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Analyzing and Forecasting the Canadian Economy through the LENS Model

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Abstract

The authors describe the key features of a new large-scale Canadian macroeconomic forecasting model developed over the past two years at the Bank of Canada. The new model, called LENS for Large Empirical and Semi-structural model, uses a methodology similar to the Federal Reserve Board's FRB/US model and the Bank of Canada's projection model of the U.S. economy (MUSE). LENS is based on a system of estimated reduced-form equations that describe the interactions among key macroeconomic variables. The model strikes a balance between theoretical structure and empirical properties, since most behavioural equations combine forward-looking expectations with adjustment costs. Compared to ToTEM, the Bank's main model for projection and policy analysis, LENS is more driven by the empirical properties of the data than economic theory and generally provides better out-of-sample forecast performance. In addition, LENS is more disaggregated, thereby allowing the analysis of a broader set of issues related to the economic outlook. These properties will make LENS a useful complement to ToTEM for constructing economic projections at the Bank of Canada.

JEL classification: E37, C53, E17, E27, F17
Bank classification: Economic models; Econometric and statistical methods

Résumé

Les auteurs exposent les principales caractéristiques d'un nouveau modèle de prévision macroéconomique de grande taille pour le Canada. Élaboré à la Banque du Canada au cours des deux dernières années, ce nouveau modèle du nom de LENS (*Large Empirical and Semi-structural model*; grand modèle empirique et semi-structurel) utilise une méthode semblable à celle du modèle FRB/US mis au point par la Réserve fédérale américaine et à celle de MUSE (modèle prévisionnel de l'économie américaine) en usage à la Banque du Canada. Le modèle LENS s'appuie sur un système d'équations estimées de forme réduite qui rendent compte des différents liens existant entre les variables macroéconomiques clés. Il réalise un équilibre entre structure théorique et propriétés empiriques, puisque la plupart des équations de comportement allient anticipations prospectives et coûts d'ajustement. Comparativement à TOTEM (principal modèle de projection et d'analyse des politiques de la Banque), le modèle LENS repose davantage sur les propriétés empiriques associées aux données que sur la théorie économique, et offre habituellement une meilleure capacité de prévision hors échantillon. En outre, son degré de désagrégation est supérieur, ce qui permet d'analyser un plus large éventail de questions relatives aux perspectives économiques. Grâce à ces propriétés, le modèle

LENS viendra compléter utilement le modèle TOTEM pour l'établissement des prévisions économiques à la Banque du Canada.

Classification JEL : E37, C53, E17, E27, F17
Classification de la Banque : Modèles économiques; Méthodes économétriques et statistiques

1. Introduction

The objective of Canadian monetary policy is to keep consumer price inflation at the 2 per cent target. Given the time delay between monetary policy actions and inflation outcomes, the Bank of Canada must take a view on the most likely evolution of the Canadian economy over the next few years. To fulfill this task, Bank staff use economic models along with judgment and information gathered from other sources such as surveys. Some of these models place monetary policy analysis at the forefront, while others put greater emphasis on forecast accuracy. Recent modelling efforts on these two distinct fronts include Alpanda et al. (2014) and Binette and Chang (2013). The main reason behind the use of multiple models stems from the fact that, being a simplification of a complex reality, no one model can answer all questions. Using multiple models is also one of the Bank's strategies to deal with economic uncertainty (Jenkins and Longworth 2002).

That said, Bank staff have been relying most heavily on one main model, ToTEM (Terms-of-Trade Economic Model), for constructing macroeconomic projections for Canada. ToTEM is a large-scale multi-sector dynamic stochastic general-equilibrium (DSGE) model that reflects the consensus view of the key macroeconomic linkages in the Canadian economy (Murchison and Rennison 2006; Dorich et al. 2013). The use of a structural model such as ToTEM reflects the desire to have a framework providing a clear representation of the economy's underlying equilibrating forces when analyzing issues related to monetary policy (Coletti and Murchison 2002). But while ToTEM has many of the micro-founded features needed for policy evaluation, it is sometimes challenging to use in a projection environment where forecasts of the *levels* of many variables are required. For instance, the cointegrating relationships implied by ToTEM are often counterfactual, leading us to detrend the data and deal with long-term trends separately. This implies that trend movements have no cyclical implications in ToTEM.

In this report, we introduce a new large-scale Canadian macroeconomic forecasting model that provides improvements along these dimensions while still having enough structure to capture key macroeconomic linkages. The new model, called LENS for Large Empirical and Semi-structural model, has a number of practical advantages relative to ToTEM. First, LENS is more driven by the historical properties of macroeconomic data and performs generally better at forecasting. Second, because of its reduced-form nature, LENS is relatively easier to use and adapt. For instance, improvements to parts of the model can be introduced without needing to re-estimate the whole model. Third, long-run trends are part of the model and respond endogenously to shocks. This is an advantage when constructing and communicating economic projections, since no data detrending is required. Fourth, while ToTEM is a multi-sector model, the structure of LENS is even more disaggregated, thereby allowing forecasts of a broader set of economic variables. For example, LENS provides an explicit decomposition of the exports forecast into non-commodity, energy and non-energy commodity exports, each of which depends on a different foreign demand variable. Overall, these properties will make LENS a useful complement to ToTEM for supporting Bank staff involved in projection and policy analysis.

The structure of LENS is similar to the Federal Reserve Board's FRB/US model (Brayton and Tinsley 1996) and the Bank of Canada's projection model of the U.S. economy (MUSE, Gosselin and Lalonde 2005), where most variables follow a rational error-correction process. In this framework, error-correction equations are used to combine long-run behavioural relationships with adjustment costs and forward-looking expectations. In some cases, economic theory is used to describe long-run relationships and adjustment costs provide flexibility to fit the data under rational expectations. In LENS, economic agents make decisions on the basis of expectations of the frictionless (or target) level of the variable of interest, but that level is not immediately attained, since moving to it entails adjustment costs. Thus, actual levels of activity will be influenced by the frictionless level as well as by expectations of how that level will evolve in the future, but with some smoothness reflecting adjustment costs.

The remainder of this report is organized as follows. Section 2 describes the specification and structure of LENS along with details about its main sectors. Section 3 covers the model's key empirical properties in terms of forecast performance and dynamic response to shocks. Section 4 offers some conclusions.

2. Model Specification and Structure

Both stocks and flows are modelled in LENS. Stationary steady-state values are assumed for business capital stock, household net wealth and government debt relative to output, and the evolution of these stocks relative to target levels has implications for the dynamics of key aggregate demand variables. GDP growth in the steady state is determined by potential output growth, which depends on exogenous assumptions regarding the growth rates of total factor productivity (TFP) and trend labour input. Real GDP is decomposed into the main National Accounts spending categories and great ratios are anchored on steady-state values that are a function of relative prices. Convergence of the dynamic model to its steady state is determined by rational error correction.

2.1 Rational error correction

Before examining the detailed structure of LENS, it is useful to review the intuition behind the rational error-correction specification that governs most behavioural equations in the model.

2.1.1 Specification

For its dynamic equations, LENS relies extensively on the rational error correction (REC) approach developed by Tinsley (2002). REC models distinguish changes in a decision variable that are induced by expectations from those that are delayed responses to previous decisions. Rational agents make decisions on the basis of forecasts of an "adjustment-cost-free" level, but this level is attained gradually owing to frictions. Decisions subject to higher adjustment costs imply a longer effective planning horizon. For instance, investment adjustment costs can reflect time to build and time to plan for the installation of capital. Deviations from target levels can be due to unanticipated shocks or can simply reflect volatility in the frictionless level.

For some variables, the frictionless level can be consistent with maximizing utility or profits in the absence of adjustment costs. For example, the frictionless level of consumption can be smooth and follow the permanent income hypothesis. In other cases, long-run demand functions can be assumed in order to determine the "no-adjustment cost" levels. As a result, the frictionless level may itself be volatile, implying that, in the presence of adjustment costs, it may not be optimal to respond to every movement in the frictionless level, especially if some are reversed. Section 2.2 provides more details on the specification of the frictionless level of each decision variable in LENS.

Under rational error correction, agents minimize the joint expected costs of diverging from the frictionless level and the costs associated with modifying spending patterns to return to it.[1] Expected future costs are discounted such that adjustment costs in the distant future have less bearing on current decisions than those in the near future. This implies the minimization of a cost function specified in terms of the discounted current and future costs of a decision variable (e.g., y):

$$y_t = \min(E_{t-1}\{COST_t\})$$

$$COST_t = \left\{\sum_{i=0}^{\infty} \beta^i \left[k_0(y_{t+i} - y^*_{t+i})^2 + k_1(\Delta y_{t+i})^2 + k_2(\Delta^2 y_{t+i})^2 + \cdots\right]\right\}, \qquad (1)$$

where $E_{t-1}\{COST_t\}$ is a forecast of the costs of diverging from the frictionless (or target) level y^*. The first squared expression is the cost of diverging from y^* in period $t+i$ with unit cost k_0. Thus, agents adjust the decision variable as a function of expected future movements in the target level. The other terms form a polynomial representation of the costs related to changes in y in future periods, $t+i$. While it is common to assume that the principal source of friction is captured by the quadratic term $k_1(\Delta y_{t+i})^2$ (see, e.g., Rotemberg 1996), REC models allow a more general representation of adjustment costs, since k_2 is the unit cost of changing the rate of change in y, k_3 the unit cost of changing the rate of acceleration in y, and so on. By minimizing this generalized cost function, it is possible to derive the following decision rule for y (also known as the rational error-correction representation):

$$\Delta y_t = -a_0(y_{t-1} - y^*_{t-1}) + \sum_{j=1}^{m-1} a_j \Delta y_{t-j} + E_{t-1}\left\{\sum_{i=0}^{\infty} f_i \Delta y^*_{t+i}\right\}, \qquad (2)$$

which stipulates that the cost-minimizing adjustment at time t (Δy_t) depends on the per cent difference between the previous period value of y and its target level $(y_{t-1} - y^*_{t-1})$, previous changes in the level of y (Δy_{t-j}) and a discounted sum of expected changes in the target level (Δy^*_{t+i}). Equation (2) is similar to an error-correction model except that the inclusion of the final term imparts a forward-looking element to the framework. The f_i coefficients are a function of the discount rate, β, and the cost parameters $(k_0, k_1, k_2, \ldots)$, whereas the order of adjustment cost (m) determines the number of lags in the decision rule.[2] A higher order of frictions implies a higher-order Euler equation, therefore providing a justification for the presence of multiple lags in the decision rule. In many cases, this is similar to what arises in DSGE models with adjustment costs, such as ToTEM. However, higher-order frictions in LENS

[1] In the case of a trending variable, adjustment costs can be interpreted as capturing costly deviations from expected trend growth (Kozicki and Tinsley 1999).

[2] The order of adjustment costs is determined empirically as part of the estimation process and is typically chosen so as to ensure that residuals are not serially correlated. Thus, there is no external source of serial persistence in these behavioural equations.

imply that the model does not require persistence in the shock processes to match persistence in the data.[3]

A key aspect of the REC methodology is that it models the relationships between the data as measured—in other words, data do not need to be detrended and trends and cycles are not modelled separately. As Cayen et al. (2009) discuss, trend determination is important in a forecasting environment where projections of the *levels* of many variables are required. Trend and cyclical behaviour are modelled jointly under rational error correction, and the adjustment process over the forecast horizon occurs through movements in both actual and frictionless (trend) levels. This is different from ToTEM (and most DSGE specifications used in central banks), where the model is used to generate forecasts for detrended variables and a separate model for trends is required to produce forecasts of levels. In practice, ToTEM's forecasts are added to trend forecasts, implying that trend movements have no cyclical implications.[4]

2.1.2 Estimation

While there are gains from simultaneous equation estimation in terms of the identification of structural shocks, this is less relevant in a forecasting environment than it is in the context of policy analysis. Moreover, the large number of parameters in LENS makes it very difficult to estimate all of its equations simultaneously, so each equation is estimated separately.

Estimating REC equations involves three main steps:

i. In the first step, the frictionless level of the variable of interest (y^*) is determined. This level is obtained from an estimated cointegrating relationship between the variable of interest and its long-run determinants. Equilibrium restrictions on long-run target levels are imposed to generate a balanced growth path in general equilibrium.
ii. In the second step, the expected future level of y^* is forecast using the estimated cointegrating parameters and a vector autoregression (VAR) model containing variables from the cointegrating space. As a result, estimation does not assume model-consistent expectations. On the other hand, the general-equilibrium version of LENS is simulated using fully model-consistent expectations.
iii. The third step is to estimate the associated dynamic equation using non-linear least squares with generalized method of moments if right-hand side variables are deemed to be endogenous. This process uses the cointegrating parameters, the VAR coefficients, a calibrated discount rate (β from equation (1)) and starting values for the a_j parameters of equation (2).

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[3] Kozicki (2012) and Brayton et al. (2014) compare the design of REC models to the DSGE modelling approach.

[4] The current trends model used by Bank staff is based on a combination of cointegrating relationships and filtering techniques. ToTEM or any DSGE model could generate forecasts for levels. The choice of using ToTEM only for cycles rests on the fact that the cointegrating relationships implied by the model are often counterfactual. Using ToTEM to forecast levels would require a high number of permanent shocks to explain trends in relative prices and great ratios present in Canadian National Accounts data.

While we could have used Bayesian estimation methods or calibrated some parameters to generate dynamic properties consistent with certain economic priors, we tried to avoid doing so, consistent with the view that LENS is more a forecasting tool than a policy analysis model.

The parameters governing the model's steady state, such as factor shares in the production function and long-run capital-output ratios, are not estimated; they are calibrated based on priors grounded in evidence on historical income shares and similar considerations.

2.2 Model structure: decomposition of aggregate demand

One of the key features of LENS is the high degree of disaggregation of output into various aggregate spending components. This allows different drivers to play a role and a broader set of economic variables to be included in the forecast.

2.2.1 Household spending

The main component of the household block in LENS is spending on non-durable goods and services (C^{nds}), which represents almost 90 per cent of total nominal consumption. The frictionless level of spending is modelled following the permanent income hypothesis, meaning that (the log of) per capita target expenditures on non-durable goods and services (C^{nds*}) depends on human wealth ($HWEALTH$) and non-human wealth ($NHWEALTH$):

$$C_t^{nds*} = -2.36 + (1 - 0.282)HWEALTH_t + 0.282NHWEALTH_t, \tag{3}$$

where human wealth is the expected discounted sum of future real disposable income flows, and non-human wealth is the sum of financial and real estate net wealth at market value.[5] In line with the permanent income hypothesis, we impose that the coefficients on human and non-human wealth sum to one. As in Macklem (1994) and Pichette and Tremblay (2003), we find that consumption is much more sensitive to changes in human wealth than non-human wealth. In LENS, projections for non-human wealth depend on business and residential capital stocks, government debt, and household debt. House prices also affect non-human wealth through their impact on the nominal residential capital stock.

We allow for deviations from permanent income along the dynamic path by following a rational error-correction process. Short-run fluctuations are driven by the error-correction term, a lag of the growth of spending on non-durable goods and services, and expected future changes in the target level.[6] In addition, real disposable income growth (ΔY^d) affects changes in spending contemporaneously, reflecting rule-of-thumb consumers. To ensure convergence to a stable ratio of C^{nds} to GDP in the steady state, the sum of the coefficients on lagged spending growth, real disposable income growth and future changes in target consumption is constrained to unity. Finally, changes in consumer confidence

[5] At estimation, human wealth was generated separately by projecting disposable income growth 40 quarters out over history using ToTEM.

[6] The order of adjustment cost for spending on non-durables and services is equal to 2.

($\Delta CCONF$) have a small contemporaneous impact on spending.[7] While this variable plays a minor role in consumption dynamics, it is included in the model to allow for a channel for confidence shocks:

$$\Delta C_t^{nds} = -0.15\left(C_{t-1}^{nds} - C_{t-1}^{nds*}\right) + 0.08\Delta C_{t-1}^{nds} + 0.78E_t\sum_{i=1}^{20} f_i\Delta C_{t+i}^{nds*} + 0.14\Delta Y_t^d + 0.01\Delta CCONF_t + \varepsilon_t^{cnds}. \quad (4)$$

The share of households that are liquidity constrained or rule-of-thumb spenders is estimated at 14 per cent; 86 per cent are estimated to be forward looking, but still face adjustment costs. This is somewhat lower than the micro-data estimate of Faruqui and Torchani (2012), who find that about 23 per cent of households in Canada were liquidity constrained over the 2000–07 period. At 15 per cent per quarter, the spending patterns of households adjust relatively slowly to differences between expenditures and their target level. In this set-up, interest rate changes affect consumption of non-durable goods and services *gradually* via permanent income (both human and non-human wealth) and *contemporaneously* through disposable income (for rule-of-thumb spenders). Forward-looking behaviour arises not only because of adjustment costs inherent to the REC framework, but also as a result of the specification of target consumption, which depends on expectations of future income.

Durable-goods consumption (C^{dur}) and residential investment (I^{res}) are modelled as complementary investment goods for which households derive utility well beyond the purchase date. Target stocks of durable and housing assets (K_i^*) depend on estimated long-run capital shares (S_i^*), demand (Y), and capital-specific user costs (UC_i):

$$K_{i,t}^* = \frac{S_{i,t}^*(1-\alpha_h)Y_t}{UC_{i,t}}, i \in \{durables, housing\}. \quad (5)$$

This specification implies stable capital-to-output ratios for durables and housing in steady state. Capital shares are anchored on cointegrating relationships with the user cost of capital, where the latter depends on the evolution of the real household effective interest rate ($fcost_i$), depreciation ($depr_i$) and the relative price (p_i) of the asset:

$$UC_{i,t} = \left[fcost_{i,t} + depr_{i,t} - \left(1 - depr_{i,t}\right)\Delta p_{i,t}\right]p_{i,t}, i \in \{durables, housing\}. \quad (6)$$

This equation implies that the return on capital must be sufficient to earn the rate of interest, recover depreciation and recoup any capital losses caused by movements in the price of the asset.

Implicit target *flows* for durable-goods consumption and residential investment are derived from the rule of perpetual inventory accumulation and feed into separate dynamic rational error-correction equations. Similar to spending on non-durable goods and services, short-run fluctuations in durable-goods consumption are also driven by contemporaneous changes in disposable income, and here again changes in consumer confidence have a simultaneous impact on both types of spending:[8]

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[7] Consumer confidence is a function of the foreign output gap in LENS.

[8] The sum of the coefficients on lagged spending, real disposable income and future changes in target levels is constrained to unity to ensure convergence to a stable ratio of durables and housing spending relative to GDP. The

$$\Delta C_t^{dur} = -0.10\left(C_{t-1}^{dur} - C_{t-1}^{dur*}\right) + 0.08\Delta C_{t-1}^{dur} + 0.02\Delta C_{t-2}^{dur} + 0.01\Delta C_{t-3}^{dur} + 0.72 E_t \sum_{i=1}^{20} f_i \Delta C_{t+i}^{dur*} + 0.17\Delta Y_t^d + 0.11\Delta CCONF_t + \varepsilon_t^{cdur}, \quad (7)$$

$$\Delta I_t^{res} = -0.08(I_{t-1}^{res} - I_{t-1}^{res*}) + 0.37\Delta I_{t-1}^{res} - 0.09\Delta I_{t-2}^{res} + 0.05\Delta I_{t-3}^{res} + 0.67 E_t \sum_{i=1}^{20} f_i \Delta I_{t+i}^{res*} + 0.15\Delta CCONF_t + \varepsilon_t^{ires}\,. \quad (8)$$

The speeds of adjustment are slower in the case of investment in durable and residential assets, with lags playing a more prominent role and smaller fractions of gaps relative to frictionless levels being closed every quarter (10 and 8 per cent, respectively). In LENS, movements in interest rates have most of their effect on domestic demand through their direct impact on the user cost of capital.

2.2.2 Business investment

As with household expenditures on durable goods and housing, capital spending by firms is modelled as a function of user cost and demand. Business fixed investment is decomposed into spending on two complementary capital goods: machinery and equipment (I^{me}) and structures (I^{str}). In the long run, the stocks of each type of investment good are determined by demand equations derived from a production function as in Gosselin and Lalonde (2005).[9] Target stocks depend on target shares, demand and user cost:[10]

$$K_{i,t}^* = \frac{S_{i,t}^*(1-\alpha)Y_t}{UC_{i,t}}, i \in \{structures, M\&E\}. \quad (9)$$

Target shares are subject to homogeneity restrictions in LENS, implying that any trending share is offset by a trend in another share and thereby maintaining a constant capital share of income. These restrictions are important in the case of business investment, given the upward trend in the share of spending on machinery and equipment (M&E) during the 1990s followed by a partial reversal in the 2000s at the expense of strong investment in structures. In particular, the share of engineering activity rose significantly since 2000 with the boom in oil prices.

Separate dynamic rational error-correction equations are specified for each type of investment. Output growth in the current period (ΔY) is included in the short-run dynamics, which could capture cash-flow effects for some subset of financially constrained firms. The change in real commodity prices ($\Delta PCOM$) is also included to help improve short-term forecast performance. Finally, business confidence ($\Delta BCONF$: year-ahead investment intentions from the Bank of Canada's *Business Outlook Survey*)

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order of adjustment costs for these components of household spending is 4. See Demers (2005) for other econometric models of Canadian residential investment.

[9] Instead of maximizing profits under a standard production-function framework, this method follows the dual approach where a translog cost function is minimized. This permits having trends in nominal capital shares.

[10] User cost is based on the same definition as in equation (6), but is constructed with business sector variables for financing costs (business effective interest rates), depreciation and relative prices. Investment prices depend on commodity prices and the exchange rate (see Appendix C).

affects M&E investment in the short run.[11] It is included in the model as a channel for confidence/uncertainty shocks:

$$\Delta I_t^{me} = -0.10(I_{t-1}^{me} - I_{t-1}^{me*}) + 0.13\Delta I_{t-1}^{me} + 0.05\Delta I_{t-2}^{me} + 0.02\Delta I_{t-3}^{me} + 0.40E_t \sum_{i=1}^{20} f_i \Delta I_{t+i}^{me*} + 0.40\Delta Y_t + 0.0015\Delta BCONF_t + 0.001\Delta PCOM_t + \varepsilon_t^{ime} , \quad (10)$$

$$\Delta I_t^{str} = -0.07(I_{t-1}^{str} - I_{t-1}^{str*}) + 0.11\Delta I_{t-1}^{str} + 0.05\Delta I_{t-2}^{str} + 0.44E_t \sum_{i=1}^{20} f_i \Delta I_{t+i}^{str*} + 0.40\Delta Y_t + 0.03\Delta PCOM_t + \varepsilon_t^{istr}. \quad (11)$$

The speed of adjustment for business capital spending (10 and 7 per cent per quarter) is similar to that for households. Of note, the coefficient on expectations of the target level is relatively low, since much of the short-term dynamics are captured by the "cash-flow" proxy.[12] This means that investment will pick up in the short term following a rise in demand, despite the increase in user cost (accelerator effect). The relatively high coefficient on cash flow could reflect the importance of small and medium-sized enterprises in Canada, which tend to be more financially constrained (Industry Canada 2013).

2.2.3 Government

LENS has a fairly detailed government block, with public spending and revenues decomposed into federal versus provincial and local components. Government expenditures are further decomposed into transfer spending, spending on goods and services (National Accounts definition), and interest payments on debt, allowing the identification of the primary and total fiscal deficits. Given such a disaggregated approach, we can consider different determinants and elasticities for fiscal policy depending on the level of government, and provide more details on the fiscal outlook.[13]

Modelling total spending and revenues is necessary because we assume that the fiscal authorities adjust the total deficit to reach a debt-to-GDP target. While fiscal policy reacts to the cyclical position of the economy in the short run, total government size (relative to output) and the tax rate adjust jointly in the long run so as to reach targets for federal and provincial government debt. Net transfers have a direct impact on household disposable income (and therefore human wealth), whereas government debt is part of household financial wealth. A detailed specification of the fiscal block is provided in Appendix A.

2.2.4 International trade

In LENS, the frictionless level of exports depends on foreign demand and relative prices. The rational error-correction framework is applied separately to non-commodity (X^{ncom}), non-energy commodity

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[11] As for consumers, business confidence depends on the foreign output gap in LENS.
[12] The sum of the coefficients on lagged spending, GDP growth and future changes in target investment are constrained to unity. The order of adjustment cost is 4 in the case of M&E investment and 3 in the case of investment in structures.
[13] Over the first couple of years of the projection, we usually anchor government spending projections on announced budgets to be consistent with official plans.

(X^{nec}) and energy (X^{ener}) exports. In addition, different measures of foreign demand are considered for each type of export. The frictionless level of (the log of) non-commodity exports is based on a long-run relationship with U.S. consumption (C^{us}), residential investment ($IRES^{us}$) and business investment (I^{us}), as well as the real effective exchange rate ($FOREX$) and a deterministic trend.[14] Non-energy commodity exports are cointegrated with U.S. residential investment, Chinese GDP (Y^{china}), the real effective exchange rate and a trend.[15] The target path for energy exports is given by a long-run relationship with U.S. GDP (Y^{us}) and real energy prices (P^{ener}):

$$X_t^{ncom*} = 4.29 + (0.22C_t^{us} + 0.26IRES_t^{us} + 0.52I_t^{us}) + 0.76FOREX_t + 0.0065TREND_t^{ncom}, \quad (12)$$

$$X_t^{nec*} = -0.62 + \left(0.32IRES_t^{us} + 0.68Y_t^{china}\right) + 0.22FOREX_t - 0.01TREND_t^{com}, \quad (13)$$

$$X_t^{ener*} = 2.0 + 1.0Y_t^{us} - 0.03P_t^{ener}. \quad (14)$$

The coefficients on U.S. demand variables in equation (12) are not very different from the Bank's foreign activity measure (Morel 2012). The dynamic models of exports follow the standard rational error-correction specification along with the contemporaneous change in the foreign output gap ($\Delta YROW^{gap}$) to help improve short-term forecast performance and capture the fact that the income elasticity of trade variables is typically higher in the short run (Hooper et al. 2000):[16]

$$\begin{aligned}\Delta X_t^{ncom} &= -0.15(X_{t-1}^{ncom} - X_{t-1}^{ncom*}) + 0.08\Delta X_{t-1}^{ncom} + 0.04\Delta X_{t-2}^{ncom} + 0.03\Delta X_{t-3}^{ncom} \\ &\quad + 0.85E_t \sum_{i=1}^{20} f_i \Delta X_{t+i}^{ncom*} + 3.42\Delta YROW_t^{gap} + \varepsilon_t^{xncom}, \quad (15)\end{aligned}$$

$$\begin{aligned}\Delta X_t^{nec} &= -0.20(X_{t-1}^{nec} - X_{t-1}^{nec*}) + 0.10\Delta X_{t-1}^{nec} + 0.02\Delta X_{t-2}^{nec} + 0.88E_t \sum_{i=1}^{20} f_i \Delta X_{t+i}^{nec*} \\ &\quad + 2.26\Delta YROW_t^{gap} + \varepsilon_t^{xnec}, \quad (16)\end{aligned}$$

$$\begin{aligned}\Delta X_t^{ener} &= -0.30(X_{t-1}^{ener} - X_{t-1}^{ener*}) + 0.05\Delta X_{t-1}^{ener} + 0.03\Delta X_{t-2}^{ener} + 0.01\Delta X_{t-3}^{ener} + 0.91E_t \sum_{i=1}^{20} f_i \Delta X_{t+i}^{ener*} \\ &\quad + 2.68\Delta YROW_t^{gap} + \varepsilon_t^{xener}. \quad (17)\end{aligned}$$

[14] A deterministic trend is required to find statistical evidence of cointegration in a context where the sum of the coefficients on foreign demand variables is restricted to one. This restriction is a necessary condition for balanced growth in LENS. While there is an upward trend in exports relative to foreign output over the sample, the unit income elasticity is not rejected in the presence of a proxy for increasing trade openness. Of course, this trend is not extended indefinitely over the forecast horizon.

[15] The trend coefficient for non-energy commodity exports is negative owing to the very strong growth in Chinese GDP over the sample.

[16] The sum of the coefficients on changes in leads of target levels and lags of actual export growth is restricted to one. The estimated order of adjustment cost is 4 for non-commodity and energy exports and 3 for non-energy commodity exports.

The speed of adjustment varies greatly across export sectors. Adjustment costs are most significant for non-commodity exports (15 per cent) followed by non-energy commodity exports (20 per cent). Convergence toward target levels is relatively fast in the case of energy exports, at 30 per cent per quarter. The relatively high coefficient on the foreign output gap reflects the fact that the short-run income elasticity of exports is much higher than its assumed long-run value of one.

Import volumes are disaggregated into non-commodity and commodity (energy and non-energy together) components and modelled as a function of aggregate demand (private domestic demand plus exports), the real effective exchange rate, and commodity prices in a rational error-correction set-up. Appendix B provides further details about the import sector in LENS.

2.2.5 Inventories

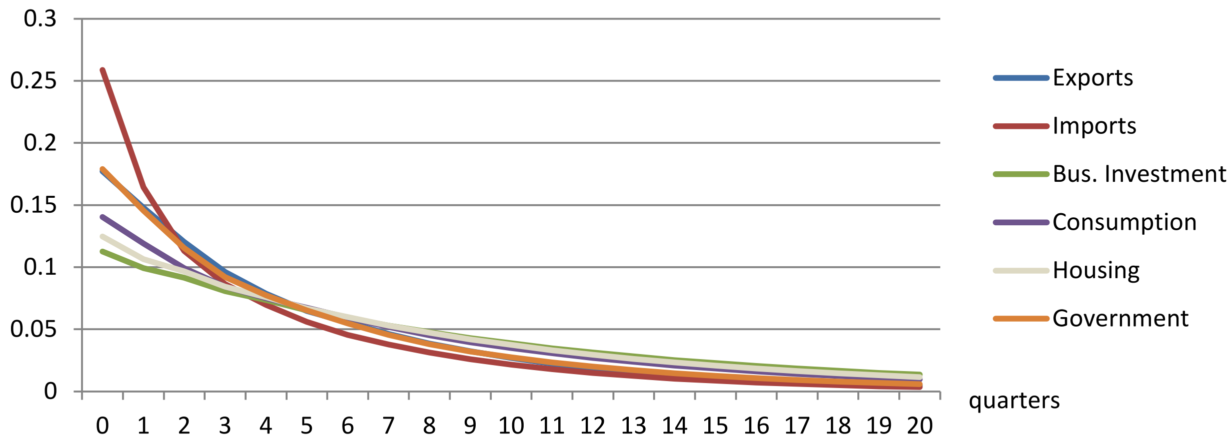
To complete the National Accounts identity, we model the swing in business inventories. In LENS, firms adjust their inventories (S) to meet a target for the stock-to-sales ratio. We assume a constant target ratio, consistent with unit root tests over the 1998–2013 period. Given the profile for sales, we back out a target level for inventories (S^*) and estimate a simple stock adjustment process:

$$\Delta S_t = -0.07(S_{t-1} - S^*_{t-1}) + 0.60\Delta S_{t-1} + (1 - 0.60)\Delta YPOT^{ss} + \varepsilon^s_t \, . \qquad (18)$$

Sales are modelled as a function of the growth rates of the main components of aggregate demand. Here $\Delta YPOT^{ss}$ is a steady-state assumption for potential output growth. This assumption, combined with the restriction that the sum of the coefficients on trending variables is one, ensures a stable ratio of inventories relative to GDP in the steady state.

To complete the discussion of estimation results for the main components of GDP, we report the distribution of lead weights. These weights correspond to the f_i coefficients in equation (2) and can be used to illustrate the relative degree of forward-looking behaviour across the model. To facilitate comparison, the lead weights are expressed as a percentage of their sum so that their integral is normalized to one. For the components of output that are further disaggregated in LENS, we compute lead weights as a weighted average of the subcomponents. As can be seen in Figure 1, lead weights converge toward zero as the planning horizon is extended, reflecting discounted expectations of the frictionless level. Adjustment is faster in the case of imports than it is for business investment.

Figure 1: Distribution of lead weights



A summary measure of the effective average length of the forward planning period for a decision variable is the *mean lead* implied by the f_i coefficients (von zur Muehlen 2001). Mean leads measure the average responsiveness to future expected events and are a reflection of the typical decision horizon for a variable.[17] As Table 1 shows, the effective planning horizon is shorter in the case of variables with lower adjustment costs.

Table 1: Comparison of forward-looking horizon

Average planning horizon (qrtrs)	
Consumption	5.65
Housing	6.05
Bus. Investment	6.33
Government	4.63
Exports	4.47
Imports	3.71

The implied mean lead varies depending on the average time it takes for the frictionless level to be reached. If adjustment costs are high, then adjustment is slow and decisions taken today are more influenced by expectations in the distant future. In LENS, average planning horizons range from 3.7 quarters (imports) to 6.3 quarters (business investment). In the REC structure, the empirical impact of expectations will depend on these leads and the persistence of the frictionless level.

[17] The mean lead can be calculated as $\overline{w} = \sum_{i=0}^{T} i\left[f_i / \sum_{i=0}^{T} f_i\right]$, where i represents the future horizon (measured in quarters) and f_i is the weight attached to it.

2.3 Model structure: supply side

A key variable in the model is potential output, which helps determine the output gap (Y^{gap}) and GDP growth, and pins down the steady-state growth of the economy. Potential output is based on a Cobb-Douglas production function with actual capital services and exogenous assumptions regarding trend labour supply and TFP.[18]

Business capital stock projections are given by the accumulation of investment forecasts, while trend labour supply projections are based on forecasts of population growth, trend employment and trend hours worked derived from separate cohort models in which household wealth is one of the key determinants (Barnett 2007). While trend labour supply is determined exogenously, labour demand depends on the cyclical position of the economy in LENS. Deviations of the main components of labour input from their trend are modelled according to simple Okun's-type relationships:

Hours worked

$$HRS_t = HRS_t^{gap} + \overline{HRS_t^{trend}}, \tag{19}$$

$$HRS_t^{gap} = 0.16Y_t^{gap} + 0.28HRS_{t-1}^{gap} + \varepsilon_t^{hrs}, \tag{20}$$

Participation rate

$$PART_t = PARt_t^{gap} + \overline{PART_t^{trend}}, \tag{21}$$

$$PART_t^{gap} = 0.04Y_t^{gap} + 0.78PART_{t-1}^{gap} + \varepsilon_t^{part}, \tag{22}$$

Unemployment rate

$$U_t = U_t^{gap} + \overline{U_t^{trend}}\,, \tag{23}$$

$$U_t^{gap} = -0.17Y_t^{gap} + 0.85U_{t-1}^{gap} - 0.26U_{t-2}^{gap} + \varepsilon_t^{u}. \tag{24}$$

The sum of these gaps and trends determines actual labour input, which behaves procyclically in this set-up. Given the profile of wages (see next section for the wage Phillips curve), labour input can be used to generate labour income. Taking into account government net transfers and interest payments on household debt (see section 2.4.3), we can construct household disposable income.

2.4 Model structure: nominal variables

In LENS, a number of equations drive the interactions between the real economy and nominal variables, such as core inflation, interest rates and the exchange rate.

[18] Over history, LENS uses the level of potential consistent with the conventional estimate of the output gap (Butler 1996).

2.4.1 Inflation, wages and relative prices

We use a hybrid Phillips curve to model quarter-over-quarter CPIX inflation (π). Core inflation is driven by the output gap and inflation in real import prices and unit labour cost. The implicit price-setting mechanism in LENS is similar in spirit to that of ToTEM (Dorich et al. 2013), whereby pricing depends on forward-looking and rule-of-thumb behaviour on the part of firms. For the forward-looking group, price reoptimization is fully rational and forward looking, while for the rule-of-thumb group it is a linear combination of past inflation and the inflation target.

Simple ordinary least squares (OLS) estimation implies slope parameters that are very small and statistically insignificant. This is not surprising given the time-series properties of core inflation since the adoption of inflation targeting in Canada. In fact, Canadian core inflation behaves like white noise around the target, so that the best forecast of inflation is the target.[19] We therefore use a Bayesian approach as in Carabenciov et al. (2008) to estimate the Phillips curve. Estimation is conducted using a small system of equations, including an IS curve and a Taylor rule, such that inflation expectations are model consistent. To replicate the properties of LENS, tight priors are used for the coefficients of the IS curve and the Taylor rule. In addition, we use a modified version of core inflation that excludes the prices of motor vehicles and regulated services, to help identify the relationship between core inflation and the output gap in the estimation:[20]

$$\pi_t = 0.65(0.24\pi_{t-1} + (1-0.24)\pi^*) + (1-0.65)\pi_{t+1} + 0.028Y_t^{gap}$$
$$+0.007\Delta PM_t^{PY} + 0.031\left(\pi_t^{ulc} - \pi^*\right) + \varepsilon_t^{\pi}. \quad (25)$$

We use priors bounded between 0 and 1 to estimate the shares of rule-of-thumb and forward-looking expectations, thereby allowing the data to play an important role.[21] However, for the slope coefficients, we use relatively tight positive priors. Bayesian estimates indicate that 65 per cent of economic agents form their expectations using a rule of thumb based on lagged inflation (24 per cent) and the target (76 per cent), while the rest are purely rational and forward looking.[22] Despite our tight priors, the slope coefficients remain relatively low.

Solving the Phillips curve forward to eliminate the lead of inflation would imply that inflation depends on the expected discounted sums of the output gap, real import price and unit labour cost inflation. While the coefficients on the economic drivers are very small, the *expected persistence* of fundamentals can have meaningful implications for current inflation.

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[19] For more details on the empirical features of the Canadian Phillips curve, see Demers (2003) and Mendes and Murchison (2009–10). The flattening of the Phillips curve has been documented internationally (IMF 2013).

[20] The inflation rates in the prices of motor vehicles and regulated prices have been at odds with the business cycle, thereby obscuring the relationship between underlying inflation and the output gap. See Box 2 in Bank of Canada (2014) for further details.

[21] The sum of the coefficients on the rule-of-thumb and forward-looking expectations is restricted to one.

[22] In ToTEM, the share of rule-of-thumb price setters is 50 per cent, and 90 per cent of this group sets their expectations based on the inflation target.

Total inflation projections are based on forecasts of core inflation along with a path for the excluded components based on assumptions for natural gas and oil prices expressed in Canadian dollars.

Turning to wages, we rely on the REC approach to model real wages in LENS. The frictionless level of real wages ($LWNR^*$) is based on a long-run relationship with average labour productivity for the total economy ($LPROD$). Dynamic OLS estimation reveals that the coefficient on labour productivity is not statistically different from one:

$$LWNR_t^* = -0.76 + 1.00 LPROD_t. \quad (26)$$

The dynamic equation for real wages is based on the lagged wage gap, a lag of real wage growth and expected future changes in the target level (with standard parameter restrictions to ensure a stable steady state):

$$\Delta LWNR_t = -0.19(LWNR_{t-1} - LWNR_{t-1}^*) + 0.11 \Delta LWNR_{t-1} + 0.89 E_t \sum_{i=1}^{20} f_i \Delta LWNR_{t+i}^* + \varepsilon_t^W. \quad (27)$$

Wages relative to productivity determine unit labour costs through an identity, which in turn affect core inflation. Many relative prices are modelled using the REC specification in LENS. Frictionless levels for the deflators of the main components of aggregate demand are anchored on the real exchange rate, commodity prices and deterministic trends, among others. These relative prices affect the cost of capital, household wealth, the current account and inflation. REC equations for the main deflators are reported in Appendix C.

2.4.2 Interest rates and the exchange rate

A number of interest rates are modelled in LENS which, in turn, influence various aspects of economic activity. They are all related, in one way or another, to the overnight rate, which is modelled with an estimated historical forward-looking policy rule (subject to the zero lower bound on nominal interest rates):

$$R1N_t = 0.83 R1N_{t-1} + (1 - 0.83) R1N^* + (1 - 0.83)[4.12(\pi_{t+2} - \pi^*) + 0.4 Y_t^{gap}] + \varphi_t\,, \quad (28)$$

$$\varphi_t = 0.25 \varphi_{t-1} + \varepsilon_t\,. \quad (29)$$

In LENS, monetary policy sets the nominal short-term interest rate ($R1N$) in response to expected deviations of quarter-over-quarter core inflation from target two quarters ahead ($\pi_{t+2} - \pi^*$) and the current output gap (Y_t^{gap}). The lagged policy rate enters the rule to reflect the fact that the authority cares about the volatility in the movements of its instrument (Cayen et al. 2006). The rule also includes φ_t, which permits temporary deviations from the feedback rule for reasons other than interest rate smoothing. Such factors could reflect any missing drivers from the rule, including considerations related to financial stability risks. This is the same policy rule as used in ToTEM. It is estimated using real-time output-gap estimates and inflation forecasts.

A term structure of interest rates is included in the model, with the long-term risk-free rate (5-year government bond rate, $R20N$) determined by the expectations hypothesis with overdiscounting. Government yields therefore depend on a geometric average of future short rates and a persistent term premium (TP) that is correlated with the U.S. term premium (TP^{us}):

$$R20N_t = \left[\left[1 + \left[\frac{\sum_{i=1}^{20} 0.97^i R1N_{t+i}}{\sum_{i=1}^{20} 0.97^i}\right]\right][1 + TP_t]\right] - 1, \qquad (30)$$

$$TP_t = 0.003 + 0.72TP_{t-1} + 0.5TP_t^{us}. \qquad (31)$$

Historical projections of the short-term rate based on ToTEM were used to estimate TP. A discount rate of 0.97 provides the best empirical fit. The estimated correlation between the Canadian and U.S. term premia is consistent with the term structure model of Bauer and Diez de los Rios (2012). Government bond yields have a direct impact on federal and provincial government finances in LENS, and also affect market interest rates. The key interest rates that drive the spending decisions of households and firms (through the $fcost$ variable in user cost) are long-term effective interest rates ($RN^{households}, RN^{firms}$).[23] They are assumed to depend on the long-term risk-free rate and a risk premium that fluctuates countercyclically around its historical average ($PREM^{households}, PREM^{firms}$):

$$RN_t^i = (1 + R20N_t)(1 + PREM_t^i), \quad i \in \{households, firms\}, \qquad (32)$$

$$PREM_t^i = 0.83PREM_{t-1}^i - 0.03Y_t^{gap}\ . \qquad (33)$$

Borrowing interest rates for the two levels of government are based on the same type of specification as for households and firms, but with constant exogenous risk premia.

Lastly, the real effective exchange rate ($FOREX$) is driven by a long-run relationship (in logs) with real energy ($ENER$) and non-energy ($BCNE$) commodity prices as well as expected real short-term interest-rate differentials (Z^{uip}):[24]

$$FOREX_t^* = 1.39 - 0.239BCNE_t - 0.071ENER_t - 14.04Z_t^{uip}, \qquad (34)$$

$$where\ Z_t^{uip} = \frac{1}{20}\sum_{i=1}^{20} R1_{t+i} - R1_{t+i}^{row}\ . \qquad (35)$$

[23] The effective interest rate for households is a weighted average of various mortgage and consumer credit interest rates. The effective interest rate for businesses is a weighted-average borrowing rate for new lending to non-financial businesses, estimated as a function of bank and market interest rates.

[24] An increase in the exchange rate corresponds to a depreciation of the Canadian dollar. At the estimation stage, expected real interest rate differentials were generated by simulating ToTEM over history. The long-run relationship between the exchange rate and commodity prices is similar to Issa, Lafrance and Murray (2006), but does not include a structural break.

Preliminary versions of LENS assumed that the exchange rate adjusted to reach a target for the net foreign asset position relative to GDP. This specification was dismissed, since it was inconsistent with the data, deteriorated forecast performance and led to counterintuitive results in the context of dynamic simulation of the model.[25] Short-term exchange rate deviations follow a simple AR(2) process.

2.4.3 Household debt

Household debt is an important driver of consumption in the model through its direct impact on net financial wealth and disposable income.[26] In LENS, we do not attempt to model household borrowing decisions using a micro-founded approach as in Alpanda et al. (2014). Instead, we follow the REC specification and try to model debt dynamics using standard macroeconomic determinants.

The frictionless level of the household debt-to-income ratio (DI^*) is based on a cointegrating relationship with existing house prices (PH) and real effective interest rates ($R^{households}$):

$$DI_t^* = 0.37 + 0.76PH_t - 15.19R_t^{households}. \tag{36}$$

Short-term dynamics of debt relative to income are based on the standard REC specification where the order of adjustment costs is equal to 2:

$$\Delta DI_t = -0.10(DI_{t-1} - DI_{t-1}^*) + 0.04\Delta DI_{t-1} + 0.72E_t \sum_{i=1}^{20} f_i \Delta DI_{t+i}^* + 0.24\Delta Y_{t-1}^d + \varepsilon_t^{di}. \tag{37}$$

In this set-up, households adjust their leverage as a function of past income and expectations of future income, house prices and interest rates.

3. Empirical Properties

In this section, we assess the forecasting ability of LENS in single-equation and general-equilibrium environments. We also describe the model's main dynamic properties in response to shocks.

3.1 Forecast performance

Given that our objective is to build a complementary model to ToTEM that is more oriented toward forecast performance, much focus was put on assessing parameter stability and medium-term forecast accuracy at the estimation stage. In addition, all equations were specified to avoid autocorrelation in the residuals, so that the forecast is not driven by the runoff of the shock terms. The out-of-sample forecast performance of each rational error-correction equation was compared to standard alternative forecasting benchmarks in the context of rolling forecast experiments conducted over the period 2002–12.[27] In most cases, REC models have better medium-term forecasting properties than autoregressive

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[25] To avoid the presence of a unit root in the consumption-to-output ratio, non-human wealth excludes net foreign assets.

[26] Disposable income is calculated to exclude interest on household credit payable in Canada.

[27] This period was chosen so as to allow for statistical testing of parameter stability and significance in differences of forecast performance. Forecasting benchmarks considered include pure error-correction models and

(AR) and error-correction models (ECM), with lower out-of-sample root mean squared forecast errors (RMSFE) at 4 and 8 quarters ahead (Table 2).

Table 2 Out-of-sample forecast performance (single equation)

Relative RMSFE*	AR 4Q ahead	AR 8Q ahead	ECM 4Q ahead	ECM 8Q ahead
Non-durables and services	0.52	0.41	0.84	0.76
Durables	0.90	1.10	1.10	0.97
Housing	0.64	0.69	0.95	0.90
Bus. investment (M&E)	0.59	0.40	0.79	0.66
Bus. investment (structures)	0.75	0.65	0.73	0.48
Federal spending	0.75	0.57	0.96	0.93
Provincial, local spending	0.99	0.93	0.97	0.93
Non-commodity exports	0.52	0.33	0.81	0.81
Non-ener. commod. exports	0.97	0.77	1.16	0.97
Energy exports	0.96	0.76	0.95	1.04
Non-commodity imports	0.35	0.34	0.98	0.87
Commodity imports	0.45	0.39	0.86	0.86

*RMSFE of REC model divided by RMSFE of competing model

The fact that the forecasting performance of our REC equations is as good as, or better than, standard error-correction models implies that REC structure can be introduced without deteriorating the empirical properties of the model. In addition, the REC equations selected for LENS exhibit greater parameter stability than alternative approaches. This characteristic is of primary importance for the model's longer-term viability.

In addition to assessing forecast accuracy relative to standard benchmarks, we conducted a more realistic forecasting exercise over the period 2005–12 using the general-equilibrium model-consistent-expectations version of LENS, and compared out-of-sample forecasting accuracy with ToTEM, combined with the trends model used in the projection, at 1, 4 and 8 quarters ahead (Table 3).[28] In most cases,

autoregressive models. Forecast performance is more representative when compared to error-correction models, since both REC and ECM forecasts assume that target levels are known. This can be an advantage compared to simple AR models.

[28] The sample for comparison with ToTEM corresponds to the period over which Bank staff started to use ToTEM for the projection. The forecast errors for ToTEM are based on the differences between actual and forecast levels, where forecast levels are the sum of forecast gaps and trends (without judgment). The forecasts of the trends are constructed by simulating the trends model over history assuming that rest-of-world variables and commodity

LENS compares favourably to ToTEM with lower RMSFEs. Of note, LENS performs substantially better at forecasting output and many of its components. Relative forecast performance is good for consumption, government spending and imports, but less so for exports. As for inflation, ToTEM and LENS perform similarly at 1 and 4 quarters ahead, but ToTEM outperforms LENS at 8 quarters ahead. The relatively weaker medium-term forecast performance of LENS for CPIX inflation could be due to the fact that the Phillips curve has been estimated using CPIX excluding the price of autos and regulated services, while forecast performance is assessed against CPIX outcomes. Although both models have similar inflation RMSFEs at shorter horizons, the sign of the errors is different, since ToTEM tends to underpredict inflation while LENS tends to overpredict inflation. This is key to understanding differences in the ability of the models to forecast interest rates during this period. The negative bias in ToTEM's inflation forecasts between 2005 and 2012 implies a lower predicted path for interest rates, which turns out to be more accurate in a context where monetary policy was more stimulative than implied by the rule.

Table 3: General-equilibrium out-of-sample forecast performance (ratio of RMSFE relative to ToTEM)

Relative RMSFE*	1Q ahead	4Q ahead	8Q ahead
GDP	0.83	0.62	0.48
Consumption	0.82	0.80	0.76
Residential	0.84	0.78	1.00
Business	1.07	0.92	0.90
Government	0.61	0.69	0.79
Exports	1.09	0.99	1.05
Imports	0.57	0.48	0.64
Core inflation	1.02	1.03	1.19
Overnight	1.26	1.40	1.33
Exchange	1.60	0.88	0.88

*RMSFE of LENS divided by RMSFE of ToTEM

3.2 Dynamic properties

To summarize the empirical properties of LENS, we examine the dynamic response of the model to various shocks that are encountered in the context of constructing an economic projection. While shocks in LENS do not really have a structural interpretation, these simulations are still useful to check that the model properly captures standard dynamic macroeconomic relationships. We focus on five temporary shocks and one permanent shock and compare results with ToTEM. In all simulations, we assume that foreign variables do not respond to shocks in Canada and that agents have perfect foresight, since expectations are fully model-consistent. In a REC model, there are an infinite number of leads. When running LENS, we use a truncated number of leads such that a significant portion of the

prices are known. Forecast errors are largely driven by the gaps (i.e., ToTEM), since the trends are relatively stable at each iteration.

asymptotic value of the infinite leads is taken into account. Monetary policy is endogenous in all simulations. Impulse-response functions are reported in Appendix D.

3.2.1 A temporary increase in consumption

We first consider a scenario in which total consumption (sum of non-durables, services and durables in LENS) increases temporarily by 1 per cent relative to a base case. This translates into a peak impact of roughly 0.3 per cent on the level of GDP in both LENS and ToTEM (Appendix D, Figure D-1). With limited impact on potential, the shock implies a similar increase in the output gap. While inflation depends on real marginal costs in ToTEM and the output gap in LENS, both models generate similar dynamic properties in the face of stronger consumption, with inflation rising very marginally. Core inflation is also driven by unit labour costs in LENS. In the case of stronger demand, there will be an increase in labour productivity (total hours worked rise proportionally less than output) and a gradual catch-up in real wages. As a result, lower unit labour costs will tend to mitigate somewhat the response of inflation to the higher output gap. The response of core inflation is lower in LENS because the demand shock is concentrated in the consumption sector. This is a key difference between the two models. A given shock to output will have the same impact on inflation in LENS regardless of whether it reflects consumption, investment or exports. Given the multi-good set-up in ToTEM, demand shocks outside of the consumption sector will tend to have very little implication for consumer price inflation, since it is marginal cost in the consumption sector that matters most for CPI inflation. If we scale the response of LENS we can see that, as a rule of thumb, a 1 per cent increase in the output gap would imply a rise in core inflation of slightly more than 0.2 percentage points (at annual rates) in general equilibrium. This rule applies for a relatively short-lived demand shock. A more persistent shock, through its effect on expectations, would imply a larger response of inflation.

In this scenario, there are three key mechanisms counteracting the impact of the increase in demand. First, tighter monetary policy leads to higher borrowing costs for households and firms. This reduces consumption, housing, and business investment, and facilitates the return of balance between aggregate supply and demand in the economy.[29] Second, the real exchange rate appreciates as a result of more favourable interest rate differentials, and this puts downward pressure on inflation through lower import prices and weaker net exports. Third, a countercyclical fiscal policy is implemented in the face of excess demand, implying a decline of net transfers from federal and provincial governments in the medium term. All else equal, lower transfers and higher interest payments on debt reduce personal income flows and human wealth, which depresses consumption. Overall, there is very little secondary cycling. In response to the positive demand shock, the monetary and fiscal rules generate very little excess supply in subsequent years. ToTEM, on the other hand, generates a negative secondary cycle in consumption, since the dynamic budget constraint requires a subsequent period of increased savings to make up for the temporary decline in households' discount rate.

[29] Business investment still increases in the short term, reflecting accelerator effects.

3.2.2 A temporary increase in the policy interest rate

To assess the transmission mechanism of monetary policy, it is useful to examine the impact of an interest rate shock. Here we consider a 100-basis-point interest rate tightening that lasts about four quarters (Appendix D, Figure D-2). The shock translates into higher borrowing rates for households, businesses, and governments, and therefore weaker domestic demand.[30] There is also an appreciation of the Canadian dollar as a result of higher interest rates, which leads to weaker net exports. The exchange rate is less sensitive to interest rate differentials in LENS: an increase in the overnight rate leads to an appreciation of the Canadian dollar that is roughly half of ToTEM's response. Overall, the decline in output is small in both models. But while the peak impact on economic activity is smaller in LENS, the overall effect is much more persistent: output is back to control after five years following the shock in ToTEM, whereas it is still equal to about half of its peak in LENS. The composition of the effect is also different between the two models, with a smaller impact on consumption in LENS but a larger impact on residential investment. Net exports are also less affected in LENS given the smaller exchange rate response. As a rule of thumb, a temporary 100-basis-point increase in short-term interest rates reduces GDP by 0.1 per cent. Here again, the impact on output would be larger in the context of a more-persistent interest rate movement, since forward-looking expectations would imply a bigger effect on long-term interest rates and therefore user cost.

The small REC model for household debt nested in LENS is useful to illustrate the impact of interest rates on household leverage in general equilibrium. In this scenario, debt to income falls by only 0.4 percentage points, because the movement in short-term rates is not persistent enough to have a significant effect on long-term rates. However, in the case of a direct shock of 100 basis points on households' effective borrowing rates, debt to income would fall by 2.5 percentage points.

3.2.3 A temporary exchange rate depreciation

Next, we consider a scenario where there is an exogenous temporary 1 per cent depreciation of the real effective exchange rate (Appendix D, Figure D-3). The weaker dollar stimulates net exports and, compared to ToTEM, output rises much more in LENS. This response is driven by a much higher short-term exchange rate sensitivity of imports that is only partly offset by a lower exchange rate elasticity of exports.[31] As a rule of thumb, a 1 per cent depreciation of the dollar pushes up GDP by about 0.1 per cent in the short term. In both models, pass-through to core inflation is negligible. The more-persistent response of inflation in ToTEM is driven by the behaviour of real wages, which imply more-persistent cost pressures. The weaker dollar, combined with higher interest rates, leads to a change in the composition of output, away from domestic demand and toward net exports. The weaker dollar also raises the cost of imported capital, thereby affecting business investment. Finally, there is a deterioration in the terms of trade as import prices increase more than export prices.

[30] Overdiscounting of expectations mitigates the response of long-term rates to changes in future short rates.

[31] The relative size of the exports response between ToTEM and LENS varies according to the persistence of the exchange rate shock. For a 1-period shock, exports in ToTEM respond more than in LENS. As the persistence of the shock increases, the response of exports in LENS increases proportionally more than in ToTEM. This is not surprising given the importance of expectations of target levels for exports in LENS (the implied mean lead is 4.47 quarters for exports – Table 1).

3.2.4 A foreign demand shock

In this scenario, we assess the implications for the Canadian economy of a 1 per cent persistent increase in foreign demand (Appendix D, Figure D-4). Stronger foreign demand is assumed to be accompanied by a 7 per cent increase in global commodity prices.[32] There are several channels through which this shock is transmitted to the Canadian economy in LENS. First, higher foreign demand stimulates exports through the direct trade channel as target levels for all types of exports rise one-for-one with the increase in foreign demand. The response of exports to foreign demand is reinforced by the impact of the foreign output gap, which roughly doubles the short-term income elasticity of exports. Second, domestic demand is stronger as a result of positive confidence effects on consumption and investment in the short term (household and business confidence are functions of the foreign output gap), and also because of higher financial wealth. Third, the exchange rate appreciation plays an important role in offsetting part of the effects on the Canadian economy.[33] This is why the increase in exports is followed by a decline as the relative price of exports rises. Overall, the stronger foreign demand leads to higher GDP and inflation in Canada. While peak impacts are similar to ToTEM, they are much less persistent in LENS.

3.2.5 A commodity-price shock

The last transitory shock we analyze is a commodity supply shock, i.e. an increase in global commodity prices that is not driven by stronger demand (Appendix D, Figure D-5). The main finding is that the impact of commodity *supply* shocks on the Canadian economy is small. For a 10 per cent commodity-price increase, the peak positive response of output is less than 0.1 per cent in LENS. Although the impact is about double in ToTEM, it remains small given the size of the shock. Of note, there is a short-term decline in output in LENS as a result of the higher sensitivity of GDP to the exchange rate. As a rule of thumb, a 10 per cent persistent commodity-price increase leads to a 2.5 per cent appreciation of the Canadian dollar in general equilibrium. Core inflation falls as a result of the higher exchange rate, but the impact is quantitatively small. In terms of the components of output, we observe stronger consumption, as a result of higher wealth, as well as stronger investment partly as a result of higher cash flows and the stronger dollar pulling down the price of imported capital. While the integral of the impact on output eventually turns positive (after about eight years), the benefits to the Canadian economy coming from higher commodity prices are likely underestimated in LENS given the absence of a commodity supply channel.

3.2.6 A permanent increase in productivity

Productivity movements are useful to illustrate the behaviour of the model in the face of permanent shocks. Here we consider an unexpected permanent increase in the level of total factor productivity that is specific to Canada (Appendix D, Figure D-6). The stronger productivity means that potential output

[32] Simulations of external shocks are based on a small rest-of-world block that is included in LENS. This block, which has dynamic properties similar to MUSE, includes an IS curve, a Phillips curve, a Taylor rule and equations linking energy and non-energy commodity prices to the global output gap.

[33] The exchange rate does not depreciate on impact, because the Canadian monetary authority reacts aggressively to counteract the effects of the shock.

increases by 1 per cent immediately. Although demand reacts quickly, adjustment costs are such that the shock initially creates a period of small excess supply (about -0.2 per cent). Of note, there is no permanent impact on the Canadian dollar in LENS, since the exchange rate does not adjust to restore the net foreign asset position following the shock (see section 2.4.2). This contrasts with ToTEM, which generates a permanent depreciation of the dollar. Consumption is positively affected by the permanent increase in human wealth, while investment rises to restore the long-run capital-output ratio. The positive supply shock has a disinflationary impact in the short term. The response of inflation is much more short lived in LENS compared to ToTEM, since real wages adjust more rapidly to the increase in productivity. On the other hand, the output gap remains negative more persistently as a result of the more sluggish increase in consumption, which is caused by a slow increase in labour income (due to a cyclical decline in labour input). LENS generates a different response depending on whether the productivity shock is expected. An expected increase in TFP would be inflationary, since it initially creates a period of excess demand: agents anticipate the shock to future income and increase spending immediately.

3.2.7 Other dynamic properties

Another important shock for monetary policy analysis is a house price shock. In LENS, house prices have a direct impact on the nominal residential capital stock and therefore on household financial wealth. The wealth channel implies that consumption falls by about 1 per cent following a 10 per cent persistent decline in real house prices, in line with ToTEM.

4. Conclusion

To help mitigate the effects of model uncertainty, the Bank of Canada uses multiple models to construct forecasts of the most likely evolution of the Canadian economy. In this report, we described the key features of LENS, a new large-scale Canadian macroeconomic forecasting model that will complement ToTEM in the projection process.

While ToTEM has a stronger theoretical structure and is better suited for monetary policy evaluation compared to LENS, LENS has a number of practical advantages that can be used in a forecasting environment. First, because of its reduced-form nature, LENS is relatively easy to update and improve. For instance, a preliminary version of LENS was completely re-estimated following historical revisions to the Canadian System of National Accounts at the end of 2012. In addition, improvements to parts of the model can be introduced on an ongoing basis without needing to re-estimate the whole model. Second, the rational error-correction set-up means that most long-run trends are determined endogenously in LENS. This is an advantage in the context of producing and communicating a projection, since no detrending is required and cyclical adjustment occurs via both stationary and trend movements. Third, further disaggregation of GDP components in LENS is useful to better understand economic developments and provide further details behind the forecast. Fourth, LENS performs generally better than error-correction models, autoregressive models and ToTEM in terms of out-of-sample forecast performance, while still having enough structure to capture key macroeconomic linkages. These properties will make LENS a useful tool for constructing economic projections at the Bank of Canada.

Future development work will focus on improving real-financial linkages in the model. For instance, while borrowing spreads are currently modelled as a function of the output gap, this framework could be improved to include balance-sheet variables. In addition, through the explicit role for expectations in rational error-correction models, we will analyze the implications of alternative assumptions regarding expectations formation, such as VAR-based forecasts. This is important given the likelihood that various economic players differ significantly in their knowledge about the workings of the economy and its future direction.

References

Alpanda, S., G. Cateau and C. Meh. 2014. "A Policy Model to Analyze Macroprudential Regulations and Monetary Policy." Bank of Canada Working Paper No. 2014-6.

Bank of Canada. 2014. *Monetary Policy Report*. April.

Barnett, R. 2007. "Trend Labour Supply in Canada: Implications of Demographic Shifts and the Increasing Labour Force Attachment of Women." *Bank of Canada Review* (Summer): 5–18.

Bauer, G. and A. Diez de los Rios. 2012. "An International Dynamic Term Structure Model with Economic Restrictions and Unspanned Risks." Bank of Canada Working Paper No. 2012-5.

Binette, A. and J. Chang. 2013. "CSI: A Model for Tracking Short-Term Growth in Canadian Real GDP." *Bank of Canada Review* (Summer): 3–12.

Brayton, F. and P. A. Tinsley. 1996. "A Guide to FRB/US: A Macroeconomic Model of the United States." Finance and Economics Discussion Series No. 96-42, U.S. Board of Governors of the Federal Reserve System.

Brayton, F., T. Laubach and D. Reifschneider. 2014. "The FRB/US Model: A Tool for Macroeconomic Policy Analysis." FEDS Notes, April.

Butler, L. 1996. "The Bank of Canada's New Quarterly Projection Model (QPM), Part 4. A Semi-Structural Method to Estimate Potential Output: Combining Economic Theory with a Time-Series Filter." Bank of Canada Technical Report No. 77.

Carabenciov, I., I. Ermolaev, C. Freedman, M. Juillard, O. Kamenik, D. Korshunov, D. Laxton and J. Laxton. 2008. "A Small Quarterly Multi-Country Projection Model." IMF Working Paper No. 08/279.

Cayen, J.-P., A. Corbett and P. Perrier. 2006. "An Optimized Monetary Policy Rule for ToTEM." Bank of Canada Working Paper No. 2006-41.

Cayen, J.-P., M.-A. Gosselin and S. Kozicki. 2009. "Estimating DSGE-Model-Consistent Trends for Use in Forecasting." Bank of Canada Working Paper No. 2009-35.

Coletti, D. and S. Murchison. 2002. "Models in Policy-Making." *Bank of Canada Review* (Summer): 19–26.

Demers, F. 2003. "The Canadian Phillips Curve and Regime Shifting." Bank of Canada Working Paper No. 2003-32.

Demers, F. 2005. "Modelling and Forecasting Housing Investment: The Case of Canada." Bank of Canada Working Paper No. 2005-41.

Dorich, J., M. Johnston, R. Mendes, S. Murchison and Y. Zhang. 2013. "ToTEM II: An Updated Version of the Bank of Canada's Quarterly Projection Model." Bank of Canada Technical Report No. 100.

Faruqui, U. and S. Torchani. 2012. "How Important Are Liquidity Constraints for Canadian Households? Evidence from Micro-Data." Bank of Canada Working Paper No. 2012-9.

Gosselin, M.-A. and R. Lalonde. 2005. "MUSE: The Bank of Canada's New Projection Model of the U.S. Economy." Bank of Canada Technical Report No. 96.

Hooper, P., K. Johnson and J. Marquez. 2000. "Trade Elasticities for the G-7 Countries." Princeton Studies in International Economics No. 87, August.

Industry Canada. 2013. "Financing Statistics, Special Edition: Key Small Business Statistics." November.

International Monetary Fund (IMF). 2013. "The Dog that Didn't Bark: Has Inflation been Muzzled or Was It Just Sleeping?" World Economic Outlook, April.

Issa, R., R. Lafrance and J. Murray. 2006. "The Turning Black Tide: Energy Prices and the Canadian Dollar." Bank of Canada Working Paper No. 2006-29.

Jenkins, P. and D. Longworth. 2002. "Monetary Policy and Uncertainty." *Bank of Canada Review* (Summer): 3–10.

Kozicki, S. 2012. "Macro has Progressed." *Journal of Macroeconomics* 34: 23–28.

Kozicki, S. and P. A. Tinsley. 1999. "Vector Rational Error Correction." *Journal of Economic Dynamics and Control* 23 (9-10): 1299–1327.

Macklem, T. 1994. "Wealth, Disposable Income and Consumption: Some Evidence for Canada." Bank of Canada Technical Report No. 71.

Mendes, R. and S. Murchison. 2009–10. "Declining Inflation Persistence in Canada: Causes and Consequences." *Bank of Canada Review* (Winter): 1–14.

Morel, L. 2012. "A Foreign Activity Measure for Predicting Canadian Exports." Bank of Canada Discussion Paper No. 2012-1.

Murchison, S. and A. Rennison. 2006. "ToTEM: The Bank of Canada's New Quarterly Projection Model." Bank of Canada Technical Report No. 97.

Pichette, L. and D. Tremblay. 2003. "Are Wealth Effects Important for Canada?" Bank of Canada Working Paper No. 2003-30.

Rotemberg, J. 1996. "Prices, Output and Hours: An Empirical Analysis Based on a Sticky Price Model." *Journal of Monetary Economics* 37 (3): 505–33.

Tinsley, P. A. 2002. "Rational Error Correction." *Computational Economics* 19 (2): 197–225.

von zur Muehlen, P. 2001. “The Effect of Past and Future Economic Fundamentals on Spending and Pricing Behavior in the FRB/US Macroeconomic Model.” Finance and Economics Discussion Series No. 2001-12, Board of Governors of the Federal Reserve System.

Appendix A: Fiscal Policy

In LENS, government spending and revenues are decomposed into federal versus provincial and local components. Total government expenditures are further decomposed into transfer spending, spending on goods and services (National Accounts definition), and interest payments. Personal income tax revenues fluctuate around a constant share of total government revenues. While fiscal policy is counter-cyclical in the short term, spending and revenues adjust jointly in the long term in order to reach assumed government debt targets. Dynamic adjustment is driven by rational error correction, with the order of adjustment cost ranging from 3 to 4. All fiscal variables are modelled in nominal terms relative to output.[34]

A.1 Federal government

The target for *total* federal spending ($GTOT^{fed*}$) is a function of the steady-state value for total federal government size ($GTOT_{ss}^{fed}$), the output gap (Y^{gap}) and the deviation of the federal debt-to-GDP ratio ($DEBT^{fed}$) from an assumed target ($DEBT^{fed*}$):

$$GTOT_t^{fed*} = GTOT_{ss}^{fed} - 0.55Y_t^{gap} - 0.01\left(DEBT_{t-1}^{fed} - DEBT^{fed*}\right).^{35} \qquad \text{(A1)}$$

The dynamic behaviour of federal spending is governed by a standard REC specification:

$$\begin{aligned}\Delta GTOT_t^{fed} &= -0.17\left(GTOT_{t-1}^{fed} - GTOT_{t-1}^{fed*}\right) - 0.29\Delta GTOT_{t-1}^{fed} - 0.06\Delta GTOT_{t-2}^{fed} \\ &+ 0.59E_t\sum_{i=1}^{20} f_i \Delta GTOT_{t+i}^{fed*} + \varepsilon_t^{gtotfed}. \qquad \text{(A2)}\end{aligned}$$

Given the profile for total government expenditures, we parse out spending on goods and services versus transfers using share equations. The share of spending on goods and services ($GNAC^{fed}$) is based on a simple AR process around the historical average ($GNAC_{ss}^{fed}$):

$$GNAC_t^{fed} = GNAC_{ss}^{fed} + 0.12GNAC_{t-1}^{fed} + \varepsilon_t^{gfednac}. \qquad \text{(A3)}$$

The target share of total federal spending going to transfers (TR^{fed*}) is given by a cointegrating relationship with the elderly dependency ratio ($EDEPR$) and the output gap:

$$TR_t^{fed*} = 0.08 + 1.04EDEPR_t - 0.44Y_t^{gap}. \qquad \text{(A4)}$$

Short-run dynamics for federal transfer spending follow a REC process:

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[34] For both levels of government, interest payments are calculated by applying the government interest rate to the previous period value of debt.

[35] For estimation, the steady-state value of total federal government size ($GTOT_{ss}^{fed}$) is assumed to evolve around a constant with structural breaks. Over the projection, it is given by the steady-state solution of the model conditional on calibrated values for the debt-to-GDP target ratio, the federal government interest rate and potential output growth. We assume that half of the steady-state fiscal adjustment to meet the debt target operates through spending while the other half operates through taxes.

$$\Delta TR_t^{fed} = -0.44\left(TR_{t-1}^{fed} - TR_{t-1}^{fed*}\right) - 0.22\Delta TR_{t-1}^{fed} + 0.02\Delta TR_{t-2}^{fed} + 0.78E_t\sum_{i=1}^{20} f_i \Delta TR_{t+i}^{fed*} - 0.01\, Y_t^{gap} \;+ \varepsilon_t^{trfed}. \quad \text{(A5)}$$

On the revenue side, the target for federal revenues (REV^{fed*}) is a function of the steady-state value for total federal government revenues (REV_{ss}^{fed}), the output gap and the federal debt gap:

$$REV_t^{fed*} = REV_{ss}^{fed} + 0.006Y_t^{gap} + 0.05\left(DEBT_{t-1}^{fed} - DEBT^{fed*}\right).^{36} \quad \text{(A6)}$$

The dynamic behaviour of federal revenues is based on a REC specification:

$$\Delta REV_t^{fed} = -0.20\left(REV_{t-1}^{fed} - REV_{t-1}^{fed*}\right) - 0.24\Delta REV_{t-1}^{fed} - 0.32\Delta REV_{t-2}^{fed} + 0.48E_t\sum_{i=1}^{20} f_i \Delta REV_{t+i}^{fed*} \;+ \varepsilon_t^{revfed}. \quad \text{(A7)}$$

A.2 Provincial and local government

As with federal spending, the target for *total* provincial and local (PL) spending ($GTOT^{pl*}$) is a function of the steady-state value for total PL government size ($GTOT_{ss}^{pl}$) and is negatively correlated with the output gap and the PL debt gap:

$$GTOT_t^{pl*} = GTOT_{ss}^{pl} - 0.69Y_t^{gap} - 0.04\left(DEBT_{t-1}^{pl} - DEBT^{pl*}\right).^{37} \quad \text{(A8)}$$

The dynamic behaviour of PL spending is governed by a standard REC specification:

$$\Delta GTOT_t^{pl} = -0.12\left(GTOT_{t-1}^{pl} - GTOT_{t-1}^{pl*}\right) - 0.02\Delta GTOT_{t-1}^{pl} + 0.08\Delta GTOT_{t-2}^{pl} + 0.21\Delta GTOT_{t-3}^{pl} + 0.79E_t\sum_{i=1}^{20} f_i \Delta GTOT_{t+i}^{pl*} \;+ \varepsilon_t^{gtotpl}\,. \quad \text{(A9)}$$

Spending on goods and services versus transfers is obtained using share equations. The target share of total PL spending going to goods and services ($GNAC^{pl*}$) is given by a cointegrating relationship with commodity prices:

$$GNAC_t^{pl*} = 0.79 + 0.16PCOM_t\,. \quad \text{(A10)}$$

Short-run dynamics for PL goods and services spending follow a REC process:

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[36] The steady-state value of federal revenues relative to output (REV_{ss}^{fed}) fluctuates around a constant with structural breaks over history. Over the projection, it is given by the steady-state solution of the model conditional on calibrated values for the debt-to-GDP target ratio, the federal government interest rate and potential output growth.

[37] $GTOT_{ss}^{pl}$ is calculated the same way as $GTOT_{ss}^{fed}$, but conditional on the steady-state calibration of the PL sector.

$$\Delta GNAC_t^{pl} = -0.07\left(GNAC_{t-1}^{pl} - GNAC_{t-1}^{pl*}\right) - 0.42\Delta GNAC_{t-1}^{pl} - 0.07\Delta GNAC_{t-2}^{pl} + 0.41E_t\sum_{i=1}^{20} f_i \Delta GNAC_{t+i}^{pl*} + \varepsilon_t^{gnacpl}. \quad \text{(A11)}$$

As in the case of the federal government, the target share of total PL spending going to transfers (TR^{pl*}) is given by a cointegrating relationship with the elderly dependency ratio ($EDEPR$) and the output gap:

$$TR_t^{pl*} = 0.03 + 0.28EDEPR_t - 0.20Y_t^{gap}. \quad \text{(A12)}$$

Short-run dynamics for PL transfer spending follow a REC process:

$$\Delta TR_t^{pl} = -0.27\left(TR_{t-1}^{pl} - TR_{t-1}^{pl*}\right) - 0.51\Delta TR_{t-1}^{pl} + 0.03\Delta TR_{t-2}^{pl} + 0.12\Delta TR_{t-3}^{pl} + 0.60E_t\sum_{i=1}^{20} f_i \Delta TR_{t+i}^{fed*} - 0.02\, Y_t^{gap} + \varepsilon_t^{trpl}. \quad \text{(A13)}$$

On the revenue side, the target for PL revenues (REV^{pl*}) is a function of the steady-state value for total PL government revenues (REV_{ss}^{pl}) and is positively correlated with the output gap and the PL debt gap:

$$REV_t^{pl*} = REV_{ss}^{pl} + 0.04Y_t^{gap} + 0.12\left(DEBT_{t-1}^{pl} - DEBT^{pl*}\right).\ ^{38} \quad \text{(A14)}$$

The dynamic behaviour of PL revenues is based on a REC specification:

$$\Delta REV_t^{pl} = -0.10\left(REV_{t-1}^{pl} - REV_{t-1}^{pl*}\right) - 0.46\Delta REV_{t-1}^{pl} - 0.17\Delta REV_{t-2}^{pl} + 0.38E_t\sum_{i=1}^{20} f_i \Delta REV_{t+i}^{pl*} + \varepsilon_t^{revpl}. \quad \text{(A15)}$$

The sensitivity of the economy to fiscal shocks is greater in LENS, largely as a result of a higher multiplier for transfer spending. This could reflect the greater importance of rule-of-thumb spending in LENS compared to ToTEM.

Fiscal multipliers (average of years 1 and 2)

	Spending	Taxes	Transfers
LENS	0.53	0.03	0.05
ToTEM	0.65	0.025	0.025

[38] REV_{ss}^{pl} is calculated the same way as REV_{ss}^{fed}, but conditional on the steady-state calibration of the PL sector.

Appendix B: Imports

Import volumes are disaggregated into commodity and non-commodity components and follow a REC specification in LENS. This allows us to take into account different driving forces behind import movements and, given the exports disaggregation, to identify the commodity versus non-commodity trade balances. The order of adjustment cost for imports is 3.

The target level of non-commodity imports (M^{ncom*}) is based on a cointegrating relationship with domestic demand (DEM), the real effective exchange rate and trade openness ($TRADE$). The target level of commodity imports (M^{com*}) is based on the same drivers, but replaces the exchange rate with real commodity prices ($PCOM$):

$$M_t^{ncom*} = -0.9 + 1.0DEM_t - 0.12FOREX_t + 0.48TRADE_t, \tag{B1}$$

$$M_t^{com*} = -2.41 + 1.0DEM_t - 0.03PCOM_t + 0.28TRADE_t.^{39} \tag{B2}$$

The dynamic model of non-commodity imports is based on the rational error-correction specification along with the contemporaneous change in the domestic output gap and the real exchange rate:

$$\Delta M_t^{ncom} = -0.14(M_{t-1}^{ncom} - M_{t-1}^{ncom*}) + 0.07\Delta M_{t-1}^{ncom} + 0.11\Delta M_{t-2}^{ncom} + 0.82E_t \sum_{i=1}^{20} f_i \Delta M_{t+i}^{ncom*} + 2.61\Delta Y_t^{gap} - 0.46\Delta FOREX_t + \varepsilon_t^{mncom}. \tag{B3}$$

Commodity imports adjust rapidly to their target level in the context of a standard REC process:

$$\Delta M_t^{com} = -0.55(M_{t-1}^{com} - M_{t-1}^{com*}) + 0.04\Delta M_{t-1}^{com} - 0.01\Delta M_{t-2}^{com} + 0.97E_t \sum_{i=1}^{20} f_i \Delta M_{t+i}^{com*} + \varepsilon_t^{mcom}. \tag{B4}$$

[39] DEM is defined as the chained sum of consumption, residential and business investment, and exports. Trade openness is measured by the share of exports among OECD countries. As in the case of exports, the coefficient on domestic demand is restricted to one so as to generate a stable imports-to-GDP ratio in the steady state. Trade openness is not extended indefinitely over the forecast horizon.

Appendix C: Relative Prices

In LENS, deflators for the main components of aggregate demand are modelled as a ratio relative to the GDP deflator and follow a REC specification. The order of adjustment cost ranges from 2 to 3. These deflators affect the cost of capital, household wealth, the current account and inflation.

C.1 Consumption

The real price of durable-goods consumption enters the calculation of user cost for durable-goods expenditures. It is a function of the exchange rate and a deterministic trend:

$$PDUR_t^* = 0.52 + 0.48FOREX_t - 0.004TREND_t^{pdur}, \tag{C1}$$

$$\Delta PDUR_t = -0.23(PDUR_t - PDUR_{t-1}^*) + 0.18\Delta PDUR_{t-1} + 0.86E_t \sum_{i=1}^{20} f_i \Delta PDUR_{t+i} + \varepsilon_t^{pdur}. \tag{C2}$$

The relative price of total consumption is backed out from the nominal National Accounts identity given the profile for the other deflators.

C.2 Residential investment

The relative price of residential investment enters the calculation of user cost for housing. It is a function of existing house prices (PH) and non-energy commodity prices:[40]

$$PRES_t^* = -1.17 + 0.33PH_t + 0.26BCNE_t, \tag{C3}$$

$$\Delta PRES_t = -0.08(PRES_t - PRES_{t-1}^*) + 0.13\Delta PRES_{t-1} + 0.65E_t \sum_{i=1}^{20} f_i \Delta PRES_{t+i} + 0.13\Delta PH_t + 0.07\Delta BCNE_t + \varepsilon_t^{pres}. \tag{C4}$$

C.3 Government

The real price of government spending on goods and services fluctuates around a deterministic trend over history, but is assumed to be constant over the projection.

C.4 Business investment

Real investment prices determine user cost for businesses. They are influenced by the real exchange rate, commodity prices, terms of trade and a deterministic trend:

[40] Existing house prices in LENS are based on a simple error-correction equation where the long-run level of real house prices depends on a deterministic trend. Stability restrictions on relative prices in the steady-state prevent us from using models of house prices that are based on non-stationary variables such as income and population. Such factors are nonetheless taken into account when constructing a forecast by using satellite models of house prices for a few quarters out.

$$PME_t^* = 0.93 + 0.39FOREX_t - 0.007TREND_t^{pme}, \tag{C5}$$

$$\Delta PME_t = -0.12(PME_t - PME_{t-1}^*) + 0.05\Delta PME_{t-1} + 0.05\Delta PME_{t-2} + 0.78E_t \sum_{i=1}^{20} f_i \Delta PME_{t+i}$$
$$+ 0.38\, \Delta FOREX_t \; + \varepsilon_t^{pme}, \tag{C6}$$

$$PNRS_t^* = -0.02 + 0.69(PX_t - PM_t) + 0.12PCOM_t, \tag{C7}$$

$$\Delta PNRS_t = -0.14(PNRS_t - PNRS_{t-1}^*) + 0.26\Delta PNRS_{t-1} + 0.86E_t \sum_{i=1}^{20} f_i \Delta PNRS_{t+i} + \varepsilon_t^{pnrs}. \tag{C8}$$

C.5 Exports and imports

International trade prices are driven by the real exchange rate, commodity prices and the real price of U.S. exports. Import prices feed into the Phillips curve for CPIX:

$$PX_t^* = -0.06 + 0.74PM_t + 0.06FOREX_t + 0.20PCOM_t, \tag{C9}$$

$$\Delta PX_t = -0.20(PX_t - PX_{t-1}^*) + 0.36\Delta PX_{t-1} - 0.09\Delta PX_{t-2} + 0.96E_t \sum_{i=1}^{20} f_i \Delta PX_{t+i} + \varepsilon_t^{px}, \tag{C10}$$

$$PM_t^* = 0.14 + 0.92PXUS_t^{cad} + 0.05PCOM_t\,, \tag{C11}$$

$$\Delta PM_t = -0.26(PM_t - PM_{t-1}^*) + 0.01\Delta PM_{t-1} + 0.04\Delta PM_{t-2} + 0.89E_t \sum_{i=1}^{20} f_i \Delta PM_{t+i} + 0.59\Delta FOREX_t$$
$$- 0.09\Delta FOREX_{t-1} + \varepsilon_t^{pm}. \tag{C12}$$

C.6 GDP

The GDP deflator is based on a simple relationship with consumer price inflation and changes in the terms of trade:

$$\Delta PY_t = \pi_t + 0.3\Delta(PX_t - PM_t) + \varepsilon_t^{py}. \tag{C13}$$

Appendix D: Impulse-Response Functions

Figure D-1: A 1 per cent consumption shock

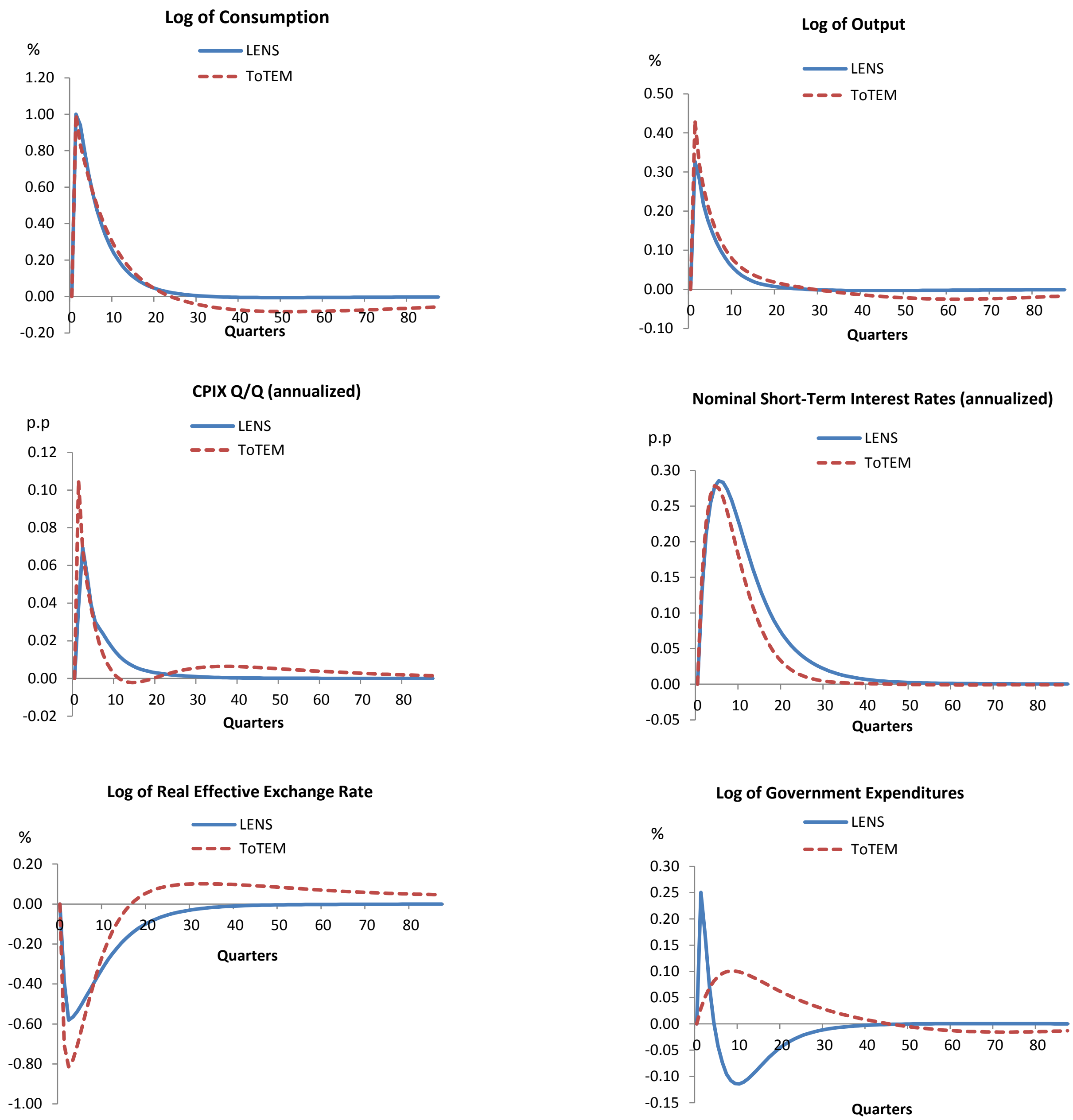

Figure D-2: A 100-basis-point increase in short-term interest rates

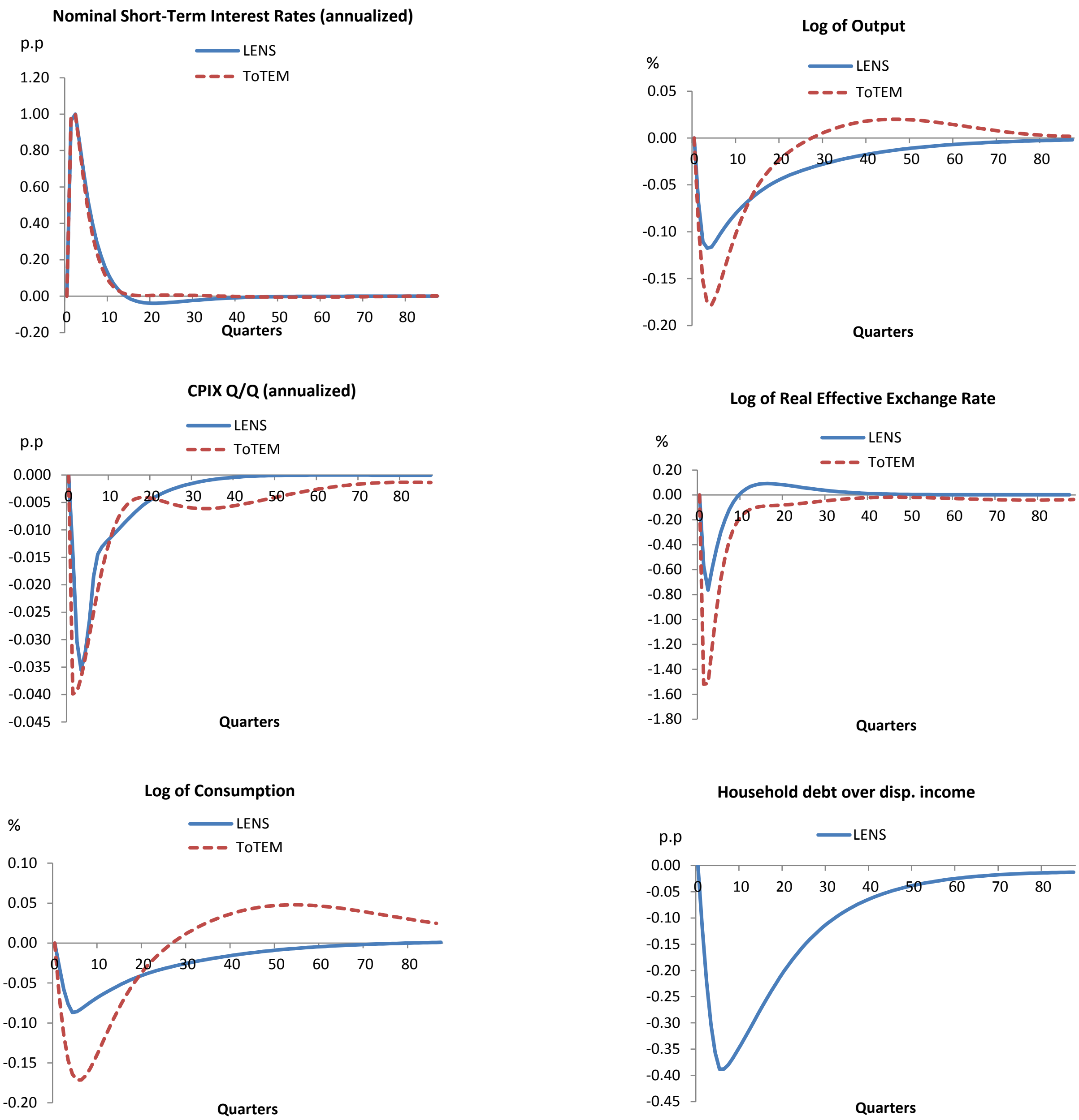

Figure D-3: A 1 per cent exchange rate depreciation

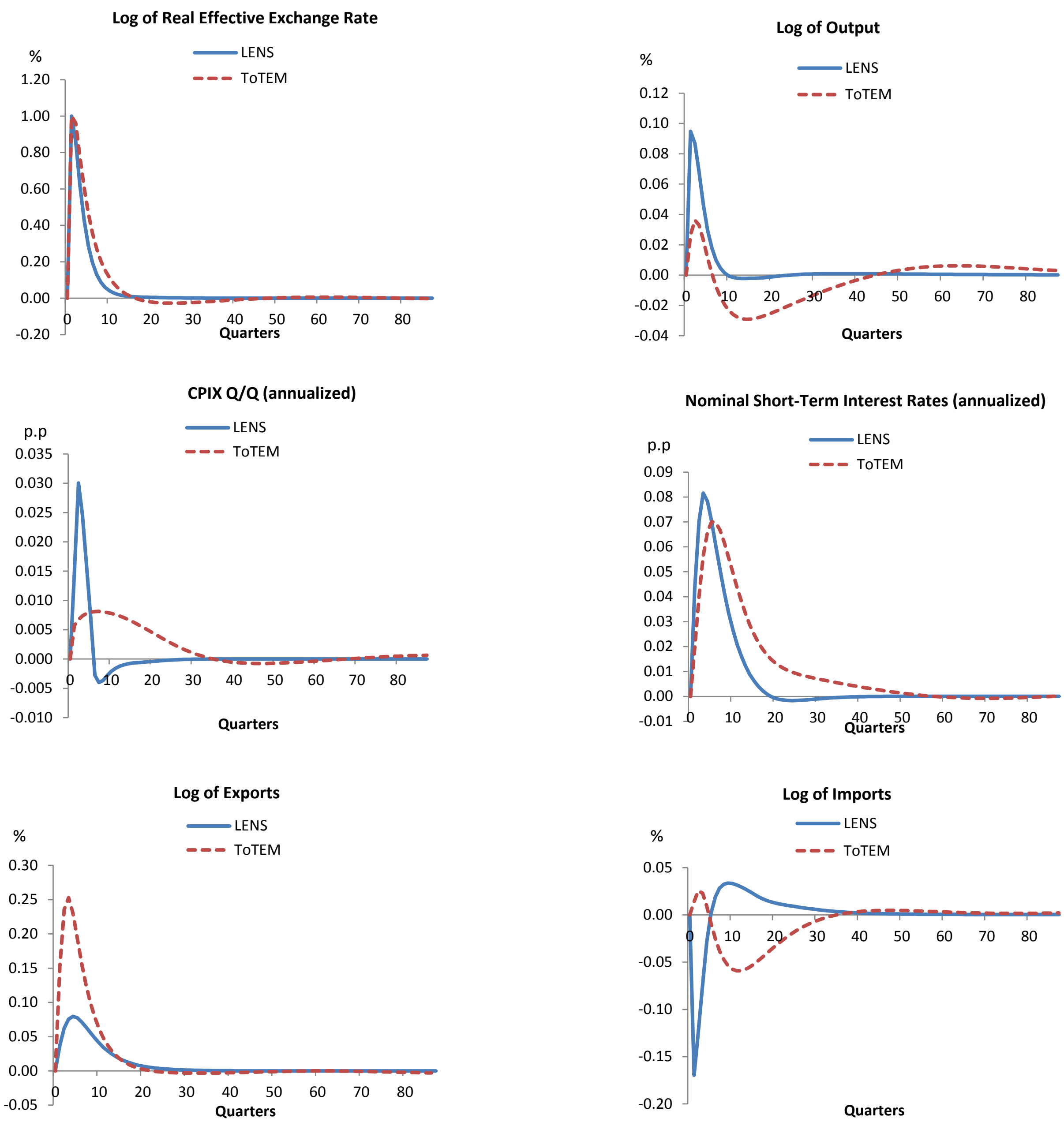

Figure D-4: A 1 per cent foreign demand shock

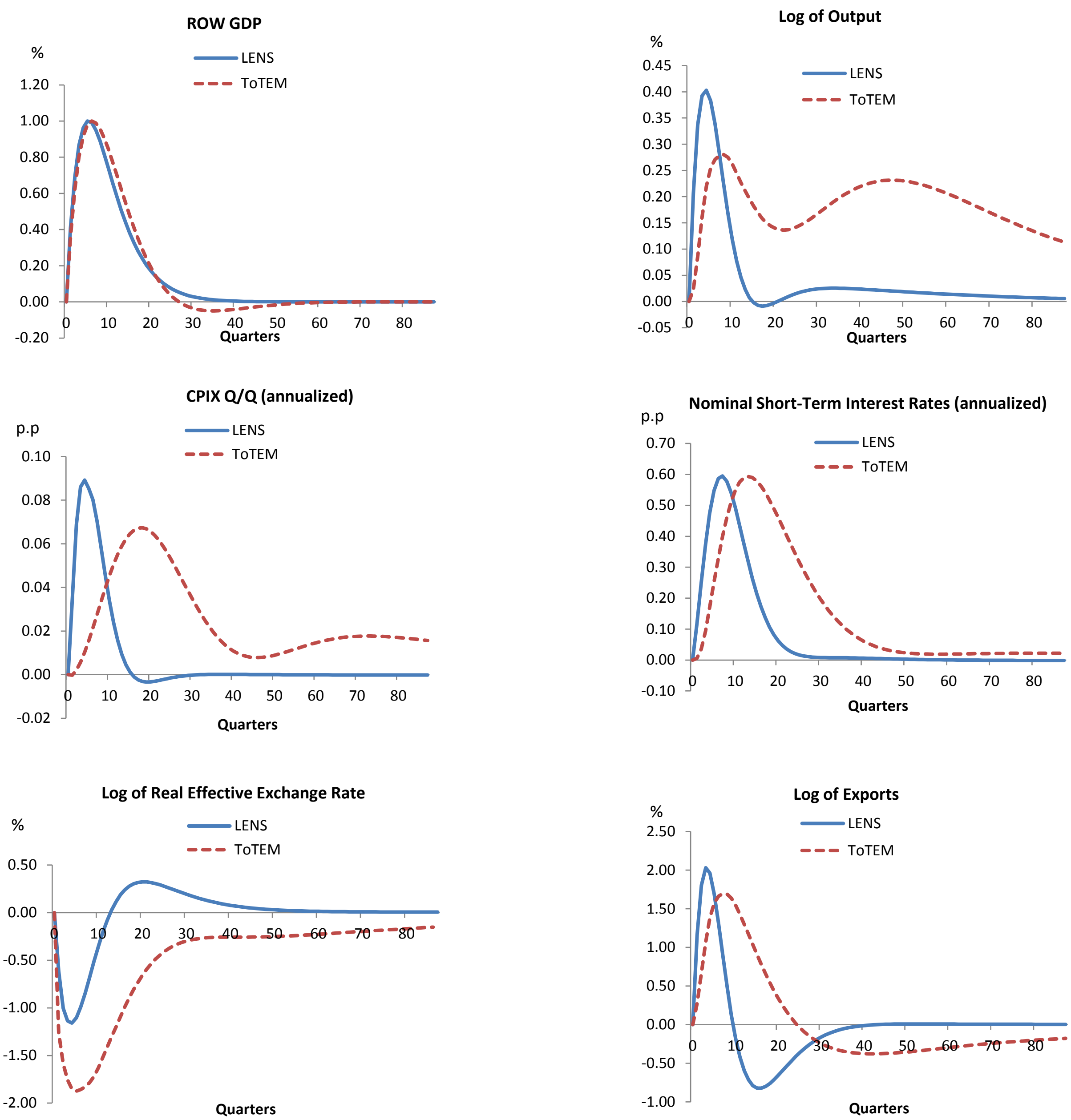

Figure D-5: A 10 per cent commodity-price increase

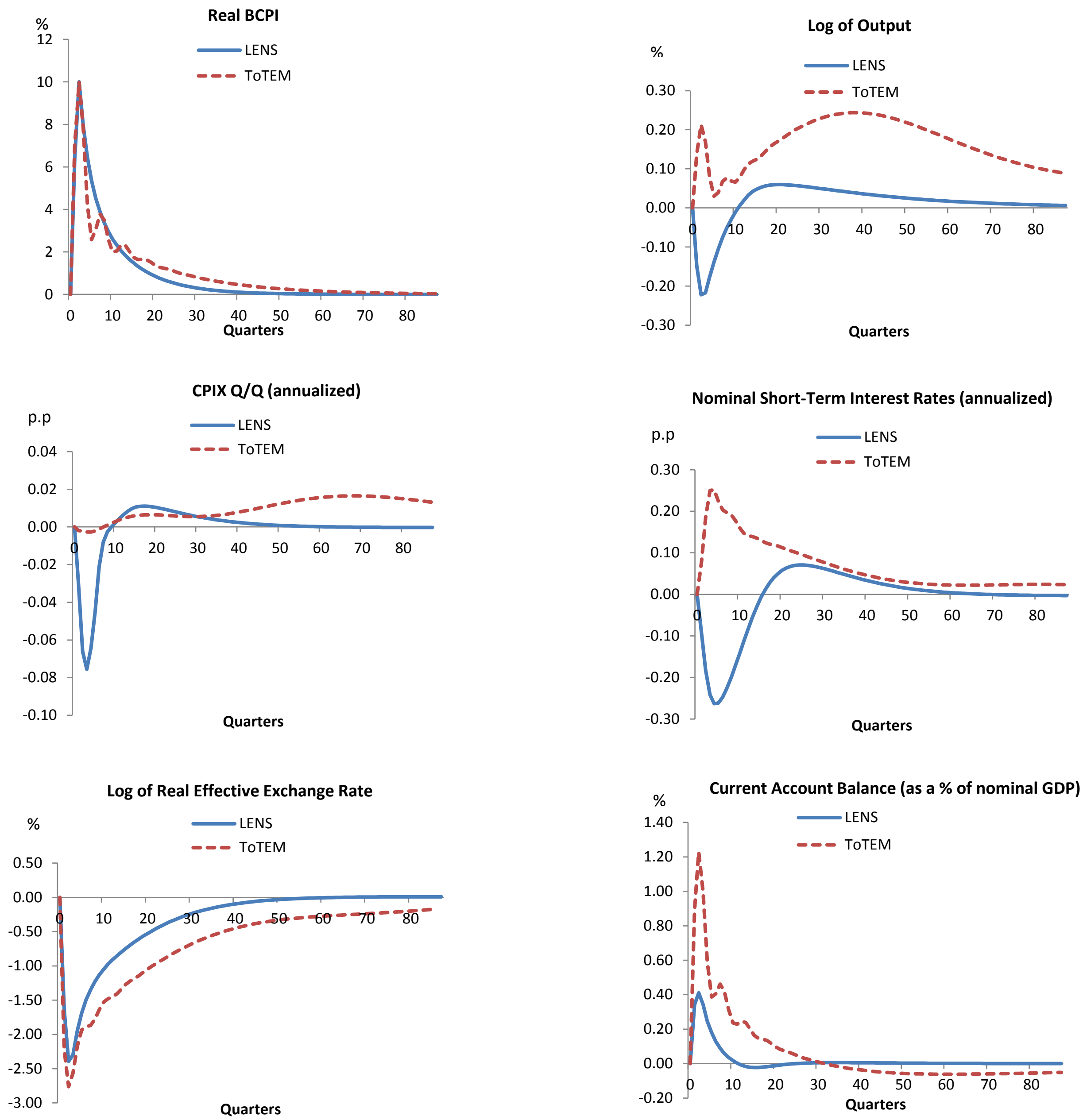

Figure D-6: A 1 per cent permanent increase in productivity