whereas VRP^U is negative (as it is the discount given by an investor to secure exposure to such shocks). Thus, the (total) variance risk premium that sums these two components mixes together market participants' (asymmetric) views about good and bad uncertainties. As a result, a positive (total) variance risk premium reflects the fact that investors are willing to pay more in order to hedge against changes in bad uncertainty than for exposure to variations in good uncertainty. Hence, focusing on the (total) variance risk premium does not provide a realistic view of the trade-off between good and bad uncertainties, as a small positive VRP quantity does not necessarily imply a lower level of risk and/or risk aversion. Rather it is an indication of a smaller difference between what agents are willing to pay for downside variation hedging versus upside variation exposure. Together, these premia point to a negatively valued SRP measured through the difference between VRP^U and VRP^D. Empirically, we show that our decomposition of VRP characterizes the role of the SRP in asset pricing. We find that, on average, and similar to results in Kozhan, Neuberger, and Schneider (2014), more than 80% of the VRP is compensation for bearing changes in downside risk.

Our findings imply that, while the VRP^D explains the empirical regularities reported by BTZ (including the hump-shaped R^2 and slope parameter patterns), the VRP^U's contribution to the results reported by BTZ is, at best, marginal. Next, we document the contribution of the SRP to the predictability of returns that takes effect beyond the one-quarter-ahead window documented by BTZ. We find that the prediction power of VRP^D and SRP increases over the term structure of equity returns. In addition, through extensive robustness testing, we establish that our findings are robust to the inclusion of a wide variety of common equity risk premium predictors. This leads to the conclusion that the in-sample predictability of aggregate returns by downside risk and skewness measures introduced here is independent from other common pricing ratios, such as the price-dividend ratio, price-earnings ratio, or default spread. Thus, we are able to close the horizon gap between short-term models such as BTZ and long-horizon predictive models such as Fama and French (1988), Campbell and Shiller (1988), Cochrane (1991), and Lettau and Ludvigson (2001). Based on the revealed in-sample predictive power of our proposed measures, and in order to address data-mining concerns raised by Goyal and Welch (2008), we conduct out-of-sample forecast ability comparisons and show that, in comparison with VRP^D and SRP, other common predictors do not have a superior forecast ability.

1.1 Related literature

This paper is related to the mounting literature on the properties of the VRP, as discussed in early works by Bakshi and Kapadia (2003), Vilkov (2008), and Carr and Wu (2009), among others. Theoretical attempts to rationalize the observed dynamics of the VRP have led to both reduced-form and general equilibrium models in the literature. Within the reduced-form framework, Todorov (2010) focuses on the temporal dependence of continuous versus discontinuous VRP components driven by a semiparametric stochastic volatility model. He documents that both components exhibit nontrivial dynamics driven by *ex ante* volatility changes over time, coupled with *unanticipated* extreme swings in the market. In a general equilibrium setting, BTZ design a simple model in which time-varying volatility-of-volatility of consumption growth is the key determinant of the VRP. Drechsler and Yaron (2011) provide an equilibrium specification that features long-run risks and discontinuities in the stochastic volatility process governing the level of uncertainty about the cash-flow process. They extend the model of Bansal and Yaron (2004) by introducing a compound Poisson jump process in the state variable specification, thus departing from BTZ's assumption of Gaussian economic shocks. Our theoretical framework also extends BTZ's model, as we specify asymmetric predictable consumption growth components and differences of centered Gamma shocks to fundamentals.

Another strand of the literature explores the explanatory ability of the VRP. Along the time-series dimension, BTZ, Drechsler and Yaron (2011), and Kelly and Jiang (2014), among others, show that the VRP can help forecast the temporal variation in the aggregate stock market returns with high (low) premia predicting high (low) future returns, especially in within-the-year time scale. Ang, Hodrick, Xing, and Zhang (2006), and Cremers, Halling, and Weinbaum (2015), among others, find that the price of variance risk successfully explains a large set of expected stock returns in the cross-section of assets.

Drawing on existing decompositions of the quadratic variation of stock returns, other studies investigate the sources of variation in the VRP. Bollerslev and Todorov (2011) assess the importance of the premium related to extreme rare events by decomposing the VRP in terms of the diffusive and jump risk compensations. The authors show that the contribution of the jump tail risk premium is sizeable, and propose a new index to assess the compensation for concerns about disastrous outcomes – when returns (r) fall below a given threshold ($-\kappa$). Namely, they disentangle the VRP into components pertaining to smooth (continuous) moves ($VRP(|r| \leq \kappa)$) and rough (discontinuous) negative ($VRP(r < -\kappa)$) and positive ($VRP(r > \kappa)$) moves. In the Bollerslev and Todorov (2011) setting, however, κ is a *strictly* positive threshold that separates small (diffusion) from large (jump) variations. Because of the lack of liquidity for deep out-of-the money options, implementation of the jump tail risk premium approach for large values of κ is challenging. Actually, this procedure necessitates an additional extreme value theory (EVT) approximation step to extrapolate tail densities, especially under the risk-neutral measure. In comparison, our decomposition (based on a threshold of $\kappa = 0$) is much simpler to implement and interpret, as it does not require any explicit model for describing the tail behavior of the distributions underlying various premia.

Barndorff-Nielsen, Kinnebrock, and Shephard (2010) propose a different decomposition of the realized variance in terms of upside and downside semi-variances obtained by summing high-frequency positive and negative squared returns, respectively. Other authors have used the same decomposition of the realized variance with either a focus on realized variance predictability (Patton and Sheppard, 2015), or on equity risk premium predictability (Guo, Wang, and Zhou, 2015). All these papers focus exclusively on realized measures and do not use options prices to infer the risk-neutral counterparts and deduce the corresponding premia. In comparison, our work clearly evaluates the premia associated with upside and downside semi-variances, both realized and risk-neutral.

The SRP captures the wedge between the objective and the risk-neutral expectation of a realized skewness measure. Our realized measure of asymmetry is simply the difference between the upside and the downside semi-variance, which turns out to be the so-called *signed jump variation* introduced in Patton and Sheppard (2015). Patton and Sheppard (2015) demonstrate that *signed jumps* improve the prediction of future realized variance. An alternative approach to computing the skewness risk premium is to construct the premium from cubic swap contracts, as proposed by Kozhan, Neuberger, and Schneider (2014). In line with our results, Kozhan, Neuberger, and

Schneider (2014) find a strong link between the VRP and the SRP. We go further by showing that the SRP can help predict future stock market returns. More importantly, our construction of the SRP only requires the existence of the second moment of stock returns, while the Kozhan, Neuberger, and Schneider (2014) approach requires the existence of the third moment. In an earlier study, Neuberger (2012) studies the properties of realized measures of skewness. In comparison with Neuberger's realized skewness measure, our proposed skewness measure only depends on the existence of the first two moments. Moreover, Nueberger's realized skewness does not time-aggregate. In contrast, our proposed skewness measure time-aggregates well, since it depends on (semi-)variance measures.

In this paper, we substantiate our claim that the difference between the upside and downside VRP is a nonparametric measure of the SRP. Bollerslev and Todorov (2011) define (up to a sign) a similar metric, which they refer to as the *Investors' Fears Index* (defined as $FI(\kappa) \equiv VRP(r < -\kappa) - VRP(r > \kappa)$). Bollerslev, Todorov, and Xu (2015) show that the *Investors' Fears Index* can help predict future stock market returns. We conduct a similar analysis using our SRP measure. However, we believe that our approach provides a straightforward interpretation of the premium originating from the asymmetric market views on *good* versus *bad* risks and is much easier to implement.

Other papers aim at decomposing the variance of macroeconomic variables. Segal, Shaliastovich, and Yaron (2015) study the impact of changes in *good* versus *bad* uncertainty on aggregate consumption growth and asset values. These authors demonstrate that these different types of uncertainties have opposite effects, with *good* (*bad*) economic risk implying a rise (decline) in future consumption growth. They characterize the role of asymmetric uncertainties in the determination of the economic activity level, whereas we empirically document and theoretically derive equilibrium compensations for fluctuations in *good* and *bad* risks.

Our study comprises two natural and linked components. First, we study the inherent asymmetry in responses of market participants to negative and positive market outcomes. To accomplish this goal, we draw on the vast existing literature on realized and risk-neutral volatility measures and their properties to construct nonparametric measures of up and down realized and risk-neutral semi-variances. We then show empirically how the stylized facts documented in the VRP literature are driven almost entirely by the contribution of VRP^D. As in Chang, Christoffersen, and Jacobs (2013), our approach avoids the traditional trade-off problem with estimates of higher moments from historical returns data needing long windows to increase precision but short windows to obtain conditional – instead of unconditional – estimates. Second, we show that, using the relative upside variance, a nonparametric measure of skewness, we can enhance the predictive power of the variance risk premium to horizons beyond one quarter ahead.

Thus, we need reliable measures for realized and risk-neutral variance and skewness. A sizeable portion of empirical finance and financial econometrics literature is devoted to measures of volatility. Canonical papers focused on the properties and construction of realized volatility include Andersen, Bollerslev, Diebold, and Ebens (2001a); Andersen, Bollerslev, Diebold, and Labys (2001b); and Andersen, Bollerslev, Diebold, and Labys (2003), among others. The construction of realized downside and upside volatilities (also known as realized semi-variances) is addressed in Barndorff-Nielsen, Kinnebrock, and Shephard (2010). We follow the consensus in the literature about the construction of these measures. Similarly, based on pioneering studies such as Carr and Madan (1998, 1999, 2001) and Bakshi, Kapadia, and Madan (2003), we have a clear view on how to construct risk-neutral measures of volatility. The construction of option-implied downside and upside volatilities is addressed in Andersen, Bondarenko, and Gonzalez-Perez (2015). Again, we follow the existing literature in this respect.

Traditional measures of skewness have well-documented empirical problems. Kim and White (2004) demonstrate the limitations of estimating the traditional third moment. Harvey and Siddique (1999, 2000) explore time variation in conditional skewness by imposing autoregressive structures. More recently, Ghysels, Plazzi, and Valkanov (2011) use Bowley's skewness measures. They overcome many problems associated with the centered third moment, such as the excessive sensitivity to outliers documented in Kim and White (2004), by using an alternative and robust measure. Amaya, Christoffersen, Jacobs, and Vasquez (2015) and Chang, Christoffersen, and Jacobs (2013) use measures similar to Neuberger's 2012 realized skewness in predicting the cross-section of returns at a weekly frequency. In addition, as Dennis and Mayhew (2009) show, traditional option-based estimates of skewness are noisy and, as a result, unreliable. Our proposed risk-neutral skewness measure, in contrast, is well-behaved and easy to build and interpret.

Our study is also related to the recent macro-finance literature on the importance of higher-order risk attitudes, such as prudence – a precautionary behavior that characterizes the aversion towards downside risk – in the determination of equilibrium asset prices, as emphasized by Chabi-Yo (2008), among others. We refer to Eeckhoudt and Schlesinger (2008) and Chabi-Yo (2012) for a discussion on necessary and sufficient conditions for an increase in savings induced by changes in higher-order risk attitudes.[3]

The rest of the paper proceeds as follows. In Section 2, we present our decomposition of the VRP and the method for construction of risk-neutral and realized semi-variances, as well as the relative upside variance, which is our measure of skewness. In Section 3, we present a simple equilibrium consumption-based asset pricing model that supports our empirical results. Section 4 details the data used in this study and the empirical construction of predictive variables used in our analysis. We present and discuss our main empirical results in Section 5. Specifically, we intuitively describe the components of variance risk and skewness risk premia, discuss their predictive ability, explore their robustness, investigate their out-of-sample forecasting performance, and link them to macroeconomic factors and policy news in Sections 5.1, 5.2, 5.3, 5.4, and 5.5, respectively. Section 6 concludes.

2 Decomposition of the variance risk premium

In what follows, we decompose equity price changes into positive and negative returns with respect to a suitably chosen threshold. In this study, we set this threshold to zero, but it can assume other values, given the questions to be answered. We sequentially build measures for upside and downside variances and for skewness. When taken to data, these measures are constructed non-parametrically.

We posit that stock prices or equity market indices such as the S&P 500, S, are defined over the physical probability space characterized by $(\Omega, \mathbb{P}, \mathcal{F})$, where $\{\mathcal{F}_t\}_{t=0}^{\infty} \in \mathcal{F}$ are progressive filters on $\mathcal{F}$. The risk-neutral probability measure $\mathbb{Q}$ is related to the physical measure $\mathbb{P}$ through Girsanov's

[3]Dionne, Li, and Okou (2015) restate a standard consumption-based capital asset pricing model (using the concept of expectation dependence) to show that consumption second-degree expectation dependence risk – a proxy for downside risk that accounts for nearly 80% of the equity premium – is a fundamental source of the macroeconomic risk driving asset prices.

change of measure relation $\frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{F}_T} = Z_T, T < \infty$. At time t, we denote total equity returns as $R_t^e = (S_t + D_t - S_{t-1})/S_{t-1}$, where D_t is the dividend paid out in period $[t-1, t]$. In high-enough sampling frequencies, D_t is effectively equal to zero. Then, we denote the log of prices by $s_t = \ln S_t$, log-returns by $r_t = s_t - s_{t-1}$, and excess log-returns by $r_t^e = r_t - r_t^f$, where r_t^f is the risk-free rate observed at time $t-1$. We obtain cumulative excess returns by summing one-period excess returns, $r_{t\to t+k}^e = \sum_{j=0}^{k} r_{t+j}^e$, where k is our prediction/forecast horizon.

We build the variance risk premium components following the steps in BTZ as the difference between option-implied and realized variances. Alternatively, these two components could be viewed as variances under risk-neutral and physical measures, respectively. In our approach, this construction requires three distinct steps: building the upside and downside realized variances, computing their expectations under the physical measure, and then doing the same under the risk-neutral measure.

2.1 Construction of the realized variance components

Following Andersen et al. (2003, 2001a), we construct the realized variance of returns on any given trading day t as $RV_t = \sum_{j=1}^{n_t} r_{j,t}^2$, where $r_{j,t}^2$ is the j^{th} intraday squared log-return and n_t is the number of intraday returns recorded on that day. We add the squared overnight log-return (the difference in log price between when the market opens at t and when it closes at $t-1$), and we scale the RV_t series to ensure that the sample average realized variance equals the sample variance of daily log-returns. For a given threshold κ, we decompose the realized variance into upside and downside realized variances following Barndorff-Nielsen, Kinnebrock, and Shephard (2010):

$$RV_t^U(\kappa) = \sum_{j=1}^{n_t} r_{j,t}^2 \mathbb{I}_{[r_{j,t} > \kappa]}, \tag{1}$$

$$RV_t^D(\kappa) = \sum_{j=1}^{n_t} r_{j,t}^2 \mathbb{I}_{[r_{j,t} \leq \kappa]}. \tag{2}$$

We add the squared overnight "positive" log-return (exceeding the threshold κ) to the upside realized variance RV_t^U, and the squared overnight "negative" log-return (falling below the threshold κ) to the downside realized variance RV_t^D. Because the daily realized variance sums the upside and the downside realized variances, we apply the same scale to the two components of the realized variance. Specifically, we multiply both components by the ratio of the sample variance of daily log-returns over the sample average of the (pre-scaled) realized variance.

For a given horizon h, we obtain the cumulative realized quantities by summing the one-day realized quantities over h periods:

$$\begin{aligned}
RV^U_{t,h}(\kappa) &= \sum_{j=1}^{h} RV^U_{t+j}(\kappa), \\
RV^D_{t,h}(\kappa) &= \sum_{j=1}^{h} RV^D_{t+j}(\kappa), \\
RV_{t,h} &= \sum_{j=1}^{h} RV_{t+j}(\kappa). \qquad (3)
\end{aligned}$$

By construction, the cumulative realized variance adds up the cumulative realized upside and downside variances:

$$RV_{t,h} \equiv RV^U_{t,h}(\kappa) + RV^D_{t,h}(\kappa). \qquad (4)$$

2.2 Disentangling upside from downside variation: A theoretical overview

This section briefly reviews the main theoretical results that allow us to separate daily positive from negative quadratic variation using intraday data. In the sequel, the threshold κ is set to 0. We largely rely on Barndorff-Nielsen, Kinnebrock, and Shephard (2010), who assume that the stock price follows a jump-diffusion of the form

$$ds_t = \mu_t dt + \sigma_t dW_t + \Delta s_t,$$

where dW_t is an increment of standard Brownian motion and $\Delta s_t \equiv s_t - s_{t-}$ refers to the jump component. The instantaneous variance can be defined as $\widetilde{\sigma}^2_t = \sigma^2_t + (\Delta s_t)^2$. Under this general assumption on the instantaneous return process, Barndorff-Nielsen, Kinnebrock, and Shephard (2010) use infill asymptotics – asymptotics as the time distance between any two records shrinks toward 0 – to demonstrate that

$$
\begin{aligned}
RV_{t,h}^{U}(0) &\overset{p}{\to} \frac{1}{2}\int_{t}^{t+h} \sigma_{\upsilon}^{2} d\upsilon + \sum_{t\leq \upsilon \leq t+h} (\Delta s_{\upsilon})^{2}\, \mathbb{I}_{[\Delta s_{\upsilon}>0]}, \\
RV_{t,h}^{D}(0) &\overset{p}{\to} \frac{1}{2}\int_{t}^{t+h} \sigma_{\upsilon}^{2} d\upsilon + \sum_{t\leq \upsilon \leq t+h} (\Delta s_{\upsilon})^{2}\, \mathbb{I}_{[\Delta s_{\upsilon}\leq 0]}.
\end{aligned}
$$

Hence, $RV_{t,h}^{D}(0)$ and $RV_{t,h}^{U}(0)$ provide a new source of information, which focuses on squared negative and positive jumps, as pointed out by Patton and Sheppard (2015).

2.3 Construction of the variance risk premium components

Next, we characterize the VRP of BTZ through premia accrued to bearing upside and downside variance risks, following these steps:

$$
\begin{aligned}
VRP_{t,h} &= \mathbb{E}_{t}^{\mathbb{Q}}[RV_{t,h}] - \mathbb{E}_{t}^{\mathbb{P}}[RV_{t,h}], \\
&= \left(\mathbb{E}_{t}^{\mathbb{Q}}[RV_{t,h}^{U}(\kappa)] - \mathbb{E}_{t}^{\mathbb{P}}[RV_{t,h}^{U}(\kappa)]\right) + \left(\mathbb{E}_{t}^{\mathbb{Q}}[RV_{t,h}^{D}(\kappa)] - \mathbb{E}_{t}^{\mathbb{P}}[RV_{t,h}^{D}(\kappa)]\right), \\
VRP_{t,h} &\equiv VRP_{t,h}^{U}(\kappa) + VRP_{t,h}^{D}(\kappa). \qquad (5)
\end{aligned}
$$

Eq. (5) represents the decomposition of the VRP of BTZ into upside and downside variance risk premia – $VRP_{t,h}^{U}(\kappa)$ and $VRP_{t,h}^{D}(\kappa)$, respectively – that lies at the heart of our analysis.

2.3.1 Construction of $\mathbb{P}$-expectation

The goal here is to evaluate $\mathbb{E}_{t}^{\mathbb{P}}[RV_{t,h}^{U}(\kappa)]$ and $\mathbb{E}_{t}^{\mathbb{P}}[RV_{t,h}^{D}(\kappa)]$. To this end, we consider three specifications:

- Random Walk

  $$\mathbb{E}_{t}^{\mathbb{P}}[RV_{t,h}^{U/D}(\kappa)] = RV_{t-h,h}^{U/D}(\kappa),$$

  where U/D stands for "U or D"; this is the model used in BTZ.

- U/D-HAR

  $$\mathbb{E}_{t}^{\mathbb{P}}[RV_{t+1}^{U/D}(\kappa)] = \omega^{U/D} + \beta_{d}^{U/D} RV_{t}^{U/D}(\kappa) + \beta_{w}^{U/D} RV_{t,5}^{U/D}(\kappa) + \beta_{m}^{U/D} RV_{t,20}^{U/D}(\kappa).$$

- M-HAR

$$\mathbb{E}_t^{\mathbb{P}}[MRV_{t+1}(\kappa)] = \omega + \beta_d MRV_t(\kappa) + \beta_w MRV_{t,5}(\kappa) + \beta_m MRV_{t,20}(\kappa),$$

where $MRV_{t,h}(\kappa) \equiv (RV_{t,h}^U(\kappa), RV_{t,h}^D(\kappa))'$.

Both U/D-HAR and M-HAR specifications mimic Corsi (2009)'s HAR-RV model. To get genuine expected values for realized measures that are not contaminated by forward bias or the use of contemporaneous data, we perform an out-of-sample forecasting exercise to predict the three realized variances, at different horizons, corresponding to 1, 2, 3, 6, 9, 12, 18, and 24 months ahead. In our subsequent analysis, we find that these alternative specifications provide similar results, probably because of the persistence in volatility. Hence, for simplicity and to save space, we only report the results based on the random walk model.

2.3.2 Construction of $\mathbb{Q}$-expectation

To build the risk-neutral expectation of $RV_{t,h}$, we follow the methodology of Andersen and Bondarenko (2007):

$$\begin{aligned} \mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^U(\kappa)] &\approx \mathbb{E}_t^{\mathbb{Q}}\Big[\int_t^{t+h} \widetilde{\sigma}_\upsilon^2 \mathbb{I}_{[\ln(F_\upsilon/F_t)>\kappa_F]} d\upsilon\Big], \\ &= \mathbb{E}_t^{\mathbb{Q}}\Big[\int_t^{t+h} \widetilde{\sigma}_\upsilon^2 \mathbb{I}_{[F_\upsilon > F_t \exp(\kappa_F)]} d\upsilon\Big], \end{aligned}$$

where κ_F is a threshold used to compute risk-neutral expectations of semi-variances.[4] Thus,

$$\begin{aligned} \mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^U(\kappa)] &\approx 2\int_{F_t \exp(\kappa_F)}^{\infty} \frac{M_0(\underline{S})}{\underline{S}^2} d\underline{S}, \\ M_0(\underline{S}) &= \min(P_t(\underline{S}), C_t(\underline{S})), \end{aligned} \quad (6)$$

where, $P_t(\underline{S}), C_t(\underline{S})$, and $\underline{S}$ are prices of European put and call options (with maturity h), and the strike price of the underlying asset, respectively. F_t is the price of a future contract at time t,

[4] Note that κ_F should be set to $(\kappa - r_t^f)h$ to get consistent thresholds when computing realized and option-implied quantities.

defined as $F_t = S_t \exp(r_t^f h)$. Similarly for $\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^D(\kappa)]$, we get

$$\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^D(\kappa)] \approx 2 \int_0^{F_t \exp(\kappa_F)} \frac{M_0(\underline{S})}{\underline{S}^2} d\underline{S}. \tag{7}$$

We simplify our notation by renaming $\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^U(\kappa)]$ and $\mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^D(\kappa)]$ as

$$IV_{t,h}^U = \mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^U(\kappa)], \tag{8}$$

$$IV_{t,h}^D = \mathbb{E}_t^{\mathbb{Q}}[RV_{t,h}^D(\kappa)]. \tag{9}$$

We refer to $IV_{t,h}^{U/D}$ as the "risk-neutral semi-variance" or "implied semi-variance" of returns. These quantities are conditioned on the threshold value κ, which we suppress to simplify notation. As evident in this section, our measures of realized and implied volatility are model-free.

2.4 Construction of the skewness risk premium

The difference between realized upside and downside variance can be perceived as a measure of (realized) skewness. To build this measure of skewness, denoted as $RSV_{t,h}$, we simply subtract downside variance from upside semi-variance:

$$RSV_{t,h}(\kappa) = RV_{t,h}^U(\kappa) - RV_{t,h}^D(\kappa). \tag{10}$$

Thus, if $RSV_{t,h}(\kappa) < 0$ the distribution is left-skewed and when $RSV_{t,h}(\kappa) > 0$ it is right-skewed.

A theoretical justification for using $RSV_{t,h}$ as a measure of skewness can be found in Feunou, Jahan-Parvar, and Tédongap (2014). Here, we elaborate the intuition behind our findings. Relying on the theoretical results of section 2.2, we get

$$RSV_{t,h}(0) \xrightarrow{p} \sum_{t<v\leq t+h} (\Delta s_v)^2 \left(\mathbb{I}_{[\Delta s_v > 0]} - \mathbb{I}_{[\Delta s_v \leq 0]} \right).$$

Now, assuming that jump sizes are *i.i.d.* and uncorrelated with the jump occurrence, the expectation of the realized skewness – which can be referred to as the conditional skewness – is

$$\mathbb{E}_t^{\mathbb{P}}\left[RSV_{t,h}(0)\right] \approx \mathbb{E}^{\mathbb{P}}\left[(\Delta s)^2\right] \sum_{t<\upsilon\leq t+h} \left(\mathbb{P}[\Delta s_\upsilon > 0] - \mathbb{P}[\Delta s_\upsilon \leq 0]\right).$$

Thus, $\mathbb{E}_t^{\mathbb{P}}\left[RSV_{t,h}(0)\right]$ captures – up to a multiplicative constant – the difference between positive and negative jump intensities. In other words, this realized skewness measure reflects the relative occurrence of positive versus negative jumps. The occurrences of directional jumps receive different weights according their sizes, which echoes the volatility jump risk analysis of Bandi and Renò (2015). Intuitively, the larger the jump size, the bigger its weight in the computation of $RSV_{t,h}(0)$.

In addition, we construct a measure of skewness risk premium (SRP), which closely resembles the variance risk premium. The SRP is defined as the difference between risk-neutral and objective expectations of the realized skewness. It can be shown that this measure of the skewness risk premium is the spread between the upside and downside components of the variance risk premium:

$$\begin{aligned} SRP_{t,h} &= \mathbb{E}_t^{\mathbb{Q}}[RSV_{t,h}] - \mathbb{E}_t^{\mathbb{P}}[RSV_{t,h}], \\ SRP_{t,h} &= VRP_{t,h}^U(\kappa) - VRP_{t,h}^D(\kappa). \end{aligned} \tag{11}$$

If $RSV_{t,h} < 0$, we view $SRP_{t,h}$ as a skewness premium – the compensation for an agent who bears downside risk. Alternatively, if $RSV_{t,h} > 0$, we view $SRP_{t,h}$ as a skewness discount – the amount that the agent is willing to pay to secure a positive return on an investment. Since this measure of the skewness risk premium is nonparametric and model-free, it is easier to implement and interpret than competing parametric or semiparametric counterparts.

3 A simple equilibrium model

We provide an equilibrium consumption-based asset pricing model that supports the proposed decomposition of the VRP in terms of upside and downside components. Our main objective is to highlight the roles that upside and downside variances play in pricing a risky asset in an otherwise standard asset pricing model. In particular, we show that the model, under standard and mild assumptions, yields closed-form solutions for VRP components and SRP that align well with empirical regularities. To save space, we only report the main results. An online appendix reports our derivations in great detail.

The intuition behind our asset pricing model is simple. We interpret the VRP as the premium a market participant is willing to pay to hedge against variation in future realized variances. It is expected to be positive, since risk-averse investors dislike large swings in volatility, especially in "bad times" – when returns turn negative. This intuition is rationalized within the general equilibrium model of BTZ, where it is shown that the variance risk premium is, in general, positive and proportional to the volatility-of-volatility.

That said, several studies, including Segal, Shaliastovich, and Yaron (2015) and Feunou, Jahan-Parvar, and Tédongap (2013), show that there are distinct good and bad uncertainties. On the one hand, market participants like good uncertainty (when returns are positive), as it signals the potential for earning higher gains. In other words, risk-averse agents like upside variations and are willing to pay to be exposed to fluctuations in the upside variance. This argument should induce a negative expected value for VRP^U. On the other hand, investors dislike bad uncertainty (when returns are negative), as it increases the likelihood of large losses. Since risk-averse agents dislike downside variations, they are willing to pay a premium to hedge against fluctuations in future downside variances. Therefore, VRP^D is expected to be positive most of the time. Empirical evidence – for example, results reported by Guo, Wang, and Zhou (2015) – support our intuition.

Upside and downside variance risk premia tend to have opposite signs. Thus, the (total) variance risk premium that sums these two components mingles market participants' asymmetric views about good and bad uncertainties. It follows that a positive (total) variance risk premium reflects investors' willingness to pay more in order to hedge against changes in bad uncertainty than for exposure to variations in good uncertainty. In other words, investors are more concerned with exposure to downside risk (and hence losses) than to upside risk (and potential large gains). Hence, focusing on the (total) variance risk premium does not provide a crisp vision of the trade-off between good and bad uncertainties.

Building on the same intuition, the sign of the SRP stems from the expected behavior of the two components of the VRP. The SRP is obtained by subtracting VRP^D from VRP^U. Given that (on average) VRP^U appears negative, whereas VRP^D tends to be positive, the SRP is expected to be negative.

In summary, we expect negative-valued VRP^U and SRP, and positive-valued VRP and VRP^D. In what follows, we show that our simple equilibrium asset pricing model delivers the expected signs for these premia.

3.1 Preferences

We consider an endowment economy in discrete time. The representative agent's preferences over the future consumption stream are characterized by Kreps and Porteus (1978) intertemporal preferences, as formulated by Epstein and Zin (1989) and Weil (1989)

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{12}$$

where C_t is the consumption bundle at time t, δ is the subjective discount factor, γ is the coefficient of risk aversion, and ψ is the elasticity of intertemporal substitution (IES). Parameter θ is defined as $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. If $\theta = 1$, then $\gamma = 1/\psi$ and Kreps and Porteus preferences collapse to expected power utility, which implies an agent who is indifferent to the timing of the resolution of the uncertainty of the consumption path. With $\gamma > 1/\psi$, the agent prefers early resolution of uncertainty. For $\gamma < 1/\psi$, the agent prefers late resolution of uncertainty. Epstein and Zin (1989) show that the logarithm of the stochastic discount factor (SDF) implied by these preferences is given by

$$\ln M_{t+1} = m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{13}$$

where $\Delta c_{t+1} = \ln\left(\frac{C_{t+1}}{C_t}\right)$ is the log growth rate of aggregate consumption, and $r_{c,t}$ is the log return of the asset that delivers aggregate consumption as dividends. This asset represents the returns on a wealth portfolio. The Euler equation states that

$$\mathbb{E}_t\left[\exp\left(m_{t+1} + r_{i,t+1}\right)\right] = 1, \tag{14}$$

where $r_{c,t}$ represents the log returns for the consumption-generating asset ($r_{c,t}$). The risk-free rate, which represents the returns on an asset that delivers a unit of consumption in the next period with certainty, is defined as

$$r_t^f = \ln\left[\frac{1}{\mathbb{E}_t(M_{t+1})}\right]. \tag{15}$$

3.2 Consumption dynamics under the physical measure

Our specification of consumption dynamics incorporates elements from Bansal and Yaron (2004), Bekaert, Engstrom, and Ermolov (2014), and especially BTZ and Segal, Shaliastovich, and Yaron (2015).

Fundamentally, we follow Bansal and Yaron (2004) in assuming that consumption growth has a predictable component. We differ from Bansal and Yaron in assuming that the predictable component is proportional to consumption growth's upside and downside volatility components. Thus, we are closer to Segal, Shaliastovich, and Yaron (2015) who maintain this structure in their formulation of the long-run risk component. As a result, we have

$$\Delta c_{t+1} = \mu_0 + \mu_1 V_{u,t} + \mu_2 V_{d,t} + \sigma_c \left(\varepsilon_{u,t+1} - \varepsilon_{d,t+1}\right), \tag{16}$$

where $\mu_1, \mu_2 \in \mathbb{R}$, $\varepsilon_{u,t+1}$ and $\varepsilon_{d,t+1}$ are two mean-zero shocks that affect both the realized and expected consumption growth.[5] $\varepsilon_{u,t+1}$ represents upside shocks to consumption growth, and $\varepsilon_{d,t+1}$ stands for downside shocks. Following Bekaert, Engstrom, and Ermolov (2014) and Segal, Shaliastovich, and Yaron (2015), we assume that these shocks follow a demeaned Gamma distribution and model them as

$$\varepsilon_{i,t+1} = \tilde{\varepsilon}_{i,t+1} - V_{i,t},\ i = \{u, d\}, \tag{17}$$

where $\tilde{\varepsilon}_{i,t+1} \sim \Gamma(V_{i,t}, 1)$. These distributional assumptions imply that volatilities of upside and downside shocks are time-varying and driven by shape parameters $V_{u,t}$ and $V_{d,t}$. In particular, we have that

$$Var_t[\varepsilon_{i,t+1}] = V_{i,t},\ i = \{u, d\}. \tag{18}$$

Naturally, the total conditional variance of consumption growth when $\varepsilon_{u,t+1}$ and $\varepsilon_{d,t+1}$ are condi-

[5]This assumption is for the sake of brevity. Violating this assumption adds to algebraic complexity but does not affect our analytical findings.

tionally independent is simply $\sigma_c^2 \left(V_{u,t} + V_{d,t}\right)$.

As a result the sign and size of μ_1 and μ_2 matter in this context. With $\mu_1 = \mu_2$, we have a stochastic volatility component in the conditional mean of the consumption growth process, similar to the classic GARCH-in-Mean structure for modeling risk-return trade-off in equity returns. With both slope parameters equal to zero, the model yields the BTZ unpredictable consumption growth.[6] If $|\mu_1| = |\mu_2|$, with $\mu_1 > 0$ and $\mu_2 < 0$, we have Skewness-in-Mean, similar in spirit to the Feunou, Jahan-Parvar, and Tédongap (2013) formulation for equity returns. With $\mu_1 \neq \mu_2$, we have free parameters that have an impact on loadings of risk factors on risky asset returns and the stochastic discount factor. Intuitively, we expect $\mu_1 > 0$: A rise in upside volatility at time t implies higher consumption growth at time $t+1$, all else being equal. By the same logic, we intuitively expect a negative-valued μ_2, implying an expected fall in consumption growth following an uptick in downside volatility – following bad economic outcomes, households curb their consumption. In what follows, we buttress our intuition with theory and derive the analytical bounds on these parameters that ensure consistency with our intuitions.

We observe that

$$\ln \mathbb{E}_t \exp\left(\nu \varepsilon_{i,t+1}\right) = f(\nu) V_{i,t}, \tag{19}$$

where $f(\nu) = -(\ln(1-\nu) + \nu)$. Both Bekaert, Engstrom, and Ermolov (2014) and Segal, Shaliastovich, and Yaron (2015) use this compact functional form for the Gamma distribution cumulant. It simply follows that $f(\nu) > 0, f''(\nu) > 0$, and $f(\nu) > f(-\nu)$ for all $\nu > 0$.

We assume that $V_{i,t}$ follows a time-varying, square root process with time-varying volatility-of-volatility, similar to the specification of the volatility process in Bollerslev, Tauchen, and Zhou

[6] A consumption-based asset pricing model with a representative agent endowed with Epstein and Zin (1989) preferences and an unpredictable consumption growth process does not support the existence of distinct upside and downside variance risk premia with the expected signs. In particular, we have found that such a setting always yields a positive upside variance risk premium.

(2009):

$$
\begin{aligned}
V_{u,t+1} &= \alpha_u + \beta_u V_{u,t} + \sqrt{q_{u,t}} z_{t+1}^u, && (20)\\
q_{u,t+1} &= \gamma_{u,0} + \gamma_{u,1} q_{u,t} + \varphi_u \sqrt{q_{u,t}} z_{t+1}^1, && (21)\\
V_{d,t+1} &= \alpha_d + \beta_d V_{d,t} + \sqrt{q_{d,t}} z_{t+1}^d, && (22)\\
q_{d,t+1} &= \gamma_{d,0} + \gamma_{d,1} q_{d,t} + \varphi_d \sqrt{q_{d,t}} z_{t+1}^2, && (23)
\end{aligned}
$$

where z_t^i are standard normal innovations, and $i = \{u, d, 1, 2\}$. The parameters must satisfy the following restrictions: $\alpha_u > 0, \alpha_d > 0, \gamma_{u,0} > 0, \gamma_{d,0} > 0, |\beta_u| < 1, |\beta_d| < 1, |\gamma_{u,1}| < 1, |\gamma_{d,1}| < 1, \varphi_u > 0, \varphi_d > 0$. In addition, we assume that $\{z_t^u\}, \{z_t^d\}, \{z_t^1\}$, and $\{z_t^2\}$ are $i.i.d. \sim N(0,1)$ and jointly independent from $\{\varepsilon_{u,t}\}$ and $\{\varepsilon_{d,t}\}$.

The assumptions above yield time-varying uncertainty and asymmetry in consumption growth. Through volatility-of-volatility processes $q_{u,t}$ and $q_{d,t}$, the setup induces additional temporal variation in consumption growth. Temporal variation in the volatility-of-volatility process is necessary for generating a sizeable variance risk premium. Asymmetry is needed to generate upside and downside variance risk premia, as we show in what follows.

We solve the model following the methodology proposed by Bansal and Yaron (2004), Bollerslev, Tauchen, and Zhou (2009), Segal, Shaliastovich, and Yaron (2015), and many others. We consider that the logarithm of the wealth-consumption ratio w_t or the price-consumption ratio ($pc_t = \ln\left(\frac{P_t}{C_t}\right)$) for the asset that pays the consumption endowment $\{C_{t+i}\}_{i=1}^{\infty}$ is affine with respect to state variables $V_{i,t}$ and $q_{i,t}$.

We then posit that the consumption-generating returns are approximately linear with respect to the log price-consumption ratio, as popularized by Campbell and Shiller (1988). That is,

$$
\begin{aligned}
r_{c,t+1} &= \kappa_0 + \kappa_1 w_{t+1} - w_t + \Delta c_{t+1},\\
w_t &= A_0 + A_1 V_{u,t} + A_2 V_{d,t} + A_3 q_{u,t} + A_4 q_{d,t},
\end{aligned}
$$

where κ_0 and κ_1 are log-linearization coefficients, and A_0, A_1, A_2, A_3, and A_4 are factor-loading coefficients to be determined. We solve for the consumption-generating asset returns, $r_{c,t}$, using the Euler equation (14). Following standard arguments, we find the equilibrium values of coefficients A_0 to A_4:

$$
\begin{aligned}
A_1 &= -\frac{f\Big[\sigma_c(1-\gamma)\Big] + (1-\gamma)\mu_1}{\theta(\kappa_1\beta_u - 1)}, && (24)\\
A_2 &= -\frac{f\Big[-\sigma_c(1-\gamma)\Big] + (1-\gamma)\mu_2}{\theta(\kappa_1\beta_d - 1)}, && (25)\\
A_3 &= \frac{(1-\kappa_1\gamma_{u,1}) - \sqrt{(1-\kappa_1\gamma_{u,1})^2 - \theta^2\varphi_u^2\kappa_1^4 A_1^2}}{\theta\kappa_1^2\varphi_u^2}, && (26)\\
A_4 &= \frac{(1-\kappa_1\gamma_{d,1}) - \sqrt{(1-\kappa_1\gamma_{d,1})^2 - \theta^2\varphi_d^2\kappa_1^4 A_2^2}}{\theta\kappa_1^2\varphi_d^2}, && (27)\\
A_0 &= \frac{\ln\delta + \big(1-\frac{1}{\psi}\big)\mu_0 + \kappa_0 + \kappa_1\big(\alpha_u A_1 + \alpha_d A_2 + \gamma_{u,0}A_3 + \gamma_{d,0}A_4\big)}{1-\kappa_1}. && (28)
\end{aligned}
$$

It is easy to see that while A_3 and A_4 are negative-valued, the signs of A_1 and A_2 depend on the signs and sizes of μ_1 and μ_2. We report the conditions that ensure $A_1 > 0$ and $A_2 < 0$ after introducing the dynamics of the model under the risk-neutral measure.

Standard algebraic manipulations yield the following representations for the conditional equity premium and innovations of the conditional equity premium:

$$
\begin{aligned}
r_{c,t+1} &= \ln\delta + \frac{\mu_0}{\psi} + \left[\frac{\mu_1}{\psi} - \frac{f\left[\sigma_c(1-\gamma)\right]}{\theta}\right] V_{u,t} + \left[\frac{\mu_2}{\psi} - \frac{f\left[-\sigma_c(1-\gamma)\right]}{\theta}\right] V_{d,t}\\
&\quad + \sigma_c(\varepsilon_{u,t+1} - \varepsilon_{d,t+1}) + (\kappa_1\gamma_{u,1} - 1)A_3 q_{u,t} + (\kappa_1\gamma_{d,1} - 1)A_4 q_{d,t} && (29)\\
&\quad + \kappa_1\left[\left(A_1 z^u_{t+1} + \varphi_u A_3 z^1_{t+1}\right)\sqrt{q_{u,t}} + \left(A_2 z^d_{t+1} + \varphi_d A_4 z^2_{t+1}\right)\sqrt{q_{d,t}}\right],\\
r_{c,t+1} - \mathbb{E}_t(r_{c,t+1}) &= \sigma_c(\varepsilon_{u,t+1} - \varepsilon_{d,t+1})\\
&\quad + \kappa_1\left[\left(A_1 z^u_{t+1} + \varphi_u A_3 z^1_{t+1}\right)\sqrt{q_{u,t}} + \left(A_2 z^d_{t+1} + \varphi_d A_4 z^2_{t+1}\right)\sqrt{q_{d,t}}\right]. && (30)
\end{aligned}
$$

It is immediately obvious that there is significant correspondence between our characterization of risky returns and equation (10) of BTZ. The differences are driven by the different distributional assumptions regarding consumption growth shocks and the fact that we model upside and downside uncertainty explicitly rather than targeting aggregate uncertainty, as in BTZ. Notice that $\left[-\frac{f[\sigma_c(1-\gamma)]}{\theta}\right] < \left[-\frac{f[-\sigma_c(1-\gamma)]}{\theta}\right]$, and both terms are positive-valued. Thus, the impact of $V_{u,t}$ and

$V_{d,t}$ on expected returns depends on μ_1 and μ_2. We subsequently provide a crisp characterization of the equity premium to complete the analysis.

Because of differences in distributional assumptions, we do not follow BTZ's or Bansal and Yaron's methods for deriving equity premium and various variance risk premia. The dynamics specified so far are all under the physical measure, $\mathbb{P}$. We need to compute the dynamics under the risk-neutral measure, $\mathbb{Q}$, to derive the formulae for upside and downside variance risk premia and skewness risk premium.

3.3 Risk-neutral dynamics and the premia

We derive the risk-neutral distribution of all the shocks, $\varepsilon_{u,t+1}$, $\varepsilon_{d,t+1}$, z^u_{t+1}, z^d_{t+1}, z^1_{t+1}, and z^2_{t+1}. Namely, we construct the characteristic functions of the shocks and exploit their salient properties to derive the expectations under the risk-neutral measure. Thus, our computations yield exact equity and risk premia measures, in contrast to the approximate values reported, for example, in equation (15) of Bollerslev, Tauchen, and Zhou (2009) or in Drechsler and Yaron (2011). Details of these derivations are available in the online Appendix.

The risk-neutral expectations of the upside and downside consumption shocks are

$$\begin{aligned}
\mathbb{E}^{\mathbb{Q}}_t\left[\varepsilon_{u,t+1}\right] &= f'(-\gamma\sigma_c)V_{u,t} = -\frac{\gamma\sigma_c}{1+\gamma\sigma_c}V_{u,t}, \\
\mathbb{E}^{\mathbb{Q}}_t\left[\varepsilon_{d,t+1}\right] &= f'(\gamma\sigma_c)V_{d,t} = \frac{\gamma\sigma_c}{1-\gamma\sigma_c}V_{d,t}.
\end{aligned}$$

Using a similar methodology, we characterize the risk-neutral distributions of Gaussian shocks z^u_{t+1}, z^d_{t+1}, z^1_{t+1}, and z^2_{t+1} as:

$$\begin{aligned}
z^u_{t+1} &\sim {}^{\mathbb{Q}}N\left((\theta-1)\kappa_1 A_1\sqrt{q_{u,t}}, 1\right) \\
z^d_{t+1} &\sim {}^{\mathbb{Q}}N\left((\theta-1)\kappa_1 A_2\sqrt{q_{d,t}}, 1\right) \\
z^1_{t+1} &\sim {}^{\mathbb{Q}}N\left((\theta-1)\kappa_1 A_3\varphi_u\sqrt{q_{u,t}}, 1\right) \\
z^2_{t+1} &\sim {}^{\mathbb{Q}}N\left((\theta-1)\kappa_1 A_4\varphi_d\sqrt{q_{d,t}}, 1\right).
\end{aligned}$$

Any premium – whether equity, variance risk, or skewness risk– can be defined as the difference between the physical and risk-neutral expectations of processes. Hence, we commence computing the premia of interest, starting with the equity risk premium:

$$\begin{aligned} ERP_t &\equiv \mathbb{E}_t\left[r_{c,t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[r_{c,t+1}\right] \\ &= \kappa_1 \left(\mathbb{E}_t\left[w_{t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[w_{t+1}\right]\right) + \mathbb{E}_t\left[\Delta c_{t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\Delta c_{t+1}\right]. \end{aligned} \tag{31}$$

It is clear from equation (31) that we need to compute both $\mathbb{E}_t\left[\Delta c_{t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\Delta c_{t+1}\right]$ and $\mathbb{E}_t\left[w_{t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[w_{t+1}\right]$. It can be shown that

$$\mathbb{E}_t\left[\Delta c_{t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\Delta c_{t+1}\right] = \gamma \sigma_c^2 \left(\frac{1}{1+\gamma\sigma_c} V_{u,t} + \frac{1}{1-\gamma\sigma_c} V_{d,t}\right).$$

Similarly,

$$\begin{aligned} \mathbb{E}_t\left[w_{t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[w_{t+1}\right] &= A_1 \left(\mathbb{E}_t\left[V_{u,t}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[V_{u,t}\right]\right) + A_2 \left(\mathbb{E}_t\left[V_{d,t}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[V_{d,t}\right]\right) \\ &\quad + A_3 \left(\mathbb{E}_t\left[q_{u,t}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[q_{u,t}\right]\right) + A_4 \left(\mathbb{E}_t\left[q_{d,t}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[q_{d,t}\right]\right). \end{aligned}$$

To compute $\mathbb{E}_t\left[w_{t+1}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[w_{t+1}\right]$, we need the premium for each risk factor ($V_{u,t}, V_{d,t}, q_{u,t}$ and $q_{d,t}$). Straightforward algebra yields

$$\begin{aligned} \mathbb{E}_t\left[V_{u,t}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[V_{u,t}\right] &= (1-\theta)\kappa_1 A_1 q_{u,t}, \\ \mathbb{E}_t\left[V_{d,t}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[V_{d,t}\right] &= (1-\theta)\kappa_1 A_2 q_{d,t}, \\ \mathbb{E}_t\left[q_{u,t}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[q_{u,t}\right] &= (1-\theta)\kappa_1 A_3 \varphi_u^2 q_{u,t}, \\ \mathbb{E}_t\left[q_{d,t}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[q_{d,t}\right] &= (1-\theta)\kappa_1 A_4 \varphi_d^2 q_{d,t}. \end{aligned}$$

Thus, it easily follows that the equity risk premium in our model is

$$ERP_t \equiv \frac{\gamma\sigma_c^2}{1+\gamma\sigma_c} V_{u,t} + \frac{\gamma\sigma_c^2}{1-\gamma\sigma_c} V_{d,t} + (1-\theta)\kappa_1^2 \left(A_1^2 + A_3^2\varphi_u^2\right) q_{u,t} + (1-\theta)\kappa_1^2 \left(A_2^2 + A_4^2\varphi_d^2\right) q_{d,t}. \tag{32}$$

This expression for the equity risk premium clearly shows that our model implies unequal loadings for upside and downside volatility factors. The slope coefficients for volatility-of-volatility factors are also, in general, unequal. We require that $\sigma_c < \frac{1}{\gamma}$ to maintain finite factor loadings.

We proceed and derive the closed form expressions for the upside and downside variance risk premia. From equation (30), we know that

$$\begin{aligned}
\sigma_{r,t}^2 &\equiv Var_t\left[r_{c,t+1}\right] \\
&= Var_t\left[\sigma_c(\varepsilon_{u,t+1} - \varepsilon_{d,t+1}) + \kappa_1\left[(A_1 z_{t+1}^u + \varphi_u A_3 z_{t+1}^1)\sqrt{q_{u,t}} + (A_2 z_{t+1}^d + \varphi_d A_4 z_{t+1}^2)\sqrt{q_{d,t}}\right]\right] \\
&= \sigma_c^2 V_{u,t} + \sigma_c^2 V_{d,t} + \kappa_1^2\left(A_1^2 + A_3^2\varphi_u^2\right) q_{u,t} + \kappa_1^2\left(A_2^2 + A_4^2\varphi_d^2\right) q_{d,t},
\end{aligned}$$

where upside and downside variances are defined as

$$\left(\sigma_{r,t}^u\right)^2 = \sigma_c^2 V_{u,t} + \kappa_1^2\left(A_1^2 + A_3^2\varphi_u^2\right) q_{u,t}, \tag{33}$$

$$\left(\sigma_{r,t}^d\right)^2 = \sigma_c^2 V_{d,t} + \kappa_1^2\left(A_2^2 + A_4^2\varphi_d^2\right) q_{d,t}. \tag{34}$$

Using the definition of the variance risk premium, we compute the upside variance risk premium as

$$\begin{aligned}
VRP_t^U &\equiv \mathbb{E}_t^{\mathbb{Q}}\left[\left(\sigma_{r,t+1}^u\right)^2\right] - \mathbb{E}_t\left[\left(\sigma_{r,t+1}^u\right)^2\right], \\
&= (\theta - 1)\left(\sigma_c^2\kappa_1 A_1 + \kappa_1^3\left(A_1^2 + A_3^2\varphi_u^2\right) A_3\varphi_u^2\right) q_{u,t}.
\end{aligned} \tag{35}$$

Similarly, we derive the following expression for the downside variance risk premium:

$$\begin{aligned}
VRP_t^D &\equiv \mathbb{E}_t^{\mathbb{Q}}\left[\left(\sigma_{r,t+1}^d\right)^2\right] - \mathbb{E}_t\left[\left(\sigma_{r,t+1}^d\right)^2\right] \\
&= (\theta - 1)\left(\sigma_c^2\kappa_1 A_2 + \kappa_1^3\left(A_2^2 + A_4^2\varphi_d^2\right) A_4\varphi_d^2\right) q_{d,t}.
\end{aligned} \tag{36}$$

As discussed before, we expect that $VRP_t^U < 0$ and $VRP_t^D > 0$; hence, it follows that

$$\sigma_c^2\kappa_1 A_1 + \kappa_1^3\left(A_1^2 + A_3^2\varphi_u^2\right) A_3\varphi_u^2 > 0, \tag{37}$$

$$\sigma_c^2\kappa_1 A_2 + \kappa_1^3\left(A_2^2 + A_4^2\varphi_d^2\right) A_4\varphi_d^2 < 0. \tag{38}$$

Since $A_4 < 0$, $A_2 < 0$ is a sufficient condition for $\sigma_c^2\kappa_1 A_2 + \kappa_1^3\left(A_2^2 + A_4^2\varphi_d^2\right) A_4\varphi_d^2 < 0$. Moreover, $A_2 < 0 \Leftrightarrow \mu_2 < \frac{f\left[-\sigma_c(1-\gamma)\right]}{\gamma - 1}$. In particular, $\mu_2 \leq 0 \Rightarrow A_2 < 0 \Rightarrow VRP_t^d > 0$. Since $A_3 < 0$, $A_1 > 0$

is a necessary condition for $\sigma_c^2\kappa_1 A_1 + \kappa_1^3\left(A_1^2 + A_3^2\varphi_u^2\right)A_3\varphi_u^2 > 0$. It is easily shown that

$$\sigma_c^2\kappa_1 A_1 + \kappa_1^3\left(A_1^2 + A_3^2\varphi_u^2\right)A_3\varphi_u^2 > 0 \Leftrightarrow A_1^L < A_1 < A_1^U$$

with

$$A_1^L = \frac{-\sigma_c^2\kappa_1 + \sqrt{\sigma_c^4\kappa_1^2 - 4\left(\kappa_1^3 A_3\varphi_u^2\right)^2 A_3^2\varphi_u^2}}{2\kappa_1^3 A_3\varphi_u^2}, \quad A_1^U = \frac{-\sigma_c^2\kappa_1 - \sqrt{\sigma_c^4\kappa_1^2 - 4\left(\kappa_1^3 A_3\varphi_u^2\right)^2 A_3^2\varphi_u^2}}{2\kappa_1^3 A_3\varphi_u^2}.$$

Both A_1^U and A_1^L are positive. In addition, it is easy to see that

$$\begin{aligned}
A_1^L &< A_1 < A_1^U \Leftrightarrow A_1^L < -\frac{f\left[\sigma_c(1-\gamma)\right] + (1-\gamma)\mu_1}{\theta(\kappa_1\beta_u - 1)} < A_1^U, \\
A_1^L &< A_1 < A_1^U \Leftrightarrow \mu_1^L < \mu_1 < \mu_1^U,
\end{aligned}$$

with

$$\begin{aligned}
\mu_1^L &= \frac{f\left[\sigma_c(1-\gamma)\right] + \theta(\kappa_1\beta_u - 1)A_1^L}{\gamma - 1} > 0, \\
\mu_1^U &= \frac{f\left[\sigma_c(1-\gamma)\right] + \theta(\kappa_1\beta_u - 1)A_1^U}{\gamma - 1} > 0,
\end{aligned}$$

which implies that

$$\mu_1 > 0.$$

Consequently, confirming our earlier intuition, we find that for the upside variance risk premium to be negative, expected consumption growth must increase with the upside variance. Similarly, a non-positive relation between expected consumption growth and the downside variance is sufficient to induce a positive downside variance risk premium.

Next, we derive the closed-form expression for the skewness risk premium. Following Feunou, Jahan-Parvar, and Tédongap (2013, 2014), we define the skewness as

$$sk_{r,t} = \left(\sigma_{r,t}^u\right)^2 - \left(\sigma_{r,t}^d\right)^2.$$

As a result, we calculate the skewness risk premium as

$$
\begin{aligned}
SRP_t &\equiv VRP_t^u - VRP_t^d \\
&= \left[\mathbb{E}_t^{\mathbb{Q}}\left[\left(\sigma_{r,t+1}^u\right)^2\right] - \mathbb{E}_t\left[\left(\sigma_{r,t+1}^u\right)^2\right]\right] - \left[\mathbb{E}_t^{\mathbb{Q}}\left[\left(\sigma_{r,t+1}^d\right)^2\right] - \mathbb{E}_t\left[\left(\sigma_{r,t+1}^d\right)^2\right]\right], \\
&= (\theta - 1)\left[\left(\sigma_c^2\kappa_1 A_1 + \kappa_1^3\left(A_1^2 + A_3^2\varphi_u^2\right)A_3\varphi_u^2\right)q_{u,t} - \left(\sigma_c^2\kappa_1 A_2 + \kappa_1^3\left(A_2^2 + A_4^2\varphi_d^2\right)A_4\varphi_d^2\right)q_{d,t}\right]. \quad (39)
\end{aligned}
$$

Based on our theoretical findings so far, it is easy to see that given $\theta < 0$ and conditions (37) and (38) – which we just verified – the skewness risk premium is negative-valued, in compliance with our earlier conjecture. Finally, since equation (32) implies that the equity risk premium loads positively on both $q_{u,t}$ and $q_{d,t}$, and because $VRP_t^U < 0$ is negatively proportional to $q_{u,t}$ and $VRP_t^D > 0$ is positively proportional to $q_{d,t}$, the equity risk premium loads positively on the downside variance risk premium and negatively on the upside variance risk premium. At this point, we have fully characterized the equity risk premium, upside and downside variance risk premia, the skewness risk premium, and, by extension, risky asset returns.

In summary, we show that first, our intuitions are naturally aligned with a simple consumption-based asset pricing model. Second, the assumptions needed to support these intuitions are mild – we require distinct and time-varying upside and downside shocks to the consumption growth process, a predictable component in conditional consumption growth proportional to these up and down shock variances, and an affine loading on risk factors. These are commonly maintained assumptions in the variance risk premium literature. Given these assumptions, we show that the upside variance risk premium is smaller in absolute terms than the downside variance risk premium, that upside and downside variance risk premia have opposite signs, and that the skewness risk premium is a negative-valued quantity. We now evaluate whether these predictions hold empirically.

4 Data

In this study, we adapt BTZ's methodology and use modified measures introduced in Section 2.3. As shown previously, these measures also lead to the construction of SRP as a byproduct. We thus need suitable data to construct excess returns, realized semi-variances ($RV^{U/D}$), and risk-neutral semi-variances ($IV^{U/D}$). In what follows, we discuss the raw data and methods we use to construct our empirical measures. Throughout the study, we set $\kappa = 0$.

4.1 Excess returns

We follow the literature in constructing market returns. Our empirical analysis is based on the S&P 500 composite index as a proxy for the aggregate market portfolio. Since our study requires reliable high-frequency data and option-implied volatilities, our sample includes data from September 1996 to December 2010. We compute the excess returns by subtracting 3-month treasury bill rates from log-differences in the S&P 500 composite index, sampled at the end of each month.

We present the summary statistics of equity returns in Panel A of Table 1. We report annualized mean, median, and standard deviations of returns in percentages. The table also reports monthly skewness, excess kurtosis, and the first-order autoregressive coefficient ($AR(1)$) for the S&P 500 monthly excess returns.

4.2 High-frequency data and realized variance components

We use daily close-to-close S&P 500 returns, realized variances data computed from five-minute intraday S&P 500 prices and 3-month treasury bill rates for the period from September 1996 to December 2010, which yields a total of 3,608 daily observations. The data are available through the Institute of Financial Markets.

To construct the daily $RV^{U/D}$s series, we use intraday S&P 500 data. We sum the five-minute squared negative returns for the downside realized variance (RV^D) and the five-minute squared positive returns for the upside realized variance (RV^U). We next add the daily squared overnight negative returns to the downside semi-variance, and the daily squared overnight positive returns to the upside realized variance. The overnight returns are computed for 4:00 pm to 9:30 am. The total realized variance is obtained by adding the downside and the upside realized variances. For the three series, we use a multiplicative scaling of the average total realized variance series to match the unconditional variance of S&P 500 returns.[7]

Our analysis is based on expectations of volatility under both physical and risk-neutral measures. Following our discussion in Section 2.3.1, to obtain expectations of realized volatility that are not

[7] Hansen and Lunde (2006) discuss various approaches to adjusting open-to-close RVs.

contaminated by forward bias or the use of contemporaneous data, we perform an out-of-sample forecasting exercise to predict the three realized variances. Forecast horizons range between 1 and 24 months ahead. Our reported results are based on the random walk forecasts of the realized volatility and its components.[8]

4.3 Options data and risk-neutral variances

Since our study hinges on decomposition of the variance process into upside and downside semi-variances, we cannot follow BTZ by using VIX as a measure of risk-neutral volatility. As a result, we construct our own measures of risk-neutral upside and downside variances ($IV^{U/D}$). We use two sources of data to construct upside and downside IV measures. First, we obtain daily data of European-style put and call options on the S&P 500 index from OptionMetrics Ivy DB. We then match these options data with return series on the underlying S&P 500 index and risk-free rates downloaded from Center for Research in Security Prices (CRSP) files.

For each day in the sample period, beginning in September 1996 and ending in December 2010, we sort call and put options data by maturity and strike price. We construct option prices by averaging the bid and ask quotes for each contract. To obtain consistent risk-neutral moments, we preprocess the data by applying the same filters as in Chang, Christoffersen, and Jacobs (2013).[9] We only consider out-of-the-money (OTM) contracts. Such contracts are the most traded, and thus the most liquid, options. Thus, we discard call options with moneyness levels – the ratios of strike prices to the underlying asset price – lower than 97% ($\underline{S}/S < 0.97$). Similarly, we discard put options with moneyness levels greater than 103% ($\underline{S}/S > 1.03$). Raw option data contain discontinuous strike prices. Therefore, on each day and for any given maturity, we interpolate implied volatilities over a finely discretized moneyness domain ($\underline{S}/S$), using a cubic spline to obtain a dense set of implied volatilities. We restrict the interpolation procedure to days that have at least two OTM call prices and two OTM put prices available.

For out-of-range moneyness levels (below or above the observed moneyness levels in the market), we extrapolate the implied volatility of the lowest or highest available strike price. We perform

[8] We explored the empirical implications of using risk premium measures constructed from HAR and M-HAR forecasts of the realized volatility and components. HAR forecasts are based on the Corsi (2009) method. These HAR results are similar to the random walk. To save space, we do not report them.

[9] That is, we discard options with zero bids, those with average quotes less than \$3/8, and those whose quotes violate common no-arbitrage restrictions.

this interpolation-extrapolation procedure to obtain a fine grid of 1,000 implied volatilities for moneyness levels between 0.01% and 300%. We then map these implied volatilities into call and put prices. Call prices are constructed for moneyness levels larger than 100% ($\underline{S}/S > 1$), whereas put prices are generated from moneyness levels smaller than 100% ($\underline{S}/S < 1$). We approximate the integrals using a recursive adaptive Lobatto quadrature. Finally, for any given future horizon of interest (1 to 24 months), we employ a linear interpolation to compute the corresponding moments, and rely on Eq. (6) and (7) to compute the upside and downside risk-neutral variance measures. We obtain 3,860 daily observations of upside/downside risk-neutral variances for maturities from 1 to 24 months.

An important issue in the construction of risk-neutral measures is the respective density of put and call contracts, especially for deep OTM contracts. Explicitly, precise computation of risk-neutral volatility components hinges on comparable numbers of OTM put and call contracts in longer-horizon maturities (18 to 24 months). Our data set provides a rich environment that supports this data construction exercise. As is clear from Table 2, while there are more OTM put contracts than OTM call contracts by any of the three measures used – moneyness, maturity, or VIX level – the respective numbers of contracts are comparable. In addition, Figure 1 shows that the growth of these contracts has continued unabated. We conclude that our construction of risk-neutral volatility components is not subject to bias because of the sparsity of data in deep OTM contracts.

Our computations are based on decompiling the variance risk premium based on realized returns to be above or below a cutoff point, $\kappa = 0$. However, as mentioned earlier, κ is not directly applicable to the risk-neutral probability space. Thus, we make the appropriate transformation to use our cutoff point by letting r^f represent the instantaneous risk-free rate, and denote time to maturity by h. Then, for the market price index at time t, we define the applicable cutoff point as $b = F_t \exp(\kappa)$ using the forward price $F_t = S_t \exp\left(r^f h\right)$. We then use b to compute the risk-neutral upside and downside variances, which can thus be viewed as a model-free *corridor* of risk-neutral volatilities as discussed in Andersen, Bondarenko, and Gonzalez-Perez (2015), Andersen and Bondarenko (2007), and Carr and Madan (1999), among others.

Panel B of Table 1 reports the summary statistics of risk-neutral volatility measures. As expected, these series are persistent – $AR(1)$ parameters are all above 0.60 – and demonstrate significant skewness and excess kurtosis. It is also clear that the main factor behind volatility behavior is the downside variance.

Figure 2 provides a stark demonstration of this point. It is immediately obvious that the contribution of upside variance to risk-neutral volatility is considerably less than that of downside variance. In fact, for most maturities, the median upside variance is about 50 to 80% smaller than the median downside variance. As time to maturity increases – a good measure for future expectations – the size of the median IV^U decreases. Notice that the size of this quantity is never as large as the median IV^D. The size of median IV^D increases uniformly over time to maturity, is close to median risk-neutral volatility values at each corresponding point in time to maturity, and demonstrates the same pattern of median risk-neutral volatility.

Thus, compared to its upside counterpart, the downside risk-neutral variance is clearly the main component of the risk-neutral volatility. We buttress this claim in the remainder of the paper through empirical analysis.

5 Empirical Results

In this section, we provide economic intuition and empirical support for our proposed decomposition of the variance risk premium. First, based on a sound financial rationale, we intuitively describe the expected behavior of the components of the variance risk premium and the skewness risk premium. We also present some empirical facts about the size and variability of these components. Since our approach is nonparametric, these facts stand as guidelines for realistic models (reduced-form and general equilibrium). Second, we provide an extensive investigation of the predictability of the equity premium, based on the variance premium and its components as well as the skewness risk premium. We empirically demonstrate the contribution of downside risk and skewness risk premia and characterize the sources of VRP predictability documented by BTZ. Third, we provide a comprehensive robustness study. Fourth, we document the out-of-sample forecast ability properties of our proposed predictors – downside variance risk and skewness risk premia. Subsequently, we establish that decomposing the variance risk premium into upside and downside variance risk premia reveals that, while VRP components are positively correlated with several macroeconomic and financial indicators, the level of spanning across these components differs. Finally, we track the reaction of variance risk premium components to macroeconomic and financial announcements. In particular, we are interested in uncovering the relationship between announcements that reduce or resolve uncertainty surrounding monetary or fiscal policy.

5.1 Description of the variance risk premium components

As mentioned earlier, we view the VRP as the premium a market participant is willing to pay to hedge against variations in future realized volatilities. It is expected to be positive, as rationalized within the general equilibrium model of BTZ, where it is shown to be in general positive and proportional to the volatility-of-volatility. We confirm these findings by reporting the summary statistics for the VRP in Table 1. We also plot the time series of the VRP, its components, and the SRP in Figure 3. We present measures based on random walk and univariate HAR forecasts of realized volatility and its components under the physical measure. Construction of quantities based on multivariate HAR (M-HAR) are virtually identical to those obtained from univariate HAR.[10] As mentioned earlier, we only report findings based on random walk forecasts of realized volatility. This approach allows us to save space, since the results obtained from the random walk or HAR methods are quite similar. The series plotted in Figure 3 demonstrate that while HAR-based quantities are more volatile than time series based on the random walk, the difference is mainly due to the magnitude of fluctuations, but not in the fluctuations themselves. This observation may explain the similarities in empirical findings. As expected, from 1996 to 2010, we can see that the variance risk premium is positive most of the time and remains high in uncertain episodes.

Similarly, we argued in Section 3 that we intuitively expect negative-valued VRP^U and positive-valued VRP^D. We also showed that, under mild assumptions, these expectations about VRP components are supported by our theoretical model. Table 1 clearly illustrates these intuitions, as the average VRP^U is about -2.60%. Moreover, Figure 3 confirms that VRP^U is usually negative through our sample period. The same table reports average VRP^D of roughly 5.21%, and in Figure 3, we observe that VRP^D is usually positive.

Building on the same intuition and methodology, we show that the sign of the SRP, which

[10] HAR and M-HAR forecasts of realized volatility and its components are based on the methodology of Corsi (2009).

stems from the expected behavior of the two components of the VRP, is negative. Indeed, Table 1 reports that the average skewness risk premium is -7.8%. In Figure 3, we clearly observe that SRP is generally negative-valued.

Table 1 also reveals highly persistent, negatively skewed, and fat-tailed empirical distributions for (down/upside) variance and skewness risk premia. Nonetheless, upside variance and skewness risk premia appear more left-skewed and leptokurtic as compared to (total) variance and downside variance risk premia.

5.2 Predictability of the equity premium

BTZ derive a theoretical model where the VRP emerges as the main driver of time variation in the equity premium. They show both theoretically and empirically that a higher VRP predicts higher future excess returns. Intuitively, the variance risk premium proxies the premium associated with the volatility-of-volatility, which not only reflects how future random returns vary but also assesses fluctuations in the tail thickness of future returns distribution.

Because the VRP sums downside and upside variance risk premia, BTZ's framework entails imposing the same coefficient on both (upside and downside) components of the VRP when they are jointly included in a predictive regression of excess returns. However, such a constraint seems very restrictive, given the asymmetric views of investors on good uncertainty – proneness to upward variability – versus bad uncertainty – aversion to downward variability. Sections 3 and 5.1 document that both VRP^D and VRP^U have intrinsically different features.

It is important to point out that by highlighting the disparities between upside and downside variance risk premia, and similar to Bollerslev and Todorov (2011) and Bollerslev, Todorov, and Xu (2015), our study intends to push the findings of BTZ further. BTZ's study is undertaken to rationalize the importance of the variance risk premium in explaining the dynamics of the equity premium. Our goal is to build on BTZ's framework, showing that introducing asymmetry in the VRP analysis provides additional flexibility to the trade-off between return and second-moment risk premia. Ultimately, our approach is intended to strengthen the concept behind the variance risk premium of BTZ.

Our results are based on a simple linear regression of k-step-ahead cumulative S&P 500 excess returns on values of a set of predictors that include the VRP, VRP^U, VRP^D, and SRP. Following the results of Ang and Bekaert (2007), reported Student's t-statistics are based on heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, as advocated by Hodrick (1992). The model used for our analysis is simply

$$r^e_{t\to t+k} = \beta_0 + \beta_1 x_t(h) + \epsilon_{t\to t+k}, \tag{40}$$

where $r^e_{t\to t+k}$ is the cumulative excess returns between time t and $t+k$, $x_t(h)$ is one of the predictors discussed in Sections 2.3 and 2.4 at time t, h is the construction horizon of $x_t(h)$, and $\epsilon_{t\to t+k}$ is a zero-mean error term. We focus our discussion on the significance of the estimated slope coefficients (β_1s), determined by the robust Student-t statistics. We report the predictive ability of regressions, measured by the corresponding adjusted R^2s. For highly persistent predictor variables, the R^2s for the overlapping multi-period return regressions must be interpreted with caution, as noted by BTZ and Jacquier and Okou (2014), among others.

We decompose the contribution of our predictors to show that (1) predictability results documented by BTZ are driven by the downside variance risk premium, (2) predictability results are mainly driven by risk-neutral expectations – thus, risk-neutral measures contribute more than realized measures – and (3) the contribution of the skewness risk premium increases as a function of both the predictability horizon (k) and the aggregation or maturity horizon (h).

Our empirical findings, presented in Tables 3 to 6, support all three claims. In Panel A of Table 3, we show that the two main regularities uncovered by BTZ, the hump-shaped increase in robust Student-t statistics and adjusted R^2s reaching their maximum at $k = 3$ (one quarter ahead), are present in the data. Both regularities are visible in the upper-left-hand-side plots in Figures 4 and 5. These effects, however, weaken as the aggregation horizon (h) increases from one month to three months or more; the predictability pattern weakens and then largely disappears for $h > 6$.

Panel B of Table 3 reports the predictability results based on using VRP^D as the predictor. A visual representation of these results is available in the upper-right-hand-side plots in Figures 4 and 5. It is immediately obvious that both regularities observed in the VRP predictive regressions are preserved. We observe the hump-shaped pattern for Student's t-statistics and the adjusted R^2s reaching their maximum between $k = 3$ or $k = 6$ months. Moreover, these results are more robust to the aggregation horizon of the predictor. We notice that, in contrast to the VRP results – where predictability is only present for monthly or quarterly constructed risk premia – the VRP^D results are largely robust to aggregation horizons; the slope parameters are statistically different from zero even for annually constructed downside variance risk premia ($h = 12$). Moreover, the VRP^D results yield higher adjusted R^2s compared with the VRP regressions at similar prediction horizons, an observation that we interpret as the superior ability of the VRP^D to explain the variation in aggregate excess returns. Last, but not least, we notice a gradual shift in prediction results from the familiar one-quarter-ahead peak of predictability documented by BTZ to 9-to-12-months-ahead peaks, once we increase the aggregation horizon h. Based on these results, we infer that the VRP^D is the likely candidate to explain the predictive power of VRP, documented by BTZ.

Our results for predictability based on the VRP^U, reported in Panel C of Table 3 and the two lower left-hand-side plots in Figures 4 and 5, are weak. The hump-shaped pattern in both robust Student's t-statistics and in adjusted R^2s, while present, is significantly weaker than the results reported by BTZ. Once we increase the aggregation horizon, h, these results are lost. We conclude that bearing upside variance risk does not appear to be an important contributor to the equity premium and, hence, is not a good predictor of this quantity. In addition, we interpret these findings as a low contribution of the VRP^U to overall VRP.

We observe a new set of interesting regularities when we use the SRP as our predictor. These results are reported in Panel D of Table 3 and the bottom-right-hand-side plots in Figures 4 and 5. It is immediately clear that the SRP displays a stronger predictive power at longer horizons than the VRP. For monthly excess returns, the SRP slope coefficient is statistically different from zero at prediction horizons of 6 months ahead or longer. At $k = 6$, the adjusted R^2 of the SRP is comparable in size with that of the VRP (2.30% against 3.65%, respectively) and is strictly greater thereafter. At $k = 6$, the adjusted R^2 for the monthly excess return regression based on the SRP is smaller than that of the VRP^D. However, their sizes are comparable at $k = 9$ and $k = 12$ months ahead. Both trends strengthen as we consider higher aggregation levels for excess returns. At the semi-annual construction level ($h = 6$), the SRP already has more predictive power than both the VRP and VRP^D at a quarter-ahead prediction horizon. The increase in adjusted R^2s of the SRP is not monotonic in the construction horizon level. We can detect a maximum at a roughly three-quarters-ahead prediction window for semi-annual and annually constructed SRP. This observation implies that the SRP is the intermediate link between one-quarter-ahead predictability using the VRP uncovered by BTZ and the long-term predictors such as the price-dividend ratio, dividend yield, or consumption-wealth ratio of Lettau and Ludvigson (2001). Given the generally unfavorable findings of Goyal and Welch (2008) regarding long-term predictors of equity premium, our findings regarding the predictive power of the SRP are particularly encouraging. As shown in Section 5.4, the SRP and VRP^D pass the out-of-sample challenge posed by Goyal and Welch (2008). We conclude that the predictability of cumulative excess returns by the SRP increases in both prediction horizon, k, and aggregation horizon, h, for the SRP.

At this point, it is natural to inquire about including both VRP components in a predictive regression. We present the empirical evidence from this estimation in Panel A of Table 6. After inclusion of the VRP^U and VRP^D in the same regression, the statistical significance of the VRP^U's slope parameters are broadly lost. We also notice a sign change in Student's t-statistics associated with the estimated slope parameters of the VRP^U and VRP^D. This observation is not surprising. As we show in our equilibrium model, and also intuitively, risk-averse investors like variability in positive outcomes of returns but dislike it in negative outcomes. Hence, in a joint regression, we expect the coefficient of VRP^D to be positive and that of VRP^U to be negative. This observation, as documented in Feunou, Jahan-Parvar, and Tédongap (2013), lends additional credibility to the role of the SRP as a predictor of aggregated excess returns.[11]

We claim that the patterns discussed earlier, and, hence, the predictive power of the VRP, VRP^D, and SRP are rooted in expectations. That is, the driving force behind our results, as well as those of BTZ, are expected risk-neutral measures of the volatility components. To show the empirical findings supporting our claim, we run predictive regressions using equation (40). Instead of using the "premia" employed so far, we use realized and risk-neutral measures of variances, up- and downside variances, and skewness for x_t, based on our discussions in Section 2, respectively.

Our empirical findings using risk-neutral volatility measures are available in Table 4. In Panel A, we report the results of running a predictive regression when the predictor is the risk-neutral

[11]Briefly, based on arguments similar to those advanced by Feunou, Jahan-Parvar, and Tédongap (2013), we expect estimated parameters of the VRP^U and VRP^D to have opposite signs, and be statistically "close." As such, they imply that the SRP is the predictor we should have included instead of these VRP components.

variance obtained from direct application of the Andersen and Bondarenko (2007) method. It is clear that the estimated slope parameters are statistically different from zero for $k \geq 3$ at most construction horizons, h. The reported adjusted R^2s also imply that the predictive regressions have explanatory power for aggregate excess return variations at $k \geq 3$. The same patterns are discernible for risk-neutral downside and upside variances (Panels B and C) and risk-neutral skewness (Panel D). Adjusted R^2s reported are lower than those reported in Table 3, and these measures of variation yield statistically significant slope parameters at longer prediction horizons than what we observe for the VRP and its components. Taken together, these observations imply that using the premium (rather than the risk-neutral variation) yields better predictions.

However, in comparison with realized (physical) variation measures, risk-neutral measures yield better results. The analysis using realized variation measures is available in Table 5. It is obvious that, by themselves, the realized measures do not yield reasonable predictability, an observation corroborated by the empirical findings of Bekaert, Engstrom, and Ermolov (2014). The majority of the estimated slope parameters are statistically indistinguishable from zero, and the adjusted R^2s are low. Inclusion of both risk-neutral or realized variance components does not change our findings dramatically, as demonstrated in Panels B and C of Table 6.

We observe in Panel D of Table 5 and in Panel C of Table 6 statistical significance and notable adjusted R^2s for realized skewness in long prediction horizons ($k \geq 6$) and for construction horizons ($h \geq 6$). By itself (as opposed to the SRP studied earlier), the realized skewness lacks predictive power in low construction or prediction horizons. Based on our results presented in Table 3, we argue that the SRP (and not the realized skewness) is a more suitable predictor, as it overcomes these two shortcomings.

A visual representation of the prediction power of risk-neutral and physical variation components is available in Figure 6. Given the weak performance of realized measures, it is easy to conclude that realized variation plays a secondary role to risk-neutral variation measures in driving the predictability results documented by BTZ or in this study. However, we need both elements in the construction of the variance or skewness risk premia, since realized or risk-neutral measures individually possess inferior prediction power.

5.3 Robustness

We perform extensive robustness exercises to document the prediction power of the VRP^D and SRP for aggregate excess returns in the presence of traditional predictor variables. The goal is to highlight the contribution of our proposed variables in a wider empirical context. Simply put, we observe that the predictive power does not disappear when we include other pricing variables, implying that the VRP^D and SRP are not simply proxies for other well-known pricing ratios.

Following BTZ and Feunou et al. (2014), among many others, we include equity pricing measures such as the log price-dividend ratio ($\log(p_t/d_t)$), lagged log price-dividend ratio ($\log(p_{t-1}/d_t)$), and log price-earnings ratio ($\log(p_t/e_t)$); yield and spread measures such as term spread (tms_t), the difference between 10-year U.S. Treasury Bond yields and 3-month U.S. Treasury Bill yields; default spread (dfs_t), defined as the difference between BBB and AAA corporate bond yields; CPI inflation ($infl_t$); and, finally, Kelly and Pruitt (2013) partial least-squares-based, cross-sectional in-sample and out-of-sample predictors ($kpis_t$ and $kpos_t$, respectively).

We consider two periods for our analysis: our full sample – September 1996 to December 2010 – and a pre-Great Recession sample, September 1996 to December 2007. The latter ends at the same point in time as the BTZ sample. We report our empirical findings in Tables 7 to 10. These results are based on semi-annually aggregated excess returns and estimated for the one-month-ahead prediction horizon.[12] In this robustness study, we scale the cumulative excess returns; we use $r^e_{t\to t+6}/6$ as the predicted value and regress it on a one-month lagged predictive variable.

Full-sample simple predictive regression results are available in Table 7. Among VRP components, only the downside variance risk premium ($dvrp_t$) and the skewness risk premium (srp_t) have slope parameters that are statistically different from zero and have adjusted R^2s comparable in magnitude with other pricing variables. Once we use $dvrp_t$ along with other pricing variables, we observe the following regularities in Table 8, which reports the joint multivariate regression results. First, the estimated slope parameter for $dvrp_t$ is statistically different from zero in all cases, except when we include srp_t. This result is not surprising, since srp_t and $dvrp_t$ are linearly dependent. Second, these regressions yield adjusted R^2s that range between 3.10% (for $dvrp_t$ and

[12] A complete set of robustness checks, including monthly, quarterly, and annually aggregated excess returns results, are available in an online Appendix.

tms_t, in line with findings of BTZ that report weak predictability for tms_t) to 25.71% (for $dvrp_t$ and $infl_t$).[13] The downside variance risk premium in conjunction with the variance risk premium or upside variance risk premium remains statistically significant and yields adjusted R^2s that are in the 7% neighborhood.

We obtain adjusted R^2s that are decidedly lower than those reported by BTZ for quarterly and annually aggregated multivariate regressions. These differences are driven by the inclusion of the Great Recession period data in our full sample. To illustrate this point, we repeat our estimation with the data set ending in December 2007. Simple predictive regression results based on this data are available in Table 9. We immediately observe that the exclusion of the Great Recession period data improves even the univariate predictive regression adjusted R^2s across the board. The estimated slope parameters are also closer to BTZ estimates and are generally statistically significant.

In Table 10, we report multivariate regression results, based on 1996 – 2007 data. We notice that once $dvrp_t$ is included in the regression model, the variance risk premium, upside variance risk premium, and skewness risk premium are no longer statistically significant. Other pricing variables – except for term spread, default spread, and inflation – yield slope parameters that are statistically significant. Thus, inflation seems to lack prediction power in this sub-sample. We do not observe statistically insignificant slope parameters for the downside variance risk premium except when we include vrp_t. Across the board, adjusted R^2s are high in this sub-sample.

5.4 Out-of-sample analysis

Our goal in this section is to compare the forecast ability of downside variance and skewness risk premia with common financial and macroeconomic variables used in equity premium predictability exercises.

To further assess the ability of downside variance risk and skewness risk premia to forecast excess returns, we follow the literature on predictive accuracy tests. We assume a benchmark model

[13] The dynamics of inflation during the Great Recession period mimic the behavior of our variance risk premia. Gilchrist et al. (2014) meticulously study the behavior of this variable in the 2007 – 2009 period. According to their study, both full and matched PPI inflation in their model display an aggregate drop in 2008 and 2009, while the reaction of financially sound and weak firms are asymmetric, with the former lowering prices and the latter raising prices in this period. Thus, the predictive power of this variable, given the inherent asymmetric responses, is not surprising.

(referred to as B) and a competitor model (referred to as C) in order to compare their predictive power for a given sample $\{y\}_{t=1}^{T}$. To generate k-period out-of-sample predictions $y_{t+k|t}$ for y_{t+k}, we split the total sample of T observations into in-sample and out-of-sample portions, where the first $1, \ldots, t_R$ in-sample observations are used to obtain the initial set of regression estimates. The out-of-sample observations span the last portion of the total sample $t = t_R + 1, \ldots, T$ and are used for forecast evaluation. The models are recursively estimated with the last in-sample observation ranging from $t = t_R$ to $t = T - k$, at each t forecasting $t + k$. That is, we use time t data to forecast the k-step-ahead value. In our analysis, we use half of the total sample for the initial in-sample estimation, that is $t_R = \lfloor \frac{T}{2} \rfloor$, where $\lfloor \chi \rfloor$ denotes the largest integer that is less than or equal to χ. In order to generate subsequent sets of forecasts, we employ a recursive scheme (expanding window), even though the in-sample period can be fixed or rolling. The forecast errors from the two models are

$$\begin{aligned} e_{t+k|t}^{B} &= y_{t+k} - y_{t+k|t}^{B}, \\ e_{t+k|t}^{C} &= y_{t+k} - y_{t+k|t}^{C}, \end{aligned}$$

where $t = t_R, \ldots, T - k$. Thus, we obtain two sets of $t = T - t_R - k + 1$ recursive forecast errors.

The accuracy of each forecast is measured by a loss function $L(\bullet)$. Among the popular loss functions are the squared error loss $L(e_{t+k|t}) = (e_{t+k|t})^2$ and the absolute error loss $L(e_{t+k|t}) = |e_{t+k|t}|$. Let $d_t^{BC} = L(e_{t+k|t})^B - L(e_{t+k|t})^C$ be the error loss differential between the benchmark and competitor models, and denote the expectation operator by $\mathbb{E}(\bullet)$. To gauge whether a model yields better forecasts than an alternative specification, a two-sided test may be run, where the null hypothesis is that the "two models have the same forecast accuracy" against the alternative hypothesis that the "two models have different forecast accuracy." Formally:

$$H_0 : \mathbb{E}(d_t^{BC}) = 0 \text{ vs. } H_A : \mathbb{E}(d_t^{BC}) \neq 0.$$

Alternatively, a one-sided test may be considered, where the null hypothesis is that "model C does not improve the forecast accuracy compared to model B" against the alternative hypothesis that "model C improves the forecast accuracy compared to model B." Formally:

$$H_0 : \mathbb{E}(d_t^{BC}) \leq 0 \text{ vs. } H_A : \mathbb{E}(d_t^{BC}) > 0.$$

In the context of our study, we apply forecast accuracy tests to non-nested models. The innovation of our analysis is to introduce two new predictors, the VRP^D and SRP. We compare the benchmark model B, which includes our proposed predictors, and the competitor C, which contains a traditional predictive variable such as the price-dividend ratio, dividend yield, or price-earnings ratio. Failure to reject the null leads us to conclude that the classical predictor does not yield more accurate forecasts than our proposed predictor. Diebold and Mariano (1995) and West (1996) provide further inference results on this class of forecast accuracy tests.

Following the influential study of Inoue and Kilian (2004), we first investigate the in-sample fit of the data by our proposed predictors – the VRP^D and SRP – and traditional predictors studied in the literature. Inoue and Kilian (2004) convincingly argue that to make a dependable out-of-sample inference, we need reasonable in-sample fit. The second column of Table 11 reports adjusted R^2s for monthly, quarterly, and semi-annually aggregated excess returns regressed on our proposed and traditional predictors. These are in-sample results and no forecasting is performed. We notice that, first, for all predictors, adjusted R^2s improve with the prediction horizon. Second, we notice that for all predictors except Kelly and Pruitt's 2013 out-of-sample cross-sectional book-to-market index, adjusted R^2s are reasonably high. The Kelly-Pruitt index is by construction an out-of-sample predictor. Thus, the seemingly poor in-sample performance is not a cause for concern for us.

Once we establish the in-sample prediction power, we move to investigate out-of-sample forecast ability. Not surprisingly, out-of-sample adjusted R^2s – reported in the third column of Table 11 – are much smaller than their in-sample counterparts, with the exception of the Kelly-Pruitt index. This observation may be due to inclusion of data from the 2007 – 2009 Great Recession period in the out-of-sample exercise. As documented in Section 5.3, most predictors lose significant prediction power once data from this period is included in the analysis.

Our task is to investigate the relative forecast performance of our proposed downside and skewness risk premia measures against other well-known predictors. To this end, we implement the Diebold and Mariano (1995) (henceforth, DM) tests of prediction accuracy. The results of performing out-of-sample forecast accuracy tests are available in the fourth and sixth columns of Table 11, where we report DM test statistics, and in the fifth and seventh columns of the same table, where we report the associated p-values. We cannot reject the null of equal or superior forecast accuracy when the benchmark is the downside variance (or skewness) risk premium and the alternative model contains one of the traditional predictors, since p-values are greater than the conventional 5% test size. We note the following important considerations. First, these results are based on the DM forecast accuracy test for non-nested models. Our findings are robust for all the horizons we consider in our analysis (1, 3, and 6 months). Second, the null hypothesis states that the mean squared forecast error of the alternative model is larger than or equal to that of the benchmark model. This is a one-sided test, and negative DM statistics indicate that the alternative model performed worse than the benchmark model. Third, we interpret the p-values cautiously, following Boyer, Jacquier, and van Norden (2012). They point out that p-values are hard to interpret because of the Lindley-Smith paradox, and, in addition, they need to be adjusted. To be precise, we produce multiple p-values in this analysis. Using unadjusted p-values in such an environment overstates the evidence against the null. Thus, following Boyer, Jacquier, and van Norden (2012), we apply a Bonferroni adjustment to the generated p-values. Our reported findings are, therefore, suitably conservative and reliable. Conventional competing variables such as the variance risk premium, price-dividend ratio, and price-earnings ratio have lower forecast accuracy than our proposed measures.

In summary, the predictive power of the downside variance risk premium and skewness risk premium are not a figment of a good in-sample fit of the data. In comparison with other pricing ratios and variables, our proposed measures have at least similar (and often superior) out-of-sample accuracy.

5.5 Links to macroeconomic variables, financial indicators, and events

Following Ludvigson and Ng (2009), we survey the correlations of variance, upside variance, downside variance, and skewness risk premia with 124 financial and economic indicators. We carry out this exercise to document the contemporaneous correlation of variance and skewness risk premia with well-known macroeconomic and financial variables. The VRP and its components are predictors of risk in financial markets; that is, an increase in VRP or VRP^D implies expectations of elevated risk levels in the future and, hence, compensation for bearing that risk. Fama and French (1989) document the counter-cyclical behavior of the equity premium: Investors demand a higher equity premium in bad times. It follows that VRP should be mildly pro-cyclical and positively correlated with cyclical macroeconomic and financial variables. The relationship between SRP and macroeconomic and financial factors is an empirically open issue that we address in this study. Finally, we are interested in the spanning of VRP, its components, and SRP by macroeconomic and financial factors. Briefly, low levels of spanning imply that the information content in the risk premium measures is nearly orthogonal to the information content of common financial or economic quantities.

The analysis and results here are based on a contemporaneous univariate regression, where the dependent variable is one of the variance or skewness risk premium measures, and the independent variable is one of the financial or economic variables.[14] Table 12 reports the 10 variables that yield the highest R^2s (wide-ranging above the 10% threshold) for each (semi-)variance risk premium component, and their respective slope parameter Student-t statistics that suggest significant relationships at conventional levels.

The estimated slope parameters for VRP and its components imply positive correlations with the mainly pro-cyclical macroeconomic variables listed in the table. Overall, payroll measures and industrial production indices exhibit strong contemporaneous links with VRP and its components. For instance, total payroll in the private sector yields an adjusted R^2 of over 50% for VRP^U, 40% for VRP, and about 25% for VRP^D.

Slope parameters for the regression model containing SRP as the predicted variable and macroeconomic and financial variables as predictors imply a negative contemporaneous correlation. The sources of predictability for the SRP – while much weaker – are diverse. The top variables with significant correlation with SRP differ from those in the other three panels of Table 12. For example, total payroll in the private sector does not have much explanatory power for the SRP; it yields an adjusted R^2 equal to 11.63% and is the 9^{th} variable in the list. Since payroll measures

[14]The complete list of these variables and supplementary results regarding our analysis are available in an online Appendix.

are pro-cyclical, these findings imply counter-cyclical behavior for the SRP.

Together, these regularities lead us to conclude that the common financial and macroeconomic indicators do not fully span the VRP components or SRP, since none explain more than 53% of the variation in these premia contemporaneously. Moreover, these indicators seem to have the least success spanning downside variance and skewness risk premia. This observation sheds further light on the success of these two variables in predicting equity premia – they contain relevant information beyond that of a large set of common macroeconomic and financial variables.

To deepen our analysis, we follow Amengual and Xiu (2014) and investigate the impact of decisions and announcements that reduce or resolve uncertainty. We use the same set of events compiled by Amengual and Xiu (2014) to study the impact of important FOMC announcements, speeches by Federal Reserve officials and the Presidents of the United States, as well as economic and political news. The events are summarized in Table 13.

Table 14 gives the changes in variance, upside variance, downside variance, and skewness risk premia as well as their end-of-the-day levels on event dates. The most striking outcome from this exercise is the observation that across the board and for all variance risk components and skewness risk premia, policy announcements that resolve financial or monetary uncertainty also reduce the premia. The impact on the SRP, however, is mixed: Announcements can increase or decrease the size of this premium. This observation, by construction, hinges on the size of the reduction imputed by the announcement on VRP^U and VRP^D. That said, in 16 out of 22 events studied, the impact of events on the SRP is negative.

To track the major changes in (up/down) variance and skewness risk premia in the temporal neighborhood of an event, we perform targeted searches in suitably chosen date intervals – in this case, a trading week before and after the event date. Most large movements are very close to the event date, consistent with the results in Table 14. These observations suggest that resolution of policy uncertainty or reduction of political tensions has a negative impact on premia demanded by the market participants to bear variance or skewness risk.

6 Conclusion

In this study, we have decomposed the celebrated variance risk premium of Bollerslev, Tauchen, and Zhou (2009) – arguably one of the most successful short-term predictors of excess equity returns – to show that its prediction power stems from the downside variance risk premium embedded in this measure. Market participants seem more concerned with market downturns and demand a premium for bearing that risk. By contrast, they seem to like upward uncertainty in the market. We support this intuition through a simple equilibrium consumption-based asset pricing model. We develop a model where consumption growth features separate upside and downside time-varying shock processes, with feedback from volatilities to future growth. We show that under mild requirements about consumption growth and upside and downside volatility processes, we can characterize the equity premium, upside and downside variance risk premia, and the skewness risk premium that support the main stylized facts observed in our empirical investigation. In particular, we observe unequal weights for upside and downside variances in the equity premium, and opposite signs for upside and downside variance risk premia.

Empirically, we demonstrate that the downside variance risk premium – the difference between option-implied, risk-neutral expectations of market downside variance and historical, realized downside variances – demonstrates significant prediction power (that is at least as powerful as the variance risk premium, and often stronger) for excess returns. We also show that the difference between upside and downside variance risk premia – our proposed measure of the skewness risk premium – is both a priced factor in equity markets and a powerful predictor of excess returns. The skewness risk premium performs well for intermediate prediction steps beyond the reach of short-run predictors such as downside variance risk or variance risk premia and long-term predictors such as price-dividend or price-earning ratios alike. The skewness risk premium constructed from one month's worth of data predicts excess returns from eight months to a year ahead. The same measure constructed from one quarter's worth of data predicts monthly excess returns from four months to one year ahead. We show that our findings demonstrate remarkable robustness to the inclusion of common pricing variables. Downside variance risk and skewness risk premia have similar or better out-of-sample forecast ability in comparison with common predictors. Finally, while these premia are connected to macroeconomic and financial indicators, they contain useful additional information. They also markedly react to decisions or announcements that modify the uncertainty anticipated by market participants.

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Table 1: **Summary Statistics**

	Mean (%)	**Median** (%)	**Std. Dev.** (%)	**Skewness**	**Kurtosis**	**AR(1)**
Panel A: Excess Returns						
Equity	1.9771	14.5157	20.9463	-0.1531	10.5559	-0.0819
Equity (1996-2007)	3.0724	12.5824	17.6474	-0.1379	5.9656	-0.0165
Panel B: Risk-Neutral						
Variance	19.3544	18.7174	6.6110	1.5650	7.6100	0.9466
Downside Variance	16.9766	16.2104	5.8727	1.6746	8.0637	0.9548
Upside Variance	9.2570	9.1825	3.1295	1.1479	6.0030	0.8991
Skewness	-7.7196	-7.0090	3.0039	-2.0380	9.6242	0.7323
Panel C: Realized						
Variance	16.7137	15.3429	5.5216	3.6748	25.6985	0.9667
Downside Variance	11.7677	10.8670	3.9857	3.9042	29.4323	0.9603
Upside Variance	11.8550	10.8683	3.8639	3.6288	25.3706	0.9609
Skewness	0.0872	0.1315	1.0911	-6.3619	170.4998	0.6319
Panel D: Risk Premium						
Variance	2.6407	2.3932	4.2538	-0.3083	6.8325	0.9265
Downside Variance	5.2089	4.8693	3.8159	0.2019	4.6310	0.9444
Upside Variance	-2.5979	-2.5730	2.5876	-2.2178	22.7198	0.8877
Skewness	-7.8068	-6.9942	3.0606	-2.0696	10.6270	0.9345

This table reports the summary statistics for the quantities investigated in this study. Mean, median, and standard deviation values are annualized and in percentages. We report excess kurtosis values. $AR(1)$ represents the values for the first auto-correlation coefficient. The full sample is from September 1996 to December 2010. We also consider a sub-sample ending in December 2007.

Table 2: **S&P 500 Index Options Data**

	OTM Put			OTM Call			
Panel A: By Moneyness	$\underline{S}/S < 0.97$	$0.97 < \underline{S}/S < 0.99$	$0.99 < \underline{S}/S < 1.01$	$1.01 < \underline{S}/S < 1.03$	$1.03 < \underline{S}/S < 1.05$	$\underline{S}/S > 1.05$	All
Number of contracts	223,579	57,188	71,879	57,522	26,154	100,121	536,443
Average price	15.08	39.44	39.67	38.47	21.97	15.50	23.90
Average implied volatility	25.68	17.05	15.88	15.58	14.30	16.31	20.06

Panel B: By Maturity	DTM < 30	30 < DTM < 60	60 < DTM < 90	90 < DTM < 120	120 < DTM < 150	DTM > 150	All
Number of contracts	115,392	140,080	83,937	36,163	22,302	138,569	536,443
Average price	10.45	14.90	20.17	24.88	26.20	45.82	23.90
Average implied volatility	19.40	20.20	20.06	21.11	20.48	20.13	20.06

Panel C: By VIX Level	VIX < 15	15 < VIX < 20	20 < VIX < 25	25 < VIX < 30	30 < VIX < 35	VIX > 35	All
Number of contracts	74,048	115,970	164,832	88,146	37,008	56,439	536,443
Average price	17.90	20.70	24.89	26.84	26.80	28.93	23.90
Average implied volatility	11.63	15.92	19.42	22.20	25.31	34.72	20.06

This table sorts S&P 500 index options data by moneyness, maturity, and VIX level. Out-of-the-money (OTM) call and put options from OptionMetrics from September 3, 1996 to December 30, 2010 are used. The moneyness is measured by the ratio of the strike price ($\underline{S}$) to underlying asset price (S). DTM is the time to maturity in number of calendar days. The average price and the average implied volatility are expressed in dollars and percentages, respectively.

Table 3: **Predictive Content of Premium Measure**

h	1		3		6		12	
	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$
k	Panel A: Variance Risk Premium							
1	2.43	2.61	2.51	2.83	1.02	0.02	0.68	-0.30
2	2.84	3.76	3.42	5.58	1.50	0.68	1.04	0.05
3	4.11	8.13	3.58	6.18	1.78	1.19	1.56	0.78
6	2.78	3.65	2.24	2.22	1.57	0.82	2.09	1.87
9	1.98	1.65	1.94	1.57	1.47	0.66	1.98	1.64
12	1.96	1.64	1.43	0.61	1.53	0.77	1.73	1.14
k	Panel B: Downside Variance Risk Premium							
1	2.57	2.99	2.68	3.30	1.27	0.34	0.95	-0.06
2	3.22	4.92	4.08	7.95	2.07	1.78	1.54	0.74
3	4.76	10.72	4.46	9.50	2.61	3.12	2.32	2.37
6	3.72	6.75	3.42	5.70	2.84	3.85	3.21	4.98
9	2.96	4.27	3.14	4.86	2.82	3.86	2.99	4.35
12	3.04	4.60	2.65	3.39	2.81	3.86	2.80	3.84
k	Panel C: Upside Variance Risk Premium							
1	2.08	1.79	1.91	1.44	0.44	-0.44	-0.04	-0.55
2	2.15	1.96	2.15	1.96	0.39	-0.47	-0.18	-0.54
3	3.05	4.41	2.07	1.79	0.26	-0.52	-0.27	-0.52
6	1.57	0.82	0.61	-0.36	-0.40	-0.48	-0.27	-0.53
9	0.83	-0.18	0.36	-0.50	-0.52	-0.42	-0.14	-0.57
12	0.74	-0.27	-0.05	-0.59	-0.33	-0.52	-0.41	-0.49
k	Panel D: Skewness Risk Premium							
1	-0.10	-0.55	0.41	-0.46	0.96	-0.04	1.25	0.30
2	0.61	-0.35	1.67	0.98	1.98	1.59	2.16	1.98
3	1.03	0.04	2.24	2.18	2.81	3.70	3.29	5.17
6	2.27	2.30	3.33	5.38	4.05	8.00	4.45	9.59
9	2.57	3.13	3.39	5.70	4.20	8.73	3.98	10.59
12	2.83	3.95	3.43	5.93	3.88	7.60	4.07	8.34

This table reports predictive regression results for prediction horizons (k) between 1 and 12 months ahead, and aggregation levels (h) between 1 and 12 months, based on a predictive regression model of the form $r_{t\to t+k} = \beta_0 + \beta_1 x_t(h) + \varepsilon_{t\to t+k}$. In this regression model, $r_{t\to t+k}$ is the cumulative excess returns between t and $t+k$; $x_t(h)$ is the proposed variance or skewness risk premia component that takes the values from variance risk, upside variance risk, downside variance risk, or skewness risk premia measures; and $\varepsilon_{t\to t+k}$ is a zero-mean error term. The reported Student's t-statistics for slope parameters are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992). $\bar{R}^2$ represents adjusted R^2s.

Table 4: **Predictive Content of Risk-Neutral Measure**

h	1		3		6		12	
	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$
k	Panel A: Risk-Neutral Variance							
1	0.28	-0.51	0.50	-0.41	0.69	-0.29	0.75	-0.24
2	1.14	0.17	1.24	0.30	1.35	0.44	1.39	0.51
3	1.30	0.38	1.52	0.72	1.83	1.28	2.13	1.92
6	2.10	1.88	2.33	2.43	2.76	3.57	3.21	4.95
9	2.32	2.44	2.55	3.05	2.95	4.22	3.15	4.85
12	2.21	2.20	2.45	2.82	2.89	4.11	3.30	5.45
k	Panel B: Risk-Neutral Downside Variance							
1	0.27	-0.51	0.57	-0.37	0.77	-0.23	0.87	-0.14
2	1.22	0.27	1.39	0.51	1.49	0.66	1.54	0.74
3	1.42	0.56	1.70	1.04	2.03	1.70	2.35	2.44
6	2.23	2.17	2.52	2.91	2.97	4.21	3.42	5.67
9	2.43	2.71	2.68	3.42	3.10	4.70	3.26	5.22
12	2.32	2.48	2.55	3.09	2.99	4.42	3.42	5.84
k	Panel C: Risk-Neutral Upside Variance							
1	0.29	-0.50	0.27	-0.51	0.36	-0.48	0.20	-0.53
2	0.93	-0.07	0.76	-0.23	0.78	-0.21	0.60	-0.35
3	0.99	-0.02	0.94	-0.06	1.04	0.05	1.00	0.00
6	1.74	1.12	1.72	1.09	1.92	1.48	2.05	1.77
9	2.01	1.71	2.09	1.89	2.31	2.42	2.38	2.59
12	1.90	1.49	2.09	1.92	2.45	2.83	2.57	3.17
k	Panel D: Risk-Neutral Skewness							
1	0.22	-0.52	0.87	-0.14	1.10	0.11	1.27	0.34
2	1.51	0.70	2.02	1.67	2.06	1.76	2.08	1.79
3	1.93	1.48	2.47	2.74	2.85	3.80	3.13	4.65
6	2.70	3.42	3.28	5.21	3.80	7.02	4.11	8.18
9	2.76	3.64	3.17	4.92	3.64	6.54	3.56	6.26
12	2.67	3.44	2.88	4.07	3.27	5.35	3.66	6.73

This table reports predictive regression results for risk-neutral variance and skewness measures. The predictive regression model, prediction horizons, aggregation levels, and notation are the same as in the results reported in Table 3. The difference is in the definition of $x_t(h)$: Instead of risk premia, we use risk-neutral measures for variance, upside variance, downside variance, and skewness. The reported Student's t-statistics for slope parameters are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992). $\bar{R}^2$ represents adjusted R^2s.

Table 5: **Predictive Content of Realized (Physical) Measure**

h	1		3		6		12	
	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$	t-Stat	$\bar{R}^2$
k	Panel A: Realized Variance							
1	-1.10	0.12	-0.99	-0.01	-0.10	-0.55	0.09	-0.55
2	-0.67	-0.30	-0.86	-0.15	0.18	-0.54	0.40	-0.47
3	-1.18	0.22	-0.75	-0.25	0.36	-0.48	0.62	-0.34
6	0.01	-0.57	0.48	-0.44	1.13	0.15	1.07	0.09
9	0.55	-0.40	0.78	-0.22	1.33	0.44	1.12	0.15
12	0.49	-0.45	0.98	-0.02	1.27	0.35	1.40	0.56
k	Panel B: Realized Downside Variance							
1	-1.05	0.06	-0.90	-0.10	-0.08	-0.55	0.09	-0.55
2	-0.53	-0.40	-0.76	-0.23	0.21	-0.53	0.39	-0.47
3	-1.04	0.05	-0.68	-0.30	0.39	-0.48	0.59	-0.36
6	0.05	-0.57	0.48	-0.44	1.08	0.10	0.99	-0.01
9	0.54	-0.41	0.75	-0.25	1.21	0.27	1.01	0.01
12	0.44	-0.48	0.89	-0.12	1.14	0.18	1.30	0.40
k	Panel C: Realized Upside Variance							
1	-1.15	0.18	-1.09	0.10	-0.13	-0.54	0.10	-0.55
2	-0.82	-0.18	-0.95	-0.05	0.14	-0.54	0.41	-0.46
3	-1.33	0.43	-0.82	-0.18	0.34	-0.49	0.66	-0.32
6	-0.05	-0.57	0.48	-0.44	1.17	0.21	1.16	0.19
9	0.39	-0.41	0.81	-0.19	1.44	0.61	1.23	0.29
12	0.54	-0.42	1.08	0.09	1.39	0.54	1.51	0.74
k	Panel D: Realized Skewness							
1	0.44	-0.45	1.58	0.81	0.63	-0.33	-0.06	-0.55
2	1.51	0.70	1.67	0.99	0.96	-0.05	-0.26	-0.52
3	1.45	0.61	1.19	0.23	0.71	-0.28	-0.89	-0.12
6	0.54	-0.40	0.11	-0.56	-1.01	0.01	-2.64	3.25
9	0.07	-0.58	-0.54	-0.41	-3.01	4.41	-3.67	6.67
12	-0.55	-0.41	-1.82	1.33	-3.37	5.70	-3.36	5.67

This table reports predictive regression results for realized variance and skewness measures. The predictive regression model, prediction horizons, aggregation levels, and notation are the same as in the results reported in Table 3. The difference is in the definition of $x_t(h)$: Instead of risk premia, we use realized (historical) measures for variance, upside variance, downside variance, and skewness. The reported Student's t-statistics for slope parameters are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992). $\bar{R}^2$ represents adjusted R^2s.

Table 6: **Joint Regression Results**

h	1			3			6			12		
	t-Stat		$\bar{R}^2$	t-Stat		$\bar{R}^2$	t-Stat		$\bar{R}^2$	t-Stat		$\bar{R}^2$
k	Up	Down		Up	Down		Up	Down		Up	Down	
Panel A: Risk Premium												
1	-0.01	1.49	2.45	-0.12	1.86	2.77	-0.58	1.32	-0.03	-0.85	1.27	-0.21
2	-0.78	2.49	4.72	-1.28	3.66	8.28	-1.38	2.46	2.26	-1.54	2.17	1.49
3	-1.28	3.79	11.04	-1.81	4.32	10.63	-2.06	3.33	4.84	-2.34	3.31	4.77
6	-2.46	4.19	9.36	-3.00	4.56	9.80	-3.31	4.39	8.98	-3.14	4.54	9.54
9	-2.75	3.99	7.76	-3.10	4.45	9.36	-3.46	4.49	9.50	-2.74	4.09	7.83
12	-3.06	4.29	9.06	-3.18	4.18	8.30	-3.13	4.24	8.61	-2.97	4.10	8.06
Panel B: Risk-Neutral Measures												
1	0.08	-0.01	-1.06	-1.13	1.24	-0.22	-1.30	1.46	0.15	-1.48	1.70	0.51
2	-1.00	1.27	0.27	-2.42	2.69	3.10	-2.27	2.61	2.90	-2.00	2.45	2.35
3	-1.59	1.89	1.39	-2.92	3.26	5.01	-3.22	3.68	6.55	-2.87	3.59	6.21
6	-1.64	2.14	3.09	-2.93	3.47	6.89	-3.25	3.99	9.12	-2.61	3.79	8.66
9	-1.32	1.88	3.12	-2.02	2.62	5.09	-2.25	3.05	6.88	-1.42	2.62	5.78
12	-1.35	1.89	2.94	-1.49	2.07	3.77	-1.39	2.18	4.93	-1.28	2.55	6.20
Panel C: Realized (Physical) Measures												
1	-0.63	0.42	-0.28	-1.82	1.72	1.17	-0.69	0.68	-0.84	0.11	-0.10	-1.10
2	-1.62	1.50	0.51	-1.92	1.83	1.24	-0.93	0.95	-0.60	0.48	-0.46	-0.90
3	-1.68	1.46	1.05	-1.41	1.34	0.25	-0.62	0.64	-0.82	1.27	-1.24	-0.02
6	-0.54	0.54	-0.97	-0.01	0.06	-1.01	1.39	-1.31	0.61	3.54	-3.49	6.14
9	0.01	0.08	-0.99	0.72	-0.64	-0.54	3.61	-3.52	6.77	4.82	-4.76	11.39
12	0.63	-0.55	-0.83	2.07	-1.98	1.77	3.99	-3.91	8.24	4.66	-4.59	11.22

This table reports predictive regression results when multiple variance components (risk premia, risk-neutral, and realized measures) are included in the regression model. The prediction horizons, aggregation levels, and notation are the same as in the results reported in Table 3. The difference is in the regression model. Both upside and downside variance components are in the model: $r_{t\to t+k} = \beta_0 + \beta_1 x_{1,t}(h) + \beta_2 x_{2,t}(h) + \varepsilon_{t\to t+k}$. $x_{1,t}(h)$ pertains to upside measures and $x_{2,t}(h)$ represents the downside measures used in the analysis. The reported Student's t-statistics for slope parameters are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992). $\bar{R}^2$ represents adjusted R^2s.

Table 7: **Semi-Annual Simple Predictive Regressions, September 1996 to December 2010**

Intercept	0.0005	-0.0026	-0.0132	0.0012	0.0756	0.0861	0.0514	0.0004	-0.0000	0.0210	-0.0105	-0.0093
	(0.1995)	(-1.1632)	(-3.0304)	(0.7230)	(2.9076)	(3.1978)	(2.0974)	(0.1293)	(-0.0067)	(6.3627)	(-3.1169)	(-2.0903)
$uvrp_t$	-0.0179											
	(-0.3917)											
$dvrp_t$		0.1135										
		(2.5900)										
srp_t			-0.1838									
			(-3.6007)									
vrp_t				0.0516								
				(1.4937)								
$log(p_t/d_t)$					-0.0419							
					(-2.8630)							
$log(p_{t-1}/d_t)$						-0.0478						
						(-3.1550)						
$log(p_t/e_t)$							-0.0384					
							(-2.0487)					
tms_t								0.7181				
								(0.4233)				
dfs_t									1.6755			
									(0.3885)			
$infl_t$										-0.8052		
										(-6.7137)		
$kpis_t$											0.1472	
											(4.0170)	
$kpos_t$												0.1203
												(2.5798)
Adj. R^2(%)	-0.5157	3.3439	6.7611	0.7406	4.1793	5.1472	1.9009	-0.5000	-0.5172	21.0807	8.4028	3.3140

This table presents predictive regressions of the semi-annually (scaled) cumulative excess return $r^e_{t\to t+6}/6 = \sum_{j=1}^{6} r^e_{t+j}/6$ on each one-period (1-month) lagged predictor from September 1996 to December 2010. The Student's t-statistics presented in parentheses below the estimated coefficients are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992).

Table 8: **Semi-Annual Multiple Predictive Regressions, September 1996 to December 2010**

$Intercept$	-0.0128	-0.0128	-0.0133	0.0787	0.0951	0.0582	-0.0045	-0.0084	0.0171	-0.0137	-0.0131
	(-2.9375)	(-2.9375)	(-2.9489)	(3.0953)	(3.6144)	(2.4206)	(-1.3553)	(-1.7752)	(4.9989)	(-3.8625)	(-2.8590)
$uvrp_t$	-0.1545										
	(-2.7123)										
$dvrp_t$	0.2100	0.0555	0.4321	0.1272	0.1392	0.1309	0.1178	0.1359	0.1289	0.1048	0.1129
	(3.7629)	(1.1549)	(3.4605)	(2.9688)	(3.2550)	(2.9980)	(2.6638)	(2.9172)	(3.3499)	(2.4918)	(2.6228)
srp_t		-0.1545									
		(-2.7123)									
vrp_t			-0.2640								
			(-2.7177)								
$log(p_t/d_t)$				-0.0462							
				(-3.2112)							
$log(p_{t-1}/d_t)$					-0.0557						
					(-3.7277)						
$log(p_t/e_t)$						-0.0471					
						(-2.5413)					
tms_t							1.2900				
							(0.7681)				
dfs_t								6.2346			
								(1.3864)			
$infl_t$									-0.8272		
									(-7.0977)		
$kpis_t$										0.1425	
										(3.9439)	
$kpos_t$											0.1197
											(2.6128)
$Adj.\ R^2(\%)$	6.9505	6.9505	6.9664	8.5372	10.3900	6.4572	3.1016	3.8843	25.7111	11.2225	6.6602

This table presents predictive regressions of the semi-annually (scaled) cumulative excess return $r^e_{t\to t+6}/6 = \sum_{j=1}^{6} r^e_{t+j}/6$ on one-period (1-month) lagged downside variance risk premium $dvrp$ and one alternative predictor in turn from September 1996 to December 2010. The Student's t-statistics presented in parentheses below the estimated coefficients are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992).

Table 9: **Semi-Annual Simple Predictive Regressions, September 1996 to December 2007**

$Intercept$	0.0129	-0.0070	-0.0124	0.0014	0.2016	0.2164	0.0913	0.0035	0.0206	0.0052	-0.0059	-0.0047
	(5.1388)	(-3.5015)	(-2.6470)	(1.0794)	(6.6115)	(7.0123)	(3.9851)	(1.5568)	(3.4844)	(1.0608)	(-2.2330)	(-1.3202)
$uvrp_t$	0.2679											
	(4.5558)											
$dvrp_t$		0.2784										
		(6.6863)										
srp_t			-0.2156									
			(-3.5176)									
vrp_t				0.2300								
				(6.5106)								
$log(p_t/d_t)$					-0.1092							
					(-6.5085)							
$log(p_{t-1}/d_t)$						-0.1176						
						(-6.9106)						
$log(p_t/e_t)$							-0.0662					
							(-3.8474)					
tms_t								-0.1990				
								(-0.1295)				
dfs_t									-25.6836			
									(-3.0118)			
$infl_t$										-0.0735		
										(-0.4029)		
$kpis_t$											0.1217	
											(4.0782)	
$kpos_t$												0.0940
												(2.4679)
$Adj.\ R^2(\%)$	13.2802	25.3069	8.1025	24.2902	24.2781	26.6030	9.6656	-0.7680	5.8882	-0.6536	10.8080	3.7964

This table presents predictive regressions of the semi-annually (scaled) cumulative excess return $r^e_{t\to t+6}/6 = \sum_{j=1}^{6} r^e_{t+j}/6$ on each one-period (1-month) lagged predictor from September 1996 to December 2007. The Student's t-statistics presented in parentheses below the estimated coefficients are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992).

Table 10: **Semi-Annual Multiple Predictive Regressions, September 1996 to December 2007**

Intercept	-0.0045	-0.0045	-0.0049	0.1736	0.1954	0.1053	-0.0087	-0.0033	-0.0142	-0.0155	-0.0162
	(-1.0099)	(-1.0099)	(-1.0492)	(6.6158)	(7.5813)	(5.7055)	(-3.2688)	(-0.4896)	(-2.8139)	(-5.8453)	(-4.7195)
$uvrp_t$	0.0454										
	(0.6200)										
$dvrp_t$	0.2554	0.3008	0.2065	0.2534	0.2629	0.3156	0.2853	0.2669	0.2980	0.2716	0.2847
	(4.5711)	(5.4521)	(1.4127)	(7.0696)	(7.6580)	(8.4728)	(6.7545)	(5.7833)	(6.8848)	(6.9931)	(7.0767)
srp_t		0.0454									
		(0.6200)									
vrp_t			0.0632								
			(0.5133)								
$log(p_t/d_t)$				-0.0990							
				(-6.8974)							
$log(p_{t-1}/d_t)$					-0.1113						
					(-7.8692)						
$log(p_t/e_t)$						-0.0855					
						(-6.1130)					
tms_t							1.3106				
							(0.9772)				
dfs_t								-4.9164			
								(-0.5838)			
$infl_t$									0.2541		
									(1.5553)		
$kpis_t$										0.1148	
										(4.5066)	
$kpos_t$											0.1050
											(3.2382)
Adj. $R^2(\%)$	24.9460	24.9460	24.8746	45.2343	49.3937	41.8336	25.2805	24.9203	26.1258	35.0975	30.4605

This table presents predictive regressions of the semi-annually (scaled) cumulative excess return $r^e_{t\to t+6}/6 = \sum_{j=1}^{6} r^e_{t+j}/6$ on one-period (1-month) lagged downside variance risk premium $dvrp$ and one alternative predictor in turn from September 1996 to December 2007. The Student's t-statistics presented in parentheses below the estimated coefficients are constructed from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions, following Hodrick (1992).

Table 11: **Out-of-Sample Analysis**

	$Adj.\ R^2(\%)\ for\ IS$	$Adj.\ R^2(\%)\ for\ OOS$	$dvrp\ vs.\ x_t$		$srp\ vs.\ x_t$	
			DM	$p-value$	DM	$p-value$
		Panel A: One Month				
$dvrp_t$	4.6723	0.6347			-0.0426	0.5170
srp_t	3.4862	-0.6055	0.0426	0.4830		
vrp_t	3.7175	-0.1087	-0.0271	0.5108	-0.0374	0.5149
$log(p_t/d_t)$	6.3871	-1.1465	-0.3716	0.6449	-0.4769	0.6833
$log(p_{t-1}/d_t)$	6.7059	-1.1123	-0.2414	0.5954	-0.3453	0.6351
$log(p_t/e_t)$	4.2430	-0.9384	0.2572	0.3985	0.2930	0.3848
$kpos_t$	-1.0697	2.0261	1.3282	0.0921	1.7998	0.0359
		Panel B: Three Months				
$dvrp_t$	24.6956	5.5674			0.0895	0.4644
srp_t	21.3847	-0.8775	-0.0895	0.5356		
vrp_t	19.8333	4.6494	0.3654	0.3574	0.2742	0.3919
$log(p_t/d_t)$	16.8502	-1.0456	0.5398	0.2947	0.6162	0.2689
$log(p_{t-1}/d_t)$	18.4235	-0.5345	0.5425	0.2937	0.6304	0.2642
$log(p_t/e_t)$	11.2493	0.2510	0.9778	0.1641	1.0725	0.1417
$kpos_t$	-0.6580	0.6473	1.7537	0.0397	1.8782	0.0302
		Panel C: Six Months				
$dvrp_t$	35.4498	0.1580			-1.2144	0.8877
srp_t	20.0010	2.3028	1.2144	0.1123		
vrp_t	31.7578	2.7778	-0.4553	0.6756	-1.0558	0.8545
$log(p_t/d_t)$	28.2580	0.4752	0.3393	0.3672	-1.2086	0.8866
$log(p_{t-1}/d_t)$	32.1452	1.2361	0.2877	0.3868	-1.2333	0.8913
$log(p_t/e_t)$	17.2860	1.0359	0.8114	0.2086	-0.6382	0.7383
$kpos_t$	2.9373	12.1162	1.7801	0.0375	1.2382	0.1078

This table presents the out-of-sample performance of predictors to forecast monthly ($r^e_{t\to t+1}$ in the top panel), quarterly ($r^e_{t\to t+3}/3$ in the middle panel) and semi-annually ($r^e_{t\to t+6}/6$ in the bottom panel) scaled cumulative excess returns, with observations spanning September 1996 to December 2010. The first two columns present the adjusted R^2 (%) for the in-sample (IS) and out-of-sample (OOS) observations – that is, the first and last half fractions of the data. The columns headed "$dvrp\ vs.\ x_t$" test the null hypothesis that "an alternative predictor (x_t) does not yield a better forecast than the downside variance risk premium ($dvrp$)." The columns headed "$srp\ vs.\ x_t$" test the null hypothesis that "an alternative predictor (x_t) does not yield a better forecast than the skewness risk premium (srp)." The reported test statistics and p-values are computed from the Diebold and Mariano (1995) model comparison procedure. Note that the Bonferroni adjustment is required when multiple p-values are produced, to avoid overstating the evidence against the null. Thus, to maintain an overall significance level of 5% (resp. 10%), one should adjust each individual test size to 0.0083 = 5%/6 (resp. 0.0167 = 10%/6) since 6 tests are performed for a given horizon.

Table 12: **Relationship between Variance Risk Premium Components and Financial and Macroeconomic Variables**

Variance Risk Premium			Downside Variance Risk Premium		
Variable	t-Stat	R^2	Variable	t-Stat	R^2
Nonfarm Payrolls, Total Private	11.44	41.94	Nonfarm Payrolls, Total Private	7.67	24.52
Nonfarm Payrolls, Wholesale Trade	10.55	38.06	IPI, Durable Goods Materials	7.08	21.70
IPI, Durable Goods Materials	9.61	33.79	Nonfarm Payrolls, Wholesale Trade	6.99	21.26
Nonfarm Payrolls, Transportation, Trade & Utilities	9.16	31.69	Industrial Production Index, Total Index	6.81	20.40
Nonfarm Payrolls, Services	8.90	30.46	IPI, Final Products and Nonindustrial Supplies	6.62	19.47
IPI, Manufacturing (SIC)	8.27	27.45	IPI, Manufacturing (SIC)	6.57	19.25
IPI, Final Products and Nonindustrial Supplies	8.20	27.08	Nonfarm Payrolls, Transportation, Trade & Utilities	6.39	18.41
Nonfarm Payrolls, Retail Trade	8.16	26.89	Nonfarm Payrolls, Services	6.25	17.77
Industrial Production Index, Total Index	7.96	25.92	Nonfarm Payrolls, Retail Trade	5.84	15.84
Nonfarm Payrolls, Construction	7.80	25.15	IPI, Final Products	5.73	15.37
Upside Variance Risk Premium			Skewness Risk Premium		
Variable	t-Stat	R^2	Variable	t-Stat	R^2
Nonfarm Payrolls, Total Private	14.24	52.82	PPI, Intermediate Materials, Supplies & Components	-5.92	16.23
Nonfarm Payrolls, Wholesale Trade	13.41	49.85	Nonfarm Payrolls, Mining and Logging	-5.67	15.09
Nonfarm Payrolls, Transportation, Trade & Utilities	10.94	39.82	Nonfarm Payrolls, Construction	-5.26	13.24
IPI, Durable Goods Materials	10.67	38.61	Nonfarm Payrolls, Wholesale Trade	-5.11	12.61
Nonfarm Payrolls, Services	10.66	38.56	1-Year Treasury	-5.06	12.39
Nonfarm Payrolls, Construction	10.38	37.31	CPI, All Items	-4.98	12.03
Nonfarm Payrolls, Retail Trade	9.51	33.33	CPI, All Items Less Medical Care	-4.95	11.94
IPI, Manufacturing (SIC)	8.49	28.50	6-Month Treasury Bill	-4.92	11.80
IPI, Final Products and Nonindustrial Supplies	8.31	27.63	Nonfarm Payrolls, Total Private	-4.88	11.63
Nonfarm Payrolls, Financial Sector	8.01	26.18	CPI, All Items Less Food	-4.82	11.36

This table reports the 10 macroeconomic variables that demonstrate high contemporaneous correlation and explanatory power for variance and skewness risk premia. The results are sorted based on the size of adjusted R^2s from performing a univariate, linear regression analysis where the dependent variable is either the variance risk premium, upside variance risk premium, downside variance risk premium, or skewness risk premium, and the independent variable is one of the 124 macroeconomic and financial variable series studied by Feunou et al. (2014). Both adjusted R^2s and Student's t-statistics for the slope parameters are reported.

Table 13: **Policy News Potentially Associated with Volatility Changes–Both Dates**

Date	ΔVariance	ΔReturn	News
08/18/98	-0.373 (-0.365)	0.013	President Clinton admits to “wrong” relationship with Ms. Lewinsky and FOMC’s decision to leave interest rates unchanged
09/01/98	-0.722 (-0.664)	0.035	Fed adds money to the banking system with Repo
09/08/98	-0.526 (-0.455)	0.021	Fed Chairman Greenspan’s statement that a rate cut might be forthcoming
09/14/98	-0.185		President Clinton advocated a coordinated global policy for economic growth in NYC
09/23/98	-0.344 (-0.280)	0.027	Fed Chairman Greenspan testimony before the Committee on the Budget, U.S. Senate
10/20/98	-0.253	-0.007	3 big US banks delivered better-than-expected earnings and bullish mood after Fed rate cut previous week
08/11/99	-0.266 (-0.276)	0.008	Fed Beige Book release shows that US economic growth remains strong
01/07/00	-0.500	0.031	Unemployment report shows the lowest unemployment rate in the past 30 years
03/16/00	-0.266	0.037	Release of Inflation Remains Tame Enough to Keep the Federal Reserve from tightening credit
04/17/00	-0.373 (-0.296)	0.032	Treasury Secretary Lawrence H. Summers Statement that fundamentals of the economy are in place
10/19/00	-0.241	0.018	Feds Greenspan Gives Keynote Speech at Cato Institute and jobless claims drop by 7,000 in the latest week
01/03/01	-0.282 (-0.179)	0.052	Fed’s Announcement of a Surprise, Inter-Meeting Rate Cut
05/17/05	-0.275 (-0.303)	0.01	John Snow calls on China to take an intermediate step in revaluing its currency
05/19/05	-0.297		Fed Chairman A. Greenspan Steps up Criticism of Fannie Mae and Freddie Mac
06/15/06	-0.549 (-0.625)	0.017	Fed Chairman B. Bernanke’s speech on inflation expectations within historical ranges
06/29/06	-0.295 (-0.325)	0.016	FOMC Statement to raise its target for the Federal Funds Rate by 25 bps
07/19/06	-0.272		Fed Chairman B. Bernanke warned that the Fed must guard against rising prices taking hold
02/28/07	-0.396		Fed Chairman B. Bernanke told a house panel that markets seem to be working well
03/06/07	-0.217		Henry Paulson in Tokyo said the global economy was as strong as he has ever seen
06/27/07	-0.271		FOMC announcement generated market rebound the previous date
08/21/07	-0.188		Senator Dodd said the Fed to deal with the turmoil after meeting with Paulson and Bernanke
09/18/07	-0.415 (-0.353)	0.024	FOMC decided to lower its target for the Federal Funds Rate by 50 bps
03/18/08	-0.216		Fed cuts the Federal Funds Rate by three-quarters of a percentage point
10/14/08	-0.489 (-0.304)	-0.048	FOMC decided to lower its target for the Federal Funds Rate by 50 bps
10/20/08	-0.426 (-0.413)	0.033	Fed Chairman B. Bernanke Testimony on the Budget, U.S. House of Representatives
10/28/08	-0.313 (-0.230)	0.075	Fed to Cut the Rate Following the Two-Day FOMC Meeting is Expected by the Market
11/13/08	-0.328 (-0.240)	0.062	President Bush’s Speech on Financial Crisis
12/19/08	-0.244		President Bush Declared that TARP Funds to be Spent on Programs Paulson Deemed Necessary
02/24/09	-0.261		President Obama’s First Speech as the President to Joint Session of U.S. Congress
05/10/10	-0.647 (-0.601)	0.003	European Policy-Makers Unveiled an Unprecedented Emergency Loan Plan
03/21/11	-0.277		Japanese nuclear reactors cooled down and situations in Libya tamed by unilateral forces
08/09/11	-0.433 (-0.370)	0.046	FOMC Statement Explicitly Stating a Duration for an Exceptionally Low Target Rate
10/27/11	-0.245 (-0.205)	0.034	European Union leaders made a bond deal to fix the Greek debt crisis
01/02/13	-0.432 (-0.427)	0.025	President Obama and Senator McConnell’s encouraging comments on the “Fiscal Cliff” issue

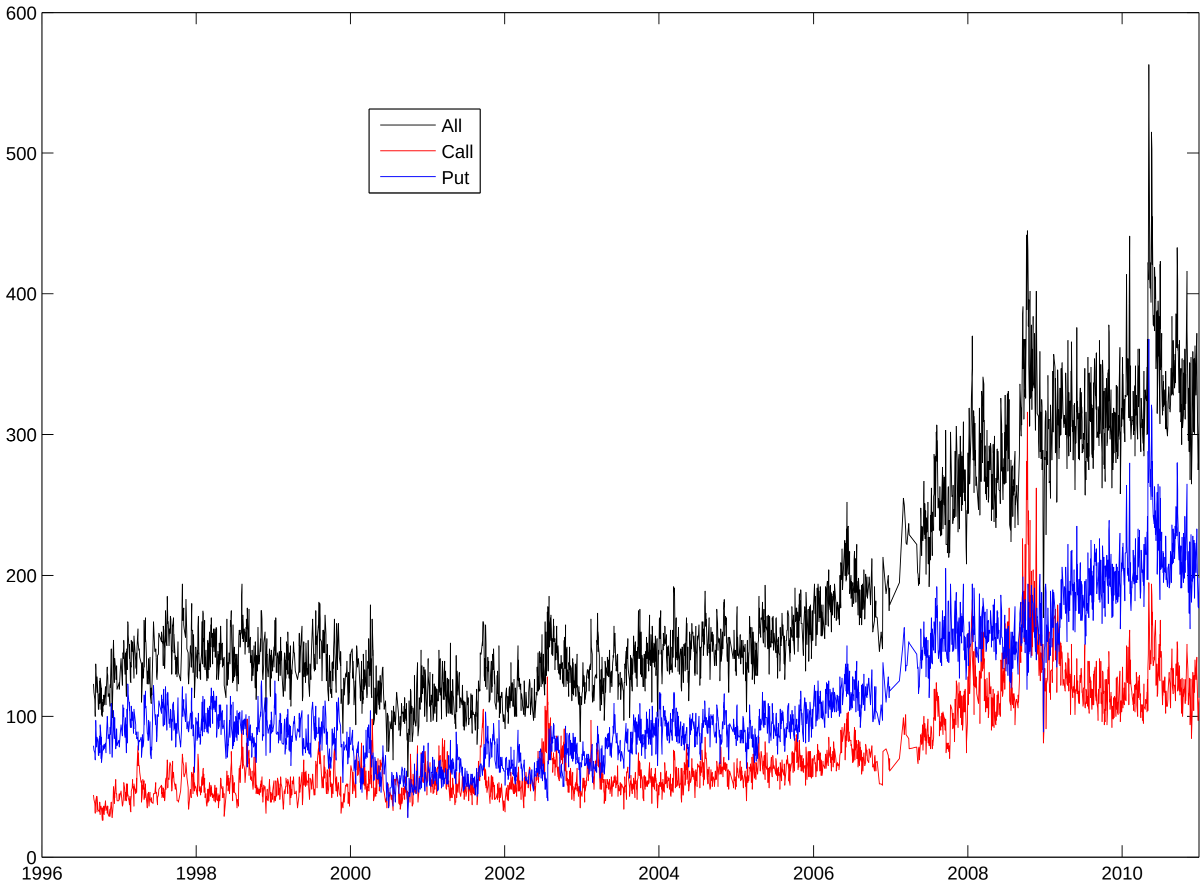
This table from Amengual and Xiu (2014) presents in the last column the events that may lead to the largest volatility drops in the sample. The first column is the date of the event. The second shows changes in estimated spot variance, whereas the third column is the returns of the index on the corresponding days.

Table 14: **Reaction of Variance and Skewness Risk Premia to Financial and Macroeconomic Announcements**

			VRP		VRP^U		VRP^D		SRP	
Booth Date	ΔVar	Δr	Change	Level	Change	Level	Change	Level	Change	Level
08/18/1998	-0.373	0.013	-0.0146	0.0964	-0.0059	-0.0045	-0.0132	0.1188	-0.0073	0.1234
09/01/1998	-0.722	0.035	-0.0292	0.1432	-0.0206	0.0066	-0.0210	0.1666	-0.0004	0.1600
09/08/1998	-0.526	0.021	-0.0404	0.1190	-0.0189	-0.0123	-0.0348	0.1498	-0.0159	0.1621
09/23/1998	-0.344	0.027	-0.0131	0.0980	-0.0105	-0.0241	-0.0088	0.1337	0.0017	0.1578
10/20/1998	-0.253	-0.007	-0.0160	0.0444	-0.0143	-0.0453	-0.0100	0.0875	0.0043	0.1328
08/11/1999	-0.266	0.008	-0.0169	0.0540	-0.0123	-0.0223	-0.0124	0.0815	-0.0001	0.1038
01/07/2000	-0.5	0.031	-0.0305	0.0137	-0.0028	-0.0341	-0.0328	0.0429	-0.0300	0.0770
03/16/2000	-0.266	0.037	-0.0174	-0.0209	-0.0118	-0.0553	-0.0130	0.0164	-0.0012	0.0717
04/17/2000	-0.373	0.032	-0.0183	0.0023	-0.0134	-0.0527	-0.0132	0.0426	0.0003	0.0953
10/19/2000	-0.241	0.018	-0.0190	0.0027	-0.0088	-0.0412	-0.0164	0.0340	-0.0076	0.0752
01/03/2001	-0.282	0.052	-0.0229	-0.0137	-0.0242	-0.0616	-0.0110	0.0285	0.0131	0.0900
05/17/2005	-0.275	0.01	-0.0063	0.0178	-0.0023	-0.0202	-0.0059	0.0372	-0.0036	0.0575
06/15/2006	-0.549	0.017	-0.0251	0.0201	-0.0141	-0.0260	-0.0209	0.0423	-0.0068	0.0683
06/29/2006	-0.295	0.016	-0.0154	0.0035	-0.0100	-0.0332	-0.0121	0.0275	-0.0021	0.0607
09/18/2007	-0.415	0.024	-0.0272	0.0059	-0.0100	-0.0357	-0.0252	0.0344	-0.0152	0.0701
10/14/2008	-0.489	-0.048	-0.0040	0.0054	-0.0106	-0.0730	0.0032	0.0641	0.0138	0.1371
10/20/2008	-0.426	0.033	-0.0628	-0.0012	-0.0280	-0.0943	-0.0558	0.0688	-0.0278	0.1631
10/28/2008	-0.313	0.075	-0.0518	0.0380	-0.0311	-0.1027	-0.0402	0.1187	-0.0091	0.2214
11/13/2008	-0.328	0.062	-0.0412	0.0071	-0.0253	-0.1270	-0.0322	0.0986	-0.0069	0.2256
05/10/2010	-0.647	0.003	-0.0631	0.0764	-0.0386	-0.0215	-0.0488	0.1058	-0.0102	0.1273
08/09/2011	-0.433	0.046	-0.0628	0.0754	-0.0365	-0.0143	-0.0504	0.0997	-0.0139	0.1140
10/27/2011	-0.245	0.034	-0.0240	-0.0447	-0.0184	-0.0953	-0.0165	0.0115	0.0019	0.1068

This table reports the reaction of the variance risk premium (VRP), upside variance risk premium (VRP^U), downside variance risk premium (VRP^D), and skewness risk premium (SRP) to the macroeconomic and financial news documented in Table 13. The table reports changes in conditional volatility (ΔVar) and S&P 500 returns (Δr) on the event day, as well as changes and levels of VRP, VRP^U, VRP^D, and SRP on the event date. A negative sign in the change of a risk premium signifies a decline on the arrival of a particular macroeconomic or financial announcement. A positive sign implies the opposite.

Figure 1: S&P 500 Put and Call Contracts per Day



This graph show the number of outstanding put and call contracts written on the S&P 500 index per day for the 1996–2010 period. In addition, it plots the sum of put and call contract numbers. Source: OptionMetrics Ivy DB accessed via WRDS.

Figure 2: The Term Structure of Risk-Neutral Variance

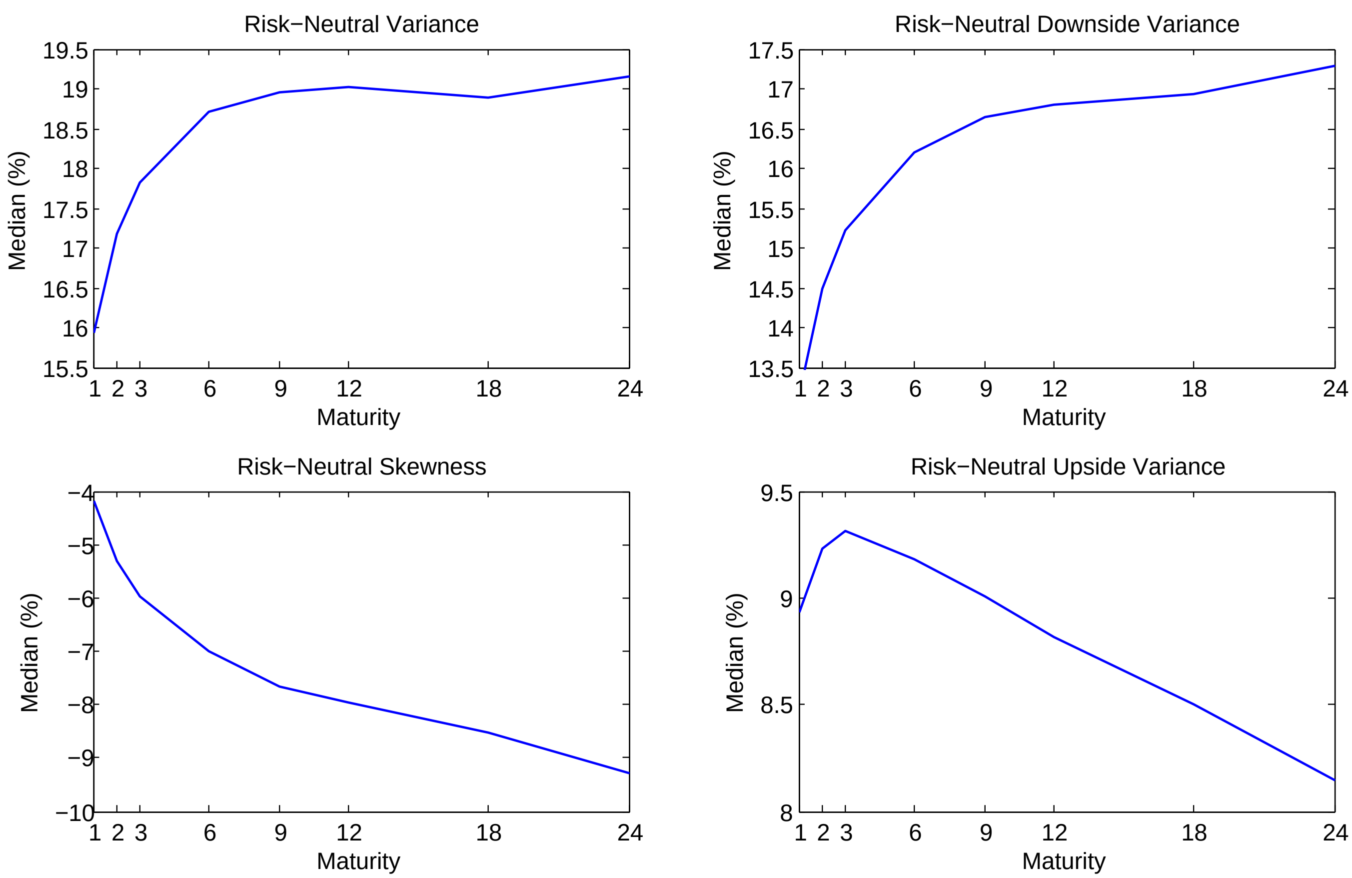
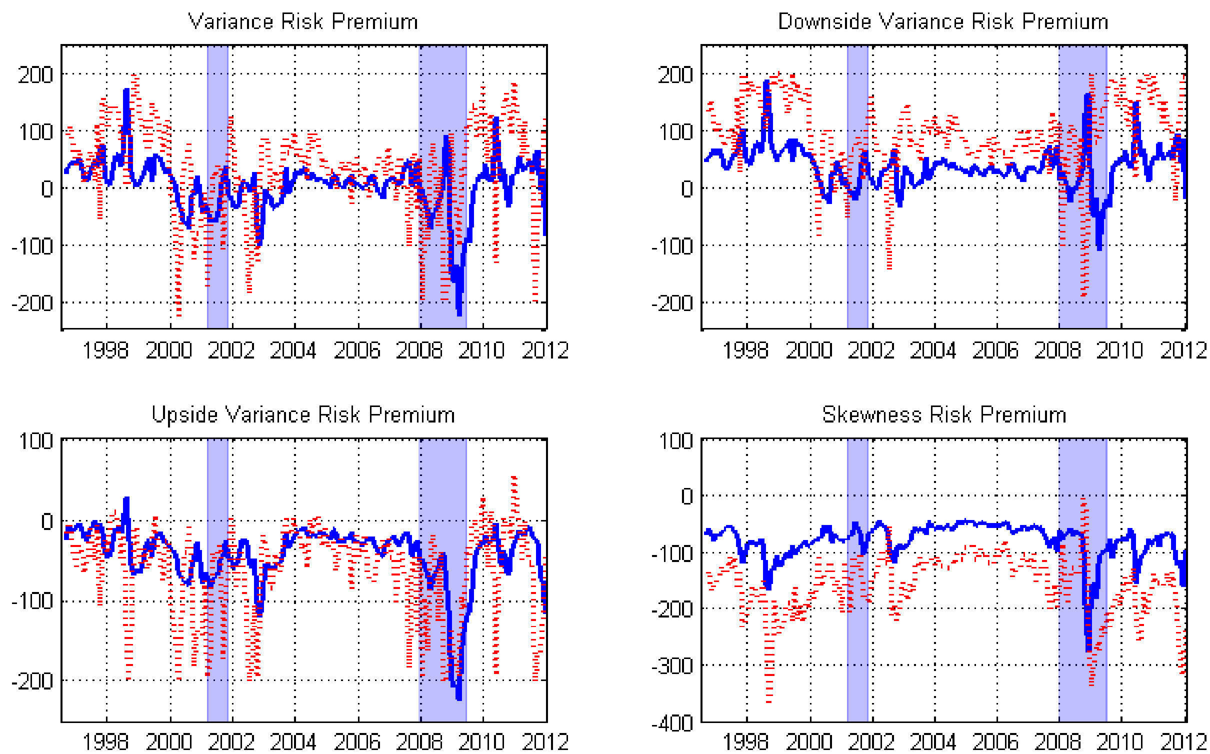
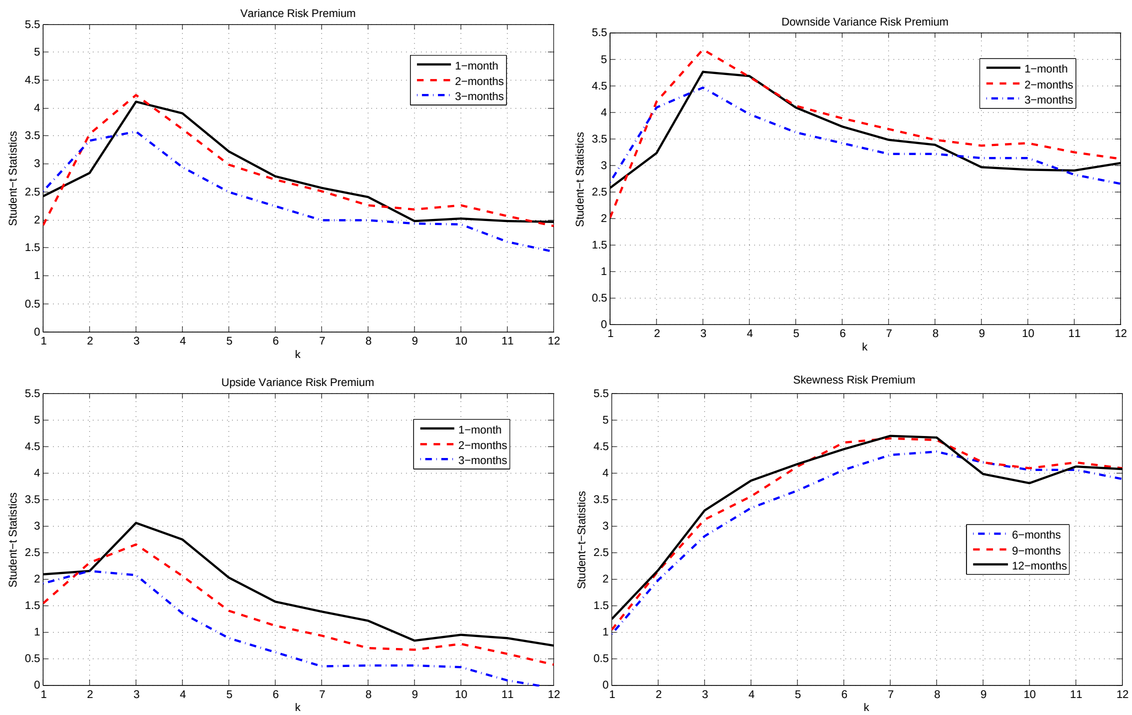


Figure 3: Time Series for Variance and Skewness Risk Premia



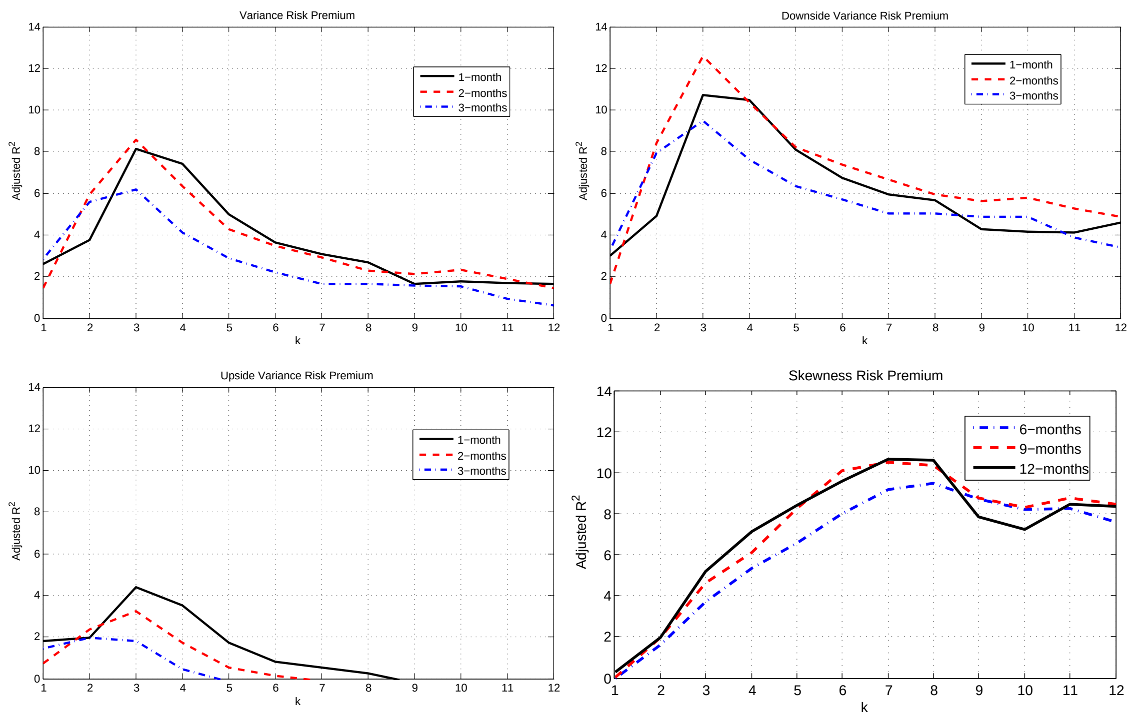
These figures plot the paths of annualized monthly values ($\times 10^3$) for the variance risk premium, upside variance risk premium, downside variance risk premium, and skewness risk premium, extracted from U.S. financial markets data for September 1996 to December 2011. Solid lines represent premia constructed from random walk forecasts of the realized volatility and components. Dotted lines represent values constructed from univariate HAR forecasts of the realized volatility. HAR methodology follows Corsi (2009). The shaded areas represent NBER recessions.

Figure 4: Student's t-Statistics for Predictive Regressions



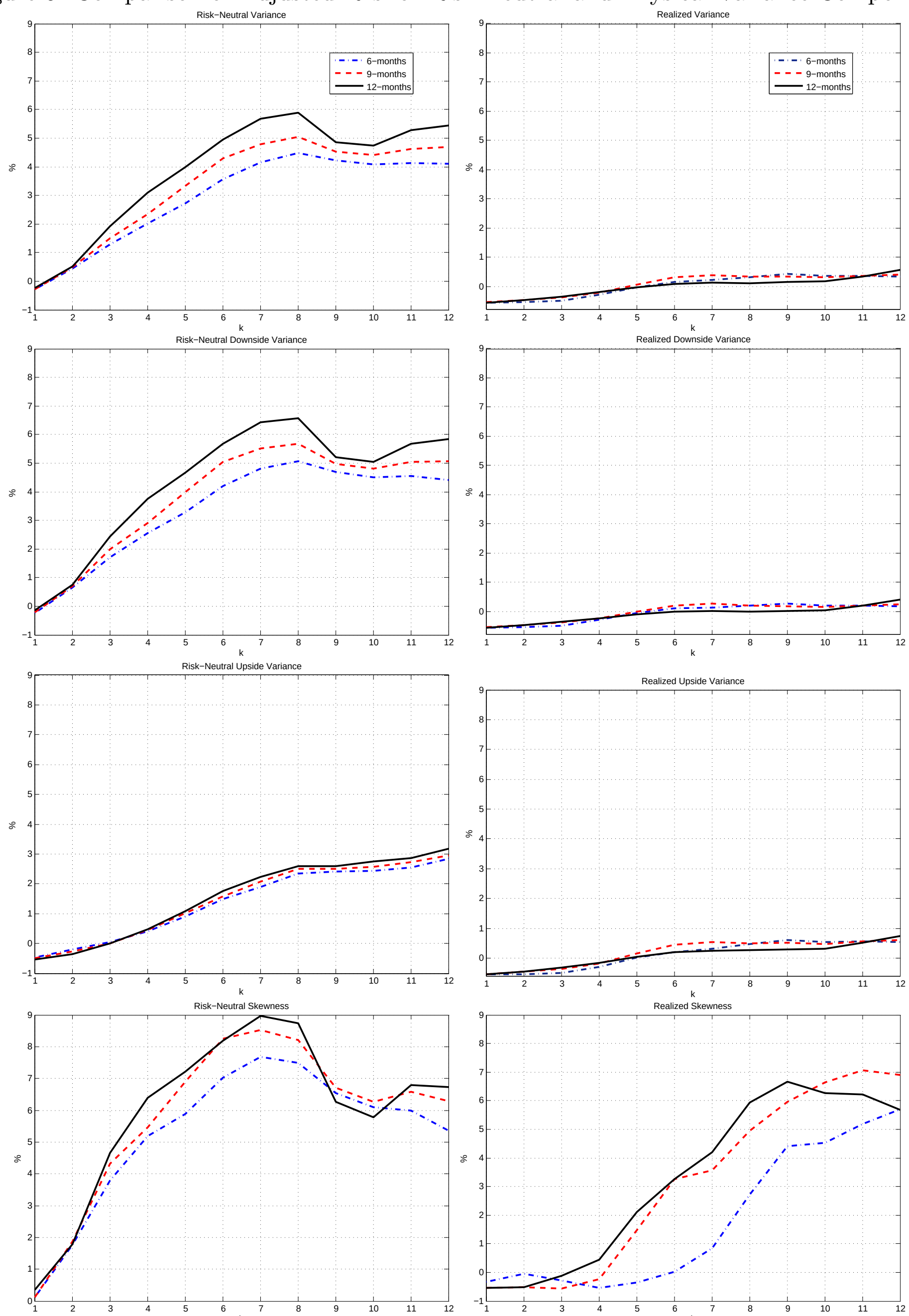
These figures plot the t-statistics for slope parameters of predictive regressions – Equation (40) – constructed following Hodrick (1992) from heteroscedasticity and serial correlation consistent standard errors that explicitly take account of the overlap in the regressions. The predictors here are variance risk, upside variance risk, downside variance risk, and skewness risk premia. In these figures, k is the prediction horizon, ranging between 1 and 12 months ahead. To simplify the figures, only three aggregation levels – h – are shown.

Figure 5: Adjusted R^2 for Predictive Regressions



These figures plot the adjusted R^2s of predictive regressions – Equation (40). The predictors here are variance risk, upside variance risk, downside variance risk, and skewness risk premia. In these figures, k is the prediction horizon, ranging between 1 and 12 months ahead. To simplify the figures, only three aggregation levels – h – are shown.

Figure 6: Comparison of Adjusted R^2s for Risk-Neutral and Physical Variance Components



These figures plot the adjusted R^2s for predictive regressions – Equation (40). The predictors here are risk-neutral and realized variance, upside variance, downside variance, and skewness. In these figures, k is the prediction horizon, ranging between 1 and 12 months ahead. To simplify the figures, only three aggregation levels – h – are shown.