BANK OF CANADA
BANQUE DU CANADA

Working Paper/Document de travail
2015-14

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www.bank-banque-canada.ca

Bank of Canada Working Paper 2015-14

April 2015

Revised April 2020

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ISSN 1701-9397

Acknowledgements

We thank our discussants Ana Babus, Denis Gorea, Harald Hau, Julian Kolm, Kristoffer Nimark, Richard Rosen, Laura Veldkamp, and Sergio Vicente. We gratefully acknowledge comments from Viral Acharya, Jason Allen, Kartik Anand, Elena Carletti, Amil Dasgupta, Michael Ehrmann, Douglas Gale, Piero Gottardi, Antonio Guarino, Christian Hellwig, Frank Heinemann, Ralf Meisenzahl, Alessandro Pavan, Morten Ravn, Wolfram Richter, Jean-Charles Rochet, Nikita Roketskiy, Lars Svensson, Michal Szkup, Dimitri Vayanos, Xin Zhang and seminar participants at FIRS 2017, ASSA 2015, Bank of Canada, Barcelona GSE Summer Forum 2014 (Information and Market Frictions; Experimental and Theoretical Macroeconomics), Behavorial Aspects in Macro and Finance conference 2014, Federal Reserve Board of Governors, CEA 2014, DIW Berlin, TU Dortmund, EEA 2013, EFA 2014, EFA Day-Ahead 2015, EUI Florence, Fed St. Louis, IMF, IWH Halle, London FIT at LBS, MPI Bonn, NBER Summer Institute 2014 in International Finance and Macroeconomics, NFA 2014, NORMAC 2013, RES 2014, Sveriges Riksbank, University College London, UB Barcelona, and Uppsala University. All remaining errors are our own. Earlier versions were circulated as "A wake-up call: information contagion and speculative currency attacks" and "A wake-up call: information contagion and strategic uncertainty". First version: June 2012. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the official views of CEPR, the Bank of Canada, or Sveriges Riksbank. We acknowledge outstanding research assistance by Jessica Redeschnig and Omar Abdelrahman.

Abstract

We offer a theory of contagion based on the information choice of investors after observing a financial crisis elsewhere. We study global coordination games of regime change in two regions with an unobserved common macro shock as the only link between regions. A crisis in the first region is a wake-up call to investors in the second region. It induces them to reassess the regional fundamental and acquire information about the macro shock. Contagion can occur even after investors learn that regions are unrelated (zero macro shock). Our results rationalize empirical evidence about contagious bank runs and currency crises after wake-up calls. We also discuss other testable implications of the model.

JEL classification: D82, F3, G01, G21
Bank classification: Exchange rates; Financial stability; International financial markets

Résumé

Nous présentons une théorie de la contagion fondée sur la décision des investisseurs d'acquérir ou non de l'information après avoir observé la survenue d'une crise financière dans un autre territoire. Nous étudions les jeux globaux de coordination autour d'un changement de régime dans deux régions qui ont pour seul lien un choc macroéconomique non observé. Une crise dans la première région envoie un signal d'alerte aux investisseurs de la seconde, qui sont alors poussés à réévaluer les facteurs fondamentaux de leur propre région et à acquérir de l'information sur le choc macroéconomique. Il peut y avoir contagion même si les investisseurs apprennent que les facteurs fondamentaux des deux régions ne sont pas reliés (absence de choc macroéconomique). Les résultats de notre étude expliquent les données empiriques sur la propagation des paniques bancaires et des crises de change après des signaux d'alerte. Nous abordons également d'autres implications vérifiables du modèle.

Classification JEL : D82, F3, G01, G21
Classification de la Banque : Taux de change; Stabilité financière; Marchés financiers internationaux

1 Introduction

Understanding the causes of financial contagion is an important question in banking and international finance. For example, Forbes (2012) distinguishes four mutually non-exclusive channels of contagion: trade, banks, portfolio investors, and wake-up calls. According to the wake-up call hypothesis—a popular explanation for contagion put forward by Goldstein (1998)—a financial crisis in region 1 is a wake-up call to investors in region 2 that induces them to re-assess and inquire about the fundamentals of region 2. Such a re-appraisal of risk can lead to a contagious spread of a crisis to region 2. In this paper, we offer a theory of contagion based on a re-assessment of local fundamentals and information acquisition after a wake-up call.

There is empirical support for wake-up call contagion both across markets and over time. Studying equity markets during the global financial crisis of 2007–09, Bekaert et al. (2014) identify wake-up calls as the key driver of contagion. Analyzing eurozone sovereign bond markets, Giordano et al. (2013) find evidence for contagion based on the wake-up call of the Greek crisis of 2009–10. For bond markets during the Asian crisis in 1997, Basu (2002) finds evidence for contagion based on the re-assessment of risks in some countries. Karas et al. (2013) find a wake-up call effect during the Russian banking panic of 2004, where deposit flows remained sensitive to bank capital, regardless of the introduction of deposit insurance. For the Panic of 1893, Ramirez and Zandbergen (2013) document contagion based on the wake-up call of newspaper reports about distant bank runs, which led to elevated deposit withdrawals in Montana. Despite this body of empirical evidence, there has been little theoretical work on the wake-up call hypothesis.

We offer a wake-up call theory of contagion based on a re-assessment of local fundamentals and information acquisition after observing a crisis elsewhere. We define contagion as an increase in the probability of a crisis in region 2 after a crisis in region 1, relative to the case of no crisis in region 1. Specifically, wake-up call contagion is the increase in the probability of a crisis in region 2 after a crisis in region 1 that prevails even if investors learn after the crisis in region 1 that regional fundamentals are uncorrelated. In our model, the wake-up call contagion effect arises from a differential information choice of investors who acquire information about the exposure of region 2 to a common macro shock only after the wake-up call of a crisis in region 1. As a result, observing no crisis is a more favorable event than if investors learn after observing a crisis that regional fundamentals are uncorrelated. Since tail events are never fully unexpected, the unfavorable news of a crisis elsewhere induces investors to acquire information about the risk of exposure (to a macro shock) that is otherwise considered unlikely.

While empirical work often measures the contribution of different contagion channels, our theoretical approach aims to isolate the wake-up call component in the transmission of financial crises. Therefore, we abstract from both common investors and balance sheet links. Building on global games (Carlsson and van Damme 1993), we study how information acquisition shapes the fundamental re-assessment and derive a set of testable implications consistent with empirical evidence.

We develop a global coordination game of regime change with incomplete information about a fundamental (Morris and Shin 2003). In contrast to the standard game, our model has two regions that move sequentially and whose only link is the exposure to an initially unobserved common macro shock. A financial crisis occurs in a given region if sufficiently many investors act against the regime (attack a currency peg, withdraw funds from a bank, or refuse to roll over debt). Investors in region 1 decide whether to act, which determines the outcome in region 1. Afterwards, investors in region 2 observe the endogenous public signal of whether a regime change occurred in region 1 and update their beliefs about the macro shock. Subsequently, investors decide whether to learn the macro shock at a cost and, thereafter, decide whether to act.

If crises are rare and the macro shock is negatively skewed, investors in region 2 have a higher incentive to acquire information after the wake-up call of observing a crisis in region 1 (Proposition 2). Intuitively, the negative skewness creates an asymmetry that makes it more valuable to acquire information after the rare event of observing a crisis. For an intermediate range of information costs, investors learn the macro shock if and only if a crisis occurred in region 1 (Proposition 1). This differential information choice is at the core of the fundamental re-assessment and shapes contagion. It arises since investors face an elevated risk of a strongly negative macro shock after a wake-up call, while the negative macro shock is less likely after no crisis. This updating result arises because observing a crisis in region 1 can whip around probabilities of tail events and focus investor attention on rarely observed downside risk. The value of information is higher after a crisis in region 1, since investors in region 2 benefit more from understanding whether regional fundamentals are linked. Specifically, an investor's benefit of tailoring its attack rule to the realized macro shock is higher after a crisis in region 1 than after no crisis.

We show that contagion can occur even if all investors learn that the macro shock is zero and regional fundamentals are unrelated (Proposition 3). That is, the probability of a crisis in region 2 after a crisis in region 1 and learning that region 2 has no exposure to region 1 is higher than the probability of a crisis in region 2 after no crisis in region 1. This result consists of two parts: endogenous information choice of investors and contagion. Endogenous information is critical for the contagion result that, in turn, is driven by Bayesian updating about the macro shock. Observing a crisis in region 1 and learning about a zero macro shock means that the crisis is unrelated to the fundamental in region 2. In contrast, absent a crisis in region 1, investors in region 2 choose not to acquire information and form a more optimistic belief about the macro shock. As a result, the probability of a crisis in region 2 is lower after no crisis in region 1.

A key assumption for the result on the differential information choice of investors is the negatively skewed distribution of the macro shock. There is an extensive empirical literature on the negative skewness of important macroeconomic variables, including GDP growth, individual stock returns, and the aggregate stock market. The literature on asymmetric business cycles studies the occurrence of sharp recessions and slow booms (for example, Neftçi 1984).[1] Barro (2006) links the high equity premium to the occurrence of rare disasters. More recent empirical research highlights the growth vulnerability dynamics, focusing on downside risks.[2] The negative skewness also plays a prominent role in the context of financial liberalization in developing countries. Rancière et al. (2003) find that financially liberalized developing countries exhibit a negatively skewed growth of both GDP and credit. Rancière et al. (2008) argue that a negative skewness captures systemic risk and is therefore a systemic component.

The wake-up call theory of contagion can be informative for a range of economic applications. It builds on negative macroeconomic outcomes adversely impacting financial conditions and is based on Bayesian learning and information choice when the macro shock is negatively skewed. For currency crises, speculators observe a currency attack and are uncertain about the magnitude of trade

[1]When measured over long periods, the negative skewness of real per capita GDP growth can be substantial, exceeding -5 for some countries (Barro 2006). Theoretical explanations for the negative skewness of output and total factor productivity include Acemoglu and Scott (1997), Veldkamp (2005), and Jovanovic (2006). Campbell and Hentschel (1992) and Bae et al. (2007) study the sources of the negative skewness of stock returns.

[2]For example, Adrian et al. (2019) study the evolution of the conditional distribution of future U.S. GDP growth and find that a build-up of negative skewness is associated with worsening financial conditions.

or financial links or institutional similarity.[3] For rollover risk and bank runs, wholesale investors observe a run elsewhere and are uncertain about interbank exposures.[4] For sovereign debt crises, bond holders observe a sovereign default elsewhere and are uncertain about the macroeconomic links, the commitment of the international lender of last resort, or the resources of multilateral bail-out funds.[5] For political regime change, activists observe a revolution, for example during the Arab spring, and are uncertain about the impact on their government's ability to stay in power.[6] The wake-up call theory of contagion is particularly relevant for currency and banking crises in developing countries, where a crisis often spreads to several countries. Institutional characteristics such as structural and policy distortions make individual countries prone to (potentially adverse) changes in the (international) macroeconomic environment (Dasgupta et al. 2011, Corsetti et al. 1999), exposing them to downside risks. Our macro shock captures such factors in a stylized way.

We derive two testable implications for information acquisition. First, information acquisition can amplify volatility measured as the increase in the dispersion of probabilities of a crisis in region 2 conditional on the realized macro shock, which forms the basis of a contagious spread of volatility. At the same time, the previously described differential information choice dampens volatility because investors refrain from tailoring their attack strategies, making them less sensitive when things are going well. Second, the extent of information acquisition about the exposure to aggregate or market-wide shocks is higher after observing a financial crisis elsewhere. This prediction is consistent with empirical evidence in Vlastakis and Markellos (2012). In a study on the U.S. stock market, they document a positive association between the demand for information on the market level, proxied by internet search intensity (Da et al. 2011) and may be interpreted as information acquisition about the common component of stock returns, with measures of volatility.

Other theories of contagion have been proposed. Regarding balance sheet links, see Allen and Gale (2000) and Dasgupta (2004) for interbank links and Kiyotaki and Moore (2002) for balance-sheet contagion. For a common discount factor channel, see Ammer and Mei (1996) and Kodres

[3] See Morris and Shin (1998) and Corsetti et al. (2004) for a one-regional global game that builds on the earlier works of Krugman (1979), Flood and Garber (1984), and Obstfeld (1986). For trade links, financial links, and institutional similarities see Glick and Rose (1999), Van Rijckeghem and Weder (2001, 2003), and Dasgupta et al. (2011).

[4] See Rochet and Vives (2004) and Goldstein and Pauzner (2005) for one-region bank runs.

[5] See Corsetti et al. (2006). See Drazen (1999) for membership contagion.

[6] For a one-regional global game of political regime change with endogenous information manipulation or dissemination, see Edmond (2013) and Shadmehr and Bernhardt (2015), respectively.

and Pritsker (2002). Regarding a common investor base, see Goldstein and Pauzner (2004) and Cole et al. (2016) for wealth effects, Pavlova and Rigobon (2008) for portfolio constraints, Taketa (2004) and Oh (2013) for learning about other investors. In terms of ex-post exposures, see Basu (1998) for a common risk factor, and Acharya and Yorulmazer (2008), Manz (2010) and Allen et al. (2012) for asset commonality among banks and information contagion. To distinguish our theory of contagion after wake-up calls more clearly from the literature, we nest a version of the standard information contagion channel as a special case.[7]

Calvo and Mendoza (2000) and Mondria and Quintana-Domeque (2013) also have endogenous information. Contagion arises in Calvo and Mendoza since globalization shifts the incentives of risk-averse investors from costly information acquisition to imitation and herding. In Mondria and Quintana-Domeque, the contagion mechanism is based on the reallocation of limited attention by risk-averse investors, where a higher relative attention allocated to one market induces a higher price volatility in another market. In contrast, we highlight a complementary channel where a wake-up call induces information acquisition about a macro shock and contagion without common investors, risk-aversion, or information processing constraints. Alternative modeling approaches include behavioral aspects (e.g. Caplin and Leahy (1994)), or the rational inattention literature.

Our modeling approach is closest to the literature on information choice in global coordination games initiated by Hellwig and Veldkamp (2009). They show that the information choices of investors inherit the strategic motive of an underlying beauty contest, which can result in multiple equilibria. Our game of regime change with complementarity in actions also yields strategic complementarity in information choices. While multiple equilibria exist, a sufficiently negatively skewed macro shock ensures a uniqueness for an intermediate range of information costs. In contrast to the acquisition of publicly available information, Szkup and Trevino (2015) examine private information acquisition in global games of regime change.

Our theory is also related to the literature on financial crises that rationalizes how small shocks have large effects. Dang et al. (2015) and Gorton and Ordoñez (2014, 2019) study how information-insensitive debt can become information-sensitive if fundamentals deteriorate, triggering adverse selection concerns and information acquisition about the collateral backing the loans. As in our pa-

[7]See Chen (1999) for a model with information contagion and uninformed junior claimants, Li and Ma (2016) for or a model on uninformed asset buyers and Chen and Suen (2013) for information contagion with model uncertainty.

per, information acquisition follows negative news but our channel differs as we analyze a Bayesian learning channel about macro fundamentals. Instead of a fear of adverse selection, there is a fear of macro downside risk with the skewed macro shock driving the differential information choice.

We proceed as follows. Section 2 describes the model. We solve for its equilibrium and describe our contagion results in section 3. Section 4 discusses testable implications. Section 5 discusses extensions and robustness issues. Section 6 concludes. All proofs are in the Online Appendix.

2 Model

We study global coordination games of regime change played sequentially in two regions $t = 1,2$. Each region has a different unit continuum of risk-neutral investors $i \in [0,1]$. Investors in region $t = 1$ move first, followed by investors in region $t = 2$.

Attack decision. In each region, investors simultaneously decide whether to attack the regime, $a_{it} = 1$, or not, $a_{it} = 0$. The outcome of the attack depends on the aggregate attack size, $A_t \equiv \int_0^1 a_{it}\, di$, and a regional fundamental $\Theta_t \in \mathbb{R}$ that measures the strength of the regime. A regime change occurs if enough investors attack, $A_t > \Theta_t$. Following Vives (2005), an attacking investor in region t receives a benefit $b_t > 0$ if a regime change occurs and otherwise incurs a loss $\ell_t > 0$, where $\gamma_t \equiv \frac{\ell_t}{b_t+\ell_t} \in (0,1)$ is the relative cost of failure:

$$u(a_{it} = 1, A_t, \Theta_t) = b_t \;\; \mathbf{1}_{\{A_t>\Theta_t\}} - \ell_t \;\; \mathbf{1}_{\{A_t\leq\Theta_t\}}. \tag{1}$$

The payoff from not attacking is normalized to zero, so the relative payoff from attacking increases in the attack size A_t (global strategic complementarity in attack decisions) and decreases in Θ_t.

A regime change is a currency attack, bank run, or debt crisis. The fundamental is interpreted as the ability of a monetary authority to defend its currency (Morris and Shin 1998; Corsetti et al. 2004), the measure of investment profitability (Rochet and Vives 2004; Goldstein and Pauzner 2005; Corsetti et al. 2006), or the resources or willingness of a debtor to repay. Investors are interpreted as currency speculators, as retail or wholesale bank creditors who withdraw funds, or as debt holders who refuse to roll over.

Macro shock. Each regional fundamental Θ_t comprises a regional component θ_t and a common component m. This common macro shock is the only link between regions:

$$\Theta_t = \theta_t + m, \tag{2}$$

where each θ_t follows an independent normal distribution with mean $\mu \in (-\infty, \infty)$ and precision $\alpha_t \in (0, \infty)$ that is independent of the macro shock. Unless stated otherwise, we consider $\alpha_1 = \alpha_2 \equiv \alpha$. Depending on its realization, the macro shock induces a positive correlation between regional fundamentals Θ_1 and Θ_2. Specifically, region 2 is exposed to region 1 if $m \neq 0$. The macro shock is assumed to take one of three values:

$$m = \begin{cases} \Delta & \quad p \\ -s\Delta \quad \text{w.p.} & \quad q \\ 0 & \quad 1-p-q, \end{cases} \tag{3}$$

where $p \in [0,1]$, $q \in [0, 1-p]$, $\Delta > 0$, $s > 0$. We impose $p = qs$ to ensure an unbiased macro shock. Its variance is $p(1+s)\Delta^2$ and its skewness is $\frac{1-s}{\sqrt{p(1+s)}}$, which is negative if and only if $s > 1$.[8]

The macro shock is initially unobserved, motivated by our applications to financial crises. For currency attacks or sovereign debt crises, this uncertainty about the macro shock reflects the unknown relevance of certain institutional similarities or of real or financial linkages across debtors. For bank runs, it reflects the uncertainty about bank portfolios and interbank exposures.

Incomplete information. Following Carlsson and van Damme (1993), there is incomplete information about the fundamental. Each investor receives a noisy private signal x_{it} before deciding whether to attack (Morris and Shin 2003):

$$x_{it} \equiv \Theta_t + \varepsilon_{it}. \tag{4}$$

Idiosyncratic noise ε_{it} is identically and independently normally distributed across investors with zero mean and precision $\beta \in (0, \infty)$. Each noise term is independent of the macro shock and the regional component.

[8]The parameter s also affects the variance of the macro shock but the effect on the skewness is key. In the robustness section 5.4, we argue that the negative skewness governed by s is at the heart of our result and study modifications to our model, including changes to Δ such that s only affects the skewness of the macro shock.

Information acquisition. An information stage precedes the coordination stage in region 2, as summarized in the timeline in Table 1. First, investors in region 2 observe whether there is a crisis in region 1. Second, investors in region 2 can acquire costly information about the macro shock.[9] Investors simultaneously decide whether to purchase a perfectly revealing signal about the macro shock at cost $c > 0$.[10] In terms of wholesale investors or currency speculators, costly information acquisition could be the hiring of analysts who assess publicly available information to gauge the relevance of institutional characteristics such as structural or policy distortions that are shared across regions and make them prone to changes in the macroeconomic environment.

Date 1	**Date 2**
1. Macro shock m and regional component θ_1 realized but unobserved	1. Regional component θ_2 realized but unobserved
Coordination stage in region 1	**Information stage in region** 2
2. Investors receive private information x_{i1} and choose whether to attack the regime, $a_{i1} \in \{0,1\}$	2. Investors choose whether to acquire information about macro shock m at cost $c > 0$
3. Payoffs to investors in region 1	**Coordination stage in region** 2
4. Outcome of regime publicly observed	3. Investors receive private information x_{i2} and choose whether to attack the regime, $a_{i2} \in \{0,1\}$
	4. Payoffs to investors in region 2

Table 1: Timeline of events.

3 Equilibrium

3.1 Region 1

We first consider the equilibrium in region 1. A Bayesian equilibrium is an attack decision a_{i1} for each investor i and an aggregate attack size A_1 that satisfy both individual optimality for all investors, $a^*_{i1} = \arg\max_{a_{i1}\in\{0,1\}} \mathbb{E}[u(a_{i1}, A_1, \Theta_1)|x_{i1}]$, and aggregation, $A^*_1 = \int_0^1 a^*_{i1} di$. Let $n_1 \in [0,1]$

[9] We abstract from information acquisition in region 1 without loss of generality (see also section 5.5).

[10] We discuss an extension to noisy signals about the macro shock in section 5.5.

be the proportion of investors in region 1 informed about the macro shock. If all investors are informed, $n_1 = 1$, the analysis is standard (see, e.g., Morris and Shin (2003), Morris and Shin (2004)). If some investors are uninformed, $n_1 < 1$, the analysis is non-standard and requires the use of mixture distributions. We focus on the case of uninformed investors, $n_1 = 0$, but the result can be readily extended to the general case, $0 < n_1 < 1$, following the same steps as in the analysis of the coordination stage in region 2.

Lemma 1 ***Equilibrium in region 1.*** *Let $n_1 = 0$. If private information is sufficiently precise, there exists a unique monotone Bayesian equilibrium. Each investor attacks when the private signal is below a signal threshold, $x_{i1} < x_1^*$. A crisis occurs when the fundamental is below a fundamental threshold, $\Theta_1 < \Theta_1^*$.*

Proof See Appendix A.1.

Lemma 1 extends the analysis in standard global games models (e.g. Morris and Shin 2003) to the case where the posterior of investors follows a mixture distribution over different macro states, comprising conditional normal distributions. The equilibrium is characterized by an indifference condition from individual optimality and by a critical mass condition which states that the proportion of attacking investors A_1^* equals the fundamental threshold Θ_1^*. The equilibrium conditions can be reduced to one equation in one unknown. Using the results of Milgrom (1981) and Vives (2005), the best-response function of individual investors are strictly increasing in the thresholds used by other investors (Appendix A.1.1). The common requirement of sufficiently precise private information suffices for uniqueness in monotone equilibrium in the case of mixture distributions.

3.2 Region 2

Considering region 2, let $n_2 \in [0,1]$ be the proportion of investors in region 2 who acquire information about the macro shock m and $d_i \in \{I,U\}$ is the information choice of investor i, with corresponding attack rules of informed and uninformed investors, $a_I \equiv a_{i2}(d_i = I)$ and $a_U \equiv a_{i2}(d_i = U)$.

Definition 1 *A pure-strategy monotone perfect Bayesian equilibrium in region 2 comprises an information choice for each investor, $d_i^* \in \{I,U\}$, an aggregate proportion of informed investors,*

$n_2^* \in [0,1]$*, an attack rule for informed and uninformed investors,* $a_I^*(m,\cdot)$ *and* $a_U^*(\cdot)$*, and an aggregate attack size,* A_2^**, such that:*

1. *At the information stage, investors optimally choose their information* d_i*.*

2. *The proportion of informed investors is consistent with individual choices,* $n_2^* = \int_0^1 d_i^*\, di$*.*

3. *At the coordination stage, attack rules are optimal, where uninformed investors use* $a_U^*(\cdot)$ *and informed investors use* $a_I^*(m,\cdot)$ *for each macro shock.*

4. *The aggregate attack size is consistent with attack rules for each macro shock:*

$$A_2^* \;=\; n_2^* \int_0^1 a_I^*(m,\cdot)di + (1-n_2^*) \int_0^1 a_U^*(\cdot)di. \tag{5}$$

To derive analytical results, we maintain the following assumption throughout.

Assumption 1 *Private information is precise,* $\beta > \underline{\beta}$*, public information is imprecise,* $\alpha < \overline{\alpha}$*, a zero macro shock is unlikely,* $1-p-q < \eta$*, crisis are rare,* $\mu > \underline{\mu}$*, the macro shock is sufficiently negatively skewed,* $s > \underline{s}$*.*

Assumption 1 states sufficient conditions for the main result on wake-up call contagion, where the bounds are described in the proofs. The rareness of crises implies a strong fundamental re-assessment after the wake-up call of a crisis in region 1 and the negative skewness is crucial for the incentives of investors to acquire information only after observing a crisis but not for the Bayesian updating channel. The assumption of a sufficiently high relative precision of private information is common in the global games literature (e.g. Vives 2005). While sufficiently imprecise public information is not required for the existence of unique attack rules, it leads to concentrated posterior beliefs about the macro shock and facilitates the analysis of how equilibrium fundamental and signal thresholds vary with the proportion of informed investors. The sufficiently low probability of a zero macro shock simplifies the analysis. It allows us to focus on the favorable and unfavorable macro states central to the re-assessment. These conditions are sufficient, but not necessary, and help with tractability and exposition. The numerical examples below show that our results also obtain under less restrictive conditions. We further discuss the robustness in section 5.

We proceed by constructing the equilibrium in region 2. Investors in region 2 observe whether a crisis occurred in region 1, and use Bayes' rule to re-assess the fundamental of region 2, specifically the macro shock m. Since only a proportion of investors may choose to acquire information, we allow for heterogeneous priors. There are three distinct fundamental thresholds – one for each realized macro shock – and thus three critical mass conditions. Similarly, there are four indifference conditions – one for uninformed investors and one for informed investors for each macro shock realization. The system of equations is derived in Appendix A.2. If some investors are informed, we denote the fundamental thresholds in region 2 as $\Theta_I^*(m)$. If all investors are uninformed, $n_2 = 0$, we denote the fundamental thresholds in region 2 as Θ_U^*.

Proposition 1 ***Equilibrium in region 2.*** *For intermediate information costs, $c \in (\underline{c}, \overline{c})$, there exists a unique monotone perfect Bayesian equilibrium. At the information stage, investors acquire information only after a wake-up call, $n_2^* = \mathbf{1}_{\{\Theta_1 < \Theta_1^*\}}$. At the coordination stage, investors use threshold strategies:*

1. *After no crisis in region 1, investors choose to be uninformed and attack whenever their private signal is sufficiently low, $x_{i2} < x_U^*$, and a crisis occurs whenever the fundamental is sufficiently low, $\Theta_2 < \Theta_U^*$.*

2. *After a crisis in region 1, investors choose to be informed and attack whenever their private signal is sufficiently low relative to a macro-shock-specific threshold $x_{i2} < x_I^*(m)$, and a crisis occurs whenever the fundamental is sufficiently low relative to a macro-shock-specific threshold, $\Theta_2 < \Theta_I^*(m)$.*

Proof See Appendix A.2.5 for a proof and Appendix A.2 for a derivation of the equilibrium conditions, as well as the required results on information acquisition discussed below.

The equilibrium is in dominant actions at the information stage. Irrespective of the information choices of other investors, each investor acquires information *only* after the wake-up call of a crisis in region 1. This occurs whenever the fundamental in region 1 is below its threshold, $\Theta_1 < \Theta_1^*$. When investors in region 2 choose to be uninformed, they use the same attack threshold, x_U^*, and there is one fundamental threshold, Θ_U^*, where both thresholds are independent of the macro shock. In contrast, when investors choose to be informed, they tailor their attack rule to the macro state, $x_I^*(m)$, and there is one fundamental threshold for each state, $\Theta_I^*(m)$.

We next build intuition for the differential information choice in Proposition 1. Examining the value of information, we trace out how investors' incentives to acquire information about the macro shock are affected by the wake-up call and other investors' information choices. Let $f \in \{0,1\}$ indicate whether a crisis occurred in region 1. After a wake-up call, $f = 1$, investors learn that the fundamental in region 1 was low, $\Theta_1 < \Theta_1^*$. Conversely for $f = 0$, the fundamental was high, $\Theta_1 \geq \Theta_1^*$. Using Bayesian updating, Lemma 2 in Appendix A.2.1 states that a less (more) favorable macro shock realization is more likely after a crisis (no crisis).

The resulting re-assessment determines the incentives of investors to acquire information, with a higher value of information after a wake-up call. Since crises are rare events, there is a strong Bayesian updating channel after a wake-up call. The negatively skewed macro shock generates an asymmetry, which assures that it is more valuable to acquire information about the realized macro shock after observing a crisis in region 1. The probability of a negative macro shock is small without a crisis in region 1, but it is substantially higher after a crisis. Hence, investors in region 2 have a high benefit from learning about the macro shock and tailoring their attack decision. This is the key effect behind the differential information choice in Proposition 1.

We proceed by discussing the value of information and how it affects the information acquisition incentives. The value of information is defined as the difference between the expected utility of an informed investor, EU_I, and an uninformed investor, EU_U, as derived in Appendix A.2.3. It depends on the proportion of informed investors and on whether a crisis occurred in region 1:

$$v(n_2, f) \equiv EU_I - EU_U. \tag{6}$$

Informed investors observe whether a crisis occurred and take into account the possible realizations of m, since these affect the signal thresholds, $x_I^*(m)$. By contrast, uninformed investors cannot tailor their attack strategy and must use the *same* signal thresholds x_U^* for all realized macro shocks. As a result, the signal thresholds of informed and uninformed investors differ and $v(n_2, f) > 0$.[11]

Information about the macro shock allows an investor to tailor her behavior and reduce two types

[11] To evaluate the incentives of investors to acquire information, we study the optimal attack behavior for any given proportion of informed investors and allow for some investors to be informed while others are uninformed, resulting in heterogeneous priors about the macro shock that follow a mixture distribution. In another global game with mixture distributions, Chen et al. (2012) develop a theory of rumors during political regime change. However, they abstract from both contagion and information choice.

of errors. First, when an investor attacks the regime although no crisis occurs, she incurs a loss (type-I error). Second, when an investor does not attack although a crisis occurs, she could have earned a benefit (type-II error). The value of information is governed by the relationship between these two types of errors. The marginal benefit of increasing $x_I^*(-s\Delta)$ above x_U^* is positive because the type-II error is relatively more costly than the type-I error. By contrast, the marginal benefit of decreasing $x_I^*(\Delta)$ below x_U^* is positive because the type-I error is more costly. In sum, informed investors attack more aggressively upon learning the low macro shock realization, $m = -s\Delta$, and less aggressively upon learning the high realization, $m = \Delta$.

Next, we turn to the strategic aspect of information acquisition. The signal thresholds of informed and uninformed investors depend on the proportion of informed investors. We find that the difference in signal thresholds increases monotonically in the proportion of informed investors, as derived in Lemma 3 in Appendix A.2.2. The divergence of signal thresholds with an increasing proportion of informed investors induces a strategic complementarity in information choice, $\frac{dv(n_2,f)}{dn_2} \geq 0$, as derived in Lemma 4 in Appendix A.2.3. Intuitively, the individual attack decision of an informed investor is more strongly adjusted the larger the proportion of informed investors, which in turn increases the value of information. In the words of Hellwig and Veldkamp (2009), investors want to know what others know in order to do what others do.

Figure 1 shows the attack threshold of informed and uninformed investors. First, informed investors attack more (less) aggressively after observing a negative (positive) macro shock. Second, conditional on observing a wake-up call, both types of investors attack more aggressively. Comparing the left and right panel, all signal thresholds are higher after a wake-up call for each macro state and for all $n_2 \in [0,1)$. When all investors are informed, $n_2 = 1$, the signal thresholds coincide irrespective of whether a crisis occurred in region 1, since region 1's outcome does not contain any information beyond the macro shock.

The relationship between the signal thresholds of informed investors is derived in Lemma 3 in Appendix 2.2.2. Strict divergence of $x_I^*(n_2,-s\Delta)$ and $x_I^*(n_2,\Delta)$ in the proportion of informed investors follows from $\frac{d\Theta_2^*(n_2,-s\Delta)}{dn_2} > 0$ and $\frac{d\Theta_2^*(n_2,\Delta)}{dn_2} < 0$. Lemma 3 shows that the signal thresholds are monotonic in the proportion of informed investors. Moreover, $x_I^*(n_2,0)$ and $x_U^*(n_2,m)$ are bounded by $x_I^*(1,-s\Delta)$ and $x_I^*(1,\Delta)$. Solving the equilibrium condition in equation (26) when

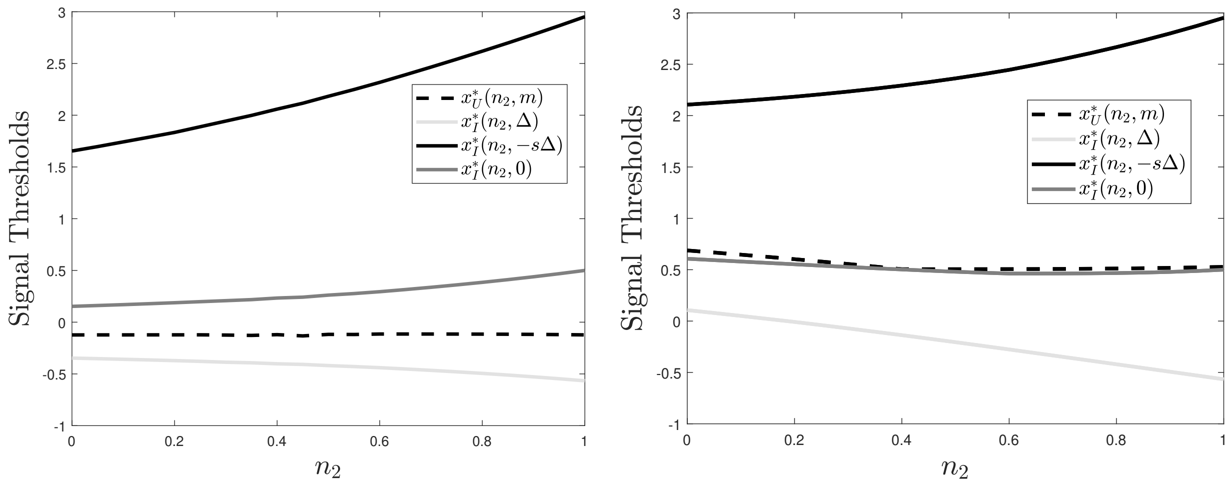

Figure 1: Signal thresholds of informed and uninformed investors, x_I^* and x_U^*, as a function of the proportion of informed investors, n_2, after no crisis (left panel) and a crisis (right panel), proven in Lemma 3. Parameter values are $\alpha = \beta = 1$, $\mu = \Delta = \gamma_{1,2} = p = 1/2$ and $s = 3$.

all investors are uninformed, we find that $[\Theta_U^* | f = 1] > [\Theta_U^* | f = 0]$ and, hence, $[x_I^*(0,m)|f = 1] > [x_I^*(0,m)|f = 0]$ for all $m \in (-s\Delta, 0, \Delta)$. The right panel of Figure 1 shows an upward shift in signal thresholds for all $n_2 < 1$, which stems from the updating of uninformed investors' belief about the macro shock (Lemma 2). This shift is stronger when more investors are uninformed, but the difference in thresholds is already noticeable for $n_2 = 0$: $[x_U^*(0,m)|f = 1] > \frac{1}{2} > [x_U^*(0,m)|f = 0]$.

Proposition 2 ***Wake-up call and the value of information.*** *The value of information is higher after a crisis in region* 1 *independent of the proportion of informed investors:*

$$v(1,1) > v(0,1) > v(1,0) > v(0,0). \tag{7}$$

Proof See Appendix A.2.4.

Proposition 2 ranks the value of information that affects the information choices of investors. The first and third inequality in (7) represent the strategic complementarity in information choices derived in Lemma 4 in Appendix A.2.3. The second inequality is due to the negatively skewed macro shock. For a sufficiently negatively skewed macro shock and rare crisis, as guaranteed by Assumption 1, we have $v(0,1) > v(1,0)$. As a result, there exists an intermediate range of information costs $c \in (\underline{c}, \overline{c})$ with $\underline{c} \equiv v(1,0)$ and $\overline{c} \equiv v(0,1)$ such that all investors choose to acquire information if and only if a crisis occurs in region 1 (the wake-up call).

Figure 2 shows that the value of information increases in the proportion of informed investors

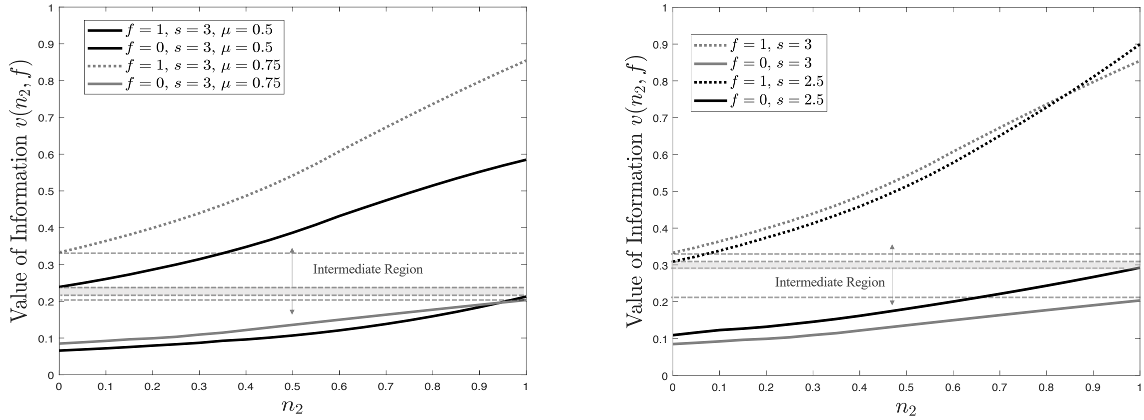

Figure 2: The value of information v and the proportion of informed investors n_2 with and without a wake-up call, $f \in \{1,0\}$. Parameters are as in Figure 1 and the left panel depicts different values of μ, while the right panel shows the effect of a higher level of s that increases the skewness of the macro shock (for $\mu = 3/4$). In both cases the intermediate range of information costs expands. Formal statements can be found in Proposition 2 and Lemma 4.

(strategic complementarity) and in the occurrence of a crisis in region 1 (Proposition 2 and Lemma 4). When crises are rare and the macro shock is sufficiently negatively skewed, the Bayesian updating channel is strong and ensures a unique equilibrium for intermediate values of information costs despite strategic complementarity in information choices. While we established the existence of the intermediate region analytically, comparative statics are difficult to obtain in general. The left panel shows the existence of an intermediate region where $v(0,1) > v(1,0)$ for $\mu = 1/2$, which implies a relatively high crisis incidence (see also Figure 4). There is a tendency for the intermediate region to expand if crisis are less frequent (left panel for higher μ) and if s increases (right panel). This resonates with Proposition 2 since the former strengthens the Bayesian updating channel (Figure 3) and the latter increases the benefits from tailoring of signal thresholds, $\frac{d(x_I^*(1,-s\Delta)-x_I^*(1,\Delta))}{ds} > 0$.[12]

We proceed by describing the contagion mechanism and build intuition for the Bayesian updating channel after the occurrence of a crisis in region 1.

3.3 Contagion

Having established a unique equilibrium for intermediate information costs, we turn to the question of contagion after a wake-up call. Contagion is defined as the increase in the likelihood of a crisis in region 2 after a crisis in region 1, compared to no crisis in region 1.

[12]In the Online Appendix A.7, we analytically show for a special case that the differential value of information increases in the parameter s, governing the negative skewness of the macro shock, $\frac{d[v(1,1)-v(1,0)]}{ds} > 0$.

Our main result is that contagion occurs even if investors learn that region 2 is not exposed to region 1 (zero macro shock). This result isolates the wake-up call component of contagion. It builds on the equilibrium information choices in Proposition 1 and holds under Assumption 1.

Proposition 3 ***Wake-up call contagion.*** *Let $c \in (\underline{c}, \overline{c})$. A financial crisis in region 2 is more likely after a crisis in region 1 when all investors acquire information and learn that the macro shock is zero, than after no crisis in region 1, when all investors choose not to acquire information:*

$$\Pr\{\Theta_2 < \Theta_I^*(m) | m = 0\} > \Pr\{\Theta_2 < \Theta_U^*\}. \tag{8}$$

Proof See Appendix A.3.

This contagion result rests on the unique equilibrium for intermediate information costs. The left-hand side of inequality (8) is the probability of a crisis in region 2 after a crisis in region 1, a wake-up call, that induces investors *to acquire information* and when they learn that the macro shock is zero. The right-hand side is the probability of a crisis in region 2 after no crisis in region 1, that induces investors *not to acquire information*. Hence, the conditional probability implicit in the right-hand side allows for any realization of the unobserved macro shock.

We find that a crisis in region 2 is more likely after a crisis in region 1 than after no crisis in region 1 *even if* all investors acquire information and learn that the macro shock is zero. Learning that the macro shock is zero implies that the crisis in region 1 is unrelated to region 2. In contrast, no crisis in region 1 implies a more favorable view about the fundamental in region 2 due to the unobserved macro shock. Hence, the decreased crisis probability after observing no crisis in region 1 is a key driver of the result. This effect tends to lower the right-hand side of inequality (8).

While Bayesian updating is fairly mechanical, the result of wake-up call contagion arises endogenously. For intermediate information costs, investors choose to acquire information only after the wake-up call. In other words, the comparison of scenarios in equation (8) hinges on the differential information choice. Critically, our mechanism of wake-up call contagion based on endogenous information is distinct from the information contagion literature with ex-post correlated fundamentals (see section 5). It allows for contagion even when the fundamentals of the two regions are uncorrelated ex-post, but potentially correlated ex-ante.

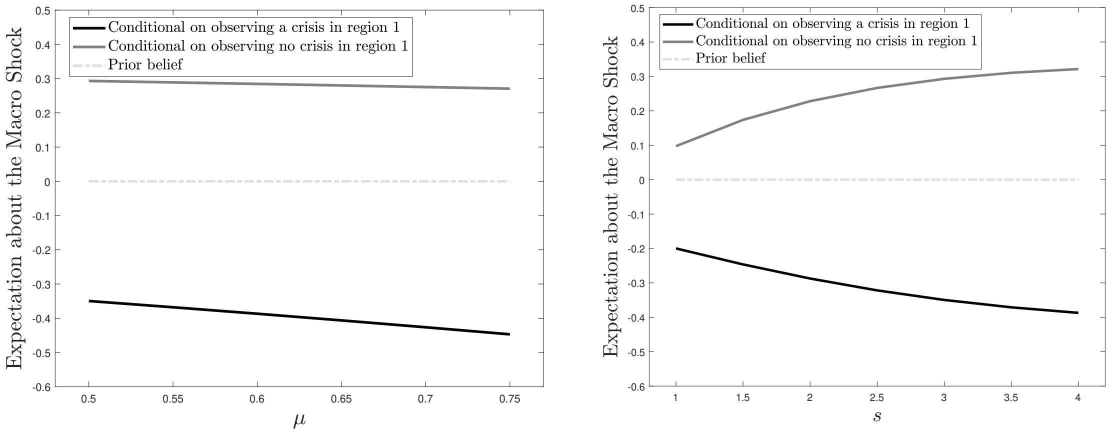

Figure 3: The two panels illustrate the updating of the belief about the mean of the macro shock for different strengths of regional fundamentals μ (left panel) and for different values of the parameter s, which governs the skewness (right panel). The parameter values are as in Figure 1.

The Bayesian updating channel is about the re-assessment of the macro shock. Intuitively, the observation of a crisis in region 1 can whip around probabilities of tail events and focus attention on rarely observed downside risk. The left panel of Figure 3 shows the re-assessment for different values of the average strength of regional fundamentals, μ. The dotted line reflects the prior about the macro shock, which has zero mean, $E[m] = 0$. After *observing a crisis* in region 1, the conditional expectation about the macro shock is negative, $E[m|f = 1] < 0$ (dark solid line). This result is due to an upward revision of the probability of a negative macro shock after bad news, $\Pr\{m = -s\Delta|f = 1\} > q$. After *observing no crisis* in region 1, by contrast, the expectation about the macro shock is positive (grey solid line), since the probability of a negative macro shock is revised down. The re-assessment is stronger—that is the difference between $E[m|f = 1]$ and $E[m|f = 0]$ is higher—when the average strength of regional fundamentals is higher and crises are rare. The right panel shows the re-assessment of the mean of the macro shock for different values of skewness, which is governed by s. The main insights are unaltered and an increase in s increases the magnitude of the re-assessment for the same reasons. However, this panel also shows that the Bayesian updating results arise even if $s = 1$ (no skewness).[13]

We wish to highlight that our contagion results do not hinge upon a common investor base or balance sheet links across regions. Therefore, Proposition 3 isolates the wake-up call component of contagion by showing that contagion occurs even if investors learn that the macro shock is zero. The assumption of a negatively skewed macro shock is inessential for the Bayesian updating

[13] In the Online Appendix A.8, we analytically describe the Bayesian updating channel under the conditions of Assumption 1, which is driven by $\frac{dE[m|f=1]}{d\mu} < 0$ and $\frac{d(E[m|f=0]-E[m|f=1])}{ds} > 0$.

channel that governs inequality (8). However, it is crucial for the information choice underlying the comparison in inequality (8), which is driven by the strong fundamental reassessment. Hence, the wake-up call effect and the associated testable implications hinge on downside risk in the form of rare but strongly negative shocks to fundamentals.

Figure 4 illustrates the magnitude of the wake-up call contagion effect. We compare the crisis probability by plotting both sides of inequality (8). In the numerical example, the wake-up call contagion effect is significant and its magnitude can exceed 15%. The bounds on the precision of private and public signals are not more stringent than the standard conditions used to assure uniqueness of equilibria in global games models (Morris and Shin 2003; Svensson 2006). Notably, the result of wake-up call contagion prevails when the probability of a zero macro shock is rather high (such as $1-p-q=1/3$) and when crises are relatively frequent (such as $\mu=1/2$), illustrating the robustness of the key results from relaxing the sufficient conditions stated in Assumption 1.

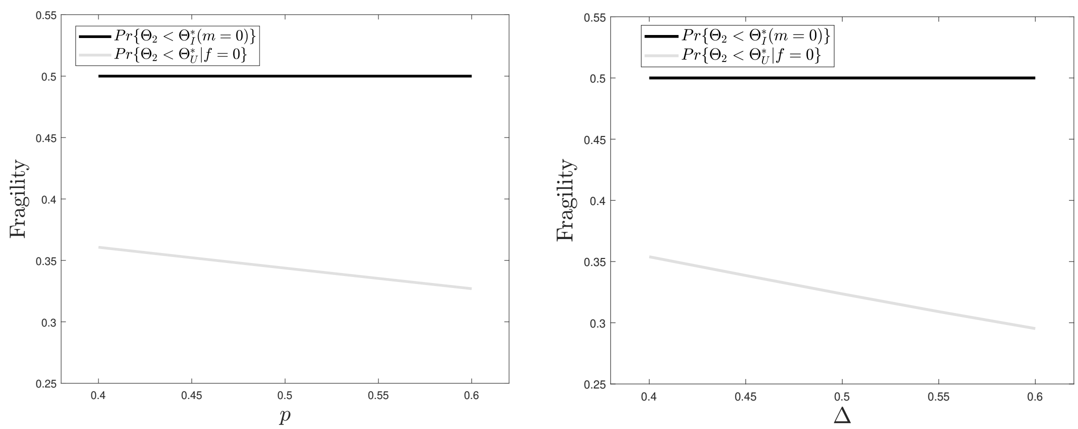

Figure 4: The magnitude of the wake-up call contagion effect in isolation, measured as the difference between the two sides of inequality (8) in Proposition 3. The left panel shows the increase in wake-up call contagion under the baseline if p increases from 0.4 to 0.6 and $\Delta = 0.4$. The right panel shows the increase in wake-up call contagion if Δ increases from 0.4 to 0.6 and $p = 0.6$. The other parameter values are as in Figure 1.

The magnitude of wake-up call contagion is governed by the strength of the Bayesian updating channel. Absent a crisis in region 1, uninformed investors place a positive probability $\Pr\{m = \Delta|f=0\} > p$ on a positive realization of the macro shock. Given that an increase in p is associated with a more favorable view about fundamentals after not observing a crisis in region 1, the difference in likelihoods of a crisis in region 2, $\Pr\{\Theta_2 < \Theta_I^*(m)|m=0, f=1\} - \Pr\{\Theta_2 < \Theta_U^*|f=0\}$, is positive and increasing in p (left panel). The magnitude of the wake-up call effect also increases in Δ (right panel), the level of the positive macro shock. Intuitively, a higher Δ is also associated with a more favorable view about fundamentals after not observing a crisis.[14]

This contagion result is further strengthened when regions are indeed related ex-post, that is when the macro shock takes a negative value.

Corollary 1 ***Negative macro shock.*** *The result of Proposition 3 is strengthened if all investors choose to acquire information after a crisis in region* 1 *and learn that the macro shock is negative:*

$$\Pr\{\Theta_2 < \Theta_I^*(m)|m = -s\Delta\} > \Pr\{\Theta_2 < \Theta_I^*(m)|m = 0\}. \tag{9}$$

Proof The proof parallels that of Proposition 3 and is therefore omitted.

The left-hand (right-hand) side of inequality (9) is the probability of a crisis in region 2 after a crisis in region 1, a wake-up call, that induces investors to acquire information and when they learn that the macro shock is negative (zero). On the left-hand side, beliefs are less favorable and the crisis is more likely.

4 Testable Implications

Our theory of contagion after a wake-up call has two sets of testable implications described in this section. We discuss how these implications have been tested or can be tested in future work.

Since the wake-up call contagion result relies on endogenous information acquisition, we first describe predictions related to information acquisition.

Prediction 1: The extent of information acquisition about the exposure to aggregate or market-wide shocks is positively associated with an increase in volatility, which forms the basis of a contagious spread of volatility after a wake-up call.

[14] Analyzing the comparative statics for the magnitude of the wake-up call contagion effect is challenging and we do it for a special case in the Online Appendix A.9. Since $s > 1$ is critical for the differential information choice—but not for the Bayesian updating channel—the analysis can be simplified by considering the case $s = 1$. Using equation (21) and the result from Lemma 2 we can show analytically that $\Pr\{\Theta_2 < \Theta_I^*|m = 0, f = 1\} > \Pr\{\Theta_2 < \Theta_U^*|f = 0\}$, $\forall p, \Delta$ and that the difference in likelihoods of a crisis is increasing in p (under the sufficient condition that α_1 is small) and increasing in Δ.

The first prediction is on financial fragility and highlights how information acquisition can amplify volatility, when measured as the increase in the dispersion of probabilities of a crisis in region 2 conditional on the macro shock realization (as may be observed by the empiricist). To see this, recall that, the occurrence of information acquisition about the exposure to aggregate or market-wide shocks hinges on observing the wake-up call of a crisis elsewhere and on the cost of information. Moreover, the acquisition of information about the macro shock induces investors to tailor their attack strategy to the observed macro shock. Hence, the total contagion effect after a wake-up call is stronger when investors learn that the macro shock realization is negative, $m = -s\Delta$ (see also Corollary 1). Conversely, the total contagion effect after a wake-up call is weaker when investors learn that the macro shock realization is positive, $m = \Delta$. In sum, the dispersion in the crisis probabilities conditional on the macro state increases as a result of the information acquisition after observing the wake-up call of a crisis elsewhere. That is, a crisis elsewhere amplifies volatility through endogenous information acquisition. Moreover, the dispersion of the probabilities of a crisis in region 2 conditional on the macro shock realization is stronger if the aggregate component of the fundamental exhibits a more severe downside risk because $\frac{d(x_I^*(1,-s\Delta)-x_I^*(1,m))}{ds} > 0$, $\forall m \in \{0, \Delta\}$.

Taken together, the wake-up call mechanism can contribute to explaining the spread of volatility in episodes of emerging market turmoil by offering a complementary contagion channel.

The second prediction focuses on the differential incentives to acquire information.

Prediction 2: The extent of information acquisition about the exposure to aggregate or market-wide shocks is higher after observing a financial crisis elsewhere than after observing no crisis.

The second prediction stems from equation (7) in Proposition 2 and is consistent with empirical evidence. In a study on the U.S. stock market, Vlastakis and Markellos (2012) find evidence for a positive association between the demand for market information, proxied by internet search intensity (Da et al. 2011), with measures of volatility. The demand for information at the market level may be interpreted as information acquisition about the common component of stock returns. In the context of our model, the market information refers to information about the aggregate component m and measures of volatility can be associated with a crisis occurring in region 1. Furthermore, Vlastakis and Markellos document that the demand for market information (information about m) is important relative to the demand for idiosyncratic information (information about θ_2). For idiosyncratic information, they find only mixed results in direction and strength. Taken together, the empirical results support our focus on information acquisition about a macro shock.

Macroeconomic news are known to play an important role in asset markets (e.g. Andersen et al. (2007)). Macro factors, or global factors in the case of sovereign debt (Longstaff et al. 2011), can explain the majority of credit risk. This suggests that information acquisition about aggregate, or market-wide, shocks plays an important role also in bond markets. Regarding sovereign debt, our theory predicts a larger extend of information acquisition about common macro risks after another country with similar characteristics enters a period of financial distress. Provided the availability of proxies for information acquisition like the internet search intensity for macro news that relate to potential common risk factors, this prediction may be tested.

The second prediction may also be testable in the corporate debt market. Consider a firm with publicly traded debt to be rolled over by investors. A crisis elsewhere refers to a spike in the credit risk of other firms in the same industry sector that may be associated with a substantial ratings downgrade or earnings warning. It is well know that institutional lenders like banks seek to insure against industry-specific risks when confronted with a significant exposure via portfolio trading or the loan book. Our theory predicts a high sensitivity of debt holders to negative news that may convey information about changes in industry-specific factors (e.g. demand factors, new trends or innovations), as well as an increase in the incentive to acquire and analyze information about potential industry shocks and vulnerabilities of firms with certain attributes and business models.

Our theory suggests that information acquisition occurs for an intermediate information cost. An empiricist can separate industries according to whether information acquisition is cheap or expensive. There are several potential proxies an empiricist may use for the information cost. Some industry sectors comprise mostly smaller firms that are not publicly listed, which makes it more difficult and costly for analysts to gauge relevant changes in industry-specific factors. Similarly, the growth prospects of high-tech industry sectors are more difficult to analyze than, for instance, utilities. Moreover, less homogeneous industry sectors also suggest to be more difficult to evaluate. Based on our theory, the corporate debt from industry sectors with lower information costs are more likely to exhibit an increase in the extend of information acquisition by investors after observing a firm in the industry that suffers a ratings downgrade or earnings warning.

Finally, the second prediction may also be tested in the market for bank commercial paper, which is rolled over frequently. Apart from a downgrade or an earnings warning, a crisis elsewhere could also be a downward revision of another bank's asset quality by the supervisor. Since macro variables appear to have a large effect on bank credit losses (see, for instance, Buncic et al. (2019)), such a downgrade of a bank or supervisory action may trigger question not only regarding the direct exposure of other banks, but also regarding the role played by negative macro risk factors that are common to all banks. Again, an empiricist would need to separate circumstances under which information about the exposure of other banks is easy to acquire from those where it may be difficult. To this end, the opaqueness and complexity of banks play an important role, which may be affected by the extend of securitization activities, cross-border linkages and the organizational structure (Flannery et al. 2013; Cetorelli and Goldberg 2014; Goldberg 2016).

5 Discussion

We start with welfare implications (section 5.1) and transparency (section 5.2). Next, we differentiate our result of contagion after a wake-up call from the information contagion literature reviewed in the introduction (section 5.3). Then, we discuss the negative skewness assumption and allow for a biased macro shock (section 5.4). Finally, we discuss other robustness issues (section 5.5).

5.1 Welfare

We discuss two measures: *(i)* utilitarian welfare and *(ii)* the ex-ante probability of regime change.

Utilitarian welfare is measured as the expected payoffs of investors. This measure is particularly relevant for the application of an investment game. For the parameters consistent with Assumption 1, the ex-ante utilitarian welfare weakly decreases in the information cost c. To see this, first observe that $v(n_2, f)$ is positive since individual investors can only gain from more information. Second, recall from Lemma 4 that there is a strategic complementarity in information choices, so it is beneficial for investors to become informed from both an individual and a social viewpoint. As a result, an increase in c has an unambiguously negative effect on utilitarian welfare.

The second welfare measure considered is the ex-ante probability of regime change, arguably a key variable of interest for a policymaker who wants to avoid a bank run or a currency attack. In general, the relationship between the information cost and ex-ante welfare is ambiguous and difficult to analyze. Specifically when comparing the scenarios where investors acquire information after observing a crisis in region 1 with the scenario where they do not acquire information, the ex-ante probability of regime change can be higher or lower, depending on parameters.

Interestingly, we find that opacity can be good in our model. There are cases where the probability of regime change is higher if investors acquire information after a crisis in region 1 than if they never acquire information. To illustrate this point, we again consider the special case of $\gamma = \mu = p = \frac{1}{2}$ (as in Figures 1-4) but also invoke stronger conditions than in Assumption 1.

Proposition 4 ***Opacity can be bliss.*** *The ex-ante probability of regime change can be higher or lower for an intermediate information cost, $c \in (\underline{c}, \overline{c})$, than for high cost, $c > \overline{c}$. An example for a higher ex-ante probability of regime change when investors acquire information arises for $\mu = \gamma = p = \frac{1}{2}$, $s = 1$ and a sufficiently small α_2.*

Proof See Appendix A.4.

This result is reminiscent of Dang et al. (2015) who show that ignorance can increase welfare. While we do not wish to draw a general policy recommendation from the special case analyzed in Proposition 4, it does show that a lower information costs can reduce a measure of welfare.

5.2 Transparency

Next, we study how the incentives to acquire information are affected by transparency, measured by α_2 (e.g. Morris and Shin (2002)). Depending on the application, such an increase in the public signal precision can, for instance, be interpreted as an increase in market disclosure standards, the precision of information provided by rating agencies or as an increase in the transparency of bank stress tests. In the context of the debate about bank stress tests, higher transparency can be seen as a commitment of the banking regulator to disclose more detailed bank-specific information.

The general case is difficult to analyze analytically and we examine a special case where the incentives to acquire information increase in transparency. In particular, we consider the special case of Proposition 4 and further simplify the analysis by imposing symmetry that can be achieved if $\alpha_1 \to 0$. Under the sufficient condition that Δ is high and α_2 is low, we find a positive association between transparency and the incentives to acquire information. Proposition 5 summarizes.

Proposition 5 ***Transparency.*** *If $\mu = \gamma = p = \frac{1}{2}$, $s = 1$, sufficiently high Δ and sufficiently low α_1, α_2 , then greater transparency increases the incentives to acquire information:*

$$\frac{dv(1,f)}{d\alpha_2} > 0, \quad f \in \{0,1\}. \tag{10}$$

Proof See Appendix A.5.

Figure 5 shows this result for parameters used in previous figures. The analytical result of Proposition 5 extends to larger values of s and to all $n_2 \in [0,1]$, suggesting that our result is not confined to the somewhat restrictive set of sufficient conditions stated in the proposition. The complementary relationship between disclosure and information acquisition established in Proposition 5 is, however, not a general result and we invoke stronger conditions as in Assumption 1.

With greater transparency, higher incentives to acquire information arise from the larger benefit of tailoring the signal thresholds to the realized macro shock. Intuitively, an increase in transparency is associated with less aggressive attacks against the regime if the prior about the fundamentals is strong, which occurs if investors observe $m = \Delta$. At the same time, greater transparency is associated with more aggressive attacks against the regime if the prior about the fundamentals is weak, which occurs if investors observe $m = -s\Delta$. Hence, signal thresholds diverge. This effect is associated with an increase in the value of information and dominates for the case considered in Proposition 5, opposing effects stemming from the curvature of the distribution functions.

While we cannot draw a general policy implication from the special case analyzed in Proposition 5, we can reject the view that more public disclosure inevitably reduces information acquisition. This observation contrasts with some of the literature that has analyzed the impact of transparency on information acquisition in coordination games. In the context of a beauty contests with private information acquisition, Colombo et al. (2014) find a crowding-out effect of public information. The incentives to acquire more precise private information decrease in the public signal precision.

In contrast, Szkup and Trevino (2015) study continuous information choice subject to a convex

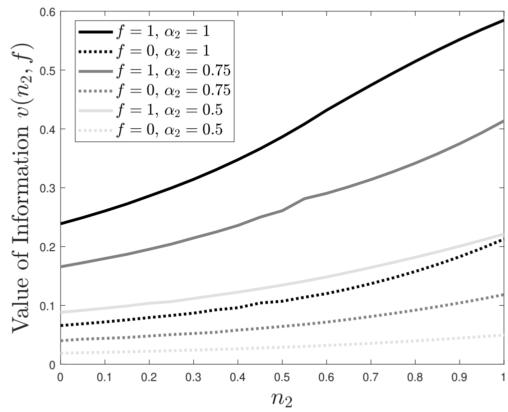

Figure 5: Value of information and the proportion of informed investors with and without a wake-up call. Solid lines are as in the left panel of Figure 2, the dashed and dotted lines show the lower value of information due to lower transparency (lower α_2). Other parameters are as in Figure 1.

information cost that is homogeneous across investors. They analyze efficiency when information choices are complements or substitutes, and the trade-off between public and private information, focusing on the precision of public information. Ahnert and Kakhbod (2017) study binary private information choice subject to heterogeneous information costs, finding that greater disclosure sometimes increases fragility. In contrast, we study the acquisition of publicly available information in a regime change game. Finally, there is an earlier literature studying the effect of transparency on the incidence of a regime change with exogenous information (Morris and Shin 1998; Heinemann and Illing 2002; Bannier and Heinemann 2005).

5.3 Information contagion

There exists a literature on information contagion. Manz (2010) establishes information contagion due to ex-post correlated fundamentals in a global games framework. Acharya and Yorulmazer (2008) show that the funding cost of one bank increases after bad news about another bank when the banks' loan portfolio returns have a common factor. To avoid information contagion ex post, banks herd their investment ex ante. Allen et al. (2012) compare the impact of information contagion on systemic risk across asset structures. Adverse news about the solvency of the banking system leads to runs on multiple banks.

We can nest a version of the standard information contagion channel as a special case of our model with endogenous information in which investors are uncertain about the common component of regional fundamentals and update their beliefs after observing a crisis elsewhere. The information contagion result purely relies on Bayesian updating of uninformed investors. That is, when the information cost is high, $c > v(1,1) \equiv \tilde{c} > \bar{c}$, information acquisition never occurs.

Proposition 6 ***Information contagion.*** *If $c > \tilde{c}$, then there exists a unique monotone perfect Bayesian equilibrium in region 2 in which no investor acquires information, $n_2^* = 0$. In this case, a crisis in region 2 is more likely after a crisis in region 1 than after no crisis in region 1:*

$$\Pr\{\Theta_2 < \Theta_U^* | f = 1\} > \Pr\{\Theta_2 < \Theta_U^* | f = 0\}. \tag{11}$$

Proof See Appendix A.6.

Proposition 6 compares the probability of a crisis in region 2 conditional on whether a crisis occurred in region 1. For a sufficiently high information cost, investors in region 2 choose not to acquire information irrespective of the occurrence of a crisis in region 1. In this case, a crisis in region 1 is unfavorable news about the fundamental in region 1. Since the macro shock is a common component of both regional fundamentals, this crisis is also unfavorable news about the fundamental in region 2. As a result, the re-assessment of the local fundamental Θ_2 via Bayesian updating increases the probability of a crisis in region 2.

Different to the result on wake-up contagion, the result of Proposition 6 only rests on Bayesian updating about the macro shock but not on investors' endogenous (differential) information choice, which is at the heart of our model and allows to isolate the wake-up call component of contagion.

5.4 Negative skewness and a biased macro shock

In this section, we further discuss the importance of the negative skewness of the macro shock as the key driver of the differential information choice (Proposition 2), which underpins our wake-up call contagion channel. Moreover, we show the robustness of our main results to two variations of our model. We proceed by first discussing a special case of the model with $s = 1$ to demonstrate that $s > 1$ is crucial for the differential information choice. Second, we consider a modified setup where we engineer offsetting changes of Δ that allow us to hold the variance of the macro shock constant when s changes. Third, we consider a setup where s and q can be varied independently.

We start with a special case of our model where $\mu = \gamma = \frac{1}{2}$, as in Figures 1-4. This simplifies the analysis and allows to discuss the role of the parameter s in a transparent way. For $s = 1$, the results in Lemma 3 continue to hold and we can show that the first and third inequality in Proposition 2 remain valid. However, the second inequality of Proposition 2 fails to hold because the value of information is identical in both scenarios when $s = 1$. The result is summarized in Corollary 2.

Corollary 2 *If* $\mu = \gamma = \frac{1}{2}$ *and* $s = 1$, *then* $v(n_2, 0) = v(n_2, 1)$, $\forall n_2 \in [0, 1]$.

Proof See Online Appendix A.10.1.

Corollary 2 highlights the role played by $s > 1$ for the differential information choice. Negative skewness drives a wedge between the relative incentives to acquire information, making information acquisition more valuable after observing a crisis. This leads to a strong Bayesian updating channel and whips around probabilities of tail events, focusing investor attention on downside risk.

Next, we study a version of the model with $\mu = \gamma = \frac{1}{2}$ in which changes in s are offset by changes in Δ in order to keep the variance of the macro shock constant at some $\chi > 0$ as s changes:

$$\Delta(s) \equiv \sqrt{\frac{\chi}{p(1+s)}} > 0, \qquad Var[m] = \chi.$$

Following an analogous argument as in Corollary 2, we find again no differential information choice if $s = 1$. Instead, under sufficient conditions akin to Assumption 1, the value of information is higher after observing a crisis, provided $s > 1$ is sufficiently high (see Online Appendix A.10.2).

Finally, we consider the case when s and q can vary independently. To be able to compare with our baseline model, we suppose that $p = q$. If $s = 1$, the argument in the proof of Corollary 2 is unchanged and we find that there is no differential information choice. For $s > 1$ the macro shock is biased, $E[m] < 0$. Under sufficient conditions akin to Assumption 1, inequality (7) of Proposition 2 continues to hold with the addition that the probability of the negative macro shock is sufficiently small (see Online Appendix A.10.3). Specifically, the value of information is higher after observing a crisis in region 1, provided $s > 1$ is sufficiently high. Regarding the wake-up call contagion result in Proposition 3, we face the challenge that the biased macro shock can lead to an opposing effect. Nevertheless, using the same parameters as in Figure 1, the left panel of

Figure 6 illustrates that the total effect has the desired sign such that inequality (8) of Proposition 3 continues to hold for all $s > 1$: we have that $\Pr\{\Theta_2 < \Theta_I^*(m)|m = 0\} > \Pr\{\Theta_2 < \Theta_U^*|f = 0\}$. If Δ is higher, the right-hand side of the inequality is lower for all values of s, because a higher positive macro shock leads to a more favorable belief about Θ_2 after not observing a crisis in region 1.

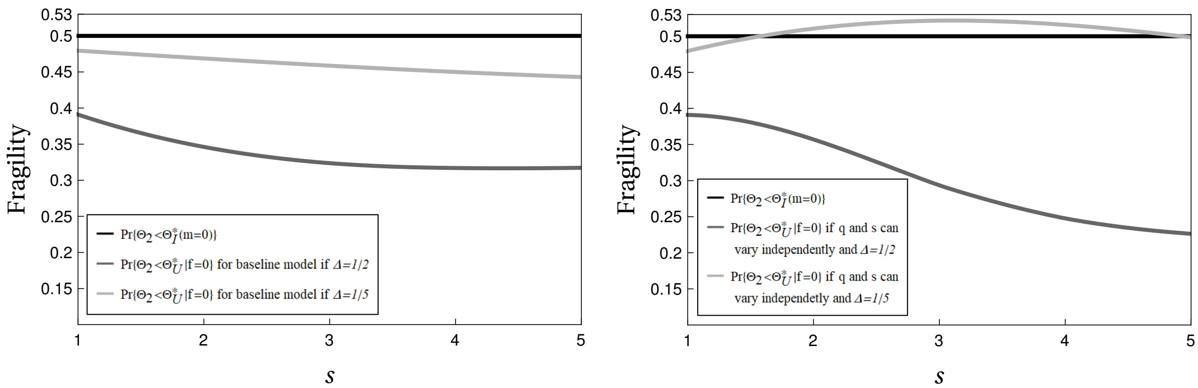

Figure 6: Wake-up call contagion for the baseline model and for a modified model where s and q can vary independently. We vary s from 1 to 5 and compare the crisis probabilities in region 2. As in Figure 4, $\Pr\{\Theta_2 < \Theta_I^*(m)|m = 0\} = 1/2$ is unaffected and identical in both models [black solid line]. In the left (right) panel we draw $\Pr\{\Theta_2 < \Theta_U^*|f = 0\}$ for the baseline (modified) model if $\Delta = 1/2$ [dark grey] and if $\Delta = 1/5$ [light grey]. For $s = 1$ the probability of regime change is identical. The other parameter values are as in Figure 1.

This result is, however, not guaranteed to hold. When reducing Δ from $1/2$ to $1/5$ in the right panel of Figure 6 we can see that inequality (8) is for some intermediate values of s violated in the modified model where s and q can vary independently (light grey line).

5.5 Other robustness issues

Our analytical results are derived under the conditions of Assumption 1. Given that the conditions might seem restrictive, it is worth noting that the conditions are sufficient but not necessary for our results (see Figure 4, for instance). Most importantly, the benchmark parameter values used for the numerical analysis provided in the figures illustrate that wake-up call contagion also holds for a high probability of the zero macro shock, suggesting that the bound η is merely relevant for analytical tractability. Also the bounds on the precision of private and public signals are not more stringent than the standard sufficient conditions for equilibrium uniqueness in global games models (Morris and Shin 2003; Svensson 2006).

Next, our model setup abstracts from information acquisition in region 1 to simplify the exposition. This allows us to focus on how the wake-up call of a crisis in region 1 affects the incentives to acquire information in region 2 and may therefore result in contagion. Allowing for information acquisition in region 1 does not affect our main insights. For some intermediate region of information costs, there is a unique equilibrium with no information acquisition in region 1 and information acquisition in region 2 only after a crisis in region 1.

Below we discuss two extensions and an alternative modeling approach. First, an important channel of our paper is how a wake-up call affects the incentives of investors in region 2 to acquire information about the macro shock. An additional channel of interest could be private information acquisition with convex costs (Szkup and Trevino 2015), whereby investors improve the precision of their private information at a cost after the wake-up call. It can be shown that the effect of wake-up call contagion is even larger when private information acquisition is also allowed.

Second, one could consider an alternative model setup, where learning is not about the realization of the macro shock but about whether region 2 is exposed to the macro shock itself. In this setup, two macro shock realizations suffice, so $1-p-q=0$, where the scenario of no exposure to the macro shock is equivalent to $m=0$. As before, both observing a crisis in region 1 and learning about an exposure to the macro shock suggest that the fundamentals in region 2 are likely to be affected by a negative macro shock that also contributed to the crisis in region 1. Conversely, learning about no exposure to the macro shock after observing a crisis in region 1 is favorable information for the local fundamentals in region 2. No crisis in region 1 would still imply a more favorable view about the fundamental in region 2 when an exposure to the macro shock has positive weight due to the Bayesian updating channel. Hence, the wake-up call component of contagion can be isolated in the same way in this alternative model setup, and the incentives to acquire information about the macro shock in such an alternative setup are similar to the present version.

Third, we have so far considered the case of a perfect signal about the macro shock. The advantage of a perfectly revealing signal is that we can cleanly isolate the wake-up call component of contagion. A generalization to noisy signals is possible without altering the key mechanisms. One approach is to assume that investors only observe the publicly available signal with probability $z \in (0,1)$ upon incurring the information acquisition cost. More concretely, the hiring of analysts only leads with a certain probability to a conclusive understanding of the institutional characteristics such as structural or policy distortions that are shared across regions. As a result, there is always a positive mass of investors who remain uninformed. This variation of our model is already captured by our analysis of the general case when $0 < n_2 < 1$. Another approach is to consider an environment where investors who incur the cost always observe a signal about the macro shock, but they do not know whether the signal is correct. In this case, we have to use the mixture distribution approach also for informed investors, which adds an additional layer of complexity. Again, in a modified setup it would not be possible to cleanly isolate the wake-up call component of contagion as in our main model in which all informed investors observe $m = 0$.

6 Conclusion

We offer a theory of contagion to explain how wake-up calls may transmit financial crises. We study global coordination games of regime change that are often applied to currency attacks, bank runs, and debt crises. There are two regions whose only link is an initially unobserved and negatively skewed macro shock. A crisis in region 1 is a wake-up call for investors in region 2 and induces them to re-assess the local fundamental in region 2. Since crises are rare events and the macro shock is negatively skewed, investors have an incentive to acquire information only after this wake-up call. The crisis probability in region 2 is higher after a crisis in region 1 than after no crisis, even if investors learn that the macro shock is zero and, hence, that there is no exposure to the crisis in region 1. In short, we isolate the wake-up call component of contagion without ex-post exposure to the crisis region, common lender effects, or balance sheet links.

A distinctive feature of our theory is that it combines information acquisition with a Bayesian updating channel. The optimal information choices of investors are driven by wake-up calls and shape the fundamental re-assessment. Information choices are strategic complements but a unique equilibrium obtains for intermediate information cost levels. We describe how the incentives to acquire information about the macro shock depend on distributional characteristics of the macro shock. Based on these results, we derive two testable implications about the information choices of investors. We argue for their consistency with empirical evidence and discuss how these can be tested in future work.

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A Online Appendix

A.1 Equilibrium in region 1

To simplify the exposition, we focus on the case of uninformed investors, $n_1 = 0$. We first discuss Bayesian updating of uninformed investors receiving a private signal x_{i1} about Θ_1 and derive the equilibrium conditions in section A.1.1. Next, we prove Lemma 1 in section A.1.2.

A.1.1 Deriving the equilibrium in region 1 for the case $n_1 = 0$

Bayesian updating. Uninformed investors in region 1 use Bayes' rule to form a belief about the macro shock, where $\hat{p} \equiv \Pr\{m = \Delta | x_{i1}\}$, and $\hat{q} \equiv \Pr\{m = -s\Delta | x_{i1}\}$:

$$\hat{p} = p \Pr\{x_{i1} | m = \Delta\} \Gamma_1^{-1} \qquad \hat{q} = q \Pr\{x_{i1} | m = -s\Delta\} \Gamma_1^{-1}, \tag{12}$$

where $\Gamma_1 = p \Pr\{x_{i1} | m = \Delta\} + q \Pr\{x_{i1} | m = -s\Delta\} + (1 - p - q) \Pr\{x_{i1} | m = 0\}$ and:

$$\Pr\{x_{i1} | m\} = \frac{1}{\sqrt{\mathrm{Var}[x_{i1} | m]}} \phi\left(\frac{x_{i1} - \mathbb{E}[x_{i1} | m]}{\sqrt{\mathrm{Var}[x_{i1} | m]}}\right) = \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i1} - (\mu + m)}{\sqrt{\frac{1}{\alpha} + \frac{1}{\beta}}}\right).$$

Using $p = qs$, we obtain $\frac{d\hat{p}}{dx_{i1}} > 0$, $\quad \frac{d\hat{q}}{dx_{i1}} < 0$, and $\frac{d(1-\hat{p}-\hat{q})}{dx_{i1}} \left[x_{i1} - \mu + \frac{1-s}{2}\Delta\right] \leq 0$, with strict inequality if $x_{i1} \neq \mu + \frac{1-s}{2}\Delta$. An investor places more weight on the probability of a positive (negative) macro shock after a higher (lower) private signal. The relationship between the posterior probability of a zero macro shock and the private signal, x_{i1}, is non-monotone. It increases if $x_{i1} > \underline{x_1}(s, \Delta) \equiv \mu + \frac{1-s}{2}\Delta$. The bound is below μ if the macro shock is negatively skewed ($s > 1$).

Equilibrium conditions. For the case of $n_1 = 0$, the system of equations comprises the critical mass and indifference condition for region 1. The critical mass condition states that the proportion of attacking investors $A_1^*(m)$ equals the fundamental threshold $\Theta_1^*(m)$ for each realized m:

$$\Theta_1^*(m) = \Phi\left(\sqrt{\beta}[x_1^* - \Theta_1^*(m)]\right), \forall m \in \{-s\Delta, 0, \Delta\}. \tag{13}$$

Given the invariant attack rule, the fundamental thresholds are equal, $\Theta_1 \equiv \Theta_1^*(m)\ \forall m$. The indifference condition states that an uninformed investor with threshold signal $x_{i1} = x_1^*$ is indifferent whether to attack:

$$\hat{p}^*\Psi(\Theta_1^*,x_1^*,\Delta)+\hat{q}^*\Psi(\Theta_1^*,x_1^*,-s\Delta)+(1-\hat{p}^*-\hat{q}^*)\Psi(\Theta_1^*,x_1^*,0)\equiv J(\Theta_1^*,x_1^*)=\gamma_1, \tag{14}$$

where $\hat{p}^*=\hat{p}(x_1^*)$, $\hat{q}^*=\hat{q}(x_1^*)$ and $\Psi(\Theta_1^*,x_1^*,m)\equiv\Phi\big(\Theta_1^*\sqrt{\alpha+\beta}-\frac{\alpha(\mu+m)+\beta x_1^*}{\sqrt{\alpha+\beta}}\big)$. Solving equation (13) for x_1^* and plugging into equation (14), we arrive at one equation in one unknown.

Monotone equilibria. Using the results of Milgrom (1981) and Vives (2005), we can show that the best-response function of an individual investor strictly increases in the threshold used by other investors. Using Proposition 1 of Milgrom (1981), we conclude that $\Pr\{\Theta_1\leq\Theta_1^*|x_{i1}\}$ monotonically decreases in x_{i1}. Hence, $\frac{d\Pr\{\Theta_1\leq\Theta_1^*|\hat{x}_1\}}{d\Theta_1^*}>0$. Equation (14) then implies:

$$0\leq\frac{d\hat{\Theta}_1(\hat{x}_1)}{d\hat{x}_1}\leq\left(1+\sqrt{2\pi\beta^{-1}}\right)^{-1}. \tag{15}$$

Thus, our focus on monotone equilibria is valid. Equation (15) is used to determine conditions sufficient for a unique monotone Bayesian equilibrium in Lemma 1.

A.1.2 Proof of Lemma 1

The proof consists of two steps. First, we show that $J(\Theta_1,x_1)\equiv J(\Theta_1)\to 1>\gamma_1$ as $\Theta_1\to 0$, and $J(\Theta_1)\to 0<\gamma_1$ as $\Theta_1\to 1$. Second, we show that $\frac{dJ(\Theta_1)}{d\Theta_1}<0$ for some sufficiently high but finite values of β, such that J strictly decreases in Θ_1. We denote this lower bound as $\underline{\beta}_1$. Therefore, if Θ_1^* exists, it is unique. Notably, this argument implicitly defines the lower and upper dominance regions of the game. However, as Θ_1 can be any real number, the limit used here is one-sided.

Step 1 (limiting behavior): We solve equation (13) for x_1^*, plug into equation (14) and let $\Psi(\Theta_1,x_1,m)\equiv\Psi(\Theta_1,m)$. Observe that $J(\Theta_1)$ is a weighted average of the $\Psi(\Theta_1,m)$'s evaluated at the different levels of m. As $\Theta_1\to 0$, then $\Psi(\Theta_1,m)\to 1$ for any $m\in\{-s\Delta,0,\Delta\}$, so $J(\Theta_1)\to 1>\gamma_1$. Likewise, as $\Theta_1\to 1$, then $\Psi(\Theta_1,m)\to 0$ for any $m\in\{-s\Delta,0,\Delta\}$, so $J(\Theta_1)\to 0<\gamma_1$.

Step 2 (strictly negative slope): The total derivative of J is:

$$\begin{aligned}
\frac{dJ(\Theta_1)}{d\Theta_1} &= \hat{p}(x_1(\Theta_1))\frac{d\Psi(\Theta_1,\Delta)}{d\Theta_1}+\hat{q}(x_1(\Theta_1))\frac{d\Psi(\Theta_1,-s\Delta)}{d\Theta_1} \\
&+ (1-\hat{p}(x_1(\Theta_1))-\hat{q}(x_1(\Theta_1)))\frac{d\Psi(\Theta_1,0)}{d\Theta_1} \\
&+ \frac{d\hat{p}(x_1(\Theta_1))}{dx_1}\frac{dx_1(\Theta_1)}{d\Theta_1}\big[\Psi(\Theta_1,\Delta)-\Psi(\Theta_1,0)\big] \\
&+ \frac{d\hat{q}(x_1(\Theta_1))}{dx_1}\frac{dx_1(\Theta_1)}{d\Theta_1}\big[\Psi(\Theta_1,-s\Delta)-\Psi(\Theta_1,0)\big]. \qquad (16)
\end{aligned}$$

The proof proceeds by inspecting the individual terms of equation (16). For the analysis of the special case where all investors are informed, $n_1 = 1$, we can use a result from standard global games models: $\frac{d\Psi(\Theta_1,m)}{d\Theta_1} < 0$ if $\beta > \frac{\alpha^2}{2\pi}$ for all m. Thus, the first three components of the sum are negative and finite for sufficiently high but finite private noise. The sign of the two terms in square brackets in the last two summands in (16) is negative and positive, respectively: $\Psi(\Theta_1^*,\Delta) \leq \Psi(\Theta_1^*,0)$ and $\Psi(\Theta_1^*,\Delta) \geq \Psi(\Theta_1^*,0)$. However, the difference vanishes in the limit when $\beta \to \infty$. The last terms to consider are $\frac{d\hat{p}(x_1(\Theta_1))}{dx_1(\Theta_1)}\frac{dx_1}{d\Theta_1}$ and $\frac{d\hat{q}(x_1(\Theta_1))}{dx_1(\Theta_1)}\frac{dx_1}{d\Theta_1}$. Given the previous sufficient conditions on the relative precision of the private signal:

$$0 < \frac{dx_1}{d\Theta_1} = 1 + \left(\sqrt{\beta}\phi(\Phi^{-1}(\Theta_1))\right) < 1 + \sqrt{2\pi\alpha^{-1}}.$$

The derivative is finite for $\beta \to \infty$. Taken together with the zero limit of the first factor of the third and forth term, this terms vanish in the limit. Note that for $\beta \to \infty$ the updated prior distribution becomes degenerate. We have $\hat{p} = 1$ for $x > \mu + \frac{\Delta}{2}$ and $\hat{p} = 0$ for $x < \mu + \frac{\Delta}{2}$. Moreover, $1 - \hat{p} - \hat{q} = 1$ for $\mu + \frac{-s\Delta}{2} < x < \mu + \frac{\Delta}{2}$ and $\hat{q} = 1$ for $x < \mu + \frac{-s\Delta}{2}$. Clearly, there are some discontinuities. At the same time, it must be that any Θ_1^* and $x_1^* \approx \Theta_1^*$ solving the system has to be very close to μ for large values of β. As a result, it is guaranteed that $\frac{d\hat{p}(x_1(\Theta_1))}{dx_1} = 0$ and $\frac{d\hat{q}(x_1(\Theta_1))}{dx_1} = 0$ is in the permissible range. Hence, by continuity, there exists a finite precision level $\beta > \underline{\beta}_1 \in (0,\infty)$ such that $\frac{dJ(\Theta_1)}{d\Theta_1} < 0$ for all $\beta > \underline{\beta}_1$. This concludes the proof of Lemma 1.

A.2 Equilibrium in region 2

To study the equilibrium in region 2, we first analyze the coordination stage in section A.2.1. The main results are on Bayesian updating and on the existence of unique attack rules are summarized in Lemma 2 and Corollary 3, respectively. Next, we analyze the information stage in sections A.2.2 and A.2.3. The main results are summarized in Lemma 3, which describes how fundamental and signal thresholds depend on the proportion of informed investors and in Lemma 4, which establishes a strategic complementarity in information choices. Finally, we prove Proposition 2 in section A.2.4 and Proposition 1 in section A.2.5.

A.2.1 Coordination stage in region 2

The optimal behavior of investors in region 2 at the coordination stage can be described by extending the results from region 1. Let $f \in \{0,1\}$ indicate whether a crisis occurred in region 1, where $f=1$ corresponds to a crisis and $f=0$ corresponds to no crisis. Investors use the information about region 1 to update their prior about their beliefs about the distribution of the macro shock, using Bayes' rule:

$$p' \equiv \Pr\{m=\Delta|f\} \quad = \quad p\Pr\{f|m=\Delta\}\,\Gamma_2^{-1} \tag{17}$$

$$q' \equiv \Pr\{m=-s\Delta|f\} \quad = \quad q\Pr\{f|m=-s\Delta\}\,\Gamma_2^{-1}, \tag{18}$$

with $\Pr\{f=1|m\} = \Pr\{\Theta_1 < \Theta_1^*|m\}$ and $\Gamma_2 \equiv p\Pr\{f|m=\Delta\} + q\Pr\{f|m=-s\Delta\} + (1-p-q)\Pr\{f|m=0\}$.

Lemma 2 states the evolution of the beliefs about the macro shock.

Lemma 2 ***Beliefs about the macro shock.*** *The wake-up call of a crisis in region* 1 *is associated with less favorable beliefs about the macro shock, while no crisis in region* 1 *is associated with more favorable beliefs about the macro shock:*

$$\begin{cases} p' < p,\ q' > q & if\ f=1 \\ p' > p,\ q' < q & if\ f=0. \end{cases}$$

Moreover, we can state that:

$$\begin{cases} \frac{p'}{1-q'} < \frac{p}{1-q}, \frac{q'}{1-p'} > \frac{q}{1-p} & if\ f=1\ and\ n_1 \in \{0,1\} \\ \frac{p'}{1-q'} > \frac{p}{1-q}, \frac{q'}{1-p'} < \frac{q}{1-p} & if\ f=0\ and\ n_1 \in \{0,1\}. \end{cases}$$

The first set of inequalities are an extension of a comparative static in Morris and Shin (2003) and Vives (2005). For the special case of $n_1 = 1$, we have $\frac{d\Theta_1^*(1,m)}{dm} < 0$. Similarly for the general case, a more favorable information about fundamentals is associated with a lower fundamental threshold. The results follow from Bayesian updating in equations (17) and (18). The second set of inequalities on the right-hand side follow from $\frac{d}{dm}\big(\Pr\{f=1|m\}-\Pr\{f=0|m\}\big)<0$. The results are immediate for $n_1 \in \{0,1\}$ and also hold for the general case, $n_1 \in [0,1]$, if the thresholds are monotone in n_t. We show this monotonicity in Lemma 3.

Using the updated p' and q' as weights, the belief about Θ_2 prior to receiving a private signal x_{i2} follows again a mixture distribution. It is an average over the cases of negative, zero and positive macro shocks with weights depending on f:

$$\Theta_2|f \equiv p'\left[\Theta_2|m=-s\Delta\right]+q'\left[\Theta_2|m=\Delta\right]+(1-p'-q')\left[\Theta_2|m=0\right]. \tag{19}$$

For the general case of $n_2 \in [0,1]$ we have seven equations in seven unknowns. Three critical mass conditions state that the proportion of attacking investors $A_2^*(m)$ equals the fundamental threshold $\Theta_2^*(m)$ for each realized $m \in \{-s\Delta,0,\Delta\}$:

$$\Theta_2^*(m)=n_2\Phi\big(\sqrt{\beta}[x_I^*(m)-\Theta_2^*(m)]\big)+(1-n_2)\Phi\big(\sqrt{\beta}[x_U^*-\Theta_2^*(m)]\big), \tag{20}$$

where the short-hands are $\Theta_2^*(m)\equiv\Theta_2^*(n_2,m)$, $x_I^*(m)\equiv x_{2I}^*(n_2,m)$, and $x_U^*\equiv x_{2U}^*(n_2)$ for the fundamental threshold and the signal thresholds of informed and uninformed investors, respectively.

The first indifference condition states for each $n_2 \in [0,1]$ that an uninformed investor with threshold signal $x_{i2}=x_U^*$ is indifferent whether to attack:

$$\underbrace{\begin{aligned}&\hat{p}'^*\Psi(\Theta_2^*(\Delta),x_U^*,\Delta)+\hat{q}'^*\Psi(\Theta_2^*(-s\Delta),x_U^*,-s\Delta)\\ &\qquad+(1-\hat{p}'^*-\hat{q}'^*)\Psi(\Theta_2^*(0),x_U^*,0)\end{aligned}}_{\equiv J(n_2,\Theta_2^*(\Delta),\Theta_2^*(-s\Delta),\Theta_2^*(0),x_U^*)} = \gamma_2 \tag{21}$$

where $\hat{p}'^*=\hat{p}'(x_U^*)$ and $\hat{q}'^*=\hat{q}'(x_U^*)$ solve equation (12) after replacing p and q with p' and q'. Moreover, $\Psi(\Theta_2^*(m),x_d^*,m)\equiv\Phi\big(\Theta_2^*\sqrt{\alpha+\beta}-\frac{\alpha(\mu+m)+\beta x_d^*}{\sqrt{\alpha+\beta}}\big)$ for $d\in\{I,U\}$ and $m\in\{-s\Delta,0,\Delta\}$.

Three additional indifference conditions, one for each realized macro shock, state that an informed investor is indifferent between attacking or not upon receiving the signal $x_{i2}=x_I^*(m)$:

$$\Psi\left(\Theta_2^*(n_2,m),x_I^*(m),m\right)=\gamma_2 \;\; \forall\, m\in\{-s\Delta,0,\Delta\}. \tag{22}$$

For the special case of the equilibrium in region 1 with $n_1 = 0$, we had two thresholds x_1^* and Θ_1^* for each m. There, the objective was to establish aggregate behavior by inserting the critical mass condition, which states x_1^* in terms of Θ_1^*, into the indifference condition. This yields one equation implicit in Θ_1^*. We pursue a similar strategy here and express the equilibrium in terms of $\Theta_2^*(-s\Delta)$, $\Theta_2^*(0)$ and $\Theta_2^*(\Delta)$ only.

To simplify the system of equations, we can use the following insight. Since uninformed investors do not observe the macro shock realization, the signal threshold must be identical across these realizations, $x_U^* \equiv x_U^*(-s\Delta) = x_U^*(0) = x_U^*(\Delta)$. In the following steps, we derive this threshold for either realization of m by using $\Theta_2^*(m)$ and equalize both expressions. First, we use the critical mass conditions in equation (20) for $\Theta_2^*(m)$ to express x_U^* as a function of each $\Theta_2^*(m)$ and $x_I^*(m)$. Second, we use the indifference condition of informed investors for each m to obtain $x_I^*(m)$ as a function of $\Theta_2^*(m)$. Thus, $\forall m$:

$$x_U^*(m) = \Theta_2^*(m) + \Phi^{-1}\left(\frac{\Theta_2^*(m) - n_2\Phi\left(\frac{\alpha(\Theta_2^*(m)-(\mu+m)) - \sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-n_2}\right)/\sqrt{\beta}. \tag{23}$$

Hence, for $m \in \{-s\Delta, 0, \Delta\}$, there exists a $\underline{\beta}_2 \in (0,\infty)$ such that for all $\beta > \underline{\beta}_2$: $\frac{dx_U^*(m)}{d\Theta_2^*(m)} > 0$.

Since the signal threshold is the same for an uninformed investor, subtracting equation (23) evaluated at $m = 0$ from the same equation evaluated at $m = -s\Delta$ or at $m = \Delta$ must yield zero. This yields the first two pair-wise implicit relationships between $\Theta_2^*(-s\Delta)$, $\Theta_2^*(0)$ and $\Theta_2^*(\Delta)$:

$$K_1(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0)) \equiv x_U^*(0) - x_U^*(-s\Delta) = 0 \tag{24}$$

$$K_2(n_2, \Theta_2^*(0), \Theta_2^*(\Delta)) \equiv x_U^*(0) - x_U^*(\Delta) = 0. \tag{25}$$

Now, we construct the third implicit relationship between the three aggregate thresholds by inserting equation (23) evaluated at each m in $\Psi(\Theta_2^*(m), x_U^*(m), m)$, respectively, and in $\hat{p}(p')$ and $\hat{q}(q')$ as used in J:

$$L(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0), \Theta_2^*(\Delta)) \equiv J(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0), \Theta_2^*(\Delta)) = \gamma_2. \tag{26}$$

Corollary 3 establishes existence and uniqueness for a given $n_2 \in [0,1]$ under the conditions of Assumption 1 by analyzing the system of equations given by (24), (25) and (26).

Corollary 3 ***Existence of unique attack rules in region 2.*** *If private information is sufficiently precise, then for any proportion of informed investors in region 2, $n_2 \in [0,1]$, there exist unique attack rules for informed investors, $a_I^*(m,\cdot)$, and for uniformed investors, $a_U^*(\cdot)$.*

Proof The first and second equation depend only on two thresholds, $K_1(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0)) = 0$ and $K_2(n_2, \Theta_2^*(0), \Theta_2^*(\Delta)) = 0$, while the third depends on all three, $L(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0), \Theta_2^*(\Delta)) = \gamma_2$. In a first step, we analyze, for a given n_2, the relationship between $\Theta_2(-s\Delta)$ and $\Theta_2(0)$, as governed by K_1. We obtain $\frac{\partial K_1}{\partial \Theta_2^*(0)} > 0$, $\frac{\partial K_1}{\partial \Theta_2(-s\Delta)} < 0$, and $\frac{\partial K_1}{\partial \Theta_2(\Delta)} = 0$. Hence, $\frac{d\Theta_2(0)}{d\Theta_2(-s\Delta)} > 0$ by the implicit function theorem. Likewise, we analyze the relationship between $\Theta_2^*(0)$ and $\Theta_2^*(\Delta)$, as governed by K_2. We obtain $\frac{\partial K_2}{\partial \Theta_2^*(0)} > 0$, $\frac{\partial K_2}{\partial \Theta_2(-s\Delta)} = 0$, and $\frac{\partial K_2}{\partial \Theta_2(\Delta)} < 0$. Hence, $\frac{d\Theta_2(0)}{d\Theta_2(\Delta)} > 0$. These results do not require a bound on the precision of private information.

In a second step, we analyze, for a given n_2, the relationship between all three fundamental thresholds, as governed by L. We know from our analysis of the case of informed investors that $\frac{d\Psi(\Theta_2,m)}{d\Theta_2} < 0$ for all m if $\beta > \frac{\alpha^2}{2\pi}$. Analogous to the argument in the proof of Lemma 1, there exists a sufficiently high but finite value of the private precision such that $\frac{\partial L}{\partial \Theta_2(m)} < 0$ for all m. Hence, in the limit $\frac{d\Theta_2(0)}{d\Theta_2(-s\Delta)} < 0$ for a given $\Theta_2(\Delta)$, $\frac{d\Theta_2(0)}{d\Theta_2(\Delta)} < 0$ for a given $\Theta_2(-s\Delta)$, and $\frac{d\Theta_2(\Delta)}{d\Theta_2(-s\Delta)} < 0$ for a given $\Theta_2(0)$. By continuity, there exists a finite precision of private information, $\underline{\beta}_2 \in (0,\infty)$, that guarantees the inequality if $\beta > \underline{\beta}_2$.

In a third step, we establish uniqueness conditional on existence. Thus suppose for now that an equilibrium exists. Then, due to the monotonicity and the opposite signs of the respective derivatives, we have that there is a single crossing of K_1 and L in the $(\Theta_2(-s\Delta), \Theta_2(0))$ space and a single crossing of K_2 and L in the $(\Theta_2(\Delta), \Theta_2(0))$ space, as shown in Figure 7. Observe that this is a "partial equilibrium" argument since the third threshold is taken as given. We now move to a "general equilibrium" argument. Building on a second feature of the system, the opposite signs of the respective derivatives are not only a sufficient condition for single crossings in the two panels of Figure 7, but they also imply that $\Theta_2(-s\Delta)$ and $\Theta_2(0)$ are each decreasing in $\Theta_2(\Delta)$ (left panel), where an increase in $\Theta_2(\Delta)$ shifts the L curve inwards. Likewise, $\Theta_2(\Delta)$ and $\Theta_2(0)$ are each decreasing in $\Theta_2(-s\Delta)$ (right panel). Hence, starting from a general equilibrium, any modification of $\Theta_2(\Delta)$ and $\Theta_2(-s\Delta)$ must lead to a violation of the system of equations. Given $\frac{\partial L}{\partial \Theta_2(\Delta)} < 0$ and $\frac{\partial L}{\partial \Theta_2(-s\Delta)} < 0$, the combination of fundamental thresholds $(\Theta_2^*(-s\Delta), \Theta_2^*(0), \Theta_2^*(\Delta))$ that satisfies K_1 and L in the $(\Theta_2(-s\Delta), \Theta_2(0))$ space and K_2 and L in the $(\Theta_2(\Delta), \Theta_2(0))$ space is unique.

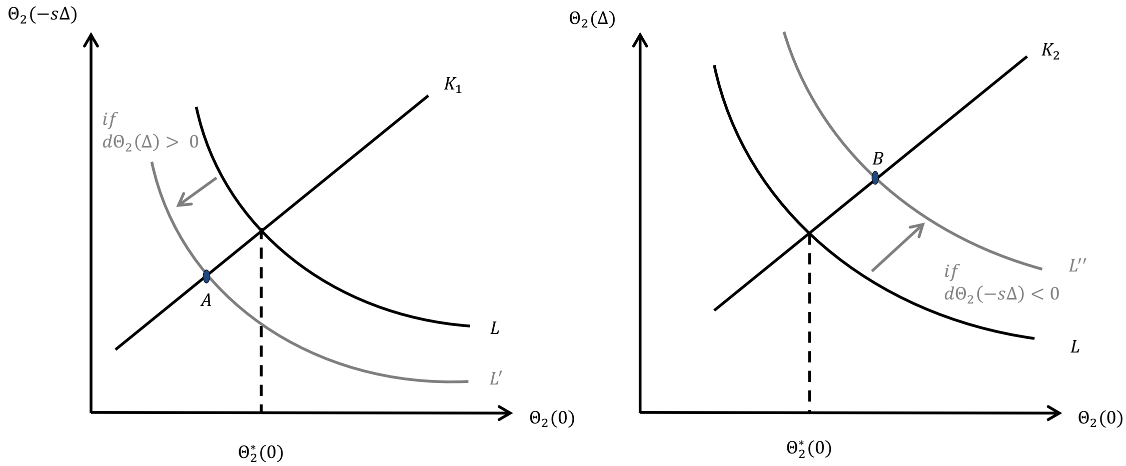

Figure 7: Single crossing.

In a fourth step, we establish the existence of a combination of fundamental thresholds. Existence can be shown by proving the following sequence of points: (i) for the highest permissible value of $\Theta_2(-s\Delta)$, the value of $\Theta_2(0)$ prescribed by K_1 is strictly larger than the value of $\Theta_2(0)$ prescribed by L; (ii) for the lowest permissible value of $\Theta_2(-s\Delta)$, the value of $\Theta_2(0)$ prescribed by K_1 is strictly smaller than the value of $\Theta_2(0)$ prescribed by L; (iii) for the highest permissible value of $\Theta_2(\Delta)$, the value of $\Theta_2(0)$ prescribed by K_2 is strictly larger than the value of $\Theta_2(0)$ prescribed by L; (iv) for the lowest permissible value of $\Theta_2(\Delta)$, the value of $\Theta_2(0)$ prescribed by K_2 is strictly smaller than the value of $\Theta_2(0)$ prescribed by L; (v) for the lowest (highest) permissible value of $\Theta_2(-s\Delta)$, also $\Theta_2(0)$ must be at its lowest (highest) permissible value from K_1 and, hence, also $\Theta_2(\Delta)$ must be at its lowest (highest) permissible value from K_2, leading to a violation of L in both the $(\Theta_2(-s\Delta), \Theta_2(0))$ space and the $(\Theta_2(\Delta), \Theta_2(0))$ space; (vi) a successive increase (decrease) in $\Theta_2(0)$ shifts L continuously inwards (outwards) in both spaces until a fixed point is reached.

Before addressing points (i)-(iv), we start by analyzing the following auxiliary step. For any $\Theta_2(m) \geq \Theta_2^*(1,m)$, it can be shown that:

$$\frac{\partial}{\partial n_2}\Phi^{-1}\left(\frac{\Theta_2(m) - n_2\Phi\left(\frac{\alpha(\Theta_2(m)-(\mu+m)) - \sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-n_2}\right) \geq 0 \tag{27}$$

because $J(1,\Theta_2) \leq \gamma_2$ for any m. Note that both the previous expression and the partial derivative hold with strict inequality if $\Theta_2(m) > \Theta_2^*(1,m)$. Inspecting the inside of the inverse of the cdf,

Φ^{-1}, we define the highest permissible values of $\Theta_2(m)$ that are labeled $\overline{\Theta}_2(n_2,m)$ for all m:

$$1 = \frac{\overline{\Theta}_2(n_2,m) - n_2\Phi\Big(\frac{\alpha(\overline{\Theta}_2(n_2,m)-(\mu+m))-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\Big)}{1-n_2}. \tag{28}$$

Hence, $1 \geq \overline{\Theta}_2(1,m) \geq \Theta_2^*(1,m)\ \forall m$, where the first (second) inequality binds iff $n_2 = 0$ ($n_2 = 1$).

We now prove points (i) and (iii). Evaluate K_1 and K_2 at the highest permissible value, $\Theta_2(0) = \overline{\Theta}_2(n_2,0)$, which yields $\overline{\Theta}_2(n_2,-s\Delta)$ and $\overline{\Theta}_2(n_2,\Delta)$, respectively. Likewise, evaluate L at the highest permissible values, $\overline{\Theta}_2(n_2,0)$ and $\overline{\Theta}_2(n_2,-s\Delta)$, which yields $\Theta_2(\Delta) < \overline{\Theta}_2(n_2,\Delta)$. Similarly, evaluate L at $\overline{\Theta}_2(n_2,0)$ and $\overline{\Theta}_2(n_2,\Delta)$, which yields $\Theta_2(-s\Delta) < \overline{\Theta}_2(n_2,-s\Delta)$. This proves points (i) and (iii). Next, we proceed with points (ii) and (iv). We can similarly define the lowest permissible value of $\Theta_2(m)$, which is labeled $\underline{\Theta}_2(n_2,m)$ for all m. Now, $0 \leq \underline{\Theta}_2(1,m) \leq \Theta_2^*(1,m)\ \forall m$, where the first (second) inequality binds if and only if $n_2 = 0$ ($n_2 = 1$). Evaluate K_1 and K_2 at the lowest permissible value, $\Theta_2(0) = \underline{\Theta}_2(n_2,0)$, which yields $\underline{\Theta}_2(n_2,-s\Delta)$ and $\underline{\Theta}_2(n_2,\Delta)$, respectively. Likewise, evaluate L at the lowest permissible values, $\underline{\Theta}_2(n_2,0)$ and $\underline{\Theta}_2(n_2,-s\Delta)$, which yields $\Theta_2(\Delta) > \underline{\Theta}_2(n_2,\Delta)$. Similarly, evaluate L at $\underline{\Theta}_2(n_2,0)$ and $\underline{\Theta}_2(n_2,\Delta)$, which yields $\Theta_2(-s\Delta) > \underline{\Theta}_2(n_2,-s\Delta)$. This proves points (ii) and (iv). The proof of points (v)-(vi) follows, which completes the overall proof of Corollary 3.

A.2.2 Information stage in region 2: proportion of informed investors and equilibrium thresholds

To characterize the value of information about the macro shock to investors in Appendix A.2.3, we first describe how the fundamental and signal thresholds depend on the proportion of informed investors, as summarized below.

Lemma 3 ***Proportion of informed investors and equilibrium thresholds.*** *If Assumption 1 holds, then:*

(A) Boundedness. *The fundamental thresholds in case of informed investors bound the fundamental thresholds in case of asymmetrically informed investors:*

$$\Theta_2^*(1,\Delta) \leq \Theta_2^*(n_2,m) \leq \Theta_2^*(1,-s\Delta)\ \ \forall m \in \{-s\Delta,0,\Delta\} \quad \forall n_2 \in [0,1]. \tag{29}$$

(B) Monotonicity in fundamental thresholds. *The fundamental threshold in the case of a negative (positive) macro shock increases (decreases) in the proportion of informed investors.*

Strict monotonicity is attained if and only if the fundamental thresholds are strictly bounded, that is $\forall n_2 \in [0,1)$*:*

$$\frac{d\Theta_2^*(n_2,-s\Delta)}{dn_2} = \begin{cases} >0 & if\ \Theta_2^*(n_2,-s\Delta) < \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) > \Theta_2^*(1,\Delta) \\ =0 & if\ \Theta_2^*(n_2,-s\Delta) = \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) = \Theta_2^*(1,\Delta), \end{cases} \tag{30}$$

$$\frac{d\Theta_t^*(n_2,\Delta)}{dn_2} = \begin{cases} <0 & if\ \Theta_2^*(n_2,-s\Delta) < \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) > \Theta_2^*(1,\Delta) \\ =0 & if\ \Theta_2^*(n_2,-s\Delta) = \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) = \Theta_2^*(1,\Delta). \end{cases} \tag{31}$$

(C) Monotonicity in signal thresholds. *As a consequence of the monotonicity in fundamental thresholds:*

$$\frac{d(x_I^*(n_2,-s\Delta) - x_I^*(n_2,\Delta)))}{dn_2} \geq 0, \ \ \forall\, n_2 \in [0,1), \tag{32}$$

where $x_I^*(n_2,-s\Delta) - x_I^*(n_2,\Delta)) > 0, \ \ \forall\, n_2 \in [0,1]$.

(D) Uninformed investors. *If* $n_2 = 0$ *then:*

$$[x_I^*(n_2,m)|f=1] > [x_I^*(n_2,m)|f=0], \ \ \forall\, m \in \{\Delta,-s\Delta,0\}. \tag{33}$$

Proof We prove the results of Lemma 3 in turn. Since the argument applies for both regions, we use the subscript t. A general observation is that the updated belief on the probability of a positive macro shock becomes degenerate: $\hat{p} \to p$ for $\alpha \to 0$. Results (A) and (B) are closely linked, so we start with them. It will be useful to consider a modified system of equations where either K_1 or K_2 are used alongside $K_3(n_t,\Theta_t^*(-s\Delta),\Theta_t^*(\Delta)) \equiv x_{tU}^*(-s\Delta) - x_{tU}^*(\Delta) = 0$.

Results (A) and (B). This proof has three steps.

Step 1: We show in the first step that for $1-p-q \to 0$ the fundamental thresholds $\Theta_t^*(-s\Delta)$ and $\Theta_t^*(\Delta)$ in the case of asymmetrically informed investors lie either both within these bounds or outside of them. As a consequence of $\hat{p} \to p$, condition $L(n_t,\Theta_t^*(-s\Delta),\Theta_t^*(\Delta)) = 0$ prescribes that, for any n_t, the thresholds $\Theta_t^*(\Delta)$ and $\Theta_t^*(-s\Delta)$ are either simultaneously within or outside of the two bounds given by the fundamental thresholds if all investors are informed, $\Theta_t^*(1,\Delta)$ and $\Theta_t^*(1,-s\Delta)$. This is proven by contradiction. First, suppose that $\Theta_t^*(\Delta) < \Theta_t^*(1,\Delta)$ and $\Theta_t^*(-s\Delta) < \Theta_t^*(1,-s\Delta)$. This leads to a violation of $L(\cdot) = 0$ because $J(\cdot) > \gamma_t \,\forall\, n_t$ if $\alpha \to 0$. Second, suppose that $\Theta_t^*(\Delta) > \Theta_t^*(1,\Delta)$ and $\Theta_t^*(-s\Delta) > \Theta_t^*(1,-s\Delta)$. Again, leading to a violation because $J(\cdot) < \gamma_t \,\forall\, n_t$ if $\alpha \to 0$. By continuity, the results continue to hold provided that $1-p-q$ is sufficiently small. That is, there exists a threshold $\eta > 0$, such that the result holds provided the sufficient condition $1-p-q < \eta$.

Step 2: We now derive the derivatives of the fundamental thresholds with respect to the proportion of informed investors, $\frac{d\Theta_t^*(m)}{dn_t}$:

$$\frac{d\Theta_t^*(n,-s\Delta)}{dn_t} = \frac{\begin{vmatrix} -\frac{\partial K_{1,2}}{\partial n_t} & \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,0)} & \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,\Delta)} \\ -\frac{\partial K_3}{\partial n_t} & \frac{\partial K_3}{\partial \Theta_t(n_t,0)} & \frac{\partial K_3}{\partial \Theta_t(n_t,\Delta)} \\ -\frac{\partial L}{\partial n_t} & \frac{\partial L}{\partial \Theta_t(n_t,0)} & \frac{\partial L}{\partial \Theta_t(n_t,\Delta)} \end{vmatrix}}{\begin{vmatrix} \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,-s\Delta)} & \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,0)} & \frac{\partial K_{1,2}}{\partial \Theta_t(n_t,\Delta)} \\ \frac{\partial K_3}{\partial \Theta_t(n_t,-s\Delta)} & \frac{\partial K_3}{\partial \Theta_t(n_t,0)} & \frac{\partial K_3}{\partial \Theta_t(n_t,\Delta)} \\ \frac{\partial L}{\partial \Theta_t(n_t,-s\Delta)} & \frac{\partial L}{\partial \Theta_t(n_t,0)} & \frac{\partial L}{\partial \Theta_t(n_t,\Delta)} \end{vmatrix}} \equiv \frac{|M_1|}{|M|} \tag{34}$$

where $|M| \equiv \det(M)$. Similarly we can derive $\frac{d\Theta_t^*(n,0)}{dn_t} = \frac{|M_2|}{|M|}$ and $\frac{d\Theta_t^*(n,\Delta)}{dn_t} = \frac{|M_3|}{|M|}$.

To find $|M|$, recall from the proof of Proposition 3 that $\frac{\partial K_1}{\partial \Theta_t(0)} > 0$, $\frac{\partial K_1}{\partial \Theta_t(-s\Delta)} < 0$ and $\frac{\partial K_1}{\partial \Theta_t(\Delta)} = 0$, while $\frac{\partial K_2}{\partial \Theta_t(0)} > 0$, $\frac{\partial K_2}{\partial \Theta_t(-s\Delta)} = 0$ and $\frac{\partial K_2}{\partial \Theta_t(\Delta)} < 0$. Furthermore, $\frac{\partial K_3}{\partial \Theta_t(0)} = 0$, $\frac{\partial K_3}{\partial \Theta_t(\Delta)} < 0$ and $\frac{\partial K_3}{\partial \Theta_t(-s\Delta)} > 0$. Finally, $\frac{\partial L}{\partial \Theta_t(m)} < 0 \; \forall m$ for a sufficiently high but finite value of β. As a result, $|M| > 0$ for a sufficiently high but finite value of β, irrespective of which of the two systems is used. That is, there exists a threshold $\beta > 0$, such that the result holds provided the sufficient condition $\beta > \underline{\beta}$.

The proof proceeds by analyzing $|M_1|$, $|M_2|$, and $|M_3|$. To do this, we first examine the derivatives $\frac{\partial K_1}{\partial n_t}$, $\frac{\partial K_3}{\partial n_t}$ and $\frac{\partial L}{\partial n_t}$. Thereafter, we combine the results to obtain the signs of the determinants $\frac{\partial K_1}{\partial n_t} = \frac{\partial x_{tU}(0)}{\partial n_t} - \frac{\partial x_{tU}(-s\Delta)}{\partial n_t}$, $\frac{\partial K_2}{\partial n_t} = \frac{\partial x_{tU}(0)}{\partial n_t} - \frac{\partial x_{tU}(\Delta)}{\partial n_t}$ and $\frac{\partial K_3}{\partial n_t} = \frac{\partial x_{tU}(-s\Delta)}{\partial n_t} - \frac{\partial x_{tU}(\Delta)}{\partial n_t}$, where:

$$\frac{\partial x_{tU}(m)}{\partial n_t} \equiv \frac{\Theta_t(m) - \Phi\big(\frac{\alpha(\Theta_t(m)-(\mu+m)) - \sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_t)}{\sqrt{\beta}}\big)}{(1-n_t)^2 \phi(\Phi^{-1}(\cdot))}. \tag{35}$$

To evaluate this partial derivatives, we can use the optimality condition in the case of symmetrically informed investors, $n_t = 1$. That is, $\Theta_t^*(1,m)$ is defined as the solution to $F_t(\Theta_t^*(1,m),m) = 0$, where uniqueness requires that F_t is strictly decreasing in the first argument. This implies:

$$\Theta_t(m) - \Phi\left(\frac{\alpha(\Theta_t(m)-(\mu+m)) - \sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_t)}{\sqrt{\beta}}\right) \lesseqqgtr 0 \; if \; \Theta_t(m) \lesseqqgtr \Theta_t(1,m).$$

Four cases are considered in turn. Case 1: $\Theta_t^*(1,\Delta) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,0) \leq \Theta_t^*(n_t,0) \leq \Theta_t^*(0,m) \leq \Theta_t^*(n_t,-s\Delta) \leq \Theta_t^*(1,-s\Delta))$. Case 2: $\Theta_t^*(1,\Delta) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(0,m) \leq \Theta_t^*(n_t,0) \leq$

$\Theta_t^*(1,0) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,-s\Delta))$. Case 3: $\Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,\Delta) \leq \Theta_t^*(1,0) \leq \Theta_t^*(0,m) \leq \Theta_t^*(1,-s\Delta) \leq \Theta_t^*(n_t,-s\Delta)$. Case 4: $\Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,\Delta) \leq \Theta_t^*(0,m) \leq \Theta_t^*(1,0) \leq \Theta_t^*(1,-s\Delta) \leq \Theta_t^*(n_t,-s\Delta)$.

Case 1: Using K_1 and K_3 we obtain $\frac{\partial K_1}{\partial n} > 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} < 0 \,\forall\, n_t \in [0,1)$.

Case 2: Using K_2 and K_3 we obtain $\frac{\partial K_2}{\partial n} < 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} < 0 \,\forall\, n_t \in [0,1)$.

Case 3: Using K_1 and K_3 we obtain $\frac{\partial K_1}{\partial n} > 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} > 0 \,\forall\, n_t \in [0,1)$.

Case 4: Using K_2 and K_3 we obtain $\frac{\partial K_2}{\partial n} < 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} > 0 \,\forall\, n_t \in [0,1)$.

After having found the partial derivative for first two equilibrium conditions ($K_{1,2}$), we turn to the other equilibrium condition (L). Here, we can invoke the envelope theorem in order to obtain $\frac{\partial L}{\partial n} = 0$. The idea is the following. Since L represents the indifference condition of an uninformed investor, the proportion of informed investors enters only indirectly via x_{tU}^* and we can write:

$$\frac{\partial L}{\partial n} = \frac{\partial J}{\partial x_{tU}^*}\frac{\partial x_{tU}^*}{\partial n} + \overbrace{\frac{\partial J}{\partial n}}^{=0}. \tag{36}$$

Since x_{tU}^* is the optimal signal threshold of an uninformed investor, it satisfies $J(\cdot, x_{tU}^*) = \gamma_t$. Thus, we must have $\frac{\partial J}{\partial x_{tU}^*} = 0$, which corresponds to a first-order optimality condition. (This implicitly uses the result that the equilibrium is unique.)

To conclude, we have for all cases that $|M| > 0$ provided that $\beta > \underline{\beta}$. It shows that $|M_1| > 0$ for case 1 and $|M_3| < 0$ for case 2, while $|M_1| < 0$ for case 1 and $|M_3| > 0$ for case 2. Furthermore, for the probability of $m = 0$, i.e. $1-p-q$, sufficiently small we have that $|M_1| > 0$ also for case 2 and $|M_3| < 0$ also for case 1, while $|M_1| < 0$ also for case 2 and $|M_3| > 0$ also for case 1. Hence, provided that $1-p-q < \eta$ and $\beta > \underline{\beta}$, we find $\forall\, n_t \in [0,1)$:

$$\frac{d\Theta_t^*(n_t,-s\Delta)}{dn_t} = \begin{cases} > 0 \quad if\ \Theta_t^*(n_t,-s\Delta) < \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) > \Theta_t^*(1,\Delta) \\ < 0 \quad if\ \Theta_t^*(n_t,-s\Delta) > \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) < \Theta_t^*(1,\Delta) \\ = 0 \quad if\ \Theta_t^*(n_t,-s\Delta) = \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) = \Theta_t^*(1,\Delta) \end{cases}$$

and $\forall n_t \in [0,1)$:

$$\frac{d\Theta_t^*(n_t,\Delta)}{dn_t} = \begin{cases} < 0 \quad if\ \Theta_t^*(n_t,-s\Delta) < \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) > \Theta_t^*(1,\Delta) \\ > 0 \quad if\ \Theta_t^*(n_t,-s\Delta) > \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) < \Theta_t^*(1,\Delta) \\ = 0 \quad if\ \Theta_t^*(n_t,-s\Delta) = \Theta_t^*(1,-s\Delta) \wedge \Theta_t^*(n_t,\Delta) = \Theta_t^*(1,\Delta) \end{cases}$$

Step 3: In this final step, we combine the results from the previous two steps to show both boundedness and monotonicity. In particular, we use the result that the derivative of the fundamental threshold w.r.t. the proportion of informed investors is zero once the boundary is hit. Therefore, the thresholds in the general case of asymmetrically informed investors are always bounded, which proves Result (A). Given boundedness, in turn, the derivatives of the fundamental threshold can be clearly signed, yielding Result (B). That is, given the result from *step 1*, the second line of each derivative drops and equations (30) and (31) follow.

We prove that $\Theta_t^*(1,\Delta) \leq \Theta_t^*(\Delta), \Theta_t^*(-s\Delta) \leq \Theta_t^*(1,-s\Delta)$ for all n_t if α sufficiently small. First, $\Theta_t^*(1,\Delta) < \Theta_t^*(\Delta) = \Theta_t^*(0) = \Theta_t^*(-s\Delta) < \Theta_t^*(1,-s\Delta)$ if $n_t = 0$, while $\Theta_t^*(1,\Delta) = \Theta_t^*(\Delta)$ and $\Theta_t^*(1,-s\Delta) = \Theta_t^*(-s\Delta)$ if $n_t = 1$. Second, $\frac{d\Theta_t^*(\Delta)}{dn_t}\big|_{n_t=0} < 0$, $\frac{d\Theta_t^*(-s\Delta)}{dn_t}\big|_{n_t=0} > 0$ and $\frac{d\Theta_t^*(\Delta)}{dn_t}\big|_{n_t=1} = \frac{d\Theta_t^*(-s\Delta)}{dn_t}\big|_{n_t=1} = 0$. Third, by continuity $\Theta_t^*(1,\Delta) \leq \Theta_t^*(\Delta), \Theta_t^*(-s\Delta) \leq \Theta_t^*(1,-s\Delta)$ and $\frac{d\Theta_t^*(\Delta)}{dn_t}\big|_{n_t=0} < 0$, $\frac{d\Theta_t^*(-s\Delta)}{dn_t}\big|_{n_t=0} > 0$ for small values of n_t. Fourth, if for any $\hat{n}_t \in (0,1]$ $\Theta_t^*(-s\Delta) \nearrow \Theta_t^*(1,-s\Delta)$ when $n_t \to \hat{n}_t$, then – for sufficiently small but positive values of α – it has to be true that $\Theta_t^*(\Delta) \searrow \Theta_t^*(1,\Delta)$ when $n_t \to \hat{n}_t$. This is because of the result in *step 1*. Fifth, given that the derivatives of the fundamental thresholds flip when both are outside of the bounds we have $\Theta_t^*(1,\Delta) = \Theta_t^*(\Delta)$ and $\Theta_t^*(1,-s\Delta) = \Theta_t^*(-s\Delta)$ for all $n_t \geq \hat{n}_t$. In conclusion, $\Theta_t^*(1,\Delta) \leq \Theta_t^*(\Delta), \Theta_t^*(-s\Delta) \leq \Theta_t^*(1,-s\Delta)$ for all $n_t \in [0,1]$ if α sufficiently small.

Result (C). From the indifference conditions for informed investors:

$$\frac{dx_{tI}^*(m)}{dn_t} = \frac{d\Theta_t^*(m)}{dn_t}\left(\frac{\beta}{\alpha+\beta}\right)^{-1}. \tag{37}$$

Therefore, by continuity, there exists a sufficiently small but positive value of α, say $\overline{\alpha}$, that implies the required inequality, taking into account the monotonicity of the fundamental thresholds. The distance between the fundamental thresholds is monotone for any $n_t > 0$, which implies

$\frac{d(x^*_{tI}(n_t,-s\Delta)-x^*_{tI}(n_t,\Delta))}{dn_t} \geq 0\ \ \forall\, n_t \in [0,1)$. Furthermore, $x^*_{tI}(n_t,-s\Delta)-x^*_{tI}(n_t,\Delta)) > 0\ \ \forall\, n_t \in [0,1]$.

Result (D). We proceed in two steps.

Step 1: We solve the equilibrium condition in equation (26) for $n_2 = 0$ to show that $[\Theta^*(n_2,m)|f = 1] > [\Theta^*(n_2,m)|f = 0]$, $\forall m \in \{\Delta, -s\Delta, 0\}$, building on the results of Lemmas 1 and 2.

Step 2: We next inspect the equilibrium adjustment of the signal thresholds. First, recall from Lemma 2 that, for sufficiently high values of β, a higher $\Theta^*_I(0,m)$ implies a higher $x^*_U(0,m)$. Second, we consider the marginal investor who becomes informed. From equation (22):

$$\Phi\big([\Theta^*_I(0,m)|f]\sqrt{\alpha+\beta} - \frac{\alpha(\mu+m)+\beta[x^*_I(n_2,m)|f]}{\sqrt{\alpha+\beta}}\big) = \gamma_2. \tag{38}$$

The result in *Step 1* implies that inequality (33) follows. This completes the proof.

A.2.3 Information stage in region 2: strategic complementarity in information choices

We next study the value of information about the macro shock. The value of information to an individual investor is defined as the difference in the expected utility between an informed and an uninformed investor before costs. These expected utilities are denoted by EU_I and EU_U, respectively. The expected utility of an informed investor writes:

$$\begin{aligned}
& \mathbb{E}[u(d_i = I, n_2)] \equiv EU_I - c \\
= \quad & -c + p' \begin{pmatrix} \int_{-\infty}^{\Theta^*_2(n_2,\Delta)} b_t \int_{x_{i2}\leq x^*_I(n_2,\Delta)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|\Delta)d\Theta_2 \\ -\int_{\Theta^*_2(n_2,\Delta)}^{+\infty} \ell_2 \int_{x_{i2}\leq x^*_I(n_2,\Delta)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|\Delta)d\Theta_2 \end{pmatrix} + \\
& q' \begin{pmatrix} \int_{-\infty}^{\Theta^*_2(n_2,-s\Delta)} b_2 \int_{x_{i2}\leq x^*_I(n_2,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|-s\Delta)d\Theta_2 \\ -\int_{\Theta^*_2(n_2,-s\Delta)}^{+\infty} \ell_2 \int_{x_{i2}\leq x^*_I(n_2,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|-s\Delta)d\Theta_2 \end{pmatrix} + \\
& (1-p'-q') \begin{pmatrix} \int_{-\infty}^{\Theta^*_2(n_2,0)} b_2 \int_{x_{i2}\leq x^*_I(n_2,0)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|0)d\Theta_2 \\ -\int_{\Theta^*_2(n_2,0)}^{+\infty} \ell_2 \int_{x_{i2}\leq x^*_I(n_2,0)} g(x_{i2}|\Theta_2)dx_{i2} f(\Theta_2|0)d\Theta_2 \end{pmatrix},
\end{aligned} \tag{39}$$

By contrast, the expected utility of an uninformed investor, $E[u(d_i = U, n_2)] \equiv EU_U$, is constructed in the same way as EU_I with the difference that all signal thresholds have to be replaced by $x^*_U(n_2)$.

Let $v \equiv EU_I - EU_U$ be the value of information conditional on the proportion of informed investors and the information set in region 2:

$$v(n_2) = \quad p' \left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(n_2,\Delta)} b_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,\Delta)} g(x_{i2}|\Theta_2) dx_{i2} f(\Theta_2|\Delta) d\Theta_2 \\ -\int_{\Theta_2^*(n_2,\Delta)}^{+\infty} \ell_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,\Delta)} g(x_{i2}|\Theta_2) dx_{i2} f(\Theta_2|\Delta) d\Theta_2 \end{array} \right) \tag{40}$$
$$+q' \left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(n_2,-s\Delta)} b_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,-s\Delta)} g(x_{i2}|\Theta_2) dx_{i2} f(\Theta_2|-s\Delta) d\Theta_2 \\ -\int_{\Theta_2^*(n_2,-s\Delta)}^{+\infty} \ell_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,-s\Delta)} g(x_{i2}|\Theta_2) dx_{i2} f(\Theta_2|-s\Delta) d\Theta_2 \end{array} \right)$$
$$+(1-p'-q') \left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(n_2,0)} b_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,0)} g(x_{i2}|\Theta_2) dx_{i2} f(\Theta_2|0) d\Theta_2 \\ -\int_{\Theta_2^*(n_2,0)}^{+\infty} \ell_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,0)} g(x_{i2}|\Theta_2) dx_{i2} f(\Theta_2|0) d\theta_2 \end{array} \right).$$

The distribution of the fundamental conditional on the realized macro shock, $f(\Theta_2|m)$, is normal with mean $\mu + m$ and precision α. The distribution of the private signal conditional on the fundamental, $g(x|\Theta_2)$, is normal with mean Θ_2 and precision β.

To build intuition, suppose that $1-p-q \to 0$. Given $\Theta_2^*(1,-s\Delta) > \Theta_2^*(1,\Delta)$ we have that $x_I^*(n_2,-s\Delta) > x_U^*(n_2) > x_I^*(n_2,\Delta)$ and marginal benefit of increasing $x_I^*(n_2,-s\Delta)$ above $x_U^*(n_2)$ is:

$$p' \left(\begin{array}{c} b_2 \int_{-\infty}^{\Theta_2^*(n_2,-s\Delta)} g(x_U^*|\Theta_2) f(\Theta_2) d\Theta_2 \\ -\ell_2 \int_{\Theta_2^*(n_2,-s\Delta)}^{+\infty} g(x_U^*|\Theta_2) f(\Theta_2) d\Theta_2 \end{array} \right) > 0, \tag{41}$$

while the marginal benefit of increasing $x_I^*(n_2,\Delta)$ above $x_U^*(n_2)$ is:

$$q' \left(\begin{array}{c} b_2 \int_{-\infty}^{\Theta_2^*(n_2,\Delta)} g(x_U^*|\Theta_2) f(\Theta_2|\Delta) d\Theta_2 \\ -\ell_2 \int_{\Theta_2^*(n_2,\Delta)}^{+\infty} g(x_U^*|\Theta_2) f(\Theta_2|\Delta) d\Theta_2 \end{array} \right) < 0. \tag{42}$$

These expressions are best understood in terms of type-I and type-II errors. Each of the expressions in equations (41) and (42) have two components. The first component in each equation represents the marginal benefit of attacking when a crisis occurs. Equivalently, this is the marginal loss from not attacking when a crisis occurs (type-I error). The second component in each equation is negative and represents the marginal cost of attacking when no crisis occurs (type-II error).

Lemma 3 together with Corollary 3 imply the following. The marginal benefit of increasing $x_I^*(n_2,-s\Delta)$ above $x_U^*(n_2)$ is positive because the type-I error is relatively more costly than the type-II error. By contrast, the marginal benefit of decreasing $x_I^*(n_2,\Delta)$ below $x_U^*(n_2)$ is positive because the type-II error is more costly. In sum, informed investors attack more aggressively upon learning that $m = -s\Delta$ and less aggressively upon learning $m = \Delta$. The value of information is governed by the relationship between the type-I and type-II errors. When the signal thresholds of informed and uninformed investors differ, the value of information is positive because the difference in thresholds increases in the proportion of informed investors. The result in Lemma 4 follows.

Lemma 4 ***Strategic complementarity in information choices.*** *If Assumption 1 holds, the value of information increases in the proportion of informed investors:*

$$\frac{dv(n_2, f)}{dn_2} \geq 0, \tag{43}$$

with strict inequality for small values of n_2.

Proof Under the sufficient conditions of Assumption 1 we have that $\Theta_2^*(n_2, -s\Delta) > \Theta_2^*(n_2, \Delta)$ and $x_I^*(n_2, -s\Delta) > x_U^*(n_2, 0) > x_I^*(n_2, \Delta)$. We will prove that $\frac{dv(n_2,f)}{dn_2} \geq 0$ and $v(n_2, f) > 0 \ \forall\, n_2 \in (0,1] \wedge f \in \{0,1\}$. Suppose that $1 - q - p \to 0$, then the last term of $\mathbb{E}[u(d_i = I, n_2)]$ and $\mathbb{E}[u(d_i = U, n_2)]$ vanishes. Given that $\Theta_2^*(n_2, -s\Delta) > \Theta_2^*(n_2, \Delta)$, the first two summands of equation (40) are strictly positive and, hence, $v(n_2) > 0 \ \ \forall\, n_2 \in (0,1]$. Furthermore, given Lemma 3, an increase in the proportion of informed investors is associated with a (weak) increase in both $\Theta_2^*(n_2, -s\Delta)$ and $x_I^*(n_2, -s\Delta)$ as well as a (weak) decrease in both $\Theta_2^*(n_2, \Delta)$ and $x_I^*(n_2, \Delta)$. For a given x_U^*, an increase in n_2 leads to a relative increase of the (positive) loss component in the first summand of equation (40) and a relative increase of the benefit component in the second summand. By continuity and monotonicity, any general equilibrium adjustment of $x_U^*(n_2)$ with n_2 cannot fully off-set the previous effects. For this reason, the left-hand side of equation (40) increases in n_2. Thus, $\frac{dv(n_2,f)}{dn_2} \geq 0$. By continuity, the results continue to hold if $1 - p - q$ is sufficiently small, that is if $1 - p - q < \eta$. This concludes the proof.

A.2.4 Proof of Proposition 2

We prove the results of the inequalities in (7). Given Assumption 1, the results of Lemma 4 apply and the first and third inequality follow. The proof of the second inequality consists of four steps.

Step 1: Suppose that $1 - p - q \to 0$ and evaluate equation (40) at $n_2 = 1$. First, observe that the first term in brackets is only affected by s through $x_U^*(1)$. Second, observe that the second term in brackets is growing strictly larger in s for a given $x_U^*(1)$, as $x_I^*(1, -s\Delta)$ grows in s because of the indifference condition of informed investors. Third, if $f=0$ observe that $x_U^*(1) \to x_I^*(1,\Delta)$ as $s \to \infty$. Given that the term in in the second bracket is finite and multiplied by $q = \frac{p}{s}$, we have that $v(1, f=0) > v(0, f=0) \to 0$ for $s \to \infty$, where the inequality is due to the result in Lemma 4.

Step 2: Now, suppose that $f=1$ and note that:

$$\lim_{\mu\to\infty}[q'|f=1]\big|_{s=\frac{\mu}{\Delta}+\iota} = \lim_{\mu\to\infty} \frac{\frac{p}{s}\Phi\big(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu+s\Delta)\big)}{\begin{pmatrix} p\Phi\big(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu-\Delta)\big) \\ +\frac{p}{s}\Phi\big(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu+s\Delta)\big) \\ +(1-p-\frac{p}{s})\Phi\big(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu)\big) \end{pmatrix}}\Big|_{s=\frac{\mu}{\Delta}+\iota} = 1,$$

where $s = \frac{\mu}{\Delta} + \iota$ with $\iota > 0$ is necessary to maintain the assumption that the prior is weak after observing a negative macro shock. Conversely, for $f=0$ we have $\lim_{\mu\to\infty}[q'|f=0]\big|_{s=\frac{\mu}{\Delta}+\iota} = 0$.

Step 3: Next, notice that for a given μ and $s > 1$, the event of a negative macro shock is never considered to be the most probable state of the world provided that s is sufficiently high. This is because $q' < p'$ holds for finite μ if s is sufficiently high: $s \geq \Pr\{f|m=-s\Delta\}(\Pr\{f|m=\Delta\})^{-1}, \forall f \in \{0,1\}$. Moreover, given *step 2* we have that $[q'|f=1] >> 0$ for μ sufficiently high such that $[p'|f=1] > [q'|f=1] > 0$, provided s is sufficiently high as well; and in the limit approaching ∞ with a higher speed of convergence. Instead, $[q'|f=0]$ is arbitrarily small.

Step 4: Given the comparative statics in *step 3*, we have for sufficiently high values of s and μ that the there is a strictly positive probability weight on the first and second bracket of $v(1,1)$, while all the probability weight is concentrated on the first bracket of $v(1,0)$. In addition, the expression in the first bracket of $v(1,1)$ is strictly larger than the expression in the first bracket of $v(1,0)$ since $x_U^* \nrightarrow x_I^*(1,\Delta)$ and $x_U^* \nrightarrow x_I^*(1,-s\Delta)$ in the former case, while $x_U^* \to x_I^*(1,\Delta)$ in the latter case. In fact, both expressions approach zero for $\mu \to \infty$, but the expression in the first bracket of $v(1,0)$ approaches zero with a higher speed of convergence. Conversely, the expression in the second bracket of $v(1,1)$ is potentially smaller than the expression in the second bracket of $v(1,0)$. Both terms approach zero with the same speed of convergence for $\mu \to \infty$. The pre-multiplied conditional probabilities $[q'|f=1]$ and $[q'|f=0]$ make the difference, where we have:

$$\lim_{\mu\to\infty}[q'|f=0]\big|_{s=\frac{\mu}{\Delta}+\iota} = \lim_{\mu\to\infty}\frac{\frac{p}{s}\big(1-\Phi\big(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu+s\Delta)\big)\big)}{\begin{pmatrix} 1-p\Phi\big(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu-\Delta)\big) \\ -\frac{p}{s}\Phi\big(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu+s\Delta)\big) \\ -(1-p-\frac{p}{s})\Phi\big(\sqrt{\alpha_1}(\Theta_1^*(\mu)-\mu)\big) \end{pmatrix}}\Big|_{s=\frac{\mu}{\Delta}+\iota} = 0.$$

In the limit $[q'|f=1]/[q'|f=0]\to\infty$ for $\mu\to\infty$ and s sufficiently high such that $0<[q'|f=1]<1$. Taken together, $v(1,1)-v(1,0)>0$ in the limit since the other terms in brackets approach zero with the same speed of convergence. By continuity, the result also holds for large, but finite, values of s and μ, as well as for sufficiently small $1-p-q$. Hence, $v(n_2=0,f=1)>v(n_2=1,f=0)$ and inequality (7) follows provided that Assumption 1 holds and s and μ are sufficiently high.

A.2.5 Proof of Proposition 1

The proof builds on the analysis of the coordination and information stages in region 2. Corollary 3 establishes the existence of unique attack rules in region 2. Proposition 2 establishes the existence of a nonempty intermediate range of information costs $c\in(\underline{c},\overline{c})$ with $\underline{c}\equiv v(1,0)$ and $\overline{c}\equiv v(0,1)$, such that all investors choose to acquire information if and only if a crisis occurs in region 1. The result in Proposition 1 follows.

A.3 Proof of Proposition 3

The proof consists of four steps. First, suppose that $s\to\infty$. Not observing a crisis in region 1 implies that $\frac{q'}{q}\to 0$ as q' goes to zero faster than q. To see this, observe that $\Pr\{f=0|m=-s\Delta\}\to 0$ if $s\to\infty$, since a Θ_2 drawn from a distribution with a highly negative mean, $\mu-s\Delta$, is increasingly unlikely to have a sufficiently high realization such that $f=0$ occurs. At the same time, $\frac{p'}{p}\nrightarrow 0$ and $\frac{1-p'-q'}{1-p-q}\nrightarrow 0$ if $s\to\infty$ and $f=0$.

Second, the right-hand side of inequality (8) has a fundamental threshold that is lower than the fundamental threshold on the left-hand side. To see this, we again use the comparative static result underlying Lemma 2. Observing $f=0$ implies that the second summand of J in equation (21) goes to zero if $s\to\infty$. Hence, $[\Theta_2^*(n_2=0,m)|f=0]<[\Theta_2^*(n_2=1,m=0)|f=1]$.

Third, given $s \to \infty$, the Θ's on the right-hand side of inequality (8) are drawn from equally favorable or, with a positive probability ($\frac{p'}{p} \nrightarrow 0$) that is away from zero, from a more favorable distribution if $f = 0$. Thus, the likelihood of a crisis in region 2 is lower if $f = 0$ and $s \to \infty$.

Fourth, by continuity, the result can be generalized to hold for a sufficiently high, but finite, value of s, say $s > \underline{s}$. This concludes the proof.

A.4 Proof of Proposition 4

We define the differential ex-ante probability of regime change, D, between when investors acquire information after observing a crisis in region 1, $n_2 = 1$, and when investors do not, $n_2 = 0$ (e.g., because of a high c). Recall that $\Theta_2^*(0,m) = \Theta_U^*$, $\forall m \in \{\Delta, -s\Delta, 0\}$, which solves equation (21). Importantly, investors do not acquire information in both scenarios after not observing a crisis in region 1, $f = 0$, which helps to simplify the expression. We also assume $1 - p - q = 0$. In sum:

$$
\begin{aligned}
D &\equiv (\Pr\{m = \Delta | f = 1\} \Pr\{\Theta_2 < \Theta_2^*(1,\Delta)\} + \Pr\{m = -s\Delta | f = 1\} \Pr\{\Theta_2 < \Theta_2^*(1,-s\Delta)\}) \\
&\quad - (\Pr\{m = \Delta | f = 1\} \Pr\{\Theta_2 < \Theta_2^*(0,\Delta) | f = 1\} + \Pr\{m = -s\Delta | f = 1\} \Pr\{\Theta_2 < \Theta_2^*(0,-s\Delta) | f = 1\}) \\
\Leftrightarrow &\quad D\,(\Pr\{\Theta_1 < \Theta_1^* | m = \Delta\} + \Pr\{\Theta_1 < \Theta_1^* | m = -s\Delta\}/s) \equiv D' \qquad (44) \\
&= (\Pr\{\Theta_2 < \Theta_2^*(1,\Delta)\} - \Pr\{\Theta_2 < \Theta_2^*(0,\Delta) | f = 1\}) \Pr\{\Theta_1 < \Theta_1^* | m = \Delta\} \\
&\quad + (\Pr\{\Theta_2 < \Theta_2^*(1,-s\Delta)\} - \Pr\{\Theta_2 < \Theta_2^*(0,-s\Delta) | f = 1\}) \Pr\{\Theta_1 < \Theta_1^* | m = -s\Delta\}/s.
\end{aligned}
$$

It is instructive to first inspect the case where $\alpha_1 \to 0$ and $\alpha_2 > 0$, which yields $\Pr\{\Theta_1 < \Theta_1^* | m = \Delta\} = \Pr\{\Theta_1 < \Theta_1^* | m = -s\Delta\} = 1/2$ and $[\Theta_2^*(0,m) | f = 1] = 1/2$ so that:

$$
2D' = \Pr\{\Theta_2 < \Theta_2^*(1,\Delta)\} - \Pr\{\Theta_2 < \tfrac{1}{2} | m = \Delta\} + \tfrac{\Pr\{\Theta_2 < \Theta_2^*(1,-s\Delta)\} - \Pr\{\Theta_2 < \frac{1}{2} | m = -s\Delta\}}{s}.
$$

We can see that $D' = D = 0$ if $\mu = \gamma = \frac{1}{2}$ and $s = 1$ due to symmetry and it can be shown that $\frac{dD'}{d\alpha_1}\big|_{\alpha_1 = 0} < 0$, meaning that $D < 0$ if $\alpha_1 > 0$. To see this, we take derivatives:

$$
\begin{aligned}
\frac{dD'}{d\alpha_1} &= -\phi\left(\frac{\Theta_2^*(0,\Delta) - \left(\frac{1}{2} + \Delta\right)}{1/\sqrt{\alpha_2}}\right)\sqrt{\alpha_2}\frac{d\Theta_2^*(0,\Delta)}{d\alpha_1}\Phi\left(\frac{-\Delta}{1/\sqrt{\alpha_1}}\right) \\
&\quad -\phi\left(\frac{\Theta_2^*(0,\Delta) - \left(\frac{1}{2} - \Delta\right)}{1/\sqrt{\alpha_2}}\right)\sqrt{\alpha_2}\frac{d\Theta_2^*(0,-\Delta)}{d\alpha_1}\Phi\left(\frac{\Delta}{1/\sqrt{\alpha_1}}\right),
\end{aligned}
$$

where we used that $d\Theta_2^*(1,\Delta)/d\alpha_1 = 0$ and $d\Theta_1^*/d\alpha_1 = 0$ for $\mu = \gamma = \frac{1}{2}$, as well as $\Theta_1^* = 1/2$. Note that $\Theta_2^*(0,m) > 1/2$ if $f = 1$ so that $d\Theta_2^*(0,m)/d\alpha_1|_{\alpha_1=0} > 0$, because $d\hat{q}'/d\alpha_1|_{\alpha_1=0} > 0$.

Conversely, when inspecting the case of $\alpha_2 \to 0$ and $\alpha_1 > 0$ we have the opposite result with $D > 0$ for sufficiently small α_2. For $\alpha_2 = 0$ we have $D' = D = 0$, because the expression in brackets in equation (44) are zero. For $\alpha_2 > 0$ the first bracket in equation (44) is negative and the second bracket is positive. If $\mu = \gamma = \frac{1}{2}$ and $s = 1$ as before, we have that $D'' \equiv \Pr\{\Theta_2 < \Theta_2^*(1,-\Delta)\} - \Pr\{\Theta_2 < \Theta_2^*(0,-\Delta)|f = 1\} - \Pr\{\Theta_2 < \Theta_2^*(1,\Delta)\} + \Pr\{\Theta_2 < \Theta_2^*(0,\Delta)|f = 1\} < 0$ so that $D > 0$ can only occur if $\Pr\{\Theta_1 < \Theta_1^*|m = -\Delta\} - \Pr\{\Theta_1 < \Theta_1^*|m = \Delta\} > 0$ is large relative to $|\Pr\{\Theta_2 < \Theta_2^*(1,-\Delta)\} - \Pr\{\Theta_2 < \Theta_2^*(0,-\Delta)|f = 1\}|$. This is the guaranteed for $\alpha_2 \to 0$ since:

$$\lim_{\alpha_2 \to 0} \frac{\Pr\{\Theta_2 < \Theta_2^*(1,\Delta)|f = 1\} - \Pr\{\Theta_2 < \Theta_2^*(0,\Delta)\}}{\Pr\{\Theta_2 < \Theta_2^*(1,-\Delta)\} - \Pr\{\Theta_2 < \Theta_2^*(0,-\Delta)|f = 1\}} = \lim_{\alpha_2 \to 0} \frac{\int_{\Theta_2^*(1,\Delta)}^{[\Theta_2^*(0,\Delta)|f=1]} \phi\left(\frac{\Theta_2-(\mu+\Delta)}{1/\sqrt{\alpha_2}}\right) d\Theta_2}{\int_{[\Theta_2^*(0,-\Delta)|f=1]}^{\Theta_2^*(1,-\Delta)} \phi\left(\frac{\Theta_2-(\mu-\Delta)}{1/\sqrt{\alpha_2}}\right) d\Theta_2} = 1.$$

In conclusion, we show by continuity that there exists a $\overline{\alpha_2} > 0$ such that, for all $\alpha_2 < \overline{\alpha_2}$, the ex-ante probability of regime change is higher for $c \in (\underline{c}, \overline{c})$ than for $c > \overline{c}$ (in the special case of $\mu = \gamma = p = \frac{1}{2}$ and $s = 1$). This concludes the proof.

A.5 Proof of Proposition 5

We analyze the role of transparency for the case of $\mu = \gamma = \frac{1}{2}$ and $1 - p - q = 0$. First, we establish some results that will be useful. It can be shown that $\frac{df(\Theta_2^*|m)}{d\alpha_2} < 0, \forall m \in \{\Delta, 0, -s\Delta\}$. Moreover, $\frac{d\Theta_2^*(m)}{d\alpha_2} < 0$ and $\frac{dx_I^*(m)}{d\alpha_2} < 0$ for $m = \Delta$, as well as $\frac{d\Theta_2^*(m)}{d\alpha_2} > 0$ and $\frac{dx_I^*(m)}{d\alpha_2} > 0$ for $m = -s\Delta$.

Next, we analyze the derivative of the value of information with respect to α_2. Observe that p' and q' only depend on α_1 and not on α_2. For $1 - p - q = 0$ we can focus on the derivatives of the first and second summand of equation (40) to describe the incentives to become informed:

$$
\begin{aligned}
\frac{dv(1,f)}{d\alpha_2} &= p'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{dx_U^*(1,f)}{d\alpha_2} g(x_U^*|\Theta_2) f(\Theta_2|\Delta) b_2 d\Theta_2 \\ +\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{dx_I^*(1,\Delta)}{d\alpha_2} g(x_I^*|\Theta_2) f(\Theta_2|\Delta) b_2 d\Theta_2 \\ +\int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{dx_U^*(1,f)}{d\alpha_2} g(x_U^*|\Theta_2) f(\Theta_2|\Delta) \ell_2 d\Theta_2 \\ -\int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{dx_I^*(1,\Delta)}{d\alpha_2} g(x_I^*|\Theta_2) f(\Theta_2|\Delta) \ell_2 d\Theta_2 \end{pmatrix} \\
&+ p'(f)\begin{pmatrix} -\frac{d\Theta_2^*(1,\Delta)}{d\alpha_2} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|\Delta) b_2 \\ -\frac{d\Theta_2^*(1,\Delta)}{d\alpha_2} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|\Delta) \ell_2 \end{pmatrix} \\
&+ p'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,\Delta)} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{df(\Theta_2|\Delta)}{d\alpha_2} b_2 \\ +\int_{\Theta_2^*(1,\Delta)}^{+\infty} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{df(\Theta_2|\Delta)}{d\alpha_2} \ell_2 \end{pmatrix} \\
&+ q'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_U^*(1,f)}{d\alpha_2} g(x_U^*|\Theta_2) f(\Theta_2|-s\Delta) b_2 d\Theta_2 \\ +\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_I^*(1,-s\Delta)}{d\alpha_2} g(x_I^*|\Theta_2) f(\Theta_2|-s\Delta) b_2 d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_U^*(1,f)}{d\alpha_2} g(x_U^*|\Theta_2) f(\Theta_2|-s\Delta) \ell_2 d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_I^*(1,-s\Delta)}{d\alpha_2} g(x_I^*|\Theta_2) f(\Theta_2|-s\Delta) \ell_2 d\Theta_2 \end{pmatrix} \\
&+ q'(f)\begin{pmatrix} -\frac{d\Theta_2^*(1,-s\Delta)}{d\alpha_2} \int_{x_I^*(1,-s\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|-s\Delta) b_2 \\ -\frac{d\Theta_2^*(1,-s\Delta)}{d\alpha_2} \int_{x_I^*(1,-s\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|-s\Delta) \ell_2 \end{pmatrix} \\
&+ q'(f)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \int_{x_I^*(1,-s\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{df(\Theta_2|-s\Delta)}{d\alpha_2} b_2 d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \int_{x_I^*(1,-s\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{df(\Theta_2|-s\Delta)}{d\alpha_2} \ell_2 d\Theta_2 \end{pmatrix}.
\end{aligned}
$$

For the special case with $\mu = \gamma = \frac{1}{2}$ and $s = 1$ the derivative simplifies, because $\frac{x_I^*(1,\Delta)}{d\alpha_2} = -\frac{x_I^*(1,-\Delta)}{d\alpha_2}$,

$\frac{d\Theta_2^*(1,\Delta)}{d\alpha_2} = -\frac{d\Theta_2^*(1,-\Delta)}{d\alpha_2}$, $\frac{1}{2} - \Theta_2^*(1,\Delta) = \Theta_2^*(1,-\Delta) - \frac{1}{2}$:

$$\begin{aligned}
\frac{dv(1,f)}{d\alpha_2}\Big|_{\mu=\gamma=\frac{1}{2},s=1} \propto\ & \frac{dx_I^*(1,\Delta)}{d\alpha_2}\left(\int_{-\infty}^{\Theta_2^*(1,\Delta)} g(x_I^*|\Theta_2) f(\Theta_2|\Delta) d\Theta_2 - \int_{\Theta_2^*(1,\Delta)}^{+\infty} g(x_I^*|\Theta_2) f(\Theta_2|\Delta) d\Theta_2 \right) \\
- & \ p'(f)\frac{d\Theta_2^*(1,\Delta)}{d\alpha_2}\left(\begin{array}{c} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2|\Delta) \\ + \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|\Delta) \end{array} \right) \\
- & \ q'(f)\frac{d\Theta_2^*(1,-\Delta)}{d\alpha_2}\left(\begin{array}{c} \int_{x_I^*(1,-\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|-\Delta) \\ + \int_{x_I^*(1,-\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2^*) dx_{i2} f(\Theta_2^*|-\Delta) \end{array} \right) \\
+ & \ p'(f)\frac{dx_U^*(1,f)}{d\alpha_2}\left(\int_{\Theta_2^*(1,\Delta)}^{+\infty} g(x_U^*|\Theta_2) f(\Theta_2|\Delta) d\Theta_2 - \int_{-\infty}^{\Theta_2^*(1,\Delta)} g(x_U^*|\Theta_2) f(\Theta_2|\Delta) d\Theta_2 \right) \\
+ & \ q'(f)\frac{dx_U^*(1,f)}{d\alpha_2}\left(\begin{array}{c} \int_{\Theta_2^*(1,-\Delta)}^{+\infty} g(x_U^*|\Theta_2) f(\Theta_2|-\Delta) d\Theta_2 \\ - \int_{-\infty}^{\Theta_2^*(1,-\Delta)} g(x_U^*|\Theta_2) f(\Theta_2|-\Delta) d\Theta_2 \end{array} \right) \\
+ & \ p'(f)\left(\begin{array}{c} - \int_{-\infty}^{\Theta_2^*(1,\Delta)} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{-\alpha_2 y(\Theta_2)}{2} f(\Theta_2|\Delta) d\Theta_2 \\ + \int_{\Theta_2^*(1,\Delta)}^{+\infty} \int_{x_I^*(1,\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{-\alpha_2 y(\Theta_2)}{2} f(\Theta_2|\Delta) d\Theta_2 \end{array} \right) \\
+ & \ q'(f)\left(\begin{array}{c} - \int_{-\infty}^{\Theta_2^*(1,-\Delta)} \int_{x_I^*(1,-\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{-\alpha_2 y(\Theta_2)}{2} f(\Theta_2|-\Delta) d\Theta_2 \\ + \int_{\Theta_2^*(1,-\Delta)}^{+\infty} \int_{x_I^*(1,-\Delta)}^{x_U^*(1,f)} g(x_{i2}|\Theta_2) dx_{i2} \frac{-\alpha_2 y(\Theta_2)}{2} f(\Theta_2|-\Delta) d\Theta_2 \end{array} \right),
\end{aligned}$$

where $y(\Theta_2) \equiv \Theta_2 - \frac{1}{2} + \Delta$. We can show that the first summand is zero. To see this, we rewrite the integrand and evaluate it at $x^*(1,\Delta)$:

$$\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{\sqrt{\alpha_2\beta}}{2\pi} e^{-\frac{1}{2}u(\Theta_2)} d\Theta_2 - \int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{\sqrt{\alpha_2\beta}}{2\pi} e^{-\frac{1}{2}u(\Theta_2)} d\Theta_2, \tag{45}$$

where $u(\Theta_2) \equiv [\beta(\Theta_2^*(1,\Delta) + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Theta_2^*(1,\Delta)) - \Theta_2)^2 + \alpha_2(\Theta_2 - \frac{1}{2} - \Delta)^2]$. From the equilibrium condition we have $\frac{1}{\sqrt{\beta}}\Phi^{-1}(\Theta_2^*(1,\Delta)) = \frac{\alpha_2}{\beta} y(\Theta_2^*(1,\Delta))$. Moreover, matching fundamental realizations that are equidistant from the fundamental equilibrium threshold we find that:

$$\begin{aligned}
\frac{d\left(e^{-\frac{1}{2}u(\Theta_2^*-\varepsilon)} - e^{-\frac{1}{2}u(\Theta_2^*+\varepsilon)}\right)}{d\varepsilon} &= -\frac{1}{2}\frac{du(\Theta_2^*-\varepsilon)}{d\varepsilon} e^{-\frac{1}{2}u(\Theta_2^*-\varepsilon)} + \frac{1}{2}\frac{du(\Theta_2^*+\varepsilon)}{d\varepsilon} e^{-\frac{1}{2}u(\Theta_2^*+\varepsilon)} \\
&= -\frac{1}{2}[2\beta(\frac{\alpha_2}{\beta}y(\Theta_2^*) + \varepsilon) - 2\alpha_2(y(\Theta_2^*) - \varepsilon)] e^{-\frac{1}{2}u(\Theta_2^*-\varepsilon)} \\
&\quad + \frac{1}{2}[-2\beta(\frac{\alpha_2}{\beta}y(\Theta_2^*) - \varepsilon) + 2\alpha_2(y(\Theta_2^*) + \varepsilon)] e^{-\frac{1}{2}u(\Theta_2^*+\varepsilon)} \\
&= (\alpha_2 + \beta)\varepsilon\left(-e^{-\frac{1}{2}u(\Theta_2^*-\varepsilon)} + e^{-\frac{1}{2}u(\Theta_2^*+\varepsilon)}\right) = 0, \forall \varepsilon \geq 0.
\end{aligned}$$

Next, the second and third summands are strictly positive since $x_I^*(1,-s\Delta) > x_U^*(1,f) > x_I^*(1,\Delta)$.

For the remaining summands we consider the limit case $\alpha_1 \to 0$, which vastly simplifies the analysis. In the limit case region 1 becomes irrelevant and $p'(f) = q'(f) = p$. Summands four and five are zero since $x_U^*(1,f) = \frac{1}{2}, \forall f \in \{0,1\}$. Finally, summands six and seven have an ambiguous sign and we know that $df(\Theta_2|\Delta)/d\alpha_2 = df(\Theta_2|-\Delta)/d\alpha_2 < 0$ for $\mu = \gamma = \frac{1}{2}$ and $s = 1$. For he limit $\alpha_1 \to 0$ we can rewrite summands six and seven as:

$$\begin{aligned}
&\frac{\alpha_2}{2}\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(1,\Delta)}\left(\int_{x_I^*(1,\Delta)}^{\Theta_2^*(1,\Delta)} g(x_{i2}|\Theta_2)dx_{i2}d\Theta_2 + \int_{\Theta_2^*(1,\Delta)}^{1/2} g(x_{i2}|\Theta_2)dx_{i2}\right) y(\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \\ -\int_{\Theta_2^*(1,\Delta)}^{+\infty}\left(\int_{x_I^*(1,\Delta)}^{\Theta_2^*(1,\Delta)} g(x_{i2}|\Theta_2)dx_{i2} + \int_{\Theta_2^*(1,\Delta)}^{1/2} g(x_{i2}|\Theta_2)dx_{i2}\right) y(\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \end{pmatrix} \qquad (46)\\
=\ &\frac{\alpha_2^{3/2}\beta^{1/2}}{4\pi}\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(1,\Delta)}\begin{pmatrix} \int_{\Theta_2^*(1,\Delta)}^{1/2} e^{-\frac{1}{2}[\beta(x_{i2}-\Theta_2)^2+\alpha_2(\Theta_2-\frac{1}{2}-\Delta)^2]}dx_{i2} \\ +\int_{\Theta_2^*(1,\Delta)+\frac{\alpha_2}{\beta}\left(\Theta_2^*(1,\Delta)-\frac{1}{2}-\Delta\right)}^{\Theta_2^*(1,\Delta)} e^{-\frac{1}{2}[\beta(x_{i2}-\Theta_2)^2+\alpha_2(\Theta_2-\frac{1}{2}-\Delta)^2]}dx_{i2}\end{pmatrix} y(\Theta_2)d\Theta_2 \\ -\int_{\Theta_2^*(1,\Delta)}^{+\infty}\begin{pmatrix} \int_{\Theta_2^*(1,\Delta)}^{1/2} e^{-\frac{1}{2}[\beta(x_{i2}-\Theta_2)^2+\alpha_2(\Theta_2-\frac{1}{2}-\Delta)^2]}dx_{i2} \\ +\int_{\Theta_2^*(1,\Delta)+\frac{\alpha_2}{\beta}\left(\Theta_2^*(1,\Delta)-\frac{1}{2}-\Delta\right)}^{\Theta_2^*(1,\Delta)} e^{-\frac{1}{2}[\beta(x_{i2}-\Theta_2)^2+\alpha_2(\Theta_2-\frac{1}{2}-\Delta)^2]}dx_{i2}\end{pmatrix} y(\Theta_2)d\Theta_2 \end{pmatrix}.
\end{aligned}$$

We first derive sufficient conditions such that a combination of the first and third summands in the last expression of equation (46) is positive. We again match equidistant fundamental realizations and evaluate at the signal $x_{i2} = \Theta_2^*(1,\Delta)$. If the resulting expression is positive at $x_{i2} = \Theta_2^*(1,\Delta)$, then also the combination of the first and third summands must be positive:

$$I(\varepsilon) = I_1(\varepsilon) + I_2(\varepsilon) \equiv (\Theta_2^* - \varepsilon - \frac{1}{2} - \Delta)e^{-\frac{1}{2}[\beta\varepsilon^2+\alpha_2(\Theta_2^*-\varepsilon-\frac{1}{2}-\Delta)^2]} - (\Theta_2^* + \varepsilon - \frac{1}{2} - \Delta)e^{-\frac{1}{2}[\beta\varepsilon^2+\alpha_2(\Theta_2^*+\varepsilon-\frac{1}{2}-\Delta)^2]}.$$

We find that $\int_0^{+\infty} I(\varepsilon)d\varepsilon > 0$ for sufficiently high values of Δ. First, $I_1(0) = I_2(0) = 0$. Taking derivatives leads to:

$$\begin{aligned}
\frac{d\left(\frac{I_1(\varepsilon)}{e^{-\frac{1}{2}\beta\varepsilon^2}}\right)}{d\varepsilon} &= (1 - \alpha_2(\Theta_2^* - \varepsilon - \frac{1}{2} - \Delta)^2)e^{-\frac{\alpha_2}{2}(\Theta_2^*-\varepsilon-\frac{1}{2}-\Delta)^2} \\
\frac{d\left(\frac{I_2(\varepsilon)}{e^{-\frac{1}{2}\beta\varepsilon^2}}\right)}{d\varepsilon} &= (\alpha_2(\Theta_2^* + \varepsilon - \frac{1}{2} - \Delta)^2 - 1)e^{-\frac{\alpha_2}{2}(\Theta_2^*+\varepsilon-\frac{1}{2}-\Delta)^2},
\end{aligned}$$

For sufficiently high Δ we have that $dI_1(\varepsilon)/d\varepsilon < 0$, while $dI_2(\varepsilon)/d\varepsilon > 0$ for small and high ε. Only for the intermedate range $\varepsilon \in \left(\hat{\varepsilon} - \sqrt{1/\alpha_2}, \hat{\varepsilon} + \sqrt{1/\alpha_2}\right)$ we have that $I_2(\varepsilon) < 0$, where $\hat{\varepsilon} \equiv$

$\frac{1}{2}+\Delta-\Theta_2^*(1,\Delta)$. Note that $\int_0^{\hat{\varepsilon}-\sqrt{1/\alpha_2}} I(\varepsilon)d\varepsilon > 0$ and $\int_{\hat{\varepsilon}+\sqrt{1/\alpha_2}}^{+\infty} I(\varepsilon)d\varepsilon > 0$ if Δ is high, while $\int_{\hat{\varepsilon}-\sqrt{1/\alpha_2}}^{\hat{\varepsilon}+\sqrt{1/\alpha_2}} I(\varepsilon)d\varepsilon < 0$. It can be shown that $\int_0^{+\infty} I_2(\varepsilon)d\varepsilon > 0$ and $\int_0^{+\infty} I_2(\varepsilon)d\varepsilon > -\int_0^{+\infty} I_1(\varepsilon)d\varepsilon$ for sufficiently high Δ and small α_2. To see this, observe that:

$$-\int_0^{\sqrt{1/\alpha_2}} e^{-\frac{\alpha_2}{2}\varepsilon'^2}d\varepsilon' + \int_{\sqrt{1/\alpha_2}}^{+\infty} e^{-\frac{\alpha_2}{2}\varepsilon'^2}d\varepsilon' > 0 \Leftarrow \alpha_2 < \left(\sqrt{\frac{\pi}{2}}erf\left(\frac{1}{\sqrt{2}}\right)\right)^{-2} \approx 0.86. \quad (47)$$

Finally, we consider the combination of the second and fourth summands in the last expression of equation (46). Following a similar arugment as before, we can show that it is positive for sufficiently high Δ. To see this, we again match equidistant fundamental realizations and evaluate at the signal $x_{i2} = \Theta_2^*(1,\Delta) + \frac{\alpha_2}{\beta}\left(\Theta_2^*(1,\Delta) - \frac{1}{2} - \Delta\right)$:

$$\begin{aligned}
I(\varepsilon) = I_3(\varepsilon) + I_4(\varepsilon) \quad &\equiv \quad (\Theta_2^* - \varepsilon - \frac{1}{2} - \Delta)e^{-\frac{1}{2}[\beta\left(\frac{\alpha_2}{\beta}(\Theta_2^*-\frac{1}{2}-\Delta)+\varepsilon\right)^2 + \alpha_2(\Theta_2^*-\varepsilon-\frac{1}{2}-\Delta)^2]} \\
&- \quad (\Theta_2^* + \varepsilon - \frac{1}{2} - \Delta)e^{-\frac{1}{2}[\beta\left(\frac{\alpha_2}{\beta}(\Theta_2^*-\frac{1}{2}-\Delta)-\varepsilon\right)^2 + \alpha_2(\Theta_2^*+\varepsilon-\frac{1}{2}-\Delta)^2]} \\
\frac{dI_3(\varepsilon)}{d\varepsilon} \quad &= \quad -(1+(\alpha_2+\beta)\varepsilon(\Theta_2^* - \varepsilon - \frac{1}{2} - \Delta))e^{-\frac{1}{2}[\beta\left(\frac{\alpha_2}{\beta}(\Theta_2^*-\frac{1}{2}-\Delta)+\varepsilon\right)^2 + \alpha_2(\Theta_2^*-\varepsilon-\frac{1}{2}-\Delta)^2]} \\
\frac{dI_4(\varepsilon)}{d\varepsilon} \quad &= \quad -(1-(\alpha_2+\beta)\varepsilon(\Theta_2^* + \varepsilon - \frac{1}{2} - \Delta))e^{-\frac{1}{2}[\beta\left(\frac{\alpha_2}{\beta}(\Theta_2^*-\frac{1}{2}-\Delta)-\varepsilon\right)^2 + \alpha_2(\Theta_2^*+\varepsilon-\frac{1}{2}-\Delta)^2]}.
\end{aligned}$$

We have that $I(0) = 0$ and $I_3(\varepsilon) < 0, \forall\varepsilon > 0$. Moreover, $I_3(\varepsilon) > 0$ for small values of ε and $I_3(\varepsilon) < 0$ for large values of ε. Similar to before, we can show that also $\int_0^{+\infty}(I_3+I_4)d\varepsilon > 0$ for sufficiently high Δ and small α_2.

Taken together, $\frac{dv(1,f)}{d\alpha_2} > 0$ for the special case with $\mu = \gamma = \frac{1}{2}$, $s = 1$, $1-p-q = 0$ and sufficiently high Δ and sufficiently small α_1 and α_2.

A.6 Proof of Proposition 6

The proof consists of two steps that deal with the impact on the fundamental thresholds and conditional distribution. First, we consider the fundamental thresholds in regions 2 after observing the outcome in region 1. From Lemma 2, we have that $f = 1$ coincides with $p' < p$, $q' > q$, and $\frac{p'}{1-q'} < \frac{p}{1-q}$, while the reverse inequalities hold if $f = 0$. Hence, observing $f = 1$ induces a lower weight on the first summand and a higher weight on the second summand of J in equation (21), while the effect of the third summand is ambiguous. Still, from $\frac{q'}{1-p'} > \frac{q}{1-p}$ it follows that the relative increase of the weight on the second summand must be higher when compared to the potential increase of the weight on the third summand.

Hence, using the comparative static result underlying Lemma 2, the $\Theta_2^*(0,m)$ that solves the version of equation (21) for region 2 and $n_2 = 0$ must be higher after observing a crisis in region 1 due to more aggressive attacks after unfavorable public information. A higher fundamental threshold is, ceteris paribus, associated with a higher conditional probability of a crisis in region 2.

Second, the distribution of the unobserved macro shock is updated after observing the outcome in region 1. Specially, the distribution of the region 2 fundamental conditional on a crisis in region 1 ($f = 1$) is less favorable than the distribution of the fundamental of region 2 conditional on no crisis in region 1 ($f = 0$). The second effect strengthens the first effect. This concludes the proof.

A.7 Figure 2: Comparative statics

We examine how the differential value of information changes in the parameter s, $\frac{d[v(1,1)-v(1,0)]}{ds}$. We consider the special case of Figures 1-4, where $\mu = \gamma = \frac{1}{2}$. To further simplify the analysis, we set $s=1$ and $1-p-q=0$. It follows that:

$$
\begin{aligned}
\frac{d[v(1,1)-v(1,0)]}{ds} \;=\;& p'(1)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{dx_U^*(1,1)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|\Delta)}{2}d\Theta_2 \\ +\int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{dx_U^*(1,1)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|\Delta)}{2}d\Theta_2 \end{pmatrix} \\
+\;& q'(1)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_U^*(1,1)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)}{2}d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_U^*(1,1)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)}{2}d\Theta_2 \end{pmatrix} \\
-\;& p'(0)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,\Delta)} \frac{dx_U^*(1,0)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|\Delta)}{2}d\Theta_2 \\ +\int_{\Theta_2^*(1,\Delta)}^{+\infty} \frac{dx_U^*(1,0)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|\Delta)}{2}d\Theta_2 \end{pmatrix} \\
-\;& q'(0)\begin{pmatrix} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_U^*(1,0)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)}{2}d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_U^*(1,0)}{ds}\frac{g(x_U^*|\Theta_2)f(\Theta_2|-s\Delta)}{2}d\Theta_2 \end{pmatrix} \\
+\;& q'(1)\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_I^*(1,-s\Delta)}{ds}g(x_I^*|\Theta_2)\frac{f(\Theta_2|-s\Delta)}{2}d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_I^*(1,-s\Delta)}{ds}g(x_I^*|\Theta_2)\frac{f(\Theta_2|-s\Delta)}{2}d\Theta_2 \end{pmatrix} \\
-\;& q'(0)\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(1,-s\Delta)} \frac{dx_I^*(1,-s\Delta)}{ds}g(x_I^*|\Theta_2)\frac{f(\Theta_2|-s\Delta)}{2}d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty} \frac{dx_I^*(1,-s\Delta)}{ds}g(x_I^*|\Theta_2)\frac{f(\Theta_2|-s\Delta)}{2}d\Theta_2 \end{pmatrix} \\
+\;& q'(1)\left(\frac{d\Theta_2^*(1,-s\Delta)}{ds}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|-s\Delta) \right) \\
-\;& q'(0)\left(\frac{d\Theta_2^*(1,-s\Delta)}{ds}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2^*)dx_{i2}f(\Theta_2^*|-s\Delta) \right) \\
+\;& q'(1)\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(1,-s\Delta)}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} \frac{g(x_{i2}|\Theta_2)dx_{i2}}{2}\frac{df(\Theta_2|-s\Delta)}{ds}d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} \frac{g(x_{i2}|\Theta_2)dx_{i2}}{2}\frac{df(\Theta_2|-s\Delta)}{ds}d\Theta_2 \end{pmatrix} \\
-\;& q'(0)\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(1,-s\Delta)}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} \frac{g(x_{i2}|\Theta_2)dx_{i2}}{2}\frac{df(\Theta_2|-s\Delta)}{ds}d\Theta_2 \\ -\int_{\Theta_2^*(1,-s\Delta)}^{+\infty}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} \frac{g(x_{i2}|\Theta_2)dx_{i2}}{2}\frac{df(\Theta_2|-s\Delta)}{ds}d\Theta_2 \end{pmatrix}, \qquad (48)
\end{aligned}
$$

where we used that, for $\mu=\frac{1}{2}$ and $s=1$, we have $\frac{1}{2}-\Theta_2^*(1,\Delta)=\Theta_2^*(1,-s\Delta)-\frac{1}{2}$ and $\frac{1}{2}-x_I^*(1,\Delta)=x_I^*(1,-s\Delta)-\frac{1}{2}$. Moreover, $\Theta_2^*(1,0)=\Theta_1=\frac{1}{2}$, $\frac{dp'(f=0)}{ds}=\frac{dq'(f=1)}{ds}$, $\frac{dp'(f=1)}{ds}=\frac{dq'(f=0)}{ds}$ and $x_U^*(n=1,f=1)-\frac{1}{2}=\frac{1}{2}-x_U^*(n=1,f=0)$ due to symmetry in the special case. Finally, together with $1-p-q=0$, we have that $\frac{dp'(f)}{ds}=-\frac{dq'(f)}{ds},\forall f\in\{0,1\}$.

Note that summands 5-6 of equation (48) are jointly positive since the expressions inside the brackets are positive, $q'(1)>q'(0)$ and $\frac{dx_I^*(1,-s\Delta)}{ds}>0$. Given that $\frac{d\Theta_1^*(1,-s\Delta)}{ds}>0$, summands 7-8 are jointly positive if:

$$\frac{q'(1)}{q(0)} > \frac{\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2^*)dx_{i2}}{\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2^*)dx_{i2}},$$

which holds for sufficiently high β. To see this, observe that a higher β shifts more mass to the upper signal threshold because $x_I^*(1,-s\Delta)$ and $\Theta_2^*(1,-s\Delta)$ approach each other faster than $\Theta_2^*(1,-s\Delta)$ and $x_U^*(1,f)$ if β increases. Formally, $x_I^*(1,-s\Delta)-\Theta_2^*(1,-s\Delta) = \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\Theta_2^*(1,-s\Delta)\right) < \Theta_2^*(1,-s\Delta)-\Theta_2^*(0,f)-\frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\Theta_2^*(0,f)\right)$ for sufficiently high β.

Similarly, summands 9-10 are jointly positive if:

$$\begin{aligned} & q'(1)\left(\begin{array}{c} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)(\Theta_2-\frac{1}{2}+s\Delta)d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty}\int_{x_U^*(1,1)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)(\Theta_2-\frac{1}{2}+s\Delta)d\Theta_2 \end{array}\right) \\ - \ & q'(0)\left(\begin{array}{c} -\int_{-\infty}^{\Theta_2^*(1,-s\Delta)}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)(\Theta_2-\frac{1}{2}+s\Delta)d\Theta_2 \\ +\int_{\Theta_2^*(1,-s\Delta)}^{+\infty}\int_{x_U^*(1,0)}^{x_I^*(1,-s\Delta)} g(x_{i2}|\Theta_2)dx_{i2}f(\Theta_2|-s\Delta)(\Theta_2-\frac{1}{2}+s\Delta)d\Theta_2 \end{array}\right) > 0. \end{aligned}$$

Notably, the second summands in both brackets are positive and the first summands are negative for values $\Theta_2 \in \left(\frac{1}{2}-s\Delta, \Theta_2^*(1,-s\Delta)\right)$ and positive for $\Theta_2 < \frac{1}{2}-s\Delta$. For a given Δ we have that the probability weight in the positive regions dominates if α sufficiently small, which makes the expressions in the brackets positive. Given that, we can use an analog argument to show that the inequality holds for sufficiently high β.

Finally, we derive conditions such that also summands 1-4 of equation (48) are jointly positive. Observe that for large β the second summands of the first bracket and the first summand of the fourth bracket dominate. Moreover, for large Δ the second summand of the second bracket, as well as the first summand of the third bracket dominate. In addition, a large Δ implies that $x_U^*(1,1) \to x_I^*(1,-s\Delta)$ and $x_U^*(1,0) \to x_I^*(1,\Delta)$ so that $\frac{dx_U^*(1,1)}{ds} > 0$ and $\frac{dx_U^*(1,0)}{ds} \to 0$. Taken together, summands 1-4 are jointly positive for sufficiently high β and Δ.

In conclusion we have shown that $\frac{d[v(1,1)-v(1,0)]}{ds} > 0\big|_{s=1}$ for the special case $\mu = \gamma = p = \frac{1}{2}$ as in Figure 2 and under the sufficient condition that β and Δ are high.

A.8 Figure 3: Comparative statics

We examine how $E[m|f=1] = \Pr\{m=\Delta|f=1\}\Delta - \Pr\{m=-s\Delta|f=1\}s\Delta$ changes with μ and with s, using the results from Lemma 2. This requires to analyze $\frac{d\Pr\{m|f=1\}}{d\mu}$ and $\frac{d\Pr\{m|f=1\}}{ds}$, $\forall m \in \{\Delta, -s\Delta\}$. We proceed in three steps.

Step 1: Taking derivatives with respect to μ leads to:

$$\frac{d\Pr\{m=\Delta|f=1\}}{d\mu} = \frac{p}{\Gamma_2(f=1)}\frac{d\Pr\{f=1|m=\Delta\}}{d\mu} - \frac{p\Pr\{f=1|m=\Delta\}}{\Gamma_2(f=1)^2}\frac{d\Gamma_2(f=1)}{d\mu}, \quad (49)$$

where $\Gamma_2(f) = p\Pr\{f|m=\Delta\} + q\Pr\{f|m=-s\Delta\} + (1-p-q)\Pr\{f|m=0\}$. Based on the results from Lemma 1 we have that $\frac{d\Pr\{f=1|m=\Delta\}}{d\mu} = \frac{d\Pr\{\Theta_1<\Theta_1^*|m=\Delta\}}{d\mu} < 0$ and $\frac{d\Gamma_2(f=1)}{d\mu} < 0$, which gives us an unclear overall effect.

Step 2: We next prove that the first summand of equation (49) outweighs the second summand. Observe that $\frac{\Pr\{f=1|m=\Delta\}}{\Gamma_2(f=1)} < 1$ and $\frac{d\Pr\{f=1|m=-s\Delta\}/d\mu}{d\Pr\{f=1|m=\Delta\}/d\mu} < 1$ since:

$$\frac{d\Phi(\sqrt{\alpha}[\Theta_1^* - (\mu - s\Delta)])/d\mu}{d\Phi(\sqrt{\alpha}[\Theta_1^* - (\mu+\Delta)])/d\mu} = \frac{\phi(\sqrt{\alpha}[\Theta_1^* - (\mu - s\Delta)])\sqrt{\alpha}(\frac{d\Theta_1^*}{d\mu} - 1)}{\phi(\sqrt{\alpha}[\Theta_1^* - (\mu+\Delta)])\sqrt{\alpha}(\frac{d\Theta_1^*}{d\mu} - 1)} < 1,$$

where a sufficiently high s as assured by Assumption 1 implies that $\phi(\sqrt{\alpha}[\Theta_1^* - (\mu - s\Delta)]) < \phi(\sqrt{\alpha}[\Theta_1^* - (\mu+\Delta)])$. As a result:

$$\begin{aligned} & -\frac{p}{\Gamma_2(f=1)}\frac{d\Pr\{f=1|m=\Delta\}}{d\mu} > -\frac{p\Pr\{f=1|m=\Delta\}}{\Gamma_2(f=1)^2}\frac{d\Gamma_2(f=1)}{d\mu} \\ = \quad & -\frac{p}{\Gamma_2(f=1)}\frac{d\Pr\{f=1|m=\Delta\}}{d\mu}\left(\begin{array}{c} p\frac{\Pr\{f=1|m=\Delta\}}{\Gamma_2(f=1)} \\ +q\frac{\Pr\{f=1|m=\Delta\}}{\Gamma_2(f=1)}\frac{d\Pr\{f=1|m=-s\Delta\}/d\mu}{d\Pr\{f=1|m=\Delta\}/d\mu} \\ +(1-p-q)\frac{\Pr\{f=1|m=\Delta\}}{\Gamma_2(f=1)}\frac{d\Pr\{f=1|m=0\}/d\mu}{d\Pr\{f=1|m=\Delta\}/d\mu} \end{array}\right), \end{aligned}$$

provided $1-p-q$ is sufficiently small, which is one of the conditions in Ass. 1. Hence, $\frac{d\Pr\{m=\Delta|f=1\}}{d\mu} < 0$. Following the same steps, $\frac{d\Pr\{m=-s\Delta|f=1\}}{d\mu} > 0$. In contrast , the signs of $\frac{d\Pr\{m=\Delta|f=0\}}{d\mu}$ and $\frac{d\Pr\{m=-s\Delta|f=0\}}{d\mu}$ are unclear since $\frac{\Pr\{f=0|m=\Delta\}}{\Gamma_2(f=0)} > 1$. To conclude, we can show that $\frac{dE[m|f=1]}{d\mu} < 0$ under sufficient conditions akin to the ones provided in Assumption 1.

Step 3: Taking derivatives with respect to s leads to:

$$\frac{dE[m|f=1]}{ds}=\frac{d\Pr\{m=\Delta|f=1\}}{ds}\Delta-\frac{d\Pr\{m=-s\Delta|f=1\}}{ds}s\Delta-\Pr\{m=-s\Delta|f=1\}\Delta \quad (50)$$

and:

$$\frac{d\Pr\{m|f=1\}}{ds}=\frac{p}{\Gamma_2(f=1)}\frac{d\Pr\{f=1|m\}}{ds}-\frac{p\Pr\{f=1|m\}}{\Gamma_2(f=1)^2}\frac{d\Gamma_2(f=1)}{ds}, \quad (51)$$

where $\frac{d\Pr\{f=1|m\}}{ds}=\phi(\sqrt{\alpha}[\Theta_1^*-(\mu+m)])\sqrt{\alpha}\left(-\frac{dJ/ds}{dJ/d\Theta_1^*}-\frac{dm}{ds}\right)$. Again there are two opposing effects. Provided $\beta>\underline{\beta}_1$ we have $\frac{dJ}{d\Theta_1^*}<0$ (Lemma 1). Next:

$$\frac{dJ}{ds} = \hat{q}\frac{d\Psi(\Theta_1^*,-s\Delta)}{ds}+\frac{d\hat{q}}{ds}\Psi(\Theta_1^*,-s\Delta)-\left(\frac{d\hat{p}}{ds}+\frac{d\hat{q}}{ds}\right)\Psi(\Theta_1^*,0)+\frac{d\hat{p}}{ds}\Psi(\Theta_1^*,\Delta),$$

where $\Psi(\Theta_1^*,-s\Delta)>\Psi(\Theta_1^*,0)>\Psi(\Theta_1^*,\Delta)$. Moreover, $\frac{d\hat{q}}{ds}>-\frac{d\hat{p}}{ds}\searrow 0$ and $\frac{d\Psi(\Theta_1^*,-s\Delta)}{ds}\searrow 0$ for $s\to\infty$. We can proof that $\frac{dJ}{ds}>0$ and $\frac{d\Theta_1^*}{ds}<0$ provided the private signal precision β and s are sufficiently high, as assured by Assumption 1. Moreover, $\frac{dE[m|f=1]}{ds}$ is negative and strictly away from zero for $s\to\infty$, because $[q'|f=1]=\Pr\{m=-s\Delta|f=1\}>>0$ under Assumption 1 as shown earlier in Proposition 2. Following similar steps, $\frac{dE[m|f=0]}{ds}\searrow 0$ for $s\to\infty$. To conclude, we can show that $\frac{dE[m|f=0]}{ds}-\frac{dE[m|f=1]}{ds}>0$ under sufficient conditions akin to the ones provided in Assumption 1.

A.9 Figure 4: Comparative statics

We prove three results. We first demonstrate in *Step 1* that $\Pr\{\Theta_2<\Theta_I^*|m=0,f=1\}>\Pr\{\Theta_2<\Theta_U^*|f=0\},\forall p,\Delta$. Thereafter, we show that $\frac{d\Pr\{\Theta_2<\Theta_U^*|f=0\}}{dp}<0$ in *Steps 2-5* and that $\frac{d\Pr\{\Theta_2<\Theta_U^*|f=0\}}{d\Delta}<0$ in *Steps 6-9*. To do so, we consider the special case where $\mu=\gamma=\frac{1}{2}$ as in Figure 4. Since $s>1$ is only key for the differential information choice, but not for the Bayesian updating channel, we further simplify the analysis by considering the case $s=1$.

Step 1: With $\mu=\gamma=\frac{1}{2}$ and $s\to 1$ we have due to symmetry that $\Theta_U^*=x_U^*=\frac{1}{2}$ and $p=q$. Form Lemma 2 we have that $p'>q'$ if a crisis in region 1 is not observed, $f=0$. As a result, ceteris paribus, $\hat{p}'>\hat{q}'$. From the equilibrium condition in equation (21) we can prove by contradiction that $\Theta_U^*<\frac{1}{2}$ if $p'>q'$. Hence, $\Pr\{\Theta_2<\Theta_I^*|m=0,f=1\}=\frac{1}{2}>\Pr\{\Theta_2<\Theta_U^*|f=0\},\forall p,\Delta$.

Step 2: An increase in p has the following implications:

$$\begin{aligned} \frac{dPr\{\Theta_2 < \Theta_U^* | f=0\}}{dp} &= \frac{dp'}{dp} Pr\{\Theta_2 < \Theta_U^* | m=\Delta\} + p' \frac{dPr\{\Theta_2 < \Theta_U^* | m=\Delta\}}{dp} \\ &+ \frac{dq'}{dp} Pr\{\Theta_2 < \Theta_U^* | m=-s\Delta\} + q' \frac{dPr\{\Theta_2 < \Theta_U^* | m=-s\Delta\}}{dp} \\ &- \left(\frac{dp'}{dp} + \frac{dq'}{dp}\right) Pr\{\Theta_2 < \Theta_U^* | m=0\} + (1-p'-q') \frac{dPr\{\Theta_2 < \Theta_U^* | m=0\}}{dp}. \end{aligned} \tag{52}$$

Step 3: We first inspect $\frac{dp'}{dp}$, noting that the symmetry property prevails when changing p in equation (14) so that Θ_1^* is unaltered and $\frac{d\Pr\{f=1|m\}|_{s=1}}{dp} < 0$ and $\frac{d\Pr\{f=0|m\}|_{s=1}}{dp} > 0, \forall m \in \{\Delta, -s\Delta, 0\}$:

$$\begin{aligned} \left.\frac{dp'}{dp}\right|_{s=1} &= \left.\frac{\left(\Pr\{f=0|m=\Delta\} + p\frac{d\Pr\{f=0|m=\Delta\}}{dp}\right)\Gamma_2(f=0) - p\Pr\{f=0|m=\Delta\}\frac{d\Gamma_2(f=0)}{dp}}{\Gamma_2^2(f=0)}\right|_{s=1} \\ \left.\frac{d\Gamma_2(f=0)}{dp}\right|_{s=1} &= \left.\left(\begin{array}{c} \Pr\{f=0|m=\Delta\} + p\frac{d\Pr\{f=0|m=\Delta\}}{dp} \\ +\frac{1}{s}\Pr\{f=0|m=-s\Delta\} + q\frac{d\Pr\{f=0|m=-s\Delta\}}{dp} \\ -\left(1+\frac{1}{s}\right)\Pr\{f=0|m=0\} + (1-p-q)\frac{d\Pr\{f=0|m=0\}}{dp} \end{array}\right)\right|_{s=1} > 0. \end{aligned}$$

Observe that $\frac{d\Pr\{f=0|m\}}{dp} = 0$ if $\alpha_1 \to 0$. Doing the same for $\frac{dq'}{dp}$ we can show that $\frac{dp'|_{s=1}}{dp} > \frac{dq'|_{s=1}}{dp} > 0$ if α_1 is sufficiently small and $\lim_{\alpha_1 \to 0}\left(\frac{dp'}{dp}\big|_{s=1}\right) = \lim_{\alpha_1 \to 0}\left(\frac{dq'}{dp}\big|_{s=1}\right) > 0$.

Step 4: Next, we inspect $sign\left(\frac{dPr\{\Theta_2 < \Theta_U^* | m\}}{dp}\right) = sign\left(\frac{d\Theta_U^*}{dp}\right)$ by analyzing the equilibrium condition in equation (20), which leads to $\frac{d\Theta_U^*}{dp} < 0$ provided p is sufficiently small. In fact, $\lim_{p\to 0}\Theta_U^* = x_U^* = \frac{1}{2}$ and $\lim_{p\to 0}\frac{d\Theta_U^*}{dp} < 0$. To see this, we apply the implicit function theorem to equation (21):

$$\frac{d\Theta_U^*}{dp} = -\frac{dJ(0, \Theta_2^*(-s\Delta), \Theta_2^*(\Delta), \Theta_2^*(0))/dp}{dJ(0, \Theta_2^*(-s\Delta), \Theta_2^*(\Delta), \Theta_2^*(0))/d\Theta_U^*}.$$

From Corollary 3 we know that $\frac{dL(\cdot)}{d\Theta_U^*} < 0$. Moreover:

$$\begin{aligned} \left.\frac{dJ(\cdot)}{dp}\right|_{s=1} &= \frac{dp'}{dp}\frac{\Pr\{x_U^*|m=\Delta\}}{\Gamma_1}\Psi(\Theta_U^*, x_U^*, \Delta)\big|_{s=1} + \frac{dq'}{dp}\frac{\Pr\{x_U^*|m=-s\Delta\}}{\Gamma_1}\big|_{s=1}\Psi(\Theta_U^*, x_U^*, -s\Delta) \\ &- \left(\frac{dp'}{dp} + \frac{dq'}{dp}\right)\frac{\Pr\{x_U^*|m=0\}}{\Gamma_1}\Psi(\Theta_U^*, x_U^*, 0)\big|_{s=1} \end{aligned}$$

is strictly negative and away from zero for sufficiently small values of α_1.

Step 5: Inspecting equation (52), there exists, by continuity, a sufficiently small positive value of α_1 such that $\frac{dPr\{\Theta_2<\Theta_U^*|f=0\}}{dp} < 0$. This concludes the proof of the second result.

Step 6: We next prove the third result. An increase in Δ has the following implications:

$$\begin{aligned}\frac{dPr\{\Theta_2 < \Theta_U^*|f=0\}}{d\Delta} &= \frac{dp'}{d\Delta}Pr\{\Theta_2 < \Theta_U^*|m=\Delta\} + p'\frac{dPr\{\Theta_2 < \Theta_U^*|m=\Delta\}}{d\Delta} \\ &+ \frac{dq'}{d\Delta}Pr\{\Theta_2 < \Theta_U^*|m=-s\Delta\} + q'\frac{dPr\{\Theta_2 < \Theta_U^*|m=-s\Delta\}}{d\Delta} \\ &- \left(\frac{dp'}{d\Delta}+\frac{dq'}{d\Delta}\right)Pr\{\Theta_2 < \Theta_U^*|m=0\} + (1-p'-q')\frac{dPr\{\Theta_2 < \Theta_U^*|m=0\}}{d\Delta}.\end{aligned} \tag{53}$$

Step 7: For $\mu=\gamma=\frac{1}{2}$ we can show that $\frac{dp'}{d\Delta}\big|_{s=1} > 0$, $\frac{dq'}{d\Delta}\big|_{s=1} < 0$ and $\frac{dp'}{d\Delta}\big|_{s=1} + \frac{dq'}{d\Delta}\big|_{s=1} = 0$.

Step 8: Analog to *Step 4* we inspect $\frac{dPr\{\Theta_2<\Theta_U^*|m\}}{d\Delta}$, which requires to examine $\frac{d\Theta_U^*}{d\Delta}$ by analyzing the equilibrium condition in equation (21). We observe that $\lim_{\Delta\to 0}\Theta_U^* = x_U^* = \frac{1}{2}$:

$$\begin{aligned}\frac{dJ(\cdot)}{d\Delta}\Big|_{s=1} &= \left(\begin{array}{c}\frac{\frac{dp'}{d\Delta}\Pr\{x_U^*|m=\Delta\}+p'\frac{d\Pr\{x_U^*|m=\Delta\}}{d\Delta}}{\Gamma_1(f=0)} \\ -p'\frac{\Pr\{x_U^*|m=\Delta\}\frac{d\Gamma_1(\cdot)}{d\Delta}}{\Gamma_1^2(f=0)}\end{array}\right)\Psi(\Theta_U^*,\Delta)\big|_{s=1} + \hat{p}'\frac{d\Psi(\Theta_U^*,\Delta)}{d\Delta}\big|_{s=1} \\ &+ \left(\begin{array}{c}\frac{\frac{dq'}{d\Delta}\Pr\{x_U^*|m=-s\Delta\}+q'\frac{d\Pr\{x_U^*|m=-s\Delta\}}{d\Delta}}{\Gamma_1(f=0)} \\ -q'\frac{\Pr\{x_U^*|m=-s\Delta\}\frac{d\Gamma_1(\cdot)}{d\Delta}}{\Gamma_1^2(f=0)}\end{array}\right)\Psi(\Theta_U^*,-s\Delta)\big|_{s=1} + \hat{q}'\frac{d\Psi(\Theta_U^*,-s\Delta)}{d\Delta}\big|_{s=1} \\ &+ \left(\frac{-\left(\frac{dp'}{d\Delta}+\frac{dq'}{d\Delta}\right)\Pr\{x_U^*|m=0\}}{\Gamma_1(f=0)} - \frac{(1-p'-q')\Pr\{x_U^*|m=0\}\frac{d\Gamma_1(\cdot)}{d\Delta}}{\Gamma_1^2(f=0)}\right)\Psi(\Theta_U^*,0)\big|_{s=1}.\end{aligned}$$

We can use the result that $\frac{d\Pr\{x_U^*|0\}}{d\Delta} = 0$ to simply the first, third and fifth summands. Moreover, we can show that these summands are jointly negative by using the symmetry properties and the fact that $p' > q'$ after not observing a crisis in region 1, $f=0$. The second summand is negative and the fourth summand is positive. Due to symmetry they are jointly negative as well since the absolute value of the derivatives of Ψ are identical. As a result, $\frac{d\Theta_U^*}{d\Delta}\big|_{s=1} < 0$.

Step 9: Observe that the fifth summand of equation (53) is zero when evaluated at $s=1$ based on *Step 7*. Moreover, the first and third summand are jointly negative. Based on *Step 8* the second and the sixth summand are negative, while the fourth summand is positive. Again using the fact that $p' > q'$, we can show that the second and forth summand are jointly negative. Taken together, $\frac{dPr\{\Theta_2<\Theta_U^*|f=0\}}{d\Delta} < 0$. This concludes the proof of the third result.

A.10 Skewness of the macro shock

This section elaborates on the discussion in section 5.4. We first consider our baseline model and revisit the key results of this paper (differential information choice and wake-up call contagion effect) for the special case of $\mu = \gamma = \frac{1}{2}$. As demonstrated in Figure 2 and in Proposition 2, the differential information choice arises for sufficiently high values of s, giving rise to the wake-up call contagion effect in Proposition 3. We first show that the differential information choice hinges on $s > 1$ by considering the case when $s = 1$. Observe that for $s = 1$ the result of Lemma 4 continues to be valid, which implies that the first and third inequality of (7) in Proposition 2 hold. However, the second inequality of (7) is violated as we show in the proof of Corollary 2.

A.10.1 Proof of Corollary 2

Consider the equilibrium condition for region 1 in equation (13) and observe that its structure is fully symmetric with $\Theta_1^* = x_1^* = \frac{1}{2}$ when $\mu = \gamma = \frac{1}{2}$ and $s = 1$. As a result, the updated prior beliefs about the macro shock distribution p' and q' are exact mirror images when observing a crisis in region 1 or not. Based on this results, the structure of the equilibrium condition for region 2 in equation (21) also shows exact mirror images when comparing the two scenarios. The same type of symmetry statement can be made for the signal thresholds. Applying these results to the value of information in equation (40), we find that there is no differential value of information: $v(n_2, 0) = v(n_2, 1)$, $\forall n_2 \in [0, 1]$. This concludes the proof of Corollary 2.

A.10.2 Offsetting changes in Δ

Following an analogous argument as in Corollary 2 we can use the symmetry properties to show that there is no differential information choice for $\mu = \gamma = \frac{1}{2}$ and $s = 1$. For the general case with $s > 1$, we revisit Proposition 2. Observe that the result of Lemma 4 is unaffected by the modification of the model. However, some steps in the proof of Proposition 2 need to be adjusted:

Step 1: The first term in brackets is now directly affected by s through $x_U^*(1)$ and indirectly through $x_I^*(1, \Delta(s))$ and $\Theta_I^*(1, \Delta(s))$. Second, observe that $\Theta_I^*(1, \Delta(s))$ is growing strictly larger in s since $\frac{ds\Delta(s)}{ds} > 0$. As before, $x_U^*(1) \to x_I^*(1, \Delta)$ as $s \to \infty$. Hence, $v(1, f = 0) \to 0$.

Steps 2, 3 and 4: After small adjustments the results in the proof of Proposition 2 go through.

Hence, the third inequality of (7) in Proposition 2 follows.

To conclude, the two key insights on the differential information choice and the wake-up call contagion effect remain valid for the modified model with offsetting changes in Δ under sufficient conditions akin to Assumption 1.

A.10.3 Independent s and q

When s and q vary independently, we have $E[m] = 0$ for $s = 1$ and $E[m] < 0$ for $s > 1$. In the former case the model is unchanged and the analysis of Corollary 2 applies. For $s > 1$, we revisit inequality (7) in Proposition 2. The result of Lemma 4 is again unaffected by the modification of the model, while the proof of Proposition 2 needs some adjustments:

Step 1: Despite q not being anymore affected by s, q' is still affected via $\Pr\{f, m = -s\Delta\}$. As before, $x_U^*(1) \to x_I^*(1,\Delta)$ as $s \to \infty$ and $v(1, f = 0) > v(0, f = 0) \to 0$.

Step 2: For $f = 1$ we now have that:

$$\frac{\partial}{\partial s}\left(\frac{q'}{p'} = \frac{q}{p}\frac{\Pr\{f = 1, m = -s\Delta\}}{\Pr\{f = 1|m = \Delta\}}\right) > 0.$$

The result flips if $f = 0$.

Step 3 and 4: Observe that, for a given μ and $s > 1$, the event of a negative macro shock is never considered to be the most probable state of the world, i.e. $q' < p'$, provided that q is sufficiently low: $\frac{p}{q} \geq \Pr\{f|m = -s\Delta\}(\Pr\{f|m = \Delta\})^{-1}$. Moreover $[q'|f = 1] >> 0$, while $[q'|f = 0]$ is arbitrarily small for high values of μ. Again, both the first and second summand of $v(0, f = 1)$ must be strictly positive and away from zero, since $x_U^* \nrightarrow x_I^*(0,\Delta)$ and $x_U^* \nrightarrow x_I^*(0,-s\Delta)$. By continuity, the result also holds for large, but finite, values of μ and s, as well as for sufficiently small $1 - p - q$. Hence, $v(n_2 = 0, f = 1) > v(n_2 = 1, f = 0)$ and inequality (7) follows.

In sum, the results go through for sufficient conditions akin to Assumption 1 with the addition that q is sufficiently low. Based on the differential information choice for $s > 1$ sufficiently high, the numerical analysis in Figure 6 demonstrates that the wake-up call contagion effect can prevail in the modified model.