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Abstract

What makes *e-money* more special than cash? Is the introduction of e-money necessarily welfare enhancing? Is an e-money system necessarily stable? What is the optimal way to design an efficient and stable e-money scheme? This paper provides a first attempt to develop a micro-founded, dynamic, general-equilibrium model of e-money for investigating these policy issues. We first identify some superior features of e-money which help mitigate informational frictions and enhance social welfare in a cash economy. A model that features both trading frictions and two-sided platforms is then built and used to compare two potential e-money schemes: (i) public provision of e-money with decentralized adoption, and (ii) private monopolistic provision of e-money. We show that, in general, both public and private provision of e-money are inefficient, and we characterize the optimal incentive scheme by addressing four potential sources of inefficiency – market powers in goods trading, network externality, liquidity constraint and monopoly distortion in e-money issuance. We show that the welfare impact of e-money depends critically on whether cash is a viable alternative to e-money as a means of payment. When it is not (e.g., for online payments where usage of money is prohibitively costly), the adoption of e-money is always welfare enhancing, albeit not welfare maximizing. However, when cash is a viable alternative (e.g., in a coffee shop), introducing e-money can sometimes reduce social welfare. Moreover, a system with public provision and decentralized adoption is inherently unstable, while a planner or a private issuer can design a pricing scheme to restore stability. Lastly, we examine an alternative e-money scheme – a hypothetical set-up with public provision through a private platform. We also compare the impact of various provision schemes on central bank seigniorage income. While this scheme may or may not improve efficiency, it can always increase seigniorage income, even though there may exist better policy options such as imposing a cash reserve requirement or collecting a charter fee.

JEL classification: E, E4, E42, E5, E58, L, L5, L51
Bank classification: Bank notes; E-money; Payment clearing and settlement systems

Résumé

Quelles sont les particularités de la monnaie électronique qui la distinguent de l'argent comptant? Celle-ci favorise-t-elle nécessairement le bien-être? Les systèmes d'émission de la monnaie électronique sont-ils forcément stables? Quel est le moyen optimal de concevoir un système d'émission efficient et stable? La présente étude constitue une première tentative en vue d'élaborer un modèle dynamique d'équilibre général de la monnaie électronique reposant sur des fondements microéconomiques qui permet d'aborder ces questions. Nous établissons d'abord une liste d'avantages propres à la monnaie électronique qui aident à atténuer les éléments de friction relatifs à l'acquisition d'information et à accroître le bien-être collectif dans une économie monétaire. Nous construisons ensuite un modèle, qui comporte des frictions accompagnant les échanges et des plateformes bilatérales, et utilisons ce modèle pour comparer deux systèmes d'émission potentiels de la monnaie électronique : 1) émission par une autorité publique et adoption décentralisée; 2) émission par un monopoleur privé. Nous montrons que, de manière générale, ces deux types d'émission sont inefficients et nous définissons le mécanisme d'incitation optimal en déterminant quatre sources possibles d'inefficience : les pouvoirs de marché lors de l'échange de biens, l'externalité de réseau, les contraintes de liquidité et la distorsion provoquée par la position monopolistique de l'émetteur. Nous montrons que l'effet de la monnaie électronique sur le bien-être dépend essentiellement du fait que l'argent comptant représente un moyen de paiement concurrent viable. Dans le cas contraire (p. ex., coût d'utilisation prohibitif de l'argent pour les paiements en ligne), l'adoption de la monnaie électronique améliore toujours le bien-être, mais ne le maximise pas pour autant. Cependant, lorsque l'argent comptant est une solution de rechange viable (p. ex., au café), le recours à la monnaie électronique peut parfois réduire le bien-être collectif. En outre, un système basé sur un émetteur public et une adoption décentralisée est intrinsèquement instable, alors qu'un planificateur ou un émetteur privé peut élaborer un régime de prix afin de rétablir la stabilité. Enfin, nous étudions une troisième possibilité : un régime hypothétique prévoyant une émission publique par l'entremise d'une plateforme privée. Nous comparons également l'incidence de divers modes d'émission sur les recettes de seigneuriage des banques centrales. On ne peut garantir que ce régime améliorera l'efficience, mais il est certain qu'il accroîtra les recettes de seigneuriage, même s'il peut exister de meilleures options pour les pouvoirs publics, comme l'imposition de réserves obligatoires de liquidités ou de droits de licence.

Classification JEL : E, E4, E42, E5, E58, L, L5, L51
Classification de la Banque : Billets de banque; Monnaie électronique; Systèmes de compensation et de règlement des paiements

1 Introduction

In recent years, a number of retail payment innovations, known as electronic money (or *e-money*), have been taking place in many countries. The development of new e-money products facilitates retail transactions for both consumers and merchants, but it also raises concerns for policy-makers over its implications for economic efficiency and financial stability, as well as its challenge for the predominant role of cash for making retail payments.[1] Unfortunately, economic theory provides little guidance regarding the potential impacts of these new payment products, and the appropriate policy response of central banks and regulators. This paper develops a micro-founded, dynamic, general-equilibrium model of e-money to help us address relevant policy issues and understand the underlying economic trade-offs.

In this paper, we will adopt the definition of e-money proposed by the Committee on Payment and Settlement Systems (CPSS, 2012): it is the "monetary value represented by a claim on the issuers which is stored on an electronic device such as a chip card or a hard drive in personal computers or servers or other devices such as mobile phones and issued upon receipt of funds in an amount not less in value than the monetary value received and accepted as a means of payment by undertakings other than the issuer."[2]

As pointed out by Fung et al. (2013), the above definition incorporates three key elements: no credit is involved; it is the liability of the e-money issuer; and it is multi-purpose.[3] Typically, a consumer needs to acquire e-money balances in advance from an e-money issuer. To finance the purchase of goods and services, these balances are transferred to merchants, who can then redeem the balances with the e-money issuer (Figure 1). Examples of e-money are multi-purpose prepaid cards, as well as stored-value cards for public transport that are also accepted at the point of sale.

Experience of introducing e-money products has been quite diverse across countries.[4] "The longest-lasting e-money schemes have tended to be concentrated in cash-intensive economies, especially those in East Asia, Africa and Europe" (Fung et al. 2013). One notable success is the Octopus card in Hong Kong (see Appendix A). The card was originated by a public transportation company, but its usage has been expanding to many other retail transactions, with almost half of the total value of transactions now

[1] For example, in the Survey of Electronic Money Developments (CPSS, 2001), the Bank for International Settlements highlighted that "Electronic money projected to take over from physical cash for most if not all small-value payments continues to evoke considerable interest both among the public and the various authorities concerned, including central banks. The electronic money developments raise policy issues for central banks as regards the possible implications for central banks' revenues, their implementation of monetary policy and their payment system oversight role."

[2] Definitions by the European Central Bank and the Financial Stability Authority in the United Kingdom are similar.

[3] The first feature rules out credit cards; the second rules out certain peer-to-peer cryptocurrency such as Bitcoins; and the third rules out single-purchase stored-value cards (e.g., Starbucks cards).

[4] The Survey of Electronic Money Developments (CPSS, 2001) notes that "in a sizeable number of the countries surveyed, card-based e-money schemes have been launched and are operating relatively successfully: Austria, Belgium, Brazil, Denmark, Finland, Germany, Hong Kong, India, Italy, Lithuania, the Netherlands, Nigeria, Portugal, Singapore, Spain, Sweden and Switzerland. In some countries the products are available on a nationwide basis and in others only within specific regions or cities .... Compared to card-based schemes, the developments of network-based or software-based e-money schemes has been much less rapid. Network-based schemes are operational or are under trial in a few countries (Australia, Austria, Colombia, Italy, the United Kingdom and the United States), but remain limited in their usage, scope and application."

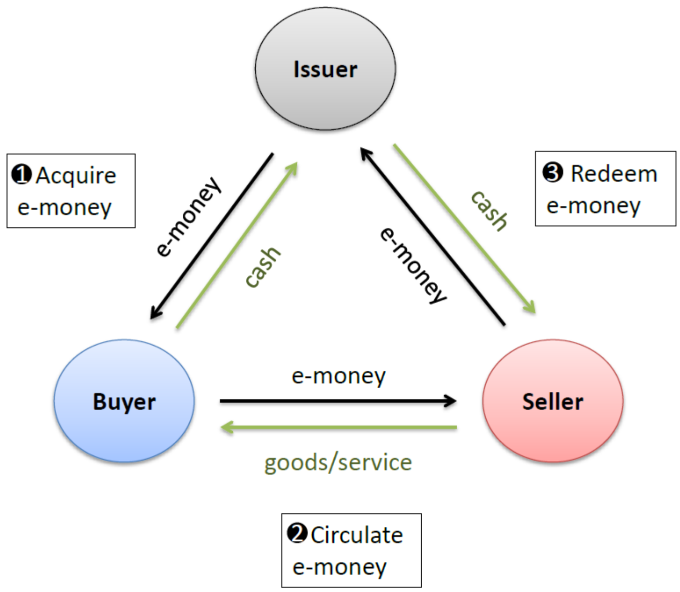

Figure 1: Acquisition, circulation and redemption of e-money

non-transport related. While there are many different variants of e-money products, this study will focus on e-money products that have features similar to those of the Octopus card. These products typically require consumers to obtain a prepaid anonymous device (e.g., prepaid cards) from the e-money issuer, and the merchants to install terminals (e.g., card reader/writer) that can operate off-line to transfer e-money balances. To emulate the perceived anonymity of traditional cash transactions, user anonymity to both the issuer and merchants during a purchase is usually reserved. As pointed out by the CPSS (1996), "this contrasts with traditional electronic payment transactions such as those with debit or credit cards which typically require online authorization and involve the debiting of the consumer's bank account after the transaction."

To the best of our knowledge, this is the first paper that develops a micro-founded, dynamic, general-equilibrium model of e-money for policy analysis. This paper investigates the following questions: What are the key superior features of e-money such as an Octopus card relative to traditional cash? Is the usage of e-money always welfare enhancing? What are the effects of the private provision of e-money on retail transactions, e-money adoption, cash usage and social welfare? Specifically, does private provision achieve efficient and stable outcomes? If not, is there any role for policy intervention such as positive or negative incentives, reserve requirement, as well as public issuance of e-money?

An underlying question behind our research is, *why does "e" matter*? In other words, does e-money possess some superior features relative to traditional paper cash?[5] It is now widely accepted that cash helps mitigate informational frictions in situations where credit is not feasible (Kocherlakota, 1998). The use of money, however, requires pre-investment by buyers, giving rise to a cash-in-advance constraint. In a decentralized economy, this constraint typically leads to an inefficient allocation: buyers hold too little cash, and hence are liquidity constrained in trading (for example, due to discounting, inflation, liquidity shocks, pricing distortion, etc.). To restore efficiency, a redistribution of trade surplus between buyers and merchants is needed to induce buyers to bring a sufficient amount of liquid balances. It may not be feasible to implement this type of arrangement in a cash-based payment system in which participants are *fully* anonymous. We argue that the introduction of e-money can relax this informational constraint because the users of e-money are only *partially* anonymous. Specifically, the technical requirement of e-money adoption allows issuers to distinguish between buyers and merchants in a payment system. This knowledge is necessary for performing the redistribution between the two types. More importantly, the e-money technology often requires e-money balances to be associated with an account (e.g., prepaid card) and hence allows the e-money issuer to restrict certain undesirable transfers or side-trades between accounts. This is important for implementing certain non-linear pricing schemes to achieve redistribution between buyers and sellers.[6,7]

We develop a dynamic, general-equilibrium model to compare two alternative e-money systems: (i) public provision of e-money with decentralized adoption, and (ii) private monopolistic provision of e-money. We show that the welfare impact of e-money depends critically on whether money is a viable alternative to e-money as a means of payment. When the usage of money is prohibitively costly (e.g., for online payments), the adoption of e-money is always welfare enhancing, albeit not welfare maximizing. However, when money is a viable alternative (e.g., in a coffee shop), e-money adoption can sometimes reduce social welfare. We show that, generally, both public and private provision of e-money are inefficient, and we characterize the optimal incentive scheme to restore efficiency by identifying four potential sources of inefficiency – market powers in goods trading, network externality, liquidity constraint and monopoly distortion in e-money issuance. Moreover, a system with public provision and decentralized adoption is inherently unstable, while a planner or a private issuer can design a pricing scheme to restore stability.

[5] If the answer is no, then perhaps the choice between money and e-money is not very different from that between U.S. and Canadian dollars, or that between government money and private money.

[6] On the one hand, "partial anonymity" gives e-money issuers an informational advantage over cash issuers. On the other hand, it imposes a tighter informational constraint on e-money issuers than debit card issuers, who possess a much richer information set, including the identities and full trading histories of users. In this regard, the model constructed in this paper may not be a good one for analyzing debit card adoption.

[7] In a companion paper, Chiu and Wong (2014) use a mechanism design approach to identify the essential technological features of e-money. Solving a general problem of e-money mechanism design, we show that the optimal mechanism features a non-linear fee structure, redistribution between types and interchange fees on merchants. In this paper, we will focus on a specific class of fee schemes and solve for the e-money issuer's optimization problem in equilibrium.

Lastly, we examine a hypothetical set-up with public provision through a private platform. While this scheme may or may not improve efficiency, it can always increase central bank seigniorage income, even though there may be better intervention such as imposing a cash reserve requirement or collecting a charter fee.

Our model builds on important lessons learnt from several lines of literature. First, the empirical literature highlights three factors determining users' choice between different payment instruments: a payment instrument's intrinsic properties (e.g., convenience, transaction speed), pricing scheme (e.g., card rewards and merchant fees), and acceptance by consumers and merchants. In the model, we introduce exogenous, heterogeneous costs to capture the private value of using a means of payment. Also, we allow the e-money issuer to design a general pricing scheme, which potentially involves fixed fees, discounts, merchant fees and contingent rewards. Furthermore, the model explicitly incorporates the channel through which an agent's incentive to adopt e-money is affected by its acceptance by other consumers and sellers.

Second, our paper is related to the two-sided market literature that emphasizes the importance of network externalities in the adoption of payment instruments. See Rochet and Tirole (2003), Armstrong (2006) and Weyl (2010) for the prototype model of platforms and their competition. There is also a large stream of research that studies electronic payment platforms such as credit cards; for example, Gowrisankaran and Stavins (2004), Humphrey, Pulley and Vesala (1996), Shy and Tarkka (2002), Shy and Wang (2011) and Wright (2003). Gans and King (2003) study a platform model of credit cards with cash users. Most of these researchers focus on the positive theories of the fee structure and of the competition among profit-maximizing platforms in a partial-equilibrium setting. Our paper complements this stream of research by highlighting the interplay between network externality and liquidity constraint in the general equilibrium. This generates interesting implications on how profit-maximizing platforms design the fee structure of e-money to compete with a non-profit-maximizing central bank in attracting users.

Third, the monetary literature underscores the fact that money and e-money are intrinsically worthless means of payment. Hence the demand for these objects is derived from the future consumption values of those goods and services for which they can be exchanged. Therefore, the liquidity values of money and e-money should be determined endogenously in a dynamic, general-equilibrium setting. This consideration helps identify new sources of inefficiency in the adoption of e-money, and illustrates how a partial-equilibrium welfare calculation can be misleading.[8]

Finally, our analysis takes into account the fact that cash is an "incumbent" in retail payment, in the sense that it has been circulating widely and is universally accepted. More importantly, this alternative

[8] See Lagos and Wright (2005) for the prototype dynamic, general-equilibrium monetary model. For models of payment in this literature, also see Li (2011), Martin, Orlando and Skeie (2008), and Monnet and Roberds (2008).

means of payment is controlled and supplied by a non-profit-maximizing central bank. Against this background, our model looks at the competition between e-money and cash issued by a central bank under alternative e-money schemes that are being discussed by policy-makers.

The rest of the paper is organized as follows. Section 2 describes the model environment. Section 3 studies public issuance of e-money and compares the allocation of decentralized and centralized adoption. Section 4 analyzes the effects of private issuance on efficiency and stability. Section 5 contrasts different systems and discusses various policy schemes. Section 6 concludes.

2 Model

Our model is based on the alternating market formulation of Lagos and Wright (2005). This model is useful because it allows us to study frictions in retail payments and agents' heterogeneity in terms of their adoption of different payment instruments, while still keeping the distribution of wealth analytically tractable. Time is indexed by $t = 0, 1....$ Each period is divided into two consecutive subperiods: day and night. There are two non-storable goods in this economy: consumption goods produced and consumed during the day, and numeraire goods at night. The economy is populated with two types of infinitely lived agents: a $[0, 1]$ continuum of buyers and a $[0, 1]$ continuum of sellers. Agents discount the future and have a discount factor $\beta \in (0, 1)$. During the day, buyers value consumption goods with utility $u(q)$. During the night, buyers have access to a linear production technology according to which l units of work generate l units of numeraire, and they are endowed with sufficiently large $\bar{l} < \infty$ units of labor per period such that the labor supply is never bounded. Buyers have the following preferences over consumption goods q_t and labor l_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(q_t) - l_t \right\}.$$

During the day, sellers have access to a linear production technology according to which q units of work generate q units of consumption goods. The marginal disutility of production is c. During the night, sellers value the numeraire according to a linear function, and they are endowed with sufficiently large $\bar{l} < \infty$ units of labor per period that the output is never bounded. A seller has the following preferences over producing consumption goods q_t and consuming numeraire l_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ -cq_t + l_t \right\}.$$

We focus on symmetric and stationary equilibria, where all agents follow identical strategies and where real allocations are constant over time.

We will call the day the *trading subperiod*, because consumption goods are traded in this subperiod. We will call the night the *loading subperiod*, because agents replenish their money/e-money balances (discussed below) in this subperiod. In the trading subperiod, sellers can produce the consumption goods but do not want to consume, while buyers want to consume but cannot produce, hence there are potential gains from trade. Trading in the trading subperiod is decentralized and bilateral, with buyers and sellers paired randomly. Agents can only observe the actions and outcomes of their trades, and are anonymous. Buyers cannot commit, and there is no enforcement or record-keeping technology. As a result, buyers cannot borrow to finance their consumption goods, because they would renege on their promise to repay their debt. This generates the need for a medium of exchange.

In this economy, there are two additional, perfectly divisible storable objects that can be used as a medium of exchange in the trading subperiod. The first storable object is called *money* (or cash). There is a central bank that maintains a fixed supply M of money (consider money growth in the extension). Money and the numeraire good are traded in a centralized competitive market in the loading subperiod. Agents take the price of money in terms of the numeraire good, ϕ, as given. So the aggregate real balance of money in the economy is ϕM. In order to hold any non-negative amount of money in period t, a buyer needs to incur a fixed cost $\kappa_C > 0$ (in terms of labor) in the previous loading subperiod. This cost is meant to capture efforts put into cash management, safekeeping against theft and loss, etc. For simplicity, we assume that sellers can costlessly store money. Therefore, in the trading subperiod, money can be used in a transaction if the buyer in the match has already paid the fixed cost.

The second storable object is called *e-money*, which is a durable asset that can be produced by its issuer in each loading subperiod and perishes in the next loading subperiod. E-money balances have to be stored in a device (e.g., prepaid card for a buyer and card reader/writer for a seller). In order to obtain this device to hold any non-negative amount of e-money in period t, a buyer needs to incur a fixed cost $\kappa_B \geq 0$ (in terms of labor) in the previous loading subperiod. This cost is random i.i.d. across agents and over time drawn from a distribution $F(\kappa_B)$ with full support on $[0, \kappa_B^{\max}]$. Similarly, in order to obtain a device to hold e-money, sellers need to incur a fixed cost $\kappa_S \sim G$, which has a full support on $[0, \kappa_S^{\max}]$.[9] Therefore, in the trading subperiod, e-money can be used in a transaction only if both the buyer and the seller in the match have paid the fixed costs. The circulation of e-money is described in Figure 2, which is motivated by the discussion in the introduction. We assume that there is an e-money issuer who, in each loading subperiod, creates and sells e-money to agents. In the next trading subperiod, e-money holders use the acquired balances to finance their transactions. In the following loading subperiod, holders of these

[9] Differential costs (i.e., non-degenerate F, G) capture heterogeneous incentives among buyers (e.g., demographics, location, income) and among merchants (e.g., types of business) to adopt electronic payments, confirmed in many surveys such as Arango et al. (2011). The asymmetry between $F(\kappa_B)$ and $G(\kappa_S)$ is motivated by the observation that consumers and merchants usually need to adopt different technical devices for using e-money.

e-money balances can ask the issuer for a redemption. Unredeemed e-money will perish automatically. Here, we assume that the issuer can commit to redeem at par (face value).

There are a few important differences between money and e-money. First, money circulates forever in the economy and the central bank does not sell new or buy old money. In contrast, e-money lasts only one period, with its issuer introducing new and retiring old e-money every period. Second, money is not redeemable on demand, while the e-money issuer promises to redeem e-money at face value. Third, while money is transferable/tradable between agents (who have already paid the fixed cost) in both subperiods, we assume that e-money is only transferable/tradable between buyers and sellers in the trading subperiod, but not among other agents or in the loading subperiod.[10] Therefore, there is one market price of money ϕ in the loading subperiod, while there is none for e-money. This difference implies that even if the central bank intends to trade money with agents in the loading subperiod, it has to buy or sell only at the competitive price, lest it create arbitrage opportunities. Owing to the lack of transferability of e-money in the loading subperiod, the e-money issuer can potentially offer different selling/buying prices to different agents. Fourth, in the loading subperiod, the e-money issuer can potentially ask buyers and sellers to pay fixed fees (in addition to the private costs κ_B, κ_S and the cost of loading e-money balances) for their adoption of the e-money device (e.g., annual membership fees). This type of non-linear pricing scheme for e-money is feasible again because of the restriction imposed on the transferability of e-money in the loading subperiod. In contrast, such a non-linear pricing scheme for money will lead to arbitrage opportunities, because of the centralized trading of money.

To focus on the adoption of the two means of payment along the extensive margin, we will shut down the intensive margin by considering indivisible consumption goods with exogenous terms of trade. Specifically, a buyer and a seller can only trade $q \in \{0, 1\}$ units of goods. The utility from consuming these two quantities are $u(0) = 0$ and $u(1) = u$. The price (in terms of the numeraire good) of each unit is $d \in (c, u)$. Below, we show that we can already gain considerable insight into the adoption of e-money by studying this simple set-up.[11]

3 Public Issuance of E-money

We start with a simple e-money scheme. Suppose the central bank is also the issuer of e-money. In the loading subperiod, it creates and sells new e-money balances to agents *at par* with cash, and redeems old e-

[10] In the real world, it is quite common globally that the transferability of e-money balances among end-users is not allowed (see CPSS, 2001). Imposing this restriction is feasible because, unlike traditional money, e-money can only be transferred by using some special technical devices (e.g., merchant terminals or card reader/writer). It will become clear below that this feature is indeed important for supporting some welfare-improving function of e-money.

[11] One can easily build on this setting to consider the general case with divisible goods and endogenous prices. This paper abstracts from this dimension because endogenous prices and quantities complicate the analysis and, while interesting, they are not issues of first-order importance for the question examined in this paper.

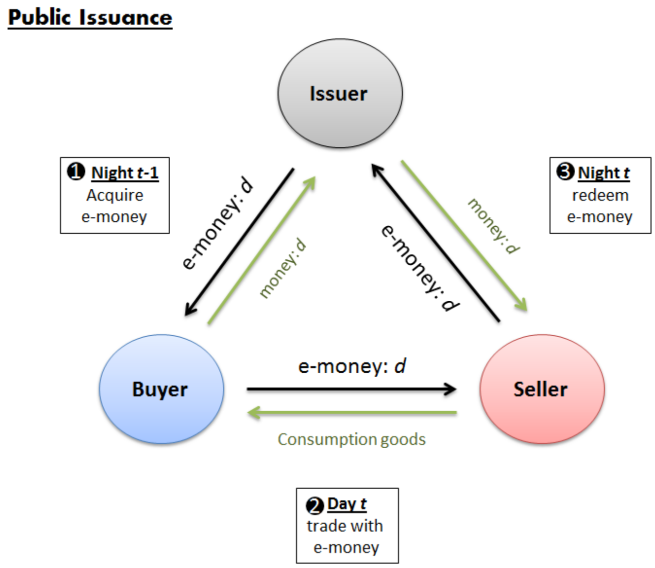

Figure 2: Usage of e-money in the model

money also *at par*, without charging any additional fees (see Figure 2). In this section, we will first consider decentralized adoption by private agents and characterize the equilibrium allocation. Then we derive the socially optimal allocation by considering centralized adoption coordinated by an e-money planner. We will also discuss some welfare implications by comparing these two cases.

3.1 Decentralized Adoption of E-money

Consider first a buyer with a real money balance z^m and real e-money balance z^e in the trading subperiod. Note that he can always trade consumption goods with money as long as $z^m \geq d$, but can only trade with e-money if he gets a seller accepting e-money (w.p. α_B) and has $z^m + z^e \geq d$. His value function is

$$W_T(z^m, z^e) = \begin{cases} u + \tilde{W}_L(z^m - d, z^e) & \text{if } z^m \geq d \\ \alpha_B[u + \tilde{W}_L(0, z^e + z^m - d)] + (1-\alpha_B)\,\tilde{W}_L(z^m, z^e) & \text{if } z^m + z^e \geq d > z^m \\ \tilde{W}_L(z^m, z^e) & \text{if } d > z^m + z^e \end{cases}, \quad (1)$$

where $\tilde{W}_L(z^m, z^e)$ is his continuation value at night (before drawing κ_B). Consider the decision of a buyer with a real money balance z^m and real e-money balance z^e in the loading subperiod after drawing κ_B. His maximization problem is

$$\max_{\varepsilon, e, z^{m\prime}, z^{e\prime}} -l - \varepsilon\kappa_C - e\kappa_B + \beta W_T(\varepsilon \cdot z^{m\prime}, e \cdot z^{e\prime}),$$

subject to

$$l + z^m + z^e \geq z^{m\prime} + z^{e\prime}.$$

Here, the buyer starts with real wealth $z^m + z^e$, and chooses whether to adopt money ($\varepsilon = 1$ if adopt, 0 otherwise), to adopt e-money ($e = 1$ if adopt, 0 otherwise), and how much money $z^{m\prime}$ and e-money $z^{e\prime}$ to be carried to the next trading period. So his expected lifetime utility after drawing κ_B is

$$W_L(z^m, z^e; \kappa_B) = \max_{\varepsilon, e, z^{m\prime}, z^{e\prime}} z^m + z^e - \varepsilon\kappa_C - e\kappa_B - z^{m\prime} - z^{e\prime} + \beta W_T(\varepsilon \cdot z^{m\prime}, e \cdot z^{e\prime}).$$

It is straightforward to show that $\tilde{W}_L(z^m, z^e)$ is linear in z^m and z^e. That is,

$$\begin{aligned} \tilde{W}_L(z^m, z^e) &= \int W_L(z^m, z^e; \kappa_B) dF(\kappa_B) \\ &= z^m + z^e + W, \end{aligned}$$

where $W \equiv \int W(0, 0; \kappa_B) dF(\kappa_B)$. This result simplifies (1) to

$$W_T(z^m, z^e) = z^m + z^e + W + (u - d)\mathbb{I}_{z^m \geq d} + \alpha_B(u - d)\mathbb{I}_{z^m + z^e \geq d > z^m}.$$

We can now use W_T to simplify the maximization problem in the loading period after drawing κ_B to

$$\begin{aligned} W(0, 0; \kappa_B) \equiv & \max_{\varepsilon, e, z^{m\prime}, z^{e\prime}} -\varepsilon\kappa_C - e\kappa_B - (1 - \beta)z^{m\prime} - (1 - \beta)z^{e\prime} \\ & +\beta W + \beta(u - d)\mathbb{I}_{\varepsilon \cdot z^{m\prime} \geq d} + \beta\alpha_B(u - d)\mathbb{I}_{\varepsilon \cdot z^{m\prime} + e \cdot z^{e\prime} \geq d > e \cdot z^{m\prime}}. \end{aligned}$$

Obviously, it is optimal to buy money or e-money if and only if its adoption cost is paid (i.e., $z^{m\prime} = 0$ iff $\varepsilon = 0$, $z^{e\prime} = 0$ iff $e = 0$). Also, if both money and e-money are adopted, then it is optimal to choose $z^{m\prime} = d$ and $z^{e\prime} = 0$, as long as $\alpha_B < 1$. But this implies that adopting both is indeed suboptimal (i.e., no *multihoming*). Because of the indivisibility of goods and the exogenous price, it is obvious that $(\varepsilon, z^{m\prime}) \in \{(0, 0), (1, d)\}$ and $(e, z^{e\prime}) \in \{(0, 0), (1, d)\}$. The optimal adoption choice is determined by comparing the following payoffs:

$$W(0, 0; \kappa_B) = \begin{cases} \beta W & \text{, for } \varepsilon = 0, e = 0 \\ \beta W - \kappa_B - (1 - \beta)d + \beta\alpha_B(u - d) & \text{, for } \varepsilon = 0, e = 1 \\ \beta W - \kappa_C - (1 - \beta)d + \beta(u - d) & \text{, for } \varepsilon = 1, e = 0 \\ \beta W - \kappa_C - \kappa_B - 2(1 - \beta)d + \beta(u - d) & \text{, for } \varepsilon = 1, e = 1 \end{cases},$$

with $\varepsilon = e = 1$ a strictly dominated option. Hence the buyer uses e-money *if and only if*

$$\kappa_B \leq \beta\alpha_B (u - d) - (1 - \beta) d - \max\{\beta u - d - \kappa_C, 0\}. \tag{2}$$

On the other hand, the buyer uses cash *if and only if*

$$\beta u - d - \kappa_C \geq \max\{\beta\alpha_B (u - d) - (1 - \beta) d - \kappa_B, 0\}. \tag{3}$$

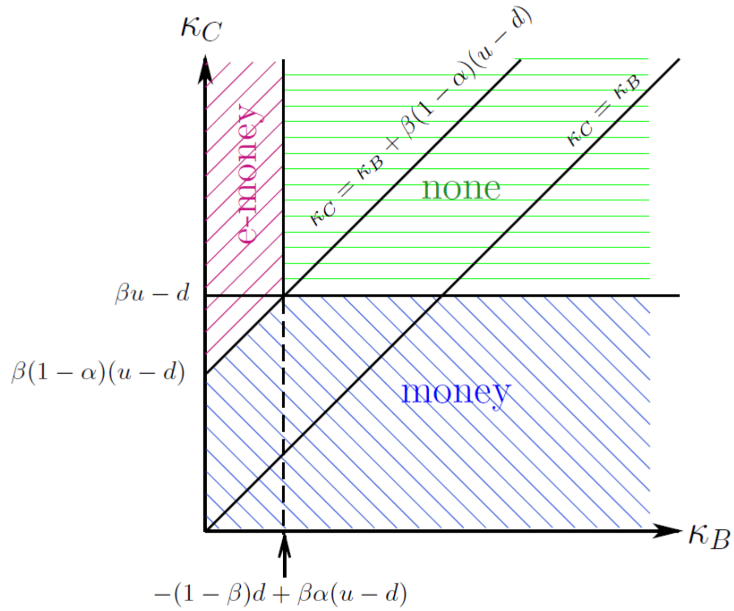

Figure 3: Adoption choice over the κ_B, κ_C space

Then the adoption choice is described in Figure 3, and we have the following lemma:

Lemma 1 *Suppose* $\alpha_B < 1$,

(a) If the buyer with κ_B *adopts e-money (money), then any buyer with* $\kappa_B' \leq \kappa_B$ $(\kappa_B' \geq \kappa_B)$ *also adopts e-money (money).*

(b) E-money (money) is never used for a sufficiently small (high) κ_C.

Proof. It is straightforward to prove (a), so we omit it here. To show (b), substitute $\kappa_C = 0$ into the right-hand side of (2). Then we have

$$\kappa_B \leq \beta\alpha_B (u-d) - (1-\beta) d - \max\{\beta (u-d) - (1-\beta) d, 0\} \leq 0.$$

Hence e-money is not used when $\kappa_C = 0$. Notice that the right-hand side of (2) is increasing in κ_C, so e-money is never used for a sufficiently small κ_C. ■

Similarly, e-money is used by a seller if

$$\kappa_S \leq \beta\alpha_S (d-c), \tag{4}$$

where α_S is the seller's probability of matching a buyer with e-money balances *only*. So e-money is used by some sellers if and only if there is a positive measure of buyers using e-money *only*. Also, if the seller with cost κ is using e-money, then any seller $\kappa' \leq \kappa$ is also using e-money.

Now we are ready to define the decentralized equilibrium:

Definition 1 *With public provision and decentralized adoption of e-money, an equilibrium is the belief* (a_B, α_S) *and the set of e-money buyers and e-money sellers* (A_B, A_S) *such that*

a. *(E-money buyer's optimization) Given* α_B*, for all* $i \in A_B$*,* $\kappa_B = \kappa_i$ *satisfies (2);*

b. *(E-money seller's optimization) Given* α_S*, for all* $i \in A_S$*,* $\kappa_S = \kappa_i$ *satisfies (4);*

c. *(rational expectation on random matching)* $\alpha_B = \Pr(A_B)$ *and* $\alpha_S = \Pr(A_S)$*.*

Lemma 1 implies that $\alpha_B = G(\kappa_S^*)$ and $\alpha_S = F(\kappa_B^*)$ in equilibrium, with the marginal buyer κ_B^* and the marginal seller κ_S^* characterized by (2) and (4) satisfied with equalities:

$$\kappa_B^* = \max\{\beta(u-d)G(\kappa_S^*) - (1-\beta)d - \max\{\beta u - d - \kappa_C, 0\}, 0\}, \quad (5)$$

$$\kappa_S^* = \beta(d-c)F(\kappa_B^*). \quad (6)$$

E-money is adopted by buyers with $\kappa_B \leq \kappa_B^*$ and by sellers with $\kappa_S \leq \kappa_S^*$. Define $\mathbb{I}_c$ as the indicator of money adoption, given by

$$\mathbb{I}_c = \begin{cases} 1 \text{ if } \beta u - d - \kappa_C \geq 0 \\ 0 \text{ otherwise} \end{cases}.$$

As shown in Figure 3, if $\mathbb{I}_c = 1$, then any buyer with $\kappa_B > \kappa_B^*$ uses money. If $\mathbb{I}_c = 0$, buyers with $\kappa_B > \kappa_B^*$ use neither money nor e-money. The aggregate demand for real money balances is thus $[1 - F(\kappa_B^*)]\mathbb{I}_c d$, which is (weakly) decreasing in the threshold κ_B^*. The aggregate demand for real balances (both money and e-money) is thus $d\{F(\kappa_B^*) + \mathbb{I}_c[1 - F(\kappa_B^*)]\}$, which is (weakly) increasing in the threshold κ_B^*. The real price of money ϕ is $[1 - F(\kappa_B^*)]\mathbb{I}_c d/M$, which is also decreasing in κ_B^*.

Equations (2) and (4), respectively, define the two best response functions, $BR_B : [0, \kappa_S^{\max}] \mapsto [0, \kappa_B^{\max}]$ and $BR_S : [0, \kappa_B^{\max}] \mapsto [0, \kappa_S^{\max}]$. As plotted in Figure 4, the equilibrium adoption (κ_B^*, κ_S^*) is determined by the intersection points between BR_B^{-1} and BR_S. Given their properties, it is easy to see that an equilibrium always exists.

3.1.1 Multiple Equilibria under Decentralized Adoption

We next study the existence of different types of equilibrium and potential multiplicity. Define V as

$$V(\kappa) \equiv \max\{\beta(u-d)G[\beta(d-c)F(\kappa)] - (1-\beta)d - \max\{\beta u - d - \kappa_C, 0\}, 0\} - \kappa.$$

Intuitively, $V(\kappa)$ measures the horizontal difference between the two curves in Figure 4. Therefore, the set of equilibrium is characterized by the roots κ_B^* of V (i.e., $V(\kappa_B^*) = 0$) and the corresponding $\kappa_S^* = BR_S(\kappa_B^*)$. Naturally, depending on the shapes of F and G, there may exist a unique equilibrium or multiple equilibria

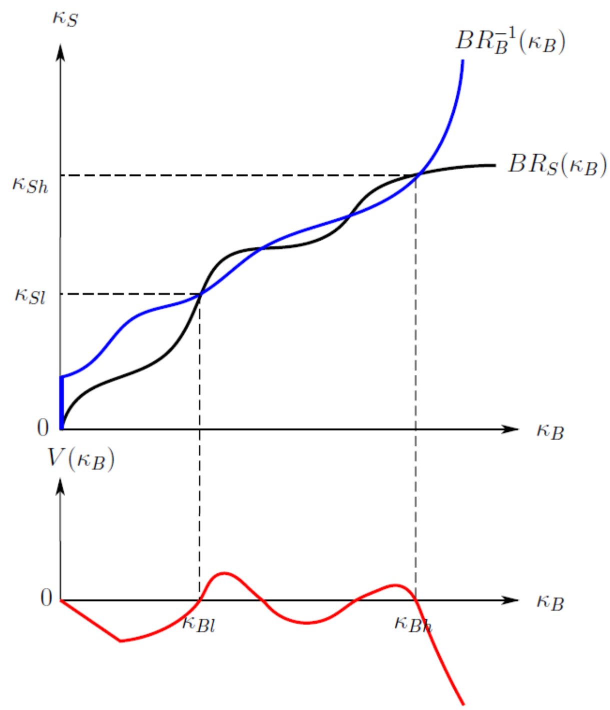

Figure 4: Determination of equilibrium adoption

while e-money may or may not be adopted. We now define different types of equilibrium as follows. A *non-monetary equilibrium* (i.e., $\phi = 0$) always exists due to coordination failure, which is a standard feature in many monetary models. A *monetary equilibrium without e-money* is one in which *all* buyers and sellers adopt *money only* (i.e., $\phi > 0, \kappa_B^* = \kappa_S^* = 0$). Note that $V(0) = 0$. Thus, there always exists an equilibrium where e-money is never used: $\kappa_B^* = \kappa_S^* = 0$. Money is used if and only if $\beta u - d - \kappa_C \geq 0$.

Next, we study *monetary equilibria with e-money* (i.e., $\phi > 0, \kappa_B^*, \kappa_S^* > 0$). Figure 4 shows that there can be three cases: (i) BR_B^{-1} and BR_S only intersect at the origin (i.e., $V(\kappa) < 0$ for all $\kappa > 0$); (ii) BR_B^{-1} and BR_S touch each other for some $\kappa_B^* > 0$ but never cross (i.e., $V(\kappa) = 0$ for some $\kappa > 0$, but $V(\kappa) \leq 0$ for all κ); and (iii) BR_B^{-1} and BR_S cross for some $\kappa_B^* > 0$ (i.e., $V(\kappa) > 0$ for some $\kappa > 0$). There is no equilibrium with e-money in case (i). There are equilibria with e-money in case (ii), but only for a measure-zero subset in the parameter space. We will thus ignore this case. In case (iii), there are equilibria with e-money. It can be shown that $V(\kappa) < 0$ for a sufficiently small, positive κ. Also, since $G[\beta(d-c)F(\kappa)]$ is bounded by 1, we have $V(\kappa) < 0$ for a sufficiently large κ. By continuity of V, if $\max_\kappa V(\kappa) > 0$, then there exist multiple $\kappa > 0$ satisfying $V(\kappa) = 0$. In other words, generically, there are multiple equilibria with e-money characterized by the set of (non-zero) roots $\{\kappa_B^*(j)\}$ of V. Moreover, condition (6) implies that the seller's equilibrium adoption is increasing in the buyer's adoption: $\kappa_S^*(j') \geq \kappa_S^*(j)$ iff $\kappa_B^*(j') \geq \kappa_B^*(j)$.

Since V is continuous, its set of roots is closed.[12] Since it is also bounded, there exists a minimum threshold $\kappa^*_{Bl} > 0$ and a maximum threshold $\kappa^*_{Bh} > \kappa^*_{Bl} > 0$. We call the equilibrium associated with κ^*_{Bh} a *monetary equilibrium with a high e-money adoption rate* (or simply *high-adoption equilibrium*), and the one with κ_{Bl} a *monetary equilibrium with a low e-money adoption rate* (or *low-adoption equilibrium*). The following proposition summarizes these results.

Proposition 1 *Given the maintained assumption,*

a. there always exists a non-monetary equilibrium;

b. there exists a monetary equilibrium without e-money iff $\beta u - d - \kappa_C \geq 0$;

c. generically, a high-adoption equilibrium and a low-adoption equilibrium coexist iff $\max_\kappa V(\kappa) \geq 0$.

The multiplicity is a familiar result due to strategic complementarity in the adoption of e-money and the acceptance of money. It has two practical implications. First, across countries, economies with similar fundamentals may adopt very different payment technologies, and for the same technologies the adoption rates can also be very different. Second, over time, the usage of a specific payment technology can change swiftly and dramatically, even without any change in fundamental factors. Changes in the belief of the e-money usage are self-fulfilling, and can trigger a fluctuation from a high-adoption equilibrium to a low-adoption equilibrium (or even to an equilibrium without e-money). This result highlights some undesirable outcomes of this provision scheme, namely the uncertain adoption of newly introduced technology and the inherent instability of existing technology.

We next perform some comparative statics exercises. Suppose there are multiple equilibria with e-money under F, G, κ_C and d. The following proposition describes the effects of changing the cost structure and goods price on the equilibrium outcome, with $\hat{\kappa}_{Bh}$, $\hat{\kappa}_{Sh}$, $\hat{\kappa}_{Bl}$, $\hat{\kappa}_{Sl}$ in the proposition denoting the new equilibrium adoption associated with the new fundamentals $\hat{F}$, $\hat{G}$, $\hat{\kappa}_C$, $\hat{d}$.

Proposition 2 *Given the maintained assumption, suppose there are multiple equilibria with e-money under* F, G, κ_C *and* d*. Then there are also multiple equilibria when*

a. $\hat{F} \geq F$,

b. $\hat{G} \geq G$,

c. $\hat{\kappa}_C \geq \kappa_C$, *or*

d. $\hat{d} \in [d, \beta u - \kappa_C)$.

Moreover, $\hat{\kappa}_{Bh} \geq \kappa_{Bh}$, $\hat{\kappa}_{Sh} \geq \kappa_{Sh}$ *and* $\hat{\kappa}_{Bl} \leq \kappa_{Bl}$, $\hat{\kappa}_{Sl} \leq \kappa_{Sl}$.

[12] By the continuity of V, the inverse image of the singleton set $\{0\}$ is closed.

Proof. Here we sketch the proof of (a)-(b), and the proof for (c) and (d) follow similar arguments. Define

$$\hat{V}(\kappa) \equiv \max\left\{\beta(u-d)\hat{G}\left[\beta(d-c)\hat{F}(\kappa)\right] - (1-\beta)d - \max\{\beta u - d - \kappa_C, 0\}, 0\right\} - \kappa,$$

that is the counterpart of V under $\hat{F}$ and $\hat{G}$. First, notice that $\hat{V}(\varepsilon) < 0$ for a sufficiently small, positive ε. Second, since $\hat{G}\left[\beta(d-c)\hat{F}(\kappa)\right]$ is bounded by 1, we have $\hat{V}(\kappa') < 0$ for a sufficiently large κ'.

Since there are multiple equilibria under F and G, there exist κ_{Bh} and κ_{Bl} such that $\kappa_{Bh} > \kappa_{Bl}$ and $V(\kappa_{Bh}) = V(\kappa_{Bl}) = 0$. Notice that $\hat{F} \geq F$ and $\hat{G} \geq G$ imply that $\hat{V}(\kappa) \geq V(\kappa)$, hence we have $\hat{V}(\kappa_{Bh}) \geq V(\kappa_{Bh}) = 0$ and $\hat{V}(\kappa_{Bl}) \geq V(\kappa_{Bl}) = 0$. Since $\hat{V}(\kappa)$ is continuous, the fact that $\hat{V}(\kappa_{Bh}) \geq V(\kappa_{Bh}) = 0$ and $\hat{V}(\kappa') < 0$ implies that there exists $\kappa'_{Bh} \in [\kappa_{Bh}, \kappa')$ such that $\hat{V}(\kappa'_{Bh}) = 0$. Therefore, we have the new maximum adoption $\hat{\kappa}_{Bh} \geq \kappa'_{Bh} \geq \kappa_{Bh}$. Define $\hat{\kappa}_{Sh} \equiv \beta(d-c)F(\hat{\kappa}_{Bh}) \geq \beta(d-c)F(\kappa_{Bh}) = \kappa_{Sh}$. Then $\kappa^*_B = \hat{\kappa}_{Bh}$ and $\kappa^*_S = \hat{\kappa}_{Sh}$ satisfy (5) and (6), hence $(\hat{\kappa}_{Bh}, \hat{\kappa}_{Sh})$ constitutes an equilibrium.

Similarly, the fact that $\hat{V}(\kappa_{Bl}) \geq V(\kappa_{Bl}) = 0$ and $\hat{V}(\varepsilon) < 0$ implies that there exists $\kappa'_{Bl} \in (\varepsilon, \kappa_{Bl}]$ such that $\hat{V}(\kappa'_{Bl}) = 0$. Therefore, we have the new minimum adoption $\hat{\kappa}_{Bl} \leq \kappa'_{Bl} \leq \kappa_{Bl}$. Define $\hat{\kappa}_{Bl} \equiv \beta(d-c)F(\hat{\kappa}_{Bl}) \leq \beta(d-c)F(\kappa_{Bl}) = \kappa_{Sl}$. Then $\kappa^*_B = \hat{\kappa}_{Bl}$ and $\kappa^*_S = \hat{\kappa}_{Sl}$ satisfy (5) and (6), hence $(\hat{\kappa}_{Bl}, \hat{\kappa}_{Sl})$ constitutes an equilibrium. In the proof we have also established that $\hat{\kappa}_{Bh} \geq \kappa_{Bh}$, $\hat{\kappa}_{Sh} \geq \kappa_{Sh}$ but $\hat{\kappa}_{Bl} \leq \kappa_{Bl}$, $\hat{\kappa}_{Sl} \leq \kappa_{Sl}$. ■

One can measure the instability of the e-money usage (or systemic risk) by the maximum loss of e-money users by comparing a high-adoption equilibrium to a low-adoption equilibrium, captured by $F(\kappa_{Bh}) - F(\kappa_{Bl})$ and $G(\kappa_{Sh}) - G(\kappa_{Sl})$. Proposition 2 states how the fundamentals of an economy affect the systemic risk of e-money usage. Propositions 2a and 2b predict that the systemic risk of e-money usage is higher when the distributions of e-money cost F and G become more stochastically dominated. A distribution $F(\kappa)$ is stochastically dominated when the entire curve of $F(\kappa)$ shifts up. This can be a result of a technological improvement that cuts the cost of e-money adoption.

To understand the source of instability in this economy, notice that the market demand for e-money from the buyers and the sellers can be captured by the first derivatives $dF(\kappa_B)/d\kappa_B$ and $dG(\kappa_S)/d\kappa_S$, respectively. So an alternative interpretation of a stochastic dominated distribution of e-money cost is the increase in the market demand for e-money. Under this interpretation, a stochastic dominated distribution of e-money cost encourages more agents to use e-money in the liquid equilibrium, so κ_{Bh} and κ_{Sh} become higher. However, since now it needs a bigger reduction in the number of e-money sellers to discourage e-money buyers not to use e-money, and vice versa, a stochastic dominated distribution of e-money cost also implies fewer agents using e-money in the low-adoption equilibrium, hence κ_{Bl} and κ_{Sl} become lower. Eventually, a technological improvement of adopting e-money, or an increase in market demand for e-money by buyers or sellers, can raise the systemic risk of e-money usage in the decentralized economy.

Proposition 2d states that a higher relative price d of goods, which is exogenously given, will increase the systemic risk of e-money usage in the decentralized economy. Now we have this implication because we model the substitution between cash and e-money. The intuition is as follows. A higher relative price d always makes sellers better off, since they have a larger share of the trade surplus. It encourages more sellers to use e-money in the liquid equilibrium. What is new here is that, actually, the buyer's surplus of using e-money increases as well. Although the buyer's private trade surplus $\beta(u-d)G(\kappa_S^*)$ decreases under a higher relative price d, the opportunity cost of using cash $\beta(u-d)-\kappa_C$ decreases as well. Since $G(\kappa_S^*)<1$, the decrease in the opportunity cost of using cash dominates the decrease in the buyer's private trade surplus, hence the buyer's surplus of using e-money increases eventually. As a result, there are more buyers to use e-money in the liquid equilibrium. So both κ_{Bh} and κ_{Sh} become higher. If we do not model the substitution with cash, then there is no longer any opportunity cost of using cash. Then there would be fewer buyers using e-money and the result would become ambiguous. On the other hand, using a similar argument as in the case of Proposition 4a, since now it needs a bigger reduction in the number of e-money buyers to discourage e-money sellers not to use e-money, and vice versa, a higher relative price d also implies fewer agents using e-money in the low-adoption equilibrium, hence both κ_{Bl} and κ_{Sl} become lower. Eventually, a rise in the relative price can raise the systemic risk of e-money usage in the decentralized economy.

3.1.2 Welfare under Decentralized Adoption

In any decentralized equilibrium, the utilitarian welfare $\mathcal{W}(\kappa_B^*,\kappa_S^*)$ is the sum of the values of buyers and of sellers, where $\mathcal{W}(\kappa_B,\kappa_S)$ is given by

$$\mathcal{W}(\kappa_B,\kappa_S)\equiv\frac{1}{1-\beta}\left[\begin{array}{l}F(\kappa_B)G(\kappa_S)\beta(u-c)-\int_0^{\kappa_B}\kappa dF(\kappa)-\int_0^{\kappa_S}\kappa dG(\kappa)\\+[1-F(\kappa_B)]\mathbb{I}_C[\beta(u-c)-\kappa_C]\end{array}\right]. \tag{7}$$

The welfare under a decentralized equilibrium consists of two parts, which correspond to the payment choices of the buyer: (i) e-money buyers (i.e., buyers using e-money), and (ii) non-e-money buyers (i.e., buyers not using e-money). For each case, the social trade surplus is $\beta(u-c)$. In case (i), there are $F(\kappa_{B0})$ e-money buyers and $G(\kappa_{S0})$ e-money sellers. As shown at the beginning of this section, these e-money buyers do not use money. So the social welfare arising from case (i) is the first line of (7). In case (ii), there is always trade for non-e-money buyers if and only if they use money, which happens if $\beta u-d-\kappa_C\geq 0$. So the social welfare arising from case (ii) is $[1-F(\kappa_{B0})]\mathbb{I}_c[\beta(u-c)-\kappa_C]$. Finally, the social cost of using e-money to the buyers and sellers is $\int_0^{\kappa_{B0}}\kappa dF(\kappa)+\int_0^{\kappa_{S0}}\kappa dG\kappa$.

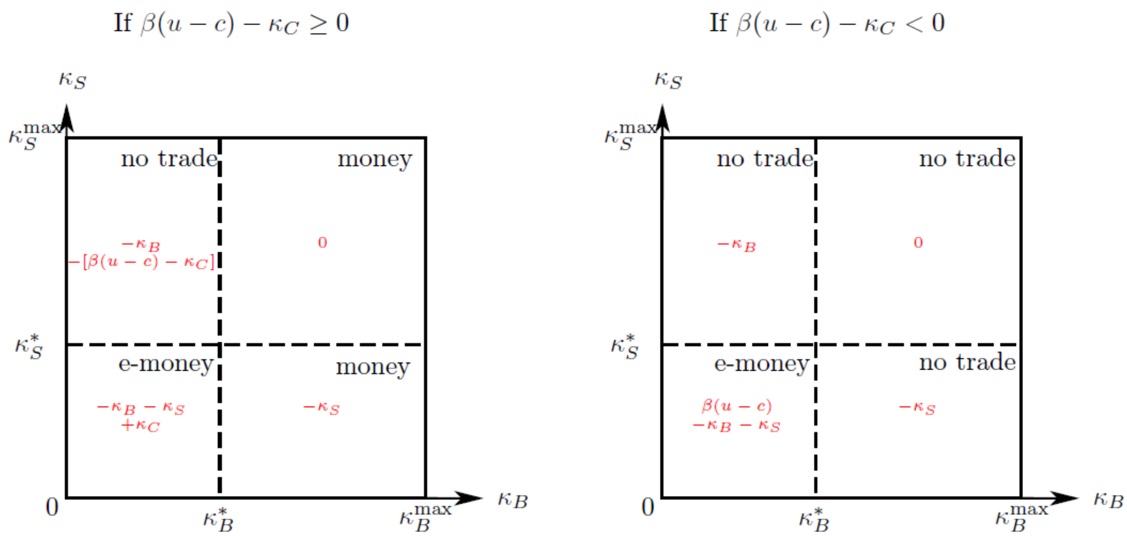

Figure 5: Welfare change $\mathcal{W}(\kappa_B^*, \kappa_S^*) - \mathcal{W}(0,0)$ over the κ_{Bl}, κ_{Sl} space

To compare the welfare of different decentralized equilibria, suppose there are multiple equilibria. The welfare difference $\mathcal{W}(\kappa_B^*, \kappa_S^*) - \mathcal{W}(0,0)$ captures the welfare gain (loss if negative) when the economy switches from an equilibrium without e-money circulating to the one with e-money circulating. As illustrated in Figure 5, the welfare difference $\mathcal{W}(\kappa_B^*, \kappa_S^*) - \mathcal{W}(0,0)$ between an equilibrium with e-money (with high or low adoption) and a trivial equilibrium (money may be used or not) can be expressed as the integral over e-money sellers:

$$
\begin{aligned}
& \mathcal{W}(\kappa_B^*, \kappa_S^*) - \mathcal{W}(0,0) \\
= \; & \frac{1}{1-\beta}\left[\int_0^{\kappa_S^*} \underbrace{\{F(\kappa_B^*)\,\beta(u-c) - \kappa\}}_{\text{seller's social e-money surplus}} dG(\kappa) - \int_0^{\kappa_B^*} \{\mathbb{I}_C[\beta(u-c) - \kappa_C] + \kappa\}\, dF(\kappa)\right].
\end{aligned}
$$

The first term on the right-hand side captures that, by adopting e-money, a seller increases welfare in the economy by $F(\kappa_B^*)\,\beta(u-c) - \kappa_S$. Using (6), we can rewrite the net welfare gain of having the marginal seller κ_S^* adopting e-money as

$$
F(\kappa_B^*)\,\beta(u-c) - \kappa_S^* = F(\kappa_B^*)\,\beta(u-d) > 0,
$$

implying underadoption, because the equilibrium adoption does not drive the marginal net social gain to zero. The intuition is that, by accepting e-money, a seller generates a positive externality to those buyers using e-money by facilitating their trade as long as $u > d$. The seller, however, does not internalize this benefit, leading to underadoption.

Similarly, we can also express the welfare difference $\mathcal{W}(\kappa_B^*, \kappa_S^*) - \mathcal{W}(0,0)$ as the integral over e-money

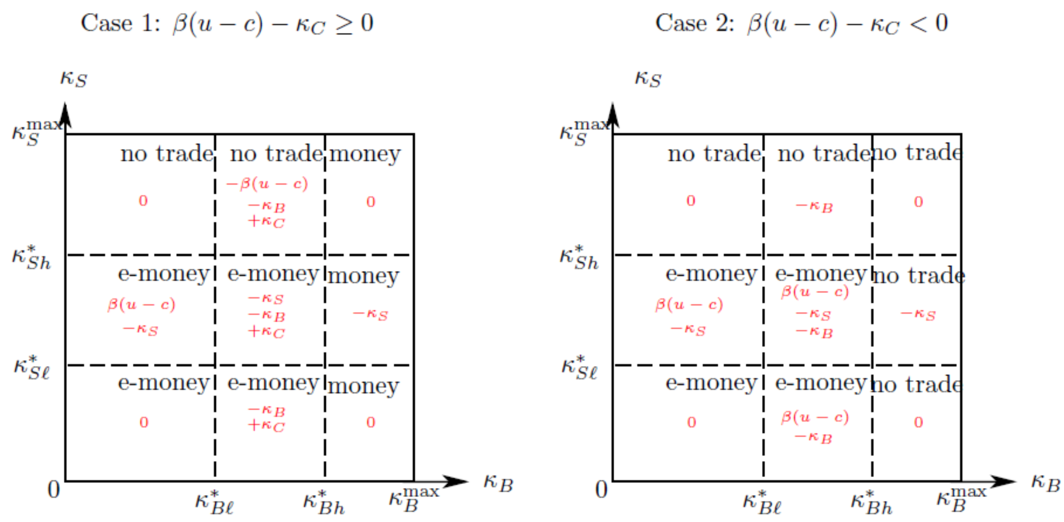

Figure 6: Welfare change $\mathcal{W}(\kappa_{Bh}, \kappa_{Sh}) - \mathcal{W}(\kappa_{Bl}, \kappa_{Sl})$ over the κ_{Bl}, κ_{Sl} space

buyers:

$$
\begin{aligned}
& \mathcal{W}(\kappa_B^*, \kappa_S^*) - \mathcal{W}(0,0) \\
= \quad & \frac{1}{1-\beta}\left[\int_0^{\kappa_B^*}\left\{\underbrace{G(\kappa_S^*)\beta(u-c)-\kappa}_{\text{buyer's social e-money surplus}} - \underbrace{\mathbb{I}_C[\beta(u-c)-\kappa_C]}_{\text{opportunity cost from using cash}}\right\}dF(\kappa) - \int_0^{\kappa_S^*}\kappa dG(\kappa)\right].
\end{aligned}
$$

The first term on the right-hand side captures that, by adopting e-money, a buyer increases total trade surplus in the economy by $G(\kappa_S^*)\beta(u-c)-\kappa_B$, but generates an opportunity cost of $\mathbb{I}_C[\beta(u-c)-\kappa_C]$ to the economy by substituting money with e-money. From the private point of view, a buyer chooses to use e-money as long as $G(\kappa_S^*)\beta(u-d)-\kappa_B^*$ is equal to the private opportunity cost $(1-\beta)d + \max\{\beta u - d - \kappa_C, 0\}$. From the social point of view, substituting (5) into the net social gain of the marginal e-money buyer κ_B^* implies

$$
\begin{aligned}
& G(\kappa_S^*)\beta(u-c) - \mathbb{I}_C[\beta(u-c)-\kappa_C] - \kappa_S^* \\
= \quad & -[\mathbb{I}_C - G(\kappa_S^*)]\beta(d-c), \qquad (8)
\end{aligned}
$$

which is negative if and only if money is used, i.e., $\beta u - d - \kappa_C \geq 0$. In this case, the equilibrium adoption drives the marginal net social gain to negative, implying overadoption. The intuition is that, as long as $d > c$, a buyer, by adopting e-money instead of money, imposes a negative externality on those sellers who accept money by destroying trades. The buyer, however, does not internalize this loss, leading to overadoption. Overall, there can be welfare loss even with e-money circulating, i.e., $\mathcal{W}(\kappa_B^*, \kappa_S^*) - \mathcal{W}(0,0) < 0$.

As shown in Figure 6, to derive the welfare difference between a high-adoption equilibrium and a low-adoption equilibrium, one can compute

$$\mathcal{W}(\kappa_{Bh},\kappa_{Sh})-\mathcal{W}(\kappa_{Bl},\kappa_{Sl})=\frac{1}{1-\beta}\left[\begin{array}{l}\beta(u-c)\left[F(\kappa_{Bh})G(\kappa_{Sh})-F(\kappa_{Bl})G(\kappa_{Sl})\right]\\-\int_{\kappa_{Bl}}^{\kappa_{Bh}}\kappa dF(\kappa)-\int_{\kappa_{Sl}}^{\kappa_{Sh}}\kappa dG(\kappa)\\-\left[F(\kappa_{Bh})-F(\kappa_{Bl})\right]\mathbb{I}_C\left[\beta(u-c)-\kappa_C\right]\end{array}\right].$$

The first term $\beta(u-c)[F(\kappa_{Bh})G(\kappa_{Sh})-F(\kappa_{Bl})G(\kappa_{Sl})]$ is the increase in the aggregate level of trade times the social trade surplus per trade. The last term $[F(\kappa_{Bh})-F(\kappa_{Bl})]\mathbb{I}_C[\beta(u-c)-\kappa_C]$ is the increase in the social opportunity cost due to the substitution of money with e-money. In general, there can be welfare loss even with wider e-money circulation, i.e., $\mathcal{W}(\kappa_B^*,\kappa_S^*)-\mathcal{W}(\kappa_{Bl},\kappa_{Sl})<0$, if the social opportunity cost dominates the social trade surplus. Nevertheless, the following proposition compares the welfare under various equilibria.

Proposition 3 *When $\mathbb{I}_C=0$, decentralized adoption of e-money is always welfare enhancing. When $\mathbb{I}_C=1$, it can be welfare reducing. Specifically, given the maintained assumptions, suppose there are multiple equilibria. When $\mathbb{I}_C=0$:*

a. $\mathcal{W}(\kappa_{Bh},\kappa_{Sh})>\mathcal{W}(\kappa_{Bl},\kappa_{Sl})>\mathcal{W}(0,0)$;

When $\mathbb{I}_C=1$:

b. if $\kappa_{Bh}>\beta(d-c)+\mathbb{E}\{\kappa_B|\kappa_B\in[\kappa_{Bl},\kappa_{Bh}]\}$, *then* $\mathcal{W}(\kappa_{Bh},\kappa_{Sh})>\mathcal{W}(\kappa_{Bl},\kappa_{Sl})$;

c. if $\kappa_{Bl}>\beta(d-c)+\mathbb{E}\{\kappa_B|\kappa_B\leq\kappa_{Bl}\}$, *then* $\mathcal{W}(\kappa_{Bl},\kappa_{Sl})>\mathcal{W}(0,0)$;

In addition, if $F''(\kappa_B),G''(\kappa_S)\leq 0$:

d. if $\kappa_{Bh}<\beta(d-c)$ *and* $\beta^2(d-c)^2G'(0)F'(0)<1$, *then* $\mathcal{W}(\kappa_{Bh},\kappa_{Sh})<\mathcal{W}(0,0)$;

e. if $\kappa_{Bl}<\beta(d-c)$ *and* $\beta^2(d-c)^2G'(0)F'(0)<1$, *then* $\mathcal{W}(\kappa_{Bl},\kappa_{Sl})<\mathcal{W}(0,0)$.

Proof. Notice that $\mathcal{W}(\kappa_B^*,\kappa_S^*)=\mathcal{W}[\kappa_B^*,\beta(d-c)F(\kappa_B^*)]$. By integrating from some $\kappa_{B0}<\kappa_B^*$ to κ_B^* with κ_S moving along the path $\kappa_S=\beta(d-c)F(\kappa_B)$, we can rewrite the welfare difference as

$$\begin{aligned}&\mathcal{W}(\kappa_B^*,\kappa_S^*)-\mathcal{W}[\kappa_{B0},\beta(d-c)F(\kappa_{B0})]\\=&\int_{\kappa_{B0}}^{\kappa_B^*}\mathcal{W}_1[\kappa,\beta(d-c)F(\kappa)]d\kappa+\beta(d-c)\int_{\kappa_{B0}}^{\kappa_B^*}\mathcal{W}_2[\kappa,\beta(d-c)F(\kappa)]dF(\kappa)\\=&\frac{1}{1-\beta}\int_{\kappa_{B0}}^{\kappa_B^*}\{\kappa_B^*-\kappa-\beta(d-c)\mathbb{I}_C\}dF(\kappa)\\&+\frac{\beta(d-c)}{1-\beta}\int_{\kappa_{B0}}^{\kappa_B^*}G[\beta(d-c)F(\kappa)]dF(\kappa)+\frac{\beta(u-d)}{1-\beta}F(\kappa_{B0})\left[G(\kappa_S^*)-G[\beta(d-c)F(\kappa_{B0})]\right],\end{aligned}\tag{9}$$

where $\mathcal{W}_1(\kappa_B,\kappa_S)\equiv\partial\mathcal{W}(\kappa_B,\kappa_S)/\partial\kappa_B$, $\mathcal{W}_2(\kappa_B,\kappa_S)\equiv\partial\mathcal{W}(\kappa_B,\kappa_S)/\partial\kappa_S$, which are given by

$$\begin{aligned}\mathcal{W}_1(\kappa_B,\kappa_S)&=(1-\beta)^{-1}F'(\kappa_B)\left[G(\kappa_S)\beta(u-c)-\kappa-\mathbb{I}_C[\beta(u-c)-\kappa_C]\right]\\\mathcal{W}_2([\kappa_B,\beta(d-c)F(\kappa_B)])&=(1-\beta)^{-1}\beta(u-d)G'[\beta(d-c)F(\kappa_B)]F(\kappa_B).\end{aligned}$$

The equality of (9) follows the fact that $\kappa_B^* = \beta(u-d)G[\beta(d-c)F(\kappa_B^*)] - (1-\beta)d - \max\{\beta u - d - \kappa_C, 0\}$ for any non-trivial solution κ_B^* of (5) and (6). First, notice that the second term and the last term of (9) are always non-negative. Second, notice that the first term on (9) can be rearranged as

$$\begin{aligned} & \int_{\kappa_{B0}}^{\kappa_B^*} \{\kappa_B^* - \kappa - \beta(d-c)\mathbb{I}_C\} dF(\kappa) \\ = \quad & [F(\kappa_B^*) - F(\kappa_{B0})][\kappa_B^* - \beta(d-c)\mathbb{I}_C - \mathbb{E}\{\kappa_B | \kappa_B \in [\kappa_{B0}, \kappa_B^*]\}], \end{aligned} \tag{10}$$

which is positive if either $\mathbb{I}_C = 0$, or $\mathbb{I}_C = 1$ and $\kappa_B^* > \beta(d-c) + \mathbb{E}\{\kappa_B | \kappa_B \in [\kappa_{B0}, \kappa_B^*]\}$. If either of these conditions hold, then we have $\mathcal{W}(\kappa_B^*, \kappa_S^*) > \mathcal{W}(\kappa_{B0}, \beta(d-c)F(\kappa_{B0}))$ for any $\kappa_{B0} < \kappa_B^*$. Third, put $\kappa_B^* = \kappa_{Bh}$ and $\kappa_{B0} = \kappa_{Bl}$, and then we prove $\mathcal{W}(\kappa_{Bh}, \kappa_{Sh}) > \mathcal{W}(\kappa_{Bl}, \kappa_{Sl})$; put $\kappa_B^* = \kappa_{Bl}$ and $\kappa_{B0} = 0$, and then we prove $\mathcal{W}(\kappa_{Bl}, \kappa_{Sl}) > \mathcal{W}(0,0)$. Thus, we prove Propositions 3a to 3c.

To prove Propositions 3d and 3e, first notice that, since $\beta(d-c)G[\beta(d-c)F(\kappa)] - \kappa$ is concave in κ, we always have

$$\beta(d-c)G[\beta(d-c)F(\kappa)] - \kappa \leq \left[\beta^2(d-c)^2 G'(0)F'(0) - 1\right]\kappa. \tag{11}$$

Second, put $\kappa_{B0} = 0$ and group the first and the second term of (9). Then we have

$$\begin{aligned} & \frac{1}{1-\beta}\int_0^{\kappa_B^*} \{\kappa_B^* - \kappa - \beta(d-c)[1 - G[\beta(d-c)F(\kappa)]]\} dF(\kappa) \\ \leq \quad & \frac{1}{1-\beta}\int_0^{\kappa_B^*} \left\{[\kappa_B^* - \beta(d-c)] + \left[\beta^2(d-c)^2 G'(0)F'(0) - 1\right]\kappa\right\} dF(\kappa), \end{aligned}$$

where the inequality follows (11). Third, the third term of (9) is zero under $\kappa_{B0} = 0$. Thus, if $\kappa_B^* < \beta(d-c)$ and $\beta^2(d-c)^2 G'(0)F'(0) < 1$, and then we have $\mathcal{W}(\kappa_B^*, \kappa_S^*) < \mathcal{W}(0,0)$. Put $\kappa_B^* = \kappa_{Bh}$ and $\kappa_B^* = \kappa_{Bl}$, then we prove Propositions 3d and 3e, respectively. ■

Proposition 3 shows that the welfare ranking amongst different decentralized equilibria depends on parameter values and derives some conditions under which different cases arise. Proposition 3a states that if money is never used in this economy, i.e., $\mathbb{I}_C = 0$, then the high-adoption equilibrium will have higher welfare than the low-adoption equilibrium. Since the adoption of e-money no longer bears an opportunity cost of forgoing money usage, increasing from the adoption of e-money users (up to a certain level) raises the welfare.

If money is a viable payment option, i.e., $\mathbb{I}_C = 1$, then it depends on parameter values. Proposition 3b states that if the cost κ_{Bh} of the marginal e-money buyer in a high-adoption equilibrium is sufficiently high (so that $\kappa_{Bh} > \beta(d-c) + \mathbb{E}\{\kappa_B | \kappa_B \in [\kappa_{Bl}, \kappa_{Bh}]\}$), then moving from a low-adoption to a high-adoption equilibrium is welfare enhancing. The intuition is as follows. In the high-adoption equilibrium, when adopting e-money, a buyer with κ_B not only receives net private gain, which equals to $-\beta[1 -$

$G(\kappa_S)](u-d)+\kappa_C-\kappa_B = \kappa_{Bh}-\kappa_B$, but also generates a social cost $\beta[1-G(\kappa_S)]\,(d-c)$, which is bounded by $-\beta\,(d-c)$. Therefore, when moving from a low-adoption to a high-adoption equilibrium, the welfare change as a result of the additional e-money buyers, $F\,(\kappa_{Bh})-F\,(\kappa_{Bl})$, is positive whenever $\int_{\kappa_{Bl}}^{\kappa_{Bh}} \kappa_{Bh}-\kappa-\beta\,(d-c)\,dF(\kappa)>0$, or simply $\kappa_{Bh}>\beta\,(d-c)+\mathbb{E}\left\{\kappa_B|\kappa_B\in[\kappa_{Bl},\kappa_{Bh}]\right\}$. This condition ensures high welfare due to higher adoption on the buyer side. On the seller side, notice that there are also more sellers using e-money in a high-adoption equilibrium, and that higher seller adoption always enhances welfare. Therefore, overall, the high-adoption equilibrium achieves a higher welfare than the low-adoption equilibrium. Similarly, Proposition 3c argues that if $\kappa_{Bl}>\beta\,(d-c)+\mathbb{E}\left\{\kappa_B|\kappa_B\leq\kappa_{Bl}\right\}$, then the low-adoption equilibrium achieves higher welfare than the monetary equilibrium without e-money.

To understand why the welfare ranking of decentralized equilibria is reversed in some scenarios, consider again the case where cash is used in this economy, i.e., $\mathbb{I}_C=1$. Proposition 3d states that if the e-money cost κ_{Bh} of the marginal buyer is sufficiently low such that $\kappa_{Bh}<\beta\,(d-c)+\mathbb{E}\left\{\kappa|\kappa\in[\kappa_{Bl},\kappa_{Bh}]\right\}$, then the high-adoption equilibrium will have a lower welfare than even the cash equilibrium. With the similar argument, in the high-adoption equilibrium, the buyer's private benefit of using e-money is simply her e-money cost κ_{Bh}. It is different from the buyer's social benefit of using e-money by (8), which is bounded by $-\beta\,(d-c)$. So if $\kappa_{Bh}<\beta\,(d-c)$, then the buyer part of welfare is lower in the high-adoption equilibrium than in the cash equilibrium. What is shown extra in the proof is that, under this condition, the change in the buyer part of welfare will dominate the change in the seller part of welfare. Overall, the high-adoption equilibrium will have lower welfare than the cash equilibrium, even when there are more agents using e-money. Similarly, we can argue that if $\kappa_{Bl}<\beta\,(d-c)$, then the low-adoption equilibrium will also have lower welfare than the cash equilibrium, as stated in Proposition 3e.

3.2 Centralized Adoption of E-money

To establish a welfare benchmark, we now consider a setting in which the adoption and holding of e-money are not chosen by individuals, but determined in a centralized fashion by a utilitarian e-money planner. Notice that while the planner can choose and enforce any pattern of e-money adoption and holdings by buyers and sellers, he can neither force agents to (or not to) use cash, nor enforce directly any goods reallocation. The planner has to partition the set of buyers into four subsets: using cash only (A_C), using e-money only (A_E), using both (A_2) and using none (A_0). He also needs to choose the subset of sellers who accept e-money (A_S).

On the buyer side, there are accordingly four participation constraints. For those A_2 buyers (i.e., "multihoming" is recommended), we need to check that they do have an incentive to use both, instead of using e-money only (recall that the e-money planner can enforce e-money adoption, but not money adoption), given that sellers' acceptance is given by κ_S:

$$\beta u - d - \kappa_C - \beta G(\kappa_S)(u-d) \geq 0. \tag{12}$$

Note that, since the planner can enforce e-money usage, the option of not adopting e-money is not available to these buyers.

For those A_E buyers (i.e., "e-money only" is recommended), we need to check that they do not have an incentive to use both instead of using e-money only (recall that the e-money planner cannot prohibit money adoption). That is, condition (12) has to be violated.

For A_C buyers (i.e., "money only" is recommended), we need to check that they will not choose to not hold any means of payments (recall that the e-money planner cannot enforce money adoption):

$$\beta u - d - \kappa_C \geq 0. \tag{13}$$

Again, in this case the planner can always enforce no e-money usage, so the deviation payoff (the right-hand side of (13)) does not involve the payoff of e-money usage (whether cash is used or not).

Finally, for those A_0 buyers (i.e., "neither" is recommended), condition (13) has to be violated. Now we are ready to define the social (e-money planner) optimum.

Definition 2 *A socially optimal allocation of e-money consists of sets* $(A_2, A_E, A_C, A_0, A_S)$, *which solves*

$$\max_{\substack{A_C, A_2, A_0, \\ A_E, A_S}} \left\{ \begin{array}{l} \Pr(A_E)\Pr(A_S)\beta(u-c) + [\Pr(A_C) + \Pr(A_2)][\beta(u-c) - \kappa_C] \\ -\int_{A_2 \cup A_E} \kappa dF(\kappa) - \int_{A_S} \kappa dG(\kappa) \end{array} \right\}, \ s.t.$$

a. (12) is satisfied for all $i \in A_2$;

b. (12) is not satisfied for all $i \in A_E$;

c. (13) is satisfied for all $i \in A_C$;

d. (13) is not satisfied for all $i \in A_0$.

The planner problem is to solve the following Ramsey problem of optimal marginal buyer κ_B^{**} and the optimal marginal seller κ_S^{**}:

$$\max_{\kappa_B^{**}, \kappa_S^{**}} \left\{ \begin{array}{l} F(\kappa_B^{**})[G(\kappa_S^{**}) + [1 - G(\kappa_{S0})]\mathbb{I}_e(\kappa_S^{**})][\beta(u-c) - \mathbb{I}_e(\kappa_S^{**})\kappa_C] \\ +[1 - F(\kappa_B^{**})]\mathbb{I}_c[\beta(u-c) - \kappa_C] - \int_0^{\kappa_B^{**}} \kappa dF(\kappa) - \int_0^{\kappa_S^{**}} \kappa dG\kappa \end{array} \right\}, \tag{14}$$

where $\mathbb{I}_e(\kappa)$ is the indicator of multihoming, given by

$$\mathbb{I}_e(\kappa) = \begin{cases} 1 \text{ if } \beta u - d - \kappa_C - \beta G(\kappa)(u-d) \geq 0 \\ 0 \text{ otherwise} \end{cases}.$$

The e-money planner can arrange two types of trade, depending on the payment choice of the buyer: (i) e-money buyers (who probably use cash as well), and (ii) non-e-money buyers. For each case, the social trade surplus is $\beta(u-c)$. In case (i), there are $F(\kappa_B^{**})$ e-money buyers and $G(\kappa_S^{**})$ e-money sellers. These e-money buyers use cash (which the planner has no direct control over) if and only if (12) is satisfied. If the e-money buyers also use cash, then they always trade. So the social welfare arising from case (i) is the first line of (14). In case (ii), there is always trade for non-e-money buyers if and only if they use cash, which happens if $\beta u - d - \kappa_C \geq 0$. So the social welfare arising from case (ii) is $[1-F(\kappa_B^{**})]\,\mathbb{I}_{\beta u-d-\kappa_C\geq 0}\,[\beta(u-c)-\kappa_C]$. Finally, the social cost of using e-money to the buyers and sellers is $\int_0^{\kappa_B^{**}} \kappa dF(\kappa) + \int_0^{\kappa_S^{**}} \kappa dG\kappa$.

The first-order condition with respect to κ_B^{**} is

$$\begin{aligned}\underbrace{\kappa_B^{**}}_{\text{marginal cost to buyer}} &= \underbrace{\{G(\kappa_S^{**})+[1-G(\kappa_S^{**})]\,\mathbb{I}_e(\kappa_S^{**})\}\,[\beta(u-c)-\mathbb{I}_e(\kappa_S^{**})\,\kappa_C]}_{\text{social marginal benefit from buyer}} \\ &- \underbrace{\mathbb{I}_c\,[\beta(u-c)-\kappa_C]}_{\text{opportunity cost from using cash}} .\end{aligned} \tag{15}$$

The left-hand side is the social marginal cost of having an additional e-money buyer, which is κ_B^{**}. The right-hand side is the social marginal benefit of having an additional e-money buyer. If $\beta u - d - \kappa_C - \beta G(\kappa_S^{**})(u-d) \geq 0$, then the additional e-money buyer will also use cash; otherwise, only e-money is used. Thus, there is probability $G(\kappa_S^{**})+[1-G(\kappa_S^{**})]\,\mathbb{I}_e(\kappa_S^{**})$ that the additional e-money buyer can complete a trade with a seller, which leads to social surplus $\beta(u-c)-\mathbb{I}_e(\kappa_S^{**})\,\kappa_C$. However, having the additional buyer using e-money also means she forgoes the other payment option. The opportunity cost is the social surplus $\beta(u-c)-\kappa_C$ forgone if cash is otherwise used, which happens if $\beta u - d - \kappa_C \geq 0$. Otherwise, the opportunity cost is zero. Similarly, the first-order condition with respect to κ_S^{**} is

$$\underbrace{\kappa_S^{**}}_{\text{marginal cost to seller}} = \underbrace{\beta\,[1-\mathbb{I}_e(\kappa_S^{**})]\,F(\kappa_B^{**})\,(u-c)}_{\text{social marginal benefit from seller}}. \tag{16}$$

Again, the left-hand side is the social marginal cost of having an additional e-money seller, which is κ_S^{**}. The right-hand side is the social marginal benefit of having an additional e-money seller. If the additional e-money buyer also uses cash, which happens if $\beta u - d - \kappa_C - \beta G(\kappa_S^{**})(u-d) \geq 0$, then having an additional e-money seller will not generate any social marginal benefit, since the buyer can always complete the trade using cash no matter which type of seller is matched. Thus, the social marginal benefit is the right-hand side of (16).

Note that the social benefit of having an e-money buyer is avoiding the cost κ_C of using money. But when (12) is satisfied, this benefit is lost, because buyers using e-money will also have incentives to adopt money. Therefore, condition (12) implies zero adoption, as shown in the following proposition.

Proposition 4 *Given the maintained assumptions, (12) is never satisfied in the non-trivial planner solution.*

Proof. Suppose not. Since $\beta u - d - \kappa_C \geq \beta u - d - \kappa_C - \beta G(\kappa_S^{**})(u-d)$, the premise that (12) is satisfied implies $\beta u - d - \kappa_C \geq 0$. Then (15) implies $\kappa_B^{**} = 0$. Also, (16) under the premise that (12) is satisfied implies $\kappa_S^{**} = 0$. So the planner solution is trivial, which is a contradiction. ■

Notice that, given the result of Proposition 5, κ_B^{**} solving (15) and (16) also solves $V_0(\kappa_B^{**}) = 0$, where

$$V_0(\kappa) \equiv \beta(u-c) G[\beta(u-c) F(\kappa)] - \mathbb{I}_c[\beta(u-c) - \kappa_C] - \kappa.$$

Since $V_0(\kappa)$ is concave in κ, then $V_0(\kappa_B^{**}) = 0$ at most have two interior solutions. The second-order condition requires

$$1 - \beta(u-c)[\beta(u-c) - \mathbb{I}_e(\kappa_S^{**})\kappa_C] F'(\kappa_B^{**}) G'(\kappa_S^{**}) \geq 0. \tag{17}$$

Since the signs on the left-hand side of (17) are opposite when it is evaluated using the distinct solutions of (15) and (16), the interior solution with the highest κ_B^{**} is the optimum, if it exists. If (15) and (16) do not have a solution, then $\kappa_B^{**} = \min \kappa_B$ and $\kappa_S^{**} = 0$ are the optimum.

3.2.1 Welfare Comparison between Centralized and Decentralized Adoption

To examine the inefficiency of the decentralized usage of e-money, we can rewrite the condition for a planner's choice of (a non-trivial) κ_B as

$$\kappa_B^{**} = -[\mathbb{I}_c - G(\kappa_S^{**})]\beta(u-c) + \mathbb{I}_c\kappa_C, \tag{18}$$

and compare it with condition (5) for a decentralized equilibrium:

$$\kappa_B^{*} = -[\mathbb{I}_c - G(\kappa_S^{*})]\beta(u-d) - (1-\mathbb{I}_c)(1-\beta)d + \mathbb{I}_c\kappa_C. \tag{19}$$

To clearly identify different sources of inefficiency, we can express this condition as

$$\begin{aligned}\kappa_B^{*} \quad = \quad & - \underbrace{\frac{u-d}{u-c}}_{\text{seller's mkt power}} \times \underbrace{\frac{\mathbb{I}_c - G(\kappa_S^{*})}{\mathbb{I}_c - G(\kappa_S^{**})}}_{\text{network externality from sellers}} [\mathbb{I}_c - G(\kappa_S^{**})]\beta(u-c) \\ & - \underbrace{(1-\mathbb{I}_c)(1-\beta)d}_{\text{liquidity constraint}} + \mathbb{I}_c\kappa_C.\end{aligned} \tag{20}$$

Comparing (18) with (20), we notice that there are three sources of distortion on the buyer side. The first source is the seller's market power, which implies that a buyer only partially internalizes the effects of his payment choice on the social trade surplus. When $\mathbb{I}_c = 0$, by using e-money, a buyer generates trade surplus $G(u-c)$ to society but only internalizes $G(u-d)$. When $\mathbb{I}_c = 1$, by using e-money, a buyer destroys the trade surplus of the cash trade $(1-G)(u-c)$ to society, but only internalizes $(1-G)(u-d)$. The second source is that inefficient acceptance of e-money by sellers affects buyers' incentive through network externality. The third source is the liquidity constraint (or the money/e-money–in-advance constraint). Buyers need to pre-invest in a medium of exchange (either money or e-money) in order to trade in the following period. The interest forgone is $(1-\beta)\,d$. But this cost does not directly affect social welfare, because it is merely a zero-sum transfer among private agents, not a resource loss to society. Moreover, this is an opportunity cost of holding e-money only when the best alternative option forgone is autarky (i.e., $\mathbb{I}_c = 0$). If the best alternative is trading with money, this opportunity cost disappears, because the interest cost has to be incurred anyway.

Similarly, we can express (6) in terms of distortion on the seller:

$$\kappa_S^* = \underbrace{\frac{d-c}{u-c}}_{\text{buyer's mkt power}} \times \underbrace{\frac{F\left(\kappa_B^*\right)}{F\left(\kappa_B^{**}\right)}}_{\text{network externality from buyers}} \times \beta F\left(\kappa_B^{**}\right)(u-c)\,. \tag{21}$$

There are still inefficiencies arising from the buyer's market power and network externality, but not from the liquidity constraint, because sellers do not need to bring e-money to complete a trade. Given these inefficiencies, the following proposition shows that the social optimum cannot be supported by decentralized adoption.

Proposition 5 *Decentralized equilibrium with e-money is generally suboptimal. The optimal policy, which yields the same outcome as the e-money planner's problem, involves adoption fees:*

$$\begin{aligned} \tau_B^{**} &= [\mathbb{I}_c - G(\kappa_S^{**})]\beta(d-c) - (1-\mathbb{I}_c)(1-\beta)d, \\ \tau_S^{**} &= -F(\kappa_B^{**})\beta(u-d). \end{aligned}$$

Proof. To see that decentralized equilibrium with e-money is suboptimal, suppose this is not the case, that is, $\kappa_B^* = \kappa_B^{**} > 0$ and $\kappa_S^* = \kappa_S^{**} > 0$. Replacing κ_B^* by κ_B^{**} in (21) and comparing that with (16), we get $d = u$, implying that $\mathbb{I}_c = 0$. Condition (20) then implies that $\kappa_B^* < 0$, which is a contradiction. To confirm that the proposed optimal policy is right, we can substitute the two fees into (20) and (21) to get (16) and (18). ■

On the seller side, the optimal policy is always to provide a positive incentive to sellers so that they can internalize their positive externality (i.e., creating more trading opportunities for the buyers). On the buyer side, the optimal policy depends on whether money is used otherwise. If money is used, then a negative incentive (i.e., a fee) is needed to discourage buyers from generating negative externality (i.e., destroying sellers' trading opportunities). If not, then a positive incentive is provided to induce the buyers to internalize their positive externality (i.e., creating more trading opportunities for the sellers) and to compensate for their interest cost of holding e-money.

Finally, we want to derive some sufficient conditions under which decentralized adoption leads to over- or underadoption.

The following proposition helps compare the optimal usage of e-money to the one in decentralized equilibria.

Proposition 6 *When $\mathbb{I}_c = 0$, a decentralized equilibrium always results in underadoption. When $\mathbb{I}_c = 1$, a decentralized equilibrium can result in overadoption.*

Proof. Define $\widetilde{\kappa}$ as the largest root solving $V(\widetilde{\kappa}) = V_0(\widetilde{\kappa})$ if a finite solution exists; otherwise, set $\widetilde{\kappa} = -\infty$. First notice that $V(\widetilde{\kappa}) = V_0(\widetilde{\kappa})$ is equivalent to

$$\mathbb{I}_c (d - \beta c) - (1 - \beta) d = \Lambda(\kappa), \tag{22}$$

where $\Lambda(\kappa) \equiv \beta (u - c) G[\beta (u - c) F(\kappa)] - \beta (u - d) G[\beta (d - c) F(\kappa)]$. Notice that $\Lambda(\kappa)$ is always positive; thus there is no solution to (22) if $\mathbb{I}_c = 0$. Therefore, we have $\widetilde{\kappa} = -\infty$ if $\mathbb{I}_c = 0$. In this case, we have $V(\kappa) \leq V_0(\kappa)$ for all κ; thus we have $\kappa_B^{**} > \kappa_{Bh}$. Therefore, a decentralized equilibrium always leads to underadoption.

Now, suppose $\mathbb{I}_c = 1$. We will show that if $(u - c) G[\beta (u - c)] - (u - d) G[\beta (d - c)] \leq d - c$ and $\widetilde{\kappa} \leq \kappa_{Bh}$, then $\kappa_B^{**} \leq \kappa_{Bh}$. Given $\Lambda(\infty) \geq d - c$ by the premise of the proposition, since $\widetilde{\kappa}$ is the largest root solving $\Lambda(\kappa) = d - c$, we must have $\Lambda(\kappa) - d + c \geq 0$ for all $\kappa \geq \widetilde{\kappa}$. Since $\widetilde{\kappa} \leq \kappa_{Bh}$, we have

$$V_0(\kappa_{Bh}) \geq V(\kappa_{Bh}) = 0.$$

Notice that $V_0(\infty) < 0$. So the fact that $V_0(\kappa_{Bh}) \geq 0$ and $V_0(\infty) < 0$ implies that there must be some $\kappa_B^{**} \in [\kappa_{Bh}, \infty)$ such that $V_0(\kappa_B^{**}) = 0$.

Given $\Lambda(\infty) \leq d - c$ by the premise of the proposition, following the above logic, we must have $V_0(\kappa) \leq V(\kappa)$ for all $\kappa \geq \widetilde{\kappa}$. To show that $\kappa_B^{**} \leq \kappa_{Bh}$, suppose that $\kappa_B^{**} > \kappa_{Bh}$. First, since we have $\kappa_B^{**} > \kappa_{Bh} > \widetilde{\kappa}$, we must have $V_0(\kappa_B^{**}) \leq V(\kappa_B^{**})$ as well. Second, notice that κ_{Bh} is the largest solution solving $V(\kappa) = 0$, so we must have $V(\kappa) < 0$ for all $\kappa > \kappa_{Bh}$. Finally, combining the above results, we have $V_0(\kappa_B^{**}) \leq V(\kappa_B^{**}) < 0$, which contradicts the fact that $V_0(\kappa_B^{**}) = 0$. ∎

Table 1: Pricing Scheme

	Loading Session t Fixed fee	Purchase e-money balance d	Loading Session $t+1$ Redeem e-money balance d
Buyer	pay τ_B	pay money $(1-\eta)d$	receive money d
Seller	pay τ_S	n.a.	receive money $(1-\tau)d$

4 Private Issuance of E-money

We next consider the private issuance of e-money by a monopolist issuer (Figure 7). This issuer is a profit-maximizing firm owned by all agents in the economy, and thus any profits will be redistributed to its shareholders. Notice that this monopolist is still subject to the informational restrictions: (i) trading histories in the morning are not observable, (ii) agents are anonymous but the type (buyer or seller) is identifiable, and (iii) costs κ_B and κ_S are private information. As discussed in the introduction, an e-money issuer can use a non-linear pricing scheme. For simplicity, we will focus on the following scheme summarized in Table 1: in the loading session, a buyer can pay a fee τ_B and a price $(1-\eta)d$ to set up an account and to load d units of e-money to his account. The buyer can redeem this balance on par in the following loading session. In the loading session, a seller can pay a fee τ_S to set up an account. The seller can redeem each unit of his e-money balances for $(1-\tau)d$ units of money in the following loading session. Here we can think of τ_B, τ_S as fixed fees (e.g., membership fees), η as a proportional discount to buyers (e.g., consumer rewards), and τ as a proportional transaction fee (e.g., merchant or interchange fees). But since we consider indivisible goods with an exogenous price d in this model, these differences are not as clear as in the general case with divisible goods and endogenous prices.[13]

As in the previous section, a buyer with money can trade only with all sellers, while a buyer with e-money can trade only with sellers accepting e-money. Now, when a buyer with both money and e-money (i.e., multihoming) meets with a seller accepting e-money, we will assume that the buyer will always pay in e-money. Here the buyer is indifferent regarding the two options, but strictly prefers to use e-money if there is any small positive incentive provided by the issuer (e.g., unspent balances have a redemption rate slightly lower than par).

To derive the optimal payment choice of a buyer, consider first the buyer's per-period payoffs of using

[13] Here, we implicitly impose a restriction that disallows a user to obtain multiple accounts. This constraint may be binding when the net fees are negative, because agents may be induced to obtain multiple accounts merely for receiving the positive incentives, but not for trading purposes. For simplicity, we abstract from this consideration here. In a more general setting, the issuer can discourage users from obtaining multiple accounts by making rewards a function of balances *spent* (instead of a function of balances *loaded*). Interestingly, this pricing scheme works like "cash rebates."

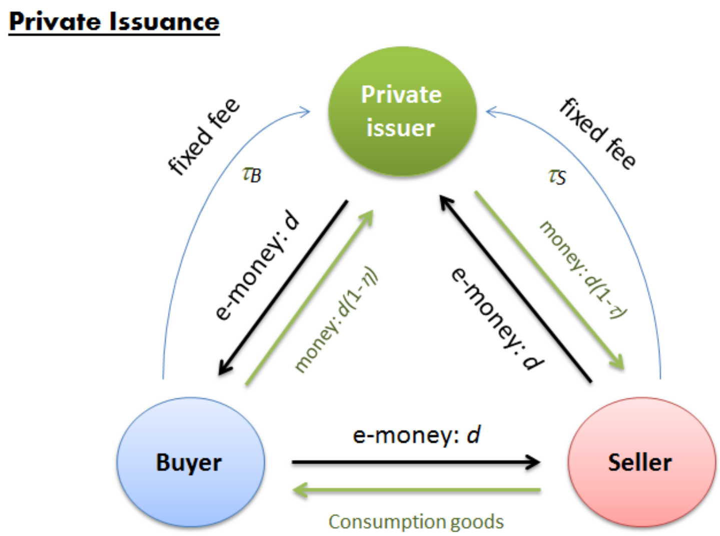

Figure 7: Private issuance

cash, e-money, both, and neither, given, respectively, by

$$\begin{aligned} v_c &= \beta u - d - \kappa_c, \\ v_e &= \beta \alpha_B (u-d) - (1-\beta)d - (\kappa_B + \tau_B - \eta d), \\ v_{ce} &= \beta u - (2-\beta)d - \kappa_c - (\kappa_B + \tau_B - \eta d), \\ v_0 &= 0. \end{aligned}$$

The interpretation of the first and the last payoff is straightforward. The second indicates the case in which the buyer, by paying a private adoption cost κ_B and an e-money fee net of discount $\tau_B - \eta d$, obtains an account with d units of e-money, which can be spent with a probability $\beta\alpha_B$. The third payoff indicates the case in which the buyer, by paying the private adoption costs $\kappa_B + \kappa_C$ and the e-money fee net of discount $\tau_B - \eta d$ and the price of money d, obtains an account with d units of e-money as well as d units of money. The buyer can trade for sure and end up with d units of money or e-money. Notice that in this simple case, what matters for payoffs is the term $\kappa_B + \tau_B - \eta d$, not the values of these two components. This property will not hold in the general case.

By comparing these payoffs, we obtain the following conditions:

$$\begin{aligned} v_c &\geq v_e \Leftrightarrow \beta(1-\alpha_B)(u-d) + (\kappa_B + \tau_B - \eta d) \geq \kappa_c \\ v_{ce} &\geq v_c \Leftrightarrow -(1-\beta)d \geq (\kappa_B + \tau_B - \eta d) \\ v_{ce} &\geq v_e \Leftrightarrow \beta u - d - \beta\alpha_B(u-d) \geq \kappa_c. \end{aligned}$$

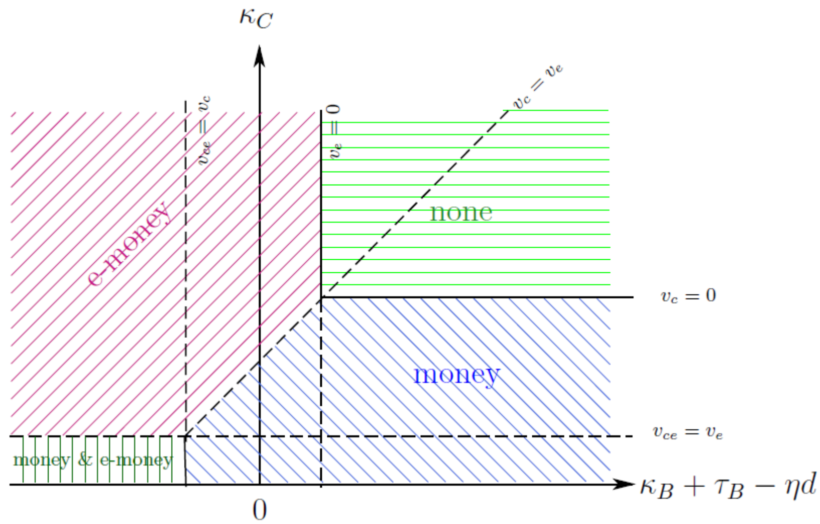

Figure 8: Buyer's payment choice

By plotting these constraints in Figure 8, we can derive buyers' payment choice as a function of κ_c and $\kappa_B + \tau_B - \eta d$. As expected, an instrument is used when it is cheap. So multihoming is chosen for low κ_c and for $\kappa_B + \tau_B - \eta d$ that is sufficiently negative. Note that negative $\kappa_B + \tau_B - \eta d$ is needed to induce multihoming, because the marginal benefit of adopting e-money (on top of holding cash) does not come from a trade surplus but from the issuer's net transfer $\eta d - \tau_B$, which has to be large enough to cover the adoption cost κ_B and the interest cost $(1 - \beta)d$. Notice that if e-money is used, it either means that all e-money users do not hold money (which happens for high κ_C), or that all e-money users multihome. This is determined by the $v_{ce} \geq v_e$ condition captured by the following indicator function:

$$\mathbb{I}_e = \begin{cases} 1 \text{ if } \beta u - d - \kappa_C - \beta\alpha_B (u - d) \geq 0 \\ 0, \text{ otherwise.} \end{cases}$$

The buyer's payoff of using e-money is given by

$$\max \{\beta\alpha_B (u - d) - (1 - \eta - \beta) d - \kappa_B - \tau_B, -(2 - \eta - \beta) d - \kappa_B - \kappa_C - \tau_B + \beta u\}.$$

The first term is the payoff of using only e-money, and the second term is the payoff of using both money and e-money. Rearranging this condition, we get the condition for buyers using e-money

$$\kappa_B \leq \max \{\beta u - d - \kappa_C - \beta\alpha_B (u - d), 0\} + \beta\alpha_B (u - d) - (1 - \beta)d - \max \{\beta u - d - \kappa_C, 0\} - (\tau_B - \eta d), \tag{23}$$

where, as in the previous analysis, the last term on the right-hand side is the higher payoff between using money only and no trade at all.

To derive the adoption choice of a seller, we first derive the payoff for not accepting e-money

$$[(1-\alpha_S)\mathbb{I}_c + \alpha_S \mathbb{I}_e]\beta\,(d-c)\,.$$

That is, the seller can trade with a buyer who has money, or who multihomes. Similarly, the payoff for accepting e-money is

$$(1-\alpha_S)\mathbb{I}_c\beta\,(d-c) + \alpha_S\beta[d(1-\tau)-c] - \kappa_S - \tau_S.$$

That is, the seller can trade with a buyer who has money, or who has e-money but subject to the fee τ in the latter case. Therefore, e-money is accepted by the seller if

$$\kappa_S \leq -(\tau_S + \alpha_S\beta d\tau) + \alpha_S(1-\mathbb{I}_e)\beta\,(d-c)\,. \tag{24}$$

Here, by paying fixed cost κ_S, τ_S and proportional cost $\alpha_S\beta d\tau$, a seller gets additional trades iff there is no multihoming (captured by $\alpha_S(1-\mathbb{I}_e)\beta\,(d-c)$). If there is multihoming, the seller receives no benefit from trade surplus and hence the net transfer to the e-money issuer has to be negative (i.e., $\tau_S+\alpha_S\beta d\tau \leq -\kappa_S$). Again, note that what matters is the net transfer $\tau_S + \alpha_S\beta d\tau$ but not the individual items.

We next study the profit-maximization problem of an e-money issuer given by

$$\max_{\substack{\tilde{\kappa}_B,\tilde{\kappa}_S\\ \tau_B,\tau_S,\eta,\\ \tau\leq 1-c/d}} G\left(\tilde{\kappa}_S\right)F\left(\tilde{\kappa}_B\right)\beta\tau d + F\left(\tilde{\kappa}_B\right)(1-\beta)d + F\left(\tilde{\kappa}_B\right)(\tau_B-\eta d) + G\left(\tilde{\kappa}_S\right)\tau_S,$$

subject to the participation constraints

$$\begin{aligned}
\forall\kappa_B &\leq \tilde{\kappa}_S, (23) \text{ is satisfied,}\\
\forall\kappa_S &\leq \tilde{\kappa}_S, (24) \text{ is satisfied,}
\end{aligned}$$

and the equilibrium condition

$$\begin{aligned}
\alpha_B &= G\left(\tilde{\kappa}_S\right),\\
\alpha_S &= F\left(\tilde{\kappa}_B\right).
\end{aligned}$$

To solve the problem, we substitute in the participation constraints to express the issuer's objective as

$$\begin{aligned}
\max_{\tilde{\kappa}_B,\tilde{\kappa}_S} \quad & F\left(\tilde{\kappa}_B\right)\beta G\left(\tilde{\kappa}_S\right)\{u-c-\mathbb{I}_e(d-c)\} - F\left(\tilde{\kappa}_B\right)\tilde{\kappa}_B - G\left(\tilde{\kappa}_S\right)\tilde{\kappa}_S\\
& -F\left(\tilde{\kappa}_B\right)\mathbb{I}_c[\beta u - d - \kappa_C] + F\left(\tilde{\kappa}_B\right)\mathbb{I}_e[\beta u - d - \kappa_C - \beta G\left(\tilde{\kappa}_S\right)(u-d)].
\end{aligned}$$

Confirming the previous discussion, the exact pricing scheme is indeterminate as long as the incentive constraints of the buyers and sellers are satisfied. Note that when $\mathbb{I}_e = 1$, the issuer's objective becomes

$$\max_{\tilde{\kappa}_B, \tilde{\kappa}_S} -F(\tilde{\kappa}_B)\,\tilde{\kappa}_B - G(\tilde{\kappa}_S)\,\tilde{\kappa}_S,$$

which implies a trivial solution $\tilde{\kappa}_B = \tilde{\kappa}_S = 0$.

Lemma 2 *Private issuance of e-money implies that buyers never multihome ($\mathbb{I}_e = 0$).*

The first-order condition with respect to $\tilde{\kappa}_B$ and $\tilde{\kappa}_S$ is given by

$$\tilde{\kappa}_B = \underbrace{\left[1 - \frac{1}{1+\varepsilon_B(\tilde{\kappa}_B)}\right]}_{\text{monopoly markup}} \left[\begin{array}{l} -\underbrace{\frac{[\mathbb{I}_c - G(\tilde{\kappa}_S)]}{[\mathbb{I}_c - G(\kappa_s^{**})]}}_{\text{network effect}}\beta[\mathbb{I}_c - G(\kappa_s^{**})](u-c) + \underbrace{\mathbb{I}_c(1-\beta)\,d}_{\text{liq. constraint}} \\ +\underbrace{\mathbb{I}_c\beta(d-c)}_{\text{seller's mkt power}} + \mathbb{I}_c\kappa_C \end{array} \right], \tag{25}$$

where the elasticities of F with respect to κ_B are given by

$$\varepsilon_B(\kappa) \equiv \frac{F'(\kappa)}{F(\kappa)}\kappa.$$

By comparing (25) with (18), we can see that there are four sources of inefficiency when $\mathbb{I}_c = 1$ (i.e., money is used by the $1 - F(\tilde{\kappa}_B)$ buyers). The first source is the "monopoly markup." As in standard monopoly pricing, the e-money monopoly limits the usage of e-money in order to extract more surplus from buyers. The second source is due to inefficient acceptance on the seller side passing through the network effect. The third source is the liquidity constraint. A buyer, by adopting e-money instead of cash, generates float interest income $(1-\beta)\,d$ to the issuer, but this is merely a transfer which does not increase social welfare. This creates an incentive to overadopt. When $\mathbb{I}_c = 0$, this float income term disappears because the issuer's interest gain is exactly offset by the buyers' interest loss, which they now have to bear since they choose to hold e-money instead of *no trade*. Finally, the fact that buyers do not extract all trade surplus (i.e., $d > c$) also leads to overadoption. Consider the marginal buyer who switches from using money to e-money. On the one hand, if he meets with sellers who do not accept e-money, he cannot trade and hence creates a welfare loss $[1 - G(\tilde{\kappa}_S)]\beta(d-c)$ by destroying the surplus of sellers, which the issuer does not internalize. On the other hand, if he meets with sellers who accept e-money, he uses e-money and creates surplus $G(\tilde{\kappa}_S)\,\beta\,(d-c)$ to the e-money scheme, even though this does not really generate any welfare gain (because trade will be carried out anyway). This source induces the issuer to overvalue the marginal user by $\beta(d-c)$ relative to the planner. When $\mathbb{I}_c = 0$, this term will disappear, because the alternative to trading with e-money is no trade.

The advantage of having a private issuer, relative to having decentralized adoption, is highlighted by comparing (19) and (25), setting $\mathbb{I}_c = 0$. With a private monopoly issuer, there are no more inefficiencies due to the seller's market power and liquidity constraint, because the e-money issuer takes these factors into account when designing the scheme. With decentralized adoption, the incentive of a buyer is lowered by the interest cost $(1-\beta)\,d$ of purchasing e-money. The issuer can undo this negative effect by providing positive incentive to the buyers, because the issuer earns float interest $(1-\beta)\,d$ from the e-money balances purchased by the buyers. Moreover, with decentralized adoption, a buyer does not internalize the effect that his adoption creates trade surplus $G(\tilde{\kappa}_S)(d-c)$ to sellers who accept e-money. The issuer can internalize this effect by extracting more from these sellers who are also his clients.

Similarly, the first-order condition with respect to $\tilde{\kappa}_S$ is given by

$$\tilde{\kappa}_S = \underbrace{\left[1 - \frac{1}{1+\varepsilon_S(\tilde{\kappa}_S)}\right]}_{\text{monopoly markup}} \left[\underbrace{\frac{F\left(\tilde{\kappa}_B\right)}{F\left(\kappa_B^{**}\right)}}_{\text{network effect}} \times \beta F\left(\kappa_B^{**}\right)(u-c)\right], \tag{26}$$

where the elasticities are given by

$$\varepsilon_S(\kappa) \equiv \frac{G'\left(\kappa\right)}{G\left(\kappa\right)}\kappa.$$

The net fees charged by the issuer are given by substituting (25) and (26) into the binding participation constraints (23) and (24) for the marginal buyer $\tilde{\kappa}_B$ and the marginal seller $\tilde{\kappa}_S$:

$$\begin{aligned}
\tau_B - \eta d &= -\underbrace{\beta G\left(\tilde{\kappa}_S\right)(d-c)}_{\text{seller's market power}} - \underbrace{(1-\beta)\,d}_{\text{liq. constraint}} + \underbrace{\frac{F\left(\tilde{\kappa}_B\right)}{F'\left(\tilde{\kappa}_B\right)}}_{\text{monopoly rent}} \\
\tau_S + \alpha_S \beta d\tau &= -\underbrace{\beta F\left(\tilde{\kappa}_B\right)(u-d)}_{\text{buyer's market power}} + \underbrace{\frac{G\left(\tilde{\kappa}_S\right)}{G'\left(\tilde{\kappa}_S\right)}}_{\text{monopoly rent}}.
\end{aligned}$$

For the buyer, a positive incentive is provided to compensate for the seller's market power and the liquidity constraint, as well as a fee for the monopoly rent. For the seller, a positive incentive is provided to compensate for the buyer's market power as well as a fee for the monopoly rent. There is a redistribution of trade surplus between the two sides. The e-money issuer charges buyers more if there are fewer e-money sellers (lower $\tilde{\kappa}_S$) or if sellers receive a smaller trade surplus $(d-c)$, and vice versa.

The outcome is that e-money could be still underused when it is arranged by a monopoly, since it is profitable for the e-money monopoly to limit the usage of e-money in order to charge a higher tariff. Overall, it is uncertain whether e-money will be more widely used than in the decentralization economy.

Proposition 7 *Private issuance by a monopolist is generally suboptimal. The optimal policy, which yields the same outcome as the e-money planner's problem, involves adoption fees*

$$\begin{aligned}
\tau_B^{**} &= \frac{-\kappa_B^{**}}{1+\varepsilon_B(\kappa_B^{**})} + \mathbb{I}_c \frac{\varepsilon_S(\kappa_S^{**})}{1+\varepsilon_S(\kappa_S^{**})}(d-\beta c), \\
\tau_S^{**} &= \frac{-\kappa_S^{**}}{1+\varepsilon_S(\kappa_S^{**})}.
\end{aligned}$$

While private issuance is generally suboptimal, the presence of an e-money monopoly can stabilize the payment system by eliminating the low-adoption equilibrium through coordination of the two-sided network effects. The basic idea is that network externality arises in a setting in which an agent's payoff depends on other agents' actions. But the issuer can make ex-post transfers to offset the external effects, so that an agent's payoff no longer depends on the actions of others. By suitably designing these ex-post transfers, the desired outcome can be uniquely implemented. Specifically, it is straightforward to check that there is a unique equilibrium that supports $\tilde{\kappa}_B, \tilde{\kappa}_S$, under the pricing scheme proposed in the following proposition.

Proposition 8 *A private issuer can uniquely support the equilibrium $\tilde{\kappa}_B, \tilde{\kappa}_S$ by setting*

$$\begin{aligned}
\tau_B &= -\beta G(\tilde{\kappa}_S)(d-c) - (1-\beta)d + \frac{F(\tilde{\kappa}_B)}{F'(\tilde{\kappa}_B)}, \\
\tau_S &= -\beta F(\tilde{\kappa}_B)(u-d) + \frac{G(\tilde{\kappa}_S)}{G'(\tilde{\kappa}_S)}, \\
\eta &= \tau = 0,
\end{aligned}$$

with additional redemption rewards (r_B, r_S) contingent on actual adoption (κ_B, κ_S):

$$\begin{aligned}
r_B &= [G(\tilde{\kappa}_S) - G(\kappa_S)]\beta(u-d), \\
r_S &= [F(\tilde{\kappa}_B) - F(\kappa_B)]\beta(d-c).
\end{aligned}$$

5 Summary and Policy Discussion

Table 2 summarizes our findings. In general, the efficiency and stability of electronic money adoption depend on market structure, the ability of users to coordinate, and whether money is a viable payment option. First, when money is not a viable alternative to e-money (e.g., online transaction), public issuance leads to underadoption on the buyer side due to the seller's market power, liquidity constraint and network externality. Private issuance can help mitigate the first two effects, but will introduce monopoly distortion. Therefore, it will still result in underadoption. There is a similar outcome on the seller side, except that inefficiency does not come from the liquidity constraint. In order to restore an optimal outcome,

Table 2: Summary of Findings

			Public Issuance	Private Issuance
$\mathbb{I}_c = 0$	**Buyer**	**Buyer/seller market powers**	$-$	0
		Liquidity constraint	$-$	0
		Network externalities	$-$	$-$
		Issuer monopoly distortion	0	$-$
		Optimal Incentive	< 0	< 0
	Seller	**Buyer/seller market powers**	$-$	0
		Liquidity constraint	0	0
		Network externalities	$-$	$-$
		Issuer monopoly distortion	0	$-$
		Optimal Incentive	< 0	< 0
$\mathbb{I}_c > 0$	**Buyer**	**Buyer/seller market powers**	$+$	$+$
		Liquidity constraint	0	$+$
		Network externalities	?	?
		Issuer monopoly distortion	0	$-$
		Optimal Incentive	$\lesseqgtr 0$	$\lesseqgtr 0$
	Seller	**Buyer/seller market powers**	$-$	0
		Liquidity constraint	0	0
		Network externalities	?	?
		Issuer monopoly distortion	0	$-$
		Optimal Incentive	< 0	< 0

positive incentives for buyers and sellers are needed. Second, when money is a viable payment option (e.g., transactions in a coffee shop), public issuance leads to inefficient adoption on the buyer side due to the seller's market power and network externality. Private issuance introduces inefficiencies due to liquidity constraint and monopoly distortion. We have similar effects on the seller side, except that inefficiency does not come from the liquidity constraint, and private issuance helps internalize buyers' market power. Overall, there can be over- or underadoption on both sides. To restore an optimal outcome, a positive incentive is provided to sellers, while the incentive provided to buyers can be positive or negative. Finally, public issuance with decentralized adoption generically creates instability. Private issuance, however, can avoid instability if contingent rewards are offered.

5.1 Public issuance of e-money and private operation of platform

So far, we have discussed and compared the cases of public issuance and private issuance. Before we conclude, let us briefly examine a hypothetical set-up: e-money is issued by the central bank and sold at par to a private operator who then acts as an intermediary providing accounts to the underlying buyers and sellers (Figure 9). In practice, this business model may improve technical efficiency by making use of a private operator's competitive advantage (e.g., financial infrastructure). In theory, this business also generates different implications in terms of efficiency and central bank seigniorage income.

Moving from private issuance to this alternative model basically adds a new term $-F(\kappa_B)(1-\beta)\,d$ to

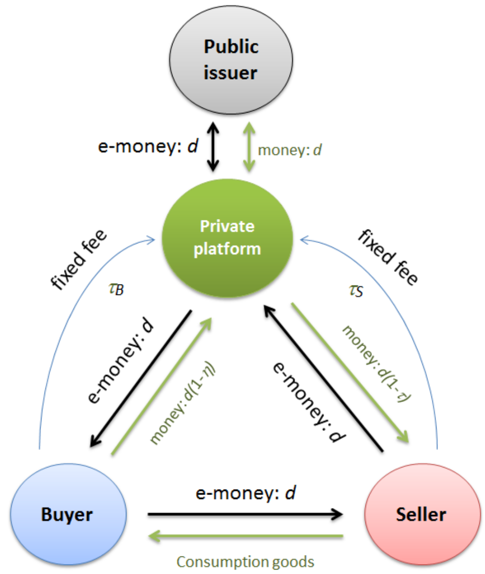

Figure 9: Central bank provision through a private platform

Table 3: Central Bank Seigniorage Income

	Trade	Central Bank Seigniorage Income
No e-money	$\mathbb{I}_{\mathbf{c}}$	$\mathbb{I}_{\mathbf{c}} d$
Public issuance	$G(\kappa_S^*)F(\kappa_B^*) + [1 - F(\kappa_B^*)]\mathbb{I}_{\mathbf{c}}$	$\{F(\kappa_B^*) + [1 - F(\kappa_B^*)]\mathbb{I}_{\mathbf{c}}\}\, d$
Private issuance & operation	$G(\tilde{\kappa}_S)F(\tilde{\kappa}_B) + [1 - F(\tilde{\kappa}_B)]\mathbb{I}_{\mathbf{c}}$	$[1 - F(\tilde{\kappa}_B)]\mathbb{I}_{\mathbf{c}} d$
Public issuance + Private operation	$G(\breve{\kappa}_S)F(\breve{\kappa}_B) + [1 - F(\breve{\kappa}_B)]\mathbb{I}_{\mathbf{c}}$	$\{F(\breve{\kappa}_B) + [1 - F(\breve{\kappa}_B)]\mathbb{I}_{\mathbf{c}}\}\, d$

the issuer's profit function. When $\mathbb{I}_c = 0$, this change reintroduces the inefficiency due to the liquidity constraint on the buyer side, leading to further underadoption. When $\mathbb{I}_c = 1$, however, this change helps offset the effect of the liquidity constraint and reduce the buyers' adoption, which can either be efficiency enhancing or not. However, this change can unambiguously increase the seigniorage income received by the central bank. Notice that the same policy outcome can be achieved by imposing a 100% cash reserve requirement, without involving the central bank in the issuance of e-money. Also, if the purpose is to raise revenue, perhaps charging a charter fee is more direct and effective. Table 3 compares the central bank's seigniorage income under different schemes.

6 Conclusion

This paper develops a dynamic, general-equilibrium model of e-money to help us investigate its efficiency, stability and optimal system design. We first identify some superior features of e-money which help mitigate informational frictions and enhance social welfare in a cash economy. Our analysis highlights that the introduction of e-money is not necessarily welfare enhancing, especially when money is a viable alternative to e-money. Laissez-faire does not ensure an efficient and stable payment system, even in this highly stylized setting. Both private and public provision can be inefficient and unstable, and can result in over- or underadoption. Efficiency depends on several factors related to liquidity constraints faced by consumers, market powers between consumers and merchants, the network externality in adoption, and monopoly distortion in e-money issuance. In theory, efficiency can be restored by providing positive or negative incentives to consumers and merchants. In practice, designing such a policy intervention can be a very complicated and information-intensive task.

There are a number of natural extensions to the current framework. First, the model considers a relatively simple goods market with indivisible goods and exogenous terms of trade. An easy extension is to allow for divisible goods with endogenous prices. Second, we focus on a simple case with only one e-money issuer. It would be interesting to examine strategic interactions between multiple issuers. Third, we assume that the e-money issuer can commit to redeem the outstanding balances. By assuming limited commitment, one can analyze the incentive-compatibility constraint faced by the e-money issuer. Fourth, in the current set-up, there is no multihoming in equilibrium. One can revisit this issue in a more general setting. We expect that multihoming can be an equilibrium outcome when it is so costly for some merchants to accept cash that they only accept e-money. These extensions are all interesting, and we leave them for future research.

Appendix: Octopus Card in Hong Kong

Reference: Fung et al. (2013)

- Contactless payment card with extremely fast transaction times.
- Operates off-line. Central clearing system reconciles all transactions on a daily basis.
- Most cards are anonymous.
- Mainly used for small-value transactions.
- Core use has been for public transportation. Still, almost half of the total value of transactions is now non-transport related.
- Octopus Cards Limited is regulated as a special-purpose deposit-taking institution by the Monetary Authority of Hong Kong. Octopus also voluntarily adheres to the Code of Practice for Multi-Purpose Stored Card Operations.

Statistics:

- In 2010, there were over 21 million Octopus cards in circulation (about 3 per capita).

    - There was a daily average of over 11 million transactions.
    - The average transaction value for the Octopus card is US$1 and the average value for a retail transaction is slightly higher, at about US$2.50.

- The average value held on an Octopus card is about US$15, which is adequate for using public transport over a two-week period.
- About 30 to 40 per cent of the transactions are non-transport related (in terms of value). The authorities have imposed a cap of 50-50 on transport and non-transport transactions.
- The cardholder has to place a refundable deposit worth about US$6 after first obtaining the card. There are no reloading or user fees.
- Octopus charges a 1 per cent fee to transport operators, and an undisclosed variable fee to merchants for the payment service.

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